CHAPTER 22

TRANSIENTS

Learning Objectives

- Introduction
- Types of Transients
- Important Differential Equations
- Transients in R-L Circuits (D.C.),
- Short Circuit Current
- Time Constant
- Transients in R-L Circuits (A.C.)
- Transients in R-C Series Circuits (D.C.)
- Transients in R-C Series Circuits (A.C)
- Double Energy Transients

Transients are the surges or spikes in electric currents and voltages which are transmitted through power or data lines. The gadget shown in the above picture is claimed to protect the equipment against such surges or spikes

22.1. Introduction

It is quite an easy job to calculate the steady current, which flows in a circuit, when it is connected to a d.c. generator or a battery. Similarly, the alternating current which flows in a circuit when connected to an alternator can also be calculated by the various method discussed in Chapters 13 and 14. These currents are known as ***steady*** currents because in such cases, it is assumed that ***(i)*** the circuit components are constant and ***(ii)*** the circuit has been connected to the generator long enough for any disturbance produced on initial switching, to resolve itself.

In general, transients disturbances are produced whenever

(*a*) an apparatus or circuit is ***suddenly*** connected to or disconnected from the supply,

(*b*) a circuit is shorted and

(*c*) there is a ***sudden*** change in the applied voltage from one finite value to another.

We will now discuss the transients produced whenever different circuits are suddenly switched on or off from the supply voltage. In each case, we will assume that the resultant current consists of two parts ***(i)*** a final steady-stage or normal current and ***(ii)*** a transient current superimposed on the steady-stage current.

It is essential to remember that the transient currents are not driven by any part of the applied voltage but are entirely associated with the changes in the stored energy in inductors and capacitors. Since there is no stored energy in resistors, ***there are no transients in pure resistive circuits.***

22.2. Types of Transients

There are ***single-energy*** transients and ***double-energy*** transients. Single-energy transients are those in which only one form of energy, either electromagnetic or electrostatic is involved as in *R-L* and *R-C* circuits.

However, double-energy transients are those in which both electromagnetic or electrostatic is involved as in *R-L-C* circuits Transient disturbances may be further classified as follows :

(*a*) Initiation Transients : These are produced when a circuit, which is originally dead, is energised.

(*b*) Subsidence Transients : These are produced when an energised circuit is rapidly de-energised and reaches an eventual steady-stage of zero current or voltage, as in the case of short-circuiting an *R-L* or *R-C* circuit suddenly.

(*c*) Transition Transients : These are due to sudden but energetic changes from one steady state to another.

(*d*) Complex Transients : These are produced in a circuit which is simultaneously subjected to two transients due to two independent disturbances or when the disturbing force producing the transients is itself variable.

(*e*) Relaxation Transients : In these transients, the transition occurs cyclically towards states, which when reached, become unstable themselves.

A distinction may also be made between free and forced transients which are produced due to the applied voltage being itself transient.

22.3. Important Differential Equations

Some of the important differential equations, used in the treatment of single and double energy transients, are given below. We will consider both first-order and second-order differential equations.

1. First Order Equations

(i) Let $\dfrac{dy}{dx} + ay = 0$ where a is a constant.

Its solution is $y = k\,e^{-ax}$ where k is the constant of integration whose value can be found from the boundary conditions *i.e.* conditions prevalent at the instant when the voltage to a circuit is applied or excluded.

(ii) If $\dfrac{dy}{dx} + ay = b$ where a and b are constants, then solution is $y = \dfrac{b}{a} + k\,e^{-ax}$

The value of k can again be found from boundary conditions.

(iii) If $\dfrac{dy}{dx} + Ay = B$

where A and B are not constants but are functions of x, then the solution is given by

$$y \quad e^{\quad A\,dx} \quad e^{A\,dx} Bdx \quad ke^{\quad A\,dx}$$

If $A = a$ = constant, then the above equation simplifies to

$$y \quad e^{\quad ax} \quad e^{ax}\,B\,dx \quad ke^{\quad ax}$$

2. Second Order Equations

(i) Suppose $\dfrac{d^2y}{dx^2} + \text{a}\dfrac{dy}{dx} + by = 0$ where a and b are constants, then the solution is

$$y \quad k_1 e^{\;_2 x} \quad k_2 e^{\;_2 x}$$

where λ_1 and λ_2 are constants of integration and whose values are,

$$_1 \quad \frac{a}{2} \quad \sqrt{\frac{a^2}{4} \quad b} \quad \text{and} \quad _2 \quad \frac{a}{2} \quad \sqrt{\frac{a^2}{4} \quad b}$$

(a) If $a^2/4 > b$, the roots are real and the above solution can be applied without any difficulty.

(b) If $a^2/4 < b$, the radicals contain a negative quantity. In that-case, the solution is given by

$$y = e^{-\frac{1}{2}ax}(k_3 \sin\lambda_0 x + k_4 \cos\lambda_0 x)$$

where k_3 and k_4 are the new constants of integration and

$$\lambda_0 = \sqrt{b - \frac{a^2}{4}}$$

(c) If $a^2/4 = b$, then both roots are equal and each is $= -\,a/2$.

Hence, in this case, the solution becomes $y = k_5\,e^{\lambda t} + k_6 t.e^{\lambda t}$

(ii) Let $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = c$

where a, b and c are constant. In this case also, the solution will again depend on the root as discussed above.

$$y \quad k_1 e \;\; 1^x \quad k_2 e \;\; _2 x \quad c/b$$

(iii) (a) Let the differential equation be given by

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = u$$

where a, b are c are constants but u is a particular function of the variable x. The solution of such an equation consists of a ***particular integral*** and a complementary function.

(*b*) Let y be a sinusoidal function of x, then

$$\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=c\sin\omega t$$

In this case, particular integral is

$$y_1 \quad \frac{c}{\sqrt{a^2 \quad ^2 \quad (\quad ^2 \quad b)^2}} \cos \quad x \quad \tan^{\ 1} \frac{^2 \quad b}{a}$$

The complementary function is given by

$$y_2 \quad k_1 e^{\ _1 x} \quad k_2 e^{\ _2 x} \text{ where } \ _1 \quad \frac{a}{2} \quad \sqrt{\frac{a^2}{4} \quad b} \text{ and } \ _2 \quad \frac{a}{2} \quad \sqrt{\frac{a^2}{4} \quad b}$$

The complete solution for the above equation is $y = y_1 + y_2$

Further treatment is the same as for case 2 (i) above.

22.4. Transients in R-L Circuits (D.C.)

If i, I_s and i_t be the resultant current, steady-state current and transient current respectively in R–L circuit of Fig. 22.1 (a), then by superimposition, the equation for the resultant current, for the duration of initiation transient, is

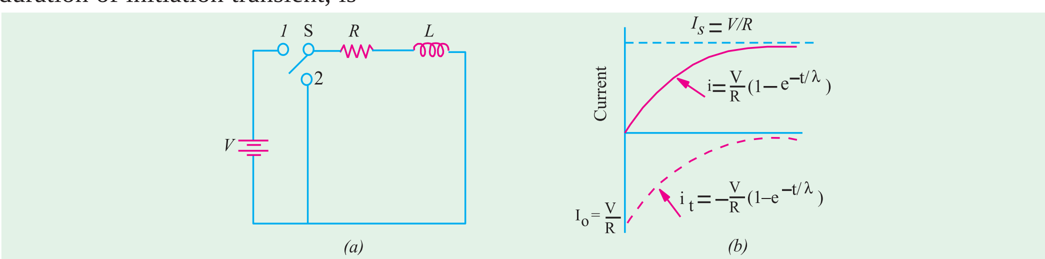


Fig. 22.1

$$i = I_s + i_t \qquad \text{... (i)}$$

Since the applied voltage V drives the steady-state current, hence

$$I_s = V/R$$

Since the transient current i_t is not associated with any voltage,

$$\therefore \qquad i_t R \quad L\frac{di_t}{dt} \quad 0 \qquad \text{... (ii)}$$

or $$\frac{di_t}{dt} \quad \frac{R}{L}dt \qquad \text{... (iii)}$$

or $$\frac{di_t}{i_t} \quad \frac{R}{L} \quad dt \therefore \text{ logh } i_t \quad \frac{R}{L}t \quad K^* \qquad \text{... (iv)}$$

where K is the constant of integration whose value may be found from the initial conditions.

Now when $t = 0$, $i_t = I_0$ (say). Then from Eq. **(*iv*)** above we get, logh $I_0 = 0 + K$

* $\log_e x$ is written as logh x. Obviously, logh $x = 2.3 \log_{10} x$.

Putting this value of K in Eq. **(*iv*)**, we have

$$\text{logh } i_t \quad \text{logh } I_0 \quad \frac{R}{L}t \text{ or } \text{logh } i_t / I_0 \quad \frac{R}{L}t \quad \frac{t}{\ }$$

$$\therefore \quad i_t = I_0\, e^{-t/\lambda} \qquad \ldots (v)$$

where $\lambda = L/R$ is called the ***time-constant*** of the circuit. Its reciprocal R/L is called the ***damping coefficient*** of the circuit. The current decreases exponentially as shown in Fig. 22.1 (*b*). From Eq. **(*i*)** and **(*v*)**, we have

$$i = I_s + I_0 e^{-t/\lambda} \qquad \ldots (vi)$$

If the time is reckoned when the voltage V is applied, so that when $t = 0$, $i = 0$, then from equation **(*vi*)**, we get

$$0 = I_s + i_0 e^{-0} = I_s + I_0 \quad \therefore \quad I_0 = -I_s = -\frac{V}{R}$$

In that case, Eq. **(*vi*)** becomes

$$i = \frac{V}{R} - \frac{V}{R} e^{-t/\lambda} \qquad \ldots (vii)$$

$$= \frac{V}{R}(1 - e^{-t/\lambda}) \qquad \ldots (viii)$$

Curves for I_s and i_t have been plotted in Fig. 22.1 (*b*). The curve for resultant current has been obtained by the superposition of steady-state current I_s (= V/R) and transient current

$$i_t = \frac{V}{R} e^{-t/\lambda}$$

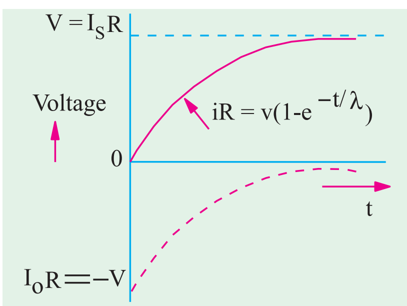


Fig. 22.2

Theoretically, the transient current i_t takes infinite time to die off but, in practice, it disappears in a very short time.

The values of resultant, steady-state and transient voltages across the resistor can be found by multiplying Eq. **(*vii*)** by R and are shown in Fig. 22.2. The e.m.f. of self-induction–$L\,di_t/dt$ is only transient in nature and equals $i_t R$ as seen from Eq. **(*ii*)** above.

22.5. Short Circuit Current

After some time, the transient current would disappear and the only current flowing in the circuit would be the steady-state current $I_s = V/R$. Let the *R-L* circuit be closed upon itself *i.e.* be short-circuited by shifting the switch [Fig. 22.1 (*a*)] to position 2. Since the voltage V has been excluded from the circuit, the trapped current I_s will immediately cease to be a steady-state current, but on the other hand, will become the initial value I_0 of a new subsidence transient current i_t.

Large-capacity short-circuit current generator

If time is measured at the instant of short-circuit, so that when t = 0, the current is $I_s = V/R$, then Eq. **(v)** becomes

$$i_t = \frac{V}{R} e^{-t/\lambda} \qquad \text{... (ix)}$$

This equation has been plotted in Fig. 22.3. The only voltage acting in the circuit is that due to self-induction *i.e.* $-L\, di_t/dt$ which equals $i_t R$.

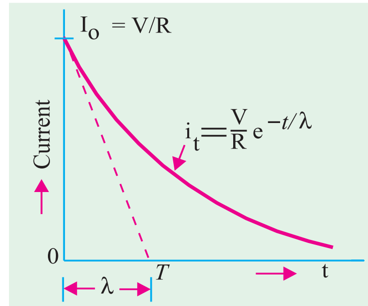


Fig. 22.3

22.6. Time Constant

The time constant of a circuit is defined as the time it would take for the transient current to decrease to zero, if the decrease were linear instead of being exponential.

In other words, it is the time during which the transient current would have decreased to zero, had it maintained its initial rate of decrease.

The initial rate of decrease can be found by differentiating Eq. (vi) and putting t = 0

$$\therefore \qquad \frac{di_t}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda} \qquad \frac{di_t}{dt}\bigg|_{t\ 0} \qquad \frac{I_0}{}$$

If the rate of decrease were constant throughout and equal to $-I_0/\lambda$, then the straight line showing the relation between i_t and t would be given by

$$i_t = -\frac{I_0}{\lambda} t$$

The time-period would be equal to the sub-tangent OT drawn to the exponential curve of Fig. 22.3 at $i_t = I_0$ *i.e.* at the beginning of the curve.

If we put $t = \lambda$ in Eq. (v), then $i_t = I_0\, e^{-1} = I_0/e = I_0/2.718 = 0.37\, I_0$

Hence, time period of a circuit is the time during which the transient current decreases to 0.37 of its initial value.

Example 22.1. *A coil having a resistance of 30 ω and an inductance of 0.09 H is connected across a battery of 20 V. Plot the current and its two components. Assume that t = 0 when the circuit is completed.* **(Electromechanic Allahabad Univ. 1992)**

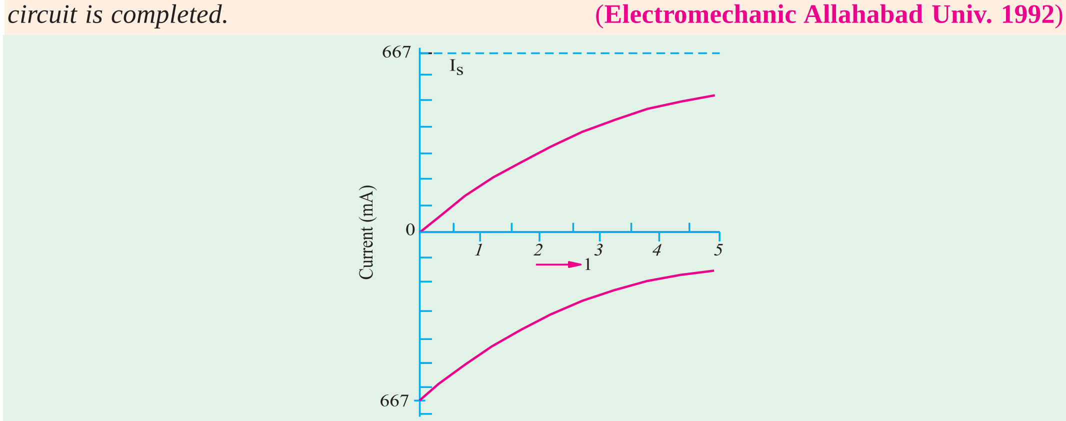


Fig. 22.4

Solution. The two components of the circuit current are **(i)** steady current $I_s = V/R = 20/30$ = 2/3 A = 667 mA and **(ii)** Transient current $i_t = -(V/R)e^{-t/\lambda}$.

Total current is $i = I_s + i_t$. Let us find the value of transient current after various time intervals. In the present case, $\lambda = L/R = 0.09/30 = 0.003$ second = 3 millisecond.

The values of i_t and i at various times are tabulated below. Value of $i = I_s + i_t$

t (ms)	$e^{-t/\lambda}$	i_t (mA)	i	t (ms)	$e^{-t/\lambda}$	i_t (mA)	i
0.0	1.000	–667	0	2.5	0.435	–290	377
0.5	0.847	–565	102	3.0	0.368	–244	423
1.0	0.716	–477	190	3.5	0.311	–208	459
1.5	0.606	–405	262	4.0	0.264	–176	491
2.0	0.514	-344	323	4.5	0.223	–148	519

It is seen that whereas transient current decreases exponentially, total circuit current increases exponentially as expected (Fig. 22.4).

Example 22.2. *A circuit of resistance 10 Ω and inductance 0.1 H in series has a direct voltage of 200 V suddenly applied to it. Find the voltage drop across the inductance at the instant of switching on and at 0.01 second. Find also the flux-linkages at these instants.*

(Basic Electricity, Bombay Univ.)

Solution. (*i*) Switching instant

At the instant of switching on, $i = 0$, so that $iR = 0$ hence all applied voltage must drop across the inductance only. Therefore, voltage drop across inductance = **200 V**.

Since at this instant $i = 0$, there are no flux-linkages of the coil.

(*ii*) When t = 0.01 second

As time passes, current grows so that the applied voltage is partly dropped across the resistance and partly across the coil. Let us first find iR drop for which purpose, we need the value of i at $t = 0.01$ second.

Now, time period of the circuit is $\lambda = L/R = 0.1/10 = 0.01$ second. Since the given time happens to be equal to time constant,

$\therefore \quad i = (200/10) \times 0.632 = 12.64$ A; $iR = 152.64 \times 10 =$ **126.4 V**

Drop across inductance = $\sqrt{200^2 - 126.4^2} =$ **155 V**

Now, $\quad L = N\Phi/i \quad$ or $\quad N\Phi = Li$

$\therefore \quad$ Flux-linkages $Li = 0.1 \times 12.64 =$ **1.264 Wb-turns.**

Example 22.3. *A coil of 10 H inductance and 5 Ω resistance is connected in parallel with a 20 Ω resistor across a 100-V d.c. supply which is suddenly disconnected. Find*

(a) *the initial rate of change of current after switching.*

(b) *the voltage across the 20 Ω resistor initially and after 0.3 s.*

(c) *the voltage across the switch contacts at the instant of separation and*

(d) *the rate at which the coil is losing stored energy 0.3 second after switching.*

Solution. (*a*) Since the steady-state current is zero, $i = I_0 \, e^{-t/\lambda}$

Now, when $t = 0$, current is = 100/5 = 20 A. It means the current flowing through the coil immediately before opening the switch is 20 A.

$\therefore \qquad I_0 = 20$ A

Hence, the above equation becomes $i = 20\,e^{-t/\lambda}$

Now $\qquad \lambda = L/R = 10/25 = 1/2.5 \qquad \therefore \quad i = 20\,e^{-2.5t}$

$$\left.\frac{di}{dt}\right|_{t=0} = (-20 \times 2.5\,e^{-2.5t})_{t=0} = \mathbf{-50\ A/s}$$

The negative sign merely shows that the current is decreasing.

(*b*) After the supply has been disconnected, the current through the $20\text{-}\Omega$ resistor is *i* since it is in series with the coil.

Initial p.d. across the $20\,\Omega$ resistor = (current at t = 0) × 20 = 20 × 20 = **400 V**

Current through the resistor after 0.3 second = $20e^{-2.5\times0.3} = 9.45$ A

$\therefore$ Voltage across the resistor after 0.3 second

= (current at t = 0.3 second) × 20 = 9.45 × 20 = **189 V**

(*c*) The e.m.f. induced in the coil at break tends to maintain the current through it in the original direction. Hence, the direction of the current through $20\,\Omega$ resistor is upwards so that the p.d. across the switch contacts will be the *sum* of supply voltage and the voltage across $20\,\Omega$ resistor.

$\therefore$ Initial voltage across switch contacts = 400 + 100 = **500 V**

(*d*) The rate of loss of energy = power = induced e.m.f. in coil × current (after 0.3 s)

$$= L\left.\frac{di}{dt}\right|_{t=0.3} \times i \quad \text{(after 0.3 second)}$$

Now, after 0.3 second, i = 9.45 A

Value of di/dt after 0.3 second = – 20 × 2.5 × $e^{-0.75}$ = – 23.6 A/second

$\therefore$Rate of loss of energy = – 10 × 23.6 × 9.45 = – 2,230 joule/second

22.7. Transients in R-L Circuits (A.C.)

Let a voltage given by $v = V_m \sin(\omega t + \Psi)$ be ***suddenly*** applied across an *R-L* circuit [Fig. 22.5 (*a*)] at a time when t = 0. It means that the voltage is applied when it is passing through the value $V_m \sin \Psi$. Since the contact may be closed at any point of the cycle, angle Ψ may have any value lying between zero and 2π radians. The resultant current, as before, is given by

$$i = i_s + i_t$$

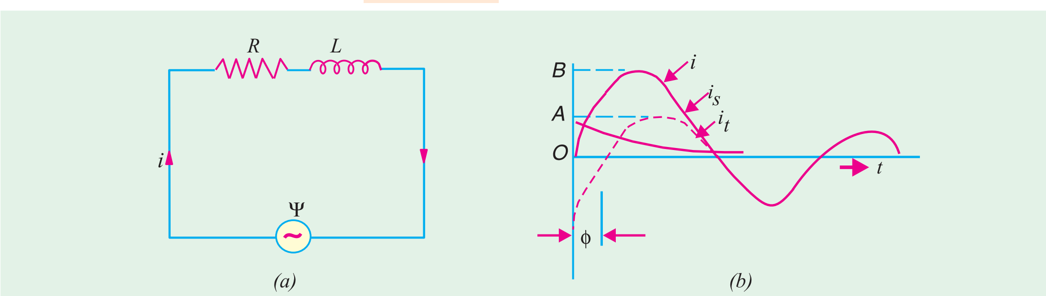


Fig. 22.5

The value of steady-state current is found by the normal circuit theory. The peak steady-state current is given by

$$I_m = \frac{V_m}{\sqrt{R^2 + X_L{}^2}} = \frac{V_m}{Z}$$

where $\sqrt{R^2 + X_L{}^2}$ is the impedance of the circuit. This current lags behind the applied voltage by an angle ϕ such that tan $\phi = X_L/R$ or $\phi = \tan^{-1}(X_L/R)$

Hence, the equation for the instantaneous value of the steady-state current becomes

$$i_s = I_m \sin(\omega t + \Psi - \phi)$$

As before, the transient current is given by

$$i_t = I_0 \, e^{-t/\lambda} \qquad \therefore \; i = I_m \sin(\omega t + \Psi - \phi) + I_0 \, e^{-t/\lambda} \qquad \dots (i)$$

Now, when $t = 0$, $i = 0$, hence putting these values in Eq. (i) above, we get

$$0 = I_m \sin(\Psi - \phi) + I_0 \qquad \therefore \; I_0 = -I_m \sin(\Psi - \phi)$$

Hence Eq. (i) can be written as

$$i = I_m \sin(\omega t + \Psi - \phi) - I_m \sin(\Psi - \phi) \, e^{-t/\lambda} \qquad \dots (ii)$$

From the above, it is seen that the value of I_0 and hence the size of the transient current depends on angle Ψ *i.e.* it depends on the instant in the cycle at which the circuit is closed. We will consider the following three cases :

Case 1

When $t = 0$, let the voltage pass through its zero value and become positive *i.e.* let $\Psi = 0$. In that case, putting this value of Ψ in Eq. (ii), we get

$$i = I_m \sin(\omega t - \phi) - I_m \sin(-\phi) \, e^{-t/\lambda} = I_m[\sin(\omega t - \phi) + \sin\phi \, e^{-t/\lambda}]$$

This is shown in Fig. 22.5 (b). It is seen that maximum instantaneous peak current OB is larger than the normal peak current OA.

Case 2

Let $t = 0$ when voltage is passing through its value V_m sin (ϕ) so that $\Psi = \phi$ or $\Psi - \phi = 0$

In that, $I_0 = 0$, there is no transient current at the time of switching on (*i.e.* $i_t = 0$). It corresponds to the contacts closing at the instant when the steady state current itself is zero.

Case 3

When $t = 0$, let the voltage be passing through

$$V_m \sin \quad \frac{\quad}{2} \quad i.e. \quad \frac{\quad}{2} \text{ and } \quad /2$$

In this case, the transient [as found from Eq. (ii)] would be given by

$$i_t \quad I_m \sin \quad \frac{\quad}{2} \, . \, e^{\;\; t/} \quad I_m e^{\;\; t/}$$

Under these conditions, the transient would have its maximum possible initial value.

Example 22.4. *A 1.0 H choke has a resistance of 50 Ω. This choke is supplied with an a.c. voltage given by e = 141 sin 314 t. Find the expression for the transient component of the current flowing through the choke after the voltage is suddenly switched on.*

(Principles of Elect. Engg-II, Jadavpur Univ.)

Solution. The equation of the transient component of the current is (Art. 22.7 Case 1)

$$i_t = I_m \sin\phi e^{-t/\lambda}$$

Here, $\lambda = L/R = 1/50 = 0.02$ second ; $\mathbf{Z} = 50 + j\,314 = 318\ \angle 80.95°$

$I_m = V_m/Z = 141/318 = 0.443$ A; $\sin 80.95° = 0.9875$

$\therefore$ $i_t = 0.443 \times 0.9875\ e^{-t/0.02} = 0.4376e^{-t/0.02}$

Example 22.5. *A 50-Hz sinusoidal voltage of maximum value of 400 V is applied to a series circuit of resistance 10 Ω and inductance 0.1 H. Find tan expression for the value of the current at any instant after the voltage is applied, assuming that voltage is zero at the instant of application. Calculate its value 0.02 second after switching on* **(Electric Circuit, Punjab Univ. 1990)**

Solution. In such cases, as seen from Art. 22.7 (Case 1), the current consists of a steady-state component and a transient component. The equation of the resultant current is

$$i = \underbrace{I_m \sin(\omega t - \phi)}_{\text{steady-state current}} + \underbrace{I_m \sin\phi\, e^{t/\lambda}}_{\text{transient current}}$$

where $I_m = V_m / Z$; $\phi = \tan^{-1}(X_L / R)$; $\lambda = L / R$ second

$$R = 10\,\Omega; X_L = 314 \times 0.1 = 31.4\,\Omega; \mathbf{Z} = 10 + j31.4 = 33\angle 72.3°$$

$$I_m = 400 / 33 = 12.1 \text{ A} ;\ \phi = 72.3° = 1.26 \text{ rad.}$$

$$\sin\phi = \sin 72.3° = 0.9527 ;\ \lambda = 0.1/10 = 1/100 \text{ second}$$

$$i = 121\ \{\sin(314\,t - 1.262) + 0.9527\ e^{100t}\}$$

Substituting $t = 0.02$ second, we get

$$i = 12.1\ (\sin(314 \times 0.02 - 1.262) + 0.9527\ e^{-2}\}$$

$$= 12.1\ (\sin 5.02 + 0.9527\ e^{-2}) = 12.1\ (\sin 288° + 0.9527\ e^{-2})$$

$$= 12.1\ (-\sin 72° + 0.9527 \times 0.1353) = 12.1\ (-0.9511 + 0.1289) = \mathbf{-9.95\ A}$$

Example 22.6. *An alternating voltage $v = 400 \sin(314\,t + \Psi)$ is suddenly applied across a coil of resistance 0.2 Ω and inductance 6.36 mH. Determine the first peak value of the resultant current when the transient current has maximum value.*

Solution. Obviously, $\omega = 314$ rad/s

$$X_L = \omega L = 314 \times 6.36 \times 10^{-3} = 2\ \Omega$$

Coil impedance $\mathbf{Z} = 0.2 + j2 \sim 2\ \angle 84.3°$

Max. value of steady-state current = 400/2 = 200 A

As seen from Art. 22.7, the maximum value of transient current will occur when

$\Psi = \phi \pm \pi/2$ where $\phi = 84.3°$ *i.e.* the phase angle of the current w.r.t. voltage

$\therefore$ $\Psi = 84.3° - 90° = -5.7°$

$\therefore$ resultant current, $i = 400 \sin(314\,t - 90°) + I_0\ e^{-31.4t}$

Now, at $t = 0$, $i = 0$ $\therefore$ $0 = 400 \sin(-90°) + I_0$

$\therefore I_0 = 400$ A

Hence, the above equation becomes

$$i = 400 \sin(\omega t - 90°) + 400\ e^{-31.4t}$$

The procedure for determining an exact solution for the first peak of the resultant current is first to differentiate the above expression, next to equate the result to zero and then to solve the resulting expression graphically for t. However, sufficiently accurate result can be obtained by determining the instant at which steady-state current reaches its first positive peak value and then to add to it the value of the transient current at this instant. The first peak value of steady-state current occurs when

$(314\,t - 90^\circ) = \pi/2$ rad ; *i.e.* when $t = \pi/314 = 0.01$ second

At this time, $i_t = 400\,e^{-0.314} = 292$ A

$\therefore$ resultant current i at this time = 200 + 292 = **492 A**

22.8. Transients in R-C Weeries Circuits (D.C.)

When a d.c. voltage V is ***suddenly*** applied to an R-C series circuit (Fig. 22.6), the voltage v_c across the capacitor rises from zero value to the steady-state value V. If v_c is the voltage across capacitor, V_{ct} the transient voltage, then

$$v_c = V + v_{ct} \qquad \text{... } (i)$$

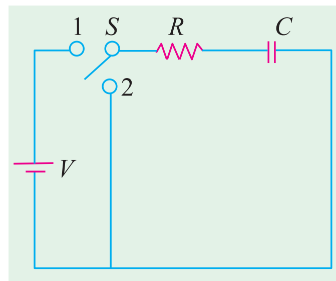


Fig. 22.6

The charging current is maximum at the beginning but then is reduced to zero so that there is no steady-state current but a transient one.

Since the transient current is not associated with any applied voltage, hence

$$i_t R + v_{ct} = 0 \qquad \text{.... } (ii)$$

Now, capacitor voltage $v_{ct} = q_1/C$

Hence, Eq. (*ii*) becomes

$$i_t R + \frac{q_t}{C} = 0$$

or $$R \cdot \frac{di_t}{dt} + \frac{1}{C} \cdot \frac{dq_t}{dt} = 0 \quad \text{or} \quad \frac{di_t}{dt} = -\frac{1}{CR}\frac{dq_t}{dt} = -\frac{1}{CR} i_t \quad (\; dq_t/dt = i_t)$$

$$\therefore \quad \frac{di_t}{i_t} = -\frac{d_t}{CR}; \qquad \text{As before } i_t = I_0 e^{-t/CR} = I_0 e^{-t/\lambda}$$

where $CR = \lambda$ = time constant. The reciprocal I/CR is known as damping coefficient.

(*i*) Charging Current

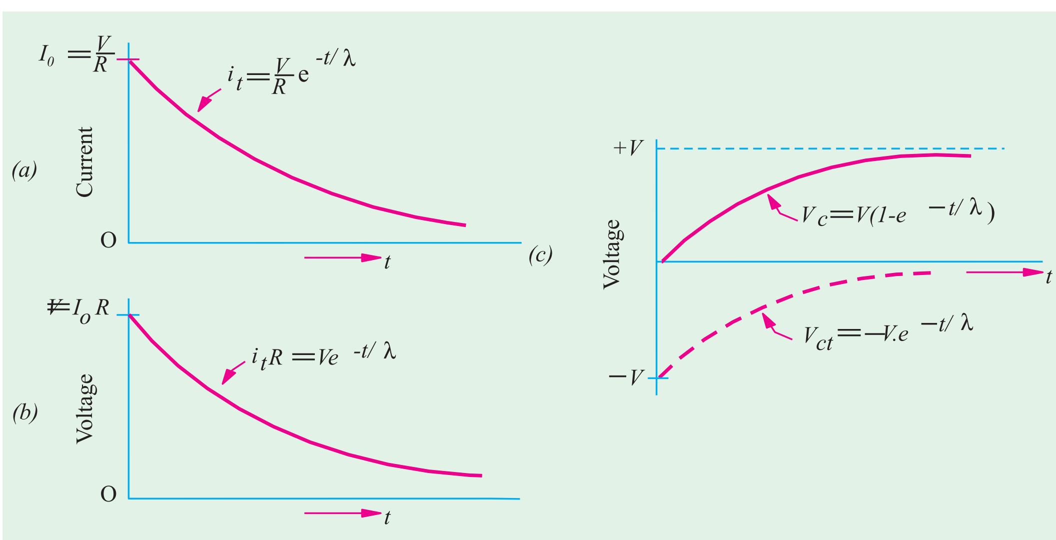


Fig. 22.7

When $t = 0$, transient current $i_t = I_0$, so that from Eq. (*ii*) $v_{ct} = -I_0R$. Moreover, when $t = 0$, $v_c = 0$, hence from Eq. **(*i*)**, $v_{ct} = -V$

Combining these results, we get

$$I_0 = V/R$$

$$\therefore \qquad i_t \quad I_0 e^{\ t/} \quad \frac{V}{R} e^{\ t/}$$

This is plotted in Fig. 22.7 (*a*)

The transient voltage across the resistor R is given by

$$i_t R = \frac{V}{R} e^{-t/\lambda} \times R$$

$$= Ve^{-t/\lambda} \quad \text{... Fig. 19.7 } (b)$$

From Eq. **(*ii*)** the value of transient voltage across the capacitor is $v_a = -i_t R$

Hence, Eq. **(*i*)** becomes

$$v_c = V - i_t R = V - Ve^{-t/\lambda}$$

or

$$v_c = V(1 - e^{-t/\lambda}) \qquad \text{... (iii)}$$

The voltage across the capacitor v_c which is the sum of the transient voltage v_{ct} and steady-state V has been plotted in Fig. 22.7 (*c*).

The charge across the capacitor is given by

$$q = v_c = CV(1 - e^{-t/\lambda}) \text{ or } q = Q(1 - e^{-t/\lambda}) \quad (\therefore Q = CV)$$

(*ii*) Discharge Current

When the capacitor has become fully charged so that charging current has ceased, then the *R-C* circuit is short-circuited by shifting the switch *S* from position 1 to position 2 (Fig. 22.6). On doing so, a transient discharge current will start flowing immediately. If time is reckoned from the instant of short-circuit, then when $t = 0$, $i_t = I_0$, hence from Eq. **(*ii*)** above $v_{ct} = -I_0$, R. Moreover, when $t = 0$, $v_c = V$. However, since there is no steady-state voltage across the capacitor, from Eq. **(*i*)**, we get $v_c = v_{ct}$. Combining these results, we get

$$I_0 = -V/R$$

$$\therefore \qquad i_t = -\frac{V}{R} e^{-t/\lambda}$$

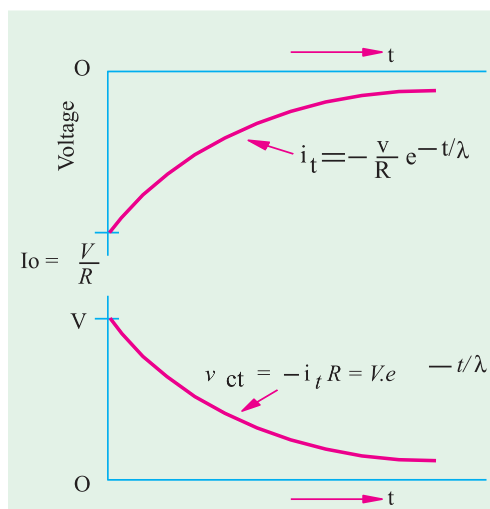


Fig. 22.8

It is plotted in Fig. 22.8 (*a*). The negative sign shows that discharge current flows in a direction opposite to that in which the charging current flows. That is why the curve has been drawn below the *X*-axis. It may be noted that the only voltage in the circuit is v_{ct} which equals $-i_t R$.

Example 22.7. *In a simple saw-tooth generator circuit with the thyratron switches on at 150 V and switches off at 10 V. If this circuit is supplied with 250 V d.c. source; find the time period of saw-tooth wave. The resistance and capacitance have the values of 10 kΩ and 1 μF respectively.* **(Principles of Elect. Engg-II, Jadavpur Univ.)**

Solution. With reference to Fig. 22.9, let

V = applied voltage

v_{cl} = switching-off voltage of the thyratron = 10 V

v_{c2} = switching-on voltage of the thyratron = 150 V

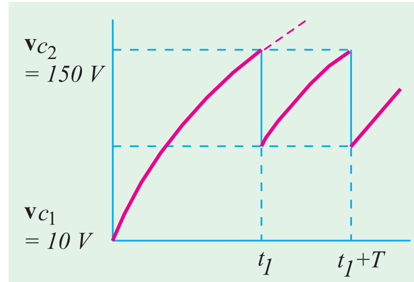


Fig. 22.9

Now, $\quad v \quad V(1 \quad e^{\,t/} \;)$

$\therefore \qquad v_{c1} = V(1-e^{-t/\lambda}) \qquad \ldots$ ***(i)***

$V(1-e^{(-t_1+T)}/\lambda \qquad \ldots$ ***(ii)***

where T is the time-period of the saw-tooth wave. From Eq. ***(i)*** and ***(ii)***, we get

$$T = \lambda \log_e (V - v_{c1})/(V - v_{c2})$$

Now $\qquad \lambda = CR = 10^4 \times 10^{-6} = 10^{-2}$ second

$$V - v_{c1} = 250 - 10 = 240\,\text{V}; V - v_{c2} = 250 - 150 = 100\,\text{V}$$

$\therefore \quad \dfrac{V - v_{c1}}{V - v_{c2}} = \dfrac{240}{100} = 2.4 \quad \therefore \; T = 10^{-2} \log_c{}^{2.4}$ = **0.00875 second**

Example 22.8. *A simple neon-tube time base for a cathode-ray oscillorgraphy employs a 300 $k\Omega$ and a 0.016 μF capacitor. The striking and extinction voltages of the neon-tube are 170 V and 140 V respectively. Calculate the frequency of the time base if the supply voltage is 200 V.*

Solution. The voltage across the capacitor increases according to the equation

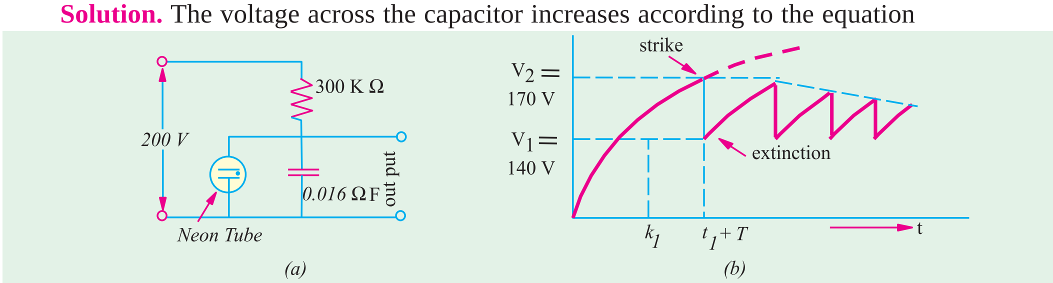


Fig. 22.10

$$v_c = V(1-e^{-t/CR})$$

It is shown in Fig. 22.10 (*b*)

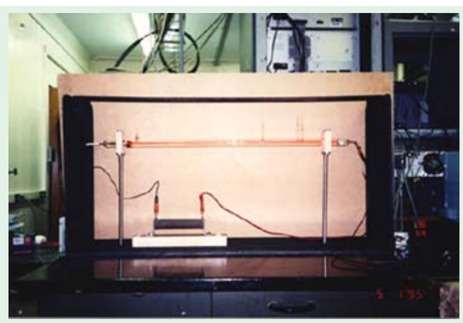
$\therefore \qquad v_{c1} = V(1-e^{-t_1/\lambda}) \qquad \ldots$ ***(i)***

$\qquad v_{c2} = V(1-e^{(-t_1+T)/\lambda}) \qquad \ldots$ ***(ii)***

From eq. ***(i)*** and ***(ii)***, we get

$$T = \lambda \text{logh}\,(V - v_{C1})/(V - v_{C2})$$

Now



A view of the neon tube experiment

$$\lambda = CR = 0.016 \times 10^{-6} \times 300 \times 10^3 = 4.8 \times 10^{-3} \text{ second}$$

$$V - v_{c1} = 200 - 140 = 60 \text{ V and } V - v_{c2} = 200 - 170 = 30 \text{ V}$$

$\therefore \qquad T = 4.8 \times 10^{-3} \log 60/30 = 1/300$ second

$\therefore$ Frequency of time base = $1/T$ = **300 Hz**

22.9. Transients in R-C Series Circuits (A.C.)

In this case, the resultant currents can be determined in the same way as for an *R-L* circuit (Art. 22.7). It is given by

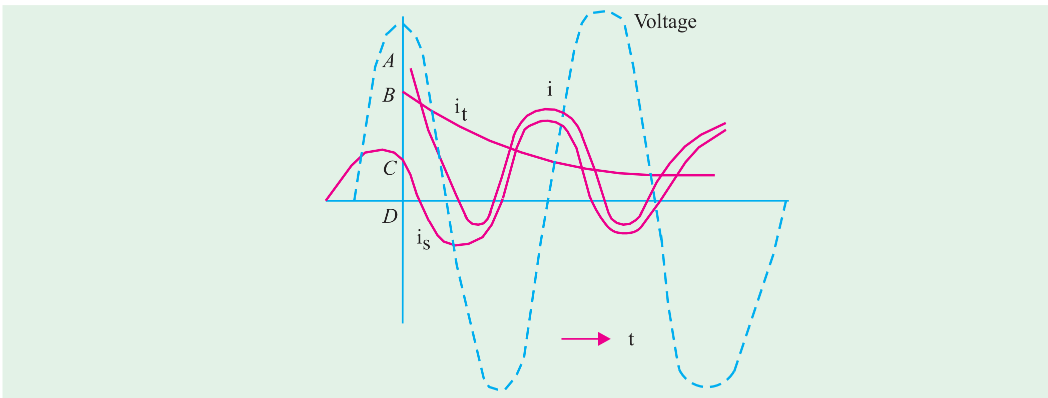


Fig. 22.11

$$i = i_s + i_t = I_m \sin(\omega t + \psi + \phi) + I_0 e^{-t/\lambda} \quad \text{where} \quad I_m = V_m / \sqrt{R^2 + X_C{}^2}$$

$$\text{and} \quad v = V_m \sin(\omega t + \psi)$$

The value of I_0 as found from initial known conditions ($t = 0$, $i = 0$) is given by $I_0 = -I_m \sin(\psi + \phi)$

Hence, the resultant current becomes

$$i = I_m \sin(\omega t + \psi + \phi) - I_m \sin(\psi + \phi) e^{-t/\lambda}$$

As shown in Fig. 22.11, the resultant current at the moment of switch closing is *OA* and is made up of steady-state current *OC* and transient current *OB*.

22.10. Double Energy Transients

In an *R-L-C* circuit, both electromagnetic and electrostatic energies are involved, hence any sudden change in the conditions of the circuit involves the redistribution of these two forms of energy. The transient currents produced due to this redistribution are known as double-energy transients. The transient current produced may be unidirectional or a decaying oscillatory current.

In an *R-L-C* circuit, the transient voltages across the three circuit parameters are $i_t, R, (L di_t / dt)$ and q_i / C. Hence, the equation of the transient voltage is

$$i_t R + L \frac{di_t}{dt} + \frac{q_t}{C} = 0 \qquad \text{... } (i)$$

Differentiating the above equation and putting i_t for dq_t/dt, we get

$$\frac{d^2 i_t}{dt^2} + \frac{R}{L} \frac{di_t}{dt} + \frac{1}{LC} i_t = 0 \qquad \text{... } (ii)$$

This is a linear differential equation of the second order with constant coefficient like 2 ***(i)*** given in Art. No. 22.3. Its solution is given by

$$i_t \quad k_1 e^{\;_1 t} \quad k_2 e^{\;_1 t} \qquad \text{... } (iii)$$

where k_1 and k_2 are constants whose values are found from the boundary conditions. The values of λ_1 and λ_2 are given by

$$\lambda_1 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \quad \text{and} \quad _2 \quad \frac{R}{2L} \quad \sqrt{\frac{R^2}{4L^2} \quad \frac{1}{LC}}$$

Depending on the value of λ_1 and λ_2, four different conditions of the circuit are distinguishable. We will now examine these four conditions in the case of an *R-L-C* circuit.

Case 1. Loss-free Circuit, *R* = 0 *i.e.* Undamped

In this case, $\lambda_1 = \sqrt{-\frac{1}{LC}} = j - 1\sqrt{LC} = -j\omega$ and $\lambda_2 = -\sqrt{-\frac{1}{LC}} = -j - 1\sqrt{LC} = -j\omega$

Hence, Eq. **(*iii*)** given above becomes

$$i_t = k_1 e^{j\omega t} + k_2 e^{-j\omega t} = k_1(\cos\omega t + j\sin\omega t) + k_2(\cos\omega t - j\sin\omega t)$$

$$= (k_1 + k_2)\cos\omega t + j(k_1 - k_2)\sin\omega t$$

or $\qquad i_t = A\cos\omega t + B\sin\omega t \qquad$ ... **(*iv*)**

where $\qquad A = k_1 + k_2$ and $B = j(k_1 - k_2)$

Eq. **(*iv*)** can be still further simplified to

$$i_t = I_m \sin(\omega t + \phi) \qquad \text{... } (v)$$

where $\qquad I_m = \sqrt{A^2 + B^2}$ and $\phi = \tan^{-1}(A/B)$

As seen from Eq. **(*v*)**, the transient current in this case is sinusoidal wave of constant peak value and frequency $f = 1/2\pi\sqrt{LC}$ as shown in Fig. 22.12 (*a*). The values of two constant terms I_m and ϕ can be determined from any *two* known initial circuit conditions which are (*i*) the initial current in the inductance and (*ii*) the initial voltage across the capacitor.

Case 2. Low-loss Circuit: $\frac{R^2}{4L^2} < \frac{1}{LC}$ *i.e.* Under-damped

In this case, λ_1 and λ_2 would be conjugate complex numbers because the term under the square root sign in each case would be negative.

$$\therefore \quad _1 \quad \frac{R}{2L} \quad j\sqrt{\frac{1}{LC} \quad \frac{R^2}{4L^2}}$$

If $\quad a = \frac{R}{2L}$ and $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ then $\lambda_1 = -a + j\omega$ and $\lambda_2 = -a - j\omega$

Putting these values in equation (*v*), we get

$$i_t = k_1 e^{(-a+j\omega)t} + k_2 e^{(-a-j\omega)t} = e^{-at}(k_1 e^{j\omega t} + k_2 e^{-j\omega t})$$

This equation can be reduced, as before, to the form

$$i_t = I_m e^{-at}\sin(\omega t + \phi) \qquad \text{... } (iv)$$

where I_m and ϕ are constants as before. Equation **(*vi*)** represents damped transient oscillatory current as shown in Fig. 22.12 (*b*).

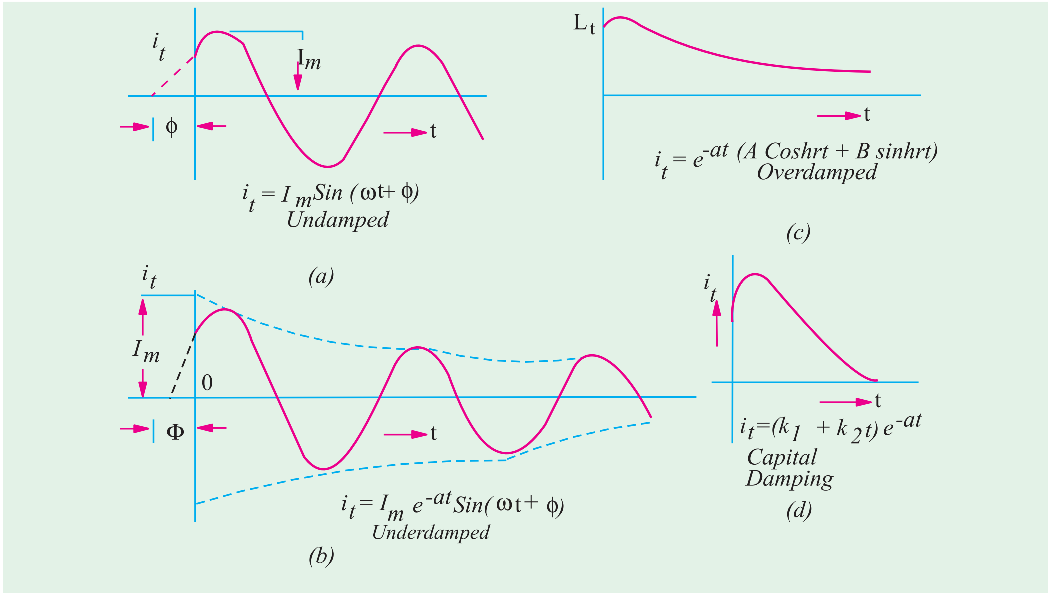


Fig. 22.12

The exponential term e^{-at} which accounts for the decay of oscillations, is called the decay or damping factor or merely ***decrement.*** It makes each current peak a definite fraction less than that preceding it. The logarithm to the Naperian base '*e*' of the ratio of peaks one cycle apart in time is $a/f = R/2fL$ and is referred to as ***logarithmic decrement.*** The frequency of damped oscillations is given by

$$f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$ and is called the natural frequency of the circuit

If $\frac{R^2}{4L^2} < \frac{1}{LC}$, then $f = \frac{1}{2\pi\sqrt{LC}}$

Case 3. High-loss Circuit: $\frac{R^2}{4L^2} \quad \frac{1}{LC}$ *i.e.* **overdamped**

In this case, λ_1 and λ_2 will pure numbers.

$$\lambda_1 - \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -a + \gamma \text{ and } \gamma_2 = -a - \gamma$$

$\therefore$
$$i_t = k_1 e^{(-a+\gamma)t} + k_2 e^{(-a-\gamma)t} = e^{-at}(k_1 e^{\gamma t} + k_2 e^{-\gamma t})$$

Now $\quad e^{rt} = \sinh\gamma t + \cosh\gamma t$

and $\quad e^{-\gamma t} = \cosh\gamma t - \sinh\gamma t$

$\therefore \quad i_t = e^{-at}\{(k_1 + k_2)\cosh\gamma t + (k_1 - k_2)\sinh\gamma t\}$

or $\quad i_t = e^{-at}(A\cosh\gamma t + B\sinh\gamma t)$

A typical curve of this equation is shown in Fig. 22.12(*c*)

Case 4. $\mathbf{\frac{R^2}{4L^2} = \frac{1}{LC}}$ ***i.e.* Critical Damping**

In this case, $\lambda_1 = \lambda_2 = -\frac{R}{2L}$

Hence, equation ***(iii)*** is reduced to

$$i_t = (k_1 + k_2 t)e^{-\frac{R}{2L}t} \text{ or } i_t = (k_1 + k_2 t)e^{-at}$$

It is a case of critical damping because current is reduced to almost zero in the shortest possible time. The above equation has been plotted in Fig. 22.12 (*d*).

Hence, we can summarize as follows:

1. Transient current is an undamped sine wave if $R = 0$

2. Transient current is non-oscillatory if $R < 2\sqrt{L/C}$

3. Transient current is non-oscillatory if $R \geq 2\sqrt{L/C}$

4. Critical damping occurs if $R = 2\sqrt{L/C}$

Example 22.9. *A 5-μF capacitor is discharged suddenly through a coil having an inductance of 2H and a resistance of 200 Ω. The capacitor is initially charged to a voltage of 10 V. Find*

(a) *an expression for the current*

(b) *the additional resistance required to give critical damping.*

Solution. Since there is no battery or generator in the circuit (Fig. 22.13), the steady-state current must be zero. It means that resultant current is simply the transient current.

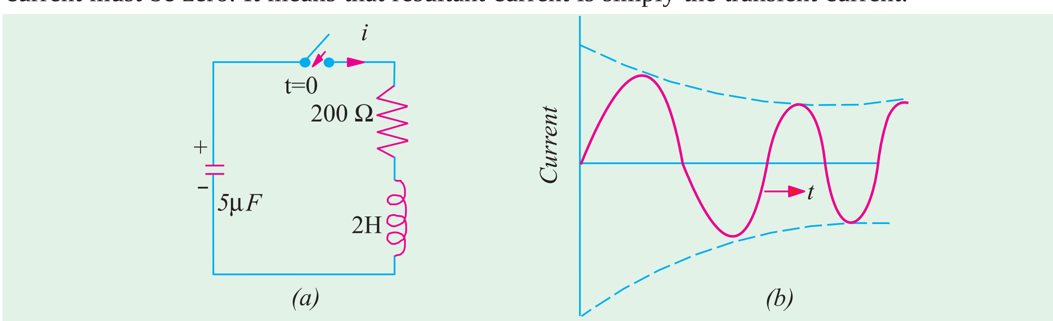


Fig. 22.13

Value of $2\sqrt{L/C} = 2\sqrt{2/5 \times 10^{-6}}$

$= 1265\ \Omega$

Since $R < 2\sqrt{L/C}$, the circuit is originally oscillatory.

(a) the expression for the transient current, therefore, is

$$i_t = I_m e^{-at} \sin(\omega t + \phi)$$

where $a = R/2L = 200/2 \times 2 = 50$

$$\sqrt{\frac{1}{LC} \quad \frac{R^2}{4L^2}} \quad \sqrt{100{,}000 \quad 2500} = 312.3 \text{ rad/s}$$

$$\therefore \qquad i_t = I_m e^{-50t} \sin(312.3t + \phi) = i \qquad \text{... (i)}$$

Two initial conditions are known from which I_m and ϕ can be found (*a*) at t = 0; i = 0 and (*b*) at t = 0; v_c = 10 V. Applying condition (*a*) to Eq. (*i*), we get

$$0 = I_m \sin\phi, \text{ hence } \phi = 0 \qquad \therefore i = I_m e^{-50t} \sin 312.3t \qquad \text{... (ii)}$$

Now, at t = 0, the voltage across the inductance must be 10 V because the current in the resistance is zero.

i.e. $(L\,di/dt)_{t=0}$ = 10 V $\therefore$ $(di/dt)_{t=0} = 10/L$ = 5 A/s ... (*iii*)

Now, from equation (*ii*), we have

$$\frac{di}{dt} = -50I_m e^{-50t} \sin 312.3t + 312.3 I_m e^{-50t} \cos 312.3t \qquad \text{... (iv)}$$

Putting t = 0, it becomes $(di/dt)_{t=0} = 312.3 I_m$

From equation (*iii*), we have

$312.3 I_m = 5$ $\qquad \therefore I_m$ = 5/312.3 = 0.016 A

Hence, the general expression for the current becomes

$$\boldsymbol{i = 0.016\, e^{-50t} \sin 312.3t}$$

It is roughly plotted (the first few cycles only) in Fig. 22.13 (*b*).

(*b*) Critical damping is achieved when $R = 2\sqrt{L/C}$

$$\therefore \quad R = 2\sqrt{2/5 \times 10^{-6}} = 1265\,\Omega$$

$\therefore$ Additional resistance reqd. = 1265 – 200 = **1065** Ω

Example 22.10. *A damped oscillation has the equation* $i = 50e^{-10t}$ *sin 628t. Find the number of oscillations which occurs before the amplitude of the oscillations decays to 1/10th of its undamped value.*

Solution. Undamped amplitude = 50 A

1/10th amplitude = (1/10) × 50 = 5A

Let the time required for this decay be t. Now, the decay of the peak of the oscillations is given by the term $50e^{-10t}$

$$\therefore \quad 5 = 50 e_1^{-10t} \quad \therefore \quad e_1^{10t_1} = 10 \text{ or } 10t_1 = \log h^{10} = 2.3 \log_{10}^{10} = 2.3$$

$\therefore \qquad t_1$ = 0.23 second

Frequency of oscillations = $628/2\pi$ = 100 Hz.

Hence, the number of oscillations which occur before the amplitude falls to 1/10th of its undamped value is = 0.23 × 100 = **23**

Example 22.11. *If, in Fig. 22.14, a break occurs at a point marked X, what would be the voltage across the break? It may be assumed that prior to the break, steady conditions existed in the circuit.*

Solution. Steady-state current through the inductance = 120/60 = 2A

Energy stored in the inductor prior to the break

$$= \frac{1}{2}LI^2 = \frac{1}{2} \times 12 \times 10^{-3} \times 4 = 24 \times 10^{-3}\text{ J}$$

Energy initially stored in the capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-8} \times 120^2 = 72 \times 10^{-6}\text{ J = 0-practically}$$

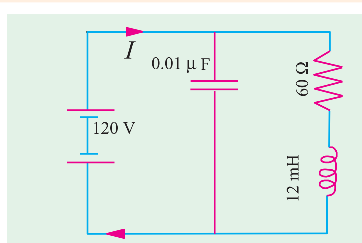


Fig. 22.14

When the break occurs, the energy stored in the inductor is transferred to the capacitor. If loss of energy during first transfer is neglected, then maximum energy stored in the capacitor is

$$= 20 \times 10^{-3}\text{ J} \quad \therefore \quad \frac{1}{2}CV_m^2 = 24 \times 10^{-3}$$

$$\therefore \qquad V_m = \sqrt{2 \times 24 \times 10^{-3} \times 10^8} = 2{,}190\text{ V}$$

Maximum voltage across the break is = 2190 + 120 = 2310 V

The voltage would be oscillatory because the energy alternates between the inductor and capacitor.

Frequency of voltage oscillation is

$$f = 1/2\pi\sqrt{LC} = 10^5/2\pi \times \sqrt{1.2} = 14{,}530\text{ Hz}$$

Decay or damping factor = e^{-at} Art. 22.10, Case 2

Here,a = $R/2L$ = 60/2 × 12 × 10^{-3} = 2500 $\therefore$ damping factor = e^{-2500t}

Hence, voltage across the break is

$$= 120 + 2190\,e^{-2500t}\sin 2\pi \times 14{,}500t = \mathbf{120 + 2190\,e^{\,2500t}\sin 91{,}290t}$$

Tutorial Problem No. 22.1

1. Deduce an expression for the growth of current in an inductive circuit.

A 15-H inductance coil of 10 Ω resistance is suddenly connected to a 20 V d.c. supply. Calculate:

(*a*) the initial rate of change of current

(*b*) the current after 2 second

(*c*) the rate of change of current after 2 second

(*d*) the energy stored in the magnetic field in this time

(*e*) the energy lost as heat in this time

(*f*) the time constant.

[(*a*) 1.33 A/s (*b*) 1.47 A (*c*) 0.352 A/s (*d*) 16.3 joules (*e*) 19.5 joules (*f*) 1.5 s]

2. A circuit consisting of a 20 Ω resistor in series with a 0.2 H inductor is supplied from 200 V (r.m.s.) 50 Hz a.c. mains. Deduce equations showing how the current varies with time if the supply is suddenly switched on (*a*) at the instant when the voltage is zero (*b*) at the instant when the voltage is a maximum.

[(*a*) 4.11 e^{-100} + 4.32 sin (314 t – 70°16') A (*b*) – 1.245 e^{-100t} + 4.32 cos (314t – 72°16') A]

3. A circuit consisting of a 20 Ω resistor, 20 mH inductor and a 100 μF capacitor in series is connected to a 200 V, d.c. supply. The capacitor is initially uncharged. Determine the equation relating the instantaneous current to the time and find the maximum instantaneous current.

[(20 e^{-500t} sin 500 t) A; 6.44 A]

4. Find an expression for the value of current at any instant after a sinusoidal voltage of amplitude 600 V at 50 Hz is applied to a series circuit of resistance 10 Ω and inductance 0.1 H, assuming that voltage is zero at the instant of switching. Also, find the value of transient current at t = 0.02 second.

[– 15.14 A; 2.17 A] (*Electric Circuits and Fields, Gujarat Univ.*)

5. A 40 Ω resistor and a 50 μF capacitor are connected in series and supplied with an alternating voltage v = 283 sin 314 t. The supply is switched on at the instant when the voltage is zero. Determine the expression for the instantaneous current at time t. **[– 3.18e^{-500t} + 3.76 sin (314 t + 57°50')]**

6. A d.c. voltage of 100 V is suddenly applied to a circuit consisting of a 100 Ω resistor, a 0.1 H inductor and a 100 μF capacitor in series. The capacitor is initially uncharged. Obtain the equation which shows how the capacitor voltage varies with time. **[100 – 115.3 e^{-500t} sin (866 t + π /3 V)]**

7. The voltage $v = 200 \sin 314t$ is suddenly applied at $t = 0$ to a circuit consisting of a 10 Ω resistor in series with a 0.1 H inductor. Deduce an equation showing how the current varies with time.

[5.78 e^{-100t} + 6.06 sin (314 t – 72°20')]

8. A 20 Ω resistor, a 0.01 H inductor and a 100 μF capacitor are connected in series. A d.c. voltage of 100 V is suddenly applied to the circuit. Obtain the equation showing how the current through the circuit varies with time. Find the maximum current and the time at which it occurs.

[10^4 e^{-100t}; 3.67 A; 0.001 second]

9. A $4 - \mu$ F capacitor is initially charged to 300 V. It is discharged through a 100 mH inductance and a resistor in series:

(*a*) find the frequency of the discharge if the resistance is zero.

(*b*) how many cycles at the above frequency will occur before the discharge oscillation decays to 1/10 of its initialy value if the resistance is 1 Ω .

(*c*) find the value of the resistance which would just prevent oscillations.

[(*a*) 796 Hz (*b*) 36.6 (*c*) 100 Ω]

OBJECTIVE TESTS – 22

1. Transient disturbance is produced in a circuit whenever
(*a*) it is suddenly connected or disconnected from the supply
(*b*) it is shorted
(*c*) its applied voltage is changed suddenly
(*d*) all of the above.

2. There are no transients in pure resistive circuits because they
(*a*) offer high resistance
(*b*) obey Ohm's law
(*c*) have no stored energy
(*d*) are linear circuits.

3. Transient currents in electrical circuit are associated with
(*a*) inductors
(*b*) capacitors
(*c*) resisters
(*d*) both (*a*) and (*b*).

4. The transients which are produced due to sudden but energetic changes from one steady state of a circuit to another are called .... transients.
(*a*) initiation (*b*) transition
(*c*) relaxation (*d*) subsidence

5. In an *R-L* circuit connected to an alternating sinusoidal voltage, size of transient current primarily depends on
(*a*) the instant in the voltage cycle at which circuit is closed
(*b*) the peak value of steady-state current
(*c*) the circuit impedance
(*d*) the voltage frequency.

6. Double-energy transients are produced in circuits consisting of
(*a*) two or more resistors
(*b*) resistance and inductance
(*c*) resistance and capacitance
(*d*) resistance, inductance and capacitance.

7. The transient current in a loss-free *L-C* circuit when excited from an ac source is a/an ..... sine wave.
(*a*) over damped (*b*) undamped
(*c*) under damped
(*d*) critically damped.

8. Transient current in an *R-L-C* circuit is oscillatory when
(*a*) $R = 0$ (*b*) $R > 2\sqrt{L/C}$
(*c*) $R < 2\sqrt{L/C}$ (*d*) $R = 2\sqrt{L/C}$

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*c*)

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