LU ICPC kladīte ;)

Contents

1. Algebra

1.1. Binary exponentiation

```
ll m_pow(ll base, ll exp, ll mod) {
    base %= mod;
    ll result = 1;
    while (exp > 0) {
        if (exp & 1) result = ((ll)result * base) % mod;
        base = ((ll)base * base) % mod;
        exp >>= 1;
    }
    return result;
}
```

1.2. Extended euclidean

$$a \cdot x + b \cdot y = \gcd(a, b)$$

```
int gcd_ext(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1; y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
```

1.3. Modular inversion & division

`gcd_ext` defined in Section 1.2.

$$\exists x(a \cdot x \equiv 1 (\operatorname{mod} m)) \Leftrightarrow \gcd(a, m) = 1$$

```
int mod_inv(int b, int m) {
    int x, y;
    int g = gcd_ext(b, m, &x, &y);
    if (g != 1) return -1;
    return (x%m + m) % m;
}
int m_divide(ll a, ll b, ll m) {
    int inv = mod_inv(b, m);
    assert(inv != -1);
    return (inv * (a % m)) % m;
}
```

1.4. Linear Diophantine equation

`gcd_ext` defined in Section 1.2.

$$a \cdot x + b \cdot y = c$$

$$\left\{x = x_0 + k \cdot \frac{b}{g}; y = y_0 - k \cdot \frac{a}{g}\right\}$$

```
bool find_x0_y0(int a, int b, int c, int &x0, int &y0, int &g) {
    g = gcd_ext(abs(a), abs(b), x0, y0);
    if (c % g) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
```

1.5. Linear sieve

```
const int N = 10000000;
vector<int> lp(N+1);
vector<int> pr;

for (int i=2; i <= N; ++i) {
    if (lp[i] == 0) {
        lp[i] = i;
        pr.push_back(i);
    }
    for (int j = 0; i * pr[j] <= N; ++j) {
        lp[i * pr[j]] = pr[j];
        if (pr[j] == lp[i]) break;
    }
}
```

1.6. FFT

```
using ld = long double;
const int N = 1<<18;
const ld PI = acos(-1.0);
struct T {
  ld x, y;
  T() : x(0), y(0) {}
  T(ld a, ld b=0) : x(a), y(b) {}

  T operator/=(ld k) { x/=k; y/=k; return (*this); }
 T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
  T operator+(T a) const { return T(x+a.x, y+a.y); }
  T operator-(T a) const { return T(x-a.x, y-a.y); }
};

void fft(T* a, int n, int s) {
  for (int i=0, j=0; i<n; i++) {
    if (i>j) swap(a[i], a[j]);
    for (int l=n/2; (j^=l) < l; l>>=1);
  }

  for(int i = 1; (1<<i) <= n; i++){
    int M = 1 << i;
    int K = M >> 1;
    T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
    for(int j = 0; j < n; j += M) {
      T w = T(1, 0);
      for(int l = j; l < K + j; ++l){
        T t = w*a[l + K];
        a[l + K] = a[l]-t;
        a[l] = a[l] + t;
        w = wn*w;
      }
    }
  }
}

void multiply(T* a, T* b, int n) {
    while (n&(n-1)) n++; // ensure n is a power of two
    fft(a,n,1);
    fft(b,n,1);
    for (int i = 0; i < n; i++) a[i] = a[i]*b[i];
    fft(a,n,-1);
    for (int i = 0; i < n; i++) a[i] /= n;
}

int main() {
  // Example polynomials: (2 + 3x) and (1 - x)
  T a[10] = {T(2), T(3)};
  T b[10] = {T(1), T(-1)};
  multiply(a, b, 4);
  for (int i = 0; i < 10; i++)
    std::cout << int(a[i].x) << " ";
}
```

2. Geometry

2.1. Miscellanea

```
ostream &operator<<(ostream &os, const point &p) {
    os << "(" << p.x << "," << p.y << ")";
    return os;
}
```

2.2. Closest pair

```
#include "basics.cpp"
//DIVIDE AND CONQUER METHOD
//Warning: include variable id into the struct point

struct cmp_y {
    bool operator()(const point & a, const point & b) const {
        return a.y < b.y;
    }
};

ld min_dist = LINF;
pair<int, int> best_pair;
vector<point> pts, stripe;
int n;

void upd_ans(const point & a, const point & b) {
     ld dist = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y -
b.y));
    if (dist < min_dist) {
        min_dist = dist;
        // best_pair = {a.id, b.id};
    }
}

void closest_pair(int l, int r) {
    if (r - l <= 3) {
        for (int i = l; i < r; ++i) {
            for (int j = i + 1; j < r; ++j) {
                upd_ans(pts[i], pts[j]);
            }
        }
        sort(pts.begin() + l, pts.begin() + r, cmp_y());
        return;
    }

    int m = (l + r) >> 1;
    type midx = pts[m].x;
    closest_pair(l, m);
    closest_pair(m, r);

      merge(pts.begin() + l, pts.begin() + m, pts.begin() + m,
pts.begin() + r, stripe.begin(), cmp_y());
    copy(stripe.begin(), stripe.begin() + r - l, pts.begin() + l);

    int stripe_sz = 0;
    for (int i = l; i < r; ++i) {
        if (abs(pts[i].x - midx) < min_dist) {
              for (int j = stripe_sz - 1; j >= 0 && pts[i].y -
stripe[j].y < min_dist; --j)
                upd_ans(pts[i], stripe[j]);
                stripe[stripe_sz++] = pts[i];
        }
    }
}

int main(){
    //read and save in vector pts
    min_dist = LINF;
    stripe.resize(n);
    sort(pts.begin(), pts.end());
    closest_pair(0, n);
}


//LINE SWEEP
int n; //amount of points
point pnt[N];

struct cmp_y {
    bool operator()(const point & a, const point & b) const {
        if(a.y == b.y) return a.x < b.x;
        return a.y < b.y;
    }
};

ld closest_pair() {
  sort(pnt, pnt+n);
  ld best = numeric_limits<double>::infinity();
  set<point, cmp_y> box;

  box.insert(pnt[0]);
  int l = 0;

  for (int i = 1; i < n; i++){
    while(l < i and pnt[i].x - pnt[l].x > best)
      box.erase(pnt[l++]);
     for(auto it = box.lower_bound({0, pnt[i].y - best}); it !=
box.end() and pnt[i].y + best >= it->y; it++)
      best = min(best, hypot(pnt[i].x - it->x, pnt[i].y - it->y));
    box.insert(pnt[i]);
  }
  return best;
}
```

3. Data structures

3.1. Treap

```
// Implicit segment tree implementation

struct Node{
    int value;
    int cnt;
    int priority;
    Node *left, *right;
    Node(int p) : value(p), cnt(1), priority(gen()), left(NULL),
right(NULL) {};
};

typedef Node* pnode;
```

```cpp
int get(pnode q){
    if(!q) return 0;
    return q->cnt;
}

void update_cnt(pnode &q){
    if(!q) return;
    q->cnt = get(q->left) + get(q->right) + 1;
}

void merge(pnode &T, pnode lef, pnode rig){
    if(!lef) {
        T=rig;
        return;
    }
    if(!rig){
        T=lef;
        return;
    }
    if(lef->priority > rig->priority){
        merge(lef->right, lef->right, rig);
        T = lef;
    }
    else{
        merge(rig->left, lef, rig->left);
        T = rig;
    }
    update_cnt(T);
}

void split(pnode cur, pnode &lef, pnode &rig, int key){
    if(!cur){
        lef = rig = NULL;
        return;
    }
    int id = get(cur->left) + 1;
    if(id <= key){
        split(cur->right, cur->right, rig, key - id);
        lef = cur;
    }
    else{
        split(cur->left, lef, cur->left, key);
        rig = cur;
    }
    update_cnt(cur);
}
```

3.2. Sparse table

```cpp
const int N;
const int M; //log2(N)
int sparse[N][M];

void build() {
  for(int i = 0; i < n; i++)
    sparse[i][0] = v[i];

  for(int j = 1; j < M; j++)
    for(int i = 0; i < n; i++)
      sparse[i][j] =
        i + (1 << j - 1) < n
        ? min(sparse[i][j - 1], sparse[i + (1 << j - 1)][j - 1])
        : sparse[i][j - 1];
}

int query(int a, int b){
  int pot = 32 - __builtin_clz(b - a) - 1;
  return min(sparse[a][pot], sparse[b - (1 << pot) + 1][pot]);
}
```

3.3. Fenwick tree point update

```cpp
struct FenwickTree {
    vector<ll> bit;  // binary indexed tree
    int n;

    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }

    FenwickTree(vector<ll> const &a) : FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i++)
            add(i, a[i]);
    }

    ll sum(int r) {
        ll ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }

   ll sum(int l, int r) { // l to r of the original array INCLUSIVE
        return sum(r) - sum(l - 1);
    }

    void add(int idx, ll delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};
```

3.4. Fenwick tree range update

```cpp
struct FenwickTree { // range update
    ll* bit1;
    ll* bit2;
    int fsize;

    void FenwickTree(int n){
        bit1 = new ll[n+1];
        bit2 = new ll[n+1];
        fsize = n;
        for (int i = 1; i <= n; i++){
            bit1[i] = 0;
            bit2[i] = 0;
        }
    }

    ll getSum(ll BITree[], int index){
        ll sum = 0;
        index++;
        while (index > 0) {
            sum += BITree[index];
            index -= index & (-index);
        }
        return sum;
    }

    void updateBIT(ll BITree[], int index, ll val){
        index++;
        while (index <= fsize) {
            BITree[index] += val;
            index += index & (-index);
        }
    }

    ll sum(ll x){
        return (getSum(bit1, x) * x) - getSum(bit2, x);
    }

   void add(int l, int r, ll val){ // add val to range l:r INCLUSIVE
        updateBIT(bit1, l, val);
        updateBIT(bit1, r + 1, -val);
        updateBIT(bit2, l, val * (l - 1));
        updateBIT(bit2, r + 1, -val * r);
    }

    ll calc(int l, int r){ // sum on range l:r INCLUSIVE
        return sum(r) - sum(l - 1);
    }
};
```

3.5. Segment tree

```
struct item {
    long long sum;
};
struct segtree {
    int size;
    vector<item> values;
    item merge(item a, item b){
        return {
            a.sum + b.sum
        };
    }
    item NEUTURAL_ELEMENT = {0};
    item single(int v){
        return {v};
    }

    void init(int n){
        size = 1;
        while (size < n){
            size *= 2;
        }
        values.resize(size*2, NEUTURAL_ELEMENT);
    }

    void build(vi &arr, int x, int lx, int rx){
        if (rx - lx == 1){
            if (lx < arr.size()){
                values[x] = single(arr[lx]);
            } else {
                values[x] = NEUTURAL_ELEMENT;
            }
            return;
        }
        int m = (lx+rx)/2;
        build(arr, 2 * x + 1, lx, m);
        build(arr, 2 * x + 2, m, rx);
        values[x] = merge(values[2*x+1], values[2*x+2]);
    }
    void build(vi &arr){
        build(arr, 0, 0, size);
    }

    void set(int i, int v, int x, int lx, int rx){
        if (rx - lx == 1){
            values[x] = single(v);
            return;
        }
        int m = (lx + rx) / 2;
        if (i < m){
            set(i, v, 2*x+1, lx, m);
        } else {
            set(i, v, 2*x+2, m, rx);
        }
        values[x] = merge(values[2*x+1], values[2*x+2]);
    }
    void set(int i, int v){
        set(i, v, 0, 0, size);
    }
    item calc(int l, int r, int x, int lx, int rx){
        if (lx >= r || rx <= l) return NEUTURAL_ELEMENT;
        if (lx >= l && rx <= r) return values[x];
        int m = (lx+rx)/2;
        item values1 = calc(l, r, 2*x+1, lx, m);
        item values2 = calc(l, r, 2*x+2, m, rx);
        return merge(values1, values2);
    }
    item calc(int l, int r){
        return calc(l, r, 0, 0, size);
    }
};
```

3.6. Trie

```
const int K = 26;

struct Vertex {
    int next[K];
    bool output = false;

    Vertex() {
        fill(begin(next), end(next), -1);
    }
};

vector<Vertex> t(1); // trie nodes

void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back();
        }
        v = t[v].next[c];
    }
    t[v].output = true;
}
```

3.7. Aho-Corasick

```
const int K = 26;

struct Vertex {
    int next[K];
    bool output = false;
    int p = -1; // parent node
    char pch; // "transition" character from parent to this node
    int link = -1; // fail link
   int go[K]; // if need more memory can delete this and use “next”

    // additional potentially useful things
    int depth = -1;
    // longest string that has an output from this vertex
    int exitlen = -1;

    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t(1);
void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch); // !!!!! ch not c
        }
        v = t[v].next[c];
    }
    t[v].output = true;
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            // !!!!! ch not c
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
```

```
// int go(int v, char ch) { // go without the go[K] variable
//     int c = ch - 'a';
//     if (t[v].next[c] == -1) {
//         // !!!!! ch not c
//         t[v].next[c] = v == 0 ? 0 : go(get_link(v), ch);
//     }
//     return t[v].next[c];
// }

// helper function
int get_depth(int v){
    if (t[v].depth == -1){
        if (v == 0) {
            t[v].depth = 0;
        } else {
            t[v].depth = get_depth(t[v].p)+1;
        }
    }
    return t[v].depth;
}
// helper function
int get_exitlen(int v){
    if (t[v].exitlen == -1){
        if (v == 0){
            t[v].exitlen = 0;
        } else if (t[v].output) {
            t[v].exitlen = get_depth(v);
        } else {
            t[v].exitlen = get_exitlen(get_link(v));
        }
    }
    return t[v].exitlen;
}
```

3.8. Disjoint set union

```
struct disjSet {
    int *rank, *parent, n;
    disjSet(int n) {
        rank = new int[n];
        parent = new int[n];
        this->n = n;
        for (int i = 0; i < n; i++) {
            parent[i] = i;
        }
    }
    int find(int a) {
        if (parent[a] != a){
            //return find(parent[a]); // no path compression
            parent[a] = find(parent[a]); // path compression
        }
        return parent[a];
    }
    void Union(int a, int b) {
        int a_set = find(a);
        int b_set = find(b);
        if (a_set == b_set) return;
        if (rank[a_set] < rank[b_set]) {
            update_union(a_set, b_set);
        } else if (rank[a_set] > rank[b_set]) {
            update_union(b_set, a_set);
        } else {
            update_union(b_set, a_set);
            rank[a_set] = rank[a_set] + 1;
        }
    }
    // change merge behaviour here
    void update_union(int a, int b){ // merge a into b
        parent[a] = b;
    }
};
```

3.9. Merge sort tree

```
struct MergeSortTree {

    int size;
    vector<vector<ll>> values;

    void init(int n){
        size = 1;
        while (size < n){
            size *= 2;
        }
        values.resize(size*2, vl(0));
    }

    void build(vl &arr, int x, int lx, int rx){
        if (rx - lx == 1){
            if (lx < arr.size()){
                values[x].pb(arr[lx]);
            } else {
                values[x].pb(-1);
            }
            return;
        }
        int m = (lx+rx)/2;
        build(arr, 2 * x + 1, lx, m);
        build(arr, 2 * x + 2, m, rx);

        int i = 0;
        int j = 0;
        int asize = values[2*x+1].size();
        while (i < asize && j < asize){
            if (values[2*x+1][i] < values[2*x+2][j]){
                values[x].pb(values[2*x+1][i]);
                i++;
            } else {
                values[x].pb(values[2*x+2][j]);
                j++;
            }
        }
        while (i < asize) {
            values[x].pb(values[2*x+1][i]);
            i++;
        }
        while (j < asize){
            values[x].pb(values[2*x+2][j]);
            j++;
        }
    }
    void build(vl &arr){
        build(arr, 0, 0, size);
    }
```

```
    int calc(int l, int r, int x, int lx, int rx, int k){
        if (lx >= r || rx <= l) return 0;

        // (elements strictly less than k currently)
        if (lx >= l && rx <= r) { // CHANGE HEURISTIC HERE
            int lft = -1;
            int rght = values[x].size();
            while (rght - lft > 1){
                int mid = (lft+rght)/2;
                if (values[x][mid] < k){
                    lft = mid;
                } else {
                    rght = mid;
                }
            }
            return lft+1;
        }

        int m = (lx+rx)/2;
        int values1 = calc(l, r, 2*x+1, lx, m, k);
        int values2 = calc(l, r, 2*x+2, m, rx, k);
        return values1 + values2;
    }
    int calc(int l, int r, int k){
        return calc(l, r, 0, 0, size, k);
    }
};
```

4. Graph algorithms

4.1. Bellman-Ford

```
void solve()
{
    vector<int> d(n, INF);
    d[v] = 0;
    for (;;) {
        bool any = false;

        for (Edge e : edges)
            if (d[e.a] < INF)
                if (d[e.b] > d[e.a] + e.cost) {
                    d[e.b] = d[e.a] + e.cost;
                    any = true;
                }

        if (!any)
            break;
    }
    // display d, for example, on the screen
}
```

4.2. Dijkstra

```
/*******************************************************************************
*   DIJKSTRA'S   ALGORITHM   (SHORTEST   PATH   TO   A   VERTEX)
*
* Time complexity: O((V+E)logE)                                               *
* Usage: dist[node]                                                           *
* Notation: m:        number of edges                                         *
*              (a, b, w):    edge between a and b with weight w
*
*         s:        starting node                                             *
*              par[v]:      parent node of u, used to rebuild the
shortest path      *
*******************************************************************************/

vector<int> adj[N], adjw[N];
int dist[N];

memset(dist, 63, sizeof(dist));
priority_queue<pii> pq;
pq.push(mp(0,0));

while (!pq.empty()) {
  int u = pq.top().nd;
  int d = -pq.top().st;
  pq.pop();

  if (d > dist[u]) continue;
  for (int i = 0; i < adj[u].size(); ++i) {
    int v = adj[u][i];
    int w = adjw[u][i];
    if (dist[u] + w < dist[v])
      dist[v] = dist[u]+w, pq.push(mp(-dist[v], v));
  }
}
```

4.3. Floyd-Warshall

```
int adj[N][N]; // no-edge = INF

for (int k = 0; k < n; ++k)
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      adj[i][j] = min(adj[i][j], adj[i][k]+adj[k][j]);
```

4.4. Bridges & articulations

```
// Articulation points and Bridges O(V+E)
int par[N], art[N], low[N], num[N], ch[N], cnt;

void articulation(int u) {
  low[u] = num[u] = ++cnt;
  for (int v : adj[u]) {
    if (!num[v]) {
      par[v] = u; ch[u]++;
      articulation(v);
      if (low[v] >= num[u]) art[u] = 1;
      if (low[v] >  num[u]) { /* u-v bridge */ }
      low[u] = min(low[u], low[v]);
    }
    else if (v != par[u]) low[u] = min(low[u], num[v]);
  }
}

for (int i = 0; i < n; ++i) if (!num[i])
  articulation(i), art[i] = ch[i]>1;
```

4.5. Dinic's max flow / matching

Time complexity:

- generally: $O(EV^2)$
- small flow: $O(F(V+E))$
- bipartite graph or unit flow: $O\left(E\sqrt{V}\right)$

Usage:

- dinic()
- add_edge(from, to, capacity)
- recover() (optional)

```
#include <bits/stdc++.h>
using namespace std;

const int N = 1e5+1, INF = 1e9;
struct edge {int v, c, f;};

int src, snk, h[N], ptr[N];
vector<edge> edgs;
vector<int> g[N];

void add_edge (int u, int v, int c) {
  int k = edgs.size();
  edgs.push_back({v, c, 0});
  edgs.push_back({u, 0, 0});
  g[u].push_back(k);
  g[v].push_back(k+1);
}

void clear() {
    memset(h, 0, sizeof h);
    memset(ptr, 0, sizeof ptr);
    edgs.clear();
    for (int i = 0; i < N; i++) g[i].clear();
    src = 0;
    snk = N-1;
}

bool bfs() {
  memset(h, 0, sizeof h);
  queue<int> q;
  h[src] = 1;
  q.push(src);
  while(!q.empty()) {
    int u = q.front(); q.pop();
    for(int i : g[u]) {
      int v = edgs[i].v;
      if (!h[v] and edgs[i].f < edgs[i].c)
        q.push(v), h[v] = h[u] + 1;
    }
  }
  return h[snk];
}

int dfs (int u, int flow) {
  if (!flow or u == snk) return flow;
  for (int &i = ptr[u]; i < g[u].size(); ++i) {
    edge &dir = edgs[g[u][i]], &rev = edgs[g[u][i]^1];
    int v = dir.v;
    if (h[v] != h[u] + 1)  continue;
    int inc = min(flow, dir.c - dir.f);
    inc = dfs(v, inc);
    if (inc) {
```

```
      dir.f += inc, rev.f -= inc;
      return inc;
    }
  }
  return 0;
}

int dinic() {
  int flow = 0;
  while (bfs()) {
    memset(ptr, 0, sizeof ptr);
    while (int inc = dfs(src, INF)) flow += inc;
  }
  return flow;
}

//Recover Dinic
void recover(){
  for(int i = 0; i < edgs.size(); i += 2){
    //edge (u -> v) is being used with flow f
    if(edgs[i].f > 0) {
      int v = edgs[i].v;
      int u = edgs[i^1].v;
    }
  }
}


int main () {
    // TEST CASE
    d::clear();
    d::add_edge(d::src,1,1);
    d::add_edge(d::src,2,1);
    d::add_edge(d::src,2,1);
    d::add_edge(d::src,2,1);

    d::add_edge(2,3,d::INF);
    d::add_edge(3,4,d::INF);

    d::add_edge(1,d::snk,1);
    d::add_edge(2,d::snk,1);
    d::add_edge(3,d::snk,1);
    d::add_edge(4,d::snk,1);
    cout<<d::dinic()<<endl; // SHOULD OUTPUT 4
    d::recover();
}
```

4.6. Flow with demands

Finding an arbitrary flow

- Assume a network with $[L;R]$ on edges (some may have $L = 0$), let's call it old network.
- Create a New Source and New Sink (this will be the src and snk for Dinic).
- Modelling network:
  1. Every edge from the old network will have cost $R - L$
  2. Add an edge from New Source to every vertex v with cost:
     - $S(L)$ for every (u, v). (sum all L that LEAVES v)
  3. Add an edge from every vertex v to New Sink with cost:
     - $S(L)$ for every (v, w). (sum all L that ARRIVES v)
  4. Add an edge from Old Source to Old Sink with cost INF (circulation problem)
- The Network will be valid if and only if the flow saturates the network (max flow == $S(L)$)

Finding Min Flow

- To find min flow that satisfies just do a binary search in the (Old Sink -> Old Source) edge
- The cost of this edge represents all the flow from old network
- Min flow = $S(L)$ that arrives in Old Sink + flow that leaves (Old Sink -> Old Source)

4.7. Kosaraju's algorithm

```
/*******************************************************************************
* KOSARAJU'S ALGORITHM (GET EVERY STRONGLY CONNECTED COMPONENTS (SCC))          *
* Description: Given a directed graph, the algorithm generates a list of every   *
* strongly connected components. A SCC is a set of points in which you can reach  *
* every point regardless of where you start from. For instance, cycles can be     *
*   a  SCC  themselves  or  part  of  a  greater  SCC.  *
* This algorithm starts with a DFS and generates an array called "ord" which      *
* stores vertices according to the finish times (i.e. when it reaches "return").  *
* Then, it makes a reversed DFS according to "ord" list. The set of points        *
*  visited  by  the  reversed  DFS  defines  a  new  SCC.  *
* One of the uses of getting all SCC is that you can generate a new DAG (Directed *
* Acyclic Graph), easier to work with, in which each SCC being a "supernode" of   *
* the DAG.                                                                        *
* Time complexity: O(V+E)                                                         *
*   Notation:   adj[i]:        adjacency  list  for  node  i  *
*                 adjt[i]:   reversed adjacency  list  for  node  i  *
*            ord:      array of vertices according to their finish time           *
*        ordn:    ord counter                                                     *
*       scc[i]:  supernode assigned to i                                          *
*              scc_cnt:   amount  of  supernodes  in  the  graph  *
*******************************************************************************/
const int N = 2e5 + 5;

vector<int> adj[N], adjt[N];
int n, ordn, scc_cnt, vis[N], ord[N], scc[N];

//Directed Version
void dfs(int u) {
  vis[u] = 1;
  for (auto v : adj[u]) if (!vis[v]) dfs(v);
  ord[ordn++] = u;
}

void dfst(int u) {
  scc[u] = scc_cnt, vis[u] = 0;
  for (auto v : adjt[u]) if (vis[v]) dfst(v);
}

// add edge: u -> v
void add_edge(int u, int v){
  adj[u].push_back(v);
  adjt[v].push_back(u);
}

//Undirected version:
/*
  int par[N];

  void dfs(int u) {
    vis[u] = 1;
    for (auto v : adj[u]) if(!vis[v]) par[v] = u, dfs(v);
    ord[ordn++] = u;
  }

  void dfst(int u) {
    scc[u] = scc_cnt, vis[u] = 0;
    for (auto v : adj[u]) if(vis[v] and u != par[v]) dfst(v);
  }

  // add edge: u -> v
  void add_edge(int u, int v){
    adj[u].push_back(v);
    adj[v].push_back(u);
  }

*/

// run kosaraju
void kosaraju(){
  for (int i = 1; i <= n; ++i) if (!vis[i]) dfs(i);
  for (int i = ordn - 1; i >= 0; --i) if (vis[ord[i]]) scc_cnt++,
dfst(ord[i]);
}
```

4.8. Lowest common ancestor (LCA)

```
// Lowest Common Ancestor <O(nlogn), O(logn)>
const int N = 1e6, M = 25;
int anc[M][N], h[N], rt;

// TODO: Calculate h[u] and set anc[0][u] = parent of node u for
each u

// build (sparse table)
anc[0][rt] = rt; // set parent of the root to itself
for (int i = 1; i < M; ++i)
  for (int j = 1; j <= n; ++j)
    anc[i][j] = anc[i-1][anc[i-1][j]];
```

```
// query
int lca(int u, int v) {
  if (h[u] < h[v]) swap(u, v);
  for (int i = M-1; i >= 0; --i) if (h[u]-(1<<i) >= h[v])
    u = anc[i][u];

  if (u == v) return u;

  for (int i = M-1; i >= 0; --i) if (anc[i][u] != anc[i][v])
    u = anc[i][u], v = anc[i][v];
  return anc[0][u];
}
```

5. String Processing

5.1. Knuth-Morris-Pratt (KMP)

```
// Knuth-Morris-Pratt - String Matching O(n+m)
char s[N], p[N];
int b[N], n, m; // n = strlen(s), m = strlen(p);

void kmppre() {
  b[0] = -1;
  for (int i = 0, j = -1; i < m; b[++i] = ++j)
    while (j >= 0 and p[i] != p[j])
      j = b[j];
}

void kmp() {
  for (int i = 0, j = 0; i < n;) {
    while (j >= 0 and s[i] != p[j]) j=b[j];
    i++, j++;
    if (j == m) {
      // match position i-j
      j = b[j];
    }
  }
}
```

5.2. Suffix Array

```
// Suffix Array O(nlogn)
// s.push('$');
vector<int> suffix_array(string &s){
  int n = s.size(), alph = 256;
  vector<int> cnt(max(n, alph)), p(n), c(n);

  for(auto c : s) cnt[c]++;
  for(int i = 1; i < alph; i++) cnt[i] += cnt[i - 1];
  for(int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
  for(int i = 1; i < n; i++)
    c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);

  vector<int> c2(n), p2(n);

  for(int k = 0; (1 << k) < n; k++){
    int classes = c[p[n - 1]] + 1;
    fill(cnt.begin(), cnt.begin() + classes, 0);

    for(int i = 0; i < n; i++) p2[i] = (p[i] - (1 << k) + n)%n;
    for(int i = 0; i < n; i++) cnt[c[i]]++;
    for(int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
    for(int i = n - 1; i >= 0; i--) p[--cnt[c[p2[i]]]] = p2[i];

    c2[p[0]] = 0;
    for(int i = 1; i < n; i++){
      pair<int, int> b1 = {c[p[i]], c[(p[i] + (1 << k))%n]};
       pair<int, int> b2 = {c[p[i - 1]], c[(p[i - 1] + (1 << k))
%n]};
      c2[p[i]] = c2[p[i - 1]] + (b1 != b2);
    }

    c.swap(c2);
  }
  return p;
}

// Longest Common Prefix with SA O(n)
vector<int> lcp(string &s, vector<int> &p){
  int n = s.size();
  vector<int> ans(n - 1), pi(n);
  for(int i = 0; i < n; i++) pi[p[i]] = i;

  int lst = 0;
  for(int i = 0; i < n - 1; i++){
    if(pi[i] == n - 1) continue;
    while(s[i + lst] == s[p[pi[i] + 1] + lst]) lst++;

    ans[pi[i]] = lst;
    lst = max(0, lst - 1);
  }

  return ans;
}

// Longest Repeated Substring O(n)
int lrs = 0;
for (int i = 0; i < n; ++i) lrs = max(lrs, lcp[i]);

// Longest Common Substring O(n)
// m = strlen(s);
// strcat(s, "$"); strcat(s, p); strcat(s, "#");
// n = strlen(s);
int lcs = 0;
for (int i = 1; i < n; ++i) if ((sa[i] < m) != (sa[i-1] < m))
  lcs = max(lcs, lcp[i]);

// To calc LCS for multiple texts use a slide window with minqueue
// The numver of different substrings of a string is n*(n + 1)/2
- sum(lcs[i])
```

5.3. Rabin-Karp

```
// Rabin-Karp - String Matching + Hashing O(n+m)
const int B = 31;
char s[N], p[N];
int n, m; // n = strlen(s), m = strlen(p)

void rabin() {
  if (n<m) return;

  ull hp = 0, hs = 0, E = 1;
  for (int i = 0; i < m; ++i)
    hp = ((hp*B)%MOD + p[i])%MOD,
    hs = ((hs*B)%MOD + s[i])%MOD,
    E = (E*B)%MOD;

  if (hs == hp) { /* matching position 0 */ }
  for (int i = m; i < n; ++i) {
    hs = ((hs*B)%MOD + s[i])%MOD;
    hhs = (hs - s[i-m]*E%MOD + MOD)%MOD;
    if (hs == hp) { /* matching position i-m+1 */ }
  }
}
```

5.4. Z-function

The Z-function of a string s is an array z where z_i is the length of the longest substring starting from s_i which is also a prefix of s.

Examples:

- "aaaaa": $[0, 4, 3, 2, 1]$
- "aaabaab": $[0, 2, 1, 0, 2, 1, 0]$
- "abacaba": $[0, 0, 1, 0, 3, 0, 1]$

```
vector<int> zfunction(const string& s){
  vector<int> z (s.size());
  for (int i = 1, l = 0, r = 0, n = s.size(); i < n; i++){
    if (i <= r) z[i] = min(z[i-l], r - i + 1);
    while (i + z[i] < n and s[z[i]] == s[z[i] + i]) z[i]++;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
  return z;
}
```

5.5. Manacher's

```
// Manacher (Longest Palindromic String) - O(n)
int lps[2*N+5];
char s[N];

int manacher() {
  int n = strlen(s);

  string p (2*n+3, '#');
  p[0] = '^';
  for (int i = 0; i < n; i++) p[2*(i+1)] = s[i];
  p[2*n+2] = '$';

  int k = 0, r = 0, m = 0;
  int l = p.length();
  for (int i = 1; i < l; i++) {
    int o = 2*k - i;
    lps[i] = (r > i) ? min(r-i, lps[o]) : 0;
    while (p[i + 1 + lps[i]] == p[i - 1 - lps[i]]) lps[i]++;
    if (i + lps[i] > r) k = i, r = i + lps[i];
    m = max(m, lps[i]);
  }
  return m;
}
```

6. Dynamic programming

6.1. Convex hull trick

```
// Convex Hull Trick

// ATTENTION: This is the maximum convex hull. If you need the
minimum
// CHT use {-b, -m} and modify the query function.

// In case of floating point parameters swap long long with long
double
typedef long long type;
struct line { type b, m; };

line v[N]; // lines from input
int n; // number of lines
// Sort slopes in ascending order (in main):
sort(v, v+n, [](line s, line t){
     return (s.m == t.m) ? (s.b < t.b) : (s.m < t.m); });

// nh: number of lines on convex hull
// pos: position for linear time search
// hull: lines in the convex hull
int nh, pos;
line hull[N];

bool check(line s, line t, line u) {
  // verify if it can overflow. If it can just divide using long
double
  return (s.b - t.b)*(u.m - s.m) < (s.b - u.b)*(t.m - s.m);
}

// Add new line to convex hull, if possible
// Must receive lines in the correct order, otherwise it won't work
void update(line s) {
  // 1. if first lines have the same b, get the one with bigger m
  // 2. if line is parallel to the one at the top, ignore
  // 3. pop lines that are worse
  // 3.1 if you can do a linear time search, use
  // 4. add new line

  if (nh == 1 and hull[nh-1].b == s.b) nh--;
  if (nh > 0  and hull[nh-1].m >= s.m) return;
  while (nh >= 2 and !check(hull[nh-2], hull[nh-1], s)) nh--;
  pos = min(pos, nh);
  hull[nh++] = s;
}

type eval(int id, type x) { return hull[id].b + hull[id].m * x; }

// Linear search query - O(n) for all queries
// Only possible if the queries always move to the right
type query(type x) {
  while (pos+1 < nh and eval(pos, x) < eval(pos+1, x)) pos++;
  return eval(pos, x);
  // return -eval(pos, x);    ATTENTION: Uncomment for minimum CHT
}

// Ternary search query - O(logn) for each query
/*
type query(type x) {
  int lo = 0, hi = nh-1;
  while (lo < hi) {
    int mid = (lo+hi)/2;
    if (eval(mid, x) > eval(mid+1, x)) hi = mid;
    else lo = mid+1;
  }
  return eval(lo, x);
  // return -eval(lo, x);     ATTENTION: Uncomment for minimum CHT
}

// better use geometry line_intersect (this assumes s and t are
not parallel)
ld intersect_x(line s, line t) { return (t.b - s.b)/(ld)(s.m -
t.m); }
ld intersect_y(line s, line t) { return s.b + s.m * intersect_x(s,
t); }
*/
```

6.2. Longest Increasing Subsequence

```
// Longest Increasing Subsequence - O(nlogn)
//
// dp(i) = max j<i { dp(j) | a[j] < a[i] } + 1
//

// int dp[N], v[N], n, lis;

memset(dp, 63, sizeof dp);
for (int i = 0; i < n; ++i) {
  // increasing: lower_bound
  // non-decreasing: upper_bound
  int j = lower_bound(dp, dp + lis, v[i]) - dp;
  dp[j] = min(dp[j], v[i]);
  lis = max(lis, j + 1);
}
```

6.3. SOS DP (Sum over Subsets)

```
// O(bits*(2^bits))

const int bits = 20;

vector<int> a(1<<bits); // initial value of each subset
vector<int> f(1<<bits); // sum over all subsets
// (at f[011] = a[011]+a[001]+a[010]+a[000])

for (int i = 0; i<(1<<bits); i++){
    f[i] = a[i];
}
for (int i = 0; i < bits; i++) {
  for(int mask = 0; mask < (1<<bits); mask++){
    if(mask & (1<<i)){
        f[mask] += f[mask^(1<<i)];
    }
  }
}
```