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CHAPTER 23

SYMMETRICAL COMPONENTS

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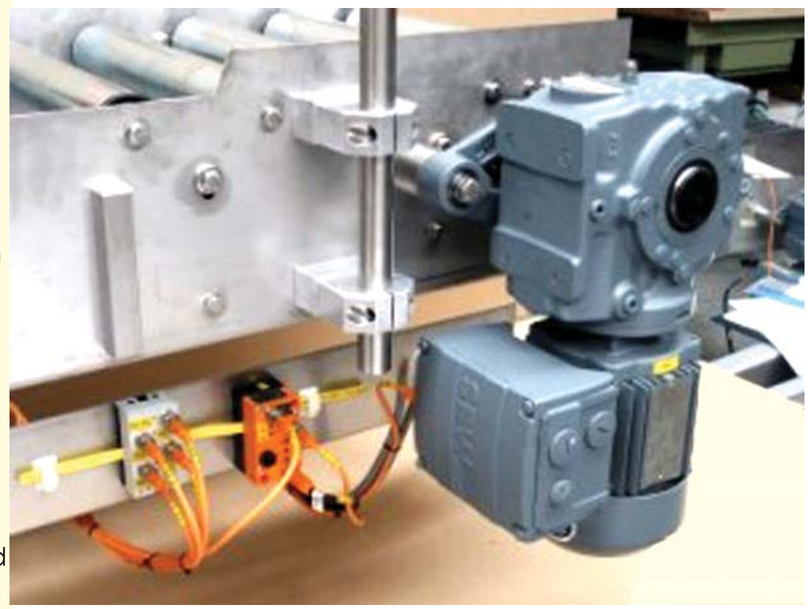

Any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three balanced systems of vectors which are called its 'symmetrical components'

23.1. Introduction

The method of symmetrical components was first proposed by C.L. Fortescue and has been found very useful in solving unbalanced polyphase circuits, for analytical determination of the

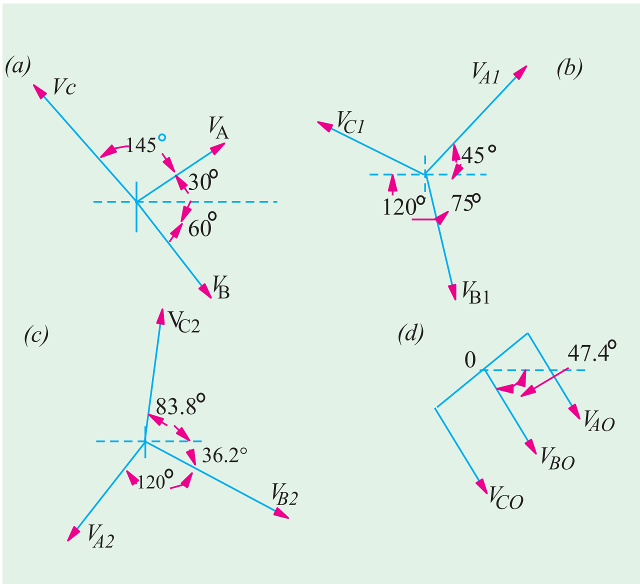

Fig. 23.1

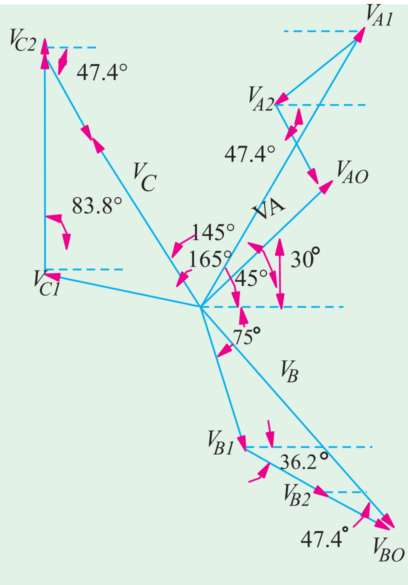

Fig. 23.2

performance of polyphase electrical machinery when operated from a system of unbalanced voltages and for calculation of currents resulting from unbalanced faults. According to Fortescue's theorem, any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three ***balanced*** systems of vectors which are called its ***'symmetrical components'***. In Fig. 23.1 (*a*) is shown a set of three unbalanced voltage vectors V_A, V_B and V_C having phase sequence $A \to B \to C$. These can be regarded as made up of the following symmetrical components :-

(*i*) A balanced system of 3-phase vectors V_{A1}, V_{B1} and V_{C1} having the phase sequence $A \to B \to C$ as the original set of three unbalanced vectors. These vectors constitute the positive-sequence components [Fig. 23.1 (*b*)].

(*ii*) A balanced system of 3-phase vectors V_{A2}, V_{B2} and V_{C2} having phase sequence $A \to C \to B$ which is opposite to that of the original unbalanced vectors. These vectors constitute the negative-sequence components [Fig. 23.1 (*c*)].

(*iii*) A system of three vectors V_{A0}, V_{B0} and V_{C0} which are equal in magnitude and are in phase with each other *i.e.* $V_{A0} = V_{B0} = V_{C0}$. These three co-phasal vectors form a uniphase system and are known as zero-sequence components [Fig. 23.1(*d*).]

3-phase monitor continuously monitors 3-phase wave lines for abnormal conditions

Hence, it means that an unbalanced 3-phase system of voltages or current can be regarded as due to the superposition of two symmetrical 3-phase systems having opposite phase sequences and a system of zero phase sequence *i.e.* ordinary single-phase current or voltage system. In Fig. 23.2, each of the original vectors has been reconstructed by the vector addition of its positive-sequence, negative - sequence and zero-sequence components. It is seen that

$$V_A = \overset{-ve}{V_{A1}} + \overset{-ve}{V_{A2}} + \overset{zero}{V_{A0}} \quad \text{... (i)}$$

$$V_B = V_{B1} + V_{B2} + V_{Bo} \quad \text{... (ii)}$$

$$V_C \quad V_{C1} \quad V_{C2} \quad V_{C0} \quad \text{... (iii)}$$

23.2. The Positive - sequence Components

As seen from above, the positive-sequence components have been designated as V_{A1}, V_{B1} and V_{C1}. The subscript 1 is meant to indicate that the vector belongs to the positive-sequence system. The letter refers to the original vector of which the positive-sequence vector is a component part.

These positive-sequence vectors are completely determined when the magnitude and phase of any one of these is known. Usually, these vectors are related to each other with the help of the operator a (for details, please refer to Art. 12.11). As seen from Fig. 23. 1 (*b*).

$$\mathbf{V}_{A1} = V_{A1};\ V_{B1} = a^2\mathrm{V}_{A1} = V_{A1}\angle -120°;\ \mathbf{V}_{C1} = a\mathbf{V}_{A1}{=}{=}V_{A1}\angle 120°$$

23.3. The Negative - sequence Components

This system has a phase sequence of $A \to C \to B$. Since this system is also balanced, it is completely determined when the magnitude and phase of one of the vectors becomes known. The suffix 2 indicates that the vector belongs to the negative-sequence system. Obviously, as seen from Fig. From 23.1 (*c*).

$$\mathbf{V}_{A2} = V_{A2};\ \mathbf{V}_{B2} = a\ V_{A2} = \mathbf{V}_{A2} = \angle 120°;\ \mathbf{V}_{C2} = a^2\mathbf{V}_{A2} = V_{A2}\angle -120°$$

23.4. The Zero - sequence Components

These three vectors are equal in magnitude and phase and hence form what is known as uniphase system. They are designated as V_{A0}, V_{B0} and V_{C0}. Since these are indentical in magnitude

$$\therefore \quad V_{A0} = V_{B0} = V_{C0}$$

23.5. Graphical Composition of Sequence Vectors

Fig. 23.2 Shows how the original vector V_A has been obtained by the addition of V_{A1}, V_{A2} and V_{A0}. The same applies to other vectors V_B and V_C.

For simplicity, let us write V_{Al} as V_1, V_{A2} as V_2 and V_{A0} as V_0. Then

$$\mathbf{V}_A = V_1 + \mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (iv)}$$

$$\mathbf{V}_B = a^2\mathbf{V}_1 + a\mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (v)}$$

$$\mathbf{V}_C = a\mathbf{V}_1 + a^2\mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (vi)}$$

23.6. Evaluation of V_{A1} or V_1

The procedure for evaluating $\mathbf{V}_1$ is as follows :

Multiplying (*v*) by *a* and (*vi*) by a^2, we get

$$a\mathbf{V}_B = a^3\mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_0; \qquad a^2\mathbf{V}_C = a^3\mathbf{V}_1 + a^4\mathbf{V}_2 + a^2\mathbf{V}_0$$

Now $\quad a^3 = 1$and $a^4 = a$, hence

$$a\mathbf{V}_B = \mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_0 \qquad \text{... (vii)}$$

$$a^2\mathbf{V}_C = \mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_0 \qquad \text{.... (viii)}$$

Adding ***(iv)***, ***(vii)*** and ***(viii)***, we get

$$\mathbf{V}_A + \mathrm{a}\mathbf{V}_B + a^2\mathbf{V}_C = 3\mathbf{V}_1 + \mathbf{V}_2\,(1 + a + a^2) + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_1$$

$$\therefore \quad \mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B\angle 120° + \mathbf{V}_C\angle -210°)$$

$$\frac{1}{3} \; V_A \quad V_B \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad V_C \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}$$

This shows that, geometrically speaking, V_1 is a vector one-third as large as the vector obtained by the vector addition of the three original vectors V_A, $V_B\angle 120°$ and $V_C\angle -120°$.

23.7. Evaluation of V_{A2} or V_2

Multiplying ***(vi)*** by a and ***(v)*** by a^2 and adding them to ***(iv)*** we get

$$a\mathbf{V}_C = a^2\mathbf{V}_1 + a^3\mathbf{V}_2 + a\mathbf{V}_0; a^2\mathbf{V}_B = a^4\mathbf{V}_1 + a^3\mathbf{V}_2 + a^2\mathbf{V}_0$$

$$\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + 3\mathbf{V}_2 + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_2 \text{ Now , } 1 + a + a^2 = 0$$

$$\therefore \quad \mathbf{V}_2 = \frac{1}{3}(\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B\angle -120° + \mathbf{V}_C\angle 120°)$$

$$= \frac{1}{3}\left[\mathbf{V}_A + \mathbf{V}_B\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \mathbf{V}_C\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

23.8. Evaluation of V_{A0} or V_0

Adding ***(iv)***, ***(v)*** and ***(vi)***, we get $\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + \mathbf{V}_2(1 + a + a^2) + 3\mathbf{V}_0 = 3\mathbf{V}_0$

$$\therefore \qquad \mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

It shows that V_0 is simply a vector one third as large as the vector obtained by adding the original vectors V_A, V_B and V_C.

To summarize the above results, we have

(i) $\mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C)$ ***(ii)*** $\mathbf{V}_2 \quad \frac{1}{3}(\mathbf{V}_A \quad a^2\mathbf{V}_B \quad a\mathbf{V}_C)$

(iii) $\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$

Note. An unbalanced system of 3-phase currents can also be likewise resolved into its symmetrical components. Hence

$$\mathbf{I}_A = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_B = a^2\mathbf{I}_1 + a\mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_C = a\mathbf{I}_1 + a^2\mathbf{I}_2 + \mathbf{I}_0$$

Also, as before $\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_A + a\mathbf{I}_B + a^2\mathbf{I}_C); \mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_A + a^2\mathbf{I}_B + a\mathbf{I}_C); \mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C)$ ... ***(ix)***

It shows that I_0 is one-third of the neutral or earth-return current and is zero for an unearthed 3-wire system. It is seen from ***(ix)*** above that I_0 is zero if the vector sum of the original current vectors is zero. This fact can be used with advantage in making numerical calculations because the original system of vectors can then be reduced to two balanced 3-phase systems having opposite phase sequences.

Example 23.1. *Find out the positive, negative and zero-phase sequence components of the following set of three unbalanced voltage vectors:*

$$V_A = 10\angle 30°; V_B = 30\angle -60°; V_C = 15\angle 145°$$

Indicate on an approximate diagram how the original vectors and their different sequence components are located. **(Principles of Elect. Engg. – I, Jadavpur Univ.)**

Solution. (*i*) Positive-sequence vectors

As seen from Art. 23.6

$$\mathbf{V}_1 \quad \frac{1}{3}(\mathbf{V}_A \quad a\mathbf{V}_B \quad a^2\mathbf{V}_C) \quad \frac{1}{3}(10 \quad 30 \quad a.30 \quad 60 \quad a^2.15 \quad 145\ )$$

$$= \frac{1}{3}(10\angle 30° + 30\angle 60° + 15\angle 25°) = 12.42 + j12.43 = 17.6\angle 45°$$

$$\therefore\ \mathbf{V}_{A|} \quad 17.6 \quad 45\ ;\ \mathbf{V}_{B|} \quad 17.6 \quad 45 \qquad 120 \quad 17.6 \qquad 75$$

$$\mathbf{V}_{C1} \quad 17.6 \quad 45 \qquad 120 \quad 17.6 \quad 165$$

These are shown in Fig. 23.1 (*b*)

(*ii*) Negative–sequence vectors

As seen from Art. 23.7,

$$\mathbf{V}_2 \quad \frac{1}{3}(\mathbf{V}_A \quad a^2\mathbf{V}_B \quad a\mathbf{V}_C) \quad \frac{1}{3}(10 \quad 30 \quad a^2.30 \quad 60 \quad a.15 \quad 145\ )$$

$$= \frac{1}{3}(10\angle 30° + 30\angle -180° + 15\angle 265°) = -7.55 - j3.32 = 8.24\angle -156.2°$$

$$\mathbf{V}_{A2} \quad 8.24 \qquad 156.2\ ; V_{B2} \quad 8.24 \qquad 156.2 \qquad 120 \quad 8.24 \qquad 36.2$$

$$\mathbf{V}_{C2} \quad 8.24 \qquad 156.2 \qquad 120 \quad 8.24 \qquad 276.2$$

These vectors are shown inFig. 23.1 (*c*)

(*iii*) Zero sequence vectors

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

$$= \frac{1}{3}(10\angle 30° + 30\angle -60° + 15\angle 145°) = 3.8 - j4.12 = 5.6\angle -47.4°$$

These vectors are shown in Fig. 23.1 (*d*).

Example 23.2. *Explain how an unsymmetrical system of 3-phase currents can be resolved into 3 symmetrical component systems.*

Determine the values of the symmetrical components of a system of currents

$$I_R = 0 + j120A; I_Y = 50 - j\,100A; I_B = -100 - j50A$$

Phase sequence is RYB. **(Elect. Engg.-I Bombay, Univ.)**

Solution. $\mathbf{I}_R \quad 0 \quad j120 \quad 120 \quad 90$

$$\mathbf{I}_Y \quad 50 \quad j100 \quad 111.8 \qquad 63.5\ ;\ \mathbf{I}_B \qquad 100 \quad j50 \quad 111.8 \qquad 153.5$$

(*i*) Positive–sequence Components

$$\mathbf{I}_1 \quad \frac{1}{3}(\mathbf{I}_R \quad a\mathbf{I}_Y \quad a^2\mathbf{I}_B) \quad \frac{1}{3}\ (0 \quad j120) \qquad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}\ (50 \quad j100) \qquad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}\ (\ \ 100 \quad j5\ 0)$$

$= 22.8 + j108.3 = 110.7\angle 78.1° \; \therefore \mathbf{I}_{R1} \quad 110.7 \quad 78.1 \; ; \mathbf{I}_{Y1} \quad 110.7 \quad 41.9 \; ; \mathbf{I}_{B1} \quad 110.7 \quad 198.1$

(*ii*) Negative–sequence components

$$\mathbf{I}_2 \quad \frac{1}{3}(\mathbf{I}_R \quad a^2\mathbf{I}_Y \quad a\mathbf{I}_B) \quad \frac{1}{3}(\; 18.3 \quad j65.1) \quad 6.1 \quad j21.7 \quad 22.5 \;\; 105.7$$

$\therefore \; \mathbf{I}_{R2} \quad 22.5 \;\; 105.7 \; ; \; \mathbf{I}_{Y2} \quad 22.5 \;\; 225 \; ; \; \mathbf{I}_{B2} \quad 22.5 \quad 14.3$

(*iii*) Zero-sequence component

$$\mathbf{I}_0 \quad \frac{1}{3}(\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad \frac{1}{3}[(0 \quad j120) \quad (50 \quad j100) \quad (\; 100 \quad j50)] \quad 16.7 \quad j10$$

As a check, it may be found that

$$\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2} + \mathbf{I}_0 ; \mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_0 ; \mathbf{I}_{B1} + \mathbf{I}_{B2} + \mathbf{I}_0$$

Example 23.3. *In a 3-phase, 4-wire system, the currents in the R, Y and B lines under abnormal conditions of loading were as follows:*

$$I_R = 100\angle 30° ; I_Y = 50\angle 300° ; I_B = 30\angle 180°$$

Calculate the positive, negative and zero-phase sequence currents in the R-line and the return current in the neutral conductor.

Solution. (*i*) The positive-sequence components of current in the *R*-line is

$$\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_R + a\mathbf{I}_Y + a^2\mathbf{I}_B)$$

Now $\mathbf{I}_R \quad 100 \quad 30 \quad 50(\sqrt{3} \quad j)$

$$\mathbf{I}_Y \quad 50 \quad 300 \quad 50 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 25(1 \quad j\sqrt{3})$$

$$\mathbf{I}_B = 30\angle 180° = (-30 + j0)$$

$$\mathbf{I}_1 \quad \frac{1}{3} \; 50(\sqrt{3} \quad j) \quad 25(1 \quad j\sqrt{3} \;) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad (\; 30) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 58 \quad 48.4$$

(*ii*) The negative–sequence components of the current in the *R*-line is

$$\mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_R + a^2\mathbf{I}_Y + a\mathbf{I}_B)$$

$$= \frac{1}{3} \; 50(\sqrt{3} \quad j) \quad 25(1 \quad j\sqrt{3}) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad (\; 30) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 18.9 \quad 24.9$$

(*iii*) The zero–sequence component of current in the *R*-line is

$$\mathbf{I}_0 \quad \frac{1}{3}(\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad \frac{1}{3}[50(\sqrt{3} \quad j) \quad 25(1 \quad j\sqrt{3}) \quad 30] \quad 27.2 \quad 4.7$$

The neutral current is

$$\mathbf{I}_N \quad \mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B \quad 3 \quad \mathbf{I}_0 \quad 3 \quad 27.2 \quad 4.7 \quad 81.6 \quad 4.7$$

Example 23.4. *A 3-phase, 4-wire system supplies loads which are unequally distributed on the three phases. An analysis of the currents flowing in the direction of the loads in the R, Y and B lines shows that in the R-line, the positive phase sequence current is 200 $\angle$ 0° A and the* *negative phase sequence current is 100 $\angle$ 60°. The total observed current flowing back to the supply in the neutral conductor is 300 $\angle$ 300° A. Calculate the currents in phase and magnitude in the three lines.*

Assuming that the 3-phase supply voltages are symmetrical and that the power factor of the load on the R-phase is $\sqrt{3/2}$ leading, determine the power factor of the loads on the two other phases.

Solution. It is given that in *R*-phase [Fig. 23.3 (*a*)]

$$\mathbf{I}_{R1} = 200\angle 0° = (200 + j0)\,\text{A}; \mathbf{I}_{R2} = 100\angle 60° = (50 + j86.6)\,A$$

$$\mathbf{I}_{R0} = \frac{1}{3}\mathbf{I}_N = (300/3)\angle 300° = (50 - j86.6)\,\text{A}$$

$$\mathbf{I}_R \quad \mathbf{I}_{R1} \quad \mathbf{I}_{R2} \quad \mathbf{I}_{R0} \quad (200 \quad j0) \quad (50 \quad j86.6) \quad (50 \quad j86.6) \quad (300 \quad j0) \quad 300 \quad 0$$

Similarly, as seen from Fig. 23.3 (*b*) for the *Y*-phase

$$\mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_{Y0} = a^2\mathbf{I}_{R1} + a\mathbf{I}_{R2} + \mathbf{I}_{R0}$$

$$= 200\angle 0° - 120° + 100\angle 60° + 120°$$

$$+\ 100\angle 300° = -100 - j173.2 - 100 + 50 - j86.6 = -150 - j259.8 = 300\angle 240°\,\text{A}$$

Similarly, as seen from Fig. 23-3 (*c*) for the *B*-phase

$$\mathbf{I}_B \quad \mathbf{I}_{B1} \quad \mathbf{I}_{B2} \quad \mathbf{I}_{B0} \quad a\mathbf{I}_{R1} \quad a^2\mathbf{I}_{R2} \quad \mathbf{I}_{R0} \quad 200 \quad 0 \quad 120 \quad 100 \quad 60 \quad 120 \quad 100 \quad 300 \quad 0$$

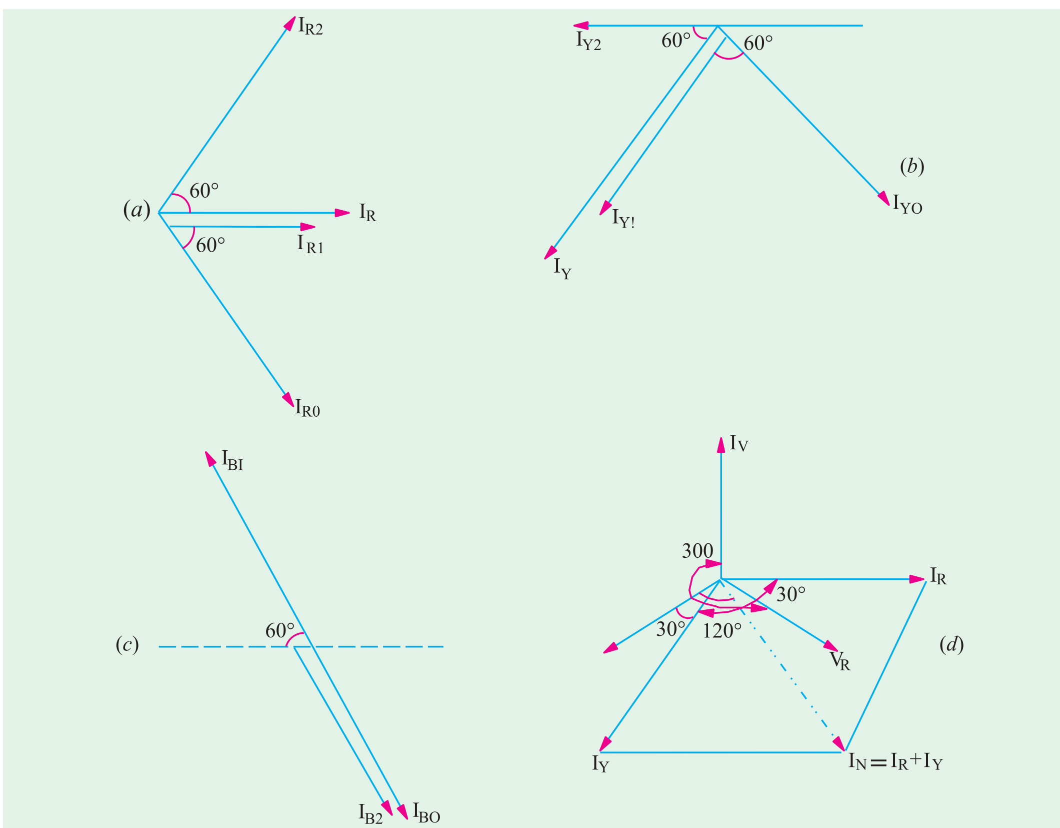

Fig. 23.3

Since the power factor of the R-phase is $\sqrt{3}/2$ leading, the current I_R leads the voltage V_R by 30° [Fig. 23.3(d)]

(d) Now, phase angle of I_Y is 240° relative to I_R so that I_Y leads its voltage V_Y by 30°. Hence, power factor of Y phase is also $\sqrt{3}/2$ leading. The power factor of B line is indeterminate because the current in this line is zero.

Example 23.5. *Prove that in a 3-phase system if V_1, V_2 and V_3 are the three balanced voltages whose phasor sum is zero, the positive and negative sequence components can be expressed as*

$$V_{1p} = \left\{\frac{1}{\sqrt{3}}(V_1 + V_2\angle 60°)\right\}\angle 30°; V_{1N} = \left\{\frac{1}{\sqrt{3}}(V_1 + V_2\angle -60°)\right\}\angle -30°$$

Phase sequence is 1-2-3.

A system of 3-phase currents is given as $I_1 = 10\ \angle 180°$, $I_2 = 14.14\ \angle -45°$ and $I_3 = 10\ \angle 90°$. Determine phasor expression for the sequence components of these currents. Phase sequence is 1-2-3. **(Elect. Engg-I, Bombay Univ.)**

Solution. As seen from Art. 23.6

$$\mathbf{V}_{1P} = \frac{1}{3}(\mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_3); \text{Now, } \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = 0 \quad \therefore \quad \mathbf{V}_3 = -(\mathbf{V}_1 + \mathbf{V}_2)$$

$$\mathbf{V}_{1P} = \frac{1}{3}[\mathbf{V}_1 + a\mathbf{V}_2 - a^2(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1-a^2) + \mathbf{V}_2(a-a^2)]$$

Now, $1 - a^2 = \frac{2}{3} + j\frac{\sqrt{3}}{2}$ and $a - a^2 = j\sqrt{3}$

$$\therefore \quad \mathbf{V}_{1P} \quad \frac{1}{3} \quad \mathbf{V}_1 \quad \frac{3}{2} \quad j\frac{\sqrt{3}}{2} \qquad j\sqrt{3}\mathbf{V}^2 \qquad \frac{1}{\sqrt{3}} \quad \mathbf{V}_1 \quad \frac{\sqrt{3}}{2} \quad j\frac{1}{2} \qquad j\mathbf{V}_2$$

$$= \frac{1}{\sqrt{3}}[\mathbf{V}_1\angle 30° + \mathbf{V}_2\angle 90°] = \left[\frac{1}{\sqrt{3}}(\mathbf{V}_1 + \mathbf{V}_2\angle 60°)\right]\angle 30°$$

Similarly, the negative-sequence component is given by

$$\mathbf{V}_{1N} = \frac{1}{3}(\mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_3) = \frac{1}{3}[\mathbf{V}_1 + a^2\mathbf{V}_2 - a(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1-a) + \mathbf{V}_2(a^2 - a)]$$

Now, $a^2 - a = -j\sqrt{3}$

$$\therefore \quad \mathbf{V}_{1N} = \frac{1}{3}\left[\mathbf{V}_1\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) - j\sqrt{3}\mathbf{V}_2\right] = \frac{1}{\sqrt{3}}\left[\mathbf{V}_1\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) - j\mathbf{V}_2\right]$$

$$= \frac{1}{\sqrt{3}}[\mathbf{V}_1\angle -30° + \mathbf{V}_2\angle -90°] = \left\{\frac{1}{\sqrt{3}}(\mathbf{V}_1 + \mathbf{V}_2\angle -60°)\right\}\angle -30°$$

Now $\mathbf{I}_1 = 10\angle 180° = -10 + j0; \mathbf{I}_2 = 14.14\angle -45° = 10 - j10$

$\mathbf{I}_3 = 10\angle 90° = j10$

$a\mathbf{I}_2 \quad 14.14 \quad 75 \qquad 3.66 \quad j13.66;\ a^2\mathbf{I}_2 \quad 14.14 \qquad 165 \qquad 13.66 \quad j3.66$

$a\mathbf{I}_3 \quad 14.14 \quad 21 \qquad 12.25 \quad j707;\ a^2\mathbf{I}_3 \quad 14.14 \qquad 30 \qquad 12.25 \quad j7.07$

$$\mathbf{I}_{1P} \quad \frac{1}{3}(\mathbf{I}_1 \quad a\mathbf{I}_2 \quad a^2\mathbf{I}_3) \quad \frac{1}{3}(5.91 \quad j6.59) \quad 1.97 \quad j2.2$$

$$\mathbf{I}_{1N} \quad \frac{1}{3}(\mathbf{I}_1 \quad a^2\mathbf{I}_2 \quad a\mathbf{I}_3) \quad \frac{1}{3}(\ \ 35.91 \quad j10.73) \quad 11.97 \quad j3.58$$

$$\mathbf{I}_{10} = \frac{1}{3}(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = 0$$

Tutorial Problems No. 23.1

1. The following currents were recorded in the *R*, *Y*, and *B* lines of a 3-phase system under abnormal conditions: $[\mathbf{I_R} = 300\angle 300°\ \text{A}; \mathbf{I_Y} = 500\angle 240°\ \text{A}; \mathbf{I_B} = 1{,}000\angle 60°\ \text{A}]$

Calculate the values of the positive, negative and zero phase-sequence components.

$[\mathbf{I_1} = 536\angle -44°20'\ \text{A}; \mathbf{I_2} = 372\angle 171°\ \text{A}; \mathbf{I_0} = 145\angle 23°20']$

2. Determine the symmetrical components of the three currents $I_0 = 10\angle 0°$; $\mathbf{I}_b = 100\angle 250°$ and $\mathbf{I}_C = 10\angle 110°\ \text{A}$ **[$\mathbf{I}_1$ = (39.45 + *j*522) A; $\mathbf{I}_2$ = (– 20.24 + *j*22.98) A; $\mathbf{I}_0$ = (– 9.21 – *j*28.19) A]**

(Elect. Meas & Measuring Instru., Madras Univ.)

3. The three current vectors of a 3-phase, four-wire system have the following values; $\mathbf{I}_A$ = 7 + j_0, $\mathbf{I}_B$ = – 12 – *j*13 and $\mathbf{I}_C$ = – 2 + *j*3. Find the symmetrical components. The phase sequence is *A*, *B*, *C*

[$\mathbf{I}_0$ = – 2.33 – *j*3.33; $\mathbf{I}_{A1}$ = (27.75 – *j*3.67); $\mathbf{I}_{B1}$ = (– 17.05 – *j*22.22); $\mathbf{I}_{C1}$ = (– 10.7 + *j*25.9); $\mathbf{I}_{A2}$ = (0.25 + *j*13.67) A; $\mathbf{I}_{B2}$ = (– 11.97 – *j*6.62) A; $\mathbf{I}_{C2}$ = (11.73 – *j*7.05) A]

23.9. Zero-Sequence Components of Current and Voltage

Any circuit which allows the flow of positive-sequence currents will also allow the flow of negative-sequence currents because the two are similar. However, a fourth wire is necessary if zero-sequence components are to flow in the lines of the 3-phase system. It follows that the line currents of 3-phase 3-wire system can contain no zero sequence components whether it is delta - or star-connected. The zero sequence components of line-to-line voltages are non-existent regardless of the degree of imbalance in these voltages. It means that a set of unbalanced 3-phase, line-to-line voltages may be represented by a positive system and a negative system of balanced voltages. This fact is of considerable importance in the analysis of 3-phase rotating machinery. For example, the operation of an induction motor when supplied from an unbalanced system of 3-phase voltages, may be analysed on the basis of two balanced systems of voltages of opposite phase sequence.

Let us consider some typical 3-phase connections with reference to zero-sequence components of current and voltage.

(*a*) Four-wire Star Connection. Due to the presence of the fourth wire, the zero sequence currents may flow. The neutral wire carries only the zero-sequence current which is the sum of the zero-sequence currents in the three lines. Since the sum of line voltages is zero, there can be no zero sequence component of line voltages.

(*b*) Three-wire Star Connection. Since there is no fourth or return wire, zero-sequence components of current cannot flow. The absence of zero-sequence currents may be explained by considering that the impedance offered to these currents is infinite and that this impedance is situated between the star points of the generator and the load. If the two star points were joined by a neutral, only zero-sequence currents will flow through it so that only zero-sequence voltage can exist between the load and generator star points. Obviously, no zero-sequence component of voltage appears across the phase load.

(*c*) Three-wire Delta Connection. Due to the absence of fourth wire, zero-sequence components of currents cannot be fed into the delta-connected load. However, though line currents have to sum up to zero (whereas phase currents need not do so) it is possible to have a zero-sequence current circulating in the delta-connected load.

Similarly, individual phase voltages will generally possess zero-sequence components though components are absent in the line-to line voltage.*

23.10. Unbalanced Star Load Supplied from Unbalanced Three-phase Three-wire System

In this case, line voltages and load currents will consist of only positive and negative-sequence components (but no zero-sequence component). But load voltages will consist of positive, negative and zero-sequence components.

Let the line voltages be denoted by $\mathbf{V}_{RY}$, $\mathbf{V}_{YB}$ and $\mathbf{V}_{BR}$, line (and load) currents by $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$, the load voltages by $\mathbf{V}_{RN}$, $\mathbf{V}_{YN}$ and $\mathbf{V}_{BN}$ and load impedances by $\mathbf{Z}_R$, $\mathbf{Z}_Y$, and $\mathbf{Z}_B$ (their values being the same for currents of any sequence).

Obviously, $\mathbf{V}_{RN} = \mathbf{I}_R\mathbf{Z}_R$; $\mathbf{V}_{YN} = \mathbf{I}_Y\mathbf{Z}_Y$ and $\mathbf{V}_{BN} = \mathbf{I}_B\mathbf{Z}_B$

If $\mathbf{V}_{RN1}$, $\mathbf{V}_{RN2}$ and $\mathbf{V}_0$ are the symmetrical components of $\mathbf{V}_{RN}$, then we have

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN}) = \frac{1}{3}(\mathbf{I}_R\mathbf{Z}_R + \mathbf{I}_Y\mathbf{Z}_Y + \mathbf{I}_B\mathbf{Z}_B)$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2}) + \mathbf{Z}_Y(\mathbf{I}_{Y1} + \mathbf{I}_{Y2}) + \mathbf{Z}_B(\mathbf{I}_{B1} + \mathbf{I}_{B2})]$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2}) + \mathbf{Z}_Y(a^2\mathbf{I}_{R1} + a\mathbf{I}_{P2}) + \mathbf{Z}_B(a\mathbf{I}_{R1} + a^2\mathbf{I}_{R2})]$$

$$= \mathbf{I}_{R1}\cdot\frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + a\mathbf{Z}_B) + \mathbf{I}_{R2.}\frac{1}{3}(\mathbf{Z}_R + a\mathbf{Z}_Y + a^2\mathbf{Z}_B) = \mathbf{I}_{R1}\mathbf{Z}_{R2} + \mathbf{I}_{R2}\mathbf{Z}_{R1} \quad \text{... (i)}$$

$$\mathbf{V}_{RN1} = \frac{1}{3}(\mathbf{V}_{RN} + a\mathbf{V}_{YN} + a^2\mathbf{V}_{BN}) = \frac{1}{3}(\mathbf{I}_R\mathbf{Z}_R + a\mathbf{I}_Y\mathbf{Z}_Y + a^2\mathbf{I}_B\mathbf{Z}_B)$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2})] + a\mathbf{Z}_Y(\mathbf{I}_{Y1} + \mathbf{I}_{Y2}) + a^2\mathbf{Z}_B(\mathbf{I}_{B1} + \mathbf{I}_{B2})$$

$$= \mathbf{I}_{R1.}\frac{1}{3}(\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B) + \mathbf{I}_{R2.}\frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + \text{a}\mathbf{Z}_B) = \mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2} \quad \text{... (ii)}$$

$$\mathbf{V}_{RN2} = \mathbf{I}_{R2}\mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}$$

Similarly, $\mathbf{V}_{YN1} = \mathbf{I}_{Y1}\mathbf{Z}_0 + \mathbf{I}_{Y2}\mathbf{Z}_{Y2} = a^2(\mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2}) = a^2\mathbf{V}_{RN1}$

$$\mathbf{V}_{BN1} = \mathbf{I}_{B1}Z_0 + \mathbf{I}_{B2}\mathbf{Z}_{B2} = a(\mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2}) = a\mathbf{V}_{RN1}$$

$$\mathbf{V}_{YN2} = \mathbf{I}_{Y2}\mathbf{Z}_0 + \mathbf{I}_{Y1}\mathbf{Z}_{Y1} = a(\mathbf{I}_{R2}\mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}) = a\mathbf{V}_{RN2}$$

$$\mathbf{V}_{BN2} = \mathbf{I}_{B2}\mathbf{Z}_0 + \mathbf{I}_{B1}\mathbf{Z}_{B1} = a^2(\mathbf{I}_{R2} + \mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}) = a^2\mathbf{V}_{RN2}$$

Now, $\mathbf{V}_{RN1}$ and $\mathbf{V}_{RN2}$ may be determined from the relation between the line and phase voltages as given below:

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = \mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{RN2} - (\mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{YN2}]$$

* However, under balanced conditions, the phase voltages will possess no zero-sequence components.

$$= \mathbf{V}_{RN1}(1-a^2)+\mathbf{V}_{RN2}(1-a) = \frac{1}{2}\sqrt{3}[\mathbf{V}_{RN1}(\sqrt{3}+j1)+\mathbf{V}_{RN2}(\sqrt{3}-j1)] \quad \dots (iii)$$

$$\mathbf{V}_{YB} = \mathbf{V}_{YN}+\mathbf{V}_{NB} = \mathbf{V}_{YN}-\mathbf{V}_{BN} = \mathbf{V}_0+\mathbf{V}_{YN1}+\mathbf{V}_{YN2}-(\mathbf{V}_0+\mathbf{V}_{BN1}+\mathbf{V}_{BN2})$$

$$= \mathbf{V}_{RN1}(a^2-a)-\mathbf{V}_{RN2}(a-a^2) = -j\sqrt{3}.\mathbf{V}_{RN1}+j\sqrt{3}\mathbf{V}_{RN2}$$

$$\therefore\ \mathbf{V}_{RN2} = \mathbf{V}_{RN1} - j\mathbf{V}_{YB}/\sqrt{3}$$

Substituting this value of $\mathbf{V}_{RN2}$ in Eq. ***(iii)*** above and simplifying, we have

$$\mathbf{V}_{RN1} = \frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1+j\sqrt{3})] \quad \dots (iv)$$

$$\mathbf{V}_{RN2} = \frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1-j\sqrt{3})] \quad \dots (v)$$

Having known $\mathbf{V}_{RN1}$ and $\mathbf{V}_{RN2}$, the currents $\mathbf{I}_{R1}$ and $\mathbf{I}_{R2}$ can be determined from the following equations:

$$\mathbf{I}_{R1} = \frac{(\mathbf{V}_{RN1}\mathbf{Z}_0 - \mathbf{V}_{RN2}\mathbf{Z}_{R2})}{\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}} \quad \dots (vi)$$

$$\mathbf{I}_{R2} = \frac{(\mathbf{Z}_{RN2}\mathbf{Z}_0 - \mathbf{V}_{RN1}\mathbf{Z}_{R1})}{\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}} \quad \dots (vii)$$

Alternatively, when $\mathbf{I}_{R1}$ becomes known, $\mathbf{I}_{R2}$ may be found from the relation

$$\mathbf{I}_{R2} = \frac{\mathbf{V}_{RN2} - \mathbf{I}_{R1}\mathbf{Z}_{R1}}{\mathbf{Z}_0} \quad \dots (viii)$$

The symmetrical components of the currents in other phases can be calculated from $\mathbf{I}_{R1}$ and $\mathbf{I}_{R2}$ by using the relations given in Art. 23.8.

The phase voltages may be calculated by any one of the two methods given below:

(i) directly by calculating the products $\mathbf{I}_R\mathbf{Z}_R$, $\mathbf{I}_Y\mathbf{Z}_Y$ and $\mathbf{I}_B\mathbf{Z}_B$.

(ii) by first calculating the zero-sequence component $\mathbf{V}_0$ of the phase or load voltages and then adding this to the appropriate positive and negative sequence components.

For example, $\mathbf{V}_{RN} = \mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{RN2}$

23.11. Unbalanced Star Load Supplied from Balanced Three-phase, Three-wire System

It is a special case of the general case considered in Art. 23.10 above. In this case, the symmetrical components of the load voltages consist only of positive and zero-sequence components. This fact may be verified by substituting the value of **VYB** in Eq. (*v*) of Art. 23.10. Now, in a balanced or symmetrical system of positive-phase sequence

$$\mathbf{V}_{YB} \quad \mathbf{V}_{RY} \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}$$

Substituting this value in Eq. ***(v)*** above, we have

$$\mathbf{V}_{RN2} = \frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1-j\sqrt{3})]$$

$$= \frac{1}{3}\left[\mathbf{V}_{RY}+\frac{1}{2}.\mathbf{V}_{RY}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)(1-j\sqrt{3})\right] = \frac{1}{3}-(\mathbf{V}_{RY}-\mathbf{V}_{RY}) = 0$$

Hence, substituting this value of $\mathbf{V}_{RN2}$ in Eq. (*iv*) and (*vi*) of Art. 23.10, we get

$$\mathbf{V}_{RN1} = j\mathbf{V}_{YB}\sqrt{3} = \frac{1}{2}\mathbf{V}_{RY} - j\frac{1}{2}\mathbf{V}_{RY}/\sqrt{3} \quad \ldots (i)$$

$$\mathbf{I}_{R1} = \mathbf{V}_{RN1}\mathbf{Z}_0/(\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}) = \mathbf{V}_{RN1}\frac{\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B}{\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R} \quad \ldots (ii)$$

$$\mathbf{I}_{R2} = -\mathbf{I}_{R1}.\mathbf{Z}_{R1}/\mathbf{Z}_0 = -\mathbf{R}1_0 = -\mathbf{I}_{R1}\frac{\mathbf{Z}_R + a\mathbf{Z}_Y + a^2\mathbf{Z}_B}{\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B} \quad \ldots (iii)$$

Example 23.6 illustrates the procedure for calculating the current in an unbalanced star-connected load when supplied from a symmetrical three-wire system.

Example 23.6. *A symmetrical 3-phase, 3-wire, 440-V system supplies an unbalanced Y-connected load of impedances $Z_R = 5\angle 30°\ \Omega$; $Z_Y = 10\angle 45°\ \Omega$, $Z_B = 10\angle 60°\ \Omega$. Phase sequence is $R \rightarrow Y \rightarrow B$. Calculate in the rectangular complex form the symmetrical components of the currents in the R-line.* **(Elect. Engg.-I, Bombay Univ.)**

Solution. As shown in Fig. 23.4, the branch impedances are

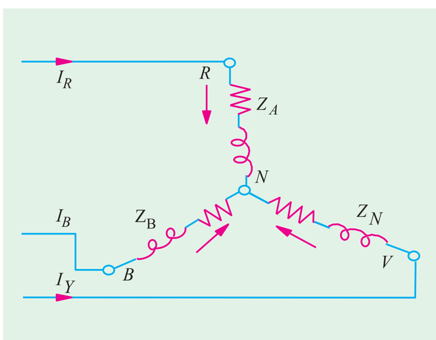

Fig. 23.4

$\mathbf{Z}_R$ 5 30 (4.33 j2.5)

$\mathbf{Z}_Y$ 10 45 (7.07 j7.07)

$\mathbf{Z}_B$ 10 60 (5 j8.66)

$a\mathbf{Z}_Y = 10\angle 165° = 10(-0.966 + j0.259) = -9.66 + j2.59)\,\Omega$

$a^2\mathbf{Z}_Y$ 10 75 10(0.259 j0.966) 2.59 j9.66

$a\mathbf{Z}_B$ 10 180 (10 j0)

$a^2\mathbf{Z}_B$ 10 60 10(0.5 j0.866) (5 j8.66)

The symmetrical component impedances required for calculation purposes are as follows:

$$\mathbf{Z}_0 \quad \frac{1}{3}(\mathbf{Z}_R \quad \mathbf{Z}_Y \quad \mathbf{Z}_B) \quad \frac{1}{3}(16.4 \quad j18.23) \quad (5.47 \quad j6.08)$$

$$\mathbf{Z}_{R1} = \frac{1}{3}(\mathbf{Z}_R + \mathbf{Z}_Y + a^2\mathbf{Z}_B)$$

$$= \frac{1}{3}(4.33 + j2.5 - 9.66 + j2.59 + 5 - j8.66)$$

$$= \frac{1}{3}(-0.33 - j3.57) = -0.11 - j1.19$$

$$\mathbf{Z}_{R2} = \frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + a\mathbf{Z}_B)$$

$$= \frac{1}{3}(4.33 + j2.5 + 2.59 - j9.66 - 10 + j0) = \frac{1}{3}(-3.08 - j7.16) = -1.03 - j2.55$$

Now

$$\mathbf{Z}_0{}^2\mathbf{Z}_{R1}\mathbf{Z}_{R2} = \frac{1}{3}(\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R)$$

$$= \frac{1}{3}[50 \quad 75 \quad 100 \quad 105 \quad 50 \quad 90\ ] \quad (\quad 4.31 \quad j65)$$

Let $\mathbf{V}_{RY}$ be taken as the reference vectors so that $\mathbf{V}_{RY} = (440 + j0)$

Then, $\mathbf{V}_{RN1} = \frac{1}{2}\mathbf{V}_{RY} - j.\frac{1}{2}\mathbf{V}_{RY}/\sqrt{3} = \frac{1}{2}(440 + j0) - j\frac{1}{2}(440 + j0)/\sqrt{3} = (220 - j127)$

Now, $\mathbf{I}_{R_1} \quad \frac{\mathbf{V}_{RN1}.\mathbf{Z}_0}{(\mathbf{Z}_0{}^2 \quad \mathbf{Z}_{R_1}\mathbf{Z}_{R_2})} \quad \frac{(220 \quad j127)(5.47 \quad j6.08)}{(\quad 4.31 \quad j65)} \quad \frac{1986 \quad j643}{(\quad 4.31 \quad j65)}$

$$= \frac{2088\angle 18^\circ}{66.4\angle 93.8^\circ} = 31.6\angle -75.8^\circ \neq (7.75 - j30.6)\text{A}$$

$$\mathbf{I}_{R_2} = -\mathbf{I}_{R_2}\mathbf{Z}_{R_1}/\mathbf{Z}_0$$

$$= \frac{-(7.75 - j30.6)(-0.11 - j1.19)}{(5.47 + j6.08)} = \frac{(37.25 + j5.88)}{(5.47 + j6.08)}$$

$$= \frac{37.75\angle 9^\circ}{8.18\angle 48^\circ} = 4.61\angle -39^\circ = (3.28 - j2.66)\text{A}$$

Hence, the symmetrical components of $\mathbf{I}_R$ are:-

Positive-sequence component = (7.75 – j30.6) A

Negative-sequence component = (3.28 – j2.66) A

Note. (*i*) $\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2}$

(*ii*) Symmetrical components of other currents are

$$\mathbf{I}_Y = a^2\mathbf{I}_{B1} \text{ and } \mathbf{I}_{Y2} = a\mathbf{I}_{R2}\ ;\ \mathbf{I}_{B1} = a\mathbf{I}_{R1} \text{ and } \mathbf{I}_{B2} = a^2\mathbf{I}_{R2}$$

Example 23.7. *A balanced star-connected load takes 75 A from a balanced 3-phase, 4-wire supply. If the fuses in two of the supply lines are removed, find the symmetrical components of the line currents before and after the fuses are removed.*

Solution. The circuit is shown in Fig. 23.5.

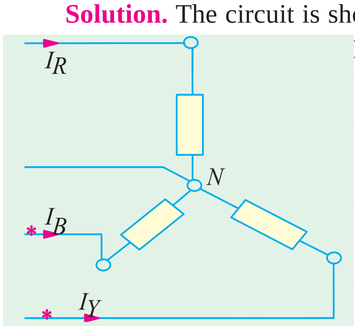

Fig. 23.5

Before uses are removed

$\mathbf{I}_R \quad 75 \quad 0\ ;\ \mathbf{I}_Y \quad 75 \quad 120\ ;\ \mathbf{I}_B \quad 75 \quad 120$

$$\mathbf{I}_1 \quad \frac{1}{3}(\mathbf{I}_R \quad a\mathbf{I}_Y \quad a^2\mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 0 \quad 75 \quad 360\ ) \quad 75 \quad 0 \quad \text{A}$$

$$\mathbf{I}_2 \quad \frac{1}{3}(\mathbf{I}_R \quad a^2\mathbf{I}_Y \quad a\mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 120 \quad 75 \quad 240\ ) \quad 0$$

$$\mathbf{I}_0 \quad \frac{1}{3}(\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 120 \quad 75 \quad 120\ ) \quad 0.$$

After fuses are removed

$$\mathbf{I}_R = 75\angle 0^\circ; \mathbf{I}_Y = \mathbf{I}_B = 0$$

$$\mathbf{I}_1 \quad \frac{1}{3}(75 \quad 0 \quad 0 \quad 0) \quad 25 \quad 0$$

$\mathbf{I}_2 = 25\angle 0°, \mathbf{I}_0 = 25\angle 0°$

Example 23.8. *Explain the terms: positive-, negative- and zero-sequence components of a 3-phase voltage system. A star-connected load consists of three equal resistors, each of 1 Ω resistance. When the load is connected to an unsymmetrical 3-phase supply, the line voltages are 200 V, 346 V and 400 V.*

Find the magnitude of the current in any one phase by the method of symmetrical components. **(Power Systems-II, A.M.I.E.)**

Solution. This question could be solved by using the following two methods:

(*a*) As shown in Fig. 23.6(*a*), the line voltages form a closed right-angled triangle with an angle = $\tan^{-1}(346/200) = \tan^{-1}(1.732) = 60°$

Hence, if $\mathbf{V}_{AB} = 200\ \angle 0°$, then $\mathbf{V}_{BC} = 346\ \angle -90° = -j346$ and $\mathbf{V}_{CA} = 400\ \angle 120°$.

As seen from Eq. (*iv*) and (*v*) of Art. 23.10

$$\mathbf{V}_{AN1} = \frac{1}{3}\left[\mathbf{V}_{AB} + \frac{1}{2}\mathbf{V}_{BC}(1 + j\sqrt{3})\right]$$

$$= \frac{1}{3}\left[200 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(-j346)\right] = 166.7 - j57.7$$

$$\mathbf{V}_{AN2} = \frac{1}{3}\left[\mathbf{V}_{AB} + \mathbf{V}_{BC}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{3}\left[200 + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(-j346)\right] = -33.3 - j57.7$$

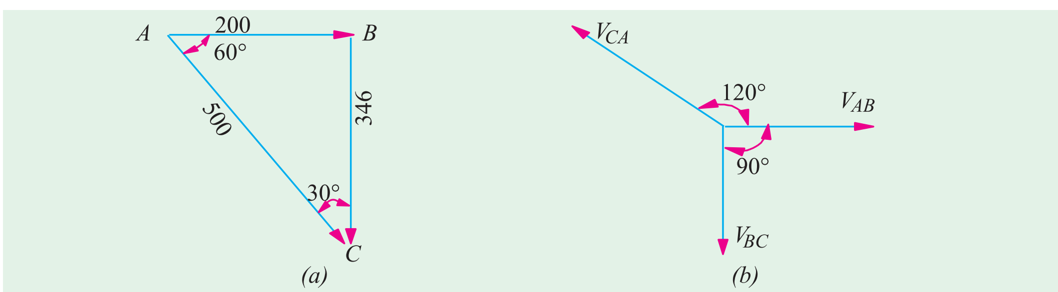

Fig. 23.6

Now, $\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C = 3$

$\mathbf{Z}_{A1} = \mathbf{Z}_A + a\mathbf{Z}_B + a^2\mathbf{Z}_C = 0$ (because $\mathbf{Z}_A \quad \mathbf{Z}_B \quad \mathbf{Z}_C$ and $1 \quad a \quad a^2 \quad 0$)

Similarly, $\mathbf{Z}_{A2} = \mathbf{Z}_A + a^2\mathbf{Z}_B + a\mathbf{Z}_C = 0$

Hence, from Eq. (***ii***) of Art. 23.10 we have $\mathbf{V}_{AN1} = \mathbf{I}_{A1}\mathbf{Z}_0 \;\therefore\; \mathbf{I}_{A1} = \mathbf{A}_{AN1}/\mathbf{Z}_0$

Now, $\mathbf{Z}_0 \quad \frac{1}{3}(\mathbf{Z}_A \quad \mathbf{Z}_B \quad \mathbf{Z}_C) \quad 3/3 \quad 1$ $\quad\therefore\; \mathbf{I}_{A1} = (166.7 - j57.7)$ A

$\mathbf{I}_{AS} = \mathbf{V}_{AN2}/\mathbf{Z}_0 = -33.3 - j57.7 \;\therefore\; \mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2}$ ($\mathbf{I}_0 \quad 0$)

$\therefore\; \mathbf{I}_A \quad (166.7 \quad j57.7) \quad (\; 33.3 \quad j57.7) \quad 133.4 \quad j115.4 \quad 176.4 \quad 40.7$ A (*b*) Using Millman's theorem and taking the line terminal *A* as reference point, the voltage between *A* and the neutral point *N* is

$$\mathbf{V}_{NA} \quad \frac{\mathbf{V}_{BA}\mathbf{Y}_B \quad \mathbf{V}_{CA}\mathbf{Y}_C}{\mathbf{Y}_A \quad \mathbf{Y}_B \quad \mathbf{Y}_C} \quad \frac{200 \quad 180 \quad 1 \quad 400 \quad 120 \quad 1}{3} \quad 133.3 \quad j115.3 \quad 176.4 \quad 139.3$$

$$\mathbf{V}_{AN} \quad 176.4 \quad 139.3 \quad 180 \quad 176.4 \quad 40.7 \quad \therefore \mathbf{I}_A \quad \mathbf{I}_{AN}/\mathbf{Z} \quad 176.4 \quad 40.7 \quad \text{... as before}$$

Example 23.9. *Three equal impedances of (8 + j6) are connected in star across a 3-phase, 3-wire supply. The phase voltages are V_A = (220 + j0), V_B = (– j220) and V_C = (– 100 + j220) V. If there is no connection between the load neutral and the supply neutral, calculate the symmetrical components of A-phase current and the three line currents.*

Solution. Since there is no fourth wire, there is no zero-component current. Moreover, $\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$. The symmetrical components of the A-phase voltages are

$$\mathbf{V}_0 \quad \frac{1}{3}(220 \quad j220 \quad 100 \quad j220) \quad 40\,\text{V}^{*}$$

$$\mathbf{V}_{A1} \quad \frac{1}{3}[220 \quad (\quad 0.5 \quad j0.866)(\quad j220) \quad (\quad 0.5 \quad j0.866)(\quad 100 \quad j220)]$$

$$= \frac{1}{3}[(660 + j86.6)] = (220 + j28.9)\,\text{V}$$

$$\mathbf{V}_{A2} \quad \frac{1}{3}[220 \quad (\quad 0.5 \quad j0.866)(\quad j220) \quad (\quad 0.5 \quad j0.866)(\quad 100 \quad j220)]$$

$$= \frac{1}{3}(-120 - j86.6) = (-40 - j28.9)\,\text{V}$$

The component currents in phase A are

$$\mathbf{I}_{A1} = \frac{\mathbf{V}_{A1}}{(8 + j6)} = \frac{220 + j28.9}{(8 + j6)} = (19.33 - j10.89)\,\text{A}$$

$$\mathbf{I}_{A2} \quad \frac{\text{V}_{A2}}{(8 \quad j6)} \quad \frac{(\quad 40 \quad j28.9)}{(8 \quad j6)} \quad (\quad 4.93 \quad j0.09)\,\text{A}$$

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} = (14.4 - j10.8)\text{A}; \mathbf{I}_B = \mathbf{I}_{B1} + \mathbf{B}_{B2} = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2}$$

$$\mathbf{I}_B \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad (19.93 \quad j10.89) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad (\quad 4.93 \quad j0.09)$$

Example 23.10. *Two equal impedance arms AB and BC are connected to the terminals A, B, C of a 3-phase supply as shown in Fig. 23.7. Each capacitor has a reactance of X = $\sqrt{3}R$. A high impedance voltmeter V is connected to the circuit at points P and Q as shown. If the supply line voltages V_{AB}, V_{BC}, V_{CA} are balanced, determine the reading of the voltmeter (a) when the phase sequence of the supply voltages is $A \rightarrow B \rightarrow C$ and (b) when the phase sequence is reversed. Hence, explain how this network could be employed to measure, respectively, the positive and negative phase sequence voltage components of an unbalanced 3-phase supply.*

* However, it is not required in the problem.

Solution. The various currents and voltages are shown in Fig. 23.7.

Phase Sequence ABC

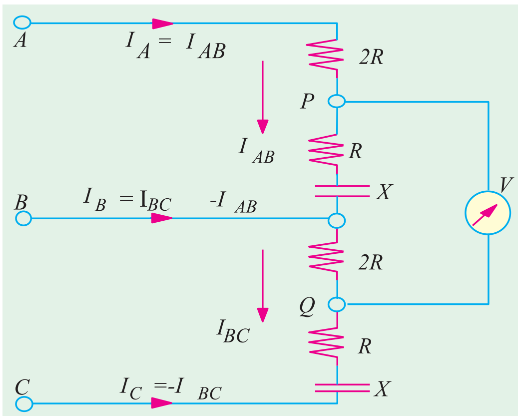

Fig. 23.7

Taking $\mathbf{V}_{AB}$ as the reference vector, we have

$$\mathbf{V}_{AB} = V(1+j0); \mathbf{V}_{BC} = a^2\mathrm{V}; \mathbf{V}_{CA} = a\mathrm{V}$$

As seen from the diagram,

$$\mathbf{I}_{AB}\quad \frac{\mathbf{V}_{AB}}{3R\quad jX}\quad \frac{V}{3R\quad j\sqrt{3}R}\quad \frac{V}{\sqrt{3}R(\sqrt{3}\quad j)}$$

$$\mathbf{I}_{BC}\quad \frac{\mathbf{V}_{BC}}{3R\quad jX}\quad \frac{a^2V}{\sqrt{3}R(\sqrt{3}\quad j)}$$

Now, $\mathbf{V}_{PO} + \mathbf{V}_{OQ} = \mathbf{V}_{PQ}$

Also, $\mathbf{V}_{PQ} = \mathbf{I}_{AB}(R - jX) + \mathbf{I}_{BC}.2R$

$$= \frac{V}{\sqrt{3}R(\sqrt{3}-j)}.R(1-j\sqrt{3}) + \frac{a^2V}{\sqrt{3}R(\sqrt{3}-j)}.2R$$

$$= \frac{V}{\sqrt{3}(\sqrt{3}-j)}.(1-j\sqrt{3}+2a^2) \qquad a^2\quad \frac{1}{2}\quad j\frac{\sqrt{3}}{2}$$

$$= \frac{V}{\sqrt{3}(\sqrt{3}-j)}.(-j2\sqrt{3}) \qquad \therefore\ V_{PQ}\quad \frac{V}{\sqrt{3}\quad 2}\quad 3\sqrt{3}\quad V$$

Hence, the voltmeter which is not phase sensitive will read the line voltage when phase sequence is $A \to B \to C$.

Phase Sequence ACB

In this case, $\mathbf{V}_{AB} = V(1+j0);\ \ \mathbf{V}_{BC} = aV;\ \mathbf{V}_{CA} = a^2V$

Also, $\qquad \mathbf{V}_{PQ} = \mathbf{V}_{PO} + \mathbf{V}_{OQ}$

$$= \mathbf{I}_{AB}(R-jX) + \mathbf{I}_{BC}.2R. = \frac{V(R-jX)}{\sqrt{3}.R(\sqrt{3}-j)} + \frac{aV.2R}{\sqrt{3}.R(\sqrt{3}-j)}$$

$$= \frac{V}{\sqrt{3}(\sqrt{3}-j)}(1-j\sqrt{3}+2a) = \frac{V}{\sqrt{3}(\sqrt{3}-j)}(1-j\sqrt{3}-1+j\sqrt{3}) = 0$$

Hence, when the phase sequence is reversed, the voltmeter reads zero.

It can be proved that with sequence *ABC*, the voltmeter reads the positive sequence component ($\mathbf{V}_1$) and with phase sequence *ACB*, it reads the negative sequence component ($\mathbf{V}_2$) with phase sequence *ABC*

$$\mathbf{V}_{PQ}\quad \frac{\mathbf{V}_{AB}}{3R\quad j\sqrt{3}.R}(R\quad j\sqrt{3}R)\quad \frac{\mathbf{V}_{BC}}{3R\quad j\sqrt{3}R}2R\quad \frac{1}{3\quad j\sqrt{3}}[(\mathbf{V}_1\quad \mathbf{V}_2)(1\quad j\sqrt{3})\quad 2(a^2\mathbf{V}_1\quad a\mathbf{V}_2)]$$

$$= \frac{1}{3-j\sqrt{3}}[\mathbf{V}_1 1-j\sqrt{3}+2a^2) + \mathbf{V}_2(1-j\sqrt{3}+2a)] = \frac{1}{3-j\sqrt{3}}\mathbf{V}_1(-j2\sqrt{3}) \qquad \therefore\ \mathbf{V}_{PQ} = \mathbf{V}_1$$

With phase sequence *ACB*

$$\mathbf{V}_{PQ} \quad \frac{1}{3 \quad j\sqrt{3}}\left[\,(\mathbf{V}_1 \quad \mathbf{V}_2)\,(1 \quad j\sqrt{3}) \quad 2(a\;\mathbf{V}_1 \quad a^2\;\mathbf{V}_2\;)\right]$$

$$= \frac{1}{3 - j\sqrt{3}}[\mathbf{V}_1(1 - j\sqrt{3} + 2a) + \mathbf{V}_2(1 - j\sqrt{3} + 2a^2)]$$

$$= \frac{1}{3 - j\sqrt{3}}[\mathbf{V}_2(-j2\sqrt{3})] \quad \therefore \quad V_{PQ} = V_2$$

23.12. Measurement of Symmetrical Components of Circuits

The apparatus consists of two identical current transformers, two impedances of the same ohmic value, one being more inductive than the other to the extent that its phase angle is 60° greater and two identical ammeters A_1 greater and two identical ammeters A_1 and A_2 as shown in Fig. 23.8 (*a*). It can be shown that A_1 reads positive-sequence current only while A_2 reads negative-sequence current only. If the turn ratio of the current transformer is *K*, then keeping in mind that zero-sequence component is zero, we have

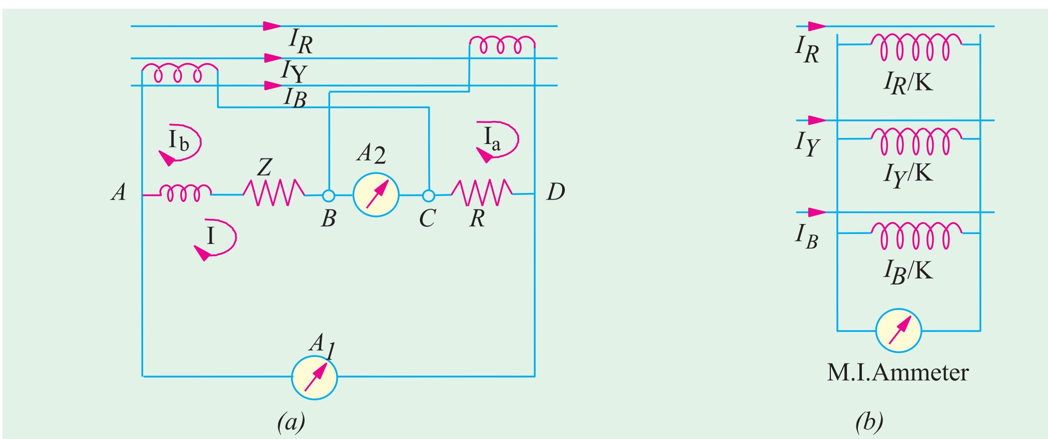

Fig. 23.8

$$\mathbf{I}_a = \mathbf{I}_R / K = (\mathbf{I}_1 + \mathbf{I}_2)/K$$

where $\mathbf{I}_1$ and $\mathbf{I}_2$ are the positive and negative-sequence components of the line current respectively.

Similarly,

$$\mathbf{I}_b = \mathbf{I}_Y / K = (a^2\mathbf{I}_1 + a\mathbf{I}_2)/K$$

If the impedance of each ammeter is $R_A + jK_A$, then impedance between points *B* and *D* is

$$\mathbf{Z}_{BD} \quad (R \quad R_A \quad jX_A)$$

The value of **Z** is so chosen that

Current transformers

$$\mathbf{Z}_{AC} \quad Z \quad R_A \quad jX_A \quad (R \quad R_A \quad jX_A) \quad 60 \quad Z_{BD} \quad 60$$

For finding the current read by A_1, imagine a break at point *X*. Thevenin voltage across the break *X* is

$$\mathbf{V}_{th} = \mathbf{I}_b\mathbf{Z}_{AC} + \mathbf{I}_a\mathbf{Z}_{BD} = \mathbf{I}_b\mathbf{Z}_{BD} = \mathbf{I}_b\mathbf{Z}_{BD}\angle 60°+\mathbf{I}a\mathbf{Z}_{BD} = (\mathbf{I}_b\angle 60°+\mathbf{I}_a)\mathbf{Z}_{BD}$$

Total impedance in series with this Thevenin voltage is

$$\mathbf{Z}_T \quad \mathbf{Z}_{AC} \quad \mathbf{Z}_{BD} \quad \mathbf{Z}_{BD} \quad 60 \quad \mathbf{Z}_{BD} \quad \frac{3}{2} \quad j\frac{\sqrt{3}}{2} \quad \mathbf{Z}_{BD}$$

The current flowing normally through the wire in which a break has been imagined is

$$\mathbf{I} \quad \frac{\mathbf{V}_{th}}{\mathbf{Z}_T} = \frac{\mathbf{I}_b\angle 60°+\mathbf{I}_a}{(3/2)+j(\sqrt{3}/2)} = \frac{1}{K}\frac{(a^2\mathbf{I}_1 + a\mathbf{I}_2)\angle 60°+\mathbf{I}_1+\mathbf{I}_2}{\sqrt{3}\angle 30°}$$

$$= \frac{1}{K}\frac{I_1(1 \quad 300 \quad 1) \quad I_2(1 \quad 180 \quad 1)}{\sqrt{3} \quad 30} \quad \frac{I_1}{K} \quad 60$$

It means that A_1 reads positive-sequence current only. The ammeter A_2 reads current which is

$$= \mathbf{I}_a \quad \mathbf{I}_b \quad \mathbf{I} = \frac{1}{K}(I_1 + I_2 + I_1\angle 240°+I_2\angle 120°-I_2\angle -60°)$$

$$= \frac{I_1}{K}\left(1-\frac{1}{2}-j\frac{\sqrt{3}}{2}-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)+\frac{I_2}{K}(\mathrm{I}+1\angle 120°) = \frac{I_2}{K}\angle 60°$$

In other words, A_2 reads negative-sequence current only.

Now, it will be shown that the reading of the moving-iron ammeter of Fig. 23-8(*b*) is proportional to the zero-sequence component.

$$\mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B) = \frac{K}{3}\left(\frac{\mathbf{I}_R}{K}+\frac{\mathbf{I}_Y}{K}+\frac{\mathbf{I}_B}{K}\right)$$

$$= \frac{K}{3} \times \text{(current through the ammeter)} = \frac{K}{3} \times \text{ammeter reading.}$$

It is obvious that for I_0 to be present, the system must be 3-phase, 4-wire. However, when the fourth wire is available, then I_0 may be found directly by finding the neutral current I_N. In that case $\mathbf{I}_0 \quad \frac{1}{3}I_N$.

23.13. Measurement of Positive and Negative-sequence *Voltages

With reference to Fig. 23.9 (*a*), it can be shown that the three voltmeters indicate only the positive-sequence component of the 3-phase system.

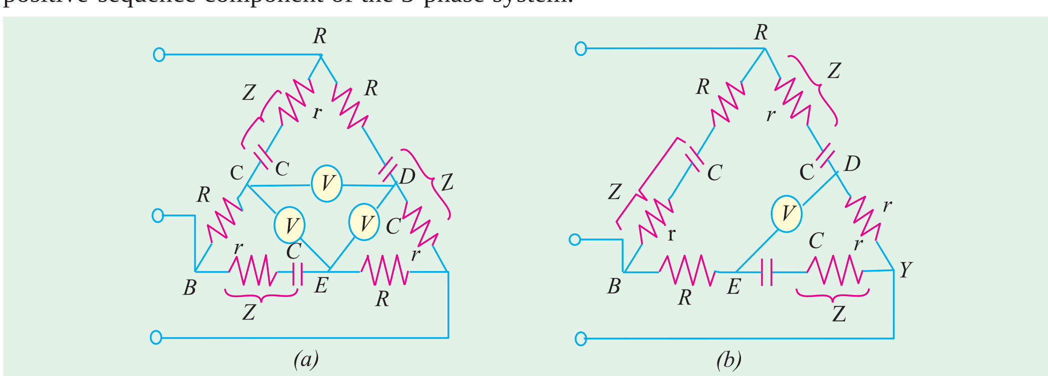

Fig. 23.9

* It is supposed to possess infinite impedance.

The line-to-neutral voltage can be written (with reference to the red phase) as

$$\mathbf{V}_{RN} = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{YN} = a^2\mathbf{V}_1 + a\mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{BN} = a\mathbf{V}_1 + a^2\mathbf{V}_2 + \mathbf{V}_0$$

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = (1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2$$

$$\mathbf{V}_{YB} = \mathbf{V}_{YN} + \mathbf{V}_{NB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = (a^2-a)\mathbf{V}_1 + (a-a^2)\mathbf{V}_2$$

$$\therefore \quad \mathbf{V}_{DY} \quad \frac{\mathbf{V}_{RY}(r \quad 1/j \quad C)}{(R \quad r \quad 1/j \quad C)} \qquad \mathbf{V}_{YE} \quad \frac{\mathbf{V}_{YB}.R}{(R \quad r \quad 1/j \quad C)}$$

$$\therefore \quad \mathbf{V}_{DE} \quad \frac{(r \quad 1/j \quad C)}{(R \quad r \quad 1/j \quad C)} \quad \mathbf{V}_{RY} \quad \frac{R}{(r \quad 1/j \quad C)}\mathbf{V}_{YB}$$

The different equation of the bridge circuit are so chosen that

$$\frac{R}{r + j1/\omega C} = -a^2 = \frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \text{or} \quad R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} + \frac{1}{j2\omega C} + j\frac{\sqrt{3}r}{2}$$

Equating the j-terms or quadrature terms on both sides, we have

$$0 = \frac{1}{j2\omega C} + j\frac{\sqrt{3}}{2} \qquad \therefore \quad \frac{1}{\omega C} = \sqrt{3}r \qquad \ldots (i)$$

Similarly, equating the reference or real terms, we have

$$R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} = \frac{r}{2} + \frac{3r}{2} = 2r \qquad \ldots (ii)$$

$$\therefore \quad \frac{r + 1/j\omega C}{R + r + 1/\omega C} = \frac{r - j\sqrt{3}r}{3r - j\sqrt{3}r} = \frac{r(1-j\sqrt{3})}{r(3-j\sqrt{3})} = \frac{1}{\sqrt{3}}\angle -30^\circ$$

and $$\mathbf{V}_{DE} = \frac{1\angle -30^\circ}{\sqrt{3}}[(1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2 - a^2(a^2-a)\mathbf{V}_1 - a^2(a-a^2)\mathbf{V}_2]$$

$$= \frac{1\angle -30^\circ}{\sqrt{3}}(1 - a^2 - a^4 + a^3)\mathbf{V}_1$$

$$[\quad 1 - a = a^2(a-a^2)] = \sqrt{3}\mathbf{V}_1\angle -30^\circ$$

Hence, the voltmeter connected between points D and E measures $\sqrt{3}$ times the positive sequence component of the phase voltage. So do the other two voltmeters.

In Fig. 23.9 (*b*), the elements have been reversed. It can be shown that provided the same relation is maintained between the elements, the high impedance voltmeter measures $\sqrt{3}$ times the negative-sequence component of phase voltage.

23.14. Measurement of Zero-sequence Component of Voltage

The zero-sequence voltage is given by

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN})$$

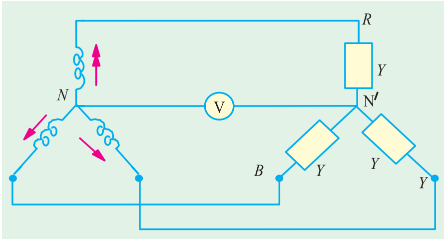

Fig. 23.10

Fig. 23.10 indicates one method of measuring $\mathbf{V}_0$. As seen

$$\mathbf{V}_{N'N} = \frac{\mathbf{V}_{RN}\mathbf{Y} + \mathbf{V}_{YN}\mathbf{Y} + \mathbf{V}_{BN}\mathbf{Y}}{3\mathbf{Y}}$$

$$= \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN}) = \mathbf{V}_0$$

Hence, voltmeter connected between the neutral points measures the zero-sequence component of the voltage.

Tutorial Problem No. 23.2

1. Explain the essential features in the representation of an unsymmetrical three-phase system of voltages or currents by symmetrical components.

In a three-phase system, the three line currents are; $\mathbf{I}_R = (30 + j50)\,\text{A}, \mathbf{I}_Y = (15 - j45)\,\text{A}, \mathbf{I}_B = (-40 + j70)$ A. Determine the values of the positive, negative and zero sequence components.

[52.2 A, 19.3 A, 25.1 A] (*London Univ.*)

2. The phase voltages of a three-phase, four-wire system are; $\mathbf{V}_{RN}$ = (200 + j0) V, $\mathbf{V}_{YN}$ = (0 – j200) V, $\mathbf{V}_{BN}$ = (– 100 + j 100) V. Show that these voltages can be replaced by symmetrical components of positive, negative and zero sequence and calculate the magnitude of each component.

Sketch vector diagrams representing the positive, negative and zero-sequence components for each of the three phases. **[176 V', 38 V, 47.1 V] (*London Univ.*)**

3. Explain, with the aid of a diagram of connections, a method of measuring the symmetrical components of the currents in an unbalanced 3-phase, 3-wire system.

If in such a system, the line currents, in amperes, are

$$\mathbf{I}_R = (10 - j2), \mathbf{I}_Y = (-2 - j4), \mathbf{I}_B = (-8 + j6)$$

Calculate their symmetrical components. **[$\mathbf{I}_{RO} = \mathbf{I}_{YO} = \mathbf{I}_{BO} = 0$; $\mathbf{I}_{R1} = 7.89 + j0.732$; $\mathbf{I}_{Y1} = a^2\,\mathbf{I}_{R1}$; $\mathbf{I}_{B1} = a\mathbf{I}_{R1}$ $\mathbf{I}_{R2} = 2.113 - j2.732$; $\mathbf{I}_{Y2} = a\mathbf{I}_{R2}$; $\mathbf{I}_{B2} = a^2\mathbf{I}_{R2}$]**

OBJECTIVE TESTS – 23

1. The method of symmetrical components is very useful for
(*a*) solving unbalanced polyphase circuits
(*b*) analysing the performance of 3-phase electrical machinery
(*c*) calculating currents resulting from unbalanced faults
(*d*) all of the above

2. An unbalanced system of 3-phase voltages having *RYB* sequence actually consists of
(*a*) a positive-sequence component
(*b*) a negative-sequence component
(*c*) a zero-sequence component
(*d*) all of the above.

3. The zero-sequence component of the unbalanced 3-phase system of vectors V_A, V_B and V_C is .... of their vector sum.
(*a*) one-third (*b*) one-half
(*c*) two-third (*d*) one-fourth

4. In the case of an unbalanced star-connected load supplied from an unbalanced 3-ϕ, 3 wire system, load currents will consist of
(*a*) positive-sequence components
(*b*) negative-sequence components
(*c*) zero-sequence components
(*d*) only (*a*) and (*b*).

ANSWERS

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*d*)

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