Report

Project 3

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Abstract:

State update for a quadrotor is done by using data form IMU for prediction and Vision based pose and velocity data for update developed by us in the project 2. In part 1 of the project we have developed a UKF based prediction step as the process model with IMU data is nonlinear and as the position and orientation update model is linear Kalman Filter based update is used. In part 2 prediction model is nonlinear same as that of part 1 and as the velocity update model is nonlinear UKF based update step is used.

Introduction:

Robots have to be able to accommodate the enormous uncertainty that exists in the physical world. Uncertainty are caused by sensors, actuator, robot software, internal models etc. State estimation addresses the problem of estimating quantities from sensor data that are not directly observable, but that can be inferred. To overcome uncertainty and accurate tracking of state we apply probabilistic robotics the idea of estimating state from sensor data, as it is important is several applications\

Probabilistic estimation techniques use a model for predicting aspects of the time behavior of a system (dynamic model) and a model of the sensor measurements (measurement model). Filter is a tool for elegantly combining multisensory fusion, filtering, and motion prediction in a single fast and accurate framework.

Background:

Unscented Kalman Filter

Let $X_t \in R^n$ be a normal Gaussian distribution that we need to propagate.

Let $Y_t \in R^n$ be a normal Gaussian distribution that we need to find(target).

Let g be the non-linear transformation

$Y = g(X) + q \qquad q \sim N(0, Q)$

Concept is to use numerical method for approximating the joint distribution of Gaussian random variable Y (target). Finding a fixed number of sigma points around X_t propagating them using the nonlinear function and then finding the weighted mean of the propagated points.

Methodology

Choose a fixed number of sigma points that captures the mean and covariance of X_t exactly. The number of sigma points are chosen depending upon the size of the state that we are propagating.

No of sigma points = 2n+1 $\qquad$ n = size of the state

Calculating the sigma points

$\sum$ = covariance matrix size will be n*n. Decompose the covariance matrix using the Cholesky Decomposition $\sum = \sqrt{\Sigma}\sqrt{\Sigma}^T$. We choose the ith column of $\sqrt{\Sigma}$

$X^{(i)} = \mu \pm \sqrt{n+\lambda}\left[\sqrt{\Sigma}\right]_i$ i = 1…………n we get 2n points $\lambda = \alpha^2(n+k) - n$

$X^{(0)} = \mu$

We get 2n+1 points where α and k determine spread of the of sigma points.

Propagate sigma points through non-linear function g which captures the mean and covariance of the transformed variable

$Y^{(i)} = g(X^{(i)})$ i = 0 …….2n

Computed the mean and covariance matrices

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{pmatrix} \mu \\ m_u \end{pmatrix}, \begin{pmatrix} \Sigma & C_u \\ {m_u}^T & S_u \end{pmatrix}\right)$$

$m_u = \sum_{i=0}^{2n} W_i^{(m)} \chi_t^{(i)}$ $W_0^{(m)} = \frac{\lambda}{n+\lambda}$ $W_i^{(m)} = \frac{\lambda}{2(n+\lambda)}$

$S_u = \sum_{i=0}^{2n} W_i^{(c)} \left(Y^{(i)} - m_u\right)\left(Y^{(i)} - m_u\right)^T + Q$ $W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$ $W_i^{(c)} = \frac{\lambda}{2(n+\lambda)}$

$C_u = \sum_{i=0}^{2n} W_i^{(c)} \left(X^{(i)} - \mu\right)\left(Y^{(i)} - m_u\right)^T$

Inertial measurement unit

The IMU consists of a tri-axial accelerometer, a tri-axial gyro, and a tri-axial magnetic sensor. The accelerometer measures the difference between the acceleration of the vehicle and gravitational acceleration. The gyroscope measures the angular velocity in the sensor frame. IMU frame is coincident with that of Micro Aerial Vehicle.

The gyroscope gives us a noisy estimate of angular velocity ‘ω_m’

$\omega_m = \omega + b_g + n_g$

where b_g = gyroscope bias $\dot{b}_g = n_{bg} \sim N(0, Q_g)$ and $n_g \sim N(0, \Sigma_g)$

The accelerometer gives us a noisy estimate of acceleration ‘a_m’

$a_m = R(q)^T(\ddot{p} - g) + b_a + n_a$

where b_a = gyroscope bias $\dot{b}_a = n_{ba} \sim N(0, Q_a)$ and $n_a \sim N(0, \Sigma_a)$

Pose and Velocity

Pose and Velocity Estimation as done in part 2 of the project using a Camera on the quadrotor. The position and orientation of the robot were obtained w.r.t the world frame whereas the linear and angular velocity were of the camera frame with respect to the world expressed in the camera frame.

<u>Part 1</u>

Goal is to update the probability distribution of the robot pose using the realization of the control input and measurement. We use last pose of the robot which is a Normal Gaussian distribution $p(x_{t-1}, \Sigma_{t-1})$. Make a process model which uses the control input applied and last state to find the probability of the current state $p(x_t | x_{t-1}, u_t)$

The states that we need to measure are:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \text{positon} \\ \text{orientation} \\ \text{velocity} \\ \text{gyroscope bias} \\ \text{accelerometer bias} \end{bmatrix} \mathcal{E} R^{15}$$

The continuous time process model is:

$\dot{x} = f(x, u, n)$

$$\dot{x} = \begin{bmatrix} \text{velocity} \\ \text{angular velocity} \\ \text{acceleration} \\ n_{bg} \\ n_{ba} \end{bmatrix} = \begin{bmatrix} x_3 \\ G(x_2)^{-1}(\omega_m - x_4 - n_g) \\ R(x_2)(a_m - x_5 - n_a) + g \\ n_{bg} \\ n_{ba} \end{bmatrix} \text{ which is nonlinear}$$

Which was discretized

$X_t = X_{t-1} + dt\, f(X_{t-1}, u_t, q_{t-1})$ which is nonlinear with non-additive noise $q_t \sim N(0, Q_{t-1})$ eq(1)

As the noise is non additive non additive UKF was formulated

$n' = n + n_q$ n = size of state n_q = size of noise

Augmented states are created

$$x_{aug} = \begin{pmatrix} x \\ q \end{pmatrix} \textit{ with mean } \mu_{aug} = \begin{pmatrix} \mu \\ 0 \end{pmatrix} \textit{ and Covariance } \Sigma_{aug} = \begin{pmatrix} \Sigma & 0 \\ 0 & Q \end{pmatrix}$$

Calculated the sigma points using as explained in methodology

$$\mu_{aug,t-1} = \begin{pmatrix} \mu_{t-1} \\ 0 \end{pmatrix} = \mu^0{}_{aug,t-1} \qquad P_{aug,t-1} = \begin{pmatrix} \Sigma_{t-1} & 0 \\ 0 & Q_{t-1} \end{pmatrix}$$

$\mu^{(i)}_{aug,t-1} = \mu^0{}_{aug,t-1} \pm \sqrt{n' + \lambda'}\left[\sqrt{P_{aug,t-1}}\right]_i$ i = 1…………n' and $\mu^0{}_{aug,t-1}$

total $2n' + 1$

$\mu^{(i)}_{aug,t-1}$ *will be of size* $n + n_q$ was broken and the first n terms were used as state and the remaining n_q were used as the noise and propagated in eq (1) and got $\mu^{(i)}_t$ which is of size n

Calculate predicted mean and covariance , $W_i^{(m)}$, $W_i^{(c)}$ calculated with n = n' as given in methodology

$$\bar{\mu}_t = \sum_{i=0}^{2n'} W_i^{(m)} \mu_t^{(i)} \qquad \bar{\Sigma}_t = \sum_{i=0}^{2n'} W_i^{(c)} \left(\mu_t^{(i)} - \bar{\mu}_t\right)\left(\mu_t^{(i)} - \bar{\mu}_t\right)^T$$

Measurement Model:

Is linear as the position are getting updated

$Z_t = C_t x_t + V_t$

Where V_t is noise from Vicon $V_t \sim N(0, R_t)$

For Part 1:

$$Z = \begin{bmatrix} p \\ q \end{bmatrix} + V_t = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ \dot{p} \\ b_g \\ b_a \end{bmatrix} + V_t = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \textit{positon} \\ \textit{orientation} \\ \textit{velocity} \\ \textit{gyroscope bias} \\ \textit{accelerometer bias} \end{bmatrix}$$

$$C_t = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix} \qquad R_t \ \mathcal{E}\ R^{6*6}$$

Update step:

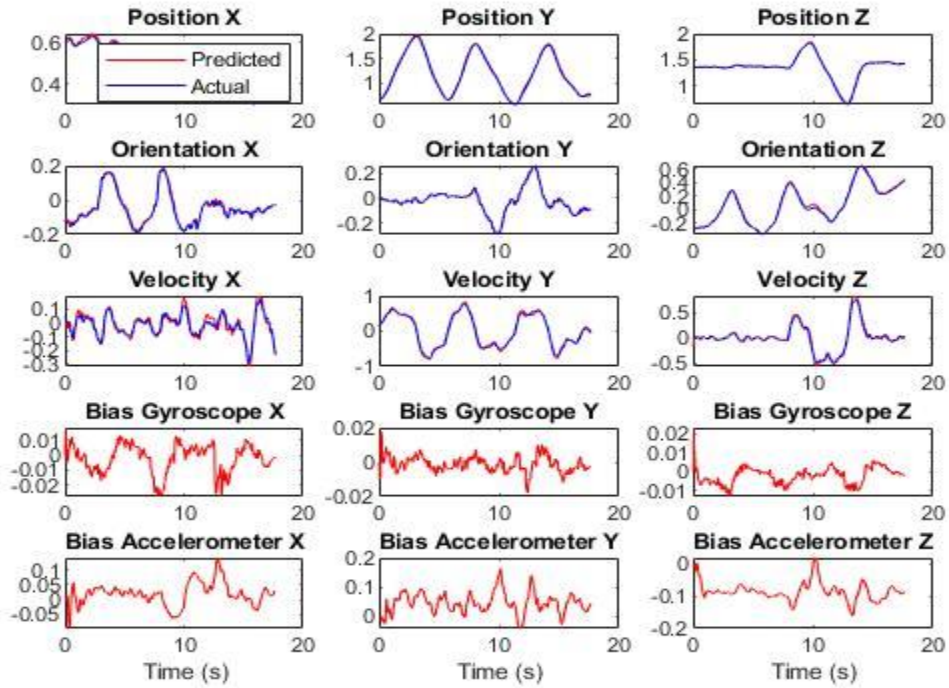
$\mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t)$

$\Sigma_t = \overline{\Sigma_t} - K_t C_t \overline{\Sigma_t}$

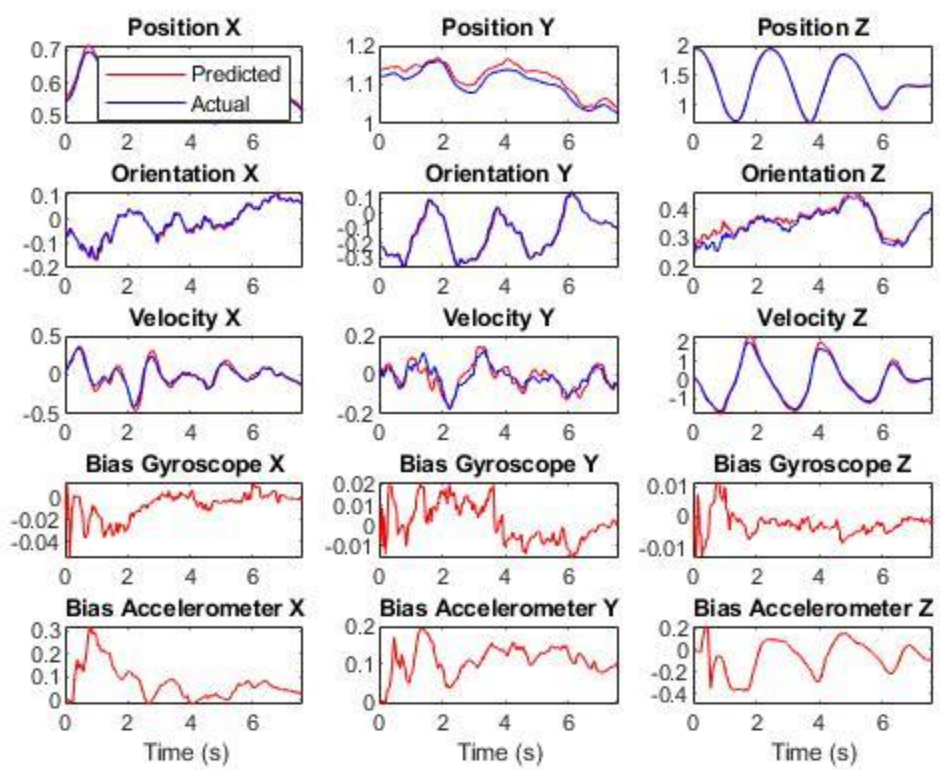
$K_t = \overline{\Sigma_t}\, C_t^T (C_t \overline{\Sigma_t}\, C_t^T + R_t)^{-1}$

Results

Data 1



Data 2



Part 2

The prediction model remains as that of the part 1 only the update model changes as we are updating the velocity and the velocity we get from the camera is of camera frame with respect to the world expressed in the camera frame.

Formulation

$z_t = {}^{C}v_C^W = g\left(x_2, x_3, {}^{B}\omega_B^W, \eta\right)$ *we are getting from camera* $\eta \sim N(0, R)$

$\begin{bmatrix} {}^{C}V_C^W \\ {}^{C}\omega_C^W \end{bmatrix} = \begin{bmatrix} R_B^C & -R_B^C S(r_{BC}^B) \\ 0 & R_B^C \end{bmatrix} \begin{bmatrix} {}^{B}V_B^W \\ {}^{B}\omega_B^W \end{bmatrix}$ C = camera frame, B = Robot frame, W = World frame

${}^{C}\omega_C^W = \textit{we get from the camera}$

$R_B^C = \textit{constant}$

$(r_{BC}^B) = \textit{constant}$

States

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \textit{positon} \\ \textit{orientation} \\ \textit{velocity} \\ \textit{gyroscope bias} \\ \textit{accelerometer bias} \end{bmatrix} \mathcal{E} R^{15}$$

${}^{C}V_C^W = R_B^C\, R(x2)^T\, x_3 - R_B^C S(r_{BC}^B)\, (R_B^C)^T\, {}^{C}\omega_C^W + \eta$ eq(2)

Which is nonlinear with additive noise

UKF with additive is applied as:

We get $\bar{\mu}_t$ and $\bar{\Sigma}_t$ from the prediction step

Sigma points were calculated as explained in methodology with n = size of ($\bar{\mu}_t$)

$\bar{\mu}_t = \bar{\mu}_t{}^{0}$ $\bar{\mu}_t{}^{(i)} = \bar{\mu}_t{}^{0} \pm \sqrt{n + \lambda}\left[\sqrt{\bar{\Sigma}_t}\right]_i$ i = 1…………n

We got total 2n+ 1 points

Which were propagated in the eq(2). We got 2n+ 1 of

$Z_t^{(i)}$ i = 0 …………2n

Calculated

$$Z_{\mu,t} = \sum_{i=0}^{2n} W_i^{(m)} Z_t^{(i)}$$

$$C_t = \sum_{i=0}^{2n} W_i^{(c)} \left(\bar{\mu}_t{}^{(i)} - \bar{\mu}_t\right)\left(Z_t^{(i)} - Z_{\mu,t}\right)^T$$

$$S_t = \sum_{i=0}^{2n} W_i^{(c)} \left(Z_t^{(i)} - Z_{\mu,t}\right)\left(Z_t^{(i)} - Z_{\mu,t}\right)^T$$

Update

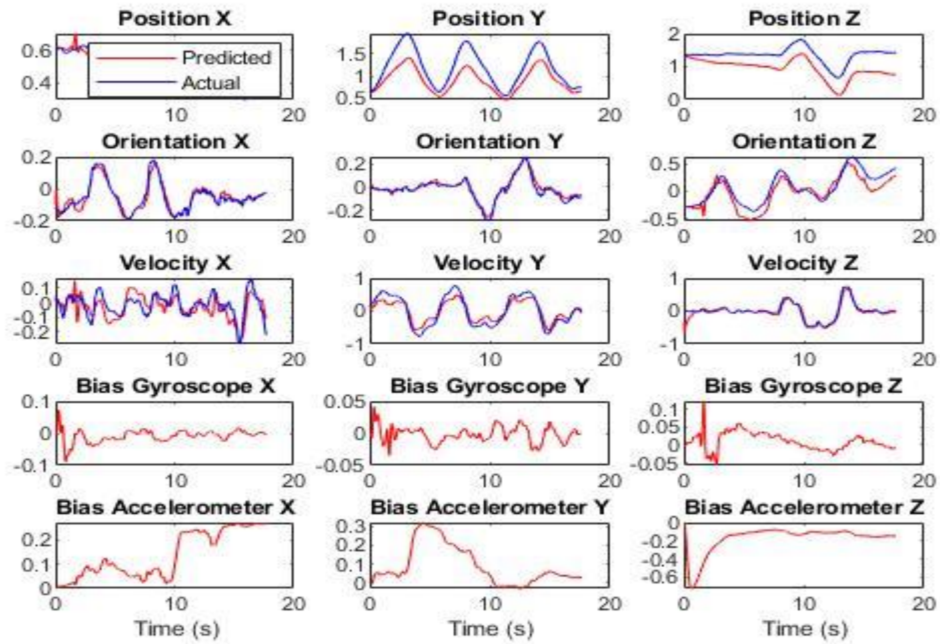
$\mu_t = \bar{\mu}_t + K_t\left(z_t - z_{\mu,t}\right)$ $z_t = {}^{C}V_C^W$

$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$

$K_t = C_t S_t^{-1}$

Results

Data1:



Data2: