140 Logic exercises 2

Hand in solutions for questions marked **(PMT)** to the SAO by Mon 23 Oct 2017
Questions marked **(PMT)** to be discussed in PMT in week 4 (23–27 Oct)

1. Translate into logic the following sentences.

   (a) James will work hard and get good grades, or he will belong to the dramatic society.

   (b) Janet likes John and she likes his brother too.

   (c) Janet likes John, but she likes his brother too.

   Express (1a) more naturally in English, using "unless".

2. Suppose that A is true, B is true and C is false in a certain situation. Which of the following evaluate to true and which to false in this situation?

   (a) $(A \to B) \to \neg B$

   (b) $(\neg A \to (\neg B \wedge C)) \vee B$

   (c) $(((A \vee \neg C) \wedge \neg B) \to A) \to (\neg B \wedge \neg C)$

3. **(PMT)** Translate into logic the following sentences (they were written by a computing person!). First decide on the atoms to use (e.g., 'Frank bought grapes' could become an atom), and then get the sentence structure correct (sort out where the $\wedge, \vee$ etc. go).

   (a) Luo Ji has white hair if Da Shi likes beer.

   (b) The house will be finished if the outstanding bill is paid or the proprietor works on it himself.

   (c) Ye Wenjie likes dim sum only if Zhang Beihai is tall.

   (d) CSG is responsible only if the computer was installed since January and is a PC.

   (e) Frank bought grapes and either apples or pears.

   (f) I'll be back by 2 p.m., and will bring the shopping if and only if it does not rain.

4. Suppose that A is false, B is false and C is true in a certain situation. Which of the following evaluate to true and which to false in this situation? Show your working.

   (a) $(\neg A \to \neg B) \to B$

   (b) $(\neg(A \to \neg B) \wedge C) \vee B$

   (c) $(((\neg A \vee \neg C) \wedge \neg B) \to \neg A) \to \neg(B \wedge \neg C)$

5. Consider a set of objects labelled $a, b, c, \ldots$ placed on a 3×3 grid, and the following atomic formulas talking about the objects:

   - $[x$ *next-to* $y]$ means (that is, it is true if) x and y are adjacent (horizontally or vertically, but not diagonally);
   - $[x$ *sees* $y]$ means x and y are in the same row or the same column;
   - $[x$ *left-of* $y]$ means x is in a column to the left of the column of y

- [x *above* y] means x is in a row above the row of y.

(a) For the placements shown in figure 1, which of the following evaluate to true, and why?

- $[A\ \textit{sees}\ B] \leftrightarrow [B\ \textit{sees}\ C]$
- $[B\ \textit{next-to}\ D] \vee [B\ \textit{next-to}\ E]$
- $\neg([A\ \textit{left-of}\ F] \wedge [F\ \textit{above}\ A])$
- $\neg([E\ \textit{left-of}\ D] \to \neg[D\ \textit{next-to}\ C]) \to \neg[A\ \textit{sees}\ E]$
- $([E\ \textit{sees}\ D] \vee [F\ \textit{sees}\ E]) \to \neg([B\ \textit{above}\ E] \leftrightarrow [B\ \textit{next-to}\ C])$

A		D
C	F	B
	E	

Figure 1: a 3×3 grid with placements of A–F

(b) Place the 6 objects $A, \ldots, F$ on the grid so that **all** the formulas in (5a) are true.

6. **(PMT)** Repeat (5b) but place the 6 objects so that all the formulas in (5a) are false. Justify your answer by explaining why each formula is false.
(Hint: work out which basic relationships need to be true or false first. E.g., to make the third formula false requires [A *left-of* F] and [F *above* A] to be true. There are many correct solutions.)

7. Show that the following pairs of formulas are logically equivalent by checking that they have the same truth value in all relevant situations (i.e., using truth tables).

   (a) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

   (b) $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$

   (c) $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $(p \to q) \wedge (q \to p)$

   (d) $p \to (q \to r)$ and $p \wedge q \to r$

   (e) $p \to (q \wedge\ r)$ and $(p \to q) \wedge\ (p \to r)$

8. Let $A_1, \ldots, A_n, B$ be any propositional formulas. Express (define) $A_1, \ldots, A_n \models B$ using (i) validity, (ii) satisfiability, (iii) logical equivalence.

Logic exercises 2 Solutions

Questions marked **(PMT)** to be discussed in PMT in week 4 (23–27 Oct)

1. (a) (J works hard $\wedge$ J gets good grades) $\vee$ J in dramsoc [brackets not really needed]

   (b) Janet likes John $\wedge$ Janet likes John's brother

   (c) Same as (1b).

   (a′) James will work hard and get good grades unless he joins the dramatic society.

2. If A, B are true and C is false, we have:

   (a) $(A \to B) \to \neg B$ = (true $\to$ true) $\to$ $\neg$true = true $\to$ false = false.

   (b) $(\neg A \to (\neg B \wedge C)) \vee B$ = ($\neg$true $\to$ ($\neg$true$\wedge$false))$\vee$true = true, because (anything$\vee$ true) = true.

   (c) $(((A \vee \neg C) \wedge \neg B) \to A) \to (\neg B \wedge \neg C)$
   = (((true $\vee$ $\neg$false) $\wedge$ $\neg$true) $\to$ true) $\to$ ($\neg$true $\wedge$ $\neg$false)
   = (((true $\vee$ true) $\wedge$ false) $\to$ true) $\to$ (false $\wedge$ true)
   = true $\to$ false [because (anything $\to$ true) = true]
   = false.

3. **(PMT)** Parts (a)–(d) are *critical* to your understanding of translations. Please ask your UTA or a helper if you are puzzled about them.

   (a) 'Luo Ji has white hair if Da Shi likes beer.' The only way it can be false is if Da Shi likes beer but Luo Ji doesn't have white hair. So it's translated as

   Da Shi likes beer $\to$ Luo Ji has white hair

   which also is false only in this case.

   (b) 'The house will be finished if the outstanding bill is paid or the proprietor works on it himself' is translated as

   (bill paid $\vee$ works himself) $\to$ house finished.

   In Computing we read the English as saying that *if* he works on it himself or the bill is paid, *then* the house will definitely be finished. We do *not* read it as saying that these are the *only* ways that the house could be finished. The *only* way the English (and the logic) can be false is when the bill is paid, or the proprietor works on it, or both, but the house does not get finished.

   (c) 'Ye Wenjie likes dim sum only if Zhang Beihai is tall.' We read this as 'the only circumstances under which Ye Wenjie likes dim sum are those in which Zhang Beihai is tall'. So the English is false in only one situation: Ye Wenjie likes dim sum, but Zhang Beihai is not tall. The following formula is false in only this situation as well, so it's a suitable translation:

   Ye Wenjie likes dim sum $\to$ Zhang Beihai is tall

   It says IF Ye Wenjie likes dim sum THEN Zhang Beihai is tall.

   (d) 'CSG is responsible only if the computer was installed since January and is a PC' is translated as CSG responsible $\to$ (installed since Jan $\wedge$ is a PC). In Computing, we read the English as 'the *only* circumstances under which CSG can be responsible are when the computer was installed since January and is a PC.' That is, if CSG *is* in fact responsible, then the computer definitely *was* installed since Jan *and* is a PC. But just because the computer was installed since January and is a PC, *this does NOT mean that CSG is responsible!*

(e) Frank bought grapes $\wedge$ (Frank bought apples $\vee$ Frank bought pears). In computing, we prefer the 'inclusive' meaning *'one or both'* of 'or'. But *'either-or'* is often used to indicate *exclusive or* (one or the other but not both), so here, grapes $\wedge$ ((apples $\vee$ pears) $\wedge$ $\neg$(apples $\wedge$ pears)) and grapes $\wedge$ (apples $\leftrightarrow$ $\neg$pears) are also acceptable.

(f) back by 2 $\wedge$ (bring shopping $\leftrightarrow$ $\neg$rains)

4. If A, B are false and C is true, then:

   (a) $(\neg A \to \neg B) \to B$
   = ($\neg$false $\to$ $\neg$false) $\to$ false
   = (true $\to$ true) $\to$ false
   = true $\to$ false = false. (These are not formulas, as they mix syntactic connectives with semantic truth values. But they're convenient for showing the working.)

   (b) $(\neg(A \to \neg B) \wedge C) \vee B$ = ($\neg$(false $\to$ $\neg$false) $\wedge$ true) $\vee$ false
   = ($\neg$(false $\to$ true) $\wedge$ true) $\vee$ false
   = ($\neg$true $\wedge$ true) $\vee$ false
   = (false $\wedge$ true) $\vee$ false
   = false $\vee$ false
   = false.

   (c) $(((\neg A \vee \neg C) \wedge \neg B) \to \neg A) \to \neg(B \wedge \neg C)$. Useful trick: when evaluating an implication $X \to Y$ where X is more complex than Y, first check if Y is true. If so, $X \to Y$ will be true, so this saves evaluating X. (Similarly, if X is simpler, evaluate it first and hope it's false.) Here, $\neg(B \wedge \neg C)$ = $\neg$(false $\wedge$ $\neg$true) = $\neg$(false $\wedge$ false) = $\neg$false = true. So the original formula is true.

5. (a) In the situation

A		D
C	F	B
	E	

   - $[A\ sees\ B] \leftrightarrow [B\ sees\ C]$ evaluates to false $\leftrightarrow$ true = false.
   - $[B\ next\text{-}to\ D] \vee [B\ next\text{-}to\ E]$ evaluates to true $\vee$ false = true.
   - $\neg([A\ left\text{-}of\ F] \wedge [F\ above\ A])$ evaluates to $\neg$(true $\wedge$ false) = $\neg$false = true.
   - $\neg([E\ left\text{-}of\ D] \to \neg[D\ next\text{-}to\ C]) \to \neg[A\ sees\ E]$ evaluates to $\neg$(true $\to$ $\neg$false) $\to$ $\neg$false = $\neg$(true $\to$ true) $\to$ true = true [because (anything $\to$ true) = true].
   - $([E\ sees\ D] \vee [F\ sees\ E]) \to \neg([B\ above\ E] \leftrightarrow [B\ next\text{-}to\ C])$ evaluates to (false $\vee$ true) $\to$ $\neg$(true $\leftrightarrow$ false) = true $\to$ $\neg$false = true $\to$ true = true.

   (b) One of several possible placements making the formulas true is:

	B	D
A		C
	F	E

6. **(PMT)** We need to ensure that

- [A *sees* B] and [B *sees* C] have different truth values;
- [B *next-to* D] and [B *next-to* E] are both false;
- [A *left-of* F] and [F *above* A] are both true;
- [E *left-of* D], [D *next-to* C], and [A *sees* E] are all true;
- at least one of [E *sees* D] and [F *sees* E] is true,
- [B *above* E] and [B *next-to* C] have the same truth value.

So one possible placement making the formulas in Q5a false is:

	F	C
A	E	D
B		

7. Show the following are logically equivalent, using truth tables. *I will write 1 for 'true' and 0 for 'false'.* In each case, the two given formulas are logically equivalent if their columns in the table are identical.

(a) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$
1	1	1	0	0	1
1	0	0	0	1	0
0	1	0	1	0	0
0	0	1	1	1	1

(b) $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg p$	$\neg p \leftrightarrow q$
1	1	1	0	0	0
1	0	0	1	0	1
0	1	0	1	1	1
0	0	1	0	1	0

(c) $X = (p \wedge q) \vee (\neg p \wedge \neg q)$ and $Y = (p \to q) \wedge (q \to p)$

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	X	$p \to q$	$q \to p$	Y
1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	0	0	0	1	0
0	1	0	1	0	0	0	1	0	0
0	0	0	1	1	1	1	1	1	1

The X and Y columns are the same, so X and Y are logically equivalent.

(d) $Z = p \to (q \to r)$ and $T = p \wedge q \to r$

p	q	r	$q \to r$	$Z : p \to (q \to r)$	$p \wedge q$	$T : p \wedge q \to r$
1	1	1	1	1	1	1
1	1	0	0	0	1	0
1	0	1	1	1	0	1
1	0	0	1	1	0	1
0	1	1	1	1	0	1
0	1	0	0	1	0	1
0	0	1	1	1	0	1
0	0	0	1	1	0	1

The Z and T columns are the same, so Z and T are logically equivalent.

(e) $p \to (q \wedge r)$ and $(p \to q) \wedge (p \to r)$

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
1	1	1	1	1	1	1	1
1	1	0	0	0	1	0	0
1	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1

8. $A_1, \ldots, A_n \models B$ can be expressed in terms of validity by '$A_1, \ldots, A_n \to B$ is valid'; in tems of satisfiability by '$A_1, \ldots, A_n \wedge \neg B$ is not saatisfiable'; and in terms of equivalence by saying '$A_1, \ldots, A_n \to B$ is logically equivalent to $\top$'.