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HISTORY OF FUZZY LOGIC

- **1964: Lotfi A. Zadeh, UC Berkeley, introduced the paper on fuzzy sets.**
  - Idea of grade of membership was born
  - Sharp criticism from academic community
    - Name!
    - Theory's emphasis on imprecision
  - Waste of government funds!

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

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HISTORY OF FUZZY LOGIC

- **1965-1975: Zadeh continued to broaden the foundation of fuzzy set theory**
  - Fuzzy multistage decision-making
  - Fuzzy similarity relations
  - Fuzzy restrictions
  - Linguistic hedges
- **1970s: research groups were form in JAPAN**

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

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HISTORY OF FUZZY LOGIC

- **1974:** Mamdani, United Kingdom, developed the first fuzzy logic controller
- **1977:** Dubois applied fuzzy sets in a comphrensive study of traffic conditions
- **1976-1987:** Industrial application of fuzzy logic in Japan and Europe
- **1987-Present:** Fuzzy Boom

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

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FUZZY LOGIC: MOTIVATIONS

- Alleviate difficulties in developing and analyzing complex systems encountered by conventional mathematical tools.
- Observing that human reasoning can utilize concepts and knowledge that do not have well-defined, sharp boundaries.

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

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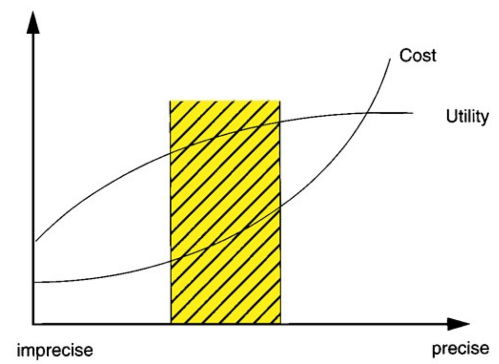
FUZZY LOGIC APPLICATIONS



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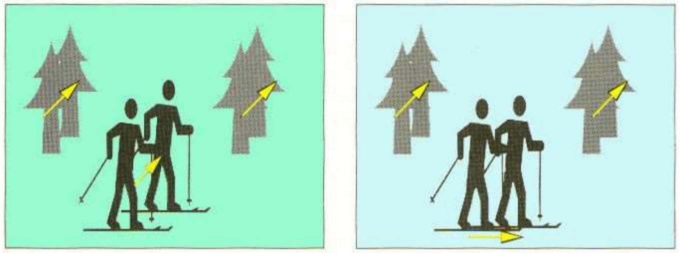
FUZZY LOGIC APPLICATIONS

Image Stabilization via Fuzzy Logic



"If all motion vectors are almost parallel and their time differential is small, then the hand jittering is detected and the direction of the hand movement is in the direction of the moving vectors".

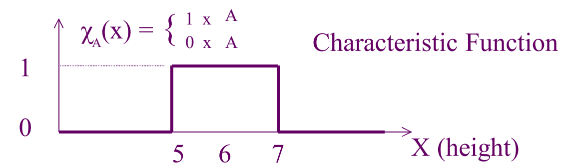
*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

CHANCE & AMBIGUITY

- **Suppose you are seated at a table on which rest two glasses of liquid.**
  - **First glass is described : "having a 95% chance Of being healthful and good"**
  - **Second glass is described : "having a .95 membership in the class of healthful and good"**
- **Which glass would you select,** keeping in mind that the first glass has a 5 % chance of being filled with nonhealthful liquids, including poisons **[Bezdek 1993]?**

CLASSICAL SET (CRISP)

- **Contain objects that satisfy precise properties of membership.**
  - **Example: Set of heights from 5 to 7 feet**



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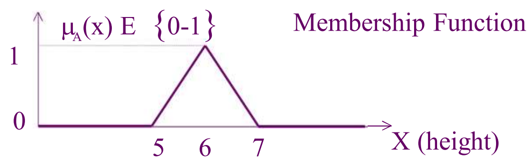
FUZZY SET

- Contain objects that satisfy imprecise properties of membership
  - Example : The set of heights in the region around 6 feet



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FUZZY: SET & MEASURE

- **Fuzzy set** provides a basic mathematical framework for dealing with vagueness

- **Fuzzy measure** provides a general framework for dealing ambiguity

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PREDICATE LOGIC

- **Logical connectives:**
  - **Union**
    - **A U B = max(χ_a (x) , χ_b (x))**
  - **Intersection**
    - **A . B = min(χ_a (x) , χ_b (x))**
  - **Complementary**
    - **A ---> χ_a (x) = 1- χ_a (x)**

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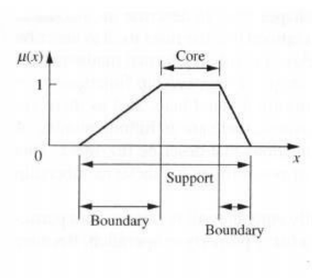
FEATURES OF THE MEMBERSHIP

Function

- **Core:** comprises those elements x of the universe such that μ_a (x) = 1.
- **Support :** region of the universe that is characterized by nonzero membership.
- **Boundary :** boundaries comprise those elements x of the universe such that 0< μ_a (x) <1



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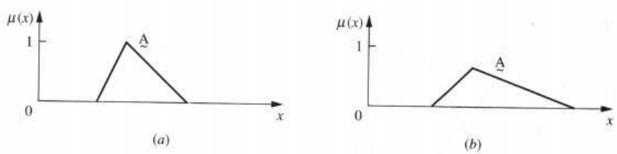
FEATURES OF THE MEMBERSHIP

Function (Cont.)

- **Normal Fuzzy Set** : at least one element x in the universe whose membership value is unity



Fuzzy sets that are normal (a) and subnormal (b).

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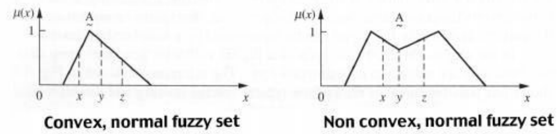
FEATURES OF THE MEMBERSHIP

Function (Cont.)

- **Convex Fuzzy set:** membership values are strictly monotonically increasing, or strictly monotonically decreasing, or strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe.



$\mu_a(y) \geq \min[\mu_a(x), \mu_a(z)]$

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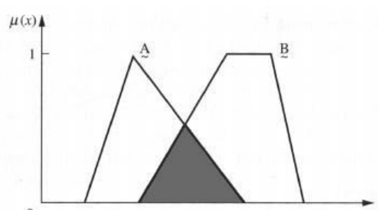
FEATURES OF THE MEMBERSHIP

Function (Cont.)

- **Special Property of two convex fuzzy set:**
  - **for A and B, which are both convex, A . B is also convex.**



The intersection of two convex fuzzy sets produces a convex fuzzy set.

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FEATURES OF THE MEMBERSHIP

Function (Cont.)

- **Cross-over points :** $\mu_a (x) = 0.5$

- **Height: defined as max** $\{\mu_a (x)\}$

BASIC CONCEPTS OF FUZZY LOGIC

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- Core technique is based on 4 basic concepts:
  - **Fuzzy sets: describe the value of variables**
  - **Linguistic variables: qualitatively and quantitatively described by fuzzy sets**
  - **Possibility distributions: constraints on the value of a linguistic variable**
  - **Fuzzy if-then rules: a knowledge**

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

DESIGN MEMBERSHIP FUNCTIONS

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- 3 ways
  - **Interview those who are familiar with the underlying concepts and later adjust**
    - **Tuned through a trial-and-error**
  - **Construct it automatically from data**
  - **Learn it based on feedback from the system performance**

*Fuzzy Logic: Intelligence, Control, and Information - J. Yen and R. Langari, Prentice Hall 1999

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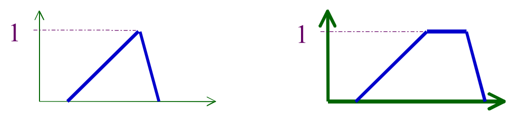
GUIDELINES FOR MEMBERSHIP

function design

- **Always use parameterizable membership functions. Do not define a membership function point by point.**
  - **Triangular and Trapezoid membership functions are sufficient for most practical applications!**



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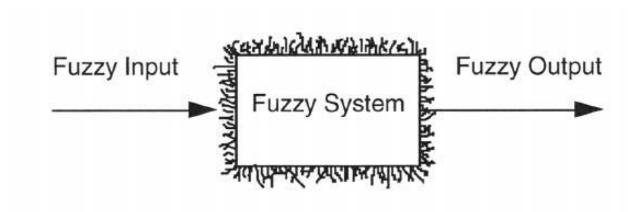
FUZZY SYSTEMS



How can fuzzy systems be used in a world where measurements and actions are expressed as crisp values?

* Fuzzy Systems Toolbox, M. Beale and H Demuth

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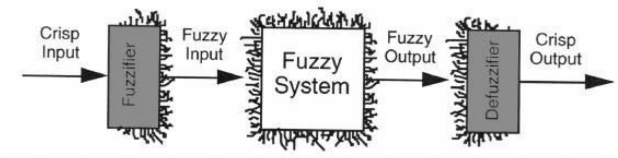
FUZZY SYSTEMS (CONT.)



* Fuzzify crisp inputs to get the fuzzy inputs
* Defuzzify the fuzzy outputs to get crisp outputs

* Fuzzy Systems Toolbox, M. Beale and H Demuth

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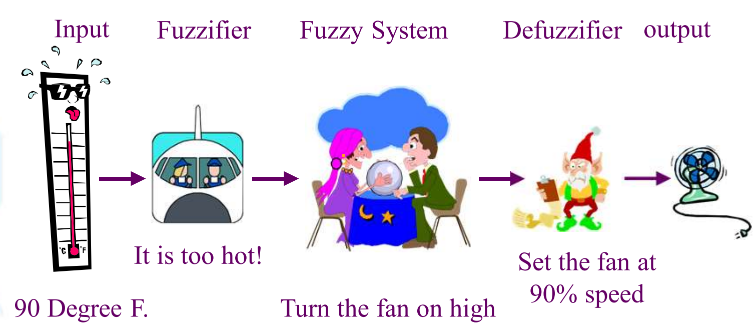
FUZZY SYSTEMS (CONT.)



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FUZZY SET: VECTOR REPRESENTATION

- Two vectors can represent fuzzy discrete sets or fuzzy continuous sets,
  - Support Vector (universe vector)
  - Grade Vector (membership vector)

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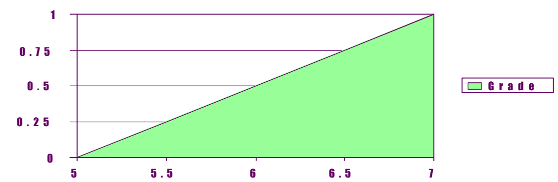
EXAMPLE: VECTOR REPRESENTATION

- Define the concept of "tall" over heights from 5 to 7 feet, using MATLAB
  - S = [5.00 5.50 6.00 6.50 7.00];
  - G = [0.00 0.25 0.50 0.75 1.00];



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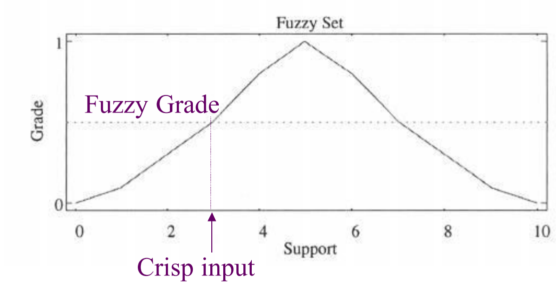
FUZZIFICATION

- Process of making a crisp quantity fuzzy
- Vector representation can be viewed as either a discrete or an approximation of a continuous set (use linear interpolation)



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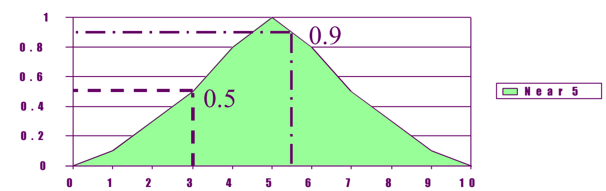
EXAMPLE: FUZZIFICATION

- Define fuzzy set "near 5"
  - S = [0:1:10];
  - G = [0.0 0.1 0.3 0.5 0.8 1 0.8 0.5 0.3 0.1 0];



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FUZZY SYSTEMS

- How do you make a machine smart?
  - Put some FAT in it!
  - A FAT enough machine can model any process
  - A FAT system can always turn inputs to outputs and turn causes to effects and turn questions to answer
- FAT stands for "Fuzzy Approximation Theorem"

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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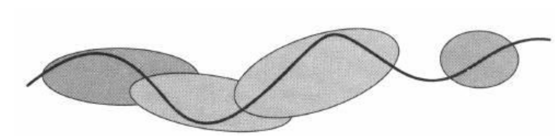
FUZZY SYSTEMS (CONT.)

- FAT idea has a simple geometry
  - Cover a curve with patches!



*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

FUZZY SYSTEMS (CONT.)

- Knowledge as rules
  - Each piece of human knowledge, each rule of the form
    - IF this then that defines a patch.
- All the rules define patches
  - Try to cover some wiggly curve
- The better the patches cover the curve, the smarter the system

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

KNOWLEDGE AS RULES

- How do you reason?
  - You want to play golf on Saturday or Sunday and you don't want to get wet when you play.
- Reach it with rules!
  - If it rains, you get wet!
  - If you get wet, you can't play golf
- If it rains on Saturday and won't rain on Sunday
  - You play golf on Sunday!

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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ARTIFICIAL INTELLIGENCE: AI

- Knowledge is rules
- Rules are in black-and-white language
  - Bivalent rules
- AI has so far, afer over 30 years of research, not produced smart machines!
  - Because they can't yet put enough rules in the computer (use 100-1000 rules, need >100k}
  - Throwing more rules at the problem

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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KNOWLEDGE AS RULES

- Fuzzy researchers have built hundreds of smart machines that work!
- Yes, we need rules
- No, we don't need a lot of rules for many tasks
- We need Fuzzy rules

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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KNOWLEDGE AS RULES

- Every term in one of our rules is Fuzzy
- Every term is vague, hazy, inexact, sloppy
- One human rule covers all these cases
  - AI rule covers one precise case

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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FUZZY SYSTEMS (CONT.)

- Fuzzy rule relates fuzzy sets
  - If X is A, then Y is B
    - A and B are fuzzy sets and subset of X and Y

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

BUILD A FUZZY SYSTEM

- 3 Steps
  - Pick the nouns or variables
    - Example: X be input and Y be output
      - Let x be temperature and Y be change in motor speed
    - Cause, effect. Stimulus, response!
  - Pick the fuzzy sets
    - Define fuzzy subsets of the nouns X and Y
  - Pick the fuzzy rules
    - Associate output to the input

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

EXAMPLE: BUILD A FUZZY SYSTEM

- Design motor speed controller for air conditioner
  - Step 1: assign input and output variables
    - Let X be temperature in Fahrenheit
    - Let Y be the change in motor speed of the air conditioner

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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EXAMPLE: BUILD A FUZZY SYSTEM

- Design motor speed controller for air conditioner
  - **Step 2: Pick fuzzy sets**
    - **Define subsets of the noun X and Y**
      - **Say 5 fuzzy sets on X**
        - » Cold, Cool, Just Right, Warm, and Hot
      - **Say 5 fuzzy sets on Y**
        - » Stop, Slow, Medium, Fast, and Blast

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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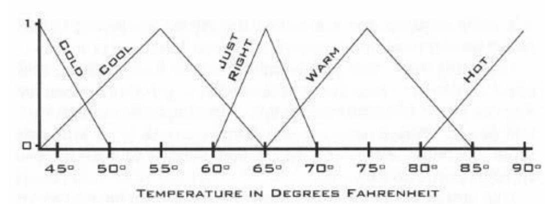
EXAMPLE: BUILD A FUZZY SYSTEM

- **Input Fuzzy set**



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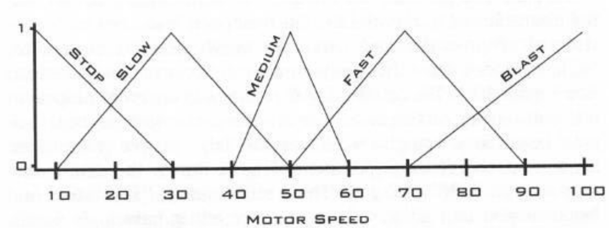
EXAMPLE: BUILD A FUZZY SYSTEM

- **Output Fuzzy set**



*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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EXAMPLE: BUILD A FUZZY SYSTEM

- Design motor speed controller for air conditioner
  - – Step 3: Assign a motor speed set to each temperature set

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EXAMPLE: BUILD A FUZZY SYSTEM

- **Rules**
  - If temperature is cold then motor speed stop
  - If temperature is cool then motor speed slows
  - If temperature is just right then motor speed is medium
  - If temperature is warm then motor speed is fast
  - If temperature is hot then motor speed blasts

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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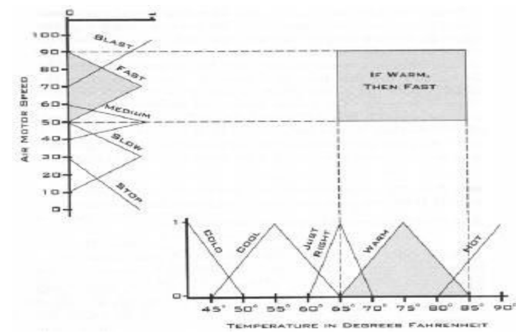
EXAMPLE: BUILD A FUZZY SYSTEM

- **Fuzzy Relation**



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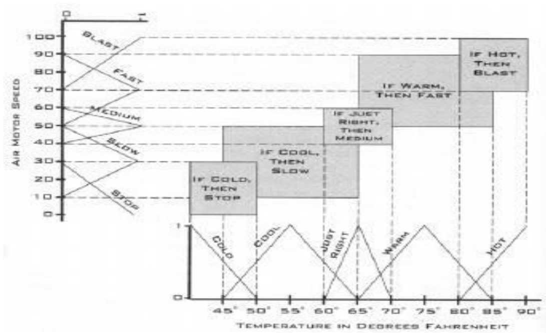
EXAMPLE: BUILD A FUZZY SYSTEM

- Fuzzy system with 5 patches



*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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FAT THEOREM

- You can always cover a curve with a finite number of fuzzy patches
  - Let the cuts overlap
  - Sloppy rules give big patches
  - Fine rules give small patches
- You pay for precision!

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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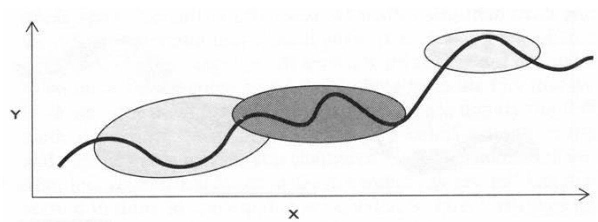
FAT THEOREM (CONT.)

- Rough cover of the non linear System



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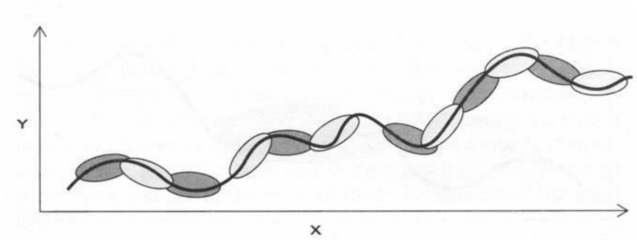
FAT THEOREM (CONT.)

- finer cover of the non linear System



*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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FUZZY ASSOCIATIVE MEMORY

- **Which rule "fires" or activates at which time?**
  - **They all fire all the time**
  - **They fire in parallel**
    - **All rules fire to some degree**
    - **Most fire to zero degree**
  - **The result is a fuzzy weighted average**

*Fuzzy Thinking:The new Science of Fuzzy Logic, Bart Kosko

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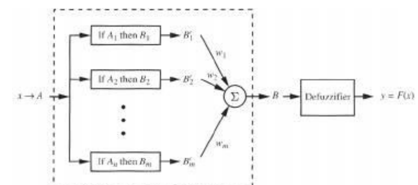
ADDITIVE FUZZY SYSTEM

- **Stores m fuzzy rules of the form**
  - **"If X = A_j then Y = B_j," then computes the output by defuzzifiy the summed (MAXed) of the partially fired then-part fuzzy sets B'_j**



*Fuzzy Engineering, Bart Kosko

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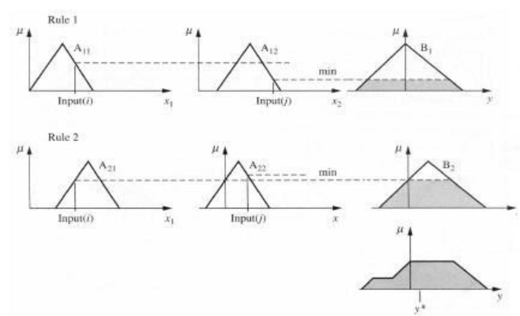
GRAPHICAL TECHNIQUE OF MAMDANI (MAX-MIN] INFERENCE

- **If x_1^k is A_1^k and x_2^k is A_2^k Then Y^k is B^k**



*Fuzzy Logic with Engineering Applications, Timothy J. Ross

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GRAPHICAL TECHNIQUE OF MAMDANI (MAX-MIN] INFERENCE

- **If x is A Then Y is B**
  - MATLAB
    - XS = support vector of X;
    - XG = grade vector of X;
    - YS = support vector of y;
    - YG = grade vector of X;
    - A = Crisp input;
    - B = ifthen_min(XS,XG,A,YG);
      - Where function B= ifthen_min (…); calculate
        - [YG_row,YG_column] = size(YG);
        - B = Min(fuzzify(XS,XG,A)*ones(1,YG_column),YG);

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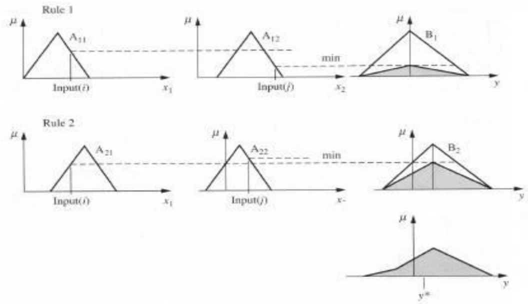
GRAPHICAL TECHNIQUE OF MAX-PRODUCT INFERENCE

- If x_1^k is A_1^k and x_2^k is A_2^k Then Y^k is B^k



*Fuzzy Logic with Engineering Applications, Timothy J. Ross

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GRAPHICAL TECHNIQUE OF MAX-PRODUCT INFERENCE

- If x is A Then Y is B
  - MATLAB
    - XS = support vector of X;
    - XG = grade vector of X;
    - YS = support vector of y;
    - YG = grade vector of X;
    - A = Crisp input;
    - B = ifthen_prod(XS,XG,A,YG);
      - Where function B= ifthen_prod (...); calculate
        - B = fuzzify(XS,XG,A) * YG;

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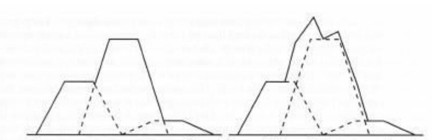
MAX: FUZZY UNION & FUZZY UNION USING ALGEBRAIC SUM

- **Variation on Fuzzy Implication**
  - When simulating human reasoning with fuzzy rules in decision making and expert systems, the **algebraic sum and product** often will give a more intuitively pleasing result



Max versus sum combination of fired then-part sets.

* Fuzzy Systems Toolbox, M. Beale and H Demuth, * *Fuzzy Engineering, Bart Kosko

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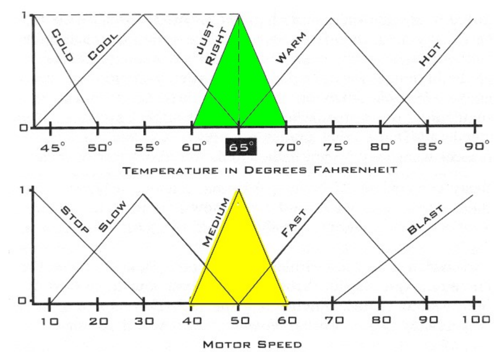
EXAMPLE: TEMP. = 65 DEGREE F.

If temperature is <u>just right</u> then motor speed is <u>medium</u>



*Fuzzy Thinkring, Bart Kosko

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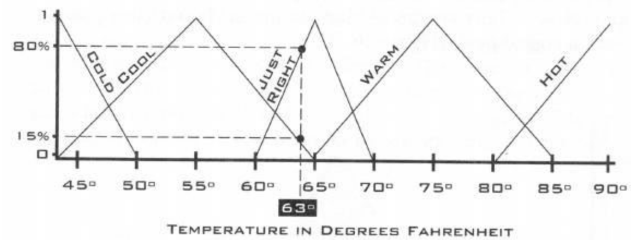
EXAMPLE: TEMP. = 63 DEGREE F.



- If temperature is <u>cool</u> then motor speed <u>slows</u>
- If temperature is <u>just right</u> then motor speed is <u>medium</u>

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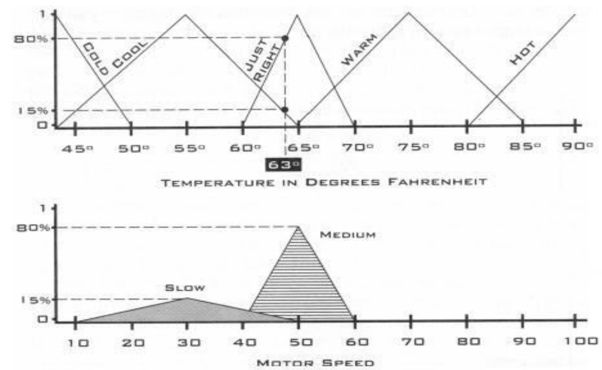
EXAMPLE: T = 63 DEGREE F. (CONT.)



*Fuzzy Thinkring, Bart Kosko

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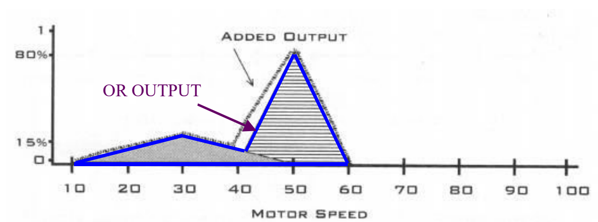
EXAMPLE: T = 63 DEGREE F. (CONT.)

- **Summed (MAXed) of the partially fired then-part fuzzy sets**



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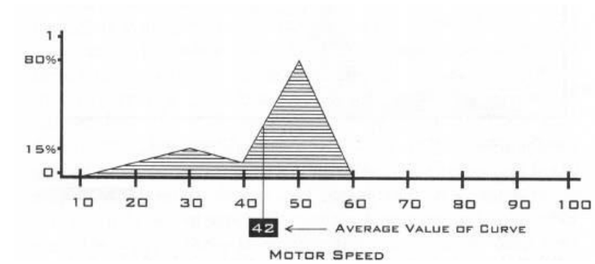
EXAMPLE: T = 63 DEGREE F. (CONT.)

- **Defuzzify to find the output motor speed**



*Fuzzy Thinkring, Bart Kosko

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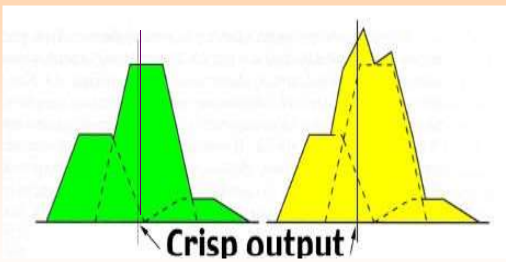
DEFUZZIFICATION

- Convert fuzzy grade to Crisp output



*Fuzzy Engineering, Bart Kosko

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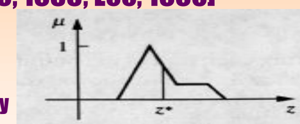
DEFUZZIFICATION (CONT.)

- Centroid Method: the most prevalent and physically appealing of all the defuzzification methods [Sugeno, 1985; Lee, 1990]
  - Often called
    - Center of area
    - Center of gravity



$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z\, dz}{\int \mu_{\underset{\sim}{C}}(z)\, dz}$$

where $\int$ denotes an algebraic integration.

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DEFUZZIFICATION (CONT.)

- **Max-membership principal**
  - **Also known as height method**

$$\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z) \qquad \text{for all } z \in Z$$

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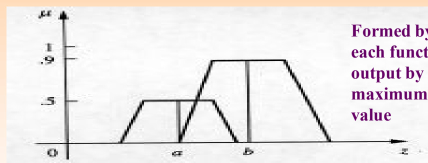
DEFUZZIFICATION (CONT.)

- **Weighted average method**
  - **Valid for symmetrical output membership functions**



Formed by weighting each functions in the output by its respective maximum membership value

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})} \qquad z^* = \frac{a(.5) + b(.9)}{.5 + .9}$$

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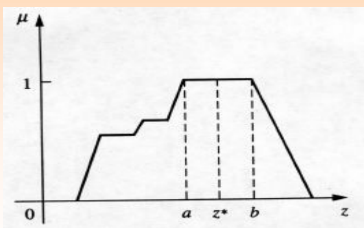
DEFUZZIFICATION (CONT.)

- **Mean-max membership (middle of maxima)**
  - **Maximum membership is a plateau**



$$Z^* = \frac{a + b}{2}$$

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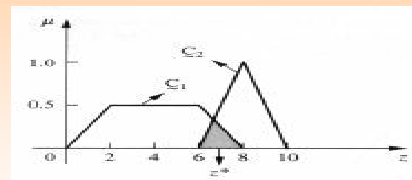
DEFUZZIFICATION (CONT.)

- **Center of sums**
  - **Faster than many defuzzification methods**



$$z^* = \frac{\int_Z z \sum_{k=1}^{n} \mu_{C_k}(z)\, dz}{\int_z \sum_{k=1}^{n} \mu_{C_k}(z)\, dz}$$

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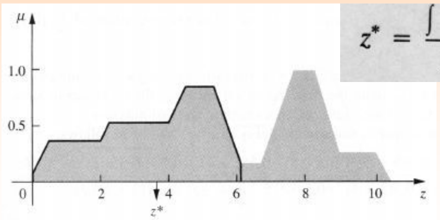
DEFUZZIFICATION (CONT.)

- **Center of Largest area**
  - **If the output fuzzy set has at least two convex subregion, defuzzify the largest area using centroid**



$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz}$$

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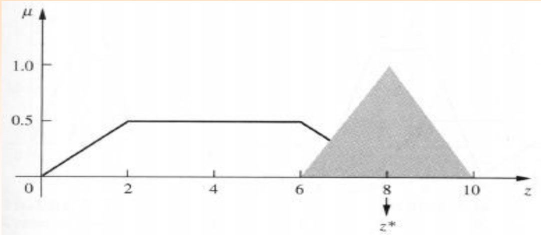
DEFUZZIFICATION (CONT.)

- **First (or last) of maxima**
  - **Determine the smallest value of the domain with maximized membership degree**



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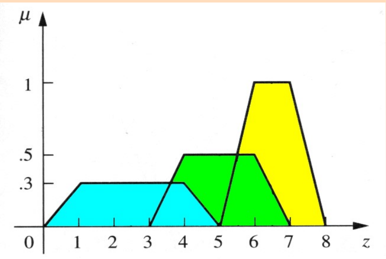
EXAMPLE: DEFUZZIFICATION

- Find an estimate crisp output from the following 3 membership functions



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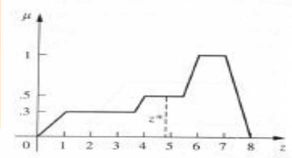
EXAMPLE: DEFUZZIFICATION

- CENTROID



$$z^* = \frac{\int \mu_B(z) \cdot z\,dz}{\int \mu_B(z)\,dz} =$$

$$\left[\int_0^1 (.3z)z\,dz + \int_1^{3.6} (.3z)\,dz + \int_{3.6}^4 \left(\frac{z-3}{2}\right) z\,dz + \int_4^{5.5} (.5)z\,dz + \int_{5.5}^6 (z-5)z\,dz + \int_6^7 z\,dz + \int_7^8 (8-z)z\,dz\right]$$

$$\div \left[\int_0^1 (.3z)\,dz + \int_1^{3.6} (.3)\,dz + \int_{3.6}^4 \left(\frac{z-3}{2}\right) dz + \int_4^{5.5} (.5)\,dz + \int_{5.5}^6 (z-5)\,dz + \int_6^7 dz + \int_7^8 (8-z)\,dz\right]$$

$$= 4.9 \text{ meters}$$

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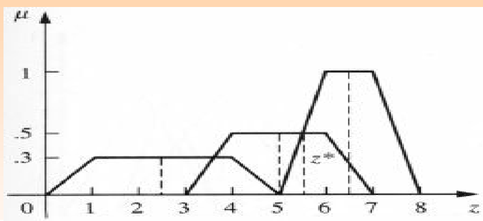
EXAMPLE: DEFUZZIFICATION

- **Weighted Average**



$$z^* = \frac{(.3 \times 2.5) + (.5 \times 5) + (1 \times 6.5)}{.3 + .5 + 1} = 5.41 \text{ meters}$$

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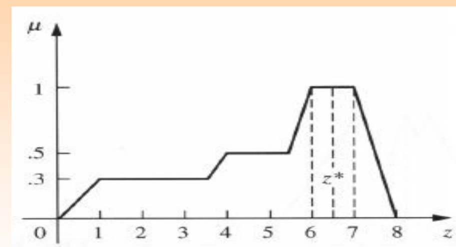
EXAMPLE: DEFUZZIFICATION

- **Mean-Max**



Z* = (6+7)/2 = 6.5

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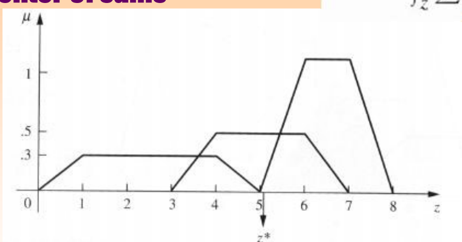
EXAMPLE: DEFUZZIFICATION

- **Center of sums**

$$z^* = \frac{\int_Z z \sum_{k=1}^{n} \mu_{C_k}(z)\,dz}{\int_z \sum_{k=1}^{n} \mu_{C_k}(z)\,dz}$$



$$z^* = \frac{\int_0^8 [2.5 \times 0.5 \times 0.3(3+5) + 5 \times 0.5 \times 0.5(2+4) + 6.5 \times 0.5 \times 1(3+1)]\,dz}{\int_0^8 [0.5 \times 0.3(3+5) + 0.5 \times 0.5(2+4) + 0.5 \times 1(3+1)]\,dz}$$

$$= 5.042 \text{ m}$$

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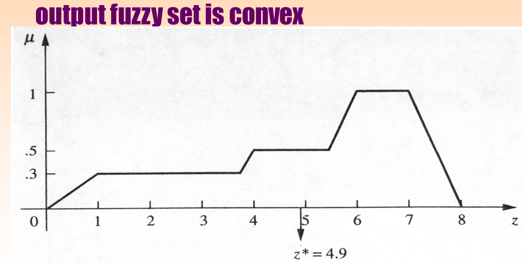
EXAMPLE: DEFUZZIFICATION

- **Center of largest area**
  - **Same as the centroid method because the complete output fuzzy set is convex**



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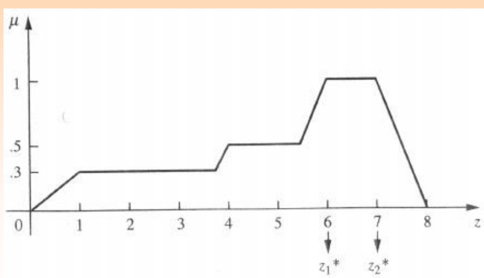
EXAMPLE: DEFUZZIFICATION

- First and Last of maxima



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DEFUZZIFICATION

- Of the seven defuzzification methods presented, which is the best?
  - It is context or problem-dependent

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DEFUZZIFICATION: CRITERIA

- Hellendoorn and Thomas specified 5 criteria against whnic to measure the methods
  - #1 Continuity
    - Small change in the input should not produce the large change in the output
  - #2 Disambiguity
    - Defuzzification method should always result in a unique value, I.e. no ambiguity
      - Not satisfied by the center of largest area!

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DEFUZZIFICATION: CRITERIA (CPNT.)

- Hellendoorn and Thomas specified 5 criteria against whnic to measure the methods
  - #3 Plausibility
    - Z* should lie approximatly in the middle of the support region and hve high degree of membership
  - #4 Computational simplicity
    - Centroid and center of sum required complex computation!
  - #5 Constitutes the difference between centroid, weighted average and center of sum
    - Problem-dependent, keep computation simplicity

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TWO FUZZY OUTPUTS

- Find the centroid using i) mean max ii) centroid and iii) center of largest area methods iv) Center of sums