

DISCRETE STRUCTURES

Example 4: An investigator interviewed 100 students to determine their preferences for the three drinks – Milk (M), Coffee (C) and Tea (T). He reported the following:
10 students had all the three drinks, 20 had 'M' and 'C', 30 had 'C' and 'T', 25 had 'M' and T
12 had 'M' only, 5 had 'C' only and 8 had 'T' only.

- (i) How many did not take any of the three drinks?
- (ii) How many take milk but not coffee?
- (iii) How many take tea and coffee but not milk?

Example 5: (i) Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations?

(ii) If the number of students who got an A in the first examination is equal to that in the second examination, if the total number of students who got an A in exactly one examination

is 40 and if 4 students did not get an A in either examination, then determine the number of students who got an A in the first examination only, who got an A in the second examination only, and who got an A in both the examinations.

Example 6: In a survey, it is reported that of 1000 programmers, 650 habitually flowchart their programs, 788 are skilled COBOL programmers, 675 are men, 278 of the women are skilled COBOL programmers, 440 programmers both habitually flowchart and are skilled in COBOL, 210 women habitually flowchart and 166 women are both skilled in COBOL and habitually flowchart. Would you accept these data as being accurately reported? Justify your answer.

Example 8: It was found that in first year of computer science of 80 students 50 know Cobol, 55 know 'C', 46 know Pascal. It was also known that 37 know 'C' and Cobol, 28 know 'C' and Pascal, 25 know Pascal and Cobol. 7 students, however, know none of the languages.

Find: How many know all three languages?

Example 11: A college record gives the following information: 119 students enrolled in Introductory Computer Science; of these 96 took Data Structures, 53 took Foundations, 39 took Assembly Language, 31 took both Foundations and Assembly Language, 32 took both Data Structures and Assembly Language, 38 took Data Structures and Foundations and 22 took all the three courses.

Is the information correct? Why?

Example 9: How many elements are in the union of five sets if the sets contain 10,000 elements each, each pair of sets has 1000 common elements, each triple of sets has 10 common elements, every four of the sets has 1 common element, and there is 1 element common in all five sets?

Example 7: 75 children went to an amusement park, where they can ride on the merry-go-round, roller coaster, and the Ferris wheel. It is known that 20 of them have taken all three rides, and 55 of them have taken at least 2. Each ride costs 5 rupees and the total collection of the park was 700 rupees. Determine the number of children who did not try any of the rides.

Ex: 4 (i) $|\overline{M} \cap \overline{C} \cap \overline{T}| = 20$

(ii) $|M - C| = 27$

(iii) $|(T \cap C) - M| = 20.$

$M \equiv$ Milk

$C \equiv$ Coffee

$T \equiv$ Tea.

Ex: 5 (i) $|F \cap S| = 14$

(ii) $|F| = |S| = 26$

$F \equiv$ Who got 'A' in first exam

$S \equiv$ Who got 'A' in second exam

$F \equiv$ the set of programs (both men & women) who habitually flow-chart their programs
 \uparrow $C \equiv$ the set of skilled 'COBOL' programmers

Ex: 6 $|M \cap (F \cup C)| = 676$

$M \equiv$ Men programmers; $F \equiv$ Female programmers

Ex: 7 No. of children who have not taken any ride = 10

Ex: 8 (i) $|B \cap C \cap P| = 12$ | $B \equiv$ COBOL

(ii) ~~$|B \cap C \cap \bar{P}| = 25$~~

Exactly two

~~Languages~~ = 54

$C \equiv C$

$P \equiv$ Pascal.

(iii) $|B| - |B \cap C| - |B \cap P| + |B \cap C \cap P| = 0$

\rightarrow (ii) $|B \cap C \cap \bar{P}| = |B \cap C| - |B \cap C \cap P|$
 $= 25$

iii) $|B \cap P \cap \bar{C}| = |B \cap P| - |B \cap C \cap P|$
 $= 13$

& $|\bar{B} \cap P \cap C| = |P \cap C| - |B \cap C \cap P|$
 $= 16$

Hence Ans = $25 + 13 + 16 = 54$

Ex: 9 $|A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5| = 40,951$

Ex: 10: $|A \cup B| = 55$

Ex: 11 = Let $D \equiv$ Data structure
 $F \equiv$ Foundations.
 $A \equiv$ Assembly Language

M T W T F S S

Page No.:

Date:

YOUVA

$$|FUDUA| = 109 < 119$$

Given information is not correct.

EXERCISE - 2.1

1. If $A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$, determine whether the following statements are true or false.

Justify your answer.

- (i) $\phi \in A$
- (ii) $\{\phi\} \subseteq A$
- (iii) $\{\phi\} \in A$
- (iv) $\{\phi, \{\phi\}\} \subseteq A$
- (v) $\{\{\phi\}\} \in A$

2. If $u = \{n \in \mathbb{N} \mid 1 \leq n \leq 9\}$,

$A = \{1, 2, 4, 6, 8\}$, $B = \{2, 4, 5, 9\}$, $C = \{x \in \mathbb{Z}^+ \mid x^2 \leq 16\}$ and $D = \{7, 8\}$,

- find
- (i) $A \oplus B, B \oplus C, C \oplus D$
 - (ii) $A - B, B - A, C - D$
 - (iii) $\overline{A \cup B}, \overline{A \cap B}$
 - (iv) $A \cap (\overline{C} \cup D)$

DISCRETE STRUCTURES

3. For $A = \{a, b, \{b, c\}, \phi\}$ determine the following sets:

(i) $A - \{a\}$, (ii) $A - \{b, c\}$, (iii) $\{\{b, c\}\} - A$, (iv) $A - \{c, \phi\}$, (v) $\{a\} - \{A\}$.

4. Give an example of sets A, B, C such that $A \in B, B \in C$ and $A \notin C$.

5. Draw Venn diagrams for the following situations.

(i) A, B, C are sets such that $A \subseteq B, A \subseteq C, (B \cap C) \subseteq A$ and $A \subseteq (B \cap C)$.

(ii) $(A \cap B \cap C) = \phi, A \cap B \neq \phi, B \cap C \neq \phi, A \cap C \neq \phi$.

6. Using Venn diagrams, prove or disprove the following:

- (i) $(A - B) - C = (A - C) - B$
- (ii) $(A - B) - C = (A - C) - (B - C)$
- (iii) $(A - B) \cap (A - C) = A - (B \cup C)$
- (iv) $(A - C) \cup (B - C) = (A \cup B) - C$
- (v) $A - (B - C) = (A - B) \cup (A \cap C)$
- (vi) $A \cap (B - C) = (A \cap B) - (A \cap C)$
- (vii) $(A \cap B) - C = (A - C) \cap (B - C)$
- (viii) $(A \oplus B) \cap C = (A \cap C) \oplus (B \cap C)$
- (ix) $A \cup (\overline{B} \cap C) = (A \cup \overline{B}) \cap (A \cup C)$.

(May 2005)

7. Using the rules of set operations, simplify the following:

- (i) $\overline{(A \cup B)} \cup (\overline{A} \cap B)$
- (ii) $[(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)] \cap B$
- (iii) $((A \cup B) \cap \overline{A}) \cup (\overline{B \cap A})$
- (iv) $\overline{[(A \cap B) \cup C]} \cap \overline{B}$.

8. What can you say about sets A and B , if

- (i) $A - B = B?$
- (ii) $A - B = B - A?$
- (iii) $A \oplus B = A?$

9. It is known that at the University, 60 percent of the professors play tennis, 50 percent of them play bridge, 70 percent jog, 20 percent play tennis and bridge, 30 percent play tennis and jog and 40 percent play bridge and jog. If someone claimed that 20 percent of the professors jog and play bridge and tennis, would you believe this claim? Why?

10. A survey was conducted among 1000 people. Of these 595 are graduates, 595 wear glasses and 550 like ice cream, 395 of them are graduates who wear glasses, 350 of

- them are graduates who like ice cream and 400 of them wear glasses and like ice cream; 250 of them are graduates who wear glasses and like ice cream. How many of them who are not graduates do not wear glasses and do not like ice cream? How many of them are graduates who do not wear glasses and do not like ice cream?
11. Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7? Also indicate how many are divisible by 3 or 7 but not by 5.
 12. How many integers between 1 and 2000 are divisible by 2, 3, 5 or 7?
 13. A college record gives the following information: 119 students enrolled in Introductory Computer Science; of these, 96 took Data Structures, 53 took Foundations, 39 took Assembly Language. Also 38 took both Data Structures and Foundations, 31 took both Foundations and Assembly Language, 32 took both Data Structures and Assembly language and 22 took all the three courses. Is the information correct? Why?
 14. A survey of 100 students of the Management Programme shows that 70 read India Today, 31 read Fortune and 54 read Business India. Also the people who read Business India do not read Fortune. Draw a Venn diagram to represent the situation.
 15. A software company writes a new package which integrates a word processing program with a spreadsheet program, and they wish it to run on a 64 K machine. The word processor requires 40 K for program and data and the spreadsheet requires 32 K for the same. If 16 K must be reserved for the code integrator, what is the minimum amount of overlapping space that will be necessary?
 16. Consider a set of integers 1 to 500. Find how many of these numbers are divisible by 3 or by 5 or by 11?
 - (i) Also indicate how many are divisible by 3 or by 11 but not by all 3, 5 and 11.
 - (ii) How many are divisible by 3 or 11 but not by 5? (May 2005)
 17. It was found that in first year of computer engineering out of 80 students, 50 know 'C' language, 55 know 'basic' and 25 know 'C++', while 8 did not know any language. Find,
 - (i) How many know all the three languages?
 - (ii) How many know exactly two languages? (May 2005)
- In the survey of 60 people, it was found that 25 read Newsweek Magazine, 26 read time, 26 read fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune and 8 read no magazine at all.
- (i) Find out the number of people who read all the three magazines.

- (ii) Fill in the correct numbers in all the regions of the Venn diagram.
- (iii) Determine number of people who reads exactly one magazine. **(Dec. 2005)**
19. Among 130 students, 60 study Mathematics, 51 Physics and 30 both Mathematics and Physics. Of the 54 students studying Chemistry, 26 study Mathematics, 21 Physics and 12 both Mathematics and Physics. All the students studying neither Mathematics nor Physics are studying Biology.
- (i) How many students are studying Biology?
- (ii) How many students not studying Chemistry are studying Mathematics but not Physics?
- (iii) How many students are studying neither Mathematics nor Physics nor Chemistry. **(May 2006)**
20. It was found that in first year of computer science of 80 students, 50 know COBOL, 55 know C language and 46 know Pascal. It was also known that 37 know C and COBOL, 28 know C and Pascal, and 25 know Pascal and COBOL. 7 students however know none of the language. Find:
- (i) How many know all the three languages?
- (ii) How many know exactly two languages?
- (iii) How many know exactly one language?
21. A survey has been taken on methods of computer travels. Each respondent was asked to check BUS, TRAIN or AUTOMOBILE as a major method of traveling to work. More than one answer was permitted. The results reported were as follows: BUS – 30 people, TRAIN – 35 people, AUTOMOBILE – 15 people, TRAIN and AUTOMOBILE – 20 people and all three methods-5 people. How many people completed the survey form? **(Dec. 2008)**
22. A survey of 500 television watchers produced the following information. 285 watch football, 195 watch hockey, 115 watch basket ball. 45 watch football and basket ball, 70 watch football and hockey, 50 watch hockey and basketball and 50 do not watch any of the three games.
- (i) How many people in the survey watch all the three games?
- (ii) How many people watch exactly one game? **(May 2008)**
23. 100 of the 120 engineering students in a college take part in at least one of the activities: group discussion, debate and quiz. Also 65 participate in group discussion, 45 participate in debate, 42 participate in quiz, 20 participate in group discussion and

debate, 25 participate in group discussion and quiz, 15 participate in quiz. Find the number of students:

- (i) Who participate in all the three activities
- (ii) Who participate in exactly one of the activities.

24. In a class of 55 students, the number of students studying different subjects follows: Maths - 23, Physics - 24, Chemistry - 19, Maths + Physics - Chemistry - 9, Physics + Chemistry - 7, all three subjects - 4. Find the number of students who have taken: (i) At least one subject, (ii) Exactly one subject, (iii) Two subjects.
25. In a survey of 100 new cars, it is found that 60 had Air Conditioner (AC), 44 had Power-Steering (PS), 36 had Power Windows (PW), 20 had AC + PS, 16 had PS + PW, 12 had AC + PW, 8 had all three. Find the number of cars that had: (i) AC and PS but not PW, (ii) AC and PW but not PS, (iii) PS and PW but not AC, (iv) AC but not PS and PW, (v) PS but not AC and PW, (vi) PW but not AC and PS, (vii) None of the three.

Problems on Power Sets:

38. Let $A = \{a, \{a\}\}$. Determine which of the following statements are true or false.
- (i) $\phi \in P(A)$
 - (ii) $\phi \subseteq P(A)$
 - (iii) $\{a\} \in P(A)$
 - (iv) $\{a, \{a\}\} \in P(A)$
 - (v) $\{\{\{a\}\}\} \subseteq P(A)$
39. Determine whether the following statements are true or false. Justify your answer.
- (i) $A \cup P(A) = P(A)$
 - (ii) $\{A\} \cup P(A) = P(A)$
 - (iii) $A - P(A) = A$
 - (iv) $P(A) - \{A\} = P(A)$
 - (v) $\{A\} \cap P(A) = A$
40. For multisets, define in brief: (May 2010)
- (i) Multisets.
 - (ii) Multiplicity of an element in a multiset.
 - (iii) Cardinality of multiset.
 - (iv) Union of multiset.
 - (v) Intersection of multiset.
 - (vi) Difference of multiset.
41. A survey has been taken on methods of computer travel. Each respondent was asked to check bus, train or automobile as a major method of travelling to work. More than one answer was permitted. The results reported were as follows:
 Bus - 30 people, train - 35 people, automobile - 100 people, bus and train - 15 people, bus and automobile - 15 people, train and automobile - 20 people and all three methods - 5 people. How many people completed a survey form? (May 2010)

debate, 25 participate in group discussion and quiz, 15 participate in debate and quiz. Find the number of students:

- (i) Who participate in all the three activities
- (ii) Who participate in exactly one of the activities.

24. In a class of 55 students, the number of students studying different subjects are as follows: Maths 23, Physics – 24, Chemistry 19, Maths + Physics – 12, Maths + Chemistry – 9, Physics + Chemistry – 7, all three subjects - 4. Find the numbers of students who have taken: (i) At least one subject, (ii) Exactly one subject, (iii) Exactly two subjects.

(May 2007)

25. In a survey of 100 new cars, it is found that 60 had Air Conditioner (AC), 48 had Power-Steering (PS), 44 had Power Windows (PW), 36 had AC + PW, 20 had AC + PS, 16 had PW + PS, 12 had all three. Find the number of cars that had: (i) Only PW, (ii) PS and PW but not AC, (iii) AC and PS but not PW.

(Dec. 2006)

Problems on Power Sets:

38. Let $A = \{a, \{a\}\}$. Determine which of the following statements are true or false.

- (i) $\phi \in P(A)$
- (ii) $\phi \subseteq P(A)$
- (iii) $\{a\} \in P(A)$
- (iv) $\{a, \{a\}\} \in P(A)$
- (v) $\{\{\{a\}\}\} \subseteq P(A)$

39. Determine whether the following statements are true or false. Justify your answer.

- (i) $A \cup P(A) = P(A)$
- (ii) $\{A\} \cup P(A) = P(A)$
- (iii) $A - P(A) = A$
- (iv) $P(A) - \{A\} = P(A)$
- (v) $\{A\} \cap P(A) = A$

40. For multisets, define in brief:

(May 2010)

- (i) Multisets.
- (ii) Multiplicity of an element in a multiset.
- (iii) Cardinality of multiset.
- (iv) Union of multiset.
- (v) Intersection of multiset.
- (vi) Difference of multiset.

41. A survey has been taken on methods of computer travel. Each respondent was asked to check bus, train or automobile as a major method of travelling to work. More than one answer was permitted. The results reported were as follows:
Bus - 30 people, train - 35 people, automobile - 100 people, bus and train - 15 people, bus and automobile - 15 people, train and automobile - 20 people and all three methods - 5 people. How many people completed a survey form? (May 2010)

42. In a survey of 260 college students, the following data were obtained: 64 had taken a Mathematical course, 94 had taken a Computer Science course, 58 had taken a Business course, 28 had taken both Mathematic and Business courses, 26 had taken both Mathematical and Computer Science course, 22 had taken both Computer Science and Business course and 14 had taken all 3 types of courses.
- (1) How many students were surveyed who had taken none of the three types of courses?
 - (2) Of the students surveyed, how many had taken only Computer Science course?

ANSWERS - 2.1

1. (i) True, (ii) True, (iii) True, (iv) True, (v) False.
2. (i) $A \oplus B = \{1, 5, 6, 8, 9\}$
(ii) $A - B = \{1, 6, 8\}$
(iii) $\overline{A \cup B} = \{3, 7\}$.
3. (i) $A - \{a\} = \{b, \{b, c\}, \phi\}$ (v) $\{a\} - \{A\} = \{a\}$.
7. (i) \bar{A} , (iii) $\bar{A} \cup \bar{B}$.
9. Claim is false.
10. 155, 100
11. 86 numbers between 1 to 250 are divisible by 3 or 7 but not by 5.
12. 1499
19. (i) 49, (ii) 16, (iii) 30.
20. (i) 12, (ii) 66, (iii) 7.
22. (i) 20

Points to Remember

- A set is a **collection** of objects.
- An object in the collection is called an **element** or member of the set.
- The term **class** is also used to denote a set.
- A set may contain **finite** number of elements or **infinite** number of elements.
- A set is called an **empty set** or a **null set** if it contains no element. An empty set is denoted by the letter ϕ .

DISCRETE STRUCTURES

- If every element of a set A is also an element of a set B, then we say A is a **subset** of B, or A is **contained** in B. This is denoted by ' $A \subseteq B$ '. This can be also denoted by ' $B \supseteq A$ '. If A is not a subset of B, this is indicated by ' $A \not\subseteq B$ '.
- If all sets, considered during a **specific discussion** are subsets of a given set, then this set is called as the **Universal Set**, and is denoted by 'U'.
- A Venn diagram (named after the British logician John Venn) is a pictorial depiction of a set.

- Let A be a given set. **Complement** of A, denoted by \bar{A} is defined as

$$\bar{A} = \{x \mid x \notin A\}$$

- The union of two sets A and B is the set consisting of all elements which are in A, or in B, or in both sets A and B. It is denoted by $A \cup B$.
- The intersection of two sets A and B, denoted by $A \cap B$ is the set consisting of elements which are in A **as well as** in B.
- If the counting of the elements of a set is interminable or impossible, then such a set is said to be infinite.
- If a set **contains multiple occurrences of an object** then such set is called '**multiset**'
- **Multisubset (or msubset):** A multiset A is said to be a multisubset of each element in A is less or equal to its multiplicity in B.

EXERCISE - II

1. Let A be the product set $\{1, 2, 3\} \times \{a, b\}$. How many relations are there on A ?
2. $A = \{1, 2, 3, 4\}$, $B = \{1, 4, 6, 8, 9\}$; aRb if and only if $b = a^2$. Find the domain, range of R . Find also its relation matrix and draw its digraph.
3. Let $A = \mathbf{R}$, set of real numbers. Consider the following relation on A ; $(a, b) \in R$ iff $2a + 3b = 6$. Find domain of R and also its range.
4. Let $A = \{1, 2, 3, 4, 5\}$ and let $R = \{(1, 1), (1, 2), (2, 1), (1, 3), (1, 4), (4, 5), (5, 1), (1, 5), (4, 1)\}$. Draw the digraph of R .

5. For a set $A = \{1, 2, 3, 4, 5\}$, the relation matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Draw its digraph.

6. Let $A = \{1, 2, 3, 4\}$.

If $R = \{(a, b) \mid (a - b) \text{ is an integral non-zero multiple of } 2\}$

and $S = \{(a, b) \mid (a - b) \text{ is an integral non-zero multiple of } 3\}$

Find $R \cup S$ and $R \cap S$.

7. For a set $A = \{1, 2, 3, 4, 5\}$ relations R_1 and R_2 are given by

$$R_1 = \{(1, 2), (3, 4), (2, 2)\} \text{ and } R_2 = \{(4, 2), (2, 5), (3, 1), (1, 3)\}$$

Find (a) $R_1 R_2$, (b) $R_2 R_1$, (c) $R_1 (R_2 R_1)$, (d) $(R_1 R) R_1$, (e) R_1^3 , (f) R_2^2 .

8. If $A = B = \{1, 2, 3\}$, $R_1 = \{(1, 1), (1, 2), (2, 3), (3, 1)\}$

$$\text{and } R_2 = \{(2, 1), (3, 1), (3, 2), (3, 3)\}$$

Compute

(a) Complement of R_1 ,

(b) Converse of R_2

(c) $R_1 \oplus R_2$.

9. Let $A = B = \{1, 2, 3, 4\}$, $R = \{(1, 1), (1, 3), (2, 3), (3, 1), (4, 2), (4, 4)\}$ and

$$S = \{(1, 2), (2, 3), (3, 1), (3, 2), (4, 3)\}$$

Compute

(a) $M_{R \cap S}$ (b) $M_{R \cup S}$ (c) M_{R^c} (d) $M_{\bar{S}}$

10. Let A be set of workers and B be a set of jobs. Let R_1 be a binary relation from A to B such that (a, b) is in R_1 if worker a is assigned to job b . (We assume that a worker might be assigned to more than one job and more than one worker might be assigned to the same job.) Let R_2 be a binary relation on A such that (a_1, a_2) is in R_2 if a_1, a_2 can get along with each other if they are assigned to the same job. State a condition in terms of R_1, R_2 and (possibly) binary relations derived from R_1 and R_2 such that an assignment of the workers to the jobs according to R_1 will not put workers that cannot get along with one another on the same job.

Example 1: Find the union and intersection of each of the following multisets:

- (a) $[a, b]$ and $[a, b, c]$
- (b) $[a, b, b]$ and $[a, b, a, b]$
- (c) $[a, a, a, b]$ and $[a, a, b, b, c]$
- (d) $[1, 1, 3, 3, 3, 4]$ and $[1, 2, 2, 4, 5, 5]$
- (e) $[a, a, (b, b), (b, b)]$ and $[a, a, b, b]$
- (f) $[a, a, (b, b), [a, (b)]]$ and $[a, a, (b), (b)]$

Example 2: Find a multiset that solves the equation

$$A \cup [a, b, b, c] = [a, a, b, b, c, c, d]$$

$$A \cap [a, b, b, c, d] = [a, b, c, d]$$

Ex 6 Let F denote the set of programmers who habitually flow-chart their programs, and let C denote the set of all skilled COBOL programmers

Let M & W denote the set of men and women programmers resply.

$$|M| = 675, \therefore |W| = 1000 - 675 = 325$$

$$|F| = 650, |C| = 788$$

$$|W \cap C| = 278, |W \cap F| = 210,$$

$$|F \cap C| = 440$$

$$|W \cap F \cap C| = 166$$

$$\therefore |M \cap C| = |C| - |W \cap C|$$

$$= 788 - 278$$

$$= 510$$

= set of all men, COBOL programmers.

$$|M \cap F| = |F| - |W \cap F|$$

$$= 650 - 210$$

$$= 440$$

$$|M \cap F \cap C| = |F \cap C| - |W \cap F \cap C|$$

$$= 440 - 166$$

$$= 274$$

Now, $|M \cap (F \cup C)|$ = the number of male programmers who habitually flowchart their programs or are COBOL programmers.

$$\begin{aligned}
 \therefore |M \cap (F \cup C)| &= |M \cap F| + |M \cap C| - |M \cap F \cap C| \\
 &= 510 + 440 - 274 \\
 &= 676
 \end{aligned}$$

Hence, there should be at least 676 men programmers.

But this contradicts the given data that there are in all only 675 men programmers.

Hence data is inaccurately reported.