第一次离散数学作业

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第一部分　1.1

1　20

1.1　(a)

inclusive or

1.2　(b)

exclusive or

1.3　(c)

inclusive or

1.4　(d)

exclusive or

2　33

2.1　(a)

$$(p \vee q) \rightarrow (p \oplus q)$$

The truth table:

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

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3.1 (c)

$$(p \to q) \vee (\neg p \to q)$$

The truth table:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

第二部分 1.2

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let p denotes ”the system is in multiuser state”.
let q denotes ”the system is operating normally”
let a denotes ”the kernel is functioning”
let b denotes ”the system is in interrupt mode”
according to the description, we know:

$$p \Leftrightarrow q, q \to a, \neg a \oplus b, \neg p \to b$$

and b is false.
assume all propositons are true.
then $\neg a$ is true, a is false.
because $q \rightarrow a$ is true, so q is false.
because $p \Leftrightarrow q$, so p is false.
so $\neg p$ is true. so $\neg p \rightarrow b$ is false, which is contradict with the assumption.
so these system specifications are not consistent.

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let p denotes "The two of us are both knights"
let q denotes "A is a knave"
the truth table:

A	B	p	q	conclusion
knight	knight	true	false	wrong
knight	knave	false	false	wrong
knave	knight	false	true	right
knave	knave	false	true	wrong

so A is a knave and B is a knight.

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let a,b,c,d,e denote Kevin,Heather,Randy,Vijay,Abby is chatting.
according to the description:
$a \vee b, c \oplus d, e \rightarrow c, a \wedge d, \neg a \wedge \neg d, b \rightarrow a \wedge e$
if a is true, d is true, and c is false, so e is false, then $a \wedge e$ is false so b is false.
in another case, if a is false, d is false, and c is true, no matter e is true or false, $a \wedge e$ is false, so b is false. So $a \vee b$ is false, which contradict with the description.
In conclusion, Kevin and Vijay are chatting, it's possible to determine who is chatting.

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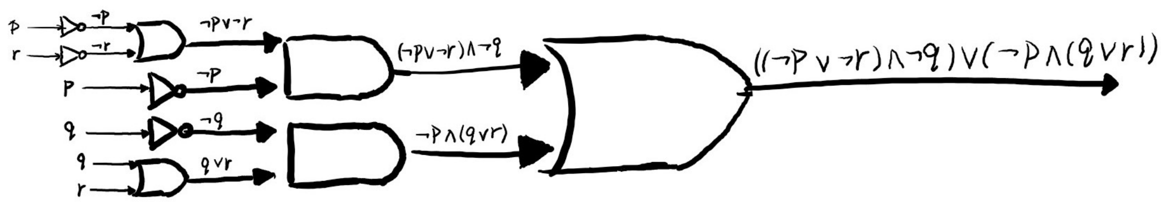

第三部分 1.3

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$(\neg p \land (p \to q)) \to \neg p$ is a tautology.
Assume $(\neg p \land (p \to q)) \to \neg p$ is false, then $(\neg p \land (p \to q))$ is true while p is false. however, if $(\neg p \land (p \to q))$ is true, then $\neg q and p \to q$ is true. Then, q is false. And because $p \to q$ is true and q is false, p is false. So $\neg p$ is true, which contradicts with the assumption.
In conclusion: $(\neg p \land (p \to q)) \to \neg p$ **is a tautology.**

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If $(p \to r) \land (q \to r)$ is true, then $(p \to r) and (q \to r)$ are true. So $(p \lor q) \to r$ is true.

If $(p \lor q) \to r$ is false, then $p \lor q$ is true and r is false. Then p and q are not false at the same time. So $(p \to r) \land (q \to r)$ is false.
$(p \to r) \land (q \to r)$ and $(p \lor q) \to r$ are of the same value. **In conclusion, they are logically equivalent.**

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Given a compound proposition p, form its truth table and write a proposition q in disjunctive normal form that is logically equivalent to p. q invloves only $\neg, \land and \lor$, so these three operators form a functionally complete set. Then according to De Morgen's law, we can replace the operator $\wedge$ with $\vee and \neg$. So $\vee and \neg$ form a functionally complete set.