MODIFIED DISTRIBUTION METHOD (MODI METHOD)

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INTRODUCTION

- In modified distribution method, cell evaluations of all the unoccupied cells are calculated simultaneously and only one closed path for the most negative cell is traced. Thus it provides considerable time saving over the stepping stone method.
- It provides a new means of finding the unused route with the largest negative improvement index.

Finding the Optimal Solution

- Once an initial solution has been found, the next step is to test that solution for optimality. The following two methods are widely used for testing the solutions:
  - ❑ Stepping Stone Method
  - ❑ Modified Distribution Method
- The two methods differ in their computational approach but give exactly the same results and use the same testing procedure.

Steps For MODI Method

1) Determine an initial basic feasible solution using any one of the three methods given below:

a) North-West corner method

b) Lest cost method

c) Vogel's approximation method

2) Determine the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$

Steps For MODI Method

3) Compute the opportunity cost using $c_{ij} - (u_i + v_j)$.
4) Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.

Steps For MODI Method

5) Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.
6) Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

Steps For MODI Method

7) Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
8) Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell.

Steps For MODI Method

9) Now, **add** this quantity to all the cells on the corner points of the closed path marked with plus signs, and **subtract** it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.
10) Repeat the whole procedure until an optimal solution is obtained.

EXAMPLE

- Building Brick Company (BBC) has orders for 80 tons of bricks at three suburban locations as follows: Northwood -- 25 tons, Westwood -- 45 tons, and Eastwood -- 10 tons. BBC has two plants, each of which can produce 50 tons per week.
- How should end of week shipments be made to fill the above orders given the following delivery cost per ton:

	Northwood	Westwood	Eastwood
Plant 1	24	30	40
Plant 2	30	40	42

- Initial Transportation Table
- Since total supply = 100 and total demand = 80, a dummy destination is created with demand of 20 and 0 unit costs.

	Northwood	Westwood	Eastwood	Dummy	Supply
Plant 1	24	30	40	0	50
Plant 2	30	40	42	0	50
Demand	25	45	10	20	

- Least Cost Starting Procedure
  - Iteration 1: Tie for least cost (0), arbitrarily select x_{14}. Allocate 20. Reduce s_1 by 20 to 30 and delete the Dummy column.
  - Iteration 2: Of the remaining cells the least cost is 24 for x_{11}. Allocate 25. Reduce s_1 by 25 to 5 and eliminate the Northwood column.
  - Iteration 3: Of the remaining cells the least cost is 30 for x_{12}. Allocate 5. Reduce the Westwood column to 40 and eliminate the Plant 1 row.
  - Iteration 4: Since there is only one row with two cells left, make the final allocations of 40 and 10 to x_{22} and x_{23}, respectively.

- Initial table

	Northwood	Westwood	Eastwood	Dummy	Supply
Plant 1	25 [24]	5 [30]	[40]	20 [0]	50
Plant 2	[30]	40 [40]	10 [42]	[0]	50
Demand	25	45	10	20	

- Total transportation cost is Rs2770

- Iteration 1,MODI Method

1) Set $u_1 = 0$
2) Since $u_1 + v_j = c_{1j}$ for occupied cells in row 1, then $v_1 = 24$, $v_2 = 30$, $v_4 = 0$.
3) Since $u_i + v_2 = c_{i2}$ for occupied cells in column 2, then $u_2 + 30 = 40$, hence $u_2 = 10$.
4) Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then,$10 + v_3 = 42$, hence $v_3 = 32$.

- Iteration 1
  - MODI Method (continued)
- Calculate the reduced costs (circled numbers on the next slide) by $c_{ij} - u_i + v_j$.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 32 = 8
(2,1)	30 - 24 -10 = -4
(2,4)	0 - 10 - 0 = -10

- Since some of the reduced cost are negative, the current solution is not optimal.
- Cell (2,4) has the most negative; therefore, it will be the basic variable that must be occupied in the next iteration.

- Iteration 1 Table

	Northwood	Westwood	Eastwood	Dummy	u_i
Plant 1	25 (24)	5 (30)	+8 (40)	20 (0)	0
Plant 2	-4 (30)	40 (40)	10 (42)	-10 (0)	10
v_j	24	30	32	0	

- Iteration 2,MODI Method
- The reduced costs are found by calculating the u_i's and v_j's for this tableau.

1) Set $u_1 = 0$.
2) Since $u_1 + v_j = c_{ij}$ for occupied cells in row 1, then $v_1 = 24$, $v_2 = 30$.
3) Since $u_i + v_2 = c_{i2}$ for occupied cells in column 2, then $u_2 + 30 = 40$, or $u_2 = 10$.
4) Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then $10 + v_3 = 42$ or $v_3 = 32$; and, $10 + v_4 = 0$ or $v_4 = -10$.

- Iteration 2
  - MODI Method (continued)
- Calculate the reduced costs (circled numbers on the next slide) by c_{ij} - $u_i + v_j$.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 32 = 8
(1,4)	0 - 0 - (-10) = 10
(2,1)	30 - 10 - 24 = -4

- Since there is still negative reduced cost for cell (2,1), the solution is not optimal.
- Cell (2,1) must be occupied

- Iteration 2 Table

	Northwood	Westwood	Eastwood	Dummy	u_i
Plant 1	25 [24]	25 [30]	+8 [40]	+10 [0]	0
Plant 2	-4 [30]	20 [40]	10 [42]	20 [0]	10
v_j	24	30	36	-6	

- Iteration 3,MODI Method
- The reduced costs are found by calculating the u_i's and v_j's for this table.

1) Set $u_1 = 0$
2) Since $u_1 + v_j = c_{1j}$ for occupied cells in row 1, then $v_1 = 24$ and $v_2 = 30$.
3) Since $u_i + v_1 = c_{i1}$ for occupied cells in column 2, then $u_2 + 24 = 30$ or $u_2 = 6$.
4) Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2,then $6 + v_3 = 42$ or $v_3 = 36$, and $6 + v_4 = 0$ or $v_4 = -6$.

- Iteration 3
  - MODI Method (continued)
- Calculate the reduced costs (circled numbers on the next slide) by $c_{ij} - u_i + v_j$.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 36 = 4
(1,4)	0 - 0 - (-6) = 6
(2,2)	40 - 6 - 30 = 4

- Since all the reduced cost are nonnegative, the current solution is optimal

- Iteration 3 Table
- Since all the reduced costs are non-negative, this is the optimal table.

	Northwood	Westwood	Eastwood	Dummy	u_i
Plant 1	24 5	30 45	40 +4	0 +6	0
Plant 2	30 20	40 +4	42 10	0 20	6
v_j	24	30	36	-6	

Optimal Solution

From	To	Amount	Cost
Plant 1	Northwood	5	120
Plant 1	Westwood	45	1,350
Plant 2	Northwood	20	600
Plant 2	Eastwood	10	420
		Total Cost =	RS.2,490