Ex/CSE/T/323/74/2012

BACHELOR OF ENGINEERING IN COMPUTER SCIENCE & ENGINEERING EXAMINATION, 2012

(3rd Year, 2nd Semester)

DESIGN AND ANALYSIS OF ALGORITHMS

Time : Three Hours — Full Marks - 100

Answer **one question** from Group A (Full Marks 20)
Answer **one question** from Group B (Full Marks 20)
Answer **six questions** from Group C (Full Marks 60)

Subparts of a question must be answered contiguously
Answers must be precise

Group - A (Marks - 20)

Answer any **one** question

1. a) State and explain the essential characteristics of a valid algorithm.

   b) Explain critically the issues of concern for designing an algorithm to solve a problem with primary focus on 'satisfiability' of a metric.

   c) Solve the following recurrences using Master Theorem :

      i) $T(n) = 9T(n/3) + n$

      ii) $T(n) = T(2n/3) + 1$

      iii) $T(n) = 3T(n/4) + nLgn$

[Turn over

d) What is meant by 'amortized cost'? 4+4+3x3+3=20

2. a) Define and illustrate Big-O, Ω and n notations for indicating times-complexity of algorithms.

b) Prove or dis-prove the following statement :

If f = O(g) and g = O(h) then f = O(h)

c) In each of the following situations, indicate with appropriate measuring, whether f = O(g) or f = W(g) or both (in which case f = n (g)

	f(n)	**g(n)**
i)	n-100	n-200
ii)	10 logn	$\log(n^2)$
iii)	$n \cdot 2^n$	3^n

2x3+5+3x3=20

Group - B (Marks - 20)

Answer any **one** question

3. a) Explain the following terms with the help of appropriate examples :

i) Decision problems and Optimization problems.

ii) Tractable problems and instrucable problems.

iii) Reducibility of problems.

11 Design an algorithm that takes as input, an indirected graph G = (V,E) and determines whether G contains a simple cycle (that is, a cycle that does not intersect itself) of length four. Its time complexity should be at most $O(|V|^3)$. You may assume that the input graph is represented either as an adjacency matrix or with adjacency lists, whichever marks your algorithm simple. 10

12. a) A long string consists of four characters A, C, G, T. They appear with frequency 31%, 20%, 9% and 40% respectively. Design an algorithm for finding the Huffman encoding of these four characters. Show the trace of your algorithm with this example.

b) Prove the following property of Huffman encoding scheme :

"If some character occurs with frequency more than 2/5 then there is guaranteed to be a codeword of length 1".

7+4=11

then 15, -30, 10 is a contiguous subsequence, but 5, 15, 40 is not. Design a linear-time algorithm for the following task :

Input : A list of integers $a_1, a_2, \ldots a_n$.

Output : The contiguous subsequence of maximum sum. (a subsequence of length zero has sum zero)

For the preceding example, the answer would be 10, -5, 40, 10 with a sum of 55. 10

10. Let us define a multiplication operation on three symbols a, b, c according to the following table :

	a	b	c
a	b	b	a
b	c	b	a
c	a	c	c

Note that the multiplication operation defined by the table is neither associative nor commutative. Design an efficient algorithm that examines a string of these symbols, say 'bbbbac', and decides whether or not it is possible to parenthesize the string in such a way that the value of the resulting operation is 'a'. For example, on inpput 'bbbbac' your algorithm should return 'yes' because $((b(bb))(ba))c = a$. 10

b) Define '3-SAT' problem and explain its importance in analysing 'hardness of problems'

c) A 'clique' is an indirected graph G - (V, E), is a subset $V' \subseteq V$ of vertices, each pair of which is connected by an edge in E. In other words, a clique is a complete subgraph of Co. The size of a clique is the number of vertices it contains. The 'clique problem' is the optimization problem of finding a clique of maximum size in a graph.

Prove that the clique problem is NP-complete.

3x3+3+8=20

4. a) Explain the following types of algorithms with their respective domains of applicability (i) Las Vigas algorithms; (ii) Monte-Carlo algorithms.

b) An array 'a' has n/4 copies of a particular unknown element x. Every other element in 'a' has at most n/8 copies. Design an O(logn) time Monte Carlo algorithm to identify x. The answer should be correct with high probability.

c) Brief explain the notion of 'Approximation algorithms' and state the circumstances when these are useful.

4x2+8+4=20

Group - C (Marks - 60)

Answer any **six** questios

All questions carry equal marks

5. A palindrome is a non-empty string over some alphabet that reads the same forward and backward. Examples of palindroms are all strings of length one' 'civic', 'racecar', etc. Design an efficient algorithm to find the longest palindrome that is a subsequence of a given input string. What is the running time of your algorithm? 10

6. Given two vectors $X = (x_1, x_2, ... x_n)$ and $Y = (y_1, y_2, ... y_n)$, X<Y if there exists an i, $1 \leq i \leq n$ such that $x_j = y_j$ for $1 \leq j \leq i$ and $x_i < y_i$. Given in such vectors each of size n, design an algorithm that determins the minimum vector. Analyze the time complexity of your algorithm. 10

7. x_1, x_2, and x_3 are thin sorted files of length 30, 20 and 10 records each. Merging x_1 and x_2 requires 50 record moves. Merging the result with x_3 requires another 60 moves. Total no. of movies required to merge x_1, x_2 and x_3 in this way is 110. If instead we first merge x_2 and x_3 (taking 30 moves) and then x_1 (taking 60 movies), total no. of movies required in this way would be 90. Design an efficient algorithm for merging n sorted files, and illustrate your algorithm with the help of a set of 10 such files whose lengths are (28, 32, 12, 05, 84, 53, 91, 35, 03 and 11). What is the time complexity of your algorithm? 10

8. A server has in customers waiting to be served. The service time required by each customer is known in advance : it is t_i minutes for customer i. So, if for example, the customers are served in order of increasing i, then the ith customer has to wait

$$\sum_{v=1}^{i} t_j \text{ minutes.}$$

We wish to minimise the total waiting time.

$$T = \sum_{i=1}^{n} (\text{timespent waiting by customer i})$$

Design an efficient algorithm for computing the optimal order in which to process the customers. What is the time complexity of your algorithm? 10

9. A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. For instance, if S is

$$5, 15, -30, 10, -5, 40, 10$$