b) Solve the recurrence relation T (1) = 1 and for all $n \geq 2$, a power of 2, $T(n) = 2T(n/2) + 6n - 1$. 10+10

5. Prove that the following algorithm for swaping two numbers is correct. 20

```
procedure swap  (x,y)   {
        x = x + y ;
        y = x – y ;
        x = x – y ;
  }
```

6. Prove that the following algorithm for the addition of natural numbers is correct. 20

```
   function add (y, z)          {
 x = 0 ; c = 0 ; d = 1 ;
while (y > 0) ∨ (z > 0) ∨ (c > 0) {
          a = y mod 2 ;
          b = z mod 2
          if a ⊕ b ⊕ c then x = x + d ;
         c = (a ∧ b) ∨ (b ∧ c) ∨ (c ∧ a);
          d = 2d ; y = [y / 2] ;
          z = [z / 2] ;
      }
      return (x) ;
}
```

———————— x ————————

Ex/CSE/T/323/76/2010

BACHELOR OF COMPUTER SC.& ENGG. EXAMINATION, 2010

(3rd Year, 2nd Semester)

DESIGN AND ANALYSIS OF ALGORITHMS

Time : Three hours Full Marks : 100

Answer any five Questions.

1. a) Prove by induction on $n \geq 0$ that

$$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$$

b) Prove by induction on $n \geq 0$ that

$$\sum_{i=1}^{n} i(i+1) = n(n+1)(n+2)/3$$

10+10

2. a) Prove by induction on $n \geq 1$ that if $x > -1$, then $(1+x)^n \geq 1 + nx$

b) Prove by the induction on $n \geq 7$ that $3^n < n!$ 10+10

3. a) Prove that any set of regions defined by n lines in the plane can be coloured with two colours so that no two regions that share on edge have the same colour.

b) Prove by induction on $n \geq 1$ that $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$ 15+5

4. a) Solve the recurrence relation

$T(1) = 8$ and for all $n \geq 2$, $T(n) = 3T(n-1) - 15$

[Turn Over]