

# CHAPTER 43

## Learning Objectives

- General
- Traction System
- Direct Steam Engine Drive
- Advantages of Electric Traction
- Saving in High Grade Coal
- Disadvantages of Electric Traction
- System of Railway Electrification
- Three Phase Low-Frequency A.C. System
- Block Diagram of an AC Locomotive
- The Tramways
- Collector Gear for OHE
- Confusion Regarding Weight and Mass of Train
- Tractive Efforts for Propulsion of a Train
- Power Output from Driving Axles
- Energy Output from Driving Axles
- Specific Energy Output
- Evaluation of Specific Energy Output
- Energy Consumption
- Specific Energy Consumption
- Adhesive Weight
- Coefficient of Adhesion

## ELECTRIC TRACTION



In electric traction, driving force (or tractive force) is generated by electricity, using electric motors. Electric trains, trams, trolley, buses and battery run cars are some examples where electric traction is employed.

### 43.1. General

By electric traction is meant locomotion in which the driving (or tractive) force is obtained from electric motors. It is used in electric trains, tramcars, trolley buses and diesel-electric vehicles etc. Electric traction has many advantages as compared to other non-electrical systems of traction including steam traction.

### 43.2. Traction Systems

Broadly speaking, all traction systems may be classified into two categories :

#### (a) non-electric traction systems

They do not involve the use of electrical energy at *any stage*. Examples are : steam engine drive used in railways and internal-combustion-engine drive used for road transport.



The above picture shows a diesel train engine. These engines are now being rapidly replaced by electric engines

#### (b) electric traction systems

They involve the use of electric energy at some stage or the other. They may be further sub-divided into two groups :

1. First group consists of self-contained vehicles or locomotives. Examples are : battery-electric drive and diesel-electric drive etc.
2. Second group consists of vehicles which receive electric power from a distribution network fed at suitable points from either central power stations or suitably-spaced sub-stations. Examples are : railway electric locomotive fed from overhead ac supply and tramways and trolley buses supplied with dc supply.

### 43.3. Direct Steam Engine Drive

Though losing ground gradually due to various reasons, steam locomotive is still the most widely-adopted means of propulsion for railway work. Invariably, the reciprocating engine is employed because

1. it is inherently simple.
2. connection between its cylinders and the driving wheels is simple.
3. its speed can be controlled very easily.

However, the steam locomotive suffers from the following disadvantages :

1. since it is difficult to install a condenser on a locomotive, the steam engine runs non-condensing and, therefore, has a very low thermal efficiency of about 6-8 percent.
2. it has strictly limited overload capacity.
3. it is available for hauling work for about 60% of its working days, the remaining 40% being spent in preparing for service, in maintenance and overhaul.

#### 43.4. Diesel-electric Drive

It is a self-contained motive power unit which employs a diesel engine for direct drive of a dc generator. This generator supplies current to traction motors which are geared to the driving axles.

In India, diesel locomotives were introduced in 1945 for shunting service on broad-gauge (BG) sections and in 1956 for high-speed main-line operations on metre-gauge (MG) sections. It was only in 1958 that Indian Railways went in for extensive main-line dieselisation.\*

Diesel-electric traction has the following advantages :

1. no modification of existing tracks is required while converting from steam to diesel-electric traction.
2. it provides greater tractive effort as compared to steam engine which results in higher starting acceleration.
3. it is available for hauling for about 90% of its working days.
4. diesel-electric locomotive is more efficient than a steam locomotive (though less efficient than an electric locomotive).

##### Disadvantages

1. for same power, diesel-electric locomotive is costlier than either the steam or electric locomotive.
2. overload capacity is limited because diesel engine is a constant-kW output prime mover.
3. life of a diesel engine is comparatively shorter.
4. diesel-electric locomotive is heavier than plain electric locomotive because it carries the main engine, generator and traction motors etc.
5. regenerative braking cannot be employed though rheostatic braking can be.

#### 43.5. Battery-electric Drive

In this case, the vehicle carries secondary batteries which supply current to dc motors used for driving the vehicle. Such a drive is well-suited for shunting in railway yards, for traction in mines, for local delivery of goods in large towns and large industrial plants. They have low maintenance cost and are free from smoke. However, the scope of such vehicles is limited because of the small capacity of the batteries and the necessity of charging them frequently.



The above picture shows a battery run car. Battery run vehicles are seen as alternatives for future transport due to their pollution-free locomotion

#### 43.6. Advantages of Electric Traction

As compared to steam traction, electric trac-

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\* The Diesel Locomotive Works at Varanasi turns out 140 locomotives of 2700 hp (2015 kW) annually. Soon it will be producing new generation diesel engines of 4000 hp (2985 kW).

tion has the following advantages :

**1. Cleanliness.** Since it does not produce any smoke or corrosive fumes, electric traction is most suited for underground and tube railways. Also, it causes no damage to the buildings and other apparatus due to the absence of smoke and flue gases.

**2. Maintenance Cost.** The maintenance cost of an electric locomotive is nearly 50% of that for a steam locomotive. Moreover, the maintenance time is also much less.

**3. Starting Time.** An electric locomotive can be started at a moment's notice whereas a steam locomotive requires about two hours to heat up.

**4. High Starting Torque.** The motors used in electric traction have a very high starting torque. Hence, it is possible to achieve higher accelerations of 1.5 to 2.5 km/h/s as against 0.6 to 0.8 km/h/s in steam traction. As a result, we are able to get the following additional advantages:

- (i) high schedule speed
- (ii) increased traffic handling capacity
- (iii) because of (i) and (ii) above, less terminal space is required—a factor of great importance in urban areas.

**5. Braking.** It is possible to use regenerative braking in electric traction system. It leads to the following advantages :

- (i) about 80% of the energy taken from the supply during ascent is returned to it during descent.
- (ii) goods traffic on gradients becomes safer and speedier.
- (iii) since mechanical brakes are used to a very small extent, maintenance of brake shoes, wheels, tyres and track rails is considerably reduced because of less wear and tear.

**6. Saving in High Grade Coal.** Steam locomotives use costly high-grade coal which is not so abundant. But electric locomotives can be fed either from hydroelectric stations or pit-head thermal power stations which use cheap low-grade coal. In this way, high-grade coal can be saved for metallurgical purposes.

**7. Lower Centre of Gravity.** Since height of an electric locomotive is much less than that of a steam locomotive, its centre of gravity is comparatively low. This fact enables an electric locomotive to negotiate curves at higher speeds quite safely.

**8. Absence of Unbalanced Forces.** Electric traction has higher coefficient of adhesion since there are no unbalanced forces produced by reciprocating masses as is the case in steam traction. It not only reduces the weight/kW ratio of an electric locomotive but also improves its riding quality in addition to reducing the wear and tear of the track rails.

### 43.7. Disadvantages of Electric Traction

**1.** The most vital factor against electric traction is the initial high cost of laying out overhead electric supply system. Unless the traffic to be handled is heavy, electric traction becomes uneconomical.

**2.** Power failure for few minutes can cause traffic dislocation for hours.

**3.** Communication lines which usually run parallel to the power supply lines suffer from electrical interference. Hence, these communication lines have either to be removed away from the rail track or else underground cables have to be used for the purpose which makes the entire system still more expensive.

**4.** Electric traction can be used only on those routes which have been electrified. Obviously, this restriction does not apply to steam traction.

**5.** Provision of a negative booster is essential in the case of electric traction. By avoiding the

flow of return currents through earth, it curtails corrosion of underground pipe work and interference with telegraph and telephone circuits.

### 43.8. Systems of Railway Electrification

Presently, following four types of track electrification systems are available :

1. Direct current system—600 V, 750 V, 1500 V, 3000 V
2. Single-phase ac system—15-25 kV,  $16\frac{2}{3}$ , 25 and 50 Hz
3. Three-phase ac system—3000-3500 V at  $16\frac{2}{3}$  Hz
4. Composite system—involving conversion of single-phase ac into 3-phase ac or dc.

### 43.9. Direct Current System

Direct current at 600-750 V is universally employed for tramways in urban areas and for many suburban railways while 1500-3000 V dc is used for main line railways. The current collection is from third rail (or conductor rail) up to 750 V, where large currents are involved and from overhead wire for 1500 V and 3000 V, where small currents are involved. Since in majority of cases, track (or running) rails are used as the return conductor, only one conductor rail is required. Both of these contact systems are fed from substations which are spaced 3 to 5 km for heavy suburban traffic and 40-50 km for main lines operating at higher voltages of 1500 V to 3000 V. These sub-stations themselves receive power from 110/132 kV, 3-phase network (or grid). At these substations, this high-voltage 3-phase supply is converted into low-voltage 1-phase supply with the help of Scott-connected or V-connected 3-phase transformers (Art. 31.9). Next, this low ac voltage is converted into the required dc voltage by using suitable rectifiers or converters (like rotary converter, mercury-arc, metal or semiconductor rectifiers). These substations are usually automatic and are remote-controlled.

The dc supply so obtained is fed via suitable contact system to the traction motors which are either dc series motors for electric locomotive or compound motors for tramway and trolley buses where regenerative braking is desired.

It may be noted that for *heavy suburban service*, low voltage dc system is undoubtedly superior to 1-phase ac system due to the following reasons :

1. dc motors are better suited for frequent and rapid acceleration of heavy trains than ac motors.
2. dc train equipment is lighter, less costly and more efficient than similar ac equipment.
3. when operating under similar service conditions, dc train consumes less energy than a 1-phase ac train.
4. the conductor rail for dc distribution system is less costly, both initially and in maintenance than the high-voltage overhead ac distribution system.
5. dc system causes no electrical interference with overhead communication lines.

The only disadvantage of dc system is the necessity of locating ac/dc conversion sub-stations at relatively short distances apart.

### 43.10. Single-Phase Low-frequency AC System

In this system, ac voltages from 11 to 15 kV at  $16\frac{2}{3}$  or 25 Hz are used. If supply is from a generating station exclusively meant for the traction system, there is no difficulty in getting the electric supply of  $16\frac{2}{3}$  or 25 Hz. If, however, electric supply is taken from the high voltage transmission lines at 50 Hz, then in addition to step-down transformer, the substation is provided with a frequency

converter. The frequency converter equipment consists of a 3-phase synchronous motor which drives a 1-phase alternator having 16  $\frac{2}{3}$  or 25 Hz frequency.

The 15 kV 16  $\frac{2}{3}$  or 25 Hz supply is fed to the electric locomotor via a single over-head wire (running rail providing the return path).

A step-down transformer carried by the locomotive reduces the 15-kV voltage to 300-400 V for feeding the ac series motors. Speed regulation of ac series motors is achieved by applying variable voltage from the tapped secondary of the above transformer.

Low-frequency ac supply is used because apart from improving the commutation properties of ac motors, it increases their efficiency and power factor. Moreover, at low frequency, line reactance is less so that line impedance drop and hence line voltage drop is reduced. Because of this reduced line drop, it is feasible to space the substations 50 to 80 km apart. Another advantage of employing low frequency is that it reduces telephonic interference.

#### 41.11. Three-phase Low-frequency AC System

It uses 3-phase induction motors which work on a 3.3 kV, 16  $\frac{2}{3}$  Hz supply. Sub-stations receive power at a very high voltage from 3-phase transmission lines at the usual industrial frequency of 50 Hz. This high voltage is stepped down to 3.3 kV by transformers whereas frequency is reduced from 50 Hz to 16  $\frac{2}{3}$  Hz by frequency converters installed at the sub-stations. Obviously, this system employs *two* overhead contact wires, the track rail forming the third phase (of course, this leads to insulation difficulties at the junctions).

Induction motors used in the system are quite simple and robust and give trouble-free operation. They possess the merits of high efficiency and of operating as a generator when driven at speeds above the synchronous speed. Hence, they have the property of automatic regenerative braking during the descent on gradients. However, it may be noted that despite all its advantages, this system has not found much favour and has, in fact, become obsolete because of its certain inherent limitations given below :

1. the overhead contact wire system becomes complicated at crossings and junctions.
2. constant-speed characteristics of induction motors are not suitable for traction work.
3. induction motors have speed/torque characteristics similar to dc shunt motors. Hence, they are not suitable for parallel operation because, even with little difference in rotational speeds caused by unequal diameters of the wheels, motors will become loaded very unevenly.

#### 43.12. Composite System

Such a system incorporates good points of two systems while ignoring their bad points. Two such composite systems presently in use are :

1. 1-phase to 3-phase system also called Kando system
2. 1-phase to dc system.

#### 43.13. Kando System

In this system, single-phase 16-kV, 50 Hz supply from the sub-station is picked up by the locomotive through the single overhead contact wire. It is then converted into 3-phase ac supply at the same frequency by means of phase converter equipment carried on the locomotives. This 3-phase supply is then fed to the 3-phase induction motors.



As seen, the complicated overhead two contact wire arrangement of ordinary 3-phase system is replaced by a single wire system. By using silicon controlled rectifier as inverter, it is possible to get variable-frequency 3-phase supply at 1/2 to 9 Hz frequency. At this low frequency, 3-phase motors develop high starting torque without taking excessive current. In view of the above, Kando system is likely to be developed further.

#### 43.14. Single-phase AC to DC System

This system combines the advantages of high-voltage ac distribution at industrial frequency with the dc series motors traction. It employs overhead 25-kV, 50-Hz supply which is stepped down by the transformer installed in the locomotive itself. The low-voltage ac supply is then converted into dc supply by the rectifier which is also carried on the locomotive. This dc supply is finally fed to dc series traction motor fitted between the wheels. The system of traction employing 25-kV, 50-Hz, 1-phase ac supply has been adopted for all future track electrification in India.

#### 43.15. Advantages of 25-kV, 50-Hz AC System

Advantages of this system of track electrification over other systems particularly the dc system are as under :

##### 1. Light Overhead Catenary

Since voltage is high (25 kV), line current for a given traction demand is less. Hence, cross-section of the overhead conductors is reduced. Since these small-sized conductors are light, supporting structures and foundations are also light and simple. Of course, high voltage needs higher insulation which increases the cost of overhead equipment (OHE) but the reduction in the size of conductors has an overriding effect.

##### 2. Less Number of Substations

Since in the 25-kV system, line current is less, line voltage drop which is mainly due to the resistance of the line is correspondingly less. It improves the voltage regulation of the line which fact makes larger spacing of 50-80 km between sub-stations possible as against 5-15 km with 1500 V dc system and 15-30 km with 3000 V dc system. Since the required number of substations along the track is considerably reduced, it leads to substantial saving in the capital expenditure on track electrification.

##### 3. Flexibility in the Location of Substations

Larger spacing of substations leads to greater flexibility in the selection of site for their proper location. These substations can be located near the national high-voltage grid which, in our country, fortunately runs close to the main railway routes. The substations are fed from this grid thereby saving the railway administration lot of expenditure for erecting special transmission lines for their substations. On the other hand, in view of closer spacing of dc substations and their far away location, railway administration has to erect its own transmission lines for taking feed from the national grid to the substations which consequently increases the initial cost of electrification.

##### 4. Simplicity of Substation Design

In ac systems, the substations are simple in design and layout because they do not have to install and maintain rotary converters or rectifiers as in dc systems. They only consist of static transformers alongwith their associated switchgear and take their power directly from the high-voltage national grid running over the length and breadth of our country. Since such sub-stations are remotely controlled, they have few attending personnel or even may be unattended.

### 5. Lower Cost of Fixed Installations

The cost of fixed installations is much less for 25 kV ac system as compared to dc system. In fact, cost is in ascending order for 25 kV ac, 3000 V dc and 1500 V dc systems. Consequently, traffic densities for which these systems are economical are also in the ascending order.

### 6. Higher Coefficient of Adhesion

The straight dc locomotive has a coefficient of adhesion of about 27% whereas its value for ac rectifier locomotive is nearly 45%. For this reason, a lighter ac locomotive can haul the same load as a heavier straight dc locomotive. Consequently, ac locomotives are capable of achieving higher speeds in coping with heavier traffic.

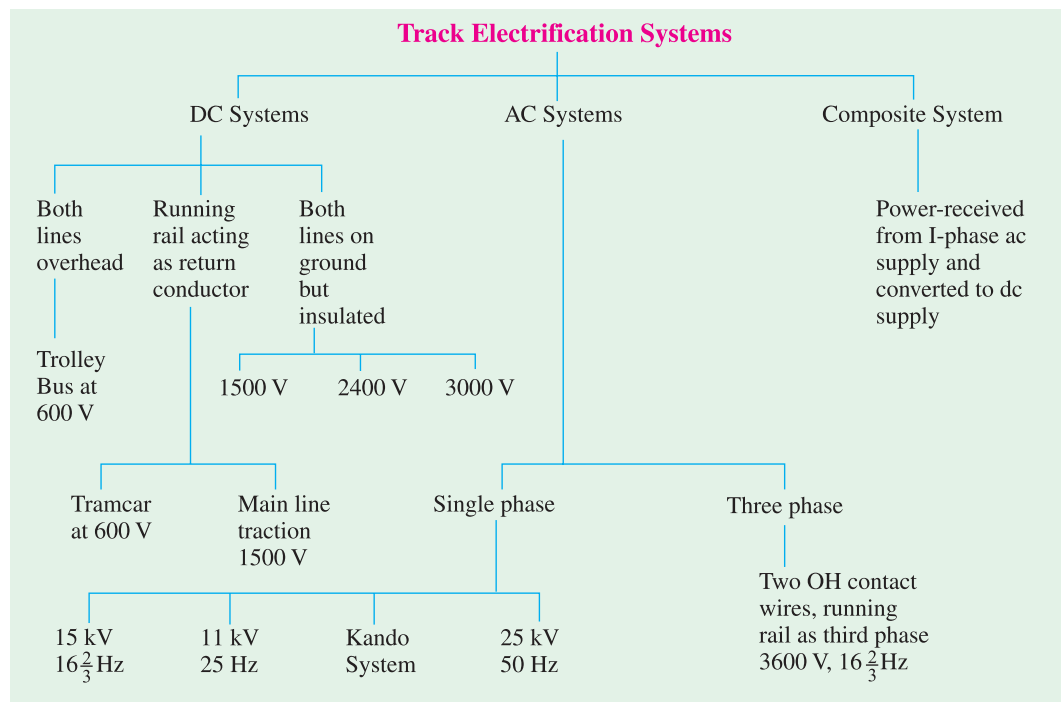
### 7. Higher Starting Efficiency

An ac locomotive has higher starting efficiency than a straight dc locomotive. In dc locomotive supply voltage at starting is reduced by means of ohmic resistors but by on-load primary or secondary tap-changer in ac locomotives.

## 43.16. Disadvantages of 25-kV AC System

1. Single-phase ac system produces both current and voltage unbalancing effect on the supply.
2. It produces interference in telecommunication circuits. Fortunately, it is possible at least to minimize both these undesirable effects.

Different track electrification systems are summarised below :



## 43.17. Block Diagram of an AC Locomotive

The various components of an ac locomotive running on single-phase 25-kV, 50-Hz ac supply are numbered in Fig. 43.1.

1. OH contact wire



2. pantograph
3. circuit breakers
4. on-load tap-changers
5. transformer
6. rectifier
7. smoothing choke
8. dc traction motors.

As seen, power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low ac voltage so obtained is converted into pulsating dc voltage by means of the rectifier. The pulsations in the dc voltage are then removed by the smoothing choke before it is fed to dc series traction motors which are mounted between the wheels.

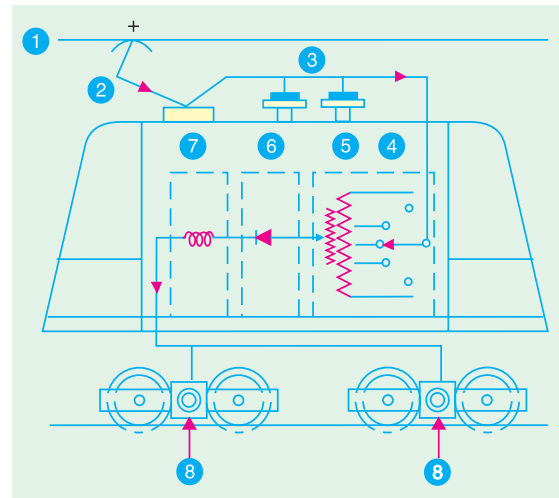


Fig. 43.1

The function of circuit breakers is to immediately disconnect the locomotive from the overhead supply in case of any fault in its electrical system. The on-load tap-changer is used to change the voltage across the motors and hence regulate their speed.

### 43.18. The Tramways

It is the most economical means of transport for very dense traffic in the congested streets of large cities. It receives power through a bow collector or a grooved wheel from an overhead conductor at about 600 V dc, the running rail forming the return conductor. It is provided with at least two driving axles in order to (i) secure necessary adhesion (ii) start it from either end and (iii) use two motors with series-parallel control. Two drum-type controllers, one at each end, are used for controlling the tramcar. Though these controllers are connected in parallel, they have suitable interlocking arrangement meant to prevent their being used simultaneously.

Tramcars are being replaced by trolley-buses and internal-combustion-engined omnibuses because of the following reasons :

1. tramcars lack flexibility of operation in congested areas.
2. the track constitutes a source of danger to other road users.

### 43.19. The Trolleybus

It is an electrically-operated pneumatic-tyred vehicle which needs *no track in the roadway*. It receives its power at 600 V dc from two overhead contact wires. Since adhesion between a rubber-tyred wheel and ground is sufficiently high, only a single driving axle and, hence, a single motor is used. The trolleybus can manoeuvre through traffic a metre or two on each side of the centre line of the trolley wires.

### 43.20. Overhead Equipment (OHE)

Broadly speaking, there are two systems of current collection by a traction unit :



Trolley Bus

(i) third rail system and (ii) overhead wire system.

It has been found that current collection from overhead wire is far superior to that from the third rail. Moreover, insulation of third rail at high voltage becomes an impracticable proposition and endangers the safety of the working personnel.

The simplest type of OHE consists of a single contact wire of hard drawn copper or silico-bronze supported either by bracket or an overhead span. To facilitate connection to the supports, the wire is grooved as shown in Fig. 43.2. Because there is appreciable sag of the wire between supports, it limits the speed of the traction unit to about 30 km/h. Hence, single contact wire system is suitable for tramways and in complicated yards and terminal stations where speeds are low and simplicity of layout is desirable.

For collection of current by high-speed trains, the contact (or trolley) wire has to be kept level without any abrupt changes in its height between the supporting structures. It can be done by using the single catenary system which consists of one catenary or messenger wire of steel with high sag and the trolley (or contact) wire supported from messenger wire by means of droppers clipped to both wires as shown in Fig. 43.3.

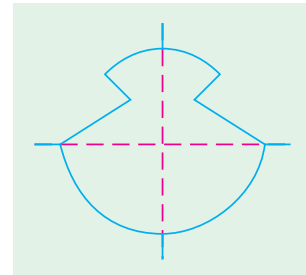


Fig. 43.2

#### 43.21. Collector Gear for OHE

The most essential requirement of a collector is that it should keep continuous contact with trolley wire at all speeds. Three types of gear are in common use :

1. trolley collector
2. bow collector
- and 3. pantograph collector.

To ensure even pressure on OHE, the gear equipment must, be flexible in order to follow variations in the sag of the contact wire. Also, reasonable precautions must be taken to prevent the collector from leaving the overhead wire at points and crossings.

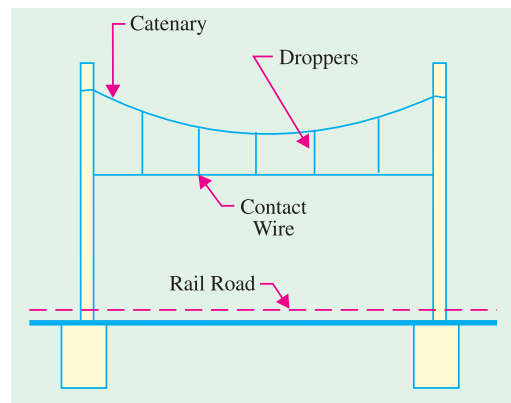


Fig. 43.3

#### 43.22. The Trolley Collector

This collector is employed on tramways and trolley buses and is mounted on the roof of the vehicle. Contact with the OH wire is made by means of either a grooved wheel or a sliding shoe carried at the end of a light trolley pole attached to the top of the vehicle and held in contact with OH wire by means of a spring. The pole is hinged to a swivelling base so that it may be reversed for reverse running thereby making it unnecessary for the trolley wire to be accurately maintained above the centre of the track. Trolley collectors always operate in the trailing position.

The trolley collector is suitable for low speeds upto 32 km/h beyond which there is a risk of its jumping off the OH contact wire particularly at points and crossing.

#### 43.23. The Bow Collector

It can be used for higher speeds. As shown in Fig. 43.4, it consists of two roof-mounted trolley poles at the ends of which is placed a light metal strip (or bow) about one metre long for current collection. The collection strip is purposely made of soft material (copper, aluminium or carbon) in order that most of the wear may occur on it rather than on the trolley wire. The bow collector also

operates in the trailing position. Hence, it requires provision of either duplicate bows or an arrangement for reversing the bow for running in the reverse direction. Bow collector is not suitable for railway work where speeds up to 120 km/h and currents up to 3000 A are encountered. It is so because the inertia of the bow collector is too high to ensure satisfactory current collection.

#### 43.24. The Pantograph Collector

Its function is to maintain link between overhead contact wire and power circuit of the electric locomotive at different speeds under all wind conditions and stiffness of OHE. It means that positive pressure has to be maintained at all times to avoid loss of contact and sparking but the pressure must be as low as possible in order to minimize wear of OH contact wire.

A 'diamond' type single-pan pantograph is shown in Fig. 43.5. It consists of a pentagonal framework of high-tensile alloy-steel tubing. The contact portion consists of a pressed steel pan fitted with renewable copper wearing strips which are forced against the OH contact wire by the upward action of pantograph springs. The pantograph can be raised or lowered from cabin by air cylinders.

#### 43.25. Conductor Rail Equipment

The conductor rails may be divided into three classes depending on the position of the contact surface which may be located at the top, bottom or side of the rail. The top contact rail is adopted universally for 600 V dc electrification. The side contact rail is used for 1200 V dc supply. The under contact rail has the advantage of being protected from snow, sleet and ice.

Fig. 43.6 shows the case when electric supply is collected from the top of an insulated conductor rail *C* (of special high-conductivity steel) running parallel to the track at a distance of 0.3 to 0.4 m from the running rail (*R*) which forms the return path. *L* is the insulator and *W* is the wooden protection used at stations and crossings.

The current is collected from top surface of the rail by flat steel shoes (200 mm × 75 mm), the necessary contact pressure being obtained by gravity. Since it is not always possible to provide conductor rail on the same side of the track, shoes are provided on both sides of the locomotive or train. Moreover two shoes are provided on each side in order to avoid current interruption at points and crossings where there are gaps in the running rail.

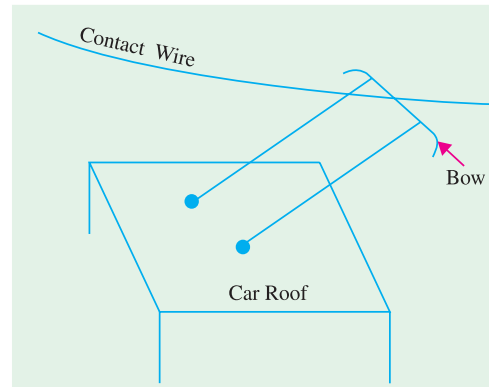


Fig. 43.4

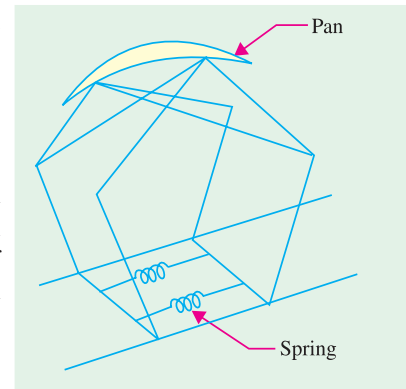


Fig. 43.5



The pantograph mechanism helps to maintain a link between the overhead contact wire and power circuit of the electric locomotive

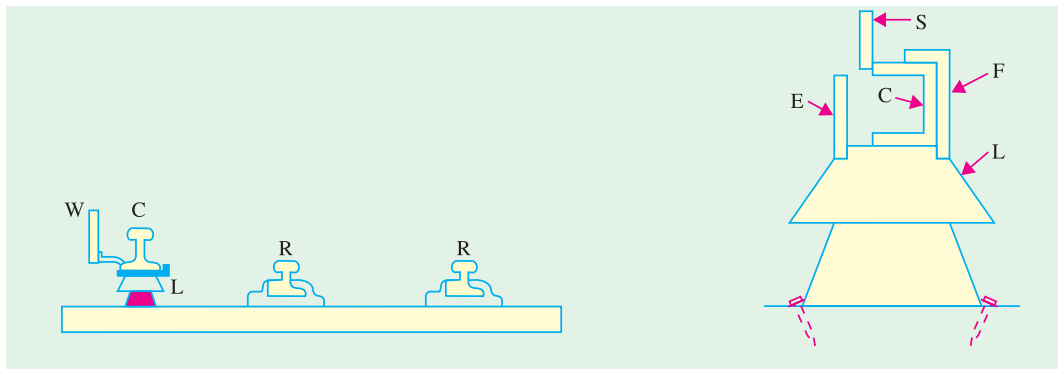


Fig. 43.6

Fig. 43.7

Fig. 43.7 shows the side contact rail and the method of the mounting. The conductor rail (C) rests upon a wooden block recessed into the top of the porcelain insulator L. Current is collected by steel shoes (S) which are kept pressed on the contact rail by springs. E and F are the guards which rest upon ledges on the insulator.

### 43.26. Types of Railway Services

There are three types of passenger services offered by the railways :

1. **City or Urban Service.** In this case, there are frequent stops, the distance between stops being nearly 1 km or less. Hence, high acceleration and retardation are essential to achieve moderately high schedule speed between the stations.
  2. **Suburban Service.** In this case, the distance between stops averages from 3 to 5 km over a distance of 25 to 30 km from the city terminus. Here, also, high rates of acceleration and retardation are necessary.
  3. **Main Line Service.** It involves operation over long routes where stops are infrequent. Here, operating speed is high and accelerating and braking periods are relatively unimportant.
- On goods traffic side also, there are three types of services (i) main-line freight service (ii) local or pick-up freight service and (iii) shunting service.

### 43.27. Train Movement

The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various *times* after the start of the run and the latter gives speed at various *distances* from the starting point. Out of the two, speed/time curve is more important because

1. its slope gives acceleration or retardation as the case may be.
2. area between it and the horizontal (*i.e.* time) axis represents the distance travelled.
3. energy required for propulsion can be calculated if resistance to the motion of train is known.

### 43.28. Typical Speed/Time Curve

Typical speed/time curve for electric trains operating on passenger services is shown in Fig. 43.8. It may be divided into the following **five** parts :

#### 1. Constant Acceleration Period (0 to $t_1$ )

It is also called notching-up or starting period because during this period, starting resistance of the motors is gradually cut out so that the motor current (and hence, tractive effort) is maintained nearly constant which produces constant acceleration alternatively called 'rheostatic acceleration' or 'acceleration while notching'.

## 2. Acceleration on Speed Curve ( $t_1$ to $t_2$ )

This acceleration commences after the starting resistance has been all cut-out at point  $t_1$  and full supply voltage has been applied to the motors. During this period, the motor current and torque decrease as train speed increases. Hence, acceleration gradually *decreases* till torque developed by motors exactly balances that due to resistance to the train motion. The shape of the portion  $AB$  of the speed/time curve depends primarily on the torque/speed characteristics of the traction motors.

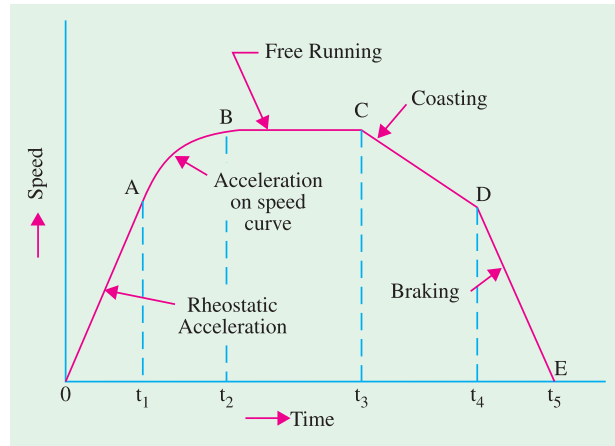


Fig. 43.8

## 3. Free-running Period ( $t_2$ to $t_3$ )

The train continues to run at the speed reached at point  $t_2$ . It is represented by portion  $BC$  in Fig. 43.8 and is a constant-speed period which occurs on level tracks.

## 4. Coasting ( $t_3$ to $t_4$ )

Power to the motors is cut off at point  $t_3$  so that the train runs under its momentum, the speed gradually falling due to friction, windage etc. (portion  $CD$ ). During this period, retardation remains practically constant. Coasting is desirable because it utilizes some of the kinetic energy of the train which would, otherwise, be wasted during braking. Hence, it helps to reduce the energy consumption of the train.

## 5. Braking ( $t_4$ to $t_5$ )

At point  $t_4$ , brakes are applied and the train is brought to rest at point  $t_5$ .

It may be noted that coasting and braking are governed by train resistance and allowable retardation respectively.

## 43.29. Speed/Time Curves for Different Services

Fig. 43.9 (a) is representative of city service where relative values of acceleration and retardation are high in order to achieve moderately high average speed between stops. Due to short

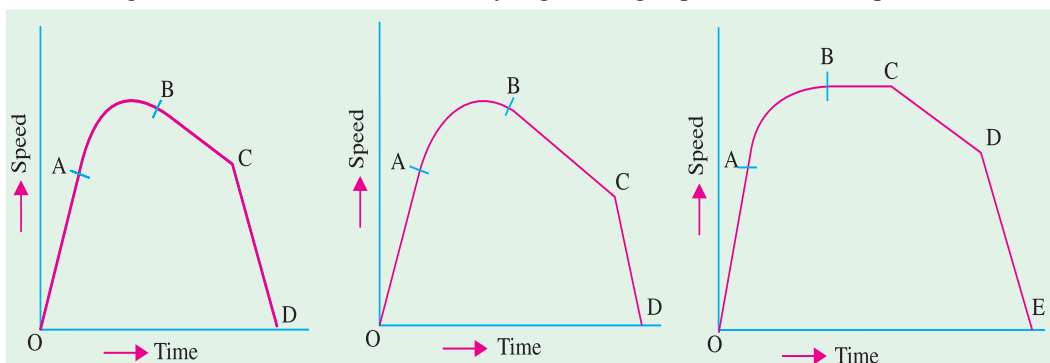


Fig. 43.9

distances between stops, there is no possibility of free-running period though a short coasting period is included to save on energy consumption.

In suburban services [Fig. 43.9 (b)], again there is no free-running period but there is comparatively *longer* coasting period because of longer distances between stops. In this case also, relatively

high values of acceleration and retardation are required in order to make the service as attractive as possible.

For main-line service [Fig. 43.9 (c)], there are long periods of free-running at high speeds. The accelerating and retardation periods are relatively unimportant.

### 43.30. Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 43.8 is replaced by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retardation, average speed and distance etc. The simple curve would be fairly accurate provided it *(i) retains the same acceleration and retardation and (ii) has the same area as the actual speed/time curve*. The simplified speed/time curve can have either of the two shapes :

(i) trapezoidal shape  $OA_1B_1C$  of Fig. 43.10 where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant-speed period.

(ii) quadrilateral shape  $OA_2B_2C$  where the same two periods are replaced by the extensions of initial constant acceleration and coasting periods.

It is found that trapezoidal diagram  $OA_1B_1C$  gives simpler relationships between the principal quantities involved in train movement and also gives closer approximation of actual energy consumed during *main-line service on level track*. On the other hand, quadrilateral diagram approximates more closely to the actual conditions in *city and suburban services*.

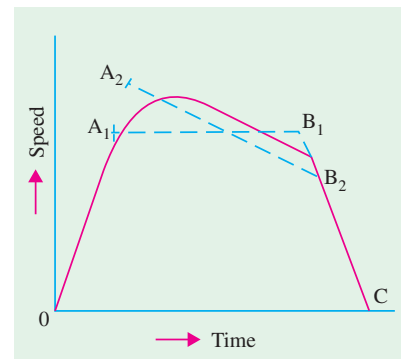


Fig. 43.10

### 43.31. Average and Schedule Speed

While considering train movement, the following three speeds are of importance :

1. **Crest Speed.** It is the maximum speed ( $V_m$ ) attained by a train during the run.

2. **Average Speed** =  $\frac{\text{distance between stops}}{\text{actual time of run}}$

In this case, only running time is considered but *not the stop time*.

3. **Schedule Speed** =  $\frac{\text{distance between stops}}{\text{actual time of run} + \text{stop time}}$

Obviously, schedule speed can be obtained from average speed by including the duration of stops. For a given distance between stations, higher values of acceleration and retardation will mean lesser running time and, consequently, higher schedule speed. Similarly, for a given distance between stations and for fixed values of acceleration and retardation, higher crest speed will result in higher schedule speed. For the same value of average speed, increase in duration of stops decreases the schedule speed.

### 43.32. SI Units in Traction Mechanics

In describing various quantities involved in the mechanics of train movement, only the latest SI system will be used. Since SI system is an 'absolute system', only absolute units will be used while gravitational units (used hitherto) will be discarded.

1. **Force.** It is measured in newton (N)
2. **Mass.** Its unit is kilogram (kg). Commonly used bigger units is tonne ( $t$ ), 1 tonne = 1000 kg



- 3. Energy.** Its basic unit is joule (J). Other units often employed are watt-hour (Wh) and kilowatt-hour (kWh).  

$$1 \text{ Wh} = 1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600 \text{ J} = 3.6 \text{ kJ}$$

$$1 \text{ kWh} = 1000 \times 1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J} = 3.6 \text{ MJ}$$
- 4. Work.** Its unit is the same as that of energy.
- 5. Power.** Its unit is watt (W) which equals 1 J/s. Other units are kilowatt (kW) and megawatt (MW).
- 6. Distance.** Its unit is metre. Other unit often used is kilometre (km).
- 7. Velocity.** Its absolute unit is metre per second (m/s). If velocity is given in km/h (or km.ph), it can be easily converted into the SI unit of m/s by multiplying it with a factor of  $(1000/3600) = 5/18 = 0.2778$ . For example,  $72 \text{ km.ph} = 72 \times 5/18 = 72 \times 0.2778 = 20 \text{ m/s}$ .
- 8. Acceleration.** Its unit is metre/second<sup>2</sup> (m/s<sup>2</sup>). If acceleration is given in km/h/s (or km.ph.ps), then it can be converted into m/s<sup>2</sup> by simply multiplying it by the factor  $(1000/3600) = 5/18 = 0.2778$  *i.e.* the same factor as for velocity. For example,  $1.8 \text{ km.ph.ps} = 1.8 \times 5/18 = 1.8 \times 0.2778 = 0.5 \text{ m/s}^2$

### 43.33. Confusion Regarding Weight and Mass of a Train

Many students often get confused regarding the correct meaning of the terms ‘weight’ and ‘mass’ and their units while solving numericals on train movement particularly when they are not expressed clearly and consistently in their absolute units. It is primarily due to the mixing up of absolute units with gravitational units. There would be no confusion at all if ***we are consistent in using only absolute units*** as required by the SI system of units which disallows the use of gravitational units.

Though this topic was briefly discussed earlier, it is worth repeating here.

- 1. Mass (M).** It is the quantity of matter contained in a body.

Its absolute unit is kilogram (kg). Other multiple in common use is tonne.

- 2. Weight (W).** It is the **force** with which earth pulls a body downwards.

The weight of a body can be expressed in (i) the **absolute** unit of newton (N) or (ii) the **gravitational** unit of kilogram-weight (kg. wt) which is often writing as ‘kgf’ in engineering literature.

Another still bigger **gravitational** unit commonly used in traction work is tonne-weight (t-wt)

$$1 \text{ t-wt} = 1000 \text{ kg-wt} = 1000 \times 9.8 \text{ N} = 9800 \text{ N}$$

#### (i) Absolute Unit of Weight

It is called newton (N) whose definition may be obtained from Newton’s Second Law of Motion.

Commonly used multiple is kilo-newton (kN). Obviously,  $1 \text{ kN} = 1000 \text{ N} = 10^3 \text{ N}$ .

For example, if a mass of 200 kg has to be given an acceleration of  $2.5 \text{ m/s}^2$ , force required is  $F = 200 \times 2.5 = 500 \text{ N}$ .

If a train of mass 500 tonne has to be given an acceleration of  $0.6 \text{ m/s}^2$ , force required is

$$F = ma = (500 \times 1000) \times 0.6 = 300,000 \text{ N} = 300 \text{ kN}$$

#### (ii) Gravitational Unit of Weight

It is ‘g’ times bigger than newton. It is called kilogram-weight (kg.wt.)

$$1 \text{ kg.wt} = g \text{ newton} = 9.81 \text{ N} \approx 9.8 \text{ N}$$

Unfortunately, the word ‘wt’ is usually omitted from kg-wt when expressing the weight of the body on the assumption that it can be understood or inferred from the language used.

Take the statement “a body has a *weight* of 100 kg”. It looks as if the weight of the body has been

expressed in terms of the mass unit 'kg'. To avoid this confusion, statement should be 'a body has a weight of 100 kg. wt.' But the first statement is justified by the writers on the ground that since the word 'weight' has already been used in the statement, it should be automatically understood by the readers that 'kg' is not the 'kg' of mass but is kg-wt. It would be mass kg if the statement is 'a body has a mass of 100 kg'. Often kg-wt is written as 'kgf' where 'f' is the first letter of the word force and is added to distinguish it from kg of mass.

Now, consider the statement "a body *weighing* 500 kg travels with a speed of 36 km/h....."

Now, weight of the body  $W = 500 \text{ kg. wt.} = 500 \times 9.8 \text{ N}$

Since we know the weight of the body, we can find its mass from the relation  $W = mg$ . But while using this equation, it is essential that we must **consistently use the absolute** units only. In this equation,  $W$  must be in newton (not in kg. wt),  $m$  in kg and  $g$  in  $\text{m/s}^2$ .

$$\therefore 500 \times 9.8 = m \times 9.8 ; \quad \therefore m = 500 \text{ kg}$$

It means that a body which *weighs* 500 kg (wt) has a *mass* of 500 kg.

As a practical rule, weight of a body in **gravitational** units is numerically equal to its mass in **absolute** units. This simple fact must be clearly understood to avoid any confusion between weight and mass of a body.

A train which weighs 500 tonne has a mass of 500 tonne as proved below :

$$\text{train weight, } W = 500 \text{ tonne-wt} = 500 \times 1000 \text{ kg-wt} = 500 \times 1000 \times 9.8 \text{ N}$$

$$\text{Now, } W = mg ; \quad \therefore 500 \times 1000 \times 9.8 = m \times 9.8$$

$$\therefore m = 500 \times 1000 \text{ kg} = 500 \times 1000/1000 = 500 \text{ tonne}$$

To avoid this unfortunate confusion, it would be helpful to change our terminology. For example, instead of saying "a train weighing 500 tonne is....." it is better to say "a 500-t train is ....." or "a train having a mass of 500 t is ....."

In order to remove this confusion, SI system of units has disallowed the use of gravitational units. There will be no confusion if **we consistently use only absolute units**.

#### 43.34. Quantities Involved in Traction Mechanics

Following principal quantities are involved in train movement :

$D$ = distance between stops	$M$ = dead mass of the train
$M_e$ = effective mass of the train	$W$ = dead weight of the train
$W_e$ = effective weight of the train	$\alpha$ = acceleration during starting period
$\beta_c$ = retardation during coasting	$\beta$ = retardation during braking
$V_a$ = average speed	$V_m$ = maximum (or crest) speed.
$t$ = total time for the run	$t_1$ = time of acceleration
$t_2$ = time of free running = $t - (t_1 + t_3)$	$t_3$ = time of braking
$F_t$ = tractive effort	$T$ = torque

#### 43.35. Relationship Between Principal Quantities in Trapezoidal Diagram

As seen from Fig. 43.11.

$$\alpha = V_m/t_1 \quad \text{or} \quad t_1 = V_m/\alpha$$

$$\beta = V_m/t_3 \quad \text{or} \quad t_3 = V_m/\beta$$

As we know, total distance  $D$  between the two stops is given by the area of trapezium  $OABC$ .

$$\begin{aligned} \therefore D &= \text{area } OABC \\ &= \text{area } OAD + \text{area } ABED + \text{area } BCE \\ &= \frac{1}{2} V_m t_1 + V_m t_2 + \frac{1}{2} V_m t_3 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} V_m t_1 + V_m [t - (t_1 + t_3)] + \frac{1}{2} V_m t_3 \\
 &= V_m \left[ \frac{t_1}{2} + t - t_1 - t_3 + \frac{t_3}{2} \right] \\
 &= V_m \left[ t - \frac{1}{2} (t_1 + t_3) \right] \\
 &= V_m \left[ t - \frac{V_m}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \right]
 \end{aligned}$$

Let,  $K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)$ . Substituting this value

of  $K$  in the above equation, we get

$$D = V_m (t - KV_m) \quad \dots (i)$$

$$\text{or } KV_m^2 - V_m t + D = 0$$

$$\therefore V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

Rejecting the positive sign which gives impracticable value, we get

$$V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

From Eq. (i) above, we get

$$KV_m^2 = V_m t - D \quad \text{or} \quad K = \frac{t}{V_m} - \frac{D}{V_m^2} = \frac{D}{V_m^2} \left( V_m \cdot \frac{t}{D} - 1 \right)$$

$$\text{Now, } V_a = \frac{D}{t} \quad \therefore K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right)$$

Obviously, if  $V_m$ ,  $V_a$  and  $D$  are given, then value of  $K$  and hence of  $\alpha$  and  $\beta$  can be found (Ex. 43.2).

### 43.36. Relationship Between Principal Quantities in Quadrilateral Diagram

The diagram is shown in Fig. 43.12. Let  $\beta_c$  represent the retardation during coasting period. As before,

$$t_1 = V_1/\alpha, t_2 = (V_2 - V_1)/\beta_c \text{ and } t_3 = V_2/\beta$$

$$D = \text{area } OABC$$

$$= \text{area } OAD + \text{area } ABED + \text{area } BCE$$

$$= \frac{1}{2} V_1 t_1 + t_2 \left( \frac{V_1 + V_2}{2} \right) + \frac{1}{2} V_2 t_3$$

$$= \frac{1}{2} V_1 (t_1 + t_2) + \frac{1}{2} V_2 (t_2 + t_3)$$

$$= \frac{1}{2} V_1 (t - t_3) + \frac{1}{2} V_2 (t - t_1)$$

$$= \frac{1}{2} t (V_1 + V_2) - \frac{V_1 t_1}{2} - \frac{V_2 t_3}{2}$$

$$= \frac{1}{2} t (V_1 + V_2) - \frac{1}{2} V_1 V_2 \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)$$

$$= \frac{1}{2} t (V_1 + V_2) - KV_1 V_2$$

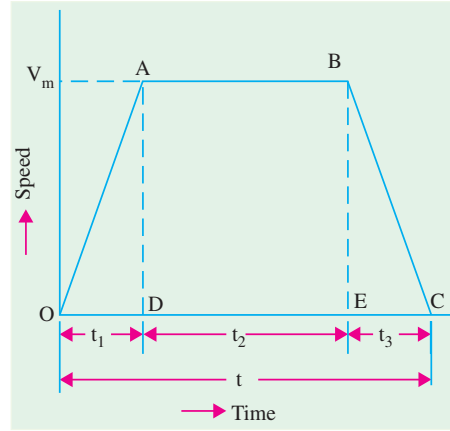


Fig. 43.11

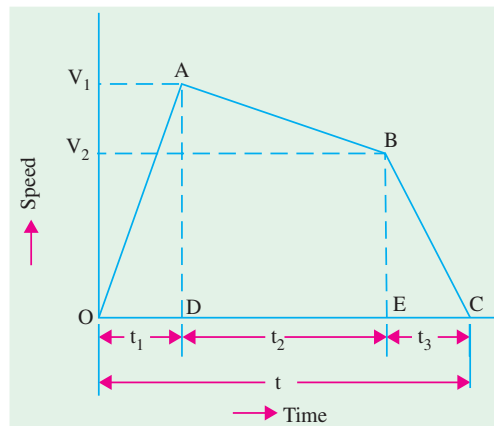


Fig. 43.12

where  $K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) = \frac{\alpha + \beta}{2\alpha\beta}$  Also,  $\beta_c = \frac{(V_1 - V_2)}{t_2}$

$$\therefore V_2 = V_1 - \beta_c t_2 = V_1 - \beta_c (t - t_1 - t_3)$$

$$= V_1 - \beta_c \left( t - \frac{V_1}{\alpha} - \frac{V_2}{\beta} \right) = V_1 \beta_c \left( t - \frac{V_1}{\alpha} \right) + \beta_c \frac{V_2}{\beta}$$

or  $V_2 \left( 1 - \frac{\beta_c}{\beta} \right) = V_1 - \beta_c \left( t - \frac{V_1}{\alpha} \right) \quad \therefore V_2 = \frac{V_1 - \beta_c (t - V_1/\alpha)}{(1 - \beta_c/\beta)}$

**Example 43.1.** A suburban train runs with an average speed of 36 km/h between two stations 2 km apart. Values of acceleration and retardation are 1.8 km/h/s and 3.6 km/h/s.

Compute the maximum speed of the train assuming trapezoidal speed/time curve.

(Electric Traction, Punjab Univ. 1994)

**Solution.** Now,  $V_a = 36 \text{ km/h} = 36 \times 5/18 = 10 \text{ m/s}$

$$\alpha = 1.8 \text{ km/h/s} = 1.8 \times 5/18 = 0.5 \text{ m/s}^2, \beta = 3.6 \text{ km/h/s} = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$$

$$t = D/V_a = 2000/10 = 200 \text{ s}; K = (\alpha + \beta)/2\alpha\beta = (0.5 + 1.0)/2 \times 0.5 \times 1 = 1.5$$

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{200 - \sqrt{200^2 - 4 \times 1.5 \times 2000}}{2 \times 1.5}$$

$$= 11 \text{ m/s} = 11 \times 18/5$$

$$= 39.6 \text{ km/h}$$

**Example 43.2.** A train is required to run between two stations 1.5 km apart at a schedule speed of 36 km/h, the duration of stops being 25 seconds. The braking retardation is 3 km/h/s. Assuming a trapezoidal speed/time curve, calculate the acceleration if the ratio of maximum speed to average speed is to be 1.25

(Elect. Power, Bombay Univ. 1980)

**Solution.** Here,  $D = 1500 \text{ m}$  ;  
schedule speed = 36 km/h =  $36 \times 5/18 = 10 \text{ m/s}$

$$\beta = 3 \text{ km/h/s} = 3 \times 5/18 = 5/6 \text{ m/s}^2$$

$$\text{Schedule time of run} = 1500/10 = 150 \text{ s}; \text{ Actual time of run} = 150 - 25 = 125 \text{ s}$$

$$\therefore V_a = 1500/125 = 12 \text{ m/s}; V_m = 1.25 \times 12 = 15 \text{ m/s}$$

$$\text{Now, } K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right) = \frac{1500}{15^2} (1.25 - 1) = \frac{5}{3}$$

$$\text{Also, } K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \quad \text{or} \quad \frac{5}{3} = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{6}{5} \right)$$

$$\therefore \alpha = 0.47 \text{ m/s}^2 = 0.47 \times 18/5 = 1.7 \text{ km/h/s}$$

**Example 43.3.** Find the schedule speed of an electric train for a run of 1.5 km if the ratio of its maximum to average speed is 1.25. It has a braking retardation of 3.6 km/h/s, acceleration of 1.8 km/h/s and stop time of 21 second. Assume trapezoidal speed/time curve.



Electric traction provides high starting torque and low maintenance costs making it the best choice for trains

**Solution.**  $\alpha = 1.8 \times 5/18 = 0.5 \text{ m/s}^2$ ,  $\beta = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$   
 $D = 1.5 \text{ km} = 1500 \text{ m}$   
 $K = \frac{1}{2} \left( \frac{1}{0.5} + \frac{1}{1} \right) = \frac{3}{2}$  Now,  $K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right)$   
or  $V_m^2 = \frac{D}{K} \left( \frac{V_m}{V_a} - 1 \right)$   $\therefore V_m^2 = \frac{1500}{3/2} (1.25 - 1) = 250$ ;  $V_m = 15.8 \text{ m/s}$   
 $V_a = V_m / 1.25 = 15.8 / 1.25 = 12.6 \text{ m/s}$   
Actual time of run  $= 1500 / 12.6 = 119 \text{ seconds}$   
Schedule time  $= 119 + 21 = 140 \text{ second}$   
 $\therefore$  Schedule speed  $= 1500 / 140 = 10.7 \text{ m/s} = \mathbf{38.5 \text{ km/h}}$

**Example 43.4.** A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.

Assume a simplified speed/time curve.

**Solution.** Given :  $D = 1.6 \text{ km} = 1600 \text{ m}$ ,  $V_a = 36 \text{ km/h} = 10 \text{ m/s}$   
 $V_1 = 72 \text{ km/h} = 20 \text{ m/s}$ ;  $\alpha = 2.7 \text{ km/h/s} = 0.75 \text{ m/s}^2$   
 $\beta_c = 0.18 \text{ km/h/s} = 0.05 \text{ m/s}^2$ ;  $\beta = 3.6 \text{ km/h/s} = 1.0 \text{ m/s}^2$

With reference to Fig. 43.12, we have

Duration of acceleration,  $t_1 = V_1 / \alpha = 20 / 0.75 = 27 \text{ s}$   
Actual time of run,  $t = 1600 / 10 = \mathbf{160 \text{ s}}$   
Duration of braking,  $t_3 = V_2 / 1.0 = V_2 \text{ second}$   
Duration of coasting,  $t_2 = (V_1 - V_2) / \beta_c = (20 - V_2) / 0.05 = (400 - 20 V_2) \text{ second}$   
Now,  $t = t_1 + t_2 + t_3$  or  $160 = 27 + (400 - 20 V_2) + V_2$   $\therefore V_2 = 14 \text{ m/s}$   
 $\therefore t_2 = (20 - 14) / 0.05 = 120 \text{ s}$ ;  $t_3 = 14 / 1.0 = \mathbf{14 \text{ s}}$

### 43.37. Tractive Effort for Propulsion of a Train

The tractive effort ( $F_t$ ) is the force developed by the traction unit at the rim of the driving wheels for moving the unit itself and its train (trailing load). The tractive effort required for train propulsion on a level track is

$$F_t = F_a + F_r$$

If gradients are involved, the above expression becomes

$$\begin{aligned} F_t &= F_a + F_g + F_r && \text{— for ascending gradient} \\ &= F_a - F_g + F_r && \text{— for descending gradient} \end{aligned}$$

where

$$\begin{aligned} F_a &= \text{force required for giving linear acceleration to the train} \\ F_g &= \text{force required to overcome the effect of gravity} \\ F_r &= \text{force required to overcome resistance to train motion.} \end{aligned}$$

#### (a) Value of $F_a$

If  $M$  is the dead (or stationary) mass of the train and  $a$  its linear acceleration, then

$$F_a = Ma$$

Since a train has rotating parts like wheels, axles, motor armatures and gearing etc., its *effective* (or accelerating) mass  $M_e$  is more (about 8 – 15%) than its stationary mass. These parts have to be given angular acceleration at the same time as the whole train is accelerated in the linear direction. Hence,  $F_e = M_e a$

(i) If  $M_e$  is in kg and  $\alpha$  in  $\text{m/s}^2$ , then  $F_a = M_e a$  newton

(ii) If  $M_e$  is in tonne and  $\alpha$  in  $\text{km/h/s}$ , then converting them into absolute units, we have

$$F_a = (1000 M_e) \times (1000/3600) a = 277.8 M_e a \text{ newton}$$

**(b) Value of  $F_g$**

As seen from Fig. 43.13,  $F_g = W \sin \theta = Mg \sin \theta$

In railway practice, gradient is expressed as the rise (in metres) a track distance of 100 m and is called percentage gradient.

$$\therefore \% G = \frac{BC}{AC/100} = 100 \frac{BC}{AC} = 100 \sin \theta$$

Substituting the value of  $\sin \theta$  in the above equation, we get

$$F_g = Mg G/100 = 9.8 \times 10^{-2} MG$$

(i) When  $M$  is in kg,  $F_g = 9.8 \times 10^{-2} MG$  newton

(ii) When  $M$  is given in tonne, then

$$F_g = 9.8 \times 10^{-2} (1000 M) G = 98 MG \text{ newton}$$

**(c) Value of  $F_r$**

Train resistance comprises all those forces which oppose its motion. It consists of mechanical resistance and wind resistance. Mechanical resistance itself is made up of internal and external resistances. The internal resistance comprises friction at journals, axles, guides and buffers etc. The external resistance consists of friction between wheels and rails and flange friction etc. Mechanical resistance is almost independent of train speed but depends on its weight. The wind friction varies directly as the square of the train speed.

If  $r$  is specific resistance of the train *i.e.* resistance offered per unit mass of the train, then  $F_r = M.r$ .

(i) If  $r$  is in newton per kg of train mass and  $M$  is the train mass in kg, then

$$F_r = M.r \text{ newton}$$

(ii) If  $r$  is in newton per tonne train mass (N/t) and  $M$  is in tonne ( $t$ ), then

$$F_r = M \text{ tonne} \times r = M_r \text{ newton}^*$$

Hence, expression for total tractive effort becomes

$$F_t = F_a \pm F_g + F_r = (277.8 \alpha M_e \pm 98 MG + Mr) \text{ newton}$$

Please remember that here  $M$  is in tonne,  $\alpha$  in  $\text{km/h/s}$ ,  $G$  is in metres per 100 m of track length (*i.e.* %  $G$ ) and  $r$  is in newton/tonne (N/t) of train mass.

The positive sign for  $F_g$  is taken when motion is along an ascending gradient and negative sign when motion is along a descending gradient.

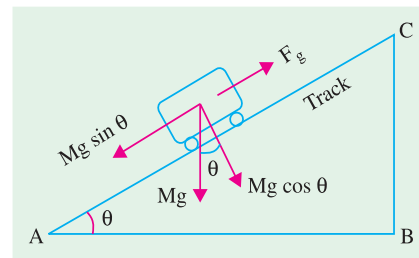


Fig. 43.13

### 43.38. Power Output from Driving Axles

If  $F_t$  is the tractive effort and  $v$  is the train velocity, then

$$\text{output power} = F_t \times v$$

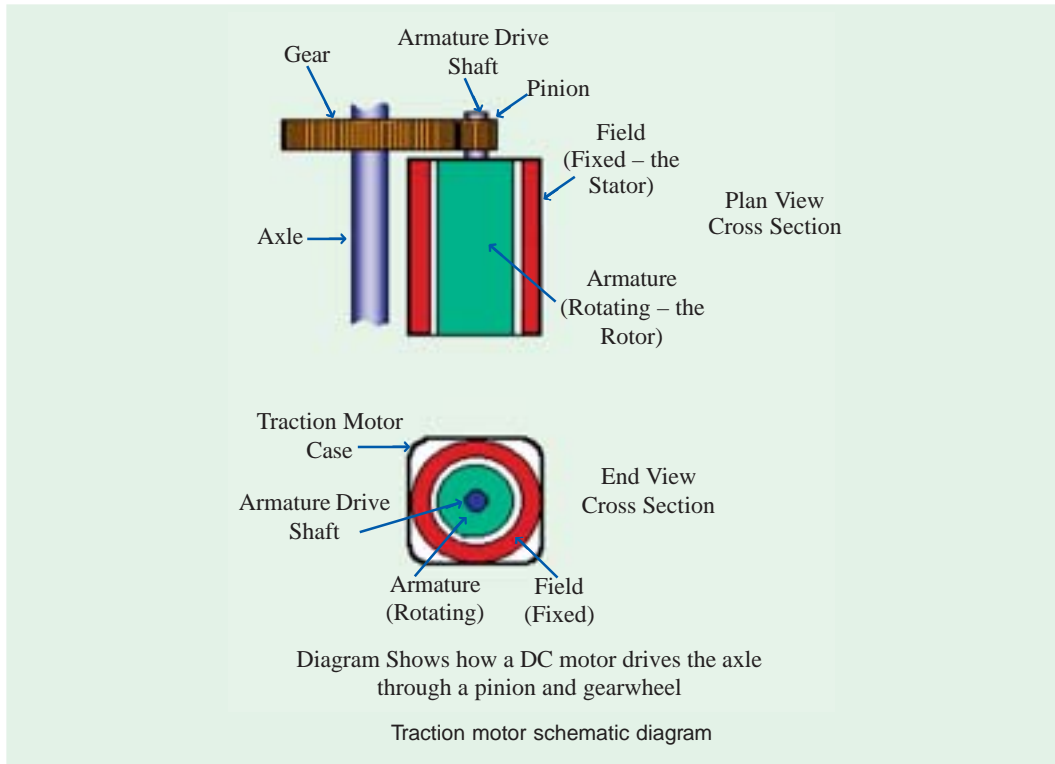
(i) If  $F_t$  is in newton and  $v$  in  $\text{m/s}$ , then

$$\text{output power} = F_t \times v \text{ watt}$$

(ii) If  $F_t$  is in newton and  $v$  is in  $\text{km/h}$ , then converting  $v$  into  $\text{m/s}$ , we have

\* If  $r$  is in kg (wt) per tonne train mass and  $M$  is in tonne, then  $F_r = M \text{ tonne} \times (r \times 9.8) \text{ newton/tonne} = 9.8 Mr \text{ newton}$ .





$$\text{output power} = F_t \times \left( \frac{1000}{3600} \right) v \text{ watt} = \frac{F_t v}{3600} \text{ kW}$$

If  $\eta$  is the efficiency of transmission gear, then power output of motors is

$$= F_t \cdot v / \eta \text{ watt} \quad \text{--- } v \text{ in m/s}$$

$$= \frac{F_t v}{3600 \eta} \text{ kW} \quad \text{--- } v \text{ in km/h}$$

### 43.39. Energy Output from Driving Axles

Energy (like work) is given by the product of power and time.

$$E = (F_t \times v) \times t = F_t \times (v \times t) = F_t \times D$$

where  $D$  is the distance travelled in the direction of tractive effort.

Total energy output from driving axles for the run is

$$E = \text{energy during acceleration} + \text{energy during free run}$$

As seen from Fig. 43.11

$$E = F_t \times \text{area } OAD + F'_t \times \text{area } ABED = F_t \times \frac{1}{2} V_m t_1 + F'_t \times \frac{1}{2} V_m t_2$$

where  $F_t$  is the tractive effort during accelerating period and  $F'_t$  that during free-running period. Incidentally,  $F_t$  will consist of all the three components given in Art. 43.37 whereas  $F'_t$  will consist of  $(98 MG + Mr)$  provided there is an ascending gradient.

### 43.40. Specific Energy Output

It is the energy output of the driving wheel expressed in watt-hour (Wh) per tonne-km ( $t$ -km) of

the train. It can be found by first converting the energy output into Wh and then dividing it by the mass of the train in tonne and route distance in km.

Hence, unit of specific energy output generally used in railway work is : Wh/tonne-km (Wh/t-km).

#### 43.41. Evaluation of Specific Energy Output

We will first calculate the total energy output of the driving axles and then divide it by train mass in tonne and route length in km to find the specific energy output. It will be presumed that :

- (i) there is a gradient of  $G$  throughout the run and
- (ii) power remains ON upto the end of free run in the case of trapezoidal curve (Fig. 43.11) and upto the accelerating period in the case of quadrilateral curve (Fig. 43.12).

Now, output of the driving axles is used for the following purposes :

1. for accelerating the train
2. for overcoming the gradient
3. for overcoming train resistance.

##### (a) Energy required for train acceleration ( $E_a$ )

As seen from trapezoidal diagram of Fig. 43.11,

$$\begin{aligned} E_a &= F_a \times \text{distance } OAD = 277.8 \alpha M_e \times \frac{1}{2} V_m t_1 \text{ joules} \\ &= 277.8 \alpha M_e \times \frac{1}{2} V_m \times \frac{V_m}{\alpha} \text{ joules} \quad \left( \text{as } t_1 = \frac{V_m}{\alpha} \right) \\ &= 277.8 \alpha M_e \times \left[ \frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha} \right] \text{ joules} \end{aligned}$$

It will be seen that since  $V_m$  is in km/h, it has been converted into m/s by multiplying it with the conversion factor of (1000/3600). In the case of ( $V_m/t$ ), conversion factors for  $V_m$  and  $\alpha$  being the same, they cancel out. Since 1 Wh = 3600 J.

$$\therefore E_a = 277.8 \alpha M_e \left[ \frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha} \right] \text{ Wh} = 0.01072 \frac{V_m^2}{M_e} \text{ Wh}$$

##### (b) Energy required for over coming gradient ( $E_g$ )

$$E_g = F_g \times D'$$

where ' $D'$ ' is the **total distance over which power remains ON**. Its maximum value equals the distance represented by the area  $OABE$  in Fig. 43.11 *i.e.* from the start to the end of free-running period in the case of trapezoidal curve [as per assumption (i) above].

Substituting the value of  $F_g$  from Art. 43.37, we get

$$E_g = 98 \text{ MG} \cdot (1000 D') \text{ joules} = 98,000 \text{ MGD}' \text{ joules}$$

It has been assumed that  $D'$  is in km.

When expressed in Wh, it becomes

$$E_g = 98,000 \text{ MGD}' \cdot \frac{1}{3600} \text{ Wh} = 27.25 \text{ MGD}' \text{ Wh}$$

##### (c) Energy required for overcoming resistance ( $E_r$ )

$$\begin{aligned} E_r &= F_r \times D' = M \cdot r \times (1000 D') \text{ joules} && \text{--- } D' \text{ in km} \\ &= \frac{1000 Mr D'}{3600} \text{ Wh} = 0.2778 Mr D' \text{ Wh} && \text{--- } D' \text{ in km} \end{aligned}$$

$\therefore$  total energy output of the driving axles is

$$\begin{aligned}
 E &= E_a + E_g + E_r \\
 &= (0.01072 V_m^2 / M_e + 27.25 MGD' + 0.2778 Mr D' \text{ Wh}
 \end{aligned}$$

Specific energy output

$$\begin{aligned}
 E_{spo} &= \frac{E}{M \times D} \quad \text{--- } D \text{ is the total run length} \\
 &= \left( 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 27.25 G \frac{D'}{D} + 0.2778 r \frac{D'}{D} \right) \text{ Wh/t-km}
 \end{aligned}$$

It may be noted that if there is no gradient, then

$$E_{spo} = \left( 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778 r \frac{D'}{D} \right) \text{ Wh/t-km}$$

### Alternative Method

As before, we will consider the trapezoidal speed/time curve. Now, we will calculate energy output not **force-wise** but **period-wise**.

#### (i) Energy output during accelerating period

$$\begin{aligned}
 E_a &= F_t \times \text{distance travelled during accelerating period} \\
 &= F_t \times \text{area } OAD \quad \text{---Fig. 43.11} \\
 &= F_t \times \frac{1}{2} V_m t_1 = \frac{1}{2} F_t \cdot V_m \cdot \frac{V_m}{\alpha} \\
 &= \frac{1}{2} \cdot F_t \left( \frac{1000}{3600} \cdot V_m \right) \cdot \frac{V_m}{\alpha} \text{ joules} \\
 &= \frac{1}{2} \cdot F_t \left( \frac{1000}{3600} \cdot V_m \right) \cdot \frac{V_m}{\alpha} \cdot \frac{1}{3600} \text{ Wh}
 \end{aligned}$$

Substituting the value of  $F_t$ , we get

$$E_a = \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (277.8 \alpha M_e + 98 MG + Mr) \text{ Wh}$$

It must be remembered that during this period, **all the three forces are at work** (Art. 43.37)

#### (ii) Energy output during free-running period

Here, work is required only against two forces *i.e.* gravity and resistance (as mentioned earlier).

Energy  $E_{fr} = F'_t \times \text{area } ABED$  ---Fig. 43.11

$$\begin{aligned}
 &= F'_t \times (V_m \times t_2) = F'_t \times \left( \frac{1000}{3600} V_m \right) \cdot t_2 \text{ joules} \\
 &= F'_t \times \left( \frac{1000}{3600} V_m \right) \times t_2 \times \frac{1}{3600} \text{ Wh} = \left( \frac{1000}{3600} \right) F'_t \times V_m t_2 \cdot \frac{1}{3600} \text{ Wh} \\
 &= \left( \frac{1000}{3600} \right) \cdot F'_t \times D_{fr} \text{ Wh} = \left( \frac{1000}{3600} \right) (98 MG + Mr) D_{fr} \text{ Wh}
 \end{aligned}$$

where  $D_{fr}$  is the distance in km travelled during the free-running period\*

Total energy required is the sum of the above two energies.

$$\begin{aligned}
 \therefore E &= E_a + E_{fr} \\
 &= \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (277.8 \alpha M_e + 98 MG + Mr) + \frac{1000}{3600} (98 MG + Mr) D_{fr} \text{ Wh}
 \end{aligned}$$

---

\*  $D_{fr} = \text{velocity in km/h} \times \text{time in hours}$   
 $= V_m \times (t_2 / 3600)$  because times are always taken in seconds.

$$\begin{aligned}
&= \frac{1000}{(3600)^2} \frac{V_m^2}{2\alpha} 277.8 \alpha M_e + \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (98 MG + Mr) + \frac{1000}{3600} (98 MG + Mr) \cdot D_{fr} \text{ Wh} \\
&= 0.01072 V_m^2 \cdot M_e + \frac{1000}{3600} (98 MG + Mr) \left( \frac{V_m^2}{2\alpha \times 3600} + D_{fr} \right) \text{ Wh}
\end{aligned}$$

$$\begin{aligned}
\text{Now,} \quad \frac{V_m^2}{2\alpha \times 3600} &= \frac{1}{2} \left( \frac{V_m}{3600} \right) \cdot \frac{V_m}{\alpha} = \frac{1}{2} \left( \frac{V_m}{3600} \right) \cdot t_1 \\
&= \text{distance travelled during accelerating period i.e. } D_a
\end{aligned}$$

$$\begin{aligned}
\therefore E &= 0.01072 V_m^2 \cdot M_e + \frac{1000}{3600} (98 MG + Mr) (D_a + D_{fr}) \text{ Wh} \\
&= 0.01072 V_m^2 \cdot M_e + (27.25 MG + 0.2778 Mr) D' \text{ Wh}
\end{aligned}$$

It is the same expression as found above.

### 43.42. Energy Consumption

It equals the total energy input to the traction motors from the supply. It is usually expressed in Wh which equals 3600 J. It can be found by dividing the energy output of the driving wheels with the combined efficiency of transmission gear and motor.

$$\therefore \text{energy consumption} = \frac{\text{output of driving axles}}{\eta_{\text{motor}} \times \eta_{\text{gear}}}$$

### 43.43. Specific Energy Consumption

It is the energy consumed (in Wh) per tonne mass of the train per km length of the run. Specific energy consumption,

$$E_{\text{spc}} = \frac{\text{total energy consumed in Wh}}{\text{train mass in tonne} \times \text{run length in km}} = \frac{\text{specific energy output}}{\eta}$$

where  $\eta$  = overall efficiency of transmission gear and motor =  $\eta_{\text{gear}} \times \eta_{\text{motor}}$

As seen from Art. 43.41, specific energy consumption is

$$E_{\text{spc}} = \left( 0.01072 \cdot \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km}$$

If no gradient is involved, then specific energy consumption is

$$E_{\text{spc}} = \left( 0.01072 \cdot \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km}$$

The specific energy consumption of a train running at a given schedule speed is influenced by

1. Distance between stops 2. Acceleration 3. Retardation 4. Maximum speed 5. Type of train and equipment 6. Track configuration.

### 43.44. Adhesive Weight

It is given by the total weight carried on the driving wheels. Its value is  $W_a = x W$ , where  $W$  is dead weight and  $x$  is a fraction varying from 0.6 to 0.8.

### 43.45. Coefficient of Adhesion

Adhesion between two bodies is due to interlocking of the irregularities of their surfaces in contact. The adhesive weight of a train is *equal to the total weight to be carried on the driving*

**wheels.** It is less than the dead weight by about 20 to 40%.

$$\text{If } x = \frac{\text{adhesive weight, } W_a}{\text{dead weight } W}, \text{ then, } W_a = x W$$

$$\begin{aligned} \text{Let, } F_t &= \text{tractive effort to slip the wheels} \\ &\text{or} \\ &= \text{maximum tractive effort possible without wheel slip} \end{aligned}$$

$$\text{Coefficient of adhesion, } \mu_a = F_t / W_a$$

$$\therefore F_t = \mu_a W_a = \mu_a x W = \mu_a x M g$$

If  $M$  is in tonne, then

$$F_t = 1000 \times 9.8 \times \mu_a M = 9800 \mu_a x M \text{ newton}$$

It has been found that tractive effort can be increased by increasing the motor torque but only upto a certain point. Beyond this point, any increase in motor torque does not increase the tractive effort but merely causes the driving wheels to slip. It is seen from the above relation that for increasing  $F_t$ , it is not enough to increase the kW rating of the traction motors alone but the weight on the driving wheels has also to be increased.

Adhesion also plays an important role in braking. If braking effort exceeds the adhesive weight of the vehicle, skidding takes place.

#### 43.46. Mechanism of Train Movement

The essentials of driving mechanism in an electric vehicle are illustrated in Fig. 43.14. The armature of the driving motor has a pinion which meshes with the gear wheel keyed to the axle of the driving wheel. In this way, motor torque is transferred to the wheel through the gear.

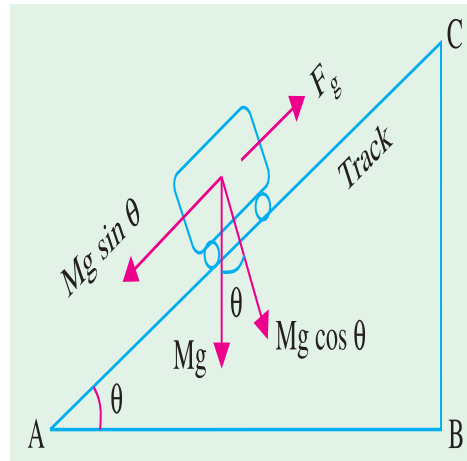


Fig. 43.14

Let,  $T$  = torque exerted by the motor

$F_1$  = tractive effort at the pinion

$F_t$  = tractive effort at the wheel

$\gamma$  = gear ratio

Here,  $d_1, d_2$  = diameters of the pinion and gear wheel respectively

$D$  = diameter of the driving wheel

$\eta$  = efficiency of power transmission from the motor to driving axle

Now,  $T = F_1 \times d_1/2$  or  $F_1 = 2T/d_1$

Tractive effort transferred to the driving wheel is

$$F_t = \eta F_1 \left( \frac{d_2}{D} \right) = \eta \cdot \frac{2T}{d_1} \left( \frac{d_2}{D} \right) = \eta T \left( \frac{2}{D} \right) \left( \frac{d_2}{d_1} \right) = 2 \gamma \eta \frac{T}{D}$$

For obtaining motion of the train without slipping,  $F_t \leq \mu_a W_a$  where  $\mu_a$  is the coefficient of adhesion (Art. 43.45) and  $W_a$  is the adhesive weight.

**Example 43.5.** The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.

**Solution.** Maximum number of revolutions per second made by armature

$$= \frac{\text{armature velocity}}{\text{armature circumference}} = \frac{44}{0.42 \pi} = \frac{100}{3} \text{ rps.}$$

Maximum number of revolutions per second made by the driving wheel

$$= \frac{100}{3} \times \frac{18}{75} = 8 \text{ rps.}$$

Maximum distance travelled by the driving wheel in one second

$$= 8 \times 0.91 \pi \text{ m/s} = 22.88 \text{ m/s}$$

Hence, limiting value of train speed

$$= 22.88 \text{ m/s} = 22.88 \times 18/5 = \mathbf{82 \text{ km/h}}$$

**Example 43.6.** A 250-tonne motor coach driven by four motors takes 20 seconds to attain a speed of 42 km/h, starting from rest on an ascending gradient of 1 in 80. The gear ratio is 3.5, gear efficiency 92%, wheel diameter 92 cm train resistance 40 N/t and rotational inertia 10 percent of the dead weight. Find the torque developed by each motor.

**Solution.**  $F_t = (277.8 \times M_e a + 98 MG + Mr)$  newton

Now,  $\alpha = V_m/t_1 = 42/20 = 2.1 \text{ km/h/s}$  Since gradient is 1 in 80, it becomes 1.25 in 100. Hence, percentage gradient  $G = 1.25$ . Also,  $M_e = 1.1 M$ . The tractive effort at the driving wheel is

$$\begin{aligned} F_t &= 277.8 \times (1.1 \times 250) \times 2.1 + 98 \times 250 \times 1.25 + 250 \times 40 \\ &= 160,430 + 30,625 + 10,000 = 201,055 \text{ N} \end{aligned}$$

Now,  $F_t = 2\gamma\eta T/D$  or  $201,055 = 2 \times 3.5 \times 0.92 \times T/0.92 \therefore T = 28,744 \text{ N-m}$

Torque developed by each motor =  $28,744/4 = \mathbf{7,186 \text{ N-m}}$

**Example 43.7.** A 250-tonne motor coach having 4 motors, each developing a torque of 8000 N-m during acceleration, starts from rest. If up-gradient is 30 in 1000, gear ratio 3.5, gear transmission efficiency 90%, wheel diameter 90 cm, train resistance 50 N/t, rotational inertia effect 10%, compute the time taken by the coach to attain a speed of 80 km/h.

If supply voltage is 3000 V and motor efficiency 85%, calculate the current taken during the acceleration period.

**Solution.** Tractive effort (Art. 43.46) at the wheel

$$= 2\gamma\eta T/D = 2 \times 3.5 \times 0.9 \times (8000 \times 4)/0.9 = 224,000 \text{ N}$$

Also,

$$\begin{aligned} F_t &= (277.8 a M_e + 98 MG + Mr) \text{ newton} \\ &= (277.8 \times (1.1 \times 250) \times a + 98 \times 250 \times 3 + 250 \times 50 \text{ N}) \\ &= (76,395 a + 86,000) \text{ N} \end{aligned}$$

Equating the two expression for tractive effort, we get

$$224,000 = 76,395 a + 86,000 ; a = 1.8 \text{ km/h/s}$$

Time taken to achieve a speed of 80 km/h is

$$t_1 = V_m/a = 80/1.8 = \mathbf{44.4 \text{ second}}$$

Power taken by motors (Art. 41.36) is

$$\begin{aligned} &= \frac{F_t \times v}{\eta} = \frac{F_t \times V_m}{\eta} = F_t \cdot \left( \frac{1000}{3600} \right) \cdot \frac{V_m}{\eta} \text{ watt} \\ &= 22,000 \times 0.2778 \times 80/0.85 = 58.56 \times 10^5 \text{ W} \end{aligned}$$

$$\text{Total current drawn} = 58.56 \times 10^5/3000 = 1952 \text{ A}$$

$$\text{Current drawn/motor} = 1952/4 = \mathbf{488 \text{ A.}}$$



**Example 43.8.** A goods train weighing 500 tonne is to be hauled by a locomotive up an ascending gradient of 2% with an acceleration of 1 km/h/s. If coefficient of adhesion is 0.25, train resistance 40 N/t and effect of rotational inertia 10%, find the weight of locomotive and number of axles if load is not to increase beyond 21 tonne/axle.

**Solution.** It should be clearly understood that a train weighing 500 tonne has a mass of 500 (Art. 43.33).

Tractive effort required is

$$F_t = (277.8 a M_e + 98 MG + Mr) \text{ newton} = M \left( 277.8 a \cdot \frac{M_e}{M} + 98 G + r \right) \text{ newton}$$

$$= M (277.8 \times 1 \times 1.1 + 98 \times 2 + 40) = 541.6 M \text{ newton}$$

If  $M_L$  is the mass of the locomotive, then

$$F_t = 541.6 (M + M_L) = 541.6 (500 + M_L) \text{ newton}$$

Maximum tractive effort (Art. 43.45) is given by

$$F_t = 1000 \mu_a M_L \cdot g = 1000 \times 0.25 M_L \times 9.8 \quad \text{--- } x = 1$$

$$\therefore 541.6 (500 + M_L) = 1000 \times 0.25 M_L \times 9.8 \quad \therefore M_L = \mathbf{142 \text{ tonne}}$$

Hence, weight of the locomotive is 142 tonne. Since, weight per axle is not to exceed 21 tonne, the number of axles required is  $= 142/21 = 7$ .

**Example 43.9.** An electric locomotive weighing 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an up-gradient of 0.1%. Train resistance is 45 N/t and rotational inertia is 10%. If this locomotive is helped by another locomotive of weight 120 tonne, find :

- (i) the trailing weight that can now be hauled up the same gradient under the same conditions.
- (ii) the maximum gradient, if the trailing hauled load remains unchanged.

Assume adhesive weight expressed as percentage of total dead weight as 0.8 for both locomotives. (Utilization of Elect. Power ; AMIE, Summer)

**Solution.** Dead weight of the train and locomotive combined  $= (100 + 500) = 600$  tonne. Same is the value of the dead mass.

$$\therefore F_t = (277.8 a M_e + 98 MG + Mr) \text{ newton}$$

$$= 277.8 \times 1 \times (1.1 \times 600) + 98 \times 600 \times 0.1 + 600 \times 45 = 216,228 \text{ N}$$

Maximum tractive effort (Art. 43.45) of the first locomotive

$$= 9800 x \mu_a M_L = 9800 \times 0.8 \times \mu_a \times 1000 = 784,000 \mu_a$$

$$\therefore 784,000 \mu_a = 216,288 ; \quad \mu_a = 0.276$$

With two locomotives,  $M_L' = (100 + 120) = 220$  tonne

$$\therefore F_t = 9800 x \mu_a M_L' = 9800 \times 0.8 \times 0.276 \times 220 = 476,045 \text{ N}$$

(i) Let trailing load which the two combined locomotives can haul be  $M$  tonne. In that case, total dead mass becomes  $M = (100 + 120 + M) = (220 + M)$  tonne. Tractive effort required is

$$= (277.8 M_e' + 98 M'G + M'r) \text{ newton}$$

$$= M' (277.8 \times 1 \times 1.1 + 98 \times 0.1 + 45) = 360.4 M' \text{ newton}$$

$$\therefore 360.4 M' = 476,045 ; M' = 1321 \text{ tonne} \quad \therefore \text{trailing load, } M = 1321 - 220 = \mathbf{1101 \text{ tonne}}$$

(ii) Total hauled load  $= 500 + 100 + 120 = 720$  tonne

Let  $G$  be the value of maximum percentage gradient. Then

$$F_t = (277.8 a M_e + 98 MG + Mr) \text{ newton} = M \left( 277.8 a \frac{M_e}{M} + 98 G + r \right) \text{ newton}$$

$$= 720 (277.8 \times 1 \times 1.1 + 98G + 45) \text{ newton} = (252,418 + 70,560 G) \text{ newton}$$

Equating it with the combined tractive effort of the two locomotive as calculated above, we have,

$$476,045 = 252,418 + 70,560 G \quad \therefore G = \mathbf{3.17 \text{ percent}}$$

**Example 43.10.** The average distance between stops on a level section of a railway is 1.25 km. Motor-coach train weighing 200 tonne has a schedule speed of 30 km/h, the duration of stops being 30 seconds. The acceleration is 1.9 km/h/s and the braking retardation is 3.2 km/h/s. Train resistance to traction is 45 N/t. Allowance for rotational inertia is 10%. Calculate the specific energy output in Wh/t-km. Assume a trapezoidal speed/time curve. (Elect. Power, Bombay Univ.)

**Solution.**  $\alpha = 1.9 \times 5/18 = 9.5/18 \text{ m/s}^2$ ;  $\beta = 3.2 \times 5/18 = 8/9 \text{ m/s}^2$   
 $K = (\alpha + \beta)/2\alpha\beta = 1.5$ ;  $D = 1.25 \text{ km} = 1250 \text{ m}$   
 Schedule time  $= 1.25 \times 3600/30 = 150 \text{ s}$ . Running time  $= 150 - 30 = 120 \text{ s}$   

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{120 - \sqrt{120^2 - 4 \times 1.5 \times 1250}}{2 \times 1.5} = 10.4 \text{ m/s} = \mathbf{37.4 \text{ km/h}}$$
  
 Braking distance  $D = V_m^2/2\beta = 10.42/2 \times (8/9) = 0.06 \text{ km}$   
 $\therefore D' = D - \text{braking distance} = 1.25 - 0.06 = 1.19 \text{ km}$   
 Specific energy output  $= 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778 r \frac{D'}{D}$   

$$= 0.01072 \times \frac{37.4^2}{1.25} \times 1.1 + 0.2778 \times 50 \times \frac{1.19}{1.25} \text{ Wh/t-km}$$
  

$$= 16.5 + 13.2 = \mathbf{29.7 \text{ Wh/t-km}}$$

**Example 43.11.** A 300-tonne EMU is started with a uniform acceleration and reaches a speed of 40 km/h in 24 seconds on a level track. Assuming trapezoidal speed/time curve, find specific energy consumption if rotational inertia is 8%, retardation is 3 km/h/s, distance between stops is 3 km, motor efficiency is 0.9 and train resistance is 40 N/tonne.

(Elect. Traction, AMIE Summer)

**Solution.** First of all, let us find  $D'$  – the distance upto which energy is consumed from the supply. It is the distance travelled upto the end of free-running period. It is equal to the total distance minus the distance travelled during braking.

Braking time,  $t_2 = V_m/\beta = 40/3 = 13.33 \text{ second}$

Distance travelled during braking period

$$= \frac{1}{2} V_m t_2 = \frac{1}{2} \times 40 \times \left( \frac{13.33}{3600} \right) = 0.074 \text{ km}$$

$$\therefore D' = D - \text{braking distance} = 3 - 0.074 = 2.926 \text{ km}$$

Since,  $M_e/M = 1.08$ , using the relation derived in Art. 43.43, we get the value of specific energy consumption as

$$= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778 r \frac{D'}{D} \right) \text{ Wh/t-km}$$

$$= \left( 0.01072 \times \frac{40^2}{0.9 \times 3} \times 1.08 + 0.2778 \times \frac{49}{0.9} \times \frac{2.926}{3} \right) = \mathbf{21.6 \text{ Wh/t-km.}}$$

**Example 43.12.** An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 70 seconds against a constant resistance of 60 N/t and is then braked to rest with uniform retardation of 3.0 km/h/s in 12 seconds. Compute

- (i) uniform acceleration (ii) coasting retardation

(iii) schedule speed if station stops are of 20-second duration

Allow 10% for rotational inertia. How will the schedule speed be affected if duration of stops is reduced to 15 seconds, other factors remaining the same ?

**Solution.** (i) As seen from Fig. 43.15,  $\alpha = V_1/t_1 = 50/25 = 2 \text{ km/h/s}$

(ii) The speeds at points B and C are connected by the relation

$$0 = V_2 + \beta t_3 \quad \text{or} \quad 0 = V_2 + (-3) \times 12 \quad \therefore V_2 = 36 \text{ km/h}$$

Coasting retardation,  $\beta_c = (V_2 - V_1)/t_2 = (36 - 50)/70 = -0.2 \text{ km/h/s}$

(iii) Distance travelled during acceleration

$$\begin{aligned} &= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 50 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h} \\ &= 0.174 \text{ km} \end{aligned}$$

Distance travelled during coasting can be found from the relation

$$\begin{aligned} V_{22} - V_{12} &= 2 \beta_c D \quad \text{or} \\ D &= (362 - 502)/2 \times -0.2 \times 3600 \\ &= 0.836 \text{ km} \end{aligned}$$

Distance covered during braking

$$\begin{aligned} &= \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 36 \frac{\text{km}}{\text{h}} \times \frac{12}{3600} \text{ h} \\ &= 0.06 \text{ km} \end{aligned}$$

Total distance travelled from start to stop

$$= 0.174 + 0.836 + 0.06 = 1.07 \text{ km}$$

Total time taken including stop time

$$= 25 + 70 + 12 + 20 = 127 \text{ second}$$

Schedule speed  $= 1.07 \times 3600/127 = 30.3 \text{ km/s}$

Schedule speed with a stop of 15 s is  $= 1.07 \times 3600/122 = 31.6 \text{ km/h}$

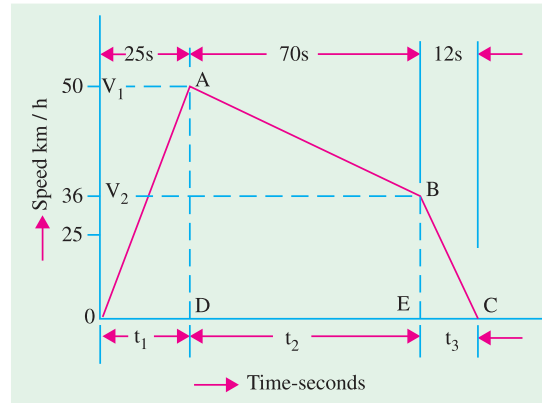


Fig. 43.15

**Example 43.13.** A 350-tonne electric train runs up an ascending gradient of 1% with the following speed/time curves :

1. uniform acceleration of 1.6 km/h/s for 25 seconds
2. constant speed for 50 seconds
3. coasting for 30 seconds
4. braking at 2.56 km/h/s to rest.

Compute the specific energy consumption if train resistance is 50 N/t, effect of rotational inertia 10%, overall efficiency of transmission gear and motor, 75%.

**Solution.** As seen from Fig. 43.16,  $V_1 = \alpha \cdot t_1 = 1.6 \times 25 = 40 \text{ km/h}$

Tractive force during coasting is

$$\begin{aligned} F_t &= (98 MG + M \cdot r) \\ &= M (98 \times 1 + 50) \\ &= 148 M \text{ newton} \end{aligned}$$

Also,  $F_t = 277.8 M_e \beta_c$  during coasting.

Equating the two expressions, we get  $277.8 M_e$

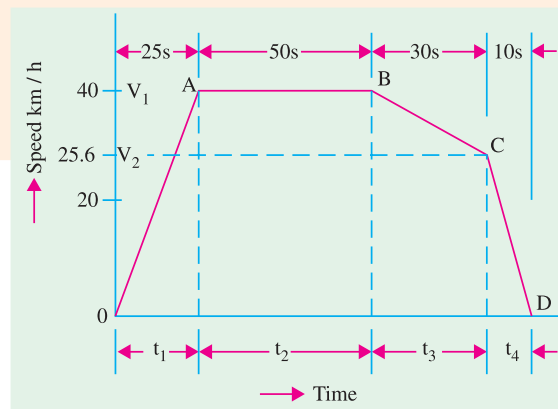


Fig. 43.16

$$\beta_c = 148 M$$

$$\therefore \beta_c = \frac{148}{277.8} \times \frac{M}{M_e} = \frac{148}{277.8} \times \frac{1}{1.1} ;$$

$$\beta_c = 0.48 \text{ km/h/s}$$

$$\begin{aligned} \text{Now, } V_2 &= V_1 + \beta_c t_3 \\ &= 40 + (-0.48) \times 30 \\ &= 25.6 \text{ km/h} \end{aligned}$$

$$t_4 = V_2 / \beta = 25.6 / 2.56 = 10 \text{ second}$$

$$\text{Distance travelled during acceleration period} = \frac{1}{2} \times 40 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h} = 0.139 \text{ km}$$

$$\text{Distance travelled during constant speed period is} = V_1 \times t_2 = 40 \times 50/3600 = 0.555 \text{ km}$$

$$\text{Distance travelled during coasting} = \left( \frac{V_1 + V_2}{2} \right) \times t_3 = \frac{40 + 25.6}{2} \times \frac{30}{3600} = 0.273 \text{ km}$$

$$\text{Distance travelled during braking} = \frac{1}{2} V_2 t_4 = \frac{1}{2} \times 25.6 \times \frac{10}{3600} = 0.035 \text{ km}$$

$$\text{Total distance between stops} = 0.139 + 0.555 + 0.273 + 0.035 = 1.002 \text{ km}$$

$$\text{Distance travelled during acceleration and free-running period is}$$

$$D' = 0.139 + 0.555 = 0.694 \text{ km}$$

Specific energy consumption (Art. 43.43) is

$$\begin{aligned} &= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km} \\ &= \left( 0.01072 \times \frac{40^2}{0.75 \times 1.002} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.694}{1.002} + 0.2778 \times \frac{50}{0.75} \times \frac{0.694}{1.002} \right) \\ &= 25.1 + 25.2 + 12.8 = \mathbf{63.1 \text{ Wh/t-km}} \end{aligned}$$

**Example 43.14.** An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 : 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.29 km/h/s. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting = 29.4 N/t, Train resistance at 30 km/h = 49 N/t

Coefficient of adhesion = 0.3, Rotational inertia = 10%

(Utilization of Elect. Power, AMIE)

**Solution.** Downward force due to gravity

$$= Mg \sin \theta = (5000 \times 1000) \times 9.8 \times 1/50 = 980,000 \text{ N}$$

$$\text{Train resistance} = 49 \times 5000 = 245,000 \text{ N}$$

$$\text{Braking force to be supplied by brakes} = 980,000 - 245,000 = 735,000 \text{ N}$$

Max. braking force which one locomotive can provide

$$= 1000 \mu_a Mg \text{ newton} \quad \text{—} M \text{ in tonne}$$

$$= 1000 \times 0.3 \times 75 \times 9.8 = 220,500 \text{ N}$$

$$\text{No. of locomotives required for braking} = 735,000/220,500 = 3.33$$

Since fraction is meaningless, it means that 4 locomotives are needed.

Tractive effort required to haul the train on level track

$$\begin{aligned}
 &= (277.8 \alpha M_e + Mr) \text{ newton} \\
 &= 277.8 \times (5000 \times 1.1) \times 0.29 + 5000 \times 29.4 = 590,090 \text{ N}
 \end{aligned}$$

No. of locomotives required =  $590,090/220,500 = 2.68 \cong 3$

It means that **4 locomotives** are enough to look after braking as well as starting.

**Example 43.15.** A 200-tonne electric train runs according to the following quadrilateral speed/time curve:

1. uniform acceleration from rest at 2 km/h/s for 30 seconds
2. coasting for 50 seconds
3. duration of braking : 15 seconds

If up-gradient is = 1%, train resistance = 40 N/t, rotational inertia effect = 10%, duration of stops = 15 s and overall efficiency of gear and motor = 75%, find

(i) schedule speed (ii) specific energy consumption (iii) how will the value of specific energy consumption change if there is a down-gradient of 1% rather than the up-gradient ?

(Electric Traction Punjab Univ. 1993)

**Solution.**  $V_1 = \alpha \cdot t_1 = 2 \times 30 = 60 \text{ km/h/s}$

During coasting, gravity component and train resistance will cause coasting retardation  $\beta_c$ .

Retarding force =  $(98 MG + Mr)$  newton =  $(98 \times 200 \times 1.0 + 200 \times 40) = 27,600 \text{ N}$

As per Art, 43.37,  $277.8 M_e \beta_c = 27,600$  or  $277.8 \times (200 \times 1.1) \times \beta_c = 27,600$

$\therefore \beta_c = 0.45 \text{ km/h/s}$

Now,  $V_2 = V_1 + \beta_c t_2$  or  $V_2 = 60 + (-0.45) \times 50 = 37.5 \text{ km/h}$

Braking retardation  $\beta = V_2/t_3 = 37.5/15 = 2.5 \text{ km/h/s}$

Distance travelled during acceleration (area OAD in Fig. 43.17)

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 60 \times \left( \frac{30}{3600} \right) = 0.25 \text{ km}$$

Distance travelled during coasting

$$= \text{area } ABED = \left( \frac{V_1 + V_2}{2} \right) \times t_2 = \left( \frac{60 + 37.5}{2} \right) \times \frac{50}{3600} = 0.677 \text{ km}$$

Distance travelled during braking

$$= \text{area } BCE = \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 37.5 \times \frac{15}{3600} = 0.078 \text{ km}$$

Total distance travelled,  $D = 0.25 + 0.677 + 0.078 = 1.005 \text{ km}$

Total schedule time =  $30 + 50 + 15 + 15 = 110 \text{ s}$

(i)  $\therefore$  Schedule speed =  $\frac{1.005}{110/3600} = 32.9 \text{ km/h}$

(ii) As per Art. 43.43, specific energy consumption

$$\begin{aligned}
 &= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \cdot \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km} \\
 &= \left( 0.01072 \times \frac{60^2}{0.75 \times 1.005} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.005} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.005} \right) \text{ Wh/t-km}
 \end{aligned}$$

(iii) the speed/time curve for this case is shown in Fig. 43.18. As before,  $V_1 = 60 \text{ km/h}$ . Here, we will take  $G = -1\%$

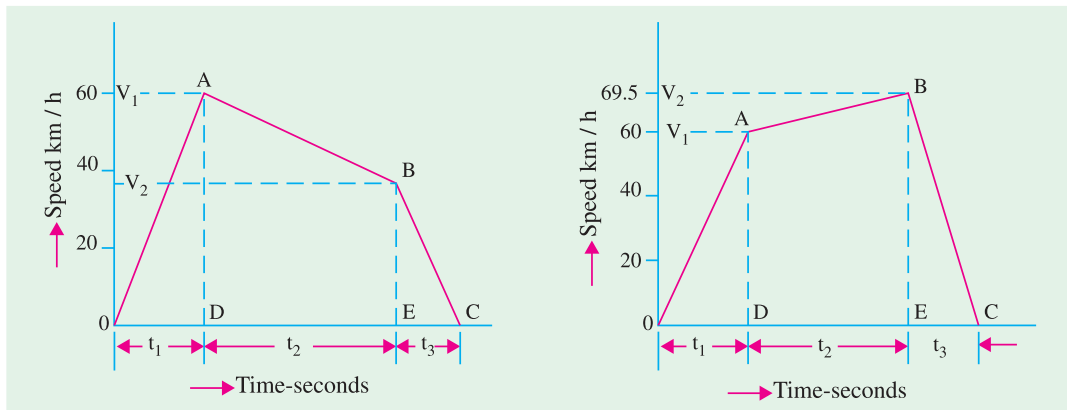


Fig. 43.17

Fig. 43.18

$\therefore$  Retarding force =  $(98 MG + Mr)$  newton =  $98 \times 200 \times (-1.0) + 200 \times 40 = -11,600$  N

The negative sign indicates that instead of being a retarding force, it is, in fact, an accelerating force. If  $\alpha_c$  is the acceleration produced, then

$$11,600 = 277.8 \times (200 \times 1.1) \times \alpha_c$$

$$\alpha_c = 0.19 \text{ km/h/s}$$

Also,  $V_2 = V_1 + \alpha_c t_2 = 60 + 0.19 \times 50 = 69.5 \text{ km/h}$

$$\beta = V_2 / t_3 = 69.5 / 15 = 4.63 \text{ km/h/s}$$

Distance travelled during acceleration = 0.25 km —as before

$$\text{Distance travelled during coasting} = \frac{60 + 69.5}{2} \times \frac{50}{3600} = 0.9 \text{ km}$$

$$\text{Distance travelled during braking} = \frac{1}{2} \times 69.5 \times \frac{15}{3600} = 0.145 \text{ km}$$

$$\therefore D = 0.25 + 0.9 + 0.145 = 1.295 \text{ km}$$

Hence, specific energy consumption is

$$= \left( 0.01072 \times \frac{60^2}{0.75 \times 1.295} \times 1.1 - 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.295} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.295} \right) \text{Wh/t-km}$$

$$= 43.7 - 7.01 + 2.86 = \mathbf{39.55 \text{ Wh/t-km}}$$

As seen, energy consumption has decreased from 69 to 39.55 Wh/t-km.

**Example 43.16.** An electric train has an average speed of 45 kmph on a level track between stops 1500 m apart. It is accelerated at 1.8 kmphps and is braked at 3 kmphps. Draw the speed - time curve for the run.

**Solution.**

Acceleration  $\alpha = 1.8 \text{ kmphps}$

Retardation  $\beta = 3.0 \text{ kmphps}$

Distance of run  $S = 1.5 \text{ km}$

Average speed  $V_a = 45 \text{ kmph}$

Time of run,  $T = \frac{S}{V_a} \times 3600 = \frac{1.5}{45} \times 3600 = 120 \text{ seconds.}$

Using equation  $V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3600 S}{K}}$



Where

$$K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2 \times 1.8} + \frac{1}{2 \times 3.0} = 0.4444$$

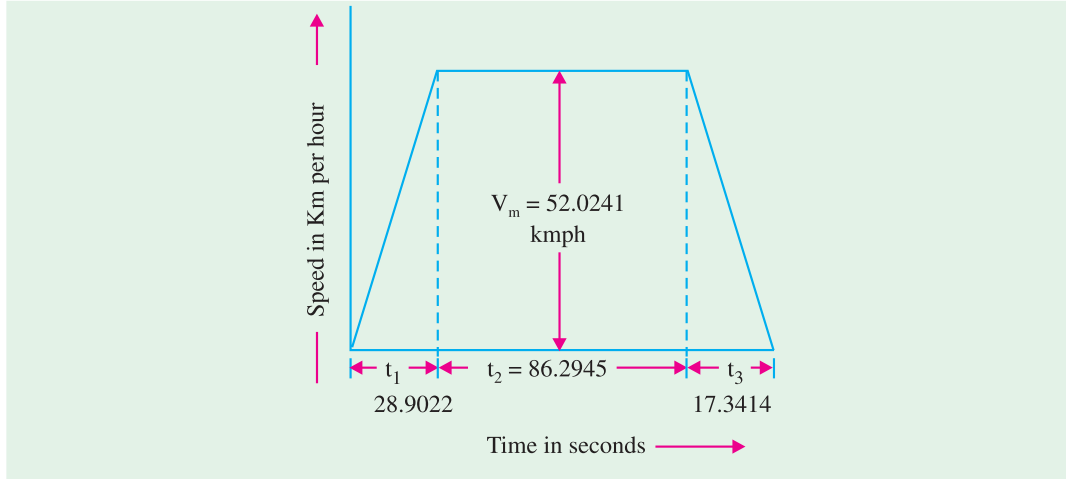


Fig. 43.19

$$\therefore \text{Maximum speed, } V_m = \frac{120}{2 \times 0.4444} - \sqrt{\frac{(120)^2}{(2 \times 0.4444)^2} - \frac{3600 \times 1.5}{0.4444}}$$

$$\therefore V_m = 52.0241 \text{ kmph}$$

$$\text{Acceleration period, } t_1 = \frac{V_m}{\alpha} = \frac{52.0241}{1.8} = 28.9022 \text{ seconds}$$

$$\text{Braking period, } t_3 = \frac{V_m}{\beta} = \frac{52.0241}{3.0} = 17.3414 \text{ seconds}$$

$$\text{Free running period, } t_2 = T - (t_1 + t_3) = 120 - (28.9622 + 17.3414) = 86.2945 \text{ seconds}$$

**Example 43.17.** A train has schedule speed of 60 km per hour between the stops which are 9 km apart. Determine the crest speed over the run, assuming trapezoidal speed – time curve. The train accelerates at 3 kmphs and retards at 4.5 kmphs. Duration of stops is 75 seconds.

**Solution.**

Acceleration  $\alpha = 3 \text{ kmphs}$

Retardation  $\beta = 4.5 \text{ kmphs}$

Distance of run,  $S = 9 \text{ km}$

Schedule speed,  $V_s = 60 \text{ kmph}$

$$\text{Schedule time, } T_s = \frac{S}{V_s} \times 3,600 \text{ seconds} = \frac{9}{60} \times 3,600 = 540 \text{ seconds}$$

$$\text{Actual time of run, } T = T_s - \text{Time of stop} = 540 - 75 = 465 \text{ seconds}$$

Using the equation

$$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3,600 S}{K}}$$

$$\text{where } K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{6} + \frac{1}{9} = 0.2777$$

$$\therefore \text{Maximum speed, } V_m = \frac{465}{2 \times 0.2777} - \sqrt{\frac{(465)^2}{4 \times (0.2777)^2} - \frac{3,600 \times 9}{0.2777}}$$

$$V_m = 837 - \sqrt{700569 - 116640}$$

$$\therefore V_m = 72.8475 \text{ kmph}$$

**Example 43.18.** An electric train is to have acceleration and braking retardation of 1.2 km/hour/sec and 4.8 km/hour/sec respectively. If the ratio of maximum to average speed is 1.6 and time for stops 35 seconds, find schedule speed for a run of 3 km. Assume simplified trapezoidal speed-time curve.

**Solution.**

$$\text{Acceleration } \alpha = 1.2 \text{ kmphps}$$

$$\text{Retardation } \beta = 4.8 \text{ kmphps}$$

$$\text{Distance of run, } S = 3 \text{ km}$$

Let the actual time of run be  $T$  seconds

$$\text{Average speed, } V_a = \frac{3,600 S}{T} = \frac{3,600 \times 3}{T} = \frac{10800}{T} \text{ kmph}$$

$$\text{Maximum speed, } V_m = 1.6 \times \frac{10800}{T} = \frac{17280}{T} \text{ kmph}$$

$$\text{Since } V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$$

$$\therefore V_m^2 = \frac{V_m T - 3,600 S}{\frac{1}{2\alpha} + \frac{1}{2\beta}} = \frac{\frac{17280}{T} T - 3600 \times 3}{\frac{1}{2 \times 1.2} + \frac{1}{2 \times 4.8}}$$

$$\text{or } V_m = 111.5419 \text{ kmph}$$

$$\text{and } V_a = \frac{V_m}{1.5} = \frac{111.5419}{1.5} = 74.3613 \text{ kmph}$$

$$\text{Actual time of run } T = \frac{3,600 S}{V_a} = \frac{3600 \times 3}{74.3613} = 145.2369 \text{ seconds}$$

$$\text{Schedule time, } T_s = \text{Actual time of run} + \text{Time of stop} = 145.2369 + 35 = 180 \text{ seconds}$$

$$\therefore \text{Schedule speed, } V_s = \frac{S \times 3,600}{T_s} = \frac{3 \times 3600}{180} = 60 \text{ kmph}$$

**Example 43.19.** An electric train has a schedule speed of 25 kmph between stations 800 m apart. The duration of stop is 20 seconds, the maximum speed is 20 percent higher than the average running speed and the braking retardation is 3 kmphps. Calculate the rate of acceleration required to operate this service.

**Solution.**

$$\text{Schedule speed, } V_s = 25 \text{ kmph}$$

$$\text{Distance of run, } S = 800 \text{ metres} = 0.8 \text{ km}$$

$$\text{Retardation, } \beta = 3 \text{ kmphps}$$

$$\text{Schedule time of run, } T_s = \frac{3,600 \times S}{V_s} = \frac{3600 \times 0.8}{25} = 115.2 \text{ seconds}$$

$$\text{Actual time of run, } T = T_s - \text{duration of stop} = 115.2 - 20 = 95.2 \text{ seconds}$$

$$\text{Average speed, } V_a = \frac{3,600 \times S}{T} = \frac{3600 \times 0.8}{95.2} = 30.25 \text{ kmphs}$$

$$\text{Maximum speed, } V_m = 1.2 V_a = 1.2 \times 30.25 = 36.3 \text{ kmph}$$

$$\text{Since } V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$$

$$\therefore \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600 S}{V_m^2}$$

$$\text{or } \frac{1}{2\alpha} + \frac{1}{2 \times 3} = \frac{36.3 \times 95.2 - 3,600 \times 0.8}{(36.3)^2} = 0.4369$$

$$\text{or } \alpha = 1.85 \text{ kmphps}$$

**Example 43.20.** A suburban electric train has a maximum speed of 80 kmph. The schedule speed including a station stop of 35 seconds is 50 kmph. If the acceleration is 1.5 kmphps, find the value of retardation when the average distance between stops is 5 km.

**Solution.**

$$\text{Schedule speed, } V_s = 50 \text{ kmph}$$

$$\text{Distance of run, } S = 5 \text{ km}$$

$$\text{Acceleration, } \alpha = 1.5 \text{ kmphps}$$

$$\text{Maximum speed, } V_m = 80 \text{ kmph}$$

$$\text{Duration of stop} = 35 \text{ seconds}$$

$$\text{Schedule time of run, } T_s = \frac{3,600 \times S}{V_s} = \frac{3,600 \times 5}{50} = 360 \text{ seconds}$$

$$\text{Actual time of run, } T = \frac{T_s - \text{duration of stop}}{V_m^2} = 360 - 30 = 330 \text{ seconds}$$

$$\text{Since } V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$$

$$\text{or } \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600 S}{V_m^2} = \frac{80 \times 330 - 3,600 \times 5}{(80)^2} = 1.3125$$

$$\text{or } \frac{1}{2\beta} = 1.3125 - \frac{1}{2\alpha} = 1.3125 - \frac{1}{2 \times 1.5} = 0.9792$$

$$\text{or } \beta = \frac{1}{2 \times 0.9792} = 0.51064 \text{ kmphps}$$

**Example 43.21.** A train is required to run between two stations 1.6 km apart at the average speed of 40 kmph. The run is to be made to a simplified quadrilateral speed-time curve. If the maximum speed is to be limited to 64 kmph, acceleration to 2.0 kmphps and coasting and braking retardation to 0.16 kmphps and 3.2 kmphps respectively, determine the duration of acceleration, coasting and braking periods.

**Solution.**

$$\text{Distance of run, } S = 1.6 \text{ km}$$

$$\text{Average speed, } V_a = 40 \text{ kmph}$$

$$\text{Maximum speed, } V_m = 64 \text{ kmph}$$

$$\text{Acceleration, } \alpha = 2.0 \text{ kmphps}$$

$$\text{Coasting retardation, } \beta_c = 0.16 \text{ kmphps}$$

$$\text{Braking retardation, } \beta = 3.2 \text{ kmphps}$$

$$\text{Duration of acceleration, } t_1 = \frac{V_m}{\alpha} = \frac{64}{2.0} = 32 \text{ seconds}$$

Actual time of run,  $T = \frac{3,600 S}{V_a} = \frac{3,600 \times 1.6}{40} = 144$  seconds

Let the speed before applying brakes be  $V_2$

then duration of coasting,  $t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - V_2}{0.16}$  seconds

Duration of braking,  $t_3 = \frac{V_2}{\beta} = \frac{V_2}{3.2}$  seconds

Since actual time of run,  $T = t_1 + t_2 + t_3$

$\therefore 144 = 32 + \frac{64 - V_2}{0.16} + \frac{V_2}{3.2}$

or  $V_2 \left( \frac{1}{0.16} - \frac{1}{3.2} \right) = 32 + 400 - 144$

or  $V_2 = \frac{288}{6.25 - 0.3125} = 48.5$  kmph

Duration of coasting,  $t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - 48.5}{0.16} = \mathbf{96.85 \text{ seconds}}$

Duration of braking,  $t_3 = \frac{V_2}{\beta} = \frac{48.5}{3.2} = \mathbf{15.15 \text{ seconds}}$

#### 43.47. General Features of Traction Motor

Electric Features

- High starting torque
- Series Speed - Torque characteristic
- Simple speed control
- Possibility of dynamic/regenerative braking
- Good commutation under rapid fluctuations of supply voltage.

Mechanical Features

- Robustness and ability to withstand continuous vibrations.
- Minimum weight and overall dimensions
- Protection against dirt and dust

No type of motor completely fulfills all these requirements. Motors, which have been found satisfactory are D.C. series for D.C. systems and A.C. series for A.C. systems. While using A.C. three phase motors are used. With the advent of Power Electronics it is very easy to convert single phase A.C. supply drawn from pantograph to three phase A.C.

#### 43.48. Speed - Torque Characteristic of D.C. Motor

$$V = E_b + I_a R_a$$

$$V \cdot I_a = E_b \cdot I_a + I_a^2 R_a$$

where  $E_b I_a$  = Power input to armature = Electrical power converted into mechanical power at the shaft of motor.

Mechanical Power  $= T \cdot \omega = T \times \frac{2\pi N}{60}$

$\therefore \frac{2\pi NT}{60} = E_b \cdot I_a \therefore T = \frac{60 E_b I_a}{2\pi N} = 9.55 \frac{E_b I_a}{N}$

But  $E_b = \frac{\phi ZNP}{60 A}$

$$\begin{aligned} \therefore T &= 9.55 \frac{\phi ZNP}{60 A} \frac{I_a}{N} = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A} \\ &= 0.1592 \times \phi \times \left[ Z \frac{I_a}{A} \right] P \text{ Nw-m} \end{aligned}$$

$\therefore$  Torque  $T = 0.1592 \times \text{flux per pole} \times \text{armature amp. conductors} \times \text{Number of poles}$

Also speed ' $N$ ' can be calculated as:

$$\begin{aligned} E_b &= \frac{\phi ZNP}{60 A} \quad \therefore N = \frac{(E_b)}{\phi ZP} 60 A \\ N &= \frac{(V - I_a R_a) 60 A}{\phi ZP} \quad \therefore N \propto \frac{V - I_a R_a}{\phi} \end{aligned}$$

But  $T = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A}$  from the equation of torque

$$\therefore \frac{T}{I_a} = \frac{9.55 \phi ZP}{60 A} \Rightarrow \frac{9.55 I_a}{T} = \frac{60 A}{\phi ZP} \text{ Put this value in the above equation of } N$$

$$\therefore N = \frac{(V - I_a R_a) \times 9.55 I_a}{T}$$

Speed

$$N = \frac{9.55 (V - I_a R_a)}{T / I_a}$$

The torque - current and speed - torque curves for D.C. motors are shown in Fig. 43.20 (a) and (b) respectively.

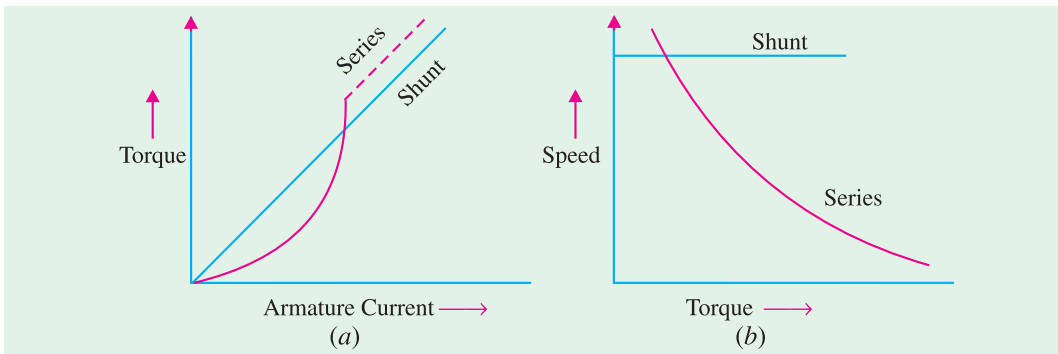


Fig. 43.20

#### 43.49. Parallel Operation of Series Motors with Unequal Wheel Diameter

An electric locomotive uses more than one motor. Each motor drives different set of axles and wheels. Due to wear and tear the diameter of wheels become different, after a long service. But the linear speed of locomotive and wheels will be the same. Therefore, motor speeds will be different due to difference in diameter of wheels driven by them as shown in Fig. 43.21. Therefore, when the motors are connected in parallel they will not share the torques equally, as the current shared by them will be different.

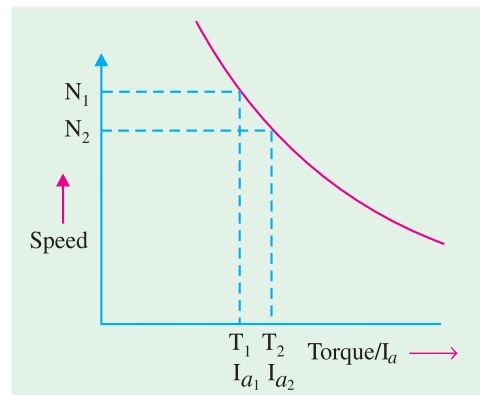


Fig. 43.21

Let the motor wheels ratio is 1.04 : 1 *i.e.* speed of rotation of motor-1 is 1.04 times that of motor-2, as shown in Fig. 43.21.

Let motor 1 drives wheel with 100 c.m. dia. and motor 2 drives wheel with 104 c.m. dia. Then speed of rotation of motor -1 will be  $\frac{104}{100} = 1.04$  times that of 2 *i.e.*  $N_2 = 1.04 N_1$ , for a given speed of locomotive.

### 43.50. Series operation of Series Motor with unequal wheel diameter

Let the motors 'A' and 'B' be identical having armature resistance  $R$  in series, as shown in Fig. 43.22.

Since they are in series, the same current ' $I$ ' will flow through both. But due to unequal wheel diameter; they deliver different loads *i.e.* voltage across each will be different

$$V = V_A + V_B \quad \text{and} \quad N \propto V - IR$$

$$\therefore \frac{N_A}{N_B} = \frac{V_A - IR}{V_B - IR}$$

$$\therefore V_A - IR = \frac{N_A}{N_B} (V_B - IR)$$

$$V_A = \frac{N_A}{N_B} (V_B - IR) + IR$$

$$V_A = \frac{N_A}{N_B} (V - V_A - IR) + IR$$

$$V_A = \frac{N_A}{N_B} (V - IR) + IR - \frac{N_A}{N_B} V_A$$

$$V_A + \frac{N_A}{N_B} V_A = \frac{N_A}{N_B} (V - IR) + IR$$

$$V_A \left( 1 + \frac{N_A}{N_B} \right) = \frac{N_A}{N_B} (V - IR) + IR$$

$$V_A = \frac{\frac{N_A}{N_B} (V - IR) + IR}{1 + \frac{N_A}{N_B}}$$

Similarly,

$$V_B = \frac{\frac{N_B}{N_A} (V - IR) + IR}{1 + \frac{N_B}{N_A}}$$

### 43.51. Series Operation of Shunt Motors with Unequal Wheel Diameter

It is similar to the case of series operation of series motors, and hence the same equation holds good.

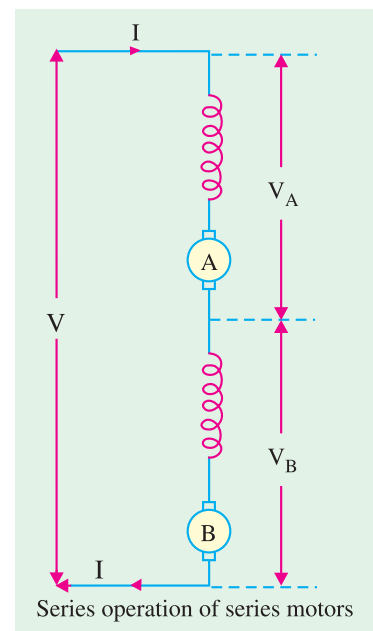


Fig. 43.22

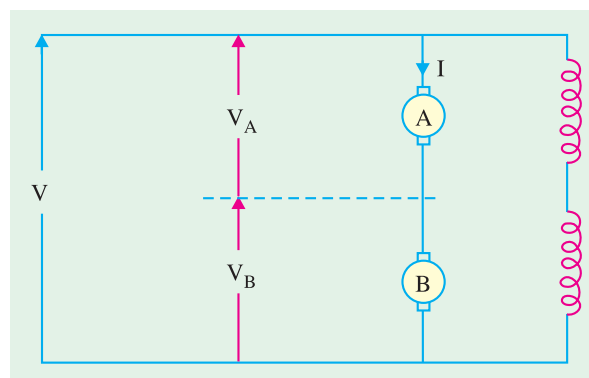


Fig. 43.23

### 43.52. Parallel Operation of Shunt Motors with Unequal Wheel Diameter

As seen from the Fig. 43.24, a small difference in speeds of two motors, causes motors to be loaded very unequally due to flat speed current curve of D.C. shunt motor.

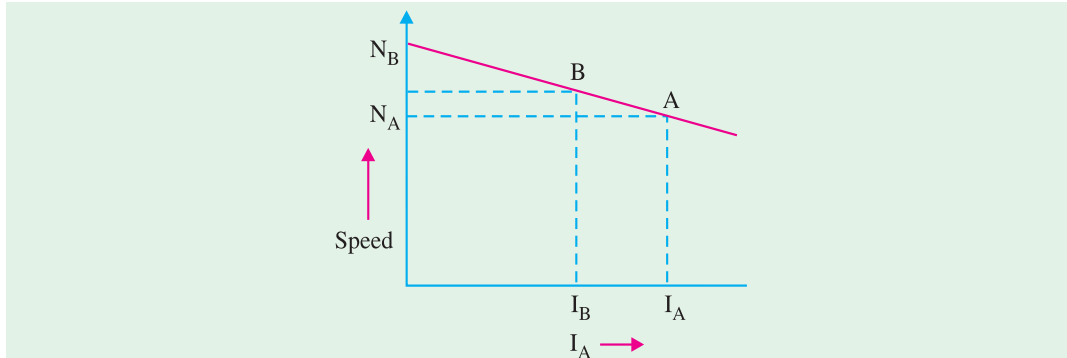


Fig. 43.24

**Example 43.22.** The torque-armature current characteristics of a series traction motor are given as:

Armature Current (amp) :	5	10	15	20	25	30	35	40
Torque (N-m) :	20	50	100	155	215	290	360	430

The motor resistance is  $0.3\Omega$ . If this motor is connected across 230 V, deduce the speed armature current characteristics.

**Solution.**

Supply voltage,  $V = 230$  V.

Total Resistance of series motor  $R_m = R_a + R_{se} = 0.3\Omega$ .

Armature current, $I_a$ in amperes	5	10	15	20	25	30	35	40
Torque, T in N-m	20	50	100	155	215	290	360	430
Back e.m.f., $E_b = (V - I_a R_m)$ in volts	228.5	227.0	225.5	224.0	222.5	221.0	219.5	218.0
Speed, $N_a$ $\frac{955 (E_b - E_{b0})}{E_b}$ in R.P.M.	545	434	323	276	247	218	204	194

The deduced speed-armature current characteristic is shown in Fig. 43.25.

**Example 43.23.** The following figures give the magnetization curve of d.c. series motor when working as a separately excited generator at 600 rpm.:

Field Current (amperes) :	20	40	60	80
E.M.F. (volts) :	215	381	485	550

The total resistance of the motor is 0.8 ohm. Deduce the speed – torque curve for this motor when operating at a constant voltage of 600 volts.

**Solution.**

Voltage applied across the motor,  $V = 600$  volts

Resistance of the motor,  $R_m = (R_a + R_{se}) = 0.8\Omega$

Speed,  $N_1 = 600$  r.p.m.



Field Current (amperes)	20	40	60	80
Back e.m.f., $E_1$ (Volts) at speed $N_1$ (600 r.p.m.) = e.m.f. generated by the armature (given)	215	381	485	550
Back e.m.f., $E$ (Volts) at speed $N$ (to be determined) = $V - I_a R_m$	584	568	552	536
Speed, $N$ (to be determined in r.p.m.) = $N_1/E_1 \times (V - I_a R_m)$	1,630	895	683	585
Torque, $T = \frac{9.55 (V - I_a R_m) I_a}{N}$ N-m	68.4	240	462	700

The deduced speed-torque curve for the motor is shown in Fig. 43.26.

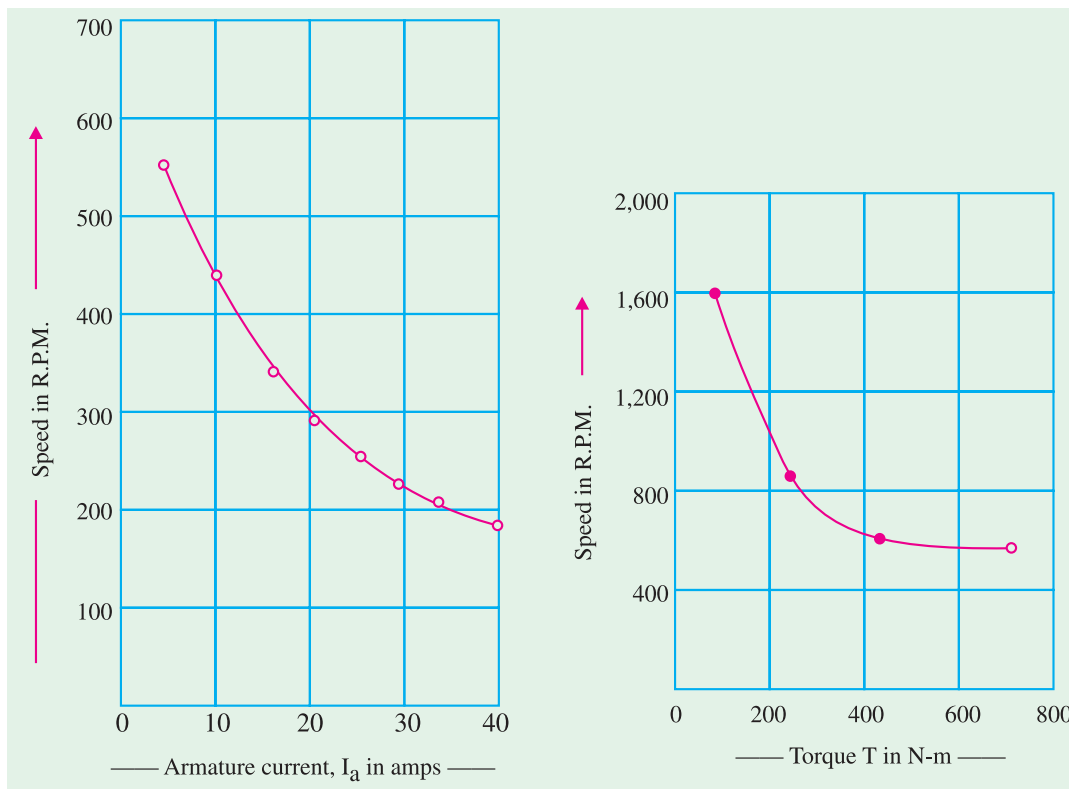


Fig. 43.25

Fig. 43.26

**Example 43.24.** Two d.c. traction motors run at a speed of 900 r.p.m. and 950 r.p.m. respectively when each takes a current of 50 A from 500 V mains. Each motor has an effective resistance of 0.3  $\Omega$ . Calculate the speed and voltage across each machine when mechanically coupled and electrically connected in series and taking a current of 50 A from 500 V mains, the resistance of each motor being unchanged.

**Solution.**

Let the two motors be A and B of speed  $N_A = 900$  rpm. And  $N_B = 950$  r.p.m. respectively. Resistance of each motor  $R_m = 0.3 \Omega$

Applied voltage across each motor,  $V = 500$  V.

Back e.m.f. of motor  $A$  when taking a current of  $50$  A

$$E_{b_A} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

Back e.m.f. of motor  $B$  when taking a current of  $50$  A

$$E_{b_B} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

When the machines are mechanically coupled and connected in series, the speed of each motor will be same, say  $N$ , current will be same and equal to  $50$  A (given) and the sum of voltage across the two motors will be equal to  $500$  V.

Let the voltage across motors  $A$  and  $B$  be  $V_A$  and  $V_B$  respectively

$$\text{Now } V_A + V_B = 500 \quad \dots(i)$$

$$\text{Back e.m.f. of motor } A, E_{b'_A} = E_{b_A} \times \frac{N}{N_A} = \frac{485}{900} \times N$$

$$\text{Voltage across motor } A, V_A = E_{b'_A} + I R_m = \frac{485}{900} \times N + 50 \times 0.3 = \frac{485}{900} \times N + 15$$

$$\text{Back e.m.f. of motor } B, E_{b'_B} = E_{b_B} \times \frac{N}{N_B} = \frac{485}{950} N$$

$$\text{Voltage across motor } B, V_B = E_{b'_B} + I R_m = \frac{485}{950} N + 15$$

$$\text{Substituting } V_A = \frac{485}{900} N + 15 \text{ and } V_B = \frac{485}{950} N + 15 \text{ in expression (i) we get}$$

$$\frac{485}{900} N + 15 + \frac{485}{950} N + 15 = 500$$

$$\text{or } \left( \frac{485}{900} + \frac{485}{950} \right) N = 470$$

$$\text{or } N \left( \frac{1}{900} + \frac{1}{950} \right) = \frac{470}{485}$$

$$\therefore N = 447.87 \text{ r.p.m.}$$

$$\text{P.D. across machine } A, V_A = \frac{485 N}{900} + 15 = 256.35 \text{ V}$$

$$\text{P.D. across machine } B, V_B = \frac{485 N}{950} + 15 = 243.65 \text{ V}$$

**Example 43.25.** A tram car is equipped with two motors which are operating in parallel. Calculate the current drawn from the supply main at  $500$  volts when the car is running at a steady speed of  $50$  kmph and each motor is developing a tractive effort of  $2100$  N. The resistance of each motor is  $0.4$  ohm. The friction, windage and other losses may be assumed as  $3500$  watts per motor.

**Solution.**

Voltage across each motor,  $V = 500$  volts

Maximum speed,  $V_m = 50$  kmph

Tractive effort,  $F_t = 2100$  Newtons

Motor resistance,  $R_m = 0.4$  W

Losses per motor =  $3500$  watts

$$\text{Power output of each motor} = \frac{F_t \times V_m \times 1000}{3600} \text{ watts}$$

$$= \frac{2100 \times 50 \times 1000}{3600} \text{ watts} = 29166.67 \text{ watts.}$$

Constant losses = 3500 watts

Copper losses =  $I^2 R_m = 0.4 I^2$

Where  $I$  is the current drawn from supply mains

Input to motor = Motor output + constant losses + copper losses

$$VI = 29166.67 + 3500 + 0.4 I^2$$

$$0.4 I^2 - 500 I + 32666.67 = 0$$

$$I = \frac{500 \pm \sqrt{(500)^2 - 4 \times 0.4 \times 32666.67}}{2 \times 0.4}$$

$$I = 69.16 \text{ A} \quad \text{or} \quad 1180.84 \text{ A}$$

Current drawn by each motor = 69.16 A

∴ 1180.84 A being unreasonably high can not be accepted

Total current drawn from supply mains =  $69.16 \times 2 = 138.32 \text{ A}$

**Example 43.26.** A motor coach is being driven by two identical d.c. series motors. First motor is geared to driving wheel having diameter of 90 cm and other motor to driving wheel having diameter of 86 cm. The speed of the first motor is 500 r.p.m. when connected in parallel with the other across 600 V supply. Find the motor speeds when connected in series across the same supply. Assume armature current to remain same and armature voltage drop of 10% at this current.

**Solution.**

Speed of first motor,  $N_1 = 500 \text{ r.p.m.}$

$$\begin{aligned} \text{Back e.m.f.,} \quad E_{b_1} &= 600 - \frac{10}{100} \times 600 \\ &= 540 \text{ volts.} \end{aligned}$$

When the motors are connected in series across 600 V supply, as shown in Fig. 43.27.

Let the supply voltage across motors I and II be  $V_1$  and  $V_2$  volts and speed  $N'_1$  and  $N'_2$  respectively.

$$\text{Since speed,} \quad N \propto \frac{V - IR}{\phi}$$

Current through the motors remains the same, therefore flux produced by it also remains the same and  $N \propto (V - IR)$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{V_1 - IR}{V_2 - IR} = \frac{V_1 - \frac{10}{100} \times 600}{V_2 - \frac{10}{100} \times 600} = \frac{V_1 - 60}{V_2 - 60} \quad \dots(i)$$

$$\text{And also} \quad N'_1 D_1 = N'_2 D_1 = N'_2 D_2$$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{D_2}{D_1} = \frac{86}{90}$$

$$\frac{V_1 - 60}{V_2 - 60} = \frac{86}{90} \quad (ii) \text{ Since peripheral speed is equal}$$

$$\text{or} \quad 90 V_1 - 5,400 = 86 V_2 - 5,160$$

$$\text{or} \quad 90 V_1 - 86 V_2 = 5,400 - 5,160 = 240 \quad \dots(iii)$$

$$\text{and also} \quad V_1 + V_2 = 600 \text{ V} \quad \dots(iv)$$

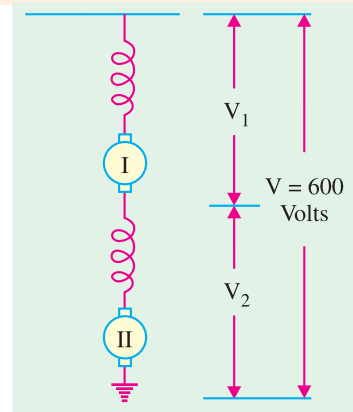


Fig. 43.27

Solving expressions (iii) and (iv)

$$V_1 = 294.55 \text{ V}$$

And

$$V_2 = 305.45 \text{ V}$$

Now the speeds of the motors can be calculated as follows :

$$\frac{N'_1}{N'_2} = \frac{x_{\phi_1}}{x_{\phi_2}}$$

$$\text{or } N'_1 = N_1 \times \frac{E_{b1}}{E_b} = 500 \times \frac{294.55 - 60}{600 - 60} = 217 \text{ r.p.m.}$$

and

$$N'_2 = N'_1 \times \frac{D_1}{D_2} = 217 \times \frac{90}{86} = \mathbf{277 \text{ r.p.m.}}$$

**Example 43.27.** Two similar series type motors are used to drive a locomotive. The supply fed to their parallel connection is 650 V. If the first motor 'A' is geared to drive wheels of radius 45 cms. and other motor 'B' to 43 cms. And if the speed of first motor 'A' when connected in parallel to 2<sup>nd</sup> motor 'B' across the main supply lines is 400 rpm., find voltages and speeds of motors when connected in series. Assume  $I_a$  to be constant and armature voltage drop of 10% at this current.

**Solution.**

$N \propto V - IR$  as flux  $\phi$  is constant, since  $I_a$  is constant

$$N_A = V_A - IR \quad N_B = V_B - IR \quad \text{Also } V = V_A + V_B$$

$$\text{Assume } \frac{N_A}{N_B} = \rho$$

$$V_A = \frac{\rho(V - IR) + IR}{1 + \rho}$$

$$\text{Armature voltage drop} = 10\% \text{ of } 650 \text{ V} \quad \therefore IR = 65$$

$$\text{But } \frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{43}{45} = \rho$$

$$V_A = \frac{43/45 (650 - 65) + 65}{1 + 43/45} = 320 \text{ V}$$

$$V_B = V - V_A = 650 - 320 = 330 \text{ V}$$

Speed  $N_A$  of motor A is 400 rpm with a supply of 650 V.

$\therefore$  Speed  $N'_A$  of motor A with supply voltage of 320 V will be

$$\frac{N'_A}{N_A} = \frac{320 - IR}{650 - IR} = \frac{320 - 65}{650 - 65} = \frac{255}{585}$$

$$\therefore N'_A = \frac{255}{585} N_A = \frac{255}{585} \times 400 = \mathbf{175 \text{ r.p.m.}}$$

$$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{N'_A}{N'_B} = \frac{43}{45}$$

$$N'_B = \frac{45}{43} N'_A = \frac{45}{43} \times 175 = \mathbf{184 \text{ r.p.m.}}$$

### 43.53. Control of D.C. Motors

The starting current of motor is limited to its normal rated current by starter during starting. At the instant of switching on the motor, back e.m.f.  $E_b = 0$

$$\therefore \text{Supply voltage} = V = IR + \text{Voltage drop across } R_s.$$

At any other instant during starting

$$V = IR + \text{Voltage across } R_s + E_b$$

At the end of accelerating period, when total  $R_s$  is cut-off

$$V = E_b + IR$$

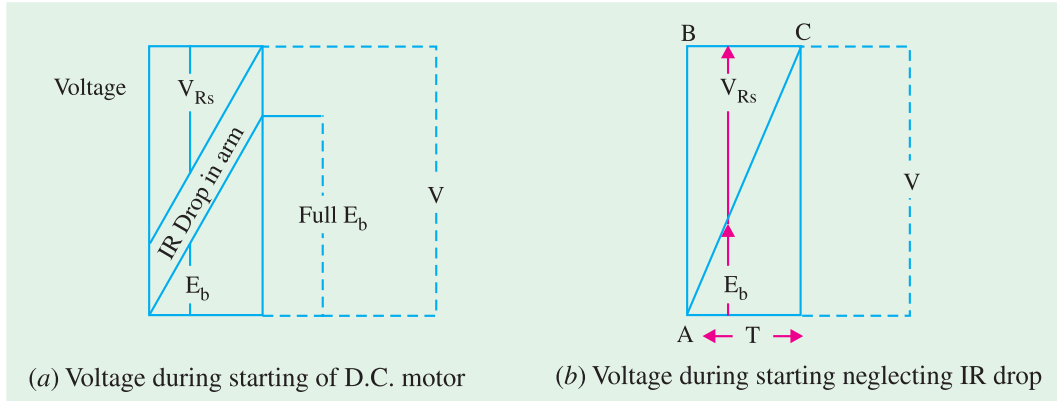


Fig. 43.28

If  $T$  is the time in sec. for starting and neglecting  $IR$  drop, total energy supplied =  $V.I.T$  watt-sec

From Fig. 43.28 (b) Energy wasted in  $R_s$  = Area of triangle  $ABC \times I = \frac{1}{2} \cdot T.V.I.$  watt - sec. =  $\frac{1}{2} VIT$  watt - sec. But total energy supplied =  $V.I.T$  watt - sec.

$\therefore$  Half the energy is wasted in starting

$\therefore \eta_{\text{starting}} = 50\%$

### 43.54. Series - Parallel Starting

With a 2 motor equipment  $\frac{1}{2}$  the normal voltage will be applied to each motor at starting as shown in Fig. 43.29 (a) (Series connection) and they will run upto approximate  $\frac{1}{2}$  speed, at which instant they are switched on to parallel and full voltage is applied to each motor.  $R_s$  is gradually cut-out, with motors in series connection and then reinserted when the motors are connected in parallel, and again gradually cut-out.

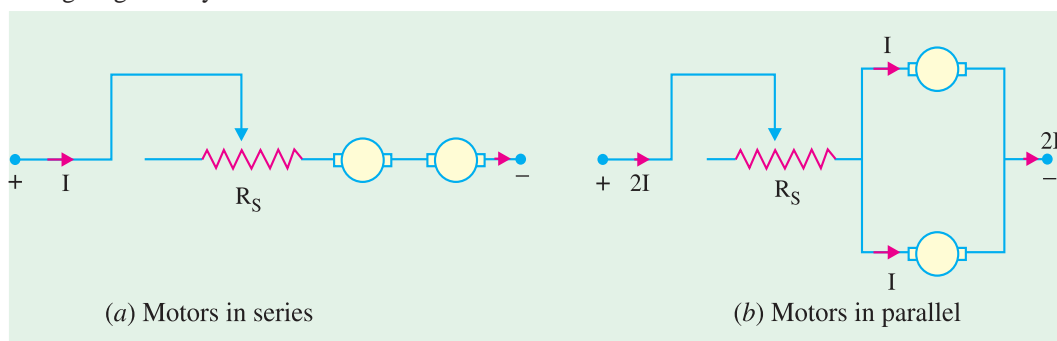


Fig. 43.29

In traction work, 2 or more similar motors are employed. Consider 2 series motors started by series parallel method, which results in saving of energy.

(a) **Series operation.** The 2 motors, are started in series with the help of  $R_s$ . The current during starting is limited to normal rated current ' $I$ ' per motor. During series operation, current ' $I$ ' is drawn

from supply. At the instant of starting  $OA = AB = IR$  drop in each motor.  $OK =$  Supply voltage ' $V$ '. The back e.m.fs. of 2 motors jointly develop along  $OM$  as shown in Fig. 43.30 (a). At point  $E$ , supply voltage  $V =$  Back e.m.fs of 2 motors  $+ IR$  drops of 2 motor. Any point on the line  $BC$  represents the sum of Back e.m.fs. of 2 motors  $+ IR$  drops of 2 motors  $+ \text{Voltage across resistance } R_s \text{ of 2 motors}$

$OE =$  time taken for series running.

At pt ' $E$ ' at the end of series running period, each motor has developed a back e.m.f.

$$= \frac{V}{2} - IR$$

$$EL = ED - LD$$

**(b) Parallel operation.** The motors are switched on in parallel at the instant ' $E$ ', with  $R_s$  reinserted as shown in Fig. 43.29 (b). Current drawn is  $2I$  from supply. Back e.m.f. across each motor  $= EL$ . So the back e.m.f. now develops along  $LG$ . At point ' $H$ ' when the motors are in full parallel, ( $R_s = 0$  and both the motors are running at rated speed)

$$\text{Supply voltage} = V = HF = HG + GF$$

$$= \text{Normal Back e.m.f. of each motor} + IR \text{ drop in each motor.}$$

#### 43.55. To find $t_s$ , $t_p$ and $\eta$ of starting

The values of time  $t_s$  during which the motors remain in series and  $t_p$  during which they are in parallel can be determined from Fig. 43.30 (a), (c). From Fig. 43.30 (a), triangles  $OLE$  and  $OGH$  are similar

$$\therefore \frac{OE}{OH} = \frac{LE}{GH} \therefore \frac{t_s}{T} = \frac{DE - DL}{FH - FG} = \frac{\frac{V}{2} - IR}{V - IR}$$

$$\therefore t_s = \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T$$

$$t_p = T - t_s = T - \left\{ \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T \right\}$$

$$t_p = T \left\{ 1 - \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) \right\}$$

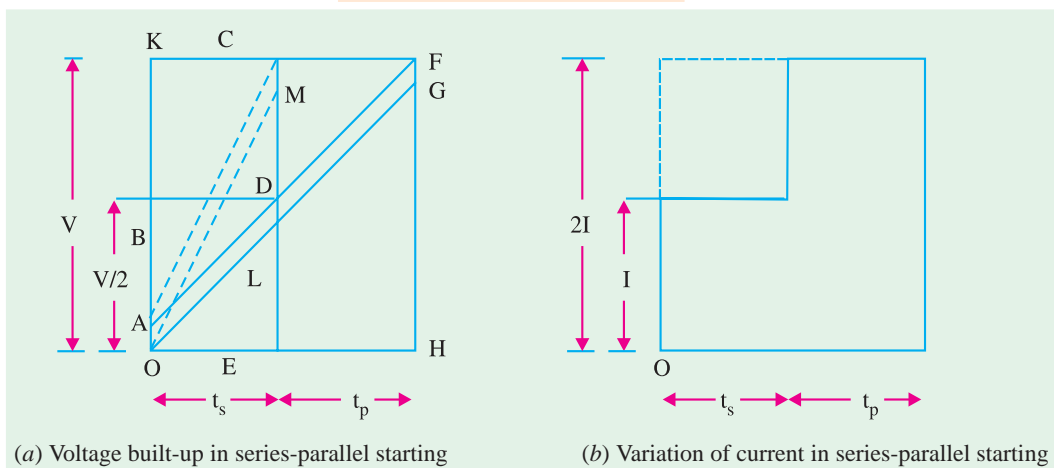


Fig. 43.30

To calculate  $\eta$  of starting, neglect  $IR$  drop in armature circuit.

This modifies Fig. 43.30 (a) to Fig. 43.30 (c). 'D' is midpoint of  $CE$  and back e.m.f. develops along  $DF$  in parallel combination.  $KC = CF$  i.e. time for series combination = time for parallel combination

i.e.  $t_s = t_p = t$  and average starting current =  $I$  per motor.

Energy lost in  $R_s$  = Area under triangle  $OKC$  + Area under triangle  $CDF$

$$= \left(\frac{1}{2} V I\right) \times t + \left(\frac{1}{2} \frac{V}{2} 2I\right) \times t = VIt$$

But total energy supplied

$$\begin{aligned} &= IVt + 2IVt \\ &\quad \text{(Series) (Parallel)} \\ &= 3VIt \end{aligned}$$

$$\begin{aligned} \therefore \eta \text{ of starting} &= \frac{3VIt - VIt}{3VIt} \\ &= \frac{2}{3} = 66.6\% \end{aligned}$$

$\therefore \eta$  is increased by 16.66% as compared to previous case. If there are 4 motors then  $\eta_{\text{starting}} = 73\%$ . So there is saving of energy lost in  $R_s$ , during starting period as compared with starting by both motors in parallel.

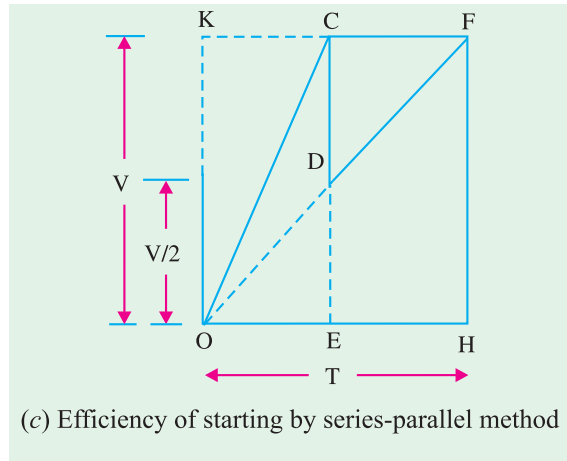


Fig. 43.30

**Example 43.28.** Two motors of a motor coach are started on series - parallel system, the current per motor being 350 A (Considered as being maintained constant) during the starting period which is 18 sec. If the acceleration during starting period is uniform, the line voltage is 600 V and resistance of each motor is 0.1  $\Omega$ . Find (a) the time during which the motors are operated in series. (b) the energy loss in the rheostat during starting period. [Nagpur University, Summer 2002]

**Solution.**

Time during which motors are in series is given by

$$t_s = \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T = \frac{1}{2} \left( \frac{600 - 2 \times 350 \times 0.1}{600 - 350 \times 0.1} \right) 18$$

$$t_s = 8.44 \text{ sec.}$$

Time during which motors are in parallel.

$$t_p = T - t_s = 18 - 8.44 = 9.56 \text{ sec.}$$

Back e.m.f.  $E_b$  of each motor, in series operation (from Fig. 43.30a)

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 350(0.1) = 265 \text{ V.}$$

When 2 motors are in series,

$$\text{Total } E_b = 265 + 265 = 530 \text{ V}$$

$$E_{b_p} = V - IR = 600 - 350(0.1) = 565 \text{ V}$$

Energy lost when motors are connected in series

$$= \frac{1}{2} E_b I t_s = \frac{1}{2} \times 530 \times 350 \times \frac{8.44}{3600} = 217 \text{ watt - hours}$$

Energy lost when motors are connected in parallel

$$\frac{1}{2} \frac{E_b}{2} 2 I t_p = \frac{1}{2} \times \frac{565}{2} \times 2 \times 350 \times \frac{9.56}{3600} = 262.5 \text{ watt - hour}$$



j 263 watt - hours

∴ Total energy lost = (217 + 263) watt - hours = **480 watt - hours**

#### 43.56. Series Parallel Control by Shunt Transition Method

The various stages involved in this method of series – parallel control are shown in Fig. 43.31

In steps 1, 2, 3, 4 the motors are in series and are accelerated by cutting out the  $R_s$  in steps. In step 4, motors are in full series. During transition from series to parallel,  $R_s$  is reinserted in circuit – step 5. One of the motors is bypassed – step 6 and disconnected from main circuit – step 7. It is then connected in parallel with other motor – step 8, giving 1<sup>st</sup> parallel position.  $R_s$  is again cut-out in steps completely and the motors are placed in full parallel.

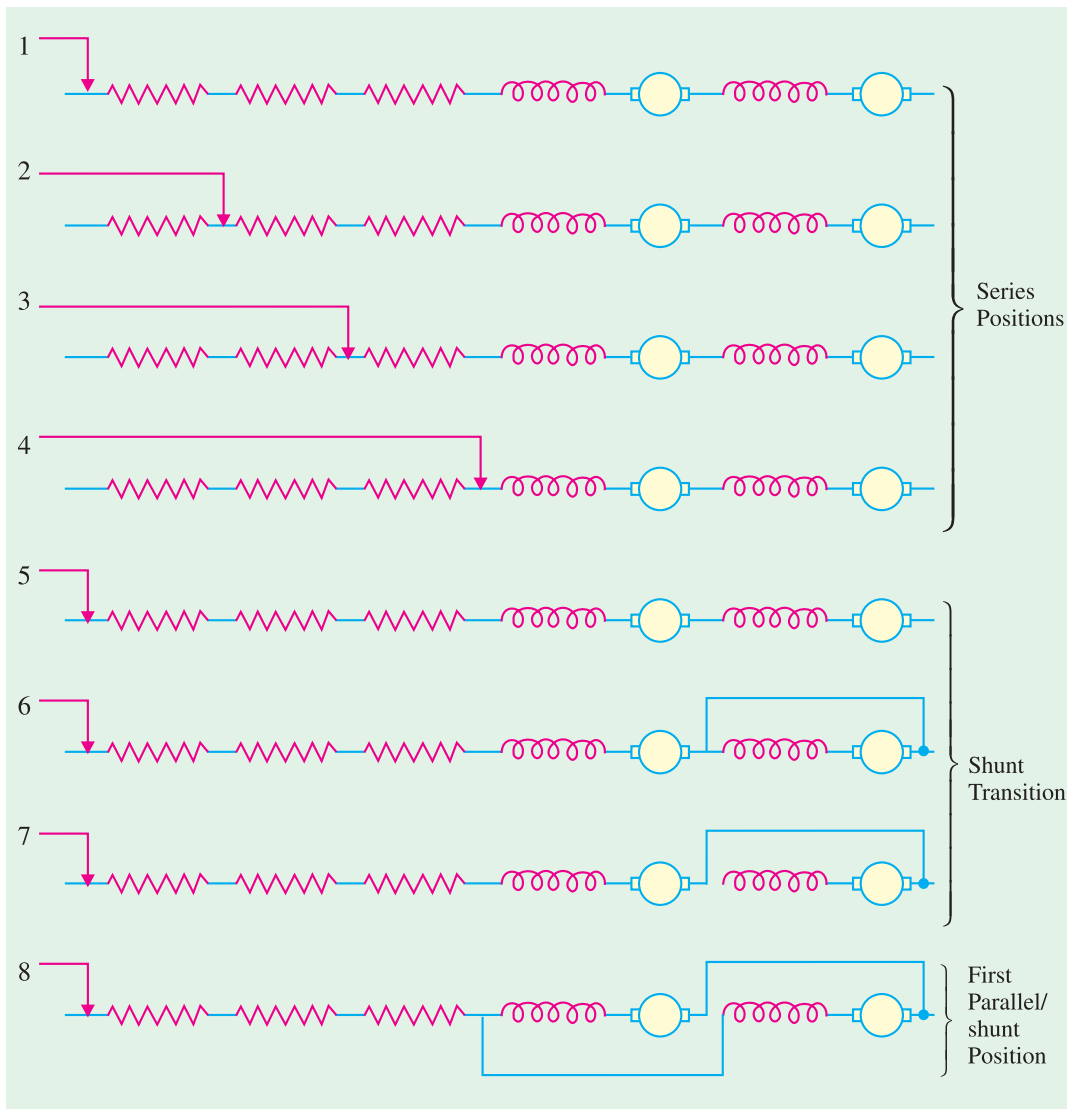


Fig. 43.31

The main difficulty with series parallel control is to obtain a satisfactory method of transition from series to parallel without interrupting the torque or allowing any heavy rushes of current.

In shunt transition method, one motor is short circuited and the total torque is reduced by about 50% during transition period, causing a noticeable jerk in the motion of vehicle.

The Bridge transition is more complicated, but the resistances which are connected in parallel with or 'bridged' across the motors are of such a value that current through the motors is not altered in magnitude and the total torque is therefore held constant and hence it is normally used for railways. So in this method it is seen that, both motors remain in circuit through-out the transition. Thus the jerks will not be experienced if this method is employed.

### 43.57. Series Parallel Control by Bridge Transition

- (a) At starting, motors are in series with  $R_s$  i.e. link  $P$  in position =  $AA'$
- (b) Motors in full series with link  $P$  in position =  $BB'$  (No  $R_s$  in the circuit)

The motor and  $R_s$  are connected in the form of Wheatstone Bridge. Initially motors are in series with full  $R_s$  as shown in Fig. 43.32 (a).  $A$  and  $A'$  are moved in direction of arrow heads. In position  $BB'$  motors are in full series, as shown in Fig. 43.32 (b), with no  $R_s$  present in the circuit.

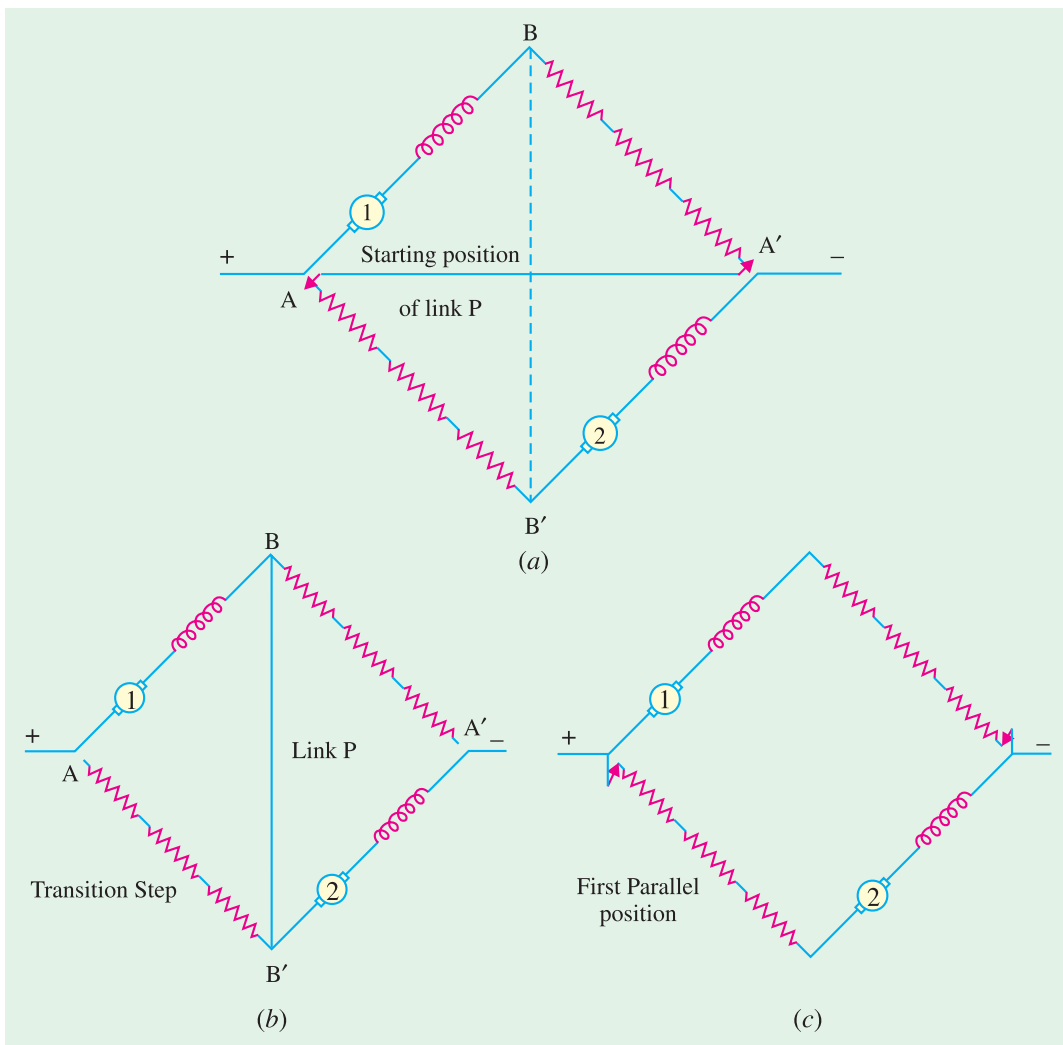


Fig. 43.32

In transition step the  $R_s$  is reinserted.

In I<sup>st</sup> parallel step, link  $P$  is removed and motors are connected in parallel with full  $R_s$  as shown in Fig. 43.32 (c). Advantage of this method is that the normal acceleration torque is available from both the motors, through - out starting period. Therefore acceleration is smoother, without any jerks, which is very much desirable for traction motors.

### 43.58. Braking in Traction

Both electrical and mechanical braking is used. Mechanical braking provides holding torque. Electric Braking reduces wear on mechanical brakes, provides higher retardation, thus bringing a vehicle quickly to rest. Different types of electrical braking used in traction are discussed.

### 43.59. Rheostatic Braking

(a) Equalizer Connection

(b) Cross Connection

(a) Equalizer Connection

For traction work, where 2 or more motors are employed, these are connected in parallel for braking, because series connection would produce too high voltage. K.E. of the vehicle is utilized in driving the machines as generators, which is dissipated in braking resistance in the form of heat.

To ensure that the 2 machines share the load equally, an equalizer connection is used as shown in Fig. 43.33 (a). If it is not used, the machine whose acceleration builds-up first would send a current through the 2<sup>nd</sup> machine in opposite direction, causing it to excite with reverse voltage. So that the 2 machines would be short circuited on themselves. The current would be dangerously high. Equalizer prevents such conditions. Hence Equalizer connection is important during braking in traction.

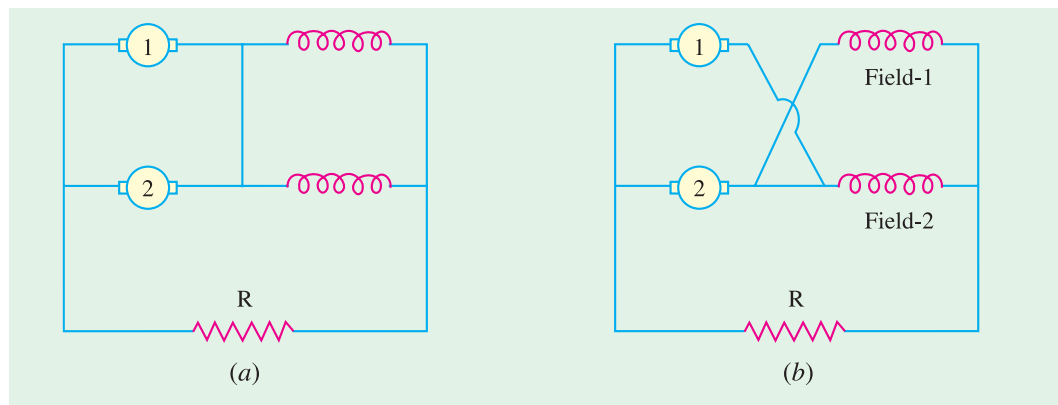


Fig. 43.33

(b) Cross Connection

In cross connection the field of machine 2 is connected in series with armature of machine 1 and the field of machine 1 is connected in series with armature of machine 2 as shown in Fig. 43.33 (b). Suppose the voltage of machine 1 is greater than that of 2. So it will send greater current through field of machine 2, causing it to excite to higher voltage. At the same time machine 1 excitation is low, because of lower voltage of machine 2. Hence machine 2 will produce more voltage and machine 1 voltage will be reduced. Thus automatic compensation is provided and the 2 machines operate satisfactorily.

Because of cross - connection during braking of traction motors, current in any of the motor will not go to a very high value.

### 43.60. Regenerative Braking with D.C. Motors

In order to achieve the regenerative braking, it is essential that (i) the voltage generated by the machine should exceed the supply voltage and (ii) the voltage should be kept at this value, irrespective of machine speed. Fig. 43.34 (a) shows the case of 4 series motors connected in parallel during normal running *i.e.* motoring.

One method of connection during regenerative braking, is to arrange the machines as shunt machines, with series fields of 3 machines connected across the supply in series with suitable resistance. One of the field winding is still kept in series across the 4 parallel armatures as shown in figure 43.34 (b).

The machine acts as a compound generator. (with slight differential compounding) Such an arrangement is quite stable; any change in line voltage produces a change in excitation which produces corresponding change in e.m.f. of motors, so that inherent compensation is provided *e.g.* let the line voltage tends to increase beyond the e.m.f. of generators. The increased voltage across the shunt circuit increases the excitation thereby increasing the generated voltage. Vice-versa is also true. The arrangement is therefore self compensating.

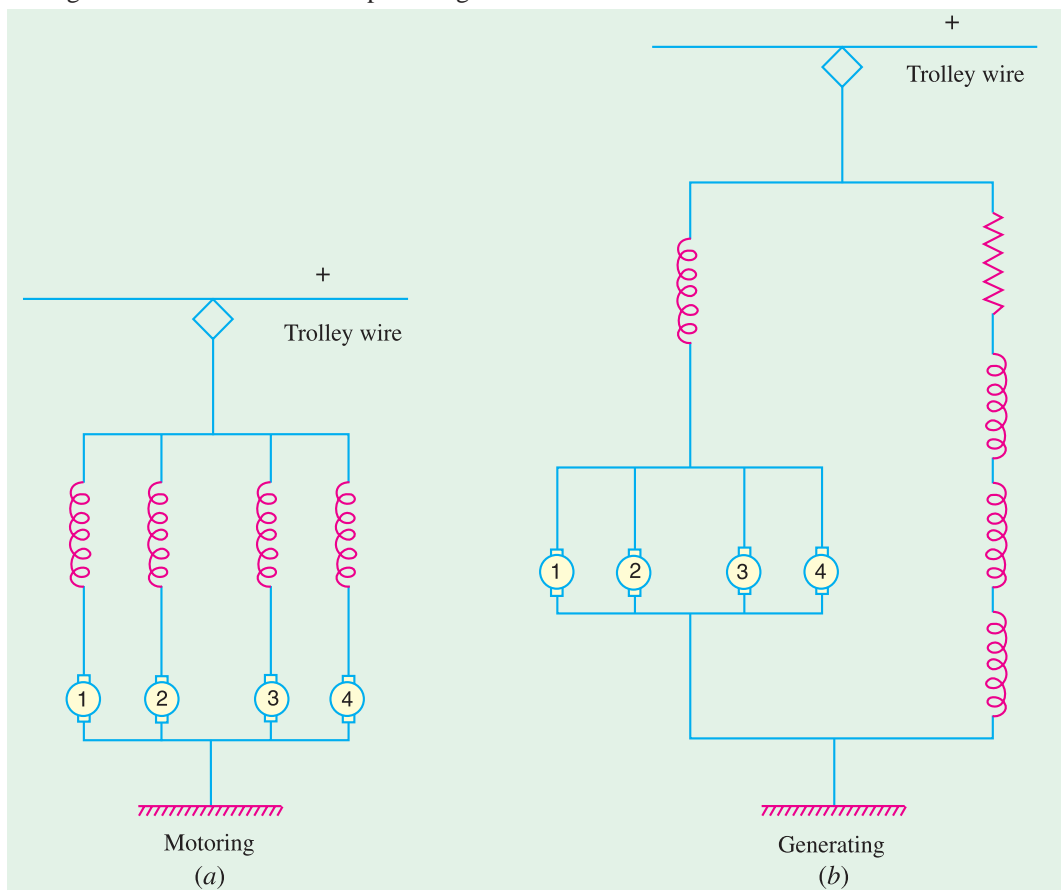


Fig. 43.34

D.C. series motor can't be used for regenerative braking without modification for obvious reasons. During regeneration current through armature reverses; and excitation has to be maintained. Hence field connection must be reversed.

**Example 43.29.** Two 750 V D.C. series motors each having a resistance of 0.1  $\Omega$  are started on series - parallel system. Mean current through out the starting period is 300 A. Starting period is 15 sec. and train speed at the end of this period is 25 km/hr. Calculate

- (i) Rheostatic losses during series and parallel combination of motors
- (ii) Energy lost in motor
- (iii) Motor output
- (iv) Starting  $\eta$
- (v) Train speed at which transition from series to parallel must be made.

[Nagpur University, Summer 2000]

**Solution.**

$$(i) \quad t_s = \frac{1}{2} \left[ \frac{V - 2IR}{V - IR} \right] T$$

$$t_s = \frac{1}{2} \left[ \frac{750 - 2(300) 0.1}{750 - (300) 0.1} \right] 15 = 7.1875 \text{ sec.}$$

$$\therefore t_p = T - t_s = 7.8125 \text{ sec.}$$

$$\begin{aligned} \text{Energy lost in Rheostat} &= \frac{1}{2} E_{b_s} I t_s + \frac{1}{2} \frac{E_{b_p}}{2} 2 I t_p \\ &= \frac{1}{2} \left[ 2 \times \left[ \frac{V}{2} - IR \right] \right] I t_s + \frac{1}{2} \left[ \frac{V - IR}{2} \right] 2 I t_p \\ &= \frac{1}{2} \left[ 2 \times \left[ \frac{750}{2} - 300(0.1) \right] \right] 300 \times 7.1875 + \frac{1}{2} \left[ \frac{750 - 300(0.1)}{2} \right] \times 2(300) \times 7.1825 \\ &= 743906.25 + 843750 \\ &= 1587656.25 \text{ watt - sec.} \\ &= \frac{1587656.25}{3600} = \mathbf{441.00 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (ii) \text{ Total Energy supplied} &= V I t_s + 2 I \cdot V \cdot t_p \\ &= 750 \times 300 (7.1875) + 2 (300) 750 (7.8125) \\ &= 1617187.5 + 3515625 \\ &= 5132812.5 \text{ watt-sec} = 1425.7812 \text{ watt - hrs.} \end{aligned}$$

$$\begin{aligned} \text{Energy lost in 2 Motors} &= (I_a^2 \times R_a) \times 2 \times 15 \\ &= (300^2 \times 0.1) \times 2 \times 15 = 270000 \text{ watt - sec.} = \mathbf{75 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (iii) \text{ Motor O/P} &= \text{Total Energy supplied} - \text{Energy lost in Rheostat} - \text{Energy lost in armature} \\ &= 1425.7812 - 441 - 75 \\ &= \mathbf{909.7812 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (iv) \quad \eta_{\text{starting}} &= \frac{\text{Total Energy Supplied} - \text{Energy lost in Rheostat}}{\text{Total Energy Supplied}} \\ &= \frac{1425.7812 - 441.00}{1425.7812} \times 100 \\ &= \mathbf{69.0605\%} \end{aligned}$$

(v) Acceleration is uniform during starting period of 15 sec. Therefore speed after which series to parallel transition must be made is given as –

$$= \frac{\text{Speed after starting period}}{\text{Total starting period}} \times t_s$$

$$= \frac{25}{15} \times 7.1875$$

$$= \mathbf{11.9791 \text{ km/hr.}}$$

**Example 43.30.** Two 600-V motors each having a resistance of  $0.1\Omega$  are started on the series-parallel system, the mean current per motor throughout the starting period being 300A. The starting period is 20 seconds and the train speed at the end of this period is 30 km per hour. Calculate (i) the rheostatic losses (in kwh) during (a) the series and (b) the parallel combinations of motors (ii) the train speed at which transition from series to parallel must be made.

**Solution.**

Number of motors operating = 2

Line voltage,  $V = 600$  volts

Current per motor,  $I = 300$  amperes

Starting period,  $T_s = 20$  seconds

Motor resistance,  $R = 0.1\Omega$

Maximum speed,  $V_m = 30$  kmph.

Back e.m.f. of each motor in full series position,

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 300 \times 0.1 = 270 \text{ volts.}$$

Back e.m.f. of each motor in full parallel position,

$$E_{b_p} = V - IR = 600 - 300 \times 0.1 = 570 \text{ volts}$$

Assuming smooth acceleration, back e.m.f. will be built up at constant rate.

Since motors take 20 seconds to build up 570 volts, therefore time taken to build up 270 volts e.m.f. will be :

$$T_{\text{series}} = 20 \times \frac{270}{570} = 9.4737 \text{ seconds}$$

$$T_{\text{parallel}} = 20 - 9.4737 = 10.5263 \text{ seconds}$$

(i) (a) Voltage drop in the starting rheostat in series combination at the starting instant  
 $= V - 2IR = 600 - 2 \times 300 \times 0.1 = 540$  volts,

which reduces to zero in full series position

Energy dissipated in starting resistance during series combination

$$= \frac{(V - 2IR) + 0}{2} \times I \times \frac{T_{\text{series}}}{3600} = \frac{540 + 0}{2} \times 300 \times \frac{9.4737}{3600}$$

$$= \mathbf{213.1579 \text{ watt - hours}}$$

(b) Voltage drop across the starting resistance in first parallel position is equal to  $V/2$  i.e. 300 volts which gradually reduces to zero.

Energy dissipated in starting resistance during parallel combination

$$= \frac{\frac{V}{2} + 0}{2} \times 2I \times \frac{T_{\text{parallel}}}{3600} = \frac{\frac{600}{2} + 0}{2} \times 2 \times 300 \times \frac{10.5263}{3600}$$

$$= \mathbf{263.1579 \text{ watt - hours}}$$

(ii) Acceleration,  $\alpha = \frac{\text{Maximum speed}}{\text{Starting period}} = \frac{V_m}{T_s} = \frac{30}{20} = 1.5 \text{ kmphps.}$

Speed at the end of series period =  $\alpha T_{\text{series}} = 1.5 \times 9.4737 = \mathbf{14.21 \text{ km/hour}}$

**Example 43.31.** Two d.c. series motors of a motor coach have resistance of  $0.1\Omega$  each. These motors draw a current of 500 A from 600 V mains during series – parallel starting period of 25 seconds. If the acceleration during starting period remains uniform, determine:

- (i) time during which the motors operate in (a) series (b) parallel.  
 (ii) the speed at which the series connections are to be changed if the speed just after starting period is 80 kmph.

**Solution.**

Number of motors operating = 2

Line voltage,  $V = 600 \text{ V}$

Current per motor,  $I = 500 \text{ A}$

Motor resistance,  $R = 0.1 \Omega$

Maximum speed,  $V_m = 80 \text{ kmph}$ .

Back e.m.f. of each motor in full series position.

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 500 \times 0.1 = 250 \text{ V}$$

Back e.m.f. each motor in full parallel operation,

$$E_{b_p} = V - IR = 600 - 500 \times 0.1 = 550 \text{ V}$$

Since motors take 25 seconds to build up 550 V, therefore, time taken to build up 250 V, will be: (assuming smooth acceleration and building up of e.m.f. at constant rate.)

(i) Period of series operation,  $T_{\text{series}} = 25 \times \frac{255}{550} = \mathbf{11.3636 \text{ seconds}}$

Period of parallel operation,  $T_{\text{parallel}} = T - T_{\text{se}} = 25 - 11.3636 = \mathbf{13.6363 \text{ seconds}}$

(iii) Speed at which the series connections are to be changed

$$= \alpha T_{\text{series}} = \frac{V_m}{T} \cdot T_{\text{series}} = \frac{80}{25} \times 11.3636 = \mathbf{36.3636 \text{ kmph}}$$

**Example 43.32.** The following figures refer to the speed-current and torque – current characteristics of a 600 V d.c. series traction motor.

Current, amperes :	50	100	150	200	250
Speed, kmph :	73.6	48	41.1	37.3	35.2
Torque, N-m :	150	525	930	1,335	1,750

Determine the braking torque at a speed of 48 kmph when operating as self excited d.c. generator. Assume resistance of motor and braking rheostat to be  $0.6\Omega$  and  $3.0 \Omega$  respectively.

**Solution.****As motor :**

Terminal voltage,  $V = 600 \text{ volts}$ .

The motor current at a speed of 48 kmph (from speed-current characteristic curve),

$$I = 100 \text{ A}$$

Back e.m.f. developed by the motor,  $E_b = V - IR_m = 600 - 100 \times 0.6 = \mathbf{540 \text{ V}}$

**As Generator:**

At the instant of applying rheostatic braking at speed of 48 kmph, the terminal voltage of machine will be equal to e.m.f. developed by the machine i.e. 540 volts.

Total resistance in the circuit =  $R_m + R_{\text{rheostat}} = 0.6 + 3 = 3.6 \Omega$

Current delivered by the machine,  $I = \frac{540}{3.6} = \mathbf{150 \text{ amps}}$

The braking torque (the torque corresponding to 150 amperes from torque-current curve)  
 $= \mathbf{930 \text{ N-m}}$



## Tutorial Problem No. 43.1

1. A train weighs 500 tonnes. What is its mass in (i) tonnes and (ii) kilograms.  
[(i) 500 t (ii) 500,000 kg]
2. A train has a mass of 200 tonnes. What is its weight in (i) newtons and (ii) kg-wt (iii) tonnes-wt.  
[(i)  $19.6 \times 10^5$  N (ii) 200,000 kg. wt (iii) 200 t-wt )]
3. A train has a speed of 100 km/h. What is its value in m/s ? [27.78 m/s]
4. A certain express train has an acceleration of 3.6 km/h/s. What is its value in  $\text{m/s}^2$  ? [1.0  $\text{m/s}^2$ ]
5. If there is an ascending gradient of 15 m in a track length of 1 km, what is the value of percentage gradient ? [1.5%]
6. A train runs at an average speed of 45 km per hour between stations 2.5 km apart. The train accelerates at 2 km/h/s and retards at 3 km/h/s. Find its maximum speed assuming a trapezoidal speed/time curve. Calculate also the distance travelled by it before the brakes are applied.  
[50.263 km/h, 2.383 km] (*Elect. Traction and Utilization B.H.U.* )
7. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20 percent higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with free running at the maximum speed.  
[2.73 km/h/s] (*Traction and Utilization of Elect. Power, Agra Univ.*)
8. A suburban electric train has a maximum speed of 65 km/h.. The schedule speed including a station stop of 30 seconds is 43.5 km/h. If the acceleration is 1.3 km/h/s, find the value of retardation when the average distance between stops is 3 km.  
[ $\beta = 1.21$  km/h/s] (*Utilization of Elect. Power and Traction, Gorakhpur Univ.,*)
9. An electric train is accelerated uniformly from rest to a speed of 40 km/h, the period of acceleration being 20 seconds. If it coasts for 60 seconds against a constant resistance of 50 N/t and is brought to rest in a further period of 10 seconds by braking, determine :  
(i) the acceleration (ii) the coasting retardation (iii) the braking retardation (iv) distance travelled and (v) schedule speed with station stops of 10 seconds duration.  
Allow 10 percent for rotational inertia.  
[ $\alpha = 2$  km/h/s,  $\beta_c = 0.1636$  km/h/s,  $\beta = 3$  km/h/s.,  $D = 0.736$  km.,  $V = 27.5$  km/h] (*Elect. Traction, Punjab Univ.*)
10. The speed-time curve of an electric train on a uniform rising gradient of 1 in 100 comprises :  
(i) uniform acceleration from rest at 2 km/h/s/ for 30 seconds.  
(ii) coasting with power off for 70 seconds.  
(iii) braking at 3 km/h/s to a standstill.  
The weight of the train is 250 tonnes, the train resistance on level track being 49 N/tonne and allowance for rotary inertia 1%.  
Calculate the maximum power developed by traction motors and total distance travelled by the train. Assume transmission efficiency as 97%.  
[3,3258 kW, 1.12 km] (*Traction and Utilization of Elect. Power, Agra Univ.*)
11. A 400-tonne goods train is to be hauled by a locomotive up a gradient of 2% with an acceleration of 1 km/h/s. Co-efficient of adhesion is 20%, track resistance 40 N/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and number of axles if the axle load is not increased beyond 22 tonnes.[152.6 tonnes, 7] (*Traction and Utilization of Elect. Power, Agra Univ.*)
12. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 20% with an acceleration of 1.2 km/h/s. Co-efficient of adhesion is 25%, track resistance 40 N/ tonne and effective rotating masses 10% of dead weight. Find the weight of the locomotive and number of axles if axle load is not to exceed 20 tonnes.  
[160 tonnes, 8] (*Utilization of Elect. Power, A.M.I.E. Winter*)

13. Determine the maximum adhesive weight of a loco required to start a 2340 tonne weight (inclusive of loco) on 1 : 150 gradient and accelerate it at 0.1 km/h/s. Assume co-efficient of adhesion as 0.25, train resistance 39.2 N/tonne and rotary inertia as 8%.  
[128.5 tonnes]  
(Elect. Traction, A.M.I.E., May)
14. Ore carrying trains weighing 5000 tonne each are to be hauled down a gradient of 1 in 60 at a maximum speed of 40 km/h and started on a level track at an acceleration of  $0.1 \text{ m/s}^2$ . How many locomotives, each weighing 75 tonne, will have to be employed ?  
Train resistance during starting = 29.4 N/tonne  
Train resistance at 40 km/h = 56.1 kg/tonne  
Co-efficient of adhesion =  $1/3$  ; Rotational inertia =  $1/10$   
[3 Loco] (Engg. Service Examination U.P.S.C.)
15. A locomotive accelerates a 400-tonne train up a gradient of 1 in 100 at 0.8 km/h/s. Assuming the coefficient of adhesion to be 0.25, determine the minimum adhesive weight of the locomotive. Assume train resistance of 60 N/tonne and allow 10% for the effect of rotational inertia.  
[65.7 t] (Elect. Traction and Utilization, Nagpur Univ.)
16. Calculate the specific energy consumption if a maximum speed of 12.20 metres/sec and for a given run of 1525 m an acceleration of  $0.366 \text{ m/s}^2$  are desired. Train resistance during acceleration is 52.6 N/1000 kg and during coasting is 6.12 N/1000 kg, 10% being allowable for rotational inertia. The efficiency of the equipment during the acceleration period is 50%. Assume a quadrilateral speed-time curve.  
[3.38 Wh/kg-m] (Util. of Elect. Power, A.M.I.E. Sec. B)
17. An electric locomotive of 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an upgradient of 1/1000. Tractive resistance of the track is 45 N per tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find, (i) the trailing weight that can be hauled up the same gradient under the same conditions and (ii) the maximum gradient, the trailing weight hauled remaining unchanged.  
Assume adhesive weight expressed as percentage of total dead weight to be the same for both the locomotive.  
[(i) 1120 t (ii) 3.15%] (Util. of Elect. Power, A.M.I.E. Sec. B.)
18. An electric train has quadrilateral speed-time curve as follows :  
(i) uniform acceleration from rest at 2 km/h/s for 30 sec,  
(ii) coasting for 50 sec.  
(iii) uniform braking to rest for 20 seconds.  
If the train is moving uniform upgradient of a 10/1000, train resistance is 40 N/tonne, rotational inertia effect 10% of dead weight and duration of stop 30 seconds, find the schedule speed.  
[28.4 km/h] (Util. of Elect. Power, A.M.I.E. Sec. B.)
19. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20% higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with the free running at the maximum speed.  
[2.73 km/h/s] (Traction & Util. of Elect. Power, Agra Univ.)
20. An electric train has an average speed of 42 km/h on a level track between stops 1,400 metre apart. It is accelerated at 1.7 km/h/s and is braked at 3.3 km/h/s. Draw the speed-time curve for the run. Estimate the sp. energy consumption. Assume tractive resistance as 50 N/t and allow 10% for rotational inertia.  
[39.48 Wh/t-km] (Util. of Elect. Power, A.M.I.E. Sec. B.)
21. An electric train weighing 200 tonne has eight motors geared to driving wheels, each wheel is 90 cm diameter. Determine the torque developed by each motor to accelerate the train to a speed of 48 km/h in 30 seconds up a gradient of 1 in 200. The tractive resistance is 50 N/t, the effect of rotational inertia is 10% of the train weight, the gear ratio is 4 to 1 and gearing efficiency is 80%.  
[2,067 N-m] (Traction & Util. of Elect. Power, Agra Univ.)

22. An electric train accelerates uniformly from rest to a speed of 48 km/h in 24 seconds. It then coasts for 69 seconds against a constant resistance of 58 N/t and is braked to rest at 3.3 km/h/s in 11 seconds.  
Calculate (i) the acceleration (ii) coasting retardation and (iii) the schedule speed, if the station stops are of 20 seconds duration. What would be the effect on schedule speed of reducing the station stops to 15 second duration, other conditions remaining the same? Allow 10% for the rotational inertia.  
**[(i) 2 km/h/s (ii) 0.19 km/h/s (iii) 30.25 km/h] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
23. An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 1 minute 10 seconds against a constant resistance of 70 N/t and is braked to rest at 4 km/h/s in 10 seconds. Calculate the schedule speed, if the station stops are of 15 second duration.  
**[31.125 km/h] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
24. An electric train has a quadrilateral speed-time curve as follows :  
(i) uniform acceleration from rest at 2.5 km/h/s for 25 second  
(ii) coasting for 50 second (iii) duration of braking 25 second.  
If the train is moving along a uniform upgradient of 1 in 100 with a tractive resistance of 45 N/t, rotational inertia 10% of dead weight, duration of stops at stations 20 second and overall efficiency of transmission gear and motor 80%, calculate the schedule speed and specific energy consumption of run.  
**[69 km/h, 26.61 Wh/t-km] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
25. An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 in 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of  $0.08 \text{ m/s}^2$ . How many locomotives, each weighing 75 tonne, will have to be employed ?  
Train resistance during starting = 3 kg/t  
Train resistance at 30 km/h = 5 kg/t  
Co-efficient of adhesion = 0.3, Rotational inertia = 10%.  
**[4 loco] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
26. A train with an electric locomotive weighing 300 tonne is to be accelerated up a gradient of 1 in 33 at an acceleration of 1 km/h/s. If the train resistance, co-efficient of adhesion and effect of rotational inertia are 80 N/t, 0.25 and 12.5% of the dead weight respectively, determine the minimum adhesive weight of the locomotive.  
**[88 t] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
27. A train weighing 400 tonne has speed reduced by regenerative braking from 40 to 20 km/h over a distance of 2 km at a down gradient of 20%. Calculate the electrical energy and average power returned to the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion 75%.  
**[324 kW/h, 4860 kW] (Util. & Traction Power, Agra Univ.)**
28. A 250-tonne motor coach having 4 motors, each developing 5,000 N-m torque acceleration, starts from rest. If upgradient is 25 in 1000, gear ratio 5, gear transmission efficiency 88%, wheel radius 44 cm, train resistance 50 N/t addition of rotational inertia 10%, calculate the time taken to reach a speed of 45 km/h.  
If the supply voltage were 1500 V d.c. and efficiency of motor is 83.4%, determine the current drawn per motor during notching period.  
**[27.25 s, 500 A] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
29. An electric train weighing 100 tonne has a rotational inertia of 10%. This train while running between two stations which are 2.5 km apart has an average speed of 50 km/h. The acceleration and retardation during braking are respectively 1 km/h/s and 2 km/h/s. The percentage gradient between these two stations is 1% and the train is to move up the incline. The track resistance is 40 N/t. If the combined efficiency of the electric train is 60%, determine (i) maximum power at driving axle (ii) total energy consumption and (iii) specific energy consumption. Assume that journey estimation is being made in simplified trapezoidal speed-time curve.  
**[(i) 875 kW (ii) 23.65 kW/h (iii) 94.6 Wh/t-km] (Util. of Elect. Power, A.M.I.E. Sec. B.)**

30. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 1 in 40 with an acceleration of 1.5 km/h/s. Determine the weight of the locomotive and number of axles, if axle load is not to exceed 24 tonne. Co-efficient of adhesion is 0.31, track resistance 45 N/t and effective rotating masses 10% of dead weight. *[7] (Util. of Elect. Power, A.M.I.E. Sec. B.)*
31. Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600V mains during series-parallel starting period of 20 seconds. If the acceleration during starting period remains uniform, determine :
- (i) time during which motor operates in (a) series, (b) parallel
- (ii) the speed at which the series connections are to be changed if the speed just after starting period is 70 km/h.
- [(i) 9.098, 10.971 Sec. (ii) 31.82 km/h] (Util. of Elect. Power and Traction, Agra Univ.)*
32. Explain how series motors are ideally suited for traction service. *(Nagpur University, Summer 2004)*
33. Explain any one method for regenerative braking of D.C. motor for traction. *(Nagpur University, Summer 2004)*
34. Discuss the effect of unequal wheel diameters on the parallel operation of traction motors. *(Nagpur University, Summer 2004)*
35. Explain the various modes of operation in traction services with neat speed-time curve. *(Nagpur University, Summer 2004)*
36. A 100 tonne motor coach is driven by 4 motors, each developing a torque of 5000 N-m during acceleration. If up-gradient is 50 in 1000, gear ratio  $a = 0.25$ , gear transmission efficiency 98%, wheel radius 0.54 M, train resistance 25 N/tonne, effective mass on account of rotational inertia is 10% higher, calculate the time taken to attain a speed of 100 kmph. *(Nagpur University, Summer 2004)*
37. What are the requirements of an ideal traction system? *(J.N. University, Hyderabad, November 2003)*
38. What are the advantages and disadvantages of electric traction? *(J.N. University, Hyderabad, November 2003)*
39. Write a brief note on the single phase a.c. series motor and comment upon its suitability for traction services. How does it compare in performance with the d.c. Services motor. *(J.N. University, Hyderabad, November 2003)*
40. Draw the speed-time curve of a main line service and explain. *(J.N. University, Hyderabad, November 2003)*
41. A train has a scheduled speed of 40 km/hr between two stops, which are 4 kms apart. Determine the crest speed over the run, if the duration of stops is 60 sec and acceleration and retardation both are 2 km/hr/sec each. Assume simplified trapezoidal speed-time curve. *(J.N. University, Hyderabad, November 2003)*
42. What are the various electric traction systems in India? Compare them. *(J.N. University, Hyderabad, November 2003)*
43. Give the features of various motors used in electric traction. *(J.N. University, Hyderabad, November 2003)*
44. Draw the speed-time curve of a suburban service train and explain. *(J.N. University, Hyderabad, November 2003)*
45. A train accelerates to a speed of 48 km/hr in 24 sec. then it coasts for 69 sec under a constant resistance of 58 newton/tonne and brakes are applied at 3.3 km/hr/sec in 11 sec. Calculate (i) the acceleration (ii) the coasting retardation (iii) the scheduled speed if station stoppage is 20 secs. What is the effect of scheduled speed if station stoppage is reduced to 15 sec duration, other conditions remaining same. Allow 10% for rotational inertia. *(J.N. University, Hyderabad, November 2003)*

46. Derive an expression for specific energy output on level track using a simplified speed-time curve. What purpose is achieved by this quantity? *(J.N. University, Hyderabad, November 2003)*
47. A 400 tonne goods train is to be hauled by a locomotive up a gradient of 2% with acceleration of 1 km/hr/sec, coefficient of adhesion 20%, track resistance 40 newtons/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and the number of axles if the axle load is not to increase beyond 22 tonnes.  
*(J.N. University, Hyderabad, November 2003)*
48. A motor has the following load cycle :  
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.  
Full speed period 15-85 sec Load constant at 600 h.p.  
Decelerating period 85-100 sec h.p. returned to line falls uniformly from 200 to zero  
Decking period 100-120 sec Motor stationary. Estimate the size of the motor.  
*(J.N. University, Hyderabad, November 2003)*
49. Explain the characteristics of series motors and also explain how they are suitable of electric traction work? *(J.N. University, Hyderabad, November 2003)*
50. For a trapezoidal speed-time curve of a electric train, derive expression for maximum speed and distance between stops. *(J.N. University, Hyderabad, November 2003)*
51. A mail is to be run between two stations 5 kms apart at an average speed of 50 km/hr. If the maximum speed is to be limited to 70 km/hr, acceleration to 2 km/hr/sec, braking retardation to 4 km/hr/sec and coasting retardation to 0.1 km/hr/sec, determine the speed at the end of coasting, duration of coasting period and braking period. *(J.N. University, Hyderabad, November 2003)*
52. Discuss the merits and demerits of the D.C. and 1- $\phi$  A.C. systems for the main and suburban line electrification of the railways. *(J.N. University, Hyderabad, April 2003)*
53. Which system do you consider to be the best for the suburban railways in the vicinity of large cities? Given reasons for your answer. *(J.N. University, Hyderabad, April 2003)*
54. Derive expression for the tractive effort for a train on a level track.  
*(J.N. University, Hyderabad, April 2003)*
55. The maximum speed of a suburbanelectric train is 60 km/hr. Its scheduled speed is 40 km/hr and duration of stops is 30 sec. If the acceleration is 2 km/hr/sec and distance between stops is 2 kms, determine the retardation. *(J.N. University, Hyderabad, April 2003)*
56. What are various types of traction motors? *(J.N. University, Hyderabad, April 2003)*
57. What are the advantages of series parallel control of D.C. motors?  
*(J.N. University, Hyderabad, April 2003)*
58. Describe about duplication of railway transmission lines.  
*(J.N. University, Hyderabad, April 2003)*
59. Write a note on feeding and distributing system on A.C. Traction and for d.c. tram ways.  
*(J.N. University, Hyderabad, April 2003)*
60. For a quadrilateral speed-time curve of a electric train, derive expression for the distance between stops and speed at the end of the coasting period. *(J.N. University, Hyderabad, April 2003)*
61. A train is required to run between stations 1.6 kms apart at an average speed of 40 km/hr. The run is to be made from a quadrilateral speed-time curve. The acceleration is 2 km/hr/sec. The coasting and braking retardations are 0.16 km/hr/sec and 3.2 km/hr/sec respectively. Determine the duration of acceleration, coasting and braking and the distance covered in each period.  
*(J.N. University, Hyderabad, April 2003)*
62. Explain the characteristics of D.C. compound motors and explain its advantage over the series motor. *(J.N. University, Hyderabad, April 2003)*
63. What are the requirements to be satisfied by an ideal traction system?  
*(J.N. University, Hyderabad, April 2003)*
64. What are the advantages and disadvantages of electrification of track?  
*(J.N. University, Hyderabad, April 2003)*

65. Discuss why a D.C. series motor is ideally suited for traction services.  
(J.N. University, Hyderabad, April 2003)
66. An electric locomotive of 100 tonnes can just accelerate a train of 500 tonnes (trailing weight) with an acceleration of 1 km/hr/sec on an up gradient 1 in 1000. Tractive resistance of the track is 45 newtons/tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find (i) the trailing weight that can be hauled up the same gradient, under the same condition (ii) the maximum gradient, the trailing hauled load remaining unchanged. Assume adhesive weight expressed as percentage of total dead weight to be same for both the locomotives.  
(J.N. University, Hyderabad, April 2003)
67. Explain how electric regeneration braking is obtained with a D.C. locomotive. How is the braking torque varied?  
(J.N. University, Hyderabad, April 2003)
68. Explain why a series motor is preferred for the electric traction.  
(J.N. University, Hyderabad, April 2003)
69. The characteristics of a series motor at 525 – V are as follows :
- |             |      |     |     |     |
|-------------|------|-----|-----|-----|
| Current (A) | 50   | 100 | 150 | 200 |
| Speed (RPM) | 1200 | 952 | 840 | 745 |
- Determine the current when working as a generator at 1000 R.P.M. and loaded with a resistance of 3 ohms. The resistance of the motor is 0.5 ohms. (J.N. University, Hyderabad, April 2003)
70. Briefly explain the a.c. motors used in traction. (J.N. University, Hyderabad, April 2003)
71. The scheduled speed of a trolley service is to be 53 km/hr. The distance between stops is 2.8 km. The track is level and each stop is of 30 sec duration. Using simplified speed-time curve, calculate the maximum speed, assuming the acceleration to be 2 km/hr/sec, retardation 3.2 km/hr/sec, the dead weight of the car as 16 tonnes, rotational inertia as 10% of the dead weight and track resistance as 40 newtons/tonne. If the overall efficiency is 80%, calculate (i) the maximum power output from the driving axles (ii) the specific energy consumption in watt-hr/tonne-km.  
(J.N. University, Hyderabad, April 2003)
72. Discuss various traction systems you know of?  
(J.N. University, Hyderabad, December 2002/January 2003)
73. Explain the requirements for ideal traction and show which drive satisfies almost all the requirements.  
(J.N. University, Hyderabad, December 2002/January 2003)
74. Define the adhesive weight of a locomotive which accelerates up a gradient of 1 in 100 at 0.8 kmphs. The self weight of locomotive is 350 Tonnes. Coefficient of adhesion is 0.25. Assume a train resistance of 45 N–m/Tonne and allow 10% for the effect of rotational inertia.  
(J.N. University, Hyderabad, December 2002/January 2003)
75. State Factors affecting specific energy consumption.  
(J.N. University, Hyderabad, December 2002/January 2003)
76. Explain with the help of a diagram, the four quadrant speed-torque characteristic of an induction motor when running in (i) forward direction (ii) reverse direction.  
(J.N. University, Hyderabad, December 2002/January 2003)
77. Explain the general features of traction motors.  
(J.N. University, Hyderabad, December 2002/January 2003)
78. A 250 tonne electric train maintains a scheduled speed of 30 kmph between stations situated 5 km apart, with station stops of 30 sec. The acceleration is 1.8 kmph ps and the braking retardation is 3 kmph ps. Assuming a trapezoidal speed-time curve, calculate (i) maximum speed of the train (ii) energy output of the motors if the tractive resistance is 40 NW per tonne.  
(J.N. University, Hyderabad, December 2002/January 2003)
79. Discuss the relative merits of electric traction and the factors on which the choice of traction system depends.  
(J.N. University, Hyderabad, December 2002/January 2003)
80. Explain the terms (i) tractive effort (ii) coefficient of adhesion (iii) specific energy consumption of train (iv) tractive resistance. (J.N. University, Hyderabad, December 2002/January 2003)



81. Existing traction systems in India. *(J.N. University, Hyderabad, December 2002/January 2003)*
82. Explain the terms tractive effort, coefficient of adhesion, train resistance and specific energy consumption of train. *(J.N. University, Hyderabad, December 2002/January 2003)*
83. An electric train maintains a scheduled speed of 40 kmph between stations situated at 1.5 km apart. It is accelerated at 1.7 kmph.ps and is braked at 3.2 kmph.ps. Draw the speed-time curve for the run. Estimate the energy consumption at the axle of the train. Assume tractive resistance constants at 50 NW per tonne and allow 10% for the effect of rotation inertia. *(J.N. University, Hyderabad, December 2002/January 2003)*
84. Explain the advantages of series parallel control of starting as compared to the rheostatic starting for a pair of dc traction motors. *(J.N. University, Hyderabad, December 2002/January 2003)*
85. Discuss the main features of various train services. What type of services correspond to trapezoidal and quadrilateral speed-time curves. *(J.N. University, Hyderabad, December 2002/January 2003)*
86. Existing electric traction system in India. *(J.N. University, Hyderabad, December 2002/January 2003)*
87. Briefly explain the controlling of D.C. Motor. *(Anna Univ., Chennai 2003)*

### OBJECTIVE TESTS – 43

- Diesel electric traction has comparatively limited overload capacity because
  - diesel electric locomotive is heavier than a plain electric locomotive
  - diesel engine has shorter life span
  - diesel engine is a constant-kW output prime mover
  - regenerative braking cannot be employed.
- The most vital factor against electric traction is the
  - necessity of providing a negative booster
  - possibility of electric supply failure
  - high cost of its maintenance
  - high initial cost of laying out overhead electric supply system.
- The direct current system used for tramways has a voltage of about .....volt.
  - 750
  - 1500
  - 3000
  - 2400
- In electric traction if contact voltage exceeds 1500 V, current collection is invariably via a
  - contact rail
  - overhead wire
  - third rail
  - conductor rail.
- For the single-phase ac system of track electrification, low frequency is desirable because of the following advantages
  - it improves commutation properties of ac motors
  - it increases ac motor efficiency
  - it increases ac motor power factor
  - all of the above.
- In Kando system of track electrification, .....is converted into .....
  - 1-phase ac, dc
  - 3-phase ac, 1-phase ac
  - 1-phase ac, 3-phase ac
  - 3-phase ac, dc.
- The main reason for choosing the composite 1-phase ac-to-dc system for all future track electrification in India is that it
  - needs less number of sub-stations
  - combines the advantages of high-voltage ac distribution at 50 Hz with dc series traction motors
  - provides flexibility in the location of sub-stations
  - requires light overhead catenary.
- Ordinary, tramway is the most economical means of transport for
  - very dense traffic of large city
  - medium traffic densities
  - rural services
  - suburban services.
- Unlike a tramway, a trolleybus requires no
  - overhead contact wire
  - driving axles
  - hand brakes
  - running rail.



10. The current collector which can be used at different speeds under all wind conditions and stiffness of OHE is called ..... collector.
- (a) trolley
  - (b) bow
  - (c) pantograph
  - (d) messenger.
11. The speed/time curve for city service has no..... period.
- (a) coasting
  - (b) free-running
  - (c) acceleration
  - (d) braking.
12. For the same value of average speed, increase in the duration of stops..... speed.
- (a) increases the schedule
  - (b) increases the crest
  - (c) decreases the crest
  - (d) decreases the schedule.
13. A train weighing 490 tonne and running at 90 km/h has a mass of ..... kg and a speed of ..... m/s.
- (a) 50,000, 25
  - (b) 490,000, 25
  - (c) 490, 25
  - (d) 50, 324.
14. A train has a mass of 500 tonne. Its weight is
- (a) 500 t.wt
  - (b) 500,000 kg-wt
  - (c) 4,900,000 newton
  - (d) all of the above
  - (e) none of the above.
15. The free-running speed of a train does NOT depend on the
- (a) duration of stops
  - (b) distance between stops
  - (c) running time
  - (d) acceleration.
16. A motor coach weighing 100 tonnes is to be given an acceleration of 1.0 km/h/s on an ascending gradient of 1 percent. Neglecting rotational inertia and train resistance, the tractive force required is ..... newton.
- (a) 109,800
  - (b) 37,580
  - (c) 28,760
  - (d) 125,780.
17. In a train, the energy output of the driving axles is used for
- (a) accelerating the train
  - (b) overcoming the gradient
  - (c) overcoming train resistance
  - (d) all of the above.
18. Longer coasting period for a train results in
- (a) higher acceleration
  - (b) higher retardation
  - (c) lower specific energy consumption
  - (d) higher schedule speed.
19. Tractive effort of an electric locomotive can be increased by
- (a) increasing the supply voltage
  - (b) using high kW motors
  - (c) increasing dead weight over the driving axles
  - (d) both (b) and (c) (e) both (a) and (b).
20. Skidding of a vehicle always occurs when
- (a) braking effort exceeds its adhesive weight
  - (b) it negotiates a curve
  - (c) it passes over points and crossings
  - (d) brake is applied suddenly.
21. Which of the following is an advantage of electric traction over other methods of traction?
- (a) Faster acceleration
  - (b) No pollution problems
  - (c) Better braking action
  - (d) All of the above
22. Which of the following is the voltage for single phase A.C. system?
- (a) 22 V
  - (b) 440 V
  - (c) 5 kV
  - (d) 15 kV
  - (e) None of the above
23. Long distance railways use which of the following?
- (a) 200 V D.C.
  - (b) 25 kV single phase A.C.
  - (c) 25 kV two phase A.C.
  - (d) 25 kV three phase A.C.
24. The speed of a locomotive is controlled by
- (a) flywheel
  - (b) gear box
  - (c) applying brakes
  - (d) regulating steam flow to engine

25. Main traction system used in India are, those using  
(a) electric locomotives  
(b) diesel engine locomotives  
(c) steam engine locomotives  
(d) diesel electric locomotives  
(e) all of the above
26. In India diesel locomotives are manufactured at  
(a) Ajmer  
(b) Varanasi  
(c) Bangalore  
(d) Jamalpur
27. For diesel locomotives the range of horsepower is  
(a) 50 to 200  
(b) 500 to 1000  
(c) 1500 to 2500  
(d) 3000 to 5000
28. .... locomotive has the highest operational availability.  
(a) Electric  
(b) Diesel  
(c) Steam
29. The horsepower of steam locomotives is  
(a) upto 1500  
(b) 1500 to 2000  
(c) 2000 to 3000  
(d) 3000 to 4000
30. The overall efficiency of steam locomotive is around  
(a) 5 to 10 percent  
(b) 15 to 20 percent  
(c) 25 to 35 percent  
(d) 35 to 45 percent
31. In tramways which of the following motors is used?  
(a) D.C. shunt motor  
(b) D.C. series motor  
(c) A.C. three phase motor  
(d) A.C. single phase capacitor start motor
32. In a steam locomotive electric power is provided through  
(a) overhead wire  
(b) battery system  
(c) small turbo-generator  
(d) diesel engine generator
33. Which of the following drives is suitable for mines where explosive gas exists?  
(a) Steam engine  
(b) Diesel engine  
(c) Battery locomotive  
(d) Any of the above
34. In case of locomotives the tractive power is provided by  
(a) single cylinder double acting steam engine  
(b) double cylinder, single acting steam engine  
(c) double cylinder, double acting steam engine  
(d) single stage steam turbine
35. Overload capacity of diesel engines is usually restricted to  
(a) 2 percent  
(b) 10 percent  
(c) 20 percent  
(d) 40 percent
36. In case of steam engines the steam pressure is  
(a) 1 to 4 kgf/cm<sup>2</sup>  
(b) 5 to 8 kgf/cm<sup>2</sup>  
(c) 10 to 15 kgf/cm<sup>2</sup>  
(d) 25 to 35 kgf/cm<sup>2</sup>
37. The steam engine provided on steam locomotives is  
(a) single acting condensing type  
(b) single acting non-condensing type  
(c) double acting condensing type  
(d) double acting non-condensing type
38. Electric locomotives in India are manufactured at  
(a) Jamalpur  
(b) Bangalore  
(c) Chittranjan  
(d) Gorakhpur
39. The wheels of a train, engine as well as bogies, are slightly tapered to  
(a) reduce friction  
(b) increase friction  
(c) facilitate braking  
(d) facilitate in taking turns
40. Automatic signalling is used for which of the following trains?

- (a) Mail and express trains
  - (b) Superfast trains
  - (c) Suburban and Urban electric trains
  - (c) All trains
41. The efficiency of diesel locomotives is nearly
- (a) 20 to 25 percent
  - (b) 30 to 40 percent
  - (c) 45 to 55 percent
  - (d) 60 to 70 percent
42. The speed of a superfast train is
- (a) 60 kmph
  - (b) 75 kmph
  - (c) 100 kmph
  - (d) more than 100 kmph
43. The number of passenger coaches that can be attached to a diesel engine locomotive on broad gauge is usually restricted to
- (a) 5
  - (b) 10
  - (c) 14
  - (d) 17
44. Which of the following state capitals is not on broad gauge track?
- (a) lucknow
  - (b) Bhopal
  - (c) Jaipur
  - (d) Chandigarh
45. Which of the following is the advantage of electric braking?
- (a) It avoids wear of track
  - (b) Motor continues to remain loaded during braking
  - (c) It is instantaneous
  - (d) More heat is generated during braking
46. Which of the following braking systems on the locomotives is costly?
- (a) Regenerative braking on electric locomotives
  - (b) Vacuum braking on diesel locomotives
  - (c) Vacuum braking on steam locomotives
  - (d) All braking systems are equally costly
47. Tractive effort is required to
- (a) overcome the gravity component of train mass
  - (b) overcome friction, windage and curve resistance
  - (c) accelerate the train mass
  - (d) do all of the above
48. For given maximum axle load tractive efforts of A.C. locomotive will be
- (a) less than that of D.C. locomotive
  - (b) more than that of D.C. locomotive
  - (c) equal to that of D.C. locomotive
  - (d) none of the above
49. Co-efficient of adhesion reduces due to the presence of which of the following?
- (a) Sand on rails
  - (b) Dew on rails
  - (c) Oil on the rails
  - (d) both (b) and (c)
50. Due to which of the following co-efficient of adhesion improves?
- (a) Rust on the rails
  - (b) Dust on the rails
  - (c) Sand on the rails
  - (d) All of the above
51. Quadrilateral speed-time curve pertains to which of the following services?
- (a) Main line service
  - (b) Urban service
  - (c) Sub-urban service
  - (d) Urban and sub-urban service
52. Which of the following is the disadvantage of electric traction over other systems of traction?
- (a) Corrosion problems in the underground pip work
  - (b) Short time power failure interrupts traffic for hours
  - (c) High capital outlay in fixed installations beside route limitation
  - (d) Interference with communication lines
  - (e) All of the above
53. Co-efficient of adhesion is
- (a) high in case of D.C. traction than in the case of A.C. traction
  - (b) low in case of D.C. traction than in the case of A.C. traction
  - (c) equal in both A.C. and D.C. traction
  - (d) any of the above
54. Speed-time curve of main line service differs from those of urban and suburban services on following account
- (a) it has longer free running period
  - (b) it has longer coasting period

- (c) accelerating and braking periods are comparatively smaller  
(d) all of the above
55. The rate of acceleration on suburban or urban services is restricted by the consideration of  
(a) engine power  
(b) track curves  
(c) passenger discomfort  
(d) track size
56. The specific energy consumption of a train depends on which of the following?  
(a) Acceleration and retardation  
(b) Gradient  
(c) Distance covered  
(d) all of the above
57. The friction at the track is proportional to  
(a)  $1/\text{speed}$   
(b)  $1/(\text{speed})^2$   
(c) speed  
(d) none of the above
58. The air resistance to the movement of the train is proportional to  
(a) speed  
(b)  $(\text{speed})^2$   
(c)  $(\text{speed})^3$   
(d)  $1/\text{speed}$
59. The normal value of adhesion friction is  
(a) 0.12  
(b) 0.25  
(c) 0.40  
(d) 0.75
60. The pulsating torque exerted by steam locomotives causes which of the following?  
(a) Jolting and skidding  
(b) Hammer blow  
(c) Pitching  
(d) All of the above
61. Which of the following braking systems is used on steam locomotives?  
(a) Hydraulic system  
(b) Pneumatic system  
(c) Vacuum system  
(d) None of the above
62. Vacuum is created by which of the following?  
(a) Vacuum pump  
(b) Ejector  
(c) Any of the above  
(d) None of the above
63. The resistance encountered by a train in motion is on account of  
(a) resistance offered by air  
(b) friction at the track  
(c) friction at various parts of the rolling stock  
(d) all of the above
64. Battery operated trucks are used in  
(a) steel mills  
(b) power stations  
(c) narrow gauge traction  
(d) factories for material transportation
65. .... method can bring the locomotive to dead stop.  
(a) Plugging braking  
(b) Rheostatic braking  
(c) Regenerative braking  
(d) None of the above
66. The value of co-efficient of adhesion will be high when rails are  
(a) greased  
(b) wet  
(c) sprayed with oil  
(d) cleaned with sand
67. The voltage used for suburban trains in D.C. system is usually  
(a) 12 V  
(b) 24 V  
(c) 220 V  
(d) 600 to 750 V
68. For three-phase induction motors which of the following is the least efficient method of speed control?  
(a) Cascade control  
(b) Pole changing  
(c) Rheostatic control  
(d) Combination of cascade and pole changing
69. Specific energy consumption becomes  
(a) more on steeper gradient  
(b) more with high train resistance  
(c) less if distance between stops is more  
(d) all of the above
70. In main line service as compared to urban and suburban service

- (a) distance between the stops is more
  - (b) maximum speed reached is high
  - (c) acceleration and retardation rates are low
  - (d) all of the above
71. Locomotive having monomotor bogies
- (a) has better co-efficient of adhesion
  - (b) are suited both for passenger as well as freight service
  - (c) has better riding qualities due to the reduction of lateral forces
  - (d) has all above qualities
72. Series motor is not suited for traction duty due to which of the following account?
- (a) Less current drain on the heavy load torque
  - (b) Current surges after temporary switching off supply
  - (c) self relieving property
  - (d) Commutating property at heavy load
73. When a bogie negotiates a curve, reduction in adhesion occurs resulting in sliding. Thus sliding is acute when
- (a) wheel base of axles is more
  - (b) degree of curvature is more
  - (c) both (a) and (b)
  - (d) none of the above
74. Energy consumption in propelling the train is required for which of the following?
- (a) Work against the resistance to motion
  - (b) Work against gravity while moving up the gradient
  - (c) Acceleration
  - (d) All of the above
75. An ideal traction system should have .....
- (a) easy speed control
  - (b) high starting tractive effort
  - (c) equipment capable of with standing large temporary loads
  - (d) all of the above
76. .... have maximum unbalanced forces
- (a) Diesel shunters
  - (b) Steam locomotives
  - (c) Electric locomotives
  - (d) Diesel locomotives
77. Specific energy consumption is affected by which of the following factors?
- (a) Retardation and acceleration values
  - (b) Gradient
  - (c) Distance between stops
  - (d) All of the above
78. In case of ..... free running and coasting periods are generally long.
- (a) main-line service
  - (b) urban service
  - (c) sub-urban service
  - (d) all of the above
79. Overhead lines for power supply to tramcars are at a minimum height of
- (a) 3 m
  - (b) 6 m
  - (c) 10 m
  - (d) 20 m
80. The return circuit for tram cars is through .....
- (a) neutral wire
  - (b) rails
  - (c) cables
  - (d) common earthing
81. Specific energy consumption is least in ..... service.
- (a) main line
  - (b) urban
  - (c) suburban
82. Locomotives with monomotor bogies have
- (a) uneven distribution of tractive effort
  - (b) suitability for passenger as well as freight service
  - (c) lot of skidding
  - (d) low co-efficient of adhesion
83. .... was the first city in India to adopt electric traction.
- (a) Delhi
  - (b) Madras
  - (c) Calcutta
  - (d) Bombay
84. .... frequency is not common in low frequency traction system
- (a) 40 Hz
  - (b) 25 Hz
  - (c) 16 Hz
85. For 25 kV single phase system power supply frequency is .....
- (a) 60 Hz
  - (b) 50 Hz
  - (c) 25 Hz
  - (d)  $16 \frac{2}{3}$  Hz

86. Power for lighting in passenger coach, in a long distance electric train, is provided
- (a) directly through overhead electric
  - (b) through individual generator of bogie and batteries
  - (c) through rails
  - (d) through locomotive
87. In India, electrification of railway track was done for the first time in which of the following years?
- (a) 1820–1825
  - (b) 1880–1885
  - (c) 1925–1932
  - (d) 1947–1954
88. Suri transmission is .....
- (a) electrical-pneumatic
  - (b) mechanical-electrical
  - (c) hydro-mechanical
  - (d) hydro-pneumatic
89. In case of a steam engine an average coal consumption per km is nearly
- (a) 150 to 175 kg
  - (b) 100 to 120 kg
  - (c) 60 to 80 kg
  - (d) 28 to 30 kg
90. Which of the following happens in Kando system?
- (a) Three phase A.C. is converted into D.C.
  - (b) Single phase A.C. is converted into D.C.
  - (c) Single phase supply is converted into three phase system
  - (d) None of the above
91. For which of the following locomotives the maintenance requirements are the least?
- (a) Steam locomotives
  - (b) Diesel locomotives
  - (c) Electric locomotives
  - (d) Equal in all of the above
92. Which of the following methods is used to control speed of 25 kV, 50 Hz single phase traction?
- (a) Reduced current method
  - (b) Tapchanging control of transformer
  - (c) Series parallel operation of motors
  - (d) All of the above
93. If the coefficient of adhesion on dry rails is 0.26, which of the following could be the value for wet rails?
- (a) 0.3
  - (b) 0.26
  - (c) 0.225
  - (d) 0.16
94. .... watt-hours per tonne km is usually the specific energy consumption for suburban services.
- (a) 15–20
  - (b) 50–75
  - (c) 120–150
  - (d) 160–200
95. The braking retardation is usually in the range
- (a) 0.15 to 0.30 km phps
  - (b) 0.30 to 0.6 km phps
  - (c) 0.6 to 2.4 km phps
  - (d) 3 to 5 km phps
  - (e) 10 to 15 km phps
96. The rate of acceleration on suburban or urban service is in the range
- (a) 0.2 to 0.5 km phps
  - (b) 1.6 to 4.0 km phps
  - (c) 5 to 10 km phps
  - (d) 15 to 25 km phps
97. The coasting retardation is around
- (a) 0.16 km phps
  - (b) 1.6 km phps
  - (c) 16 km phps
  - (d) 40 km phps
98. which of the following track is electrified
- (a) Delhi–Bombay
  - (b) Delhi–Madras
  - (c) Delhi–Howrah
  - (d) Delhi–Ahmedabad
99. .... is the method of braking in which motor armature remains connected to the supply and draws power from it producing torque opposite to the direction of motion.
- (a) Rheostatic braking
  - (b) Regenerative braking
  - (c) Plugging
100. For 600 V D.C. line for tramcars, track is connected to .....
- (a) positive of the supply
  - (b) negative of the supply
  - (c) mid voltage of 300 V
  - (d) none of the above

## ANSWERS

- |         |         |         |         |         |         |         |         |         |          |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| 1. (c)  | 2. (d)  | 3. (a)  | 4. (b)  | 5. (d)  | 6. (c)  | 7. (b)  | 8. (a)  | 9. (d)  | 10. (c)  |
| 11. (b) | 12. (d) | 13. (b) | 14. (d) | 15. (a) | 16. (b) | 17. (d) | 18. (c) | 19. (d) | 20. (a)  |
| 21. (d) | 22. (d) | 23. (b) | 24. (d) | 25. (e) | 26. (b) | 27. (c) | 28. (a) | 29. (a) | 30. (a)  |
| 31. (b) | 32. (c) | 33. (c) | 34. (c) | 35. (b) | 36. (c) | 37. (d) | 38. (c) | 39. (d) | 40. (c)  |
| 41. (a) | 42. (d) | 43. (d) | 44. (c) | 45. (a) | 46. (a) | 47. (d) | 48. (b) | 49. (d) | 50. (d)  |
| 51. (d) | 52. (e) | 53. (b) | 54. (d) | 55. (c) | 56. (d) | 57. (c) | 58. (b) | 59. (b) | 60. (a)  |
| 61. (c) | 62. (c) | 63. (d) | 64. (d) | 65. (a) | 66. (d) | 67. (d) | 68. (c) | 69. (d) | 70. (d)  |
| 71. (d) | 72. (b) | 73. (c) | 74. (d) | 75. (d) | 76. (b) | 77. (d) | 78. (a) | 79. (c) | 80. (b)  |
| 81. (a) | 82. (b) | 83. (d) | 84. (a) | 85. (b) | 86. (b) | 87. (c) | 88. (c) | 89. (d) | 90. (c)  |
| 91. (c) | 92. (b) | 93. (d) | 94. (b) | 95. (d) | 96. (b) | 97. (a) | 98. (c) | 99. (c) | 100. (b) |

## ROUGH WORK