Indexing

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Index Definition in SQL

- Create an index

  create index <index-name> **on** <relation-name> (<attribute-list>)

  E.g.: **create index** *b-index* **on** *branch(branch_name)*

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key is a candidate key.
  - Not really required if SQL **unique** integrity constraint is supported
- To drop an index

  drop index <index-name>

- Most database systems allow specification of type of index, and clustering.

Basic Concepts

- Indexing mechanisms used to speed up access to desired data.
  - E.g., author catalog in library
- **Search Key** - attribute to set of attributes used to look up records in a file.
- An **index file** consists of records (called **index entries**) of the form

search-key	pointer

- Index files are typically much smaller than the original file
- Two basic kinds of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across "buckets" using a "hash function".

Index Evaluation Metrics

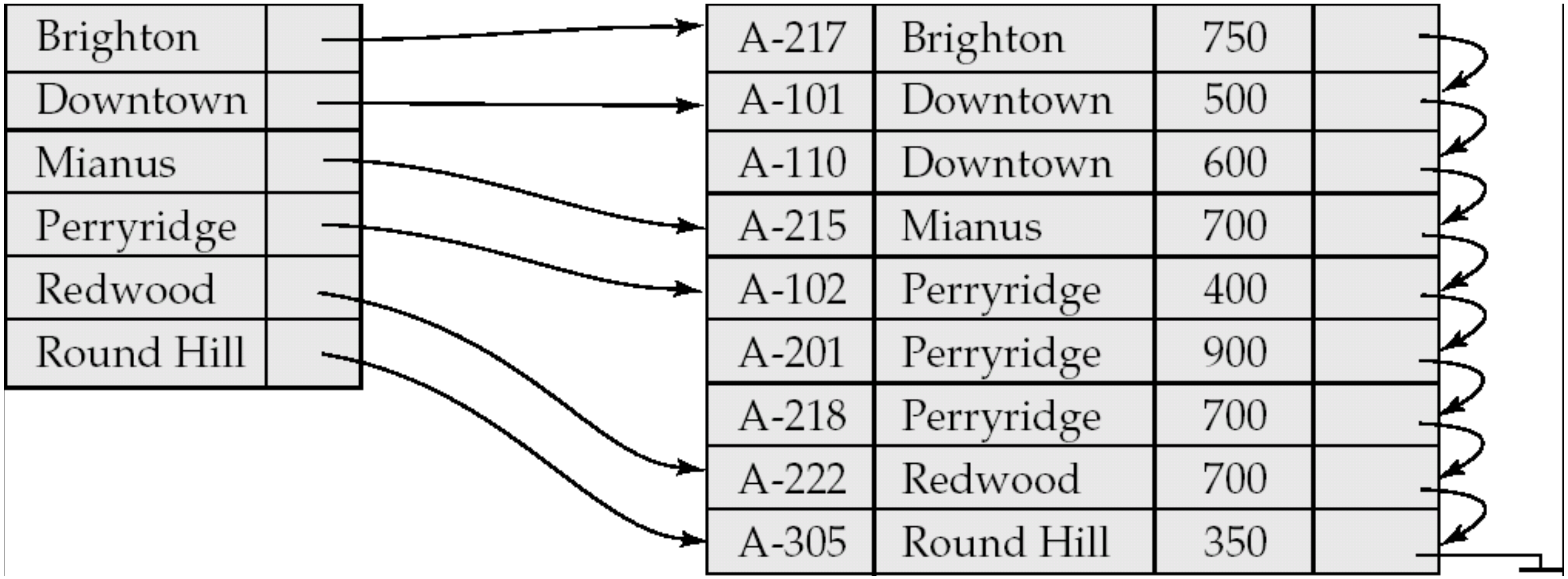
- Access types supported efficiently. E.g.,
  - records with a specified value in the attribute
  - or records with an attribute value falling in a specified range of values (e.g. 10000 < *salary* < 40000)
- Access time
- Insertion time
- Deletion time
- Space overhead

Ordered Indices

- In an **ordered index,** index entries are stored sorted on the search key value. E.g., author catalog in library.
- **Primary index:** in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
  - Also called **clustering index**
  - The search key of a primary index is usually but not necessarily the primary key.
- **Secondary index**: an index whose search key specifies an order different from the sequential order of the file. Also called non-clustering index**.**
- Index-sequential file**:** ordered sequential file with a primary index.

Dense Index Files

Dense index — Index record appears for every search-key value in the file.



Sparse Index Files

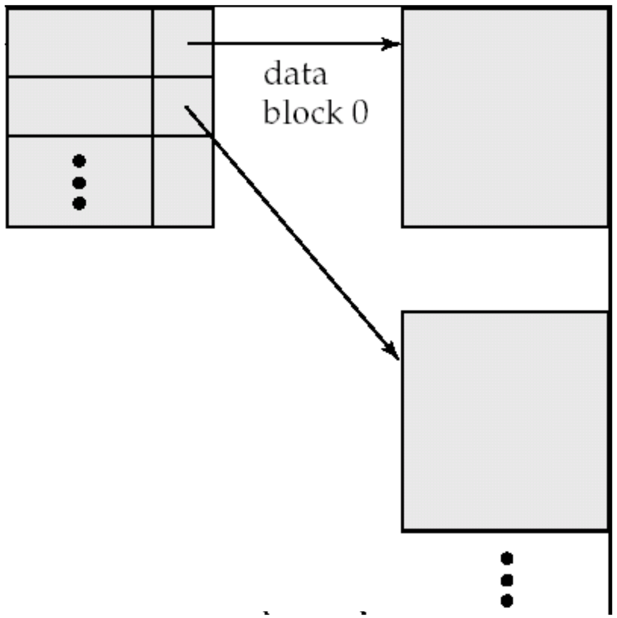
- Sparse Index: contains index records for only some search-key values.
  - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value *K* we:
  - Find index record with largest search-key value < *K*
  - Search file sequentially starting at the record to which the index record points

Brighton	
Mianus	
Redwood	

A-217	Brighton	750	
A-101	Downtown	500	
A-110	Downtown	600	
A-215	Mianus	700	
A-102	Perryridge	400	
A-201	Perryridge	900	
A-218	Perryridge	700	
A-222	Redwood	700	

Sparse Index Files (Cont.)

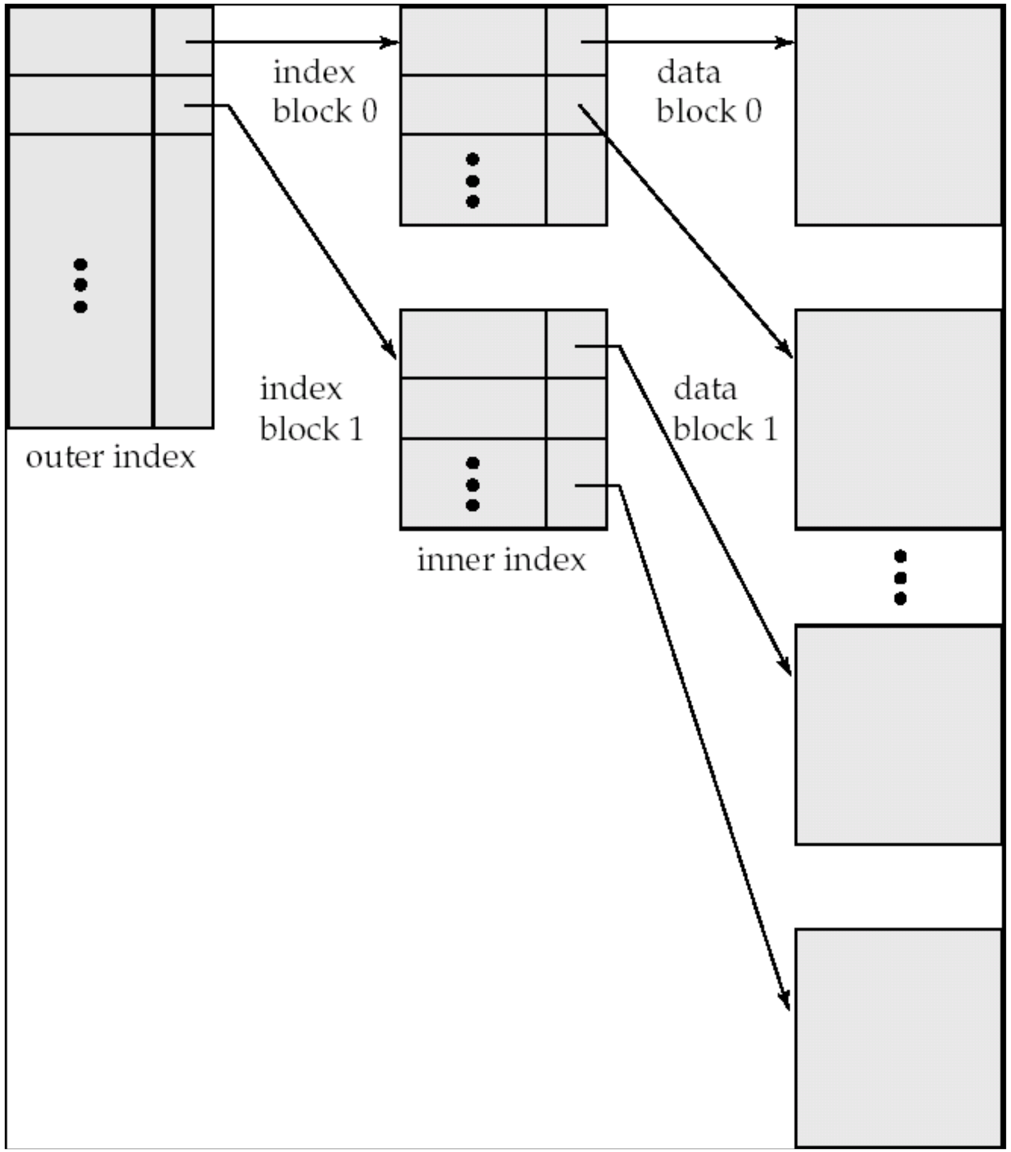
- Compared to dense indices:
  - Less space and less maintenance overhead for insertions and deletions.
  - Generally slower than dense index for locating records.
- **Good tradeoff**: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.



Multilevel Index

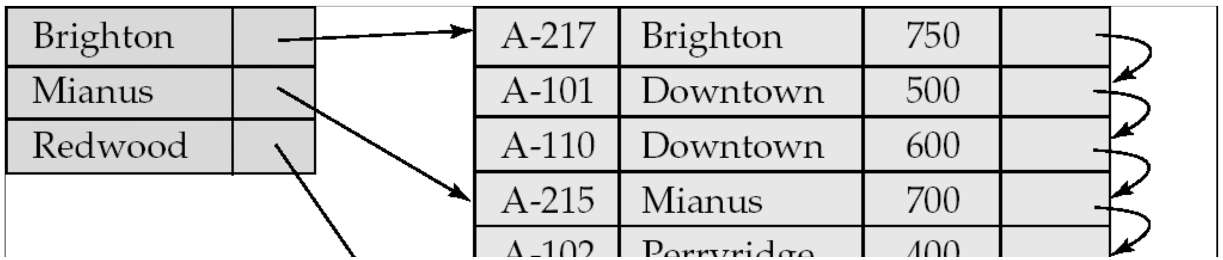
- If primary index does not fit in memory, access becomes expensive.
- Solution: treat primary index kept on disk as a sequential file and construct a sparse index on it.
  - outer index – a sparse index of primary index
  - inner index – the primary index file
- If even outer index is too large to fit in main memory, yet another level of index can be created, and so on.
- Indices at all levels must be updated on insertion or deletion from the file.

Multilevel Index (Cont.)



Index Update: Record Deletion

- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.
- Single-level index deletion:
  - **Dense indices** – deletion of search-key: similar to file record deletion.
  - **Sparse indices**:
    - if deleted key value exists in the index, the value is replaced by the next search-key value in the file (in search-key order).
    - If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

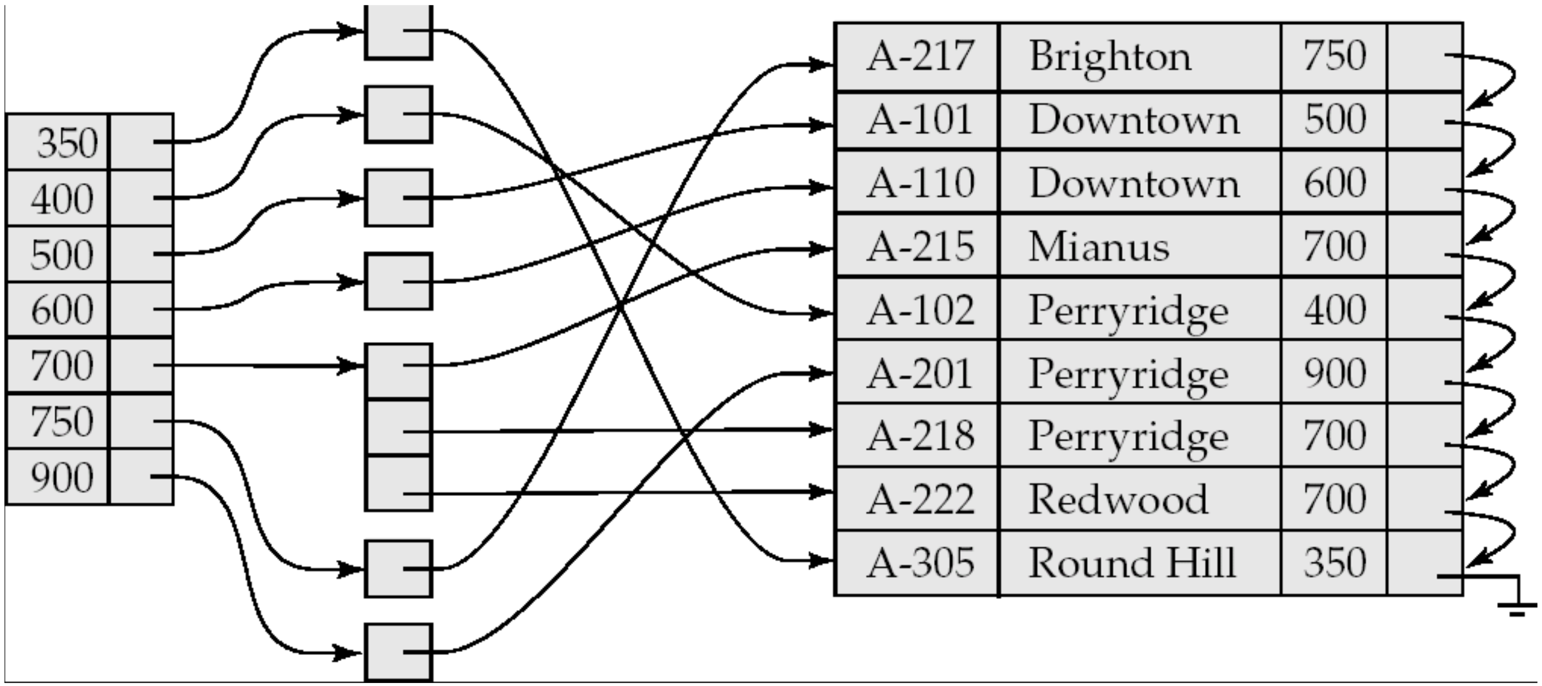


Index Update: Record Insertion

- Single-level index insertion:
  - Perform a lookup using the key value from inserted record
  - **Dense indices** – if the search-key value does not appear in the index, insert it.
  - **Sparse indices** – if index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created.
    - If a new block is created, the first search-key value appearing in the new block is inserted into the index.
- Multilevel insertion (as well as deletion) algorithms are simple extensions of the single-level algorithms

Secondary Indices Example

- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense



Secondary index on *balance* field of *account*

Primary and Secondary Indices

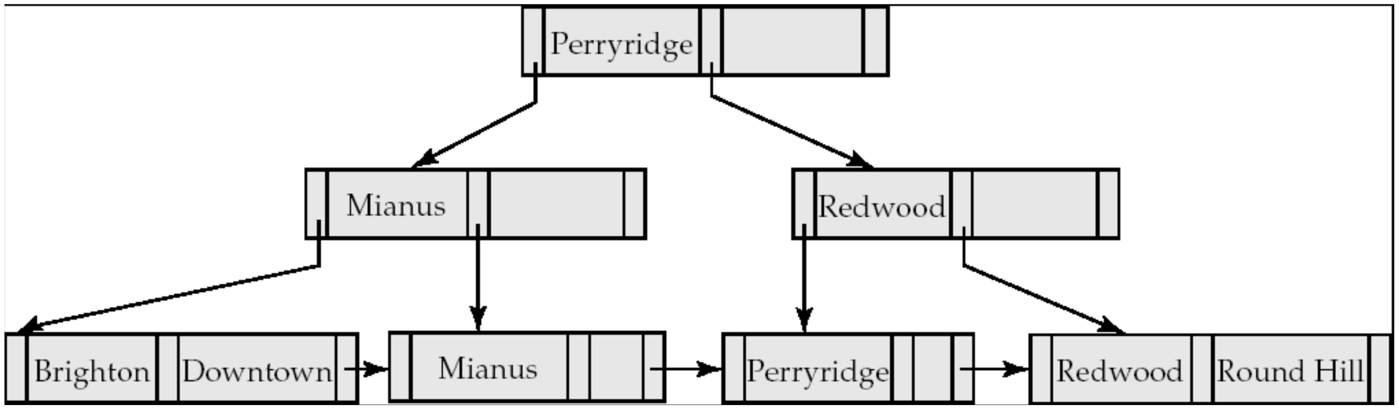
- Indices offer substantial benefits when searching for records.
- BUT: Updating indices imposes overhead on database modification -- when a file is modified, every index on the file must be updated,
- Sequential scan using primary index is efficient, but a sequential scan using a secondary index is expensive
  - Each record access may fetch a new block from disk
  - Block fetch requires about 5 to 10 micro seconds, versus about 100 nanoseconds for memory access

B^+-Tree Index Files

- Disadvantage of indexed-sequential files
  - performance degrades as file grows, since many overflow blocks get created.
  - Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files:
  - automatically reorganizes itself with small, local, changes, in the face of insertions and deletions.
  - Reorganization of entire file is not required to maintain performance.
- (Minor) disadvantage of B^+-trees:
  - extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages
  - B^+-trees are used extensively

B+-Tree Index Files (Cont.)

- A B+-tree is a rooted tree satisfying the following properties:
- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n–1)/2 \rceil$ and $n–1$ values
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n–1)$ values.



B^+-Tree Node Structure

- Typical node

P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).

- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

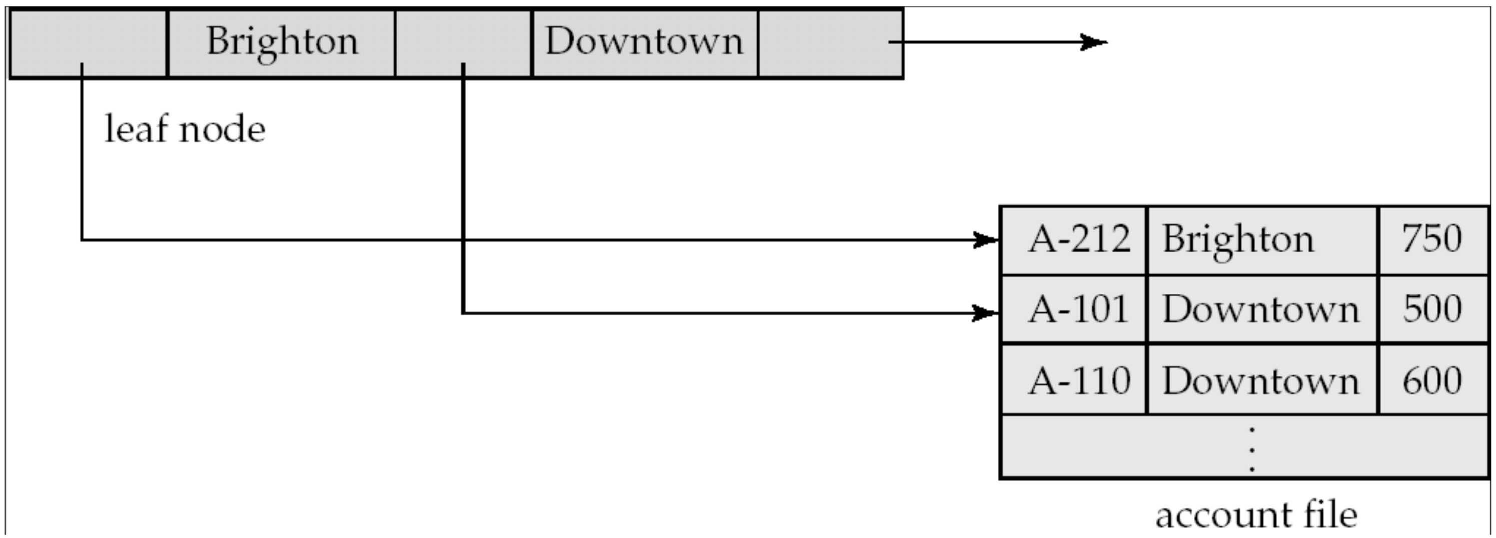
Leaf Nodes in B+-Trees

Properties of a leaf node:

•For i = 1, 2, . . ., n–1, pointer P_i either points to a file record with search-key value K_i, or to a bucket of pointers to file records, each record having search-key value K_i. Only need bucket structure if search-key does not form a primary key.

•If L_i, L_j are leaf nodes and $i < j$, L_i' s search-key values are less than L_j' s search-key values

•P_n points to next leaf node in search-key order



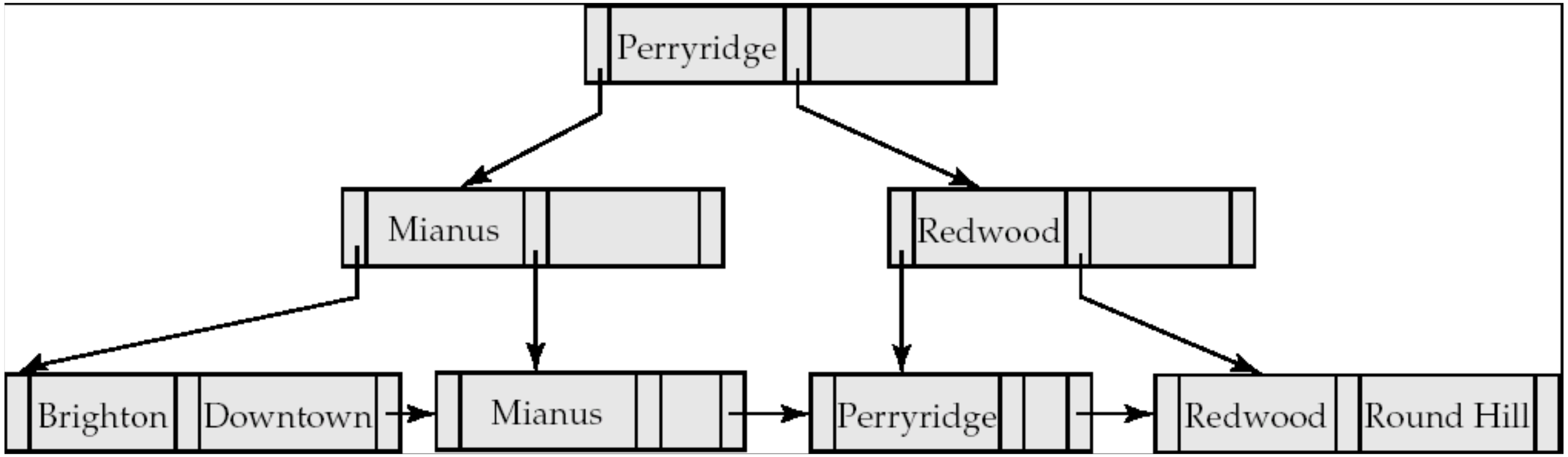
Non-Leaf Nodes in B+-Trees

Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *m* pointers:

- All the search-keys in the subtree to which P_1 points are less than K_1
- For $2 \leq i \leq n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
- All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}

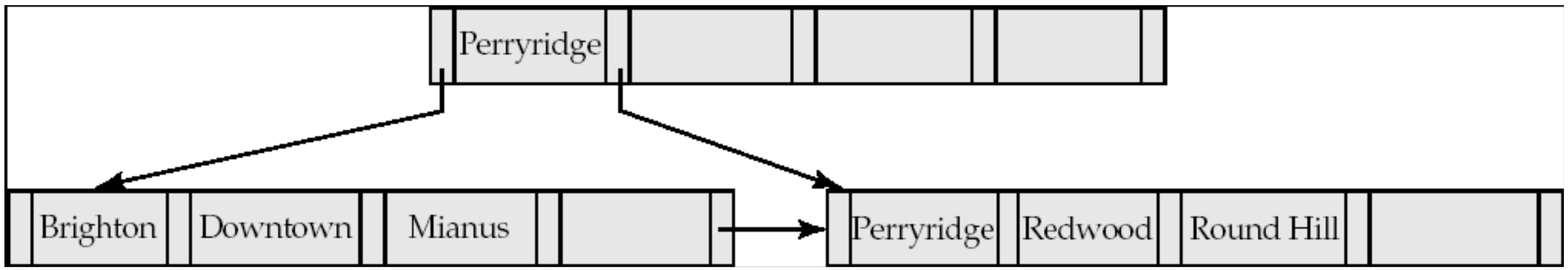
P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

Example of a B⁺-tree



B⁺-tree for *account* file ($n = 3$)

Example of B+-tree



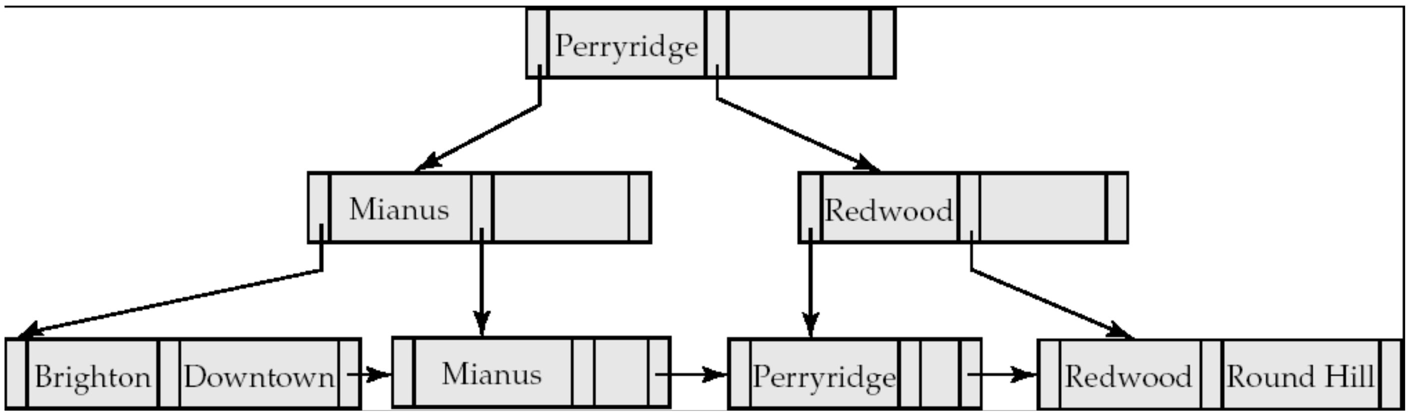
B+-tree for *account* file (n = 5)

Queries on B+-Trees

Find all records with a search-key value of *k.*

1. *N=root*
2. Repeat
    1. Examine *N* for the smallest search-key value > *k.*
    2. If such a value exists, assume it is K_i. Then set $N = P_i$
    3. Otherwise $k \geq K_{n-1}$. Set $N = P_n$

    Until *N* is a leaf node
3. If for some *i*, key $K_i = k$ follow pointer P_i to the desired record or bucket.
4. Else no record with search-key value *k* exists.



Updates on B+-Trees: Insertion

Find the leaf node in which the search-key value would appear

If the search-key value is already present in the leaf node

1. Add record to the file

If the search-key value is not present, then

1. Add the record to the main file (and create a bucket if necessary)
2. If there is room in the leaf node, insert (key-value, pointer) pair in the leaf node
3. Otherwise, split the node (along with the new (key-value, pointer) entry) as discussed in the next slide.

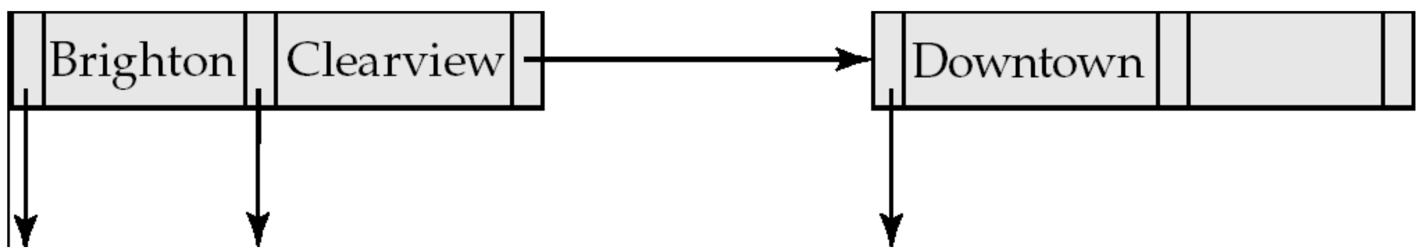
Updates on B^+-Trees: Insertion (Cont.)

Splitting a leaf node:

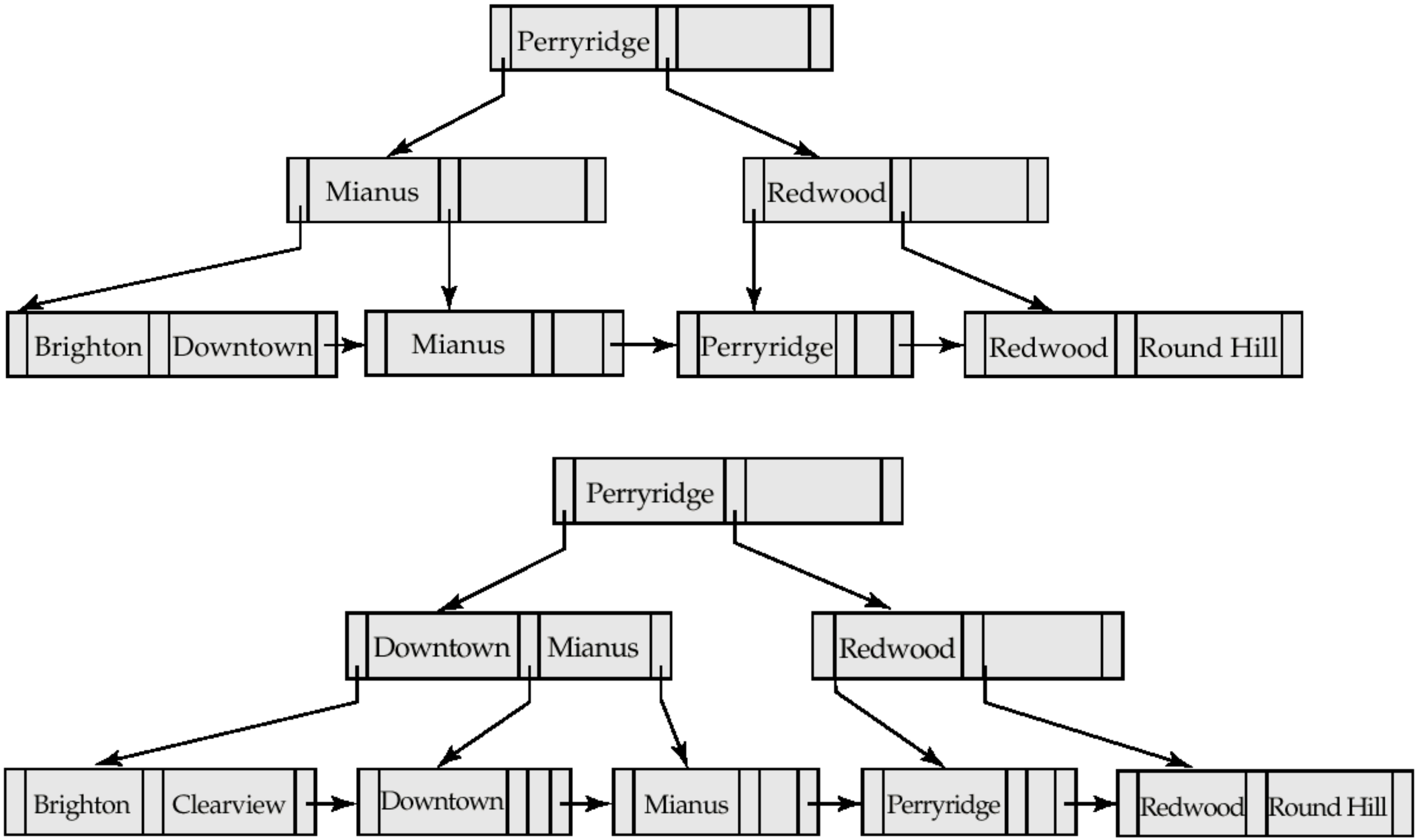
- take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first in the original node, and the rest in a new node.
- let the new node be *p,* and let *k* be the least key value in *p.* Insert (*k,p*) in the parent of the node being split.
- If the parent is full, split it and **propagate** the split further up.

Splitting of nodes proceeds upwards till a node that is not full is found.

- In the worst case the root node may be split increasing the height of the tree by 1.



Updates on B$^+$-Trees: Insertion (Cont.)



B$^+$-Tree before and after insertion of "Clearview"

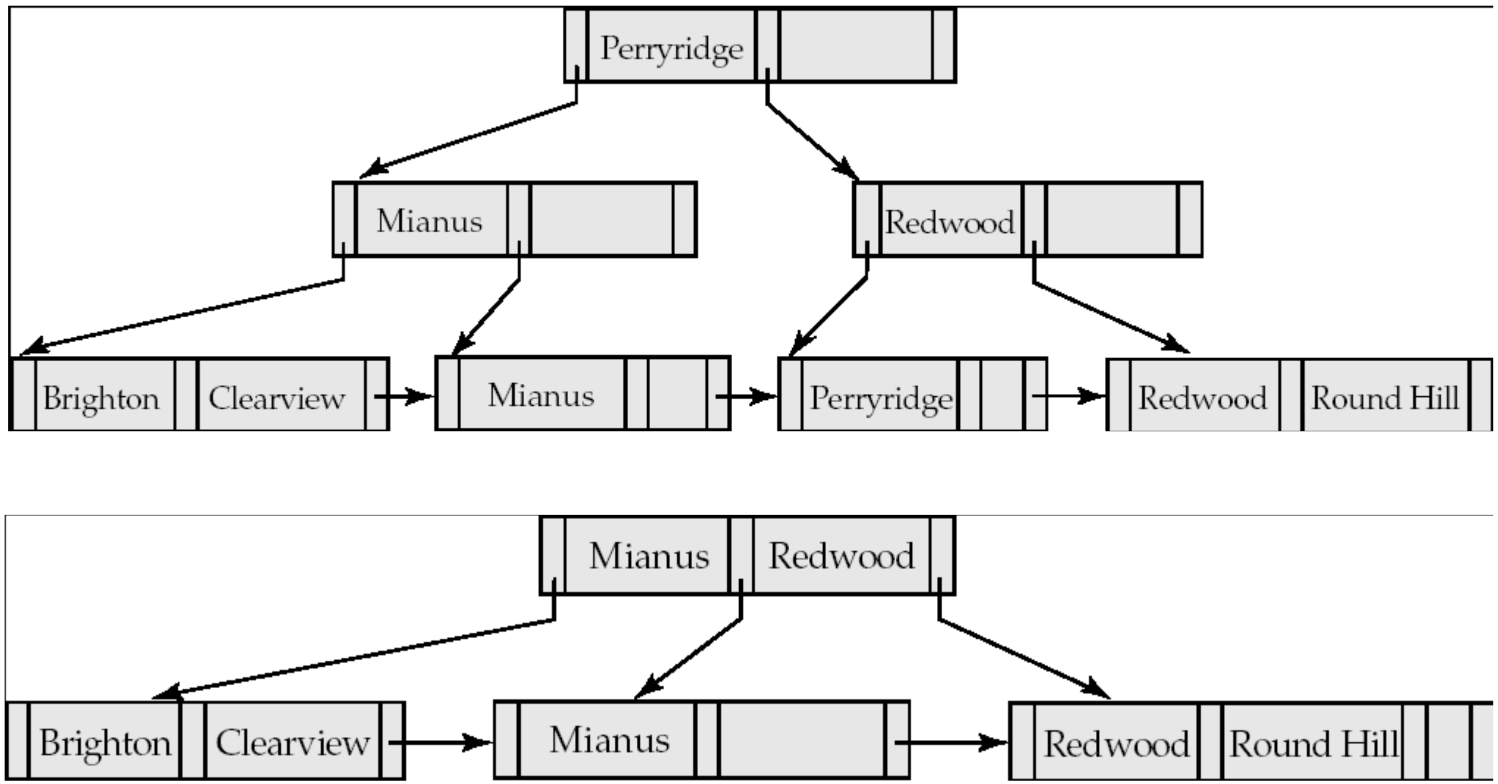
Updates on B^+-Trees: Deletion

- Find the record to be deleted, and remove it from the main file and from the bucket (if present)
- Remove (search-key value, pointer) from the leaf node if there is no bucket or if the bucket has become empty
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings:***
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B$^+$-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards until a node which several pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Examples of B⁺-Tree Deletion (Cont.)



Before and After deletion of "Perryridge" from result of previous example

B^+-Tree File Organization

- Index file degradation problem is solved by using B^+-Tree indices.
- Data file degradation problem is solved by using B^+-Tree File Organization.
- The leaf nodes in a B^+-tree file organization store records, instead of pointers.
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B^+-tree index.

Indexing Strings

- Variable length strings as keys
  - Variable fanout
  - Use space utilization as criterion for splitting, not number of pointers
- **Prefix compression**
  - Key values at internal nodes can be prefixes of full key
    - Keep enough characters to distinguish entries in the subtrees separated by the key value
      - E.g. "Silas" and "Silberschatz" can be separated by "Silb"
  - Keys in leaf node can be compressed by sharing common prefixes

Multiple-Key Access

- Use multiple indices for certain types of queries.
- Example:

  select *account_number*

  from *account*

  where *branch_name* = "Perryridge" **and** *balance* = 1000
- Possible strategies for processing query using indices on single attributes:
  1. Use index on *branch_name* to find accounts with branch name Perryridge; test *balance = 1000*
  2. Use index on *balance* to find accounts with balances of $1000; test *branch_name =* "Perryridge".
  3. Use *branch_name* index to find pointers to all records pertaining to the Perryridge branch. Similarly use index on *balance*. Take intersection of both sets of pointers obtained.

Hashing

Static Hashing

- A **bucket** is a unit of storage containing one or more records (a bucket is typically a disk block).
- In a **hash file organization** we obtain the bucket of a record directly from its search-key value using a **hash function.**
- Hash function h is a function from the set of all search-key values K to the set of all bucket addresses B.
- Hash function is used to locate records for access, insertion as well as deletion.
- Records with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate a record.

Example of Hash File Organization

- Hash file organization of *account* file, using *branch_name* as key (See figure in next slide.)
- There are 10 buckets,
- The binary representation of the *i*th character is assumed to be the integer *i.*
- The hash function returns the sum of the binary representations of the characters modulo 10
  - E.g. h(Perryridge) = 5 h(Round Hill) = 3 h(Brighton) = 3

Example of Hash File Organization

Hash file organization of *account* file, using *branch_name* as key (see previous slide for details).

bucket 0

bucket 1

bucket 2

bucket 3

A-217	Brighton	750
A-305	Round Hill	350

bucket 4

A-222	Redwood	700

bucket 5

A-102	Perryridge	400
A-201	Perryridge	900
A-218	Perryridge	700

bucket 6

bucket 7

A-215	Mianus	700

bucket 8

A-101	Downtown	500
A-110	Downtown	600

bucket 9

Hash Functions

- Worst hash function maps all search-key values to the same bucket; this makes access time proportional to the number of search-key values in the file.
- An ideal hash function is **uniform**, i.e., each bucket is assigned the same number of search-key values from the set of *all* possible values.
- Ideal hash function is **random**, so each bucket will have the same number of records assigned to it irrespective of the *actual distribution* of search-key values in the file.
- Typical hash functions perform computation on the internal binary representation of the search-key.
  - For example, for a string search-key, the binary representations of all the characters in the string could be added and the sum modulo the number of buckets could be returned. .