Ramakrishna Mission Vivekananda Centenary College

NON-PARAMETRIC RECONSTRUCTION OF THE Hubble Parameter with LISA using GAUSSIAN PROCESS REGRESSION

Presentation by Krishnanjan Sil

HUBBLE PARAMETER

- H(z) quantifies the expansion rate of the Universe at any given redshift z.
- At the present epoch, H(z=0) = H_0 (the current expansion rate).

$$H \equiv \frac{\dot{a}}{a}$$

Wienberg, Cosmology

Dissertation –Krishnanjan Sil

HUBBLE TENSION

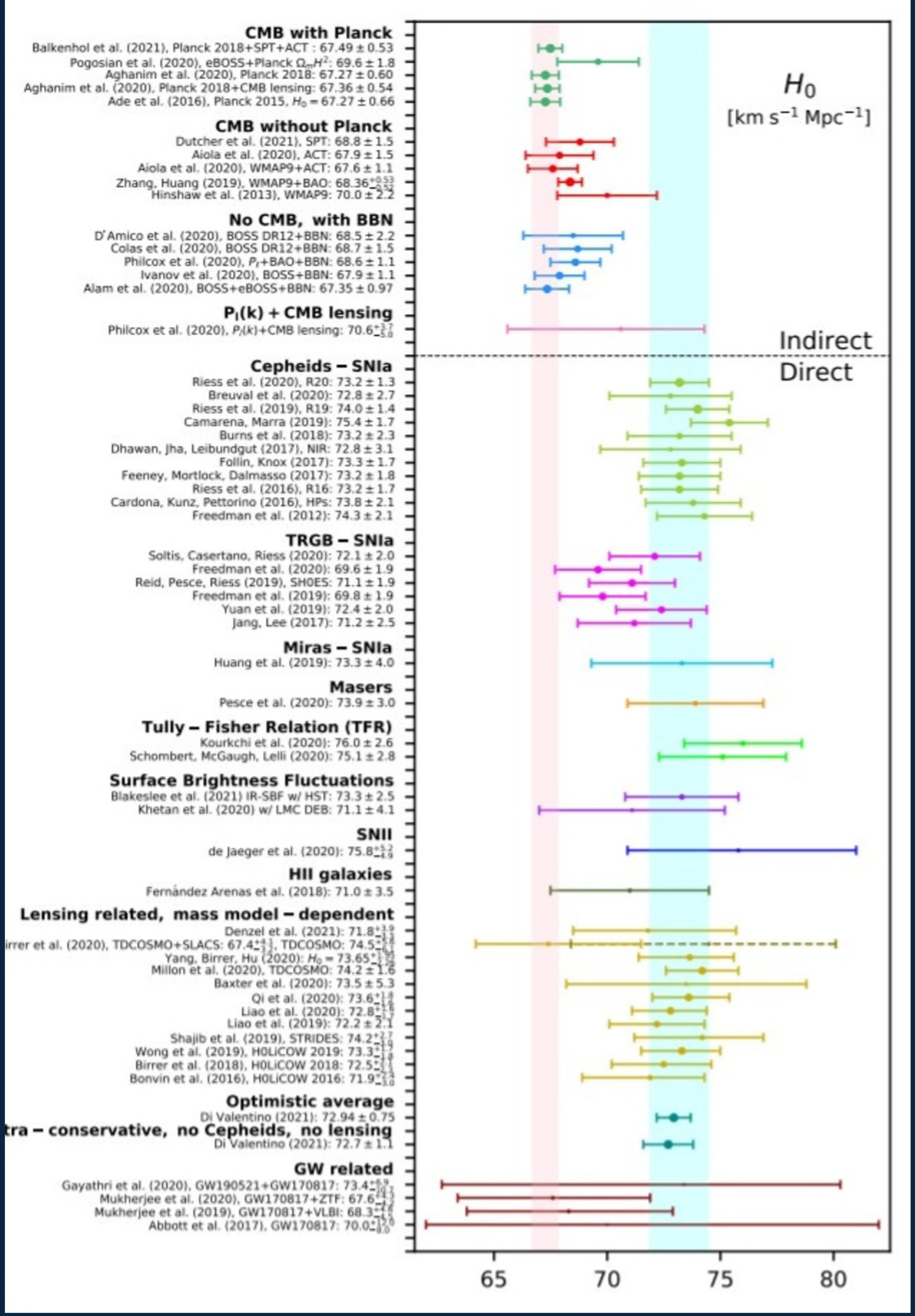
• Hubble tension: Conflicting Hubble constant (H_0) measurements—~74 km/s/Mpc from local supernovae vs. ~67 km/s/Mpc from CMB—exceed uncertainties.

• Physics challenge: The discrepancy questions the ΛCDM model, suggesting modified gravity, evolving dark energy, or inhomogeneous universe models.

• Complexities: H_0 variations may arise from galaxy age, position, or redshift misinterpretations.

~5σ Tension!

Di Valentino et al., 2021



Di Valentino et al., 2021

IMPORTANCE

- Age of the Universe,
- Masses of clusters and galaxies,
- Energy budget of luminous sources,
- Physical scales of cosmic objects,
- Constraints on dark matter,
- Neutrino mass constraints,
- Constraints on ΛCDM extension parameter(s),
- Model independent cross-checks on the Hubble tension.

GRAVITATIONAL WAVES

• Gravitational waves: Spacetime ripples from massive cosmic events (e.g., black hole mergers), predicted by Einstein, travel at light speed.
• Detection: LIGO/Virgo interferometers detect tiny distortions via laser beams. First direct detection in 2015 (GW150914).
• 1974: Indirect proof from pulsar orbital decay.
• Advantage: Pass through matter, revealing "dark" events invisible to light telescopes.
• Future: Advanced detectors (Einstein Telescope, LISA) will explore early universe and extreme physics.

Image Credit: Caltech

Abbott et al., 2021

LIGO

https://www.ligo.caltech.edu/#

LIGO India

CAD drawing of the proposed
LIGO India observatory

LISA & ET

– Sub-percent precision: LISA achieves $<1\%$ Hubble constant accuracy with bright sirens at $3<z<8$, minimizing statistical errors.

– Higher redshift coverage: LISA probes $z\sim1$–8, resolving Hubble constant–dark energy degeneracies; ET covers $z\leq2$, testing local vs. high-z discrepancies.

– Model-independent tests: LISA/ET use Gaussian Process to validate early dark energy and H0-tension solutions without cosmological assumptions.

– Multi-messenger synergy: ET's $<5\%$ luminosity distance errors, combined with galaxy surveys, enhance dark siren redshift accuracy for H0.

– Early Universe probes: Both test pre-recombination physics (e.g., early dark energy) as H0 tension source, weakly constrained by current data.

Mukherjee et al., 2024

LISA

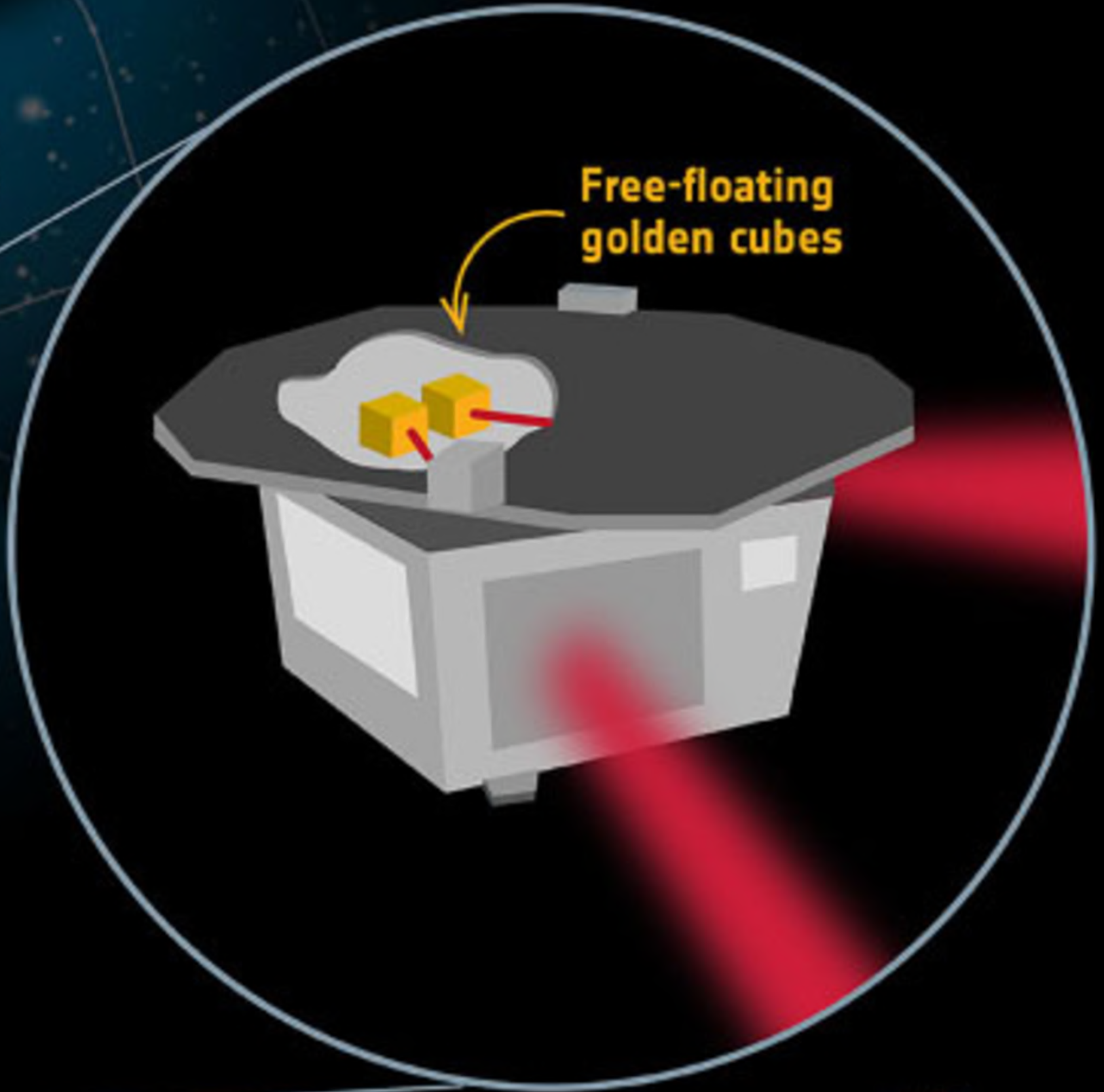
LISA - LASER INTERFEROMETER SPACE ANTENNA

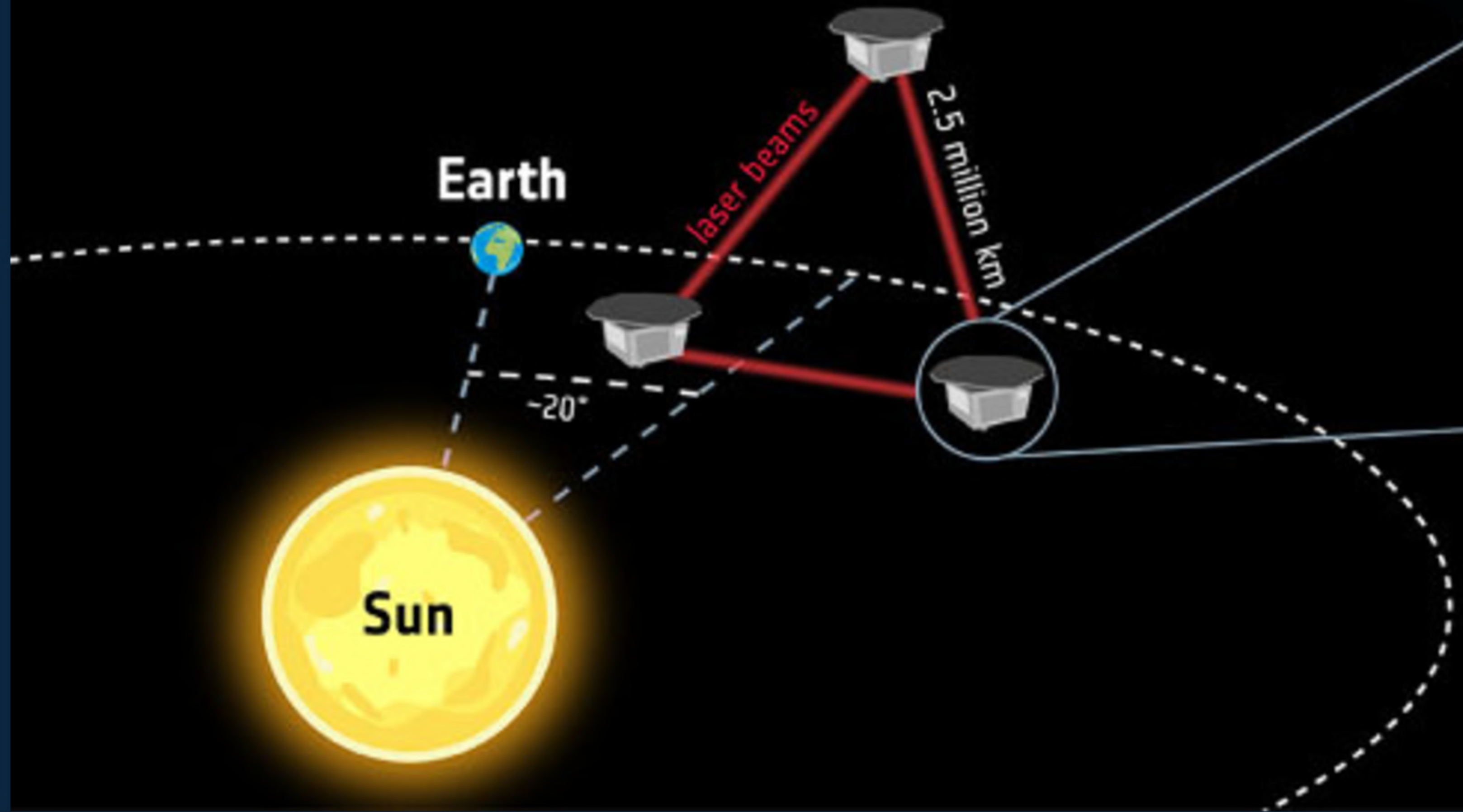
Gravitational waves are ripples in spacetime that alter the distances between objects. LISA will detect them by measuring subtle changes in the distances between **free-floating cubes** nestled within its three spacecraft.

3 **identical spacecraft** exchange **laser beams**. Gravitational waves change the distance between the **free-floating cubes** in the different spacecraft. This tiny change will be measured by the laser beams.

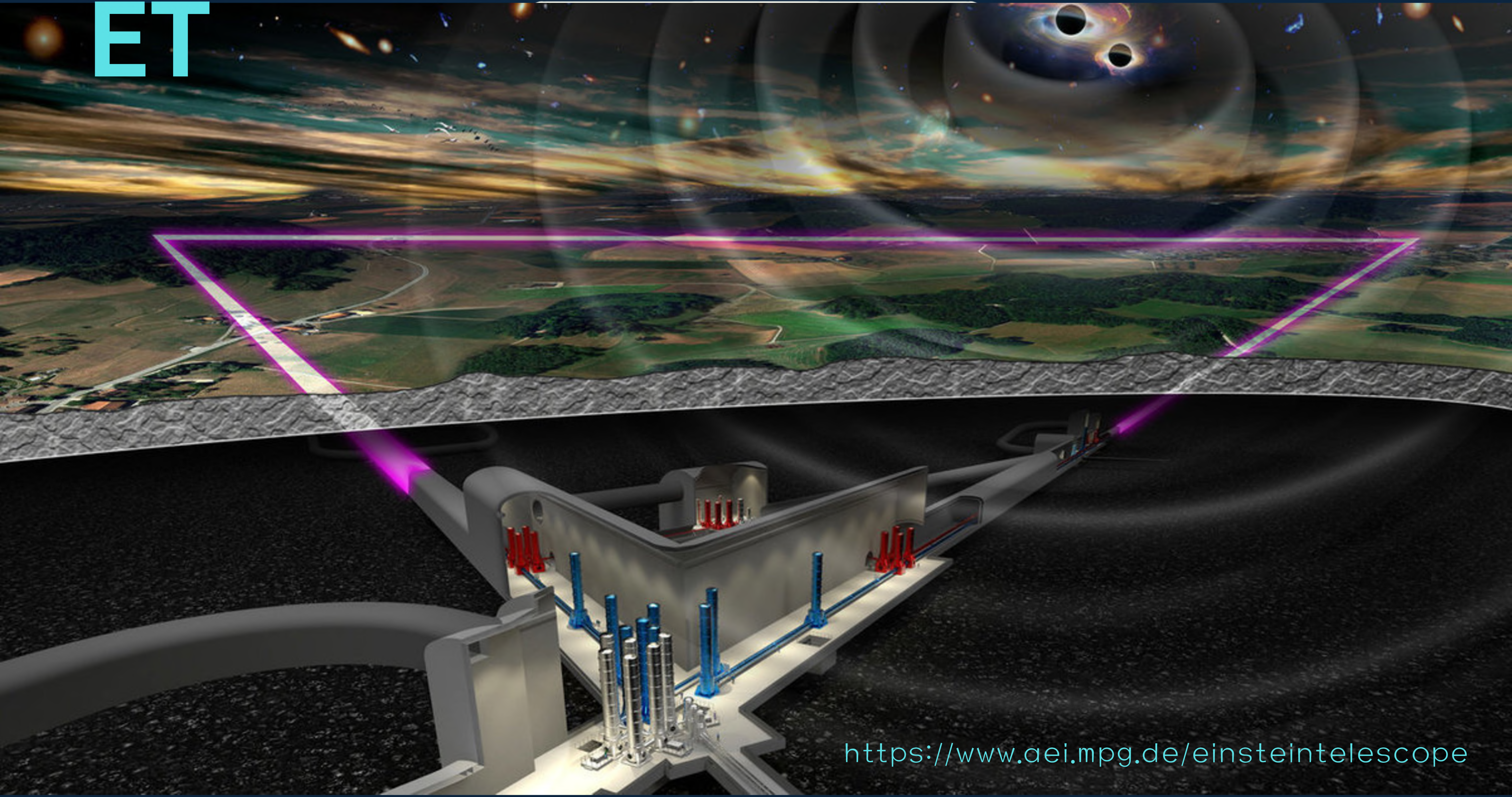
* *Changes in distances travelled by the laser beams are not to scale and extremely exaggerated*

Powerful events such as **colliding black holes** shake the fabric of spacetime and cause gravitational waves





ET



https://www.aei.mpg.de/einsteintelescope

GPR

•Non-parametric modeling: Gaussian Process Regression (GPR) reconstructs H(z) from data without dark energy or gravity assumptions, preserving uncertainties in eLISA/ET mock catalogs.

•Sparse, noisy data: GPR interpolates GW-derived H(z) data, smoothing gaps using covariance kernels to handle errors.

• Derivative reconstruction: GPR calculates derivatives via covariance matrices.

•Future missions: GPR on ET (1,000 events) or eLISA (80 events) data constrains H0, rivaling current probes and aiding Hubble tension resolution.

•Kernel insights: GPR kernel choice (e.g., RBF, Matern) affects high-redshift H(z) and acceleration, guiding model selection.

Rasmussen and Williams, 2005

MOCK CATALOGUE -ET

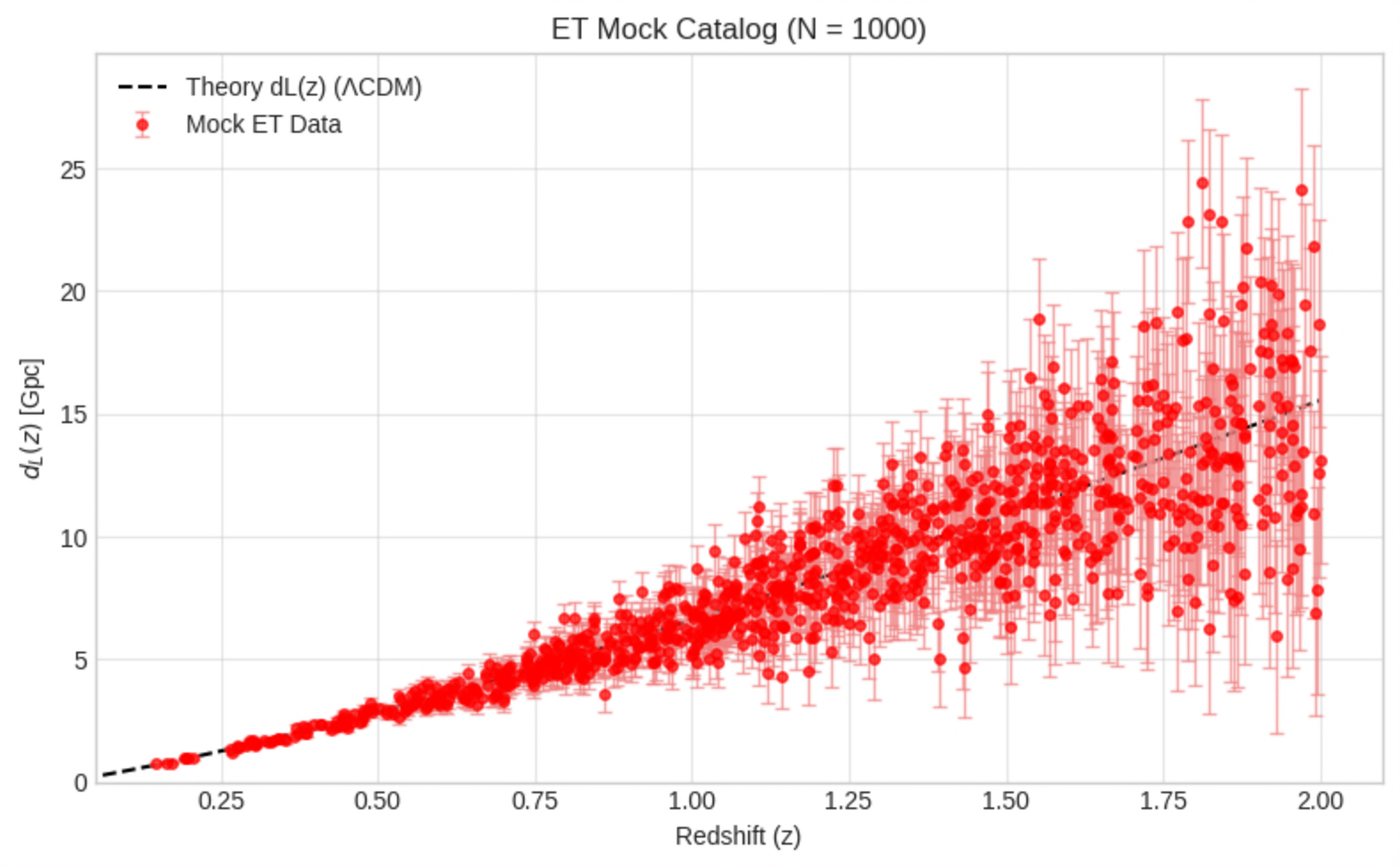
- Adopted Vanilla ΛCDM. $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$ & $\Omega_\Lambda = 0.7$.

- z lies within [0.07,2.0].

- Error models: Instrumental noise, Lensing scatter & Systematic floor.

Ferreira et al., 2022

MOCK CATALOGUE -ET



Mock Catalogue - LISA

- Adopted Vanilla ΛCDM. H_0= 70 km/s/Mpc, Ω_m= 0.3 & Ω_Λ=0.7.

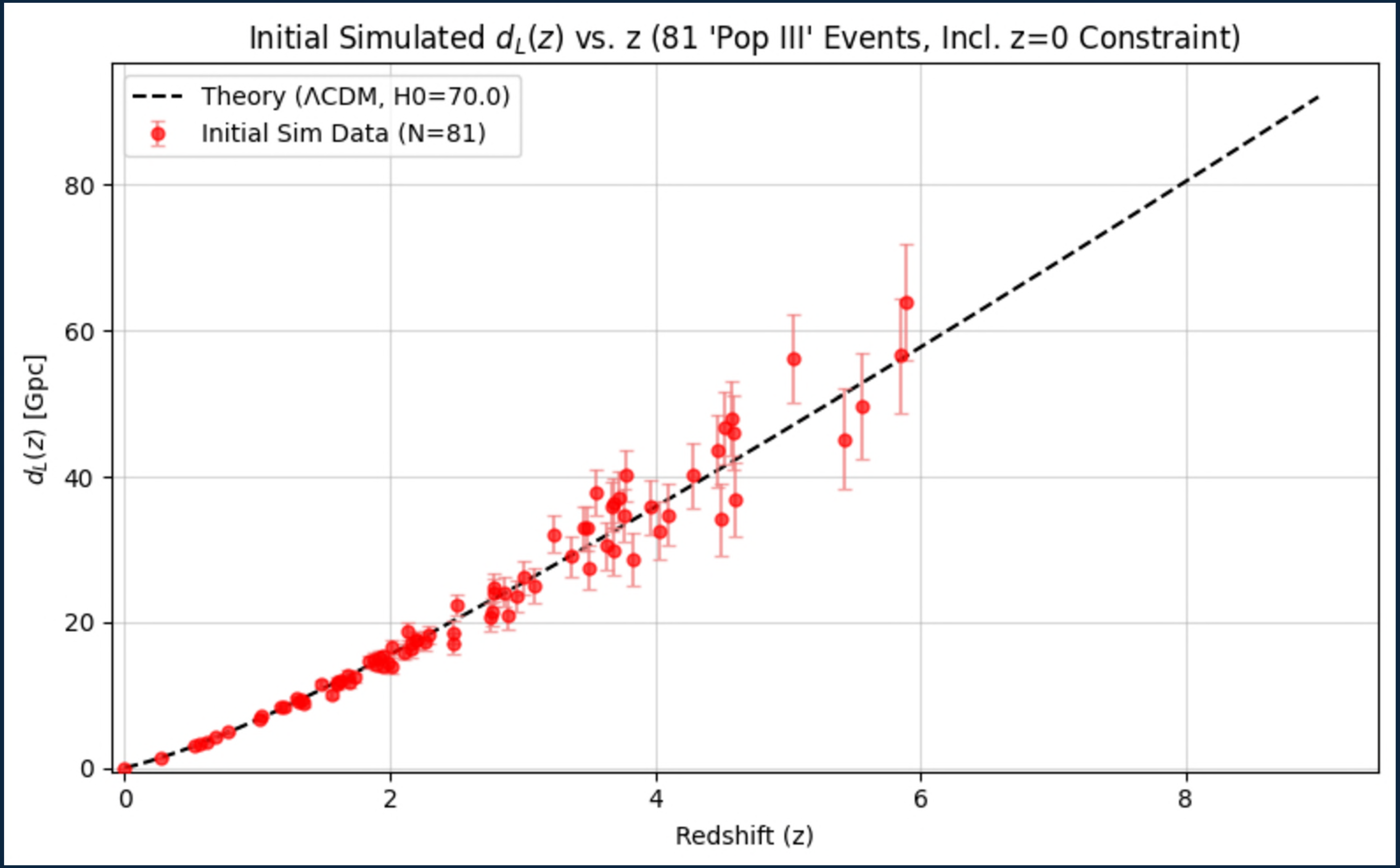
- z upto 9. Added an event at z=0.

- Error models: Weak lensing & Peculiar velocity.

(2.42, 3.84), Delay
(2.64, 6.03), Pop III
(2.14, 4.70), No Delay.

Ferreira et al., 2022

Mock Catalogue - LISA



GPR - LISA

1. Log-posterior: Define $\theta = [\log l, \log \sigma_x]$ with uniform priors. Returns log marginal likelihood ($-\infty$ if unstable/out-of-bounds).

2. Bayesian Optimization (BO): Search $l \in [0.1, 25]$, $\sigma_x \in [\hat{\sigma}/100, 100\ \hat{\sigma}]$. 60 calls (15 random starts) using Expected Improvement.

3. BO hyperparameters: Convert best (l, σ_x) to $\theta_0 = [\log l, \log \sigma_x]$ for MCMC starting point.

4. MCMC refinement: 40 walkers, 5,000 steps, 2,000 burn-in, initialized at θ_0. Target acceptance fraction 0.2-0.5, check autocorrelation.

5. Final hyperparameters: Use median l, σ_x from post-burn-in chains (16/84 percentiles for errors). Fallback to BO if MCMC fails.

6. GP kernel: Set kernel = $\sigma_x^2 \cdot \mathrm{Matern}(\ell, \nu=9/2)$ with MCMC-derived ℓ, σ_x. Zero mean.

GPR - LISA

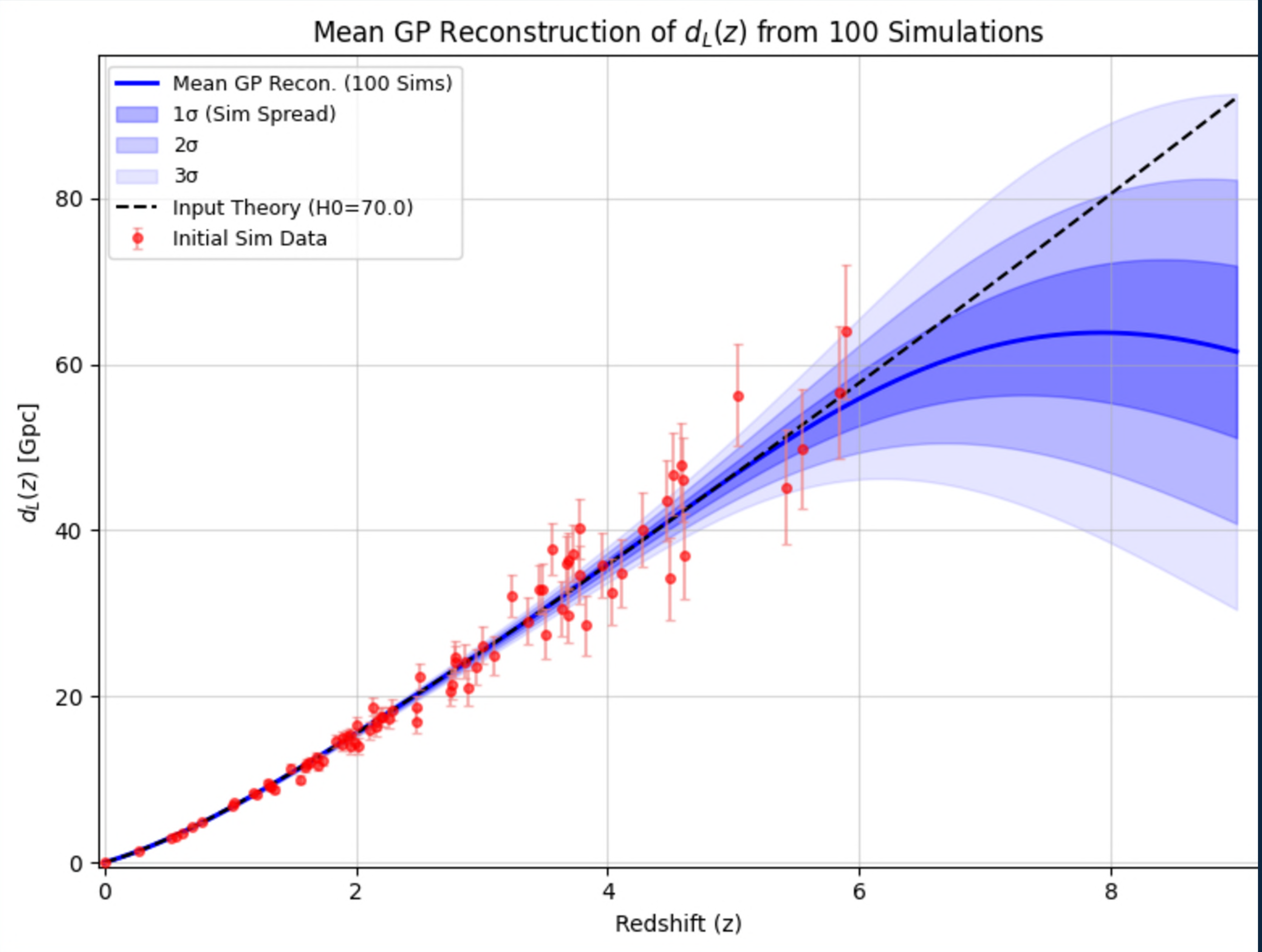
7. Main simulation loop (N simulations):
For each sim: generate mock z-dL data, enforce dL(0)=0 constraint
Fit GP with fixed kernel to each mock dataset

8. GP prediction and differentiation:
Predict dL(z) on a dense z-grid ($0 \to z_{max}$)
Compute dL′(z) via finite differences

9. Reconstruct H(z) per simulation:
$H(z)=c\,(1+z)^2 / [dL'(z)(1+z)-dL(z)]$, with special handling at z=0
Store each sim's dL, dL′, H(z)

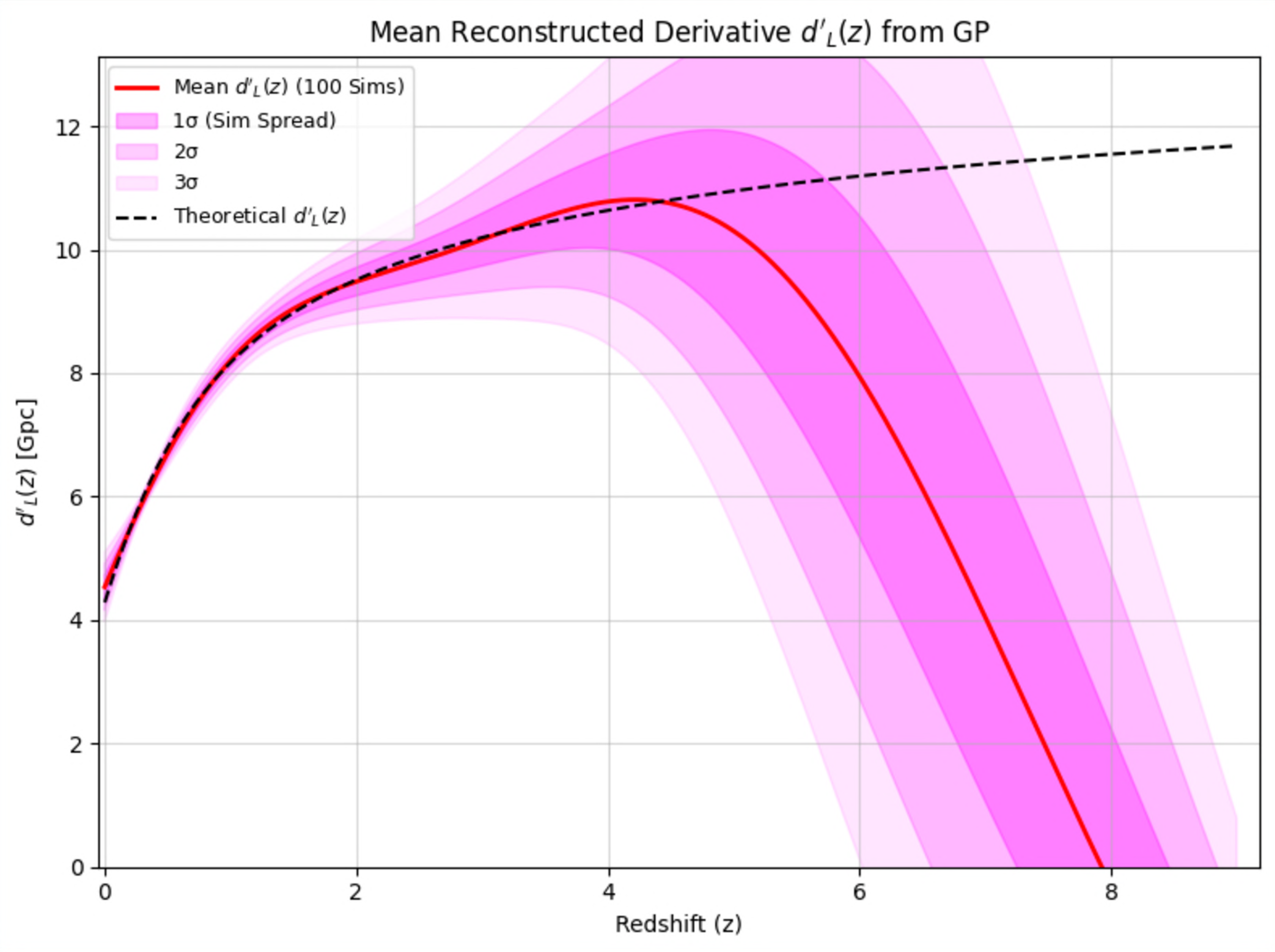
10. Ensemble statistics & plots:
Compute mean$\pm\sigma$ across all sims for dL, dL′, H(z)
Report H_0 reconstruction (mean, std, relative error)
Visualize GP reconstructions with 1-3 σ bands vs. theoretical curves

d_L vs z - LISA



Mean GP Reconstruction of $d_L(z)$ from 100 Simulations
Mean GP Recon. (100 Sims)
1σ (Sim Spread)
2σ
3σ
Input Theory (H0=70.0)
Initial Sim Data
$d_L(z)$ [Gpc]
0
20
40
60
80
0
2
4
6
8
Redshift (z)

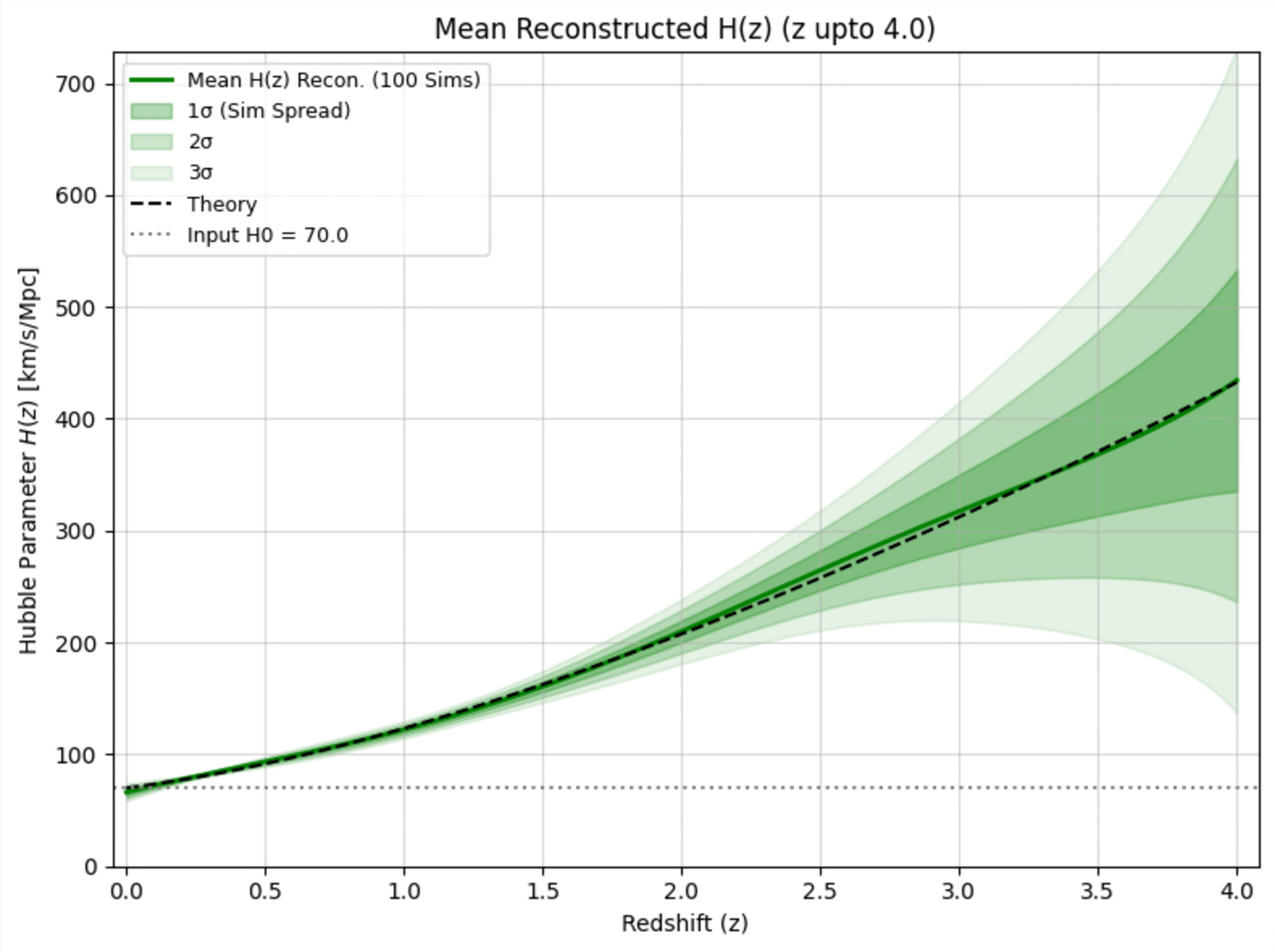
d'_L vs z - LISA


Mean Reconstructed Derivative $d'_L(z)$ from GP
Mean $d'_L(z)$ (100 Sims)
1σ (Sim Spread)
2σ
3σ
Theoretical $d'_L(z)$
$d'_L(z)$ [Gpc]
Redshift (z)
0
2
4
6
8
10
12

Formula

$$H(z) = \frac{c(1+z)^2}{d_L'(1+z) - d_L}$$

H(z) vs z - LISA

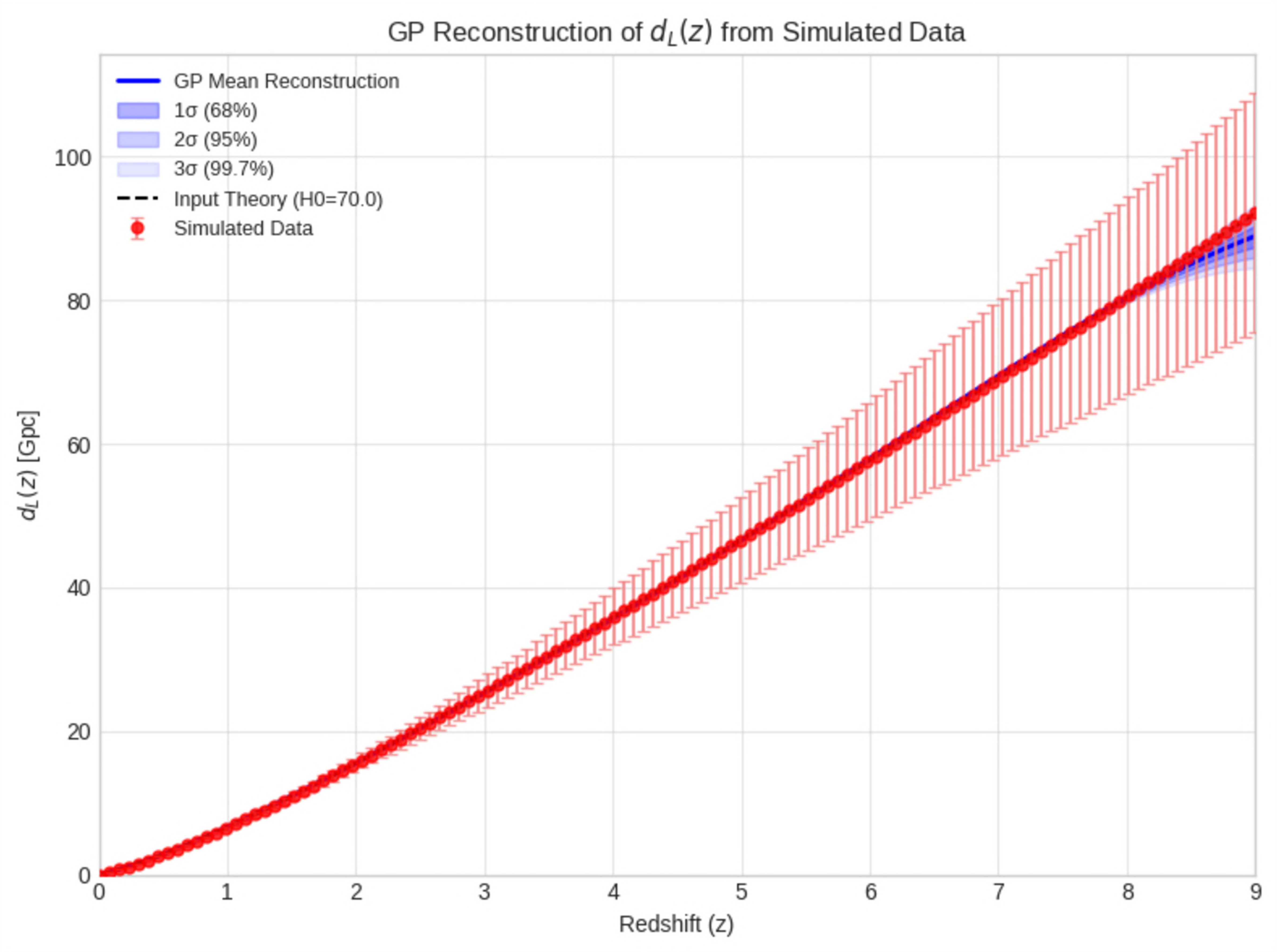


Mean Reconstructed H(z) (z upto 4.0)
Mean H(z) Recon. (100 Sims)
1σ (Sim Spread)
2σ
3σ
Theory
Input H0 = 70.0
Hubble Parameter H(z) [km/s/Mpc]
Redshift (z)
0
100
200
300
400
500
600
700
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

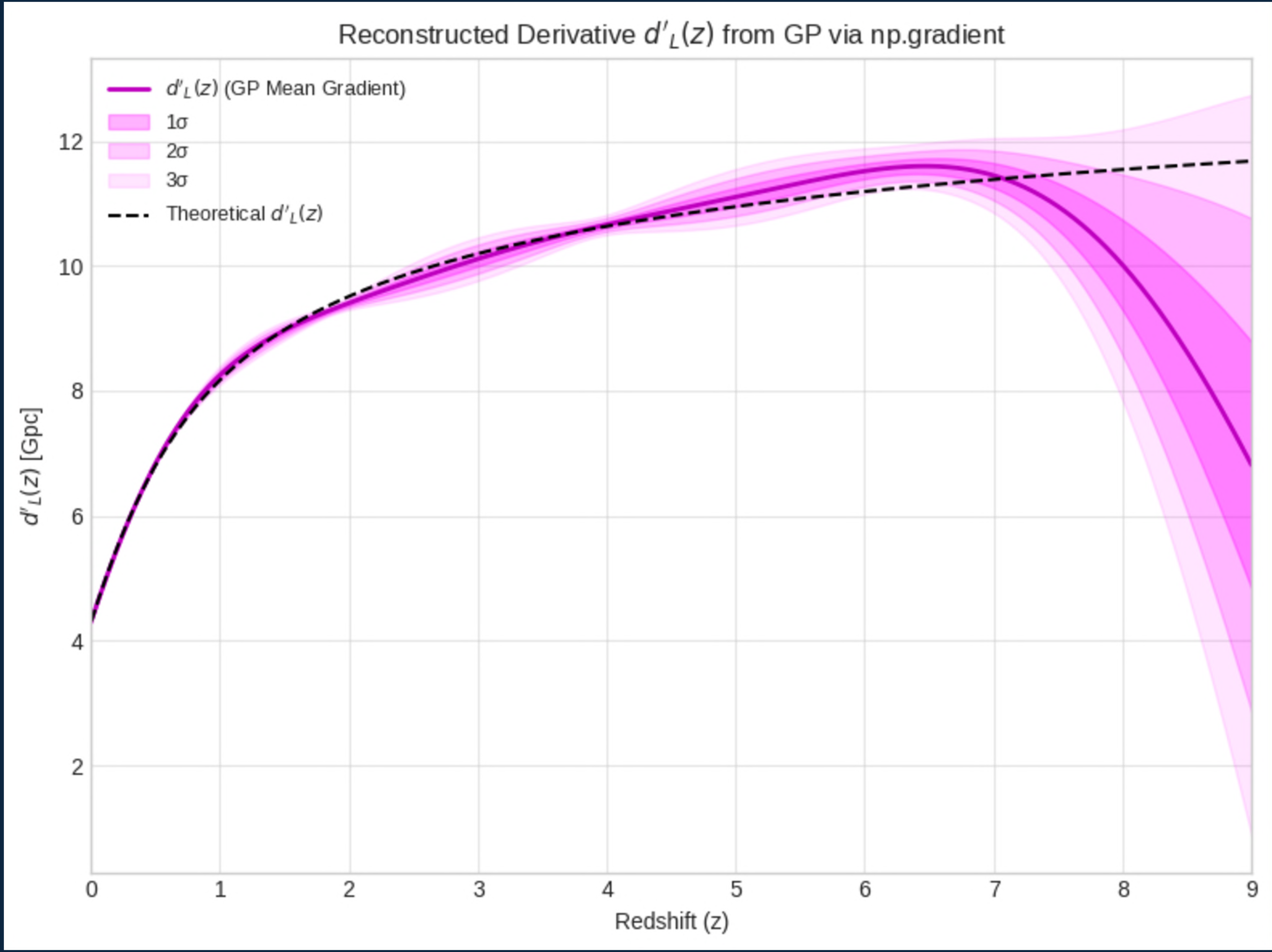
Cross Verification

Used noise-free mean to check the GPR

d_L vs z - LISA

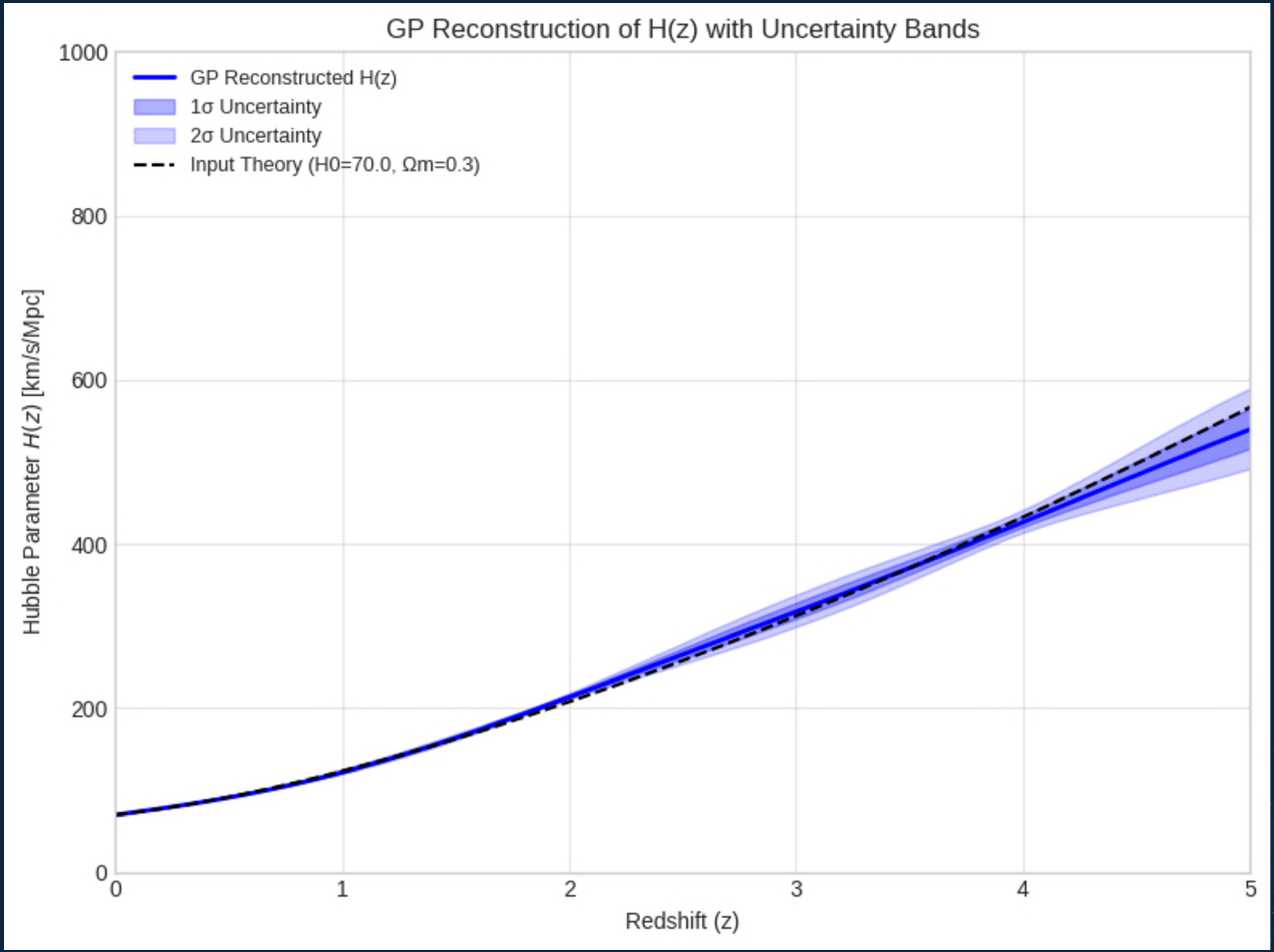

GP Reconstruction of $d_L(z)$ from Simulated Data
GP Mean Reconstruction
1σ (68%)
2σ (95%)
3σ (99.7%)
Input Theory (H0=70.0)
Simulated Data
$d_L(z)$ [Gpc]
Redshift (z)
0
20
40
60
80
100
0
1
2
3
4
5
6
7
8
9

d'_L vs z - LISA



Reconstructed Derivative $d'_L(z)$ from GP via np.gradient
$d'_L(z)$ (GP Mean Gradient)
1σ
2σ
3σ
Theoretical $d'_L(z)$
$d'_L(z)$ [Gpc]
Redshift (z)
0
1
2
3
4
5
6
7
8
9
2
4
6
8
10
12

H(z) vs z - LISA



GP Reconstruction of H(z) with Uncertainty Bands
GP Reconstructed H(z)
1σ Uncertainty
2σ Uncertainty
Input Theory (H0=70.0, Ωm=0.3)
Hubble Parameter H(z) [km/s/Mpc]
Redshift (z)
1000
800
600
400
200
0
0
1
2
3
4
5

Pros:

Model-independent

Uncertainty quantification

Smooth reconstruction

Flexible

Cons:

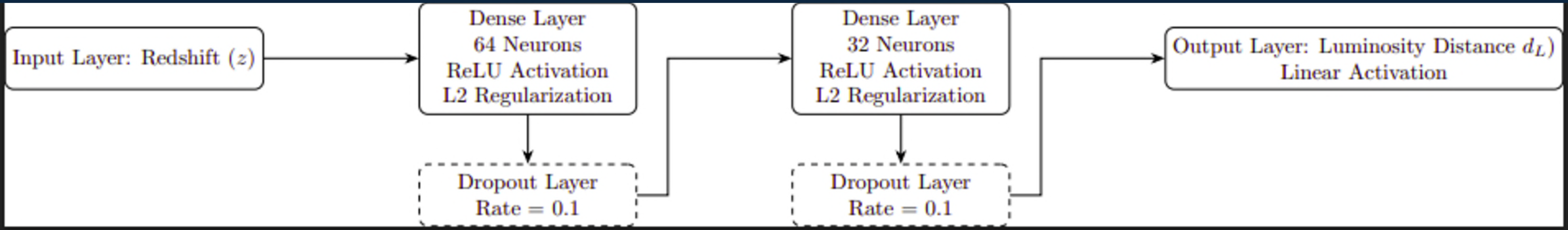
Computationally expensive

Derivative noise

Boundary effects

Gaussian bias

ANN



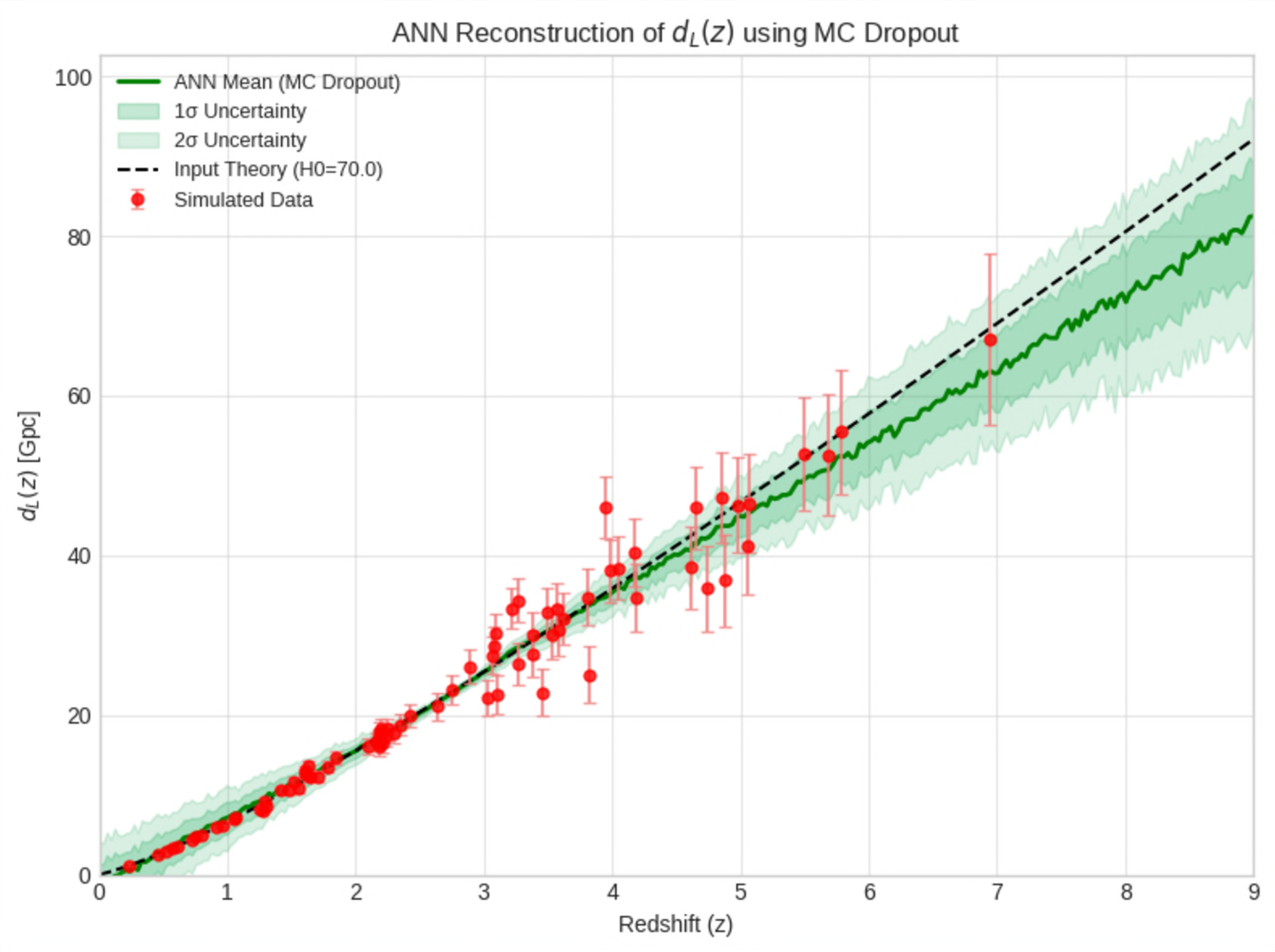
Scalable

Non-linear patterns

Better predictor

Uncertainty (MC dropout)

d_L vs z - LISA-ANN



Challenges

• GaPP Incompatibility: Legacy GaPP code to compute derivative via properties of GP kernels requires Python 2.7, necessitating a custom GPR implementation in Python 3.

• Derivative Noise: Finite-difference derivatives via np.gradient introduce high-z scatter in H(z).

• Computational Cost: Running GPR and MCMC on 1000-event ET catalogs is time-consuming;
convergence demands significant CPU time.

• GPU Limitations: Restricted Google Colab GPU resources slow down deep-learning experiments, limiting ANN architecture exploration.

In a Nutshell

My key achievements to date include:

- LISA Mock Catalogue GPR: 100 astrophysically realistic mock catalogs generated for (80+1) events for each scenario: "No Delay", "Delay", and "Pop III". Stable reconstruction of $H(z)$ up to $z = 4$ with uncertainties 5% at $z < 2$; cross-checked against noise-free mean catalogs.

- ET Mock Data: Full generation of 1000-event catalogs.

- ANN for LISA: Achieved $d_L(z)$ reconstruction up to $z = 9$ with mean squared error $< 10^{-4}$ on validation sets, demonstrating feasibility of alternative machine-learning approaches.

Future Work

- Additional Detectors: Incorporate forecasts for next-generation observatories such as DECIGO (Mandel, Sesana, and Vecchio, 2018), the Big Bang Observer (BBO), Cosmic Explorer (CE) (Reitze et al., 2019), and TianQin, to evaluate their combined leverage on $H(z)$.

- Dark Siren Analyses: Develop statistical redshift inference techniques (e.g. galaxy catalog cross-correlations) to include dark sirens without electromagnetic counterparts, thereby multiplying the available event sample.

- Advanced Machine Learning: Investigate tools like LADDER (Shah et al., 2024) architectures for deep-learning reconstruction, which may better capture non-Gaussian features in noisy standard siren datasets.

- Deep Gaussian Processes: Combine Deep GPs with ANN outputs to enforce smoothness, improve extrapolation, and mitigate finite-difference noise in derivative estimation.

- Ef-KAN Kernels: Explore the recently proposed Ef-KAN kernel families for nonparametric inference (Cui et al., 2025), which offer adaptive, data-driven covariance structures that may further tighten constraints on $H(z)$.

THANK YOU

1krishnanjansil1@gmail.com