

## Problems

1. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $A \rightarrow B$
  - $C \rightarrow B$
  - $B \rightarrow A$
  - $C \rightarrow A$
2. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $C \rightarrow B$
  - $B \rightarrow A$
  - $A \rightarrow C$
3. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $C \rightarrow B$
  - $B \rightarrow A$
4. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $B \rightarrow C$
  - $A \rightarrow C$
  - $C \rightarrow A$
5. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $A \rightarrow B$
  - $C \rightarrow B$
  - $B \rightarrow C$
  - $B \rightarrow A$
6. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $A \rightarrow B$
  - $C \rightarrow B$
  - $A \rightarrow C$
  - $C \rightarrow A$

7. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $B \rightarrow C$
  - $A \rightarrow C$
8. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $C \rightarrow A$
9. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $C \rightarrow B$
  - $A \rightarrow C$
  - $C \rightarrow A$
10. Given the relation  $R(A, B, C)$ , compute a canonical cover based on the following functional dependencies:
  - $C \rightarrow B$
  - $B \rightarrow A$
11. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
  - $D \rightarrow B$
  - $C \rightarrow AB$
  - $A \rightarrow D$
  - $D \rightarrow C$
  - $CD \rightarrow B$
  - $A \rightarrow C$
12. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
  - $A \rightarrow B$
  - $A \rightarrow C$
  - $BD \rightarrow A$
  - $D \rightarrow A$
13. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
  - $AB \rightarrow C$

- $A \rightarrow C$
  - $C \rightarrow D$
14. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $C \rightarrow B$
  - $B \rightarrow D$
  - $A \rightarrow D$
15. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $C \rightarrow AB$
  - $A \rightarrow D$
  - $D \rightarrow A$
  - $C \rightarrow B$
  - $AC \rightarrow D$
  - $AB \rightarrow C$
16. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $C \rightarrow B$
  - $B \rightarrow C$
17. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $B \rightarrow AC$
  - $B \rightarrow D$
  - $A \rightarrow D$
  - $B \rightarrow A$
18. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $D \rightarrow B$
  - $D \rightarrow C$
  - $A \rightarrow D$
  - $A \rightarrow B$
  - $B \rightarrow A$
  - $A \rightarrow C$

19. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $A \rightarrow D$
  - $D \rightarrow A$
  - $B \rightarrow D$
  - $C \rightarrow A$
20. Given the relation  $R(A, B, C, D)$ , compute a canonical cover based on the following functional dependencies:
- $B \rightarrow AD$
  - $D \rightarrow B$
  - $B \rightarrow A$
21. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $B \rightarrow ACE$
  - $A \rightarrow BDE$
  - $A \rightarrow E$
  - $AD \rightarrow CE$
  - $CE \rightarrow BD$
  - $B \rightarrow D$
  - $A \rightarrow B$
  - $C \rightarrow ABD$
  - $AE \rightarrow C$
22. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $E \rightarrow B$
  - $B \rightarrow C$
23. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $E \rightarrow B$
  - $A \rightarrow BDE$
  - $DE \rightarrow A$
  - $BD \rightarrow A$
  - $E \rightarrow AB$
  - $BE \rightarrow A$

- $D \rightarrow E$
24. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $A \rightarrow BD$
  - $BD \rightarrow E$
  - $C \rightarrow AB$
  - $D \rightarrow A$
  - $CDE \rightarrow A$
  - $AB \rightarrow CD$
25. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $E \rightarrow A$
  - $A \rightarrow B$
  - $E \rightarrow B$
  - $ABC \rightarrow E$
26. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $B \rightarrow ACE$
  - $ABE \rightarrow C$
  - $DE \rightarrow C$
  - $C \rightarrow D$
  - $B \rightarrow E$
  - $AC \rightarrow D$
  - $E \rightarrow C$
  - $B \rightarrow C$
  - $A \rightarrow C$
27. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $BC \rightarrow A$
  - $A \rightarrow D$
  - $B \rightarrow AE$
28. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $E \rightarrow A$

- $BE \rightarrow A$
  - $E \rightarrow C$
  - $B \rightarrow AD$
29. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $AC \rightarrow DE$
  - $BD \rightarrow AC$
  - $B \rightarrow E$
  - $C \rightarrow BDE$
  - $CE \rightarrow AD$
30. Given the relation  $R(A, B, C, D, E)$ , compute a canonical cover based on the following functional dependencies:
- $ACD \rightarrow B$
  - $C \rightarrow BD$
  - $ABE \rightarrow D$
  - $DE \rightarrow A$
  - $AC \rightarrow BD$
  - $CE \rightarrow D$
  - $B \rightarrow C$
31. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $F \rightarrow C$
  - $F \rightarrow A$
  - $E \rightarrow B$
  - $A \rightarrow E$
  - $AF \rightarrow C$
  - $A \rightarrow D$
  - $E \rightarrow C$
  - $B \rightarrow C$
32. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $AF \rightarrow D$
  - $B \rightarrow F$
  - $BC \rightarrow DE$

- $F \rightarrow B$
- $E \rightarrow A$
- $C \rightarrow AF$

33. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:

- $A \rightarrow F$
- $AB \rightarrow D$
- $B \rightarrow CE$
- $E \rightarrow F$
- $B \rightarrow D$

34. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:

- $C \rightarrow BD$
- $D \rightarrow B$
- $BDE \rightarrow AF$
- $CDE \rightarrow BF$
- $BD \rightarrow C$
- $AF \rightarrow DE$
- $BF \rightarrow AD$
- $BE \rightarrow A$
- $A \rightarrow B$
- $A \rightarrow CDF$
- $DF \rightarrow E$

35. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:

- $C \rightarrow AD$
- $F \rightarrow A$
- $A \rightarrow BD$
- $DF \rightarrow A$
- $BC \rightarrow A$
- $F \rightarrow BC$
- $ABF \rightarrow DE$
- $ACE \rightarrow D$

36. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $B \rightarrow AC$
  - $B \rightarrow E$
  - $D \rightarrow E$
  - $D \rightarrow AC$
37. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $DF \rightarrow A$
  - $BE \rightarrow ACF$
  - $A \rightarrow D$
  - $E \rightarrow D$
  - $D \rightarrow A$
38. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $CE \rightarrow A$
  - $BD \rightarrow F$
  - $D \rightarrow B$
  - $B \rightarrow AF$
  - $F \rightarrow AD$
  - $DE \rightarrow CF$
  - $E \rightarrow D$
  - $ACD \rightarrow EF$
  - $F \rightarrow D$
  - $BF \rightarrow D$
39. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $A \rightarrow EF$
  - $AEF \rightarrow C$
  - $B \rightarrow EF$
40. Given the relation  $R(A, B, C, D, E, F)$ , compute a canonical cover based on the following functional dependencies:
- $F \rightarrow AD$
  - $D \rightarrow ACF$



- $F \rightarrow CDE$
- $D \rightarrow BE$
- $BD \rightarrow C$
- $C \rightarrow F$
- $F \rightarrow D$

41. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $F \rightarrow A$
- $AF \rightarrow BD$
- $C \rightarrow E$
- $E \rightarrow CD$
- $AB \rightarrow G$
- $B \rightarrow CG$
- $ABE \rightarrow CFG$
- $BD \rightarrow EF$

42. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $A \rightarrow DE$
- $CG \rightarrow A$
- $CD \rightarrow E$
- $B \rightarrow D$
- $EG \rightarrow B$
- $E \rightarrow A$
- $FG \rightarrow D$
- $DF \rightarrow AC$

43. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $B \rightarrow AF$
- $G \rightarrow C$
- $BF \rightarrow C$
- $FG \rightarrow A$
- $BDE \rightarrow C$
- $CG \rightarrow AF$
- $AG \rightarrow DE$

- $E \rightarrow F$
- $E \rightarrow C$

44. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $D \rightarrow AE$
- $D \rightarrow C$
- $BF \rightarrow A$
- $CE \rightarrow D$
- $E \rightarrow A$
- $C \rightarrow DEG$
- $G \rightarrow BC$

45. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $BE \rightarrow D$
- $EF \rightarrow G$
- $BE \rightarrow G$
- $G \rightarrow ACF$
- $C \rightarrow G$
- $F \rightarrow AG$
- $EF \rightarrow ACG$
- $G \rightarrow AC$
- $ACG \rightarrow DE$
- $BF \rightarrow G$
- $BG \rightarrow AD$

46. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:

- $E \rightarrow CFG$
- $G \rightarrow C$
- $DF \rightarrow A$
- $DF \rightarrow C$
- $CDG \rightarrow E$
- $E \rightarrow A$
- $G \rightarrow F$

47. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:
- $C \rightarrow AD$
  - $E \rightarrow B$
  - $A \rightarrow C$
48. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:
- $E \rightarrow ADF$
  - $B \rightarrow CDF$
  - $CD \rightarrow AF$
49. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:
- $DFG \rightarrow CE$
  - $F \rightarrow G$
  - $A \rightarrow F$
  - $B \rightarrow C$
  - $A \rightarrow B$
  - $CDG \rightarrow E$
  - $E \rightarrow F$
  - $E \rightarrow BFG$
  - $AG \rightarrow C$
  - $F \rightarrow B$
  - $D \rightarrow ABG$
50. Given the relation  $R(A, B, C, D, E, F, G)$ , compute a canonical cover based on the following functional dependencies:
- $A \rightarrow BDF$
  - $A \rightarrow BE$
  - $BEF \rightarrow ACG$
  - $DE \rightarrow B$

## Solutions

1. **Solution:**  $C \rightarrow B \ A \rightarrow B \ B \rightarrow A$

Example resolution:

- Merging  $C \rightarrow A$  and  $C \rightarrow B$  into  $C \rightarrow AB$
- Removed extraneous component:  $C \rightarrow AB$  gives  $C \rightarrow B$

2. **Solution:**  $C \rightarrow B \ B \rightarrow A \ A \rightarrow C$  - There is nothing to do

3. **Solution:**  $C \rightarrow B \ B \rightarrow A$  - There is nothing to do

4. **Solution:**  $B \rightarrow C \ A \rightarrow C \ C \rightarrow A$  - There is nothing to do

5. **Solution:**  $B \rightarrow AC \ A \rightarrow B \ C \rightarrow B$

Example resolution:

- Merging  $B \rightarrow A$  and  $B \rightarrow C$  into  $B \rightarrow AC$

6. **Solution:**  $C \rightarrow A \ A \rightarrow BC$

Example resolution:

- Merging  $A \rightarrow C$  and  $A \rightarrow B$  into  $A \rightarrow BC$
- Merging  $C \rightarrow A$  and  $C \rightarrow B$  into  $C \rightarrow AB$
- Removed extraneous component:  $C \rightarrow AB$  gives  $C \rightarrow A$

7. **Solution:**  $B \rightarrow C \ A \rightarrow C$  - There is nothing to do

8. **Solution:**  $C \rightarrow A$  - There is nothing to do

9. **Solution:**  $C \rightarrow AB \ A \rightarrow C$

Example resolution:

- Merging  $C \rightarrow A$  and  $C \rightarrow B$  into  $C \rightarrow AB$

10. **Solution:**  $C \rightarrow B \ B \rightarrow A$  - There is nothing to do

11. **Solution:**  $D \rightarrow BC \ A \rightarrow D \ C \rightarrow A$

Example resolution:

- Merging  $D \rightarrow C$  and  $D \rightarrow B$  into  $D \rightarrow BC$
- Merging  $A \rightarrow C$  and  $A \rightarrow D$  into  $A \rightarrow CD$
- Removed extraneous component:  $A \rightarrow CD$  gives  $A \rightarrow D$
- Removed extraneous component:  $D \rightarrow BC$  gives  $D \rightarrow C$
- Removed extraneous component:  $C \rightarrow AB$  gives  $C \rightarrow A$
- Removed extraneous component:  $CD \rightarrow B$  gives  $D \rightarrow B$
- Merging  $D \rightarrow B$  and  $D \rightarrow C$  into  $D \rightarrow BC$

12. **Solution:**  $D \rightarrow A \ A \rightarrow BC$   
 Example resolution:
- Merging  $A \rightarrow C$  and  $A \rightarrow B$  into  $A \rightarrow BC$
  - Removed extraneous component:  $BD \rightarrow A$  gives  $D \rightarrow A$
  - Merging  $D \rightarrow A$  and  $D \rightarrow A$  into  $D \rightarrow A$
13. **Solution:**  $A \rightarrow C \ C \rightarrow D$   
 Example resolution:
- Removed extraneous component:  $AB \rightarrow C$  gives  $A \rightarrow C$
  - Merging  $A \rightarrow C$  and  $A \rightarrow C$  into  $A \rightarrow C$
14. **Solution:**  $C \rightarrow B \ B \rightarrow D \ A \rightarrow D$  - There is nothing to do
15. **Solution:**  $C \rightarrow BD \ A \rightarrow D \ D \rightarrow A \ AB \rightarrow C$   
 Example resolution:
- Merging  $C \rightarrow B$  and  $C \rightarrow AB$  into  $C \rightarrow AB$
  - Removed extraneous component:  $AC \rightarrow D$  gives  $C \rightarrow D$
  - Merging  $C \rightarrow D$  and  $C \rightarrow AB$  into  $C \rightarrow ABD$
  - Removed extraneous component:  $C \rightarrow ABD$  gives  $C \rightarrow BD$
16. **Solution:**  $C \rightarrow B \ B \rightarrow C$  - There is nothing to do
17. **Solution:**  $B \rightarrow AC \ A \rightarrow D$   
 Example resolution:
- Merging  $B \rightarrow D$  and  $B \rightarrow AC$  into  $B \rightarrow ACD$
  - Merging  $B \rightarrow A$  and  $B \rightarrow ACD$  into  $B \rightarrow ACD$
  - Removed extraneous component:  $B \rightarrow ACD$  gives  $B \rightarrow AC$
18. **Solution:**  $A \rightarrow D \ D \rightarrow BC \ B \rightarrow A$   
 Example resolution:
- Merging  $D \rightarrow C$  and  $D \rightarrow B$  into  $D \rightarrow BC$
  - Merging  $A \rightarrow B$  and  $A \rightarrow D$  into  $A \rightarrow BD$
  - Merging  $A \rightarrow C$  and  $A \rightarrow BD$  into  $A \rightarrow BCD$
  - Removed extraneous component:  $A \rightarrow BCD$  gives  $A \rightarrow CD$
  - Removed extraneous component:  $A \rightarrow CD$  gives  $A \rightarrow D$
19. **Solution:**  $A \rightarrow D \ D \rightarrow A \ B \rightarrow D \ C \rightarrow A$  - There is nothing to do
20. **Solution:**  $B \rightarrow AD \ D \rightarrow B$   
 Example resolution:
- Merging  $B \rightarrow A$  and  $B \rightarrow AD$  into  $B \rightarrow AD$

21. **Solution:**  $A \rightarrow CE \quad C \rightarrow ABD \quad B \rightarrow C$

Example resolution:

- Merging  $A \rightarrow E$  and  $A \rightarrow BDE$  into  $A \rightarrow BDE$
- Merging  $B \rightarrow D$  and  $B \rightarrow ACE$  into  $B \rightarrow ACDE$
- Merging  $A \rightarrow B$  and  $A \rightarrow BDE$  into  $A \rightarrow BDE$
- Removed extraneous component:  $A \rightarrow BDE$  gives  $A \rightarrow DE$
- Removed extraneous component:  $A \rightarrow DE$  gives  $A \rightarrow E$
- Removed extraneous component:  $B \rightarrow ACDE$  gives  $B \rightarrow CDE$
- Removed extraneous component:  $B \rightarrow CDE$  gives  $B \rightarrow CE$
- Removed extraneous component:  $B \rightarrow CE$  gives  $B \rightarrow C$
- Removed extraneous component:  $AD \rightarrow CE$  gives  $A \rightarrow CE$
- Merging  $A \rightarrow CE$  and  $A \rightarrow E$  into  $A \rightarrow CE$
- Removed extraneous component:  $A \rightarrow CE$  gives  $A \rightarrow E$
- Removed extraneous component:  $CE \rightarrow BD$  gives  $C \rightarrow BD$
- Merging  $C \rightarrow ABD$  and  $C \rightarrow BD$  into  $C \rightarrow ABD$
- Removed extraneous component:  $AE \rightarrow C$  gives  $A \rightarrow C$
- Merging  $A \rightarrow C$  and  $A \rightarrow E$  into  $A \rightarrow CE$

22. **Solution:**  $E \rightarrow B \quad B \rightarrow C$  - There is nothing to do

23. **Solution:**  $E \rightarrow AB \quad D \rightarrow E \quad A \rightarrow D$

Example resolution:

- Merging  $E \rightarrow AB$  and  $E \rightarrow B$  into  $E \rightarrow AB$
- Removed extraneous component:  $E \rightarrow AB$  gives  $E \rightarrow B$
- Removed extraneous component:  $A \rightarrow BDE$  gives  $A \rightarrow DE$
- Removed extraneous component:  $A \rightarrow DE$  gives  $A \rightarrow D$
- Removed extraneous component:  $DE \rightarrow A$  gives  $E \rightarrow A$
- Merging  $E \rightarrow A$  and  $E \rightarrow B$  into  $E \rightarrow AB$
- Removed extraneous component:  $E \rightarrow AB$  gives  $E \rightarrow B$
- Removed extraneous component:  $BD \rightarrow A$  gives  $D \rightarrow A$
- Merging  $D \rightarrow E$  and  $D \rightarrow A$  into  $D \rightarrow AE$
- Removed extraneous component:  $D \rightarrow AE$  gives  $D \rightarrow E$
- Removed extraneous component:  $BE \rightarrow A$  gives  $E \rightarrow A$
- Merging  $E \rightarrow A$  and  $E \rightarrow B$  into  $E \rightarrow AB$

24. **Solution:**  $A \rightarrow BCD \quad D \rightarrow AE \quad C \rightarrow A$

Example resolution:

- Removed extraneous component:  $A \rightarrow BD$  gives  $A \rightarrow B$
- Removed extraneous component:  $BD \rightarrow E$  gives  $D \rightarrow E$
- Merging  $D \rightarrow A$  and  $D \rightarrow E$  into  $D \rightarrow AE$
- Removed extraneous component:  $C \rightarrow AB$  gives  $C \rightarrow A$
- Removed extraneous component:  $CDE \rightarrow A$  gives  $DE \rightarrow A$
- Removed extraneous component:  $D \rightarrow AE$  gives  $D \rightarrow E$
- Removed extraneous component:  $DE \rightarrow A$  gives  $D \rightarrow A$
- Merging  $D \rightarrow A$  and  $D \rightarrow E$  into  $D \rightarrow AE$
- Removed extraneous component:  $AB \rightarrow CD$  gives  $A \rightarrow CD$
- Merging  $A \rightarrow CD$  and  $A \rightarrow B$  into  $A \rightarrow BCD$

25. **Solution:**  $E \rightarrow A \ A \rightarrow B \ AC \rightarrow E$

Example resolution:

- Merging  $E \rightarrow B$  and  $E \rightarrow A$  into  $E \rightarrow AB$
- Removed extraneous component:  $E \rightarrow AB$  gives  $E \rightarrow A$
- Removed extraneous component:  $ABC \rightarrow E$  gives  $AC \rightarrow E$

26. **Solution:**  $C \rightarrow D \ E \rightarrow C \ B \rightarrow AE \ A \rightarrow C$

Example resolution:

- Merging  $B \rightarrow E$  and  $B \rightarrow ACE$  into  $B \rightarrow ACE$
- Merging  $B \rightarrow C$  and  $B \rightarrow ACE$  into  $B \rightarrow ACE$
- Removed extraneous component:  $B \rightarrow ACE$  gives  $B \rightarrow AE$
- Removed extraneous component:  $ABE \rightarrow C$  gives  $BE \rightarrow C$
- Removed extraneous component:  $BE \rightarrow C$  gives  $E \rightarrow C$
- Merging  $E \rightarrow C$  and  $E \rightarrow C$  into  $E \rightarrow C$
- Removed extraneous component:  $DE \rightarrow C$  gives  $E \rightarrow C$
- Merging  $E \rightarrow C$  and  $E \rightarrow C$  into  $E \rightarrow C$
- Removed extraneous component:  $AC \rightarrow D$  gives  $C \rightarrow D$
- Merging  $C \rightarrow D$  and  $C \rightarrow D$  into  $C \rightarrow D$

27. **Solution:**  $B \rightarrow AE \ A \rightarrow D$

Example resolution:

- Removed extraneous component:  $BC \rightarrow A$  gives  $B \rightarrow A$
- Merging  $B \rightarrow AE$  and  $B \rightarrow A$  into  $B \rightarrow AE$

28. **Solution:**  $E \rightarrow AC \ B \rightarrow AD$

Example resolution:

- Merging  $E \rightarrow C$  and  $E \rightarrow A$  into  $E \rightarrow AC$

- Removed extraneous component:  $BE \rightarrow A$  gives  $E \rightarrow A$
- Merging  $E \rightarrow A$  and  $E \rightarrow AC$  into  $E \rightarrow AC$

29. **Solution:**  $C \rightarrow ABD \quad BD \rightarrow C \quad B \rightarrow E$

Example resolution:

- Removed extraneous component:  $AC \rightarrow DE$  gives  $C \rightarrow DE$
- Merging  $C \rightarrow BDE$  and  $C \rightarrow DE$  into  $C \rightarrow BDE$
- Removed extraneous component:  $C \rightarrow BDE$  gives  $C \rightarrow BE$
- Removed extraneous component:  $C \rightarrow BE$  gives  $C \rightarrow B$
- Removed extraneous component:  $BD \rightarrow AC$  gives  $BD \rightarrow C$
- Removed extraneous component:  $CE \rightarrow AD$  gives  $C \rightarrow AD$
- Merging  $C \rightarrow AD$  and  $C \rightarrow B$  into  $C \rightarrow ABD$

30. **Solution:**  $C \rightarrow BD \quad B \rightarrow C \quad DE \rightarrow A$

Example resolution:

- Removed extraneous component:  $ACD \rightarrow B$  gives  $CD \rightarrow B$
- Removed extraneous component:  $CD \rightarrow B$  gives  $C \rightarrow B$
- Merging  $C \rightarrow BD$  and  $C \rightarrow B$  into  $C \rightarrow BD$
- Removed extraneous component:  $ABE \rightarrow D$  gives  $BE \rightarrow D$
- Removed extraneous component:  $BE \rightarrow D$  gives  $B \rightarrow D$
- Merging  $B \rightarrow C$  and  $B \rightarrow D$  into  $B \rightarrow CD$
- Removed extraneous component:  $B \rightarrow CD$  gives  $B \rightarrow C$
- Removed extraneous component:  $AC \rightarrow BD$  gives  $C \rightarrow BD$
- Merging  $C \rightarrow BD$  and  $C \rightarrow BD$  into  $C \rightarrow BD$
- Removed extraneous component:  $CE \rightarrow D$  gives  $C \rightarrow D$
- Merging  $C \rightarrow D$  and  $C \rightarrow BD$  into  $C \rightarrow BD$

31. **Solution:**  $F \rightarrow A \quad E \rightarrow B \quad A \rightarrow DE \quad B \rightarrow C$

Example resolution:

- Merging  $F \rightarrow A$  and  $F \rightarrow C$  into  $F \rightarrow AC$
- Merging  $A \rightarrow D$  and  $A \rightarrow E$  into  $A \rightarrow DE$
- Merging  $E \rightarrow C$  and  $E \rightarrow B$  into  $E \rightarrow BC$
- Removed extraneous component:  $E \rightarrow BC$  gives  $E \rightarrow B$
- Removed extraneous component:  $F \rightarrow AC$  gives  $F \rightarrow A$
- Removed extraneous component:  $AF \rightarrow C$  gives  $F \rightarrow C$
- Merging  $F \rightarrow C$  and  $F \rightarrow A$  into  $F \rightarrow AC$
- Removed extraneous component:  $F \rightarrow AC$  gives  $F \rightarrow A$



32. **Solution:**  $C \rightarrow EF \quad AF \rightarrow D \quad B \rightarrow F \quad F \rightarrow B \quad E \rightarrow A$

Example resolution:

- Removed extraneous component:  $BC \rightarrow DE$  gives  $C \rightarrow DE$
- Merging  $C \rightarrow AF$  and  $C \rightarrow DE$  into  $C \rightarrow ADEF$
- Removed extraneous component:  $C \rightarrow ADEF$  gives  $C \rightarrow DEF$
- Removed extraneous component:  $C \rightarrow DEF$  gives  $C \rightarrow EF$

33. **Solution:**  $B \rightarrow CDE \quad A \rightarrow F \quad E \rightarrow F$

Example resolution:

- Merging  $B \rightarrow D$  and  $B \rightarrow CE$  into  $B \rightarrow CDE$
- Removed extraneous component:  $AB \rightarrow D$  gives  $B \rightarrow D$
- Merging  $B \rightarrow D$  and  $B \rightarrow CDE$  into  $B \rightarrow CDE$

34. **Solution:**  $A \rightarrow DF \quad D \rightarrow BC \quad C \rightarrow D \quad BF \rightarrow D \quad BE \rightarrow A \quad DF \rightarrow E$

Example resolution:

- Merging  $A \rightarrow CDF$  and  $A \rightarrow B$  into  $A \rightarrow BCDF$
- Removed extraneous component:  $A \rightarrow BCDF$  gives  $A \rightarrow CDF$
- Removed extraneous component:  $A \rightarrow CDF$  gives  $A \rightarrow DF$
- Removed extraneous component:  $A \rightarrow DF$  gives  $A \rightarrow F$
- Removed extraneous component:  $C \rightarrow BD$  gives  $C \rightarrow D$
- Removed extraneous component:  $BDE \rightarrow AF$  gives  $DE \rightarrow AF$
- Removed extraneous component:  $DE \rightarrow AF$  gives  $DE \rightarrow F$
- Removed extraneous component:  $DE \rightarrow F$  gives  $DE \rightarrow$
- Removed extraneous component:  $DE \rightarrow$  gives  $E \rightarrow$
- Removed extraneous component:  $E \rightarrow$  gives  $\rightarrow$
- Removed extraneous component:  $CDE \rightarrow BF$  gives  $DE \rightarrow BF$
- Removed extraneous component:  $DE \rightarrow BF$  gives  $DE \rightarrow F$
- Removed extraneous component:  $DE \rightarrow F$  gives  $DE \rightarrow$
- Removed extraneous component:  $DE \rightarrow$  gives  $E \rightarrow$
- Removed extraneous component:  $E \rightarrow$  gives  $\rightarrow$
- Removed extraneous component:  $BD \rightarrow C$  gives  $D \rightarrow C$
- Merging  $D \rightarrow C$  and  $D \rightarrow B$  into  $D \rightarrow BC$
- Removed extraneous component:  $AF \rightarrow DE$  gives  $A \rightarrow DE$
- Merging  $A \rightarrow DE$  and  $A \rightarrow F$  into  $A \rightarrow DEF$
- Removed extraneous component:  $A \rightarrow DEF$  gives  $A \rightarrow DF$
- Removed extraneous component:  $BF \rightarrow AD$  gives  $BF \rightarrow D$

35. **Solution:**  $C \rightarrow A \quad F \rightarrow CE \quad A \rightarrow BD$

Example resolution:

- Merging  $F \rightarrow BC$  and  $F \rightarrow A$  into  $F \rightarrow ABC$
- Removed extraneous component:  $F \rightarrow ABC$  gives  $F \rightarrow BC$
- Removed extraneous component:  $F \rightarrow BC$  gives  $F \rightarrow C$
- Removed extraneous component:  $C \rightarrow AD$  gives  $C \rightarrow A$
- Removed extraneous component:  $DF \rightarrow A$  gives  $F \rightarrow A$
- Merging  $F \rightarrow A$  and  $F \rightarrow C$  into  $F \rightarrow AC$
- Removed extraneous component:  $F \rightarrow AC$  gives  $F \rightarrow C$
- Removed extraneous component:  $BC \rightarrow A$  gives  $C \rightarrow A$
- Merging  $C \rightarrow A$  and  $C \rightarrow A$  into  $C \rightarrow A$
- Removed extraneous component:  $ABF \rightarrow DE$  gives  $BF \rightarrow DE$
- Removed extraneous component:  $BF \rightarrow DE$  gives  $F \rightarrow DE$
- Merging  $F \rightarrow DE$  and  $F \rightarrow C$  into  $F \rightarrow CDE$
- Removed extraneous component:  $F \rightarrow CDE$  gives  $F \rightarrow CE$
- Removed extraneous component:  $ACE \rightarrow D$  gives  $CE \rightarrow D$
- Removed extraneous component:  $CE \rightarrow D$  gives  $C \rightarrow D$
- Merging  $C \rightarrow D$  and  $C \rightarrow A$  into  $C \rightarrow AD$
- Removed extraneous component:  $C \rightarrow AD$  gives  $C \rightarrow A$

36. **Solution:**  $D \rightarrow ACE \quad B \rightarrow ACE$

Example resolution:

- Merging  $B \rightarrow E$  and  $B \rightarrow AC$  into  $B \rightarrow ACE$
- Merging  $D \rightarrow AC$  and  $D \rightarrow E$  into  $D \rightarrow ACE$

37. **Solution:**  $D \rightarrow A \quad BE \rightarrow CF \quad A \rightarrow D \quad E \rightarrow D$

Example resolution:

- Removed extraneous component:  $DF \rightarrow A$  gives  $D \rightarrow A$
- Merging  $D \rightarrow A$  and  $D \rightarrow A$  into  $D \rightarrow A$
- Removed extraneous component:  $BE \rightarrow ACF$  gives  $BE \rightarrow CF$

38. **Solution:**  $F \rightarrow D \quad E \rightarrow CF \quad D \rightarrow B \quad B \rightarrow AF \quad CD \rightarrow E$

Example resolution:

- Merging  $F \rightarrow D$  and  $F \rightarrow AD$  into  $F \rightarrow AD$
- Removed extraneous component:  $F \rightarrow AD$  gives  $F \rightarrow D$
- Removed extraneous component:  $CE \rightarrow A$  gives  $E \rightarrow A$
- Merging  $E \rightarrow D$  and  $E \rightarrow A$  into  $E \rightarrow AD$

- Removed extraneous component:  $E \rightarrow AD$  gives  $E \rightarrow D$
- Removed extraneous component:  $BD \rightarrow F$  gives  $D \rightarrow F$
- Merging  $D \rightarrow B$  and  $D \rightarrow F$  into  $D \rightarrow BF$
- Removed extraneous component:  $D \rightarrow BF$  gives  $D \rightarrow B$
- Removed extraneous component:  $DE \rightarrow CF$  gives  $E \rightarrow CF$
- Merging  $E \rightarrow CF$  and  $E \rightarrow D$  into  $E \rightarrow CDF$
- Removed extraneous component:  $E \rightarrow CDF$  gives  $E \rightarrow CF$
- Removed extraneous component:  $ACD \rightarrow EF$  gives  $CD \rightarrow EF$
- Removed extraneous component:  $CD \rightarrow EF$  gives  $CD \rightarrow E$
- Removed extraneous component:  $BF \rightarrow D$  gives  $F \rightarrow D$
- Merging  $F \rightarrow D$  and  $F \rightarrow D$  into  $F \rightarrow D$

39. **Solution:**  $A \rightarrow CEF \ B \rightarrow EF$

Example resolution:

- Removed extraneous component:  $AEF \rightarrow C$  gives  $AF \rightarrow C$
- Removed extraneous component:  $AF \rightarrow C$  gives  $A \rightarrow C$
- Merging  $A \rightarrow C$  and  $A \rightarrow EF$  into  $A \rightarrow CEF$

40. **Solution:**  $D \rightarrow ABCE \ F \rightarrow D \ C \rightarrow F$

Example resolution:

- Merging  $F \rightarrow CDE$  and  $F \rightarrow AD$  into  $F \rightarrow ACDE$
- Merging  $D \rightarrow BE$  and  $D \rightarrow ACF$  into  $D \rightarrow ABCEF$
- Merging  $F \rightarrow D$  and  $F \rightarrow ACDE$  into  $F \rightarrow ACDE$
- Removed extraneous component:  $F \rightarrow ACDE$  gives  $F \rightarrow CDE$
- Removed extraneous component:  $F \rightarrow CDE$  gives  $F \rightarrow DE$
- Removed extraneous component:  $F \rightarrow DE$  gives  $F \rightarrow D$
- Removed extraneous component:  $D \rightarrow ABCEF$  gives  $D \rightarrow ABEF$
- Removed extraneous component:  $D \rightarrow ABEF$  gives  $D \rightarrow ABE$
- Removed extraneous component:  $BD \rightarrow C$  gives  $D \rightarrow C$
- Merging  $D \rightarrow C$  and  $D \rightarrow ABE$  into  $D \rightarrow ABCE$

41. **Solution:**  $B \rightarrow EFG \ F \rightarrow AB \ C \rightarrow E \ E \rightarrow CD$

Example resolution:

- Removed extraneous component:  $AF \rightarrow BD$  gives  $F \rightarrow BD$
- Merging  $F \rightarrow BD$  and  $F \rightarrow A$  into  $F \rightarrow ABD$
- Removed extraneous component:  $F \rightarrow ABD$  gives  $F \rightarrow AB$
- Removed extraneous component:  $AB \rightarrow G$  gives  $B \rightarrow G$

- Merging  $B \rightarrow CG$  and  $B \rightarrow G$  into  $B \rightarrow CG$
  - Removed extraneous component:  $B \rightarrow CG$  gives  $B \rightarrow C$
  - Removed extraneous component:  $ABE \rightarrow CFG$  gives  $BE \rightarrow CFG$
  - Removed extraneous component:  $BE \rightarrow CFG$  gives  $B \rightarrow CFG$
  - Merging  $B \rightarrow CFG$  and  $B \rightarrow C$  into  $B \rightarrow CFG$
  - Removed extraneous component:  $B \rightarrow CFG$  gives  $B \rightarrow CG$
  - Removed extraneous component:  $BD \rightarrow EF$  gives  $B \rightarrow EF$
  - Merging  $B \rightarrow EF$  and  $B \rightarrow CG$  into  $B \rightarrow CEFG$
  - Removed extraneous component:  $B \rightarrow CEFG$  gives  $B \rightarrow EFG$
42. **Solution:**  $A \rightarrow DE \quad CG \rightarrow A \quad CD \rightarrow E \quad B \rightarrow D \quad EG \rightarrow B \quad E \rightarrow A \quad FG \rightarrow D \quad DF \rightarrow C$   
Example resolution:
- Removed extraneous component:  $DF \rightarrow AC$  gives  $DF \rightarrow C$
43. **Solution:**  $G \rightarrow ADE \quad E \rightarrow CF \quad B \rightarrow ACF$   
Example resolution:
- Merging  $E \rightarrow C$  and  $E \rightarrow F$  into  $E \rightarrow CF$
  - Removed extraneous component:  $BF \rightarrow C$  gives  $B \rightarrow C$
  - Merging  $B \rightarrow C$  and  $B \rightarrow AF$  into  $B \rightarrow ACF$
  - Removed extraneous component:  $FG \rightarrow A$  gives  $G \rightarrow A$
  - Merging  $G \rightarrow A$  and  $G \rightarrow C$  into  $G \rightarrow AC$
  - Removed extraneous component:  $G \rightarrow AC$  gives  $G \rightarrow C$
  - Removed extraneous component:  $BDE \rightarrow C$  gives  $DE \rightarrow C$
  - Removed extraneous component:  $DE \rightarrow C$  gives  $E \rightarrow C$
  - Merging  $E \rightarrow C$  and  $E \rightarrow CF$  into  $E \rightarrow CF$
  - Removed extraneous component:  $CG \rightarrow AF$  gives  $G \rightarrow AF$
  - Merging  $G \rightarrow AF$  and  $G \rightarrow C$  into  $G \rightarrow ACF$
  - Removed extraneous component:  $G \rightarrow ACF$  gives  $G \rightarrow AF$
  - Removed extraneous component:  $G \rightarrow AF$  gives  $G \rightarrow A$
  - Removed extraneous component:  $AG \rightarrow DE$  gives  $G \rightarrow DE$
  - Merging  $G \rightarrow DE$  and  $G \rightarrow A$  into  $G \rightarrow ADE$
44. **Solution:**  $C \rightarrow DEG \quad D \rightarrow C \quad BF \rightarrow A \quad E \rightarrow A \quad G \rightarrow BC$   
Example resolution:
- Merging  $D \rightarrow C$  and  $D \rightarrow AE$  into  $D \rightarrow ACE$
  - Removed extraneous component:  $D \rightarrow ACE$  gives  $D \rightarrow CE$

- Removed extraneous component:  $D \rightarrow CE$  gives  $D \rightarrow C$
- Removed extraneous component:  $CE \rightarrow D$  gives  $C \rightarrow D$
- Merging  $C \rightarrow DEG$  and  $C \rightarrow D$  into  $C \rightarrow DEG$

45. **Solution:**  $G \rightarrow CDEF \quad F \rightarrow AG \quad BE \rightarrow G \quad C \rightarrow G$

Example resolution:

- Merging  $BE \rightarrow G$  and  $BE \rightarrow D$  into  $BE \rightarrow DG$
- Merging  $EF \rightarrow ACG$  and  $EF \rightarrow G$  into  $EF \rightarrow ACG$
- Merging  $G \rightarrow AC$  and  $G \rightarrow ACF$  into  $G \rightarrow ACF$
- Removed extraneous component:  $G \rightarrow ACF$  gives  $G \rightarrow CF$
- Removed extraneous component:  $EF \rightarrow ACG$  gives  $F \rightarrow ACG$
- Merging  $F \rightarrow AG$  and  $F \rightarrow ACG$  into  $F \rightarrow ACG$
- Removed extraneous component:  $F \rightarrow ACG$  gives  $F \rightarrow AG$
- Removed extraneous component:  $BE \rightarrow DG$  gives  $BE \rightarrow G$
- Removed extraneous component:  $ACG \rightarrow DE$  gives  $CG \rightarrow DE$
- Removed extraneous component:  $CG \rightarrow DE$  gives  $G \rightarrow DE$
- Merging  $G \rightarrow DE$  and  $G \rightarrow CF$  into  $G \rightarrow CDEF$
- Removed extraneous component:  $BF \rightarrow G$  gives  $F \rightarrow G$
- Merging  $F \rightarrow G$  and  $F \rightarrow AG$  into  $F \rightarrow AG$
- Removed extraneous component:  $BG \rightarrow AD$  gives  $G \rightarrow AD$
- Merging  $G \rightarrow AD$  and  $G \rightarrow CDEF$  into  $G \rightarrow ACDEF$
- Removed extraneous component:  $G \rightarrow ACDEF$  gives  $G \rightarrow CDEF$

46. **Solution:**  $G \rightarrow CF \quad E \rightarrow AG \quad DF \rightarrow AC \quad DG \rightarrow E$

Example resolution:

- Merging  $DF \rightarrow C$  and  $DF \rightarrow A$  into  $DF \rightarrow AC$
- Merging  $E \rightarrow A$  and  $E \rightarrow CFG$  into  $E \rightarrow ACFG$
- Merging  $G \rightarrow F$  and  $G \rightarrow C$  into  $G \rightarrow CF$
- Removed extraneous component:  $E \rightarrow ACFG$  gives  $E \rightarrow AFG$
- Removed extraneous component:  $E \rightarrow AFG$  gives  $E \rightarrow AG$
- Removed extraneous component:  $CDG \rightarrow E$  gives  $DG \rightarrow E$

47. **Solution:**  $C \rightarrow AD \quad E \rightarrow B \quad A \rightarrow C$  - There is nothing to do

48. **Solution:**  $E \rightarrow ADF \quad B \rightarrow CD \quad CD \rightarrow AF$

Example resolution:

- Removed extraneous component:  $B \rightarrow CDF$  gives  $B \rightarrow CD$

49. **Solution:**  $A \rightarrow F \ D \rightarrow AE \ F \rightarrow BG \ E \rightarrow F \ B \rightarrow C$

Example resolution:

- Merging  $A \rightarrow B$  and  $A \rightarrow F$  into  $A \rightarrow BF$
- Merging  $E \rightarrow BFG$  and  $E \rightarrow F$  into  $E \rightarrow BFG$
- Merging  $F \rightarrow B$  and  $F \rightarrow G$  into  $F \rightarrow BG$
- Removed extraneous component:  $E \rightarrow BFG$  gives  $E \rightarrow FG$
- Removed extraneous component:  $E \rightarrow FG$  gives  $E \rightarrow F$
- Removed extraneous component:  $A \rightarrow BF$  gives  $A \rightarrow F$
- Removed extraneous component:  $DFG \rightarrow CE$  gives  $DG \rightarrow CE$
- Removed extraneous component:  $DG \rightarrow CE$  gives  $D \rightarrow CE$
- Merging  $D \rightarrow ABG$  and  $D \rightarrow CE$  into  $D \rightarrow ABCEG$
- Removed extraneous component:  $D \rightarrow ABCEG$  gives  $D \rightarrow ACEG$
- Removed extraneous component:  $D \rightarrow ACEG$  gives  $D \rightarrow AEG$
- Removed extraneous component:  $D \rightarrow AEG$  gives  $D \rightarrow AG$
- Removed extraneous component:  $D \rightarrow AG$  gives  $D \rightarrow A$
- Removed extraneous component:  $CDG \rightarrow E$  gives  $DG \rightarrow E$
- Removed extraneous component:  $DG \rightarrow E$  gives  $D \rightarrow E$
- Merging  $D \rightarrow E$  and  $D \rightarrow A$  into  $D \rightarrow AE$
- Removed extraneous component:  $AG \rightarrow C$  gives  $A \rightarrow C$
- Merging  $A \rightarrow C$  and  $A \rightarrow F$  into  $A \rightarrow CF$
- Removed extraneous component:  $A \rightarrow CF$  gives  $A \rightarrow F$

50. **Solution:**  $A \rightarrow DEF \ BEF \rightarrow ACG \ DE \rightarrow B$

Example resolution:

- Merging  $A \rightarrow BE$  and  $A \rightarrow BDF$  into  $A \rightarrow BDEF$
- Removed extraneous component:  $A \rightarrow BDEF$  gives  $A \rightarrow DEF$