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DMGT. Tutorial 10

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PA 20
Batch A1

Q.1. $(V, \times)$ is abelian $V = \{1, -1, i, -i\} \subset \mathbb{C}$

w.r.t. $(*)$

→ (P1) $a * b = c \quad \forall\, a, b, c \in V$

$1 \times -1 = -1 \in V$

$i \times -i = -i^2 = 1 \in V$

$\cancel{i^n \in \{-1, 1, -i\}}$

so cosed under multiplication.

(P2) $(a * b) * c = a * (b * c)$

$\forall\, a, b, c \in V$

eg. $(1 \times 1) \times -1 = (1) \times (1 \times -1)$

$-1 = -1 \quad \forall\, V.$

(P3) multiplicative Identity exists

$a * e = a$

$\boxed{e = 1}$

$i \times 1 = i \; ; \; -i \times 1 = -i$

$1 \times 1 = 1 \; ; \; -1 \times 1 = -1$

P (4) Multiplicative Inverse

$$a \times * y = e$$

$$y = \frac{e}{a} = \frac{1}{a}$$

So $$i \times \frac{1}{i} = 1$$

$$-i \times \frac{1}{i} = 1$$

$$\frac{1}{1} = \frac{-1}{1} = 1$$

P (5) ~~Clos~~ Commutative

$$a * b = b * a$$

$$1 \times (-1) = (-1) \times 1 = (-1)$$

$$i \times -i = -1 = -i \times i$$

$$i \times 1 = 1 \times i$$

$$-1 \times i = i \times -1$$

as all 5 properties are satisfied,

V is abelian.

Q.2 Semigroup but not group.

eg. Set of odd numbers wrt ($*$) multiplication

let $S = \{ a : a = 2n + 1 \ \forall n \in \mathbb{Z} \}$

so $(S, *)$

1. $a * b = c \ \forall a, b, c \in S$

So eg. $3 \times 5 = 15 \in S$

$3 \times -1 = -3 \in S$

2. $(a * b) * c = a * (b * c)$

$\forall a, b, c \in S$

3. multiplication Identity exists.

$a * e = a \quad \forall a, e \in S$

$e = \frac{a}{a} = 1$

eg. $-3 \times 1 = -3$

$4 \times 1 = 4$

4. Inverse doesnt exist

$a \times \frac{1}{a} = e = 1$ But $1/a \notin S$

So only subgroup not group.

Q.3. Considering $\left(\mathbb{Z}/4\mathbb{Z}, *, +\right)$
$= \{0, 1, 2, 3\}$

(1) $a \times b = c \quad \forall \; a, b, c \in \mathbb{Z}$

(2) $(a \times b) \times c = a \times (b \times c)$

$\forall \; a, b, c \in \mathbb{Z}$

(3) $a \times e = a$;

$e = 1 \quad \forall \; a, e \in \mathbb{Z}$

(4) $a \times \frac{1}{a} \cancel{e} = a$ but

$1/a \notin \mathbb{Z}/4\mathbb{Z}$

Not all properties of $*$ are satisfied.

For (+) ; ~~a + b =~~ ~~3 = 7~~ $(a = b$ iff $a - b \in \mathbb{Z}/4$

(1) $a + b = c \quad \forall \; a, b, c \in \mathbb{Z}$

$3 + 4 = 7$

$7/4 = 3$ as remainder.

So $7 \cancel{e} = 3 \in \mathbb{Z}/4\mathbb{Z}$

(2) $(a + b) + c = a + (b + c)$

$\forall \; a, b, c \in \mathbb{Z}/4\mathbb{Z}$

(3) $a + e = a$; $e = 0$

$\forall \; a, \cancel{e} \in \mathbb{Z}/4\mathbb{Z}$

(4) $a + (-a) = e$

So inverse exists

as $-a \in Z/4Z$

$3 + (-3) = 0$

$-3/4 = 3$ as remainder

and $3 \in Z/4Z$

So $-3 \in Z/4Z$

(5) $a + b = b + a$

$\forall (a, b) \in Z/4Z$

So its an ~~g~~ abelian group wrt $(+)$

$(Z/4Z, +, *)$ is a ring.

(Q.4) $G =$ odd numbers

~~$4 \in$~~ $= \{ a : a = 2n+1 \; \forall n \in Z \}$

~~S~~

So Subgroup $G_s = \{ a : a = 2n+1 \; \forall n \in Z^+ \}$

So $G_s = \{$ all positive odd no $\}$

is Subgroup of $G = \{$ all odd numbers $\}$

Q.5: ~~Ver~~ Verify if set of polynomials is a group wrt multiplication.

Def $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2}$

$\dots \quad a_1 x + a_0$

let $S = \{ P(x) \}$

$p(x) * q(x) = r(x)$

$\forall$ ~~B~~ $p(x), q(x), r(x)$

$\in S$

this products would yield a higher power polynomial.

(2) $(p(x) * q(x)) * r(x)$

$= p(x) \times (q(x) \times r(x))$

it is associative.

$\forall \; p(x), q(x), r(x) \in S.$

(3) $p(x) \times e(x) = p(x)$

$\boxed{e(x) = 0x + 1}$ is identity.

(4) $p(x) \times \frac{1}{p(x)} = 0x + 1$

$\forall\, p(x) \in S$

So it is a group.

if $p(x)$ contains

$(a, x \in R)$