

BACHELOR OF COMPUTER SCIENCE ENGG. EXAMINATION, 2009
(Third Year, Second Semester)

DESIGN AND ANALYSIS OF ALGORITHMS

Time : Three hours

Full Marks : 100

Answer any *five* questions

1. a) Prove by induction on $n \geq 0$ that

$$\sum_{i=1}^n i^2 2^i = n^2 2^{n+1} - n 2^{n+2} + 3 \cdot 2^{n+1} - 6 \text{ and also by geometry.}$$

- b) Prove by induction on $n \geq 7$ that $3^n < n!$.

12 + 08

2. a) ~~Prove by induction that~~ the sum of the cubes of three successive natural numbers is divisible by 9.

- b) Prove by induction that n circles divide the plane into $n^2 - n + 2$ regions if every pair of circles intersect in exactly two points and no three circles intersect in a common point.

10+10

3. a) Solve the recurrence exactly:

$$T(1) = 8 \text{ and for all } n \geq 2, 3T(n-1) = 15.$$

- b) Solve the recurrence exactly:

$$T(1) = 2 \text{ and for all } n \geq 2, T(n) = T(n-1) + n - 1.$$

10+10

4. a) Prove that the following algorithm for swapping two numbers is correct.

$$x = x + y; y = x - y; x = x - y;$$

b) Prove that the following algorithm for computing the maximum value in an array $a[1..n]$ is correct.

```
Function max (A){
    m = A[1];
    for i = 2 to n do
        If A[i] > m then m = A[i]
    return (m)
```

10 + 10

5. a) Prove correctness of the algorithm

```
function add(y, z){ // Add y + z, where y, z ∈ N
    x = 0; c = 0; d = 1;
    while (y > 0) ∨ (z > 0) ∨ (c > 0){
        a = y mod 2;    b = z mod 2;
        if a ⊕ b ⊕ c then x = x + d;
        c = (a ∧ b) ∨ (b ∧ c) ∨ (c ∧ a);
        d = 2d; y = ⌊y/2⌋; z = ⌊z/2⌋;
    }
    return (x);
}
```

b) Prove correctness of the algorithm:

```
function power(y, z){ //  $y^z$  where  $y \in \mathbb{R}$ ,  $z \in \mathbb{N}$ 
    x = 1;
    while (z > 0) {
        x = xy; z = z - 1;
    }
    return (x);
}
```

10+10

6. a) Prove that the following recursive algorithm computes $3^n - 2^n$ for all $n \geq 0$:

```
function g(n){ //  $3^n - 2^n$ ,  $n \geq 0$ 
    if n ≤ 1 then return (n);
    else return (5g(n - 1) - 6g(n - 2));
}
```

b) Prove that the following algorithm is correct:

```
procedure bubblesort(A){ // into ascending order
```

```

    if n > 1 then
        for j = 1 to n - 1
            if A[j] > A[j + 1] then
                swap A[j] and A[j + 1]
            bubblesort(n - 1);
    }

```

10+10

7. a) What is the value returned by the following function? Express your answer as a function of n . Give, using the big- O notation, the worst-case running time.

```

function mystery(n){
    r=0;
    for i=1 to n-1
        for j=i+1 to n
            for k=1 to j
                r=r+1;
    return(r);
}

```

- b) What is the value returned by the following function? Express your answer as a function of n . Give, using the big- O notation, the worst-case running time.

```

function pesky(n){
    r=0;
    for i=1 to n
        for j=1 to i
            for k=j to i+j
                r=r+1;
    return(r);
}

```

10 + 10