2016/17 Past Paper Q2

a) i) factors (1) = []

ii) factors (4) = [2,2]

iii) factors (7) = [7,0,0]

iv) factors (10) = [2,5,0,0,0]

note 0 elements are important (as is the correct length of each array) xx

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b) i) $i = -2$

note any negative int ≤ -2 is also fine :)
-1 does not raise an exception as $-1/2 = 0$ (returns [] in this case) xx

ii) factors will fault on line 5 `int [] fs = new int[n/2];` if it tries to create an array of negative size.

iii) $n \geq 0$

note the Precondition should rule out all negative inputs :)
(although $n \geq -1$ or $n > -2$ also acceptable since -2 is first case to produce an error)
($n > 1$ or $n \geq 2$ are overkill and rule out perfectly valid inputs) xx

iv) The size of the output array is normally larger than necessary to hold the factors of n.
The extra 0 elements cannot satisfy the postcondition if $n \neq 0$.
This is because $n \neq 0 \rightarrow \neg \exists m \in \mathbb{N}.\ m * 0 = n$.

note pos not defined in post condition xx
(it is internal variable in method)

v) $\forall y \in [0..r.length).\ [(\exists m \in \mathbb{N}.\ m * r[y] = n) \vee r[y] = 0]$

many other possibilities too

or $\exists x \in [0..r.length].\ [\forall y \in [0..x).\ \exists m \in \mathbb{N}.\ m * r[y] = n \wedge \forall y \in [x..r.length).\ r[y] = 0]$

note must be possible for $x = r.length$ so we don't force 0s in r.

c) $R \to Q$

Given:

(1) $\prod_{k=0}^{r.length-1} r[k] = n$ (R)

To show:

(α) $\forall y \in [0..r.length).\ \exists m \in \mathbb{N}.\ m * r[y] = n$ (Q)

Proof:

two cases to consider for when result array is empty or not

note if $r.length = 0$ and $n \neq 1$ then R is actually false. However $R \to Q$ is true if R is false. :)

Case 1:

(ass 1) $r.length = 0$

The range $[0..0)$ is empty so (α) holds vacuously.

Case 2:

(ass 2) $r.length \neq 0$

Take $y \in [0..r.length)$ arbitrary (such y exists since array not empty)

Now show: (β) $\exists m \in \mathbb{N}.\ m * r[y] = n$

(2) $\left(\prod_{k=0}^{y-1} r[k]\right) * \left(\prod_{k=y}^{r.length-1} r[k]\right) = n$ from (1), (ass 2) and def. $\prod$

(3) $\left(\prod_{k=0}^{y-1} r[k]\right) * r[y] * \left(\prod_{k=y+1}^{r.length-1} r[k]\right) = n$ from (2), (ass 2) and def. $\prod$

(4) $\left(\left(\prod_{k=0}^{y-1} r[k]\right) * \left(\prod_{k=y+1}^{r.length-1} r[k]\right)\right) * r[y] = n$ from (3) and arith

(let) Let $m = \left(\prod_{k=0}^{y-1} r[k]\right) * \left(\prod_{k=y+1}^{r.length-1} r[k]\right)$ well defined from (ass 2) and def. $\prod$

(6) $m * r[y] = n$ from (4) and (let)

(β) follows from (6) and (let) then (α) follows from (β) and arb. choice of y.

d) i) $I \Leftrightarrow 0 \leq curr \leq n$
$\wedge\ 2 \leq cand \leq n$
$\wedge\ 0 \leq pos \leq fs.length$
$\wedge\ curr * \prod_{k=0}^{pos-1} fs[k] = n$

note $fs.length = n/2$ ☺

note choice of precondition in (b) iii) may change your lower bound for curr. ☺

ii) $V = curr - cand$

or any similar

note $curr/cand$ is not a valid variant

e.g. $7/4 = 1$, $7/5 = 1$ } does not always decrease! ☹

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e) i) e.g. $R \wedge \forall y \in [0..r.length).\ isPrime(r[y])$

or $R \wedge \forall y \in r[0..r.length).\ isPrime(y)$

where $isPrime(y) \Leftrightarrow \forall z \in (1..y).\ \neg \exists x \in \mathbb{N}.\ y = z * x$

note care to say contents of r are prime and not its indicies. ☹

remember: 1 is not a prime no.! (many people get this wrong) ☹

ii) The JMC student is correct, because if any number x in the output array were not prime, then it would be expressible as the product of two integers, say a and b, that must both be bigger than 1 and smaller than x. However, the algorithm would already have covered these values in an earlier loop iteration and found such factors, fully dividing them out of curr. Thus, it is only possible to find x as a factor of curr if it is only divisible by 1 and itself (i.e. it is prime)

note simply being a JMC student does not automatically make them right (even if often the case) ☺