DMGT - Tutorial - 4

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$A = \{1, 2, 3, 4, 5\}$

$R = \{x, y \mid |x - y| = 1\}$

$R = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4)\}$

$$W_0 = \begin{array}{c|c} & 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \hline \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

1^{st} col C_1 is at R_2

1^{st} row R_1 is at C_2

(R_2, C_2)

$$W_1 = \begin{array}{c|c} & 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \hline \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

2^{nd} col C_2 = 1 is at $R_1 R_2 R_3$

2^{nd} row R_2 = 1 is at (C_1, C_2)

(R_1, C_1) (R_2, C_1) (R_3, C_1), (R_1, C_2), (R_2, C_2), (R_3, C_2)

$$W_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

3^{rd} col C_3 = 1 is at $R_4 R_2 R_4$

3^{rd} row R_3 = 1 is at C_1 C_2 C_4

(R_4, C_1) (R_4, C_2), (R_4, C_4), (R_2, C_1) (R_2, C_2) (R_2, C_4)

$$W_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

4^{th} col $C_4 = 1$ is at $R_3; R_4; R_5$

4^{th} row $R_4 = 1$ is at $(C_1, C_2, C_3, C_4, C_5)$

: $(R_3, C_1)(R_3, C_2)(R_3, C_4)(R_3, C_5)$
$(R_4, C_1)(R_4, C_2)(R_4, C_3)(R_4, C_4)(R_4, C_5)$
$(R_5, C_1)(R_5, C_2)(R_5, C_3)(R_5, C_4)(R_5, C_5)$

$$W_4 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

5^{th} col C_5;

1 is at (R_3, R_4, R_5, R_2)

5^{th} row R_5;

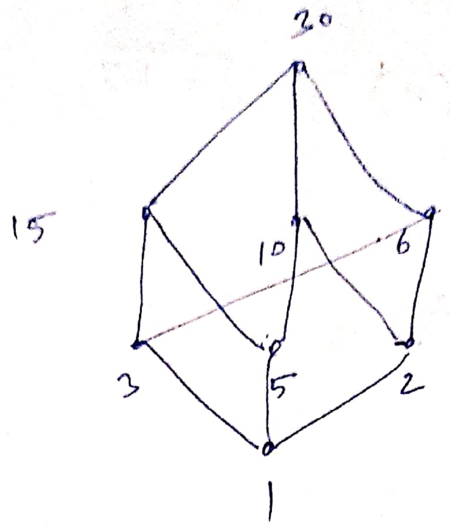
1 is at $(C_1, C_2, C_3, C_4, C_5)$

$$W_5 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = R^*.$$

Q-2.

Divisors of 30 =

$\{1, 2, 3, 5, 6, 10, 15, 30\}$



Maximal = 30

Minimal = 1

lub = 30

glb = 1

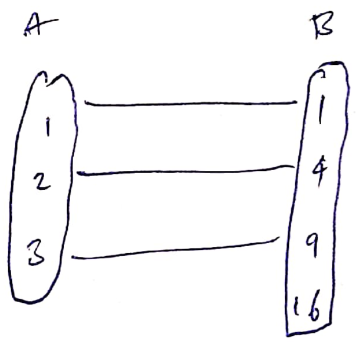
it is a lattice ;

but not a complemented lattice

Q.3. (i) Surjective but not injective function

eg. $f(x) = x^3 + 2x^2 - x + 1$

(ii) Into function



$\boxed{f(x) = x^2}$

(iii) Neither injective ~~not~~ nor surjective : $f(x) = 1$

(iv) R is Symmetric and antisymmetric

$R = \{(1,2),(2,1)(1,1)\}$

(v) R is not ~~a~~ symmetric ; but it's transitive.

$R = \{(1,1),(1,2),(1,3)\}$

Q.4. $f(x) = 2x+5$ $\qquad g(x) = x^2 - 3$

$f^{-1}(x) = \frac{x-5}{2}$; $g^{-1}(x) = \sqrt{x+3}$

$$g \circ f(x) = (2x+5)^2 - 3$$

$g \circ f(4) = (8+5)^2 - 3$

$= 169 - 3 = \underline{\underline{166}}$

$f(g(x)) = 2x^2 - 1$

$f \circ g(4) = 2(4)^2 - 1$

$= 2(16) - 1 = 32 - 1 = \underline{\underline{31}}$

$f^{-1}(f(x)) = \frac{2x+5-5}{2}$

$f^{-1}(f(x)) = x$

$f^{-1} \circ f(2) = 2$

$f^2(x) = 2(2x+5)+5$

$= 4x + 15$

$f^2(a+3) = 4(a+3) + 15$

$= 4a + 12 + 15$

$= 4a + 27.$

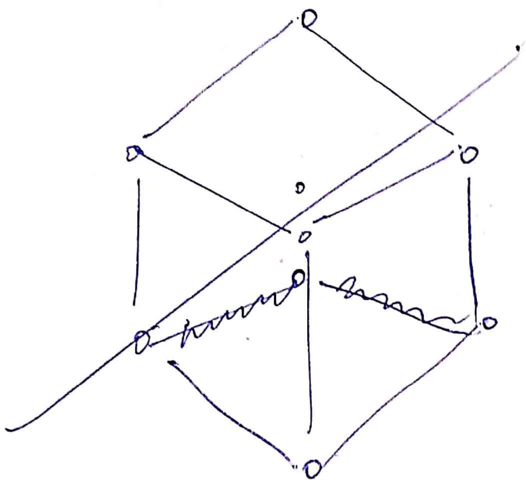
$g(f(a+2)) = (2(a+2)+5)^2 - 3$

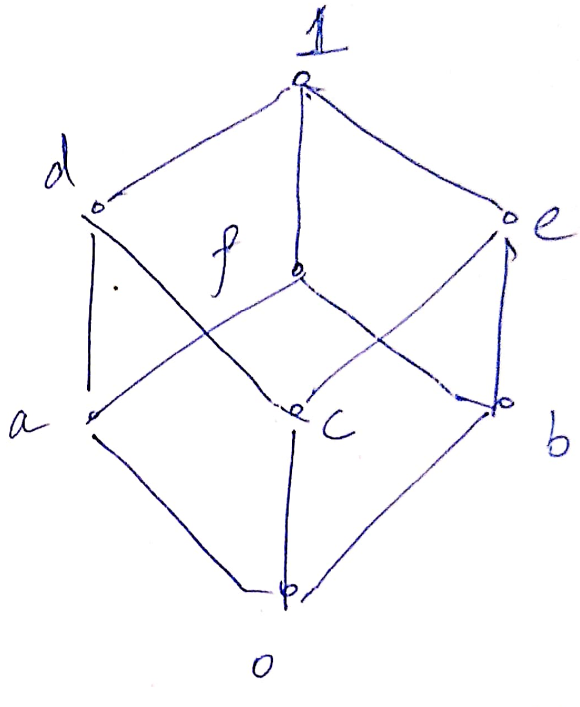
$= (4a + 4 + 5)^2 - 3$

$= (16a^2 + 81 + 72) - 3$

$= 16a^2 + 150$

Q.5· Lattice which is both complemented and distributive





Q.6- any subset of set of R is uncountable they simply cannot be represented as a function that takes natural number as an input & hence making them uncountable

(0, 1) includes (π/4), (e/3), 0.00000111...

that cannot be expressed as natural numbers.