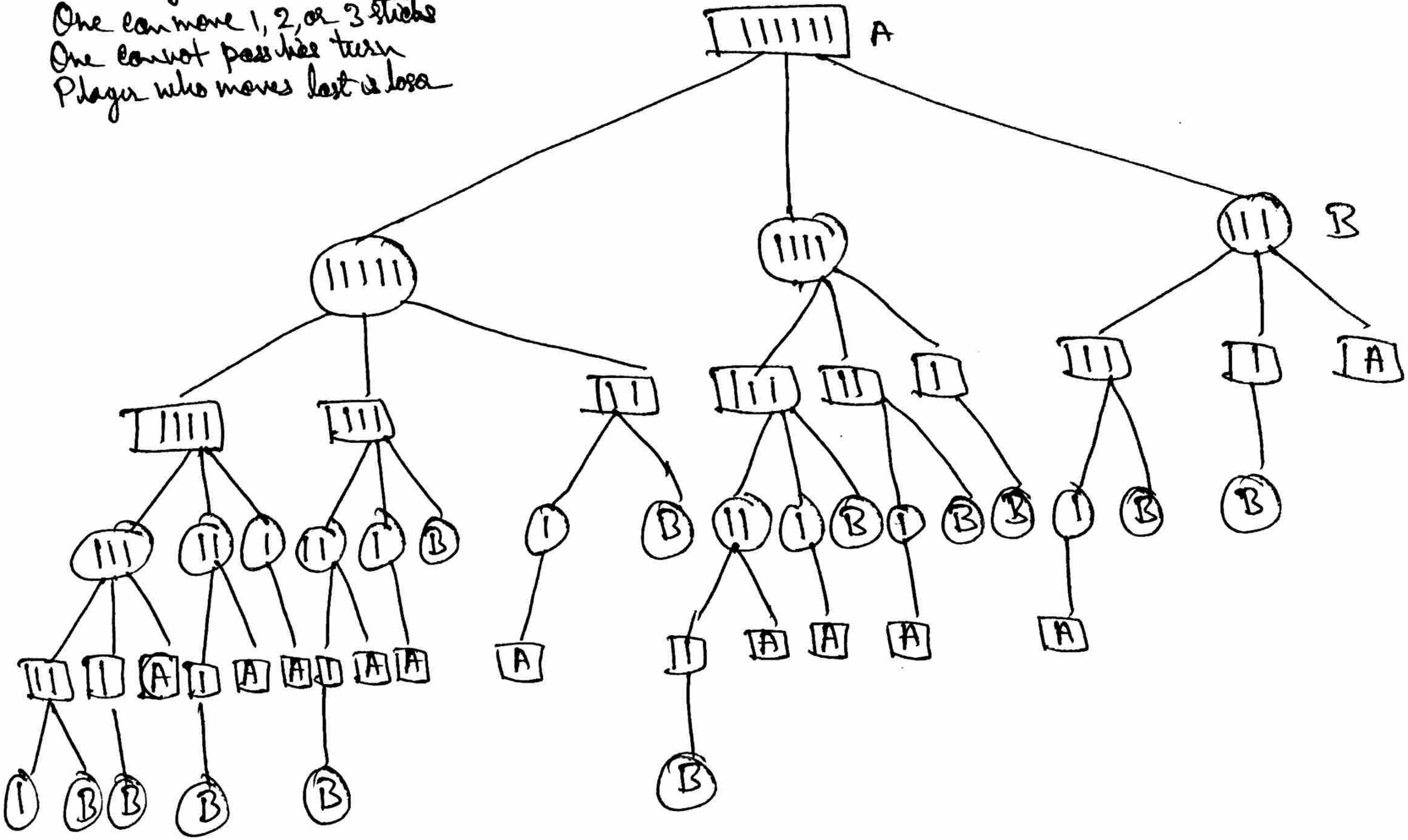
Algorithm — VI

Game of Nim & Nim Tree with six sticks

□ Player A to move
○ Player B to move
One can move 1, 2, or 3 sticks
One cannot pass his turn
Player who moves last is loser



Player in node is loser

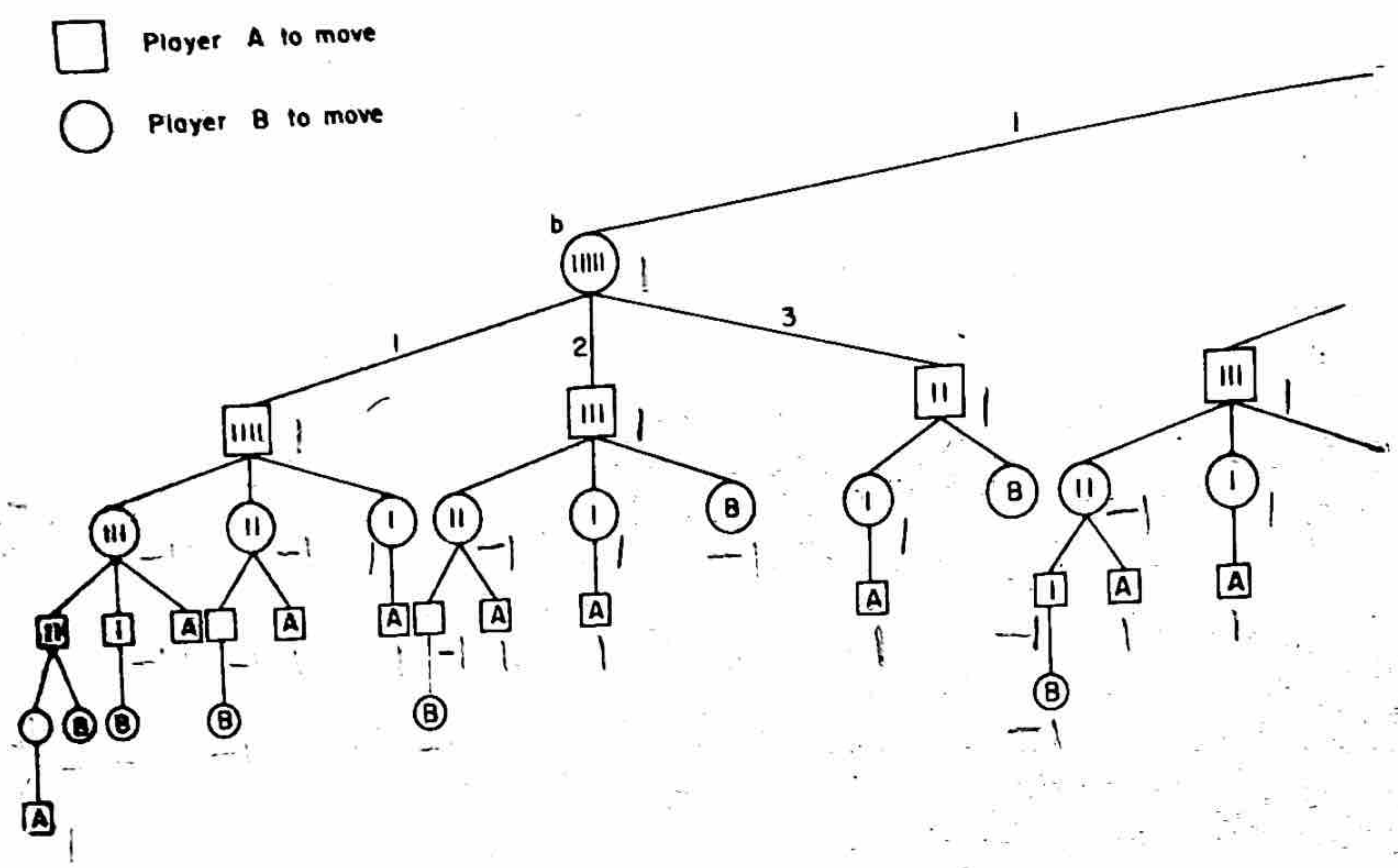
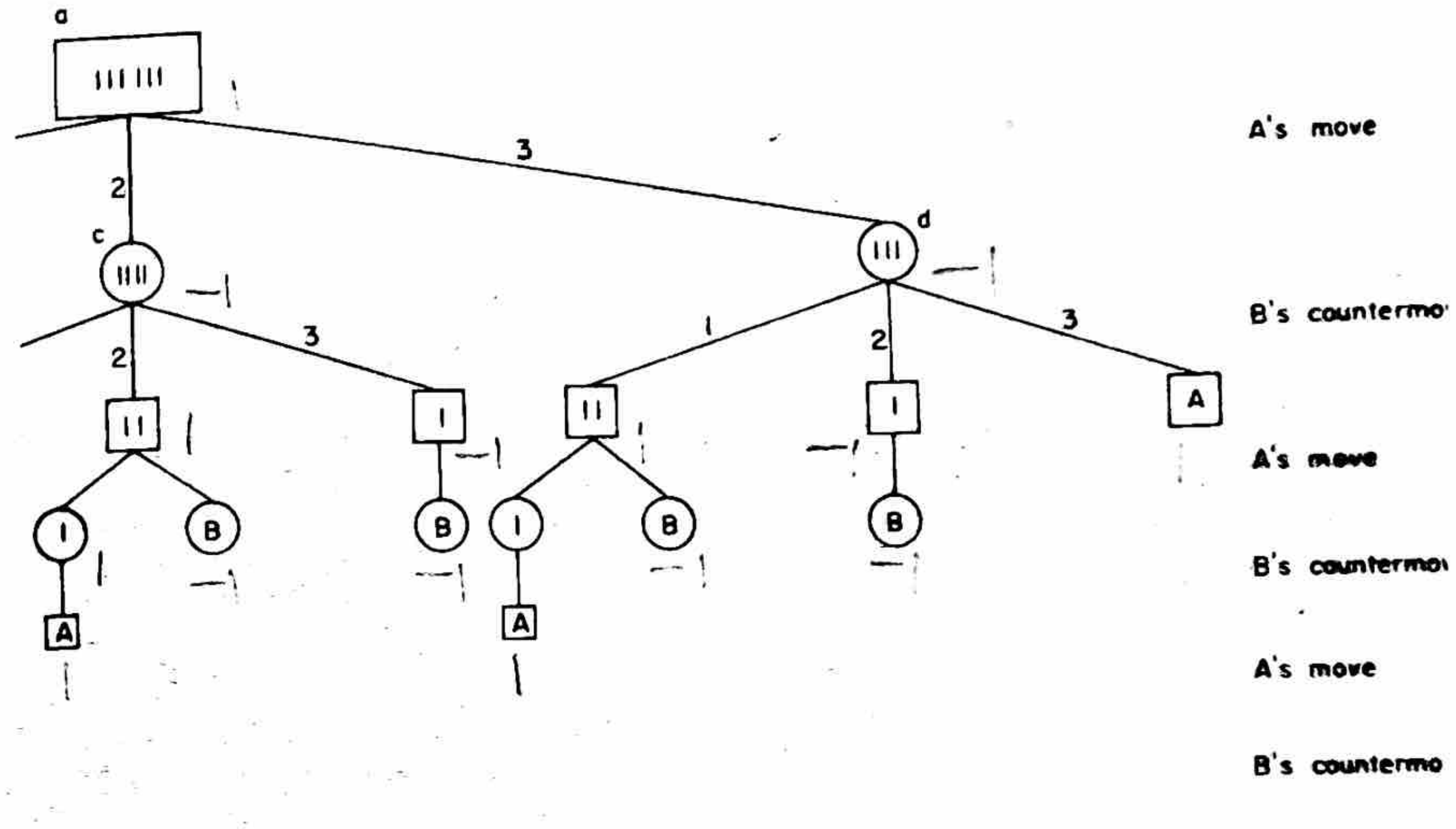


Figure 5.17 Complete Game Tree for Nim with $n = 6$.

should make. Starting at the initial configuration represented by the root of figure 5.17 player A is faced with the choice of making any one of three possible moves. Which one should he make? Assuming that player A wants to win the game, he should make the move that maximizes his chances of winning. For the simple tree of figure 5.17 this move is not too difficult to determine. We can use an evaluation function $E(X)$ which assigns a numeric value to the board configuration X. This function is a measure of the value or worth of configuration X to player A. So, $E(X)$ is high for a configuration from which A has a good chance of winning and low for a configuration from which A has a good chance of losing. $E(X)$ has its maximum value for configurations that are either winning terminal configurations for A or configurations from which A is guaranteed to win regardless of B's countermoves. $E(X)$ has its minimum value for configurations from which B is guaranteed to win.

For a game such as nim with $n = 6$, whose game tree has very few nodes, it is sufficient to define $E(X)$ only for terminal configurations. We could define $E(X)$ as:

$$E(X) = \begin{cases} 1 & \text{if } X \text{ is a winning configuration for } A \\ -1 & \text{if } X \text{ is a losing configuration for } A \end{cases}$$



Using this evaluation function we wish to determine which of the configurations b, c, d player A should move the game into. Clearly, the choice is the one whose value is max $\{V(b), V(c), V(d)\}$ where $V(x)$ is the value of configuration x. For leaf nodes x, $V(x)$ is taken to be $E(x)$. For all other nodes x let $d \geq 1$ be the degree of x and let $c_1, c_2, \ldots, c_d$ be the configurations represented by the children of x. Then $V(x)$ is defined by:

$$V(x) = \begin{cases} \max\limits_{1 \leq i \leq d} \{V(c_i)\} & \text{if } x \text{ is a square node} \\ \min\limits_{1 \leq i \leq d} \{V(c_i)\} & \text{if } x \text{ is a circular node} \end{cases} \tag{5.3}$$

The justification for (5.3) is fairly simple. If x is a square node, then it is at an odd level and it will be A's turn to move from here if the game ever reaches this node. Since A wants to win he will move to that child node with maximum value. In case x is a circular node it must be on an even level and if the game ever reaches this node, then it will be B's turn to move. Since B is out to win the game for himself, he will (barring mistakes) make a move that will minimize A's chances of winning. In this case the