DMGT Tutorial - 3

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PA 34

SY CSF

8/4/22

Q.1. Let $A = \{1, 2, 3, 4\}$

$R_1 = \{(x, y) \mid x + y = 5\}$

$R_2 = \{(x, y) \mid y - x = 1\}$

$R_1 = \{(2,3), (4,1), (1,4), (3,2)\}$

$R_2 = \{(4,3), (3,2), (2,1)\}$

$R_1 \circ R_2 = \{(2,1), (4,2), (3,3)\}$

$(R_1 \cdot R_2)^{-1} = \{(1,2), (2,4), (3,3)\}$ — (1)

$R_1^{-1} = \{(3,2), (1,4), (4,1), (2,3)\}$

$R_2^{-1} = \{(3,4), (2,3), (1,2)\}$

$R_2^{-1} \cdot R_1^{-1} = \{(3,3), (2,4), (1,2)\}$ — (2)

as (1) = (2),

$(R_1 \cdot R_2)^{-1} = R_2^{-1} \cdot R_1^{-1}$

Q. 2. If $A = \{1, 2, 3, 4, 5\}$

$R = \{(x, y) : 2x + 3y > 5\}$

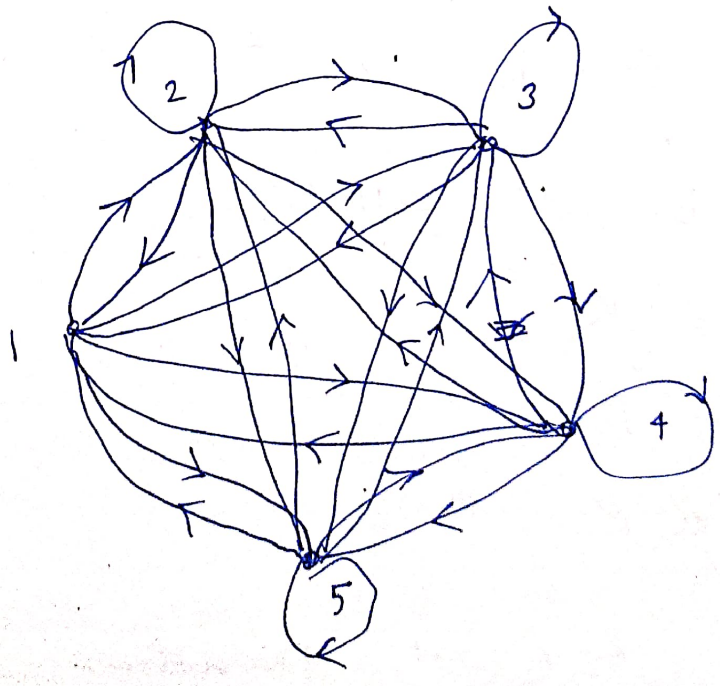
$R = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3)$
$(4,5), (5,4), (1,3), (3,1), (1,4), (4,1)$
$(1,5), (5,1), (2,4), (4,2), (2,5), (5,2)$
$(3,5), (5,3)\}$

is the Range set.

Matrix:

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Diagram:



Vertex 1

In degree = 4

Out degree = 4

V_2

In degree = 5

Out degree = $\neq 5$

V_3

In degree = 5

Out degree = $\neq 5$

V_4

In degree = 5

Out degree = $\neq 5$

V_5

In degree = 5

Out degree = $\neq 5$

$R(3) = \{(3,1), (3,2), (3,3), (3,4), (3,5)\}$

Q.3. $R = \{(1,3), (2,4), (3,5), (4,6), (3,1), (4,2), (5,3), (6,4)\}$

$(1,1) \notin R$

So R is not Reflexive

$(1,3) \in R$

$(3,5) \in R$

But $(1,5) \notin R$

So R is not Transitive.

$\forall (x,y) \in R$,

$\exists (y,x) \in R$

So R is reflexive

as if $(x,y) \in R$,

$$|x-y| = 2$$

$$\Rightarrow |y-x| = 2$$

$$\Rightarrow (y,x) \in R.$$

Q. 4. (i) $R = \{(1,1), (2,2), (3,3)\}$

$R = \{a = b \ \forall \ a,b \in \{1,2,3\}\}$

(ii) $R = \{(1,2), (2,1)\}$

(iii) $R = \{(1,1), (2,2), (3,3)\}$

(iv) $R = \{(1,2), (2,3), (1,3)\}$

Q. 5. $R_1 = \{(1,1), (2,2), (3,3)\}$

$(1,2) \in R$ but $(2,1) \notin R$ so not symmetric

T ✓ as $(a,a) \in R \ \forall \ a \in A$

S ✗ ✓ R is Reflexive

R ✓ ∴ as $(b,a) \in R \ \forall \ (a,b) \in R$,

R is Transitive

as $\forall (a,a) \in R \quad \forall a \in A$,

it is also symmetric.

S ✓ T ✓ R ✓

(ii) $R = \{(1,1),(1,2),(2,3),(3,3),(4,4)\}$

→ as $(1,1) \in R$

But $(2,2) \notin R$

Not Reflexive

→ $(1,2) \in R$

But $(2,1) \notin R$

So not symmetric

→ $(1,2) \in R$

$(2,3) \in R$

$(1,3) \in R$ so it is transitive

T ✗ S ✗ R ✗