

Q1.(a) (10 points) Adjacency matrix

$$M = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

Since $\vec{h} = M\vec{a}$, $\vec{a} = M^T\vec{h}$, we can first compute

$$MM^T = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \quad M^TM = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then we can update the hub weight using $\vec{h} = MM^T\vec{h}$ by iterations:

Iteration No.	1	2	3	4
$\vec{h}(A)$	1	2	2.286	2.4
$\vec{h}(B)$	1	0	0	0
$\vec{h}(C)$	1	0.667	0.286	0.114
$\vec{h}(D)$	1	1.333	1.428	1.486

Similarly, we can compute the authority weight using $\vec{a} = M^TM\vec{a}$ by iterations:

Iteration No.	1	2	3	4
$\vec{a}(A)$	1	0	0	0
$\vec{a}(B)$	1	1.333	1.428	1.486
$\vec{a}(C)$	1	2	2.286	2.4
$\vec{a}(D)$	1	0.667	0.286	0.114

Suggested Marking Scheme: Students are not required to show detailed steps. Full mark should be granted if the final output is correct. If students do show the correct steps, but the final calculation is wrong, deduct 2 points. Otherwise, deduct 5 points.

Q1.(b) (2 points) The ranking scores are

	$\vec{h} + \vec{a}$
A	$2.4+0=2.4$
B	$0+1.486=1.486$
C	$0.114+2.4=2.514$
D	$1.486+0.114=1.6$

So these sites are ranked (in descending order) as C, A, D, B

Q2. (8 *points*)

Node	PageRank Score		
	Initialization	Iteration 1	Iteration 2
A	1	0	0
B	1	$\frac{1}{3}$	0
C	1	$\frac{4}{3}$	$\frac{1}{3}$
D	1	$\frac{4}{3}$	$\frac{4}{3}$

Suggested Marking Scheme: 1 point per cell for Iteration 1 and 2.