1. Correctly parenthesize each of these lambda expressions:
   (a) (λx . x) (λy . y) λx . x (λy . y) z

   (b) (λf . λy . λz . f z y z) p x

   (c)λx . x λy . y λz . z λw . w z y x

2. Find the set of free variables for each of the following lambda expressions:
a) λx . x y λz . x z
b) (λx . x y) λz . w λw . w z y x
c) x λz . x λw . w z y
d) λx . x y λx . y x

3. Carry out the following substitutions:
a) (f (λx . x y) λz . x y z)[x→g] b) (λx . λy . f x y)[y→x]
c) ((λx . f x) λf . f x)[f→g x] d) (λf . λy . f x y)[x→f y]

4. Using the function Twice and the successor function succ, define a function that
a) adds four to its argument.
b) adds sixteen to its argument.

5. Use both normal order reduction and applicative order reduction to reduce the following lambda expressions. Reach a normal form representation if possible.
a) (λg . g 5) (λx . (add x 3))
b) (λx . (λy z . z y) x) p (λx . x)
c) (λx . x x x) (λx . x x x)
d) (λx . λy . (add x ((λx . (sub x 3)) y))) 5 6
e) (λc . c (λa . λb . b)) ((λa . λb . λf . f a b) p q)
f) Twice (λn . (mul 2 (add n 1))) 5
g) Twice (Twice (λn . (mul 2 (add n 1)))) 5
h) Twice Twice sqr 2
i) (λx . ((λz . (add x x)) ((λx . λz . (z 13 x)) 0 div))) ((λx . (x 5)) sqr)

6. Use call by value semantics to reduce the following lambda expressions:
a) (λf . f add (f mul (f add 5))) (λg . λx . g x x)
b) (λx . λf . f (f x)) ((λy . (add y 2)) ((λz . (sqr z)) ((λy . (succ y)) 1))) sqr