A New Filter Scheme for the Filtering of Fault Currents

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***Abstract* — In protection relaying schemes, the digital filter unit plays the essential roles to calculate the accurate phasor. However, the decaying DC components in fault currents always cause false operations of relay systems. This paper presents a new filter scheme which can remove such components from fault currents. Using the proposed scheme, the Full-Cycle DFT (FCDFT) only needs one-cycle samples to obtain an accurate fundamental phasor. The decaying DC component is removed by an iterative computation. The proposed filter scheme can help digital filters achieve accurate results rapidly. Simulations results illustrate the effectiveness of this new algorithm for distance relaying applications.**

***Keywords* – digital filter, decaying DC component, FCDFT, iterative computation**

I. INTRODUCTION

Digital filter units are the most important parts in computer relays. Usually, the Full-Cycle Discrete Fourier Transform (FCDFT) [1-2] is widely used in these units to obtain the fundamental phasors of fault measurements. When fault current waveforms are purely sinusoidal, the FCDFT only requires one-cycle-samples to obtain an accurate fundamental phasor. However, majority of fault currents contain large amounts of decaying DC components. Since the said components contain non-integer harmonics, the accurate fundamental phasors of fault currents cannot be obtained by FCDFT. As a result, the inaccurate current phasor will cause false operations of the relaying schemes.

In recent decades, many papers [3-6] have been proposed with the goal of overcoming the problems caused by decaying DC components. These studies aim to find methods for removing decaying DC components in fault currents as fast as possible. In [3], Benmouyal proposed a digital mimic filter technique to remove decaying DC component by a pre-defined line time constant. However, this technique cannot exactly remove decaying DC components. If the pre-defined line time constant is far from the exact line time constant, then the performance of this technique will be degraded.. In [4-5], Gu and Yang proposed the DFT-based algorithms which require one-cycle-plus-two samples to acquire an accurate fundamental phasor. Using these algorithms, the decaying DC parameters can be accurately removed by three consecutive phasor equations. However, these algorithms need two extra samples to obtain an accurate fundamental phasor. In [6], a simpler method was proposed, requiring two extra samples in attaining an accurate fundamental phasor. In [7], an algorithm needing one extra sample to obtain the fundamental phasors for distance protections was also proposed. In [8], Sidhu further suggested a technique which does not call for extra samples in obtaining an accurate fundamental phasor. This algorithm needs two extra phasor obtained by different frequency. In order for this algorithm to work properly, it is necessary to deal with the harmonics containing the fault measurements and the cut-off frequency of the anti-aliasing filters carefully.

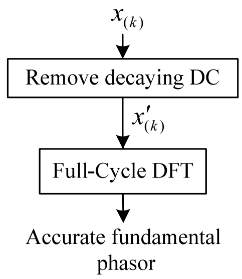


Fig. 1 Signal flow of the proposed filter scheme

This paper presents a new filter scheme without the intervention of extra samples in removing decaying DC components from fault currents. Fig.1 shows the proposed filter scheme which contains two stage computations. The first stage is an iterative computation which is used to remove the decaying DC component from the samples $x_{(k)}$. After that, the new samples $x'_{(k)}$ are sent to the FCDFT stage for obtaining an accurate fundamental phasor. Since this algorithm does not necessitate extra samples in removing decaying DC components, the response time of Fig.1 is quite fast. In comparison with [3-7], the response time of the proposed algorithm is faster. Contrary to [8], the proposed algorithm does not limit the harmonics in fault measurements.

II. THE PROPOSED ALGORITHM

The signal $x(t)$ which contains the decaying DC component is used to describe the proposed algorithm. The signal $x(t)$ is as follows:

$$x(t) = \sum_{m=1}^{N/2} A_m \cos(\omega_m t + \theta_m) + Be^{-t/\tau} \qquad (1)$$

where A_m, θ_m, and $\omega_m = 2\pi \times m$ are the magnitude, phase angle and angular frequency of the m^{th} order harmonic component, respectively. B and τ are the magnitude and time constant of the decaying DC component. N samples are taken per cycle (the sampling frequency $f_S = 60N$), and the k^{th} sample of $x(t)$ is presented by variable $x_{(k)}$ as follows:

$$x_{(k)} = x(k\Delta t) = \sum_{m=1}^{N/2} A_m \cos(\frac{2km\pi}{N} + \theta_m) + B\Gamma^k \tag{2}$$

where the decaying constant $\Gamma = e^{-\Delta t/\tau}$ and the sampling interval $\Delta t = 1/(60N)$.

In order to remove the decaying DC component, the parameters B and Γ need to be obtained. As described in Fig.1, the proposed algorithm is a pre-filter of the FCDFT. Since the FCDFT needs N samples to obtain a phasor, the same N samples are used to obtain the parameters B and Γ . Computing the summation of the N samples, the following equation is found

$$\sum_{i=1}^{N} x_{(k)} = \sum_{i=1}^{N} B\Gamma^k = B \times \frac{\Gamma^{(N+1)} - 1}{\Gamma - 1} \tag{3}$$

The summation of N samples only contain the information of the parameters B and Γ . Since the N samples are known, the result of (3) is acknowledged as well. If the result of (3) equals zero, it can be concluded that the fault current does not contain the decaying DC component. In contrast, if the result is not equal to zero, then the result can be used to get parameters B and Γ . However, since one equation cannot solve two unknowns, another equation is needed to obtain B and Γ . In order to achieve this, an iterative computation is strongly proposed.

At the onset, Γ_1 is assumed to be the decaying constant of the decaying DC components. Equation (3) can be used to obtain the magnitude of the decaying DC component as B_1. Using these two variables B_1 and Γ_1 , the decaying DC samples can be rebuiltas follows:

$$\text{The decaying DC samples} = B_1\Gamma_1^k, \qquad k=1, 2, ..., \text{N} \tag{4}$$

We can subtract (4) from $x_{(k)}$ to acquire new current samples $x'_{(k)}$ as follows:

$$x'_{(k)} = x_{(k)} - B_1\Gamma_1^k, \qquad k = 1, 2, ..., \text{N} \tag{5}$$

If the obtained B_1 and Γ_1 match the actual B and Γ accurately, the obtained $x'_{(k)}$ is purely sinusoidal and will satisfy the following equation

$$\sum_{k=1}^{N} x'_{(k)} = 0 \tag{6}$$

Thus, (6), the new equation, can help (3) in finding B and Γ . In the simulation experiences, the simple summation of (6) is not robust for some cases. In order to increase robustness, the simple summation of (6) is modified as follows:

$$f(\Gamma_1) = \left| \sum_{k=1,3,..}^{N-1} x'_{(k)} \right| + \left| \sum_{k=2,4,..}^{N} x'_{(k)} \right| = 0 \tag{7}$$

In (7), the summation of (6) is split into two items. The first item is the absolute value of the summation of odd samples while the second is the absolute value of the summation of even samples. Equation (7) can be considered as a function of the decaying constant Γ_1 . If the preceived decaying constant Γ_1 matches its actual value Γ , then the solution of (7) will be equal to zero. In order to converge Γ_1 to Γ , the secant method [9] is employed.

The two decaying constants Γ_1 and Γ_2 are initially estimated. In using (3), (4), and (7), two results of $f(\Gamma_1)$ and $f(\Gamma_2)$ can be obtained. Using the secant method, the new decaying constant Γ_{NEW} can be attained as follows:

$$\Gamma_{NEW} = \Gamma_2 - f(\Gamma_2) \times \frac{\Gamma_1 - \Gamma_2}{f(\Gamma_1) - f(\Gamma_2)} \tag{8}$$

Then, Γ_1 and Γ_2 is modified as follows:

$$\begin{aligned} \Gamma_1 &= \Gamma_2 \\ \Gamma_2 &= \Gamma_{NEW} \end{aligned} \tag{9}$$

Repeating the computations of (8-9), a converged decaying constant Γ_2 can be found. The obtained decaying constant Γ_2 and magnitude B_2 will converge to their actual value. B and Γ .

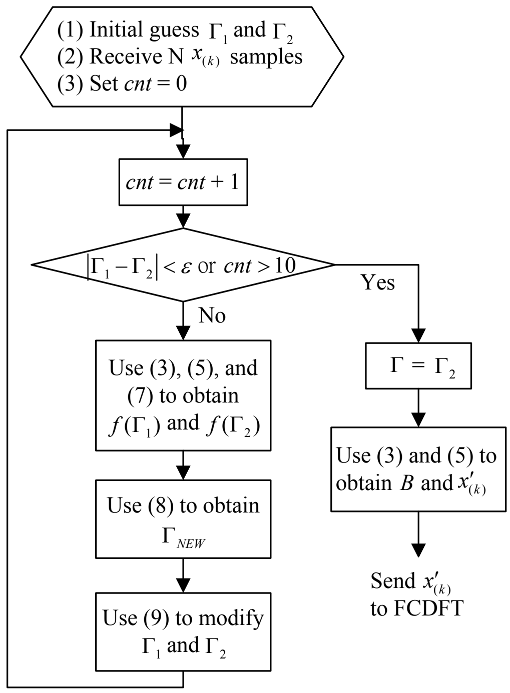


Fig. 2 Flowchart of the proposed algorithm

Notably, the summation computation of (3) can remove all the integer harmonics, and yet maintain the decaying DC component. However, if the fault current contains a pure DC component, the result of (3) will also contain the information of this DC component. It must be noted that the specified condition is not a general situation. In the previous simulations, the proposed iterative computations will make the converged decaying constant Γ_2 fit the sum of both the pure and the decaying DC component. Thus, both DC components will not be contained in the new current samples $x'_{(k)}$. In the next section, a test case is employed to demonstrate this phenomenon.

The flowchart of the proposed algorithm is shown in Fig.2. This algorithm operates sample-by-sample. At the start of each operation, N $x_{(k)}$ samples are received and the initial values of Γ_1 and Γ_2 are set. Afterwards, the secant method is used to obtain a convergent Γ_2. In its computations, $|\Gamma_1 - \Gamma_2| < \varepsilon$ is utilized to be the convergent criterion. Meanwhile, a counter *cnt* is employed to limit the number of iterations. If $cnt \geq 10$, this operation is stopped altogether. When this occurs, $\Gamma = \Gamma_2$ is set and (3) is used to obtain B. Finally, (5) is employed in obtaining N $x'_{(k)}$ samples. Since the decaying DC components in $x'_{(k)}$ have been removed, then the FCDFT can use them to obtain an accurate fundamental phasor.

III. SIMULATION RESULTS

In this section, the performances of the proposed algorithm are evaluated by some specific test signals and one power system example. The sampling frequency is $f_S = 960Hz$ (N=16, and $\Delta t = 0.001\sec$). When using the secant method, the initial decaying constants are $\Gamma_1 = 0.7$ and $\Gamma_2 = 0.75$ for all simulation cases.

A. *Test Signals Cases*

The two signals $x_1(t)$ and $x_2(t)$ are specified to verify the effectiveness of the proposed algorithm in an ideal environment. Likewise, all of the test signals are simulated by the FCDFT and the digital mimic filter + FCDFT for comparison. The time constant set for the digital mimic filter is $\tau = 0.01\sec$.

a) Standard signal test

$$x_1(t) = 10\cos(120\pi \times t + 1) + 5\cos(240\pi \times t + 2) \\ +2\cos(360\pi \times t + 3) + 10e^{-40t}$$

The signal $x_1(t)$ denotes the standard fault signal that contains some integer harmonics and a decaying DC component. Fig.3 depicts the fundamental phasor magnitude responses of different filter algorithms. Since the decaying DC components can accurately be removed by the proposed filter scheme, the accurate magnitude can only be attained using one cycle samples. In contrast, the conventional FCDFT cannot obtain accurate results, and the mimic filter + FCDFT can only get results with less error.

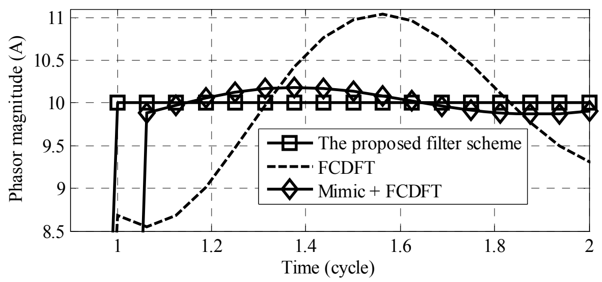


Fig. 3 Fundamental phasor magnitude responses of different filters

b) Pure DC component test

$$x_2(t) = 10 + 10\cos(120\pi \times t + 1) + 10e^{-40t}$$

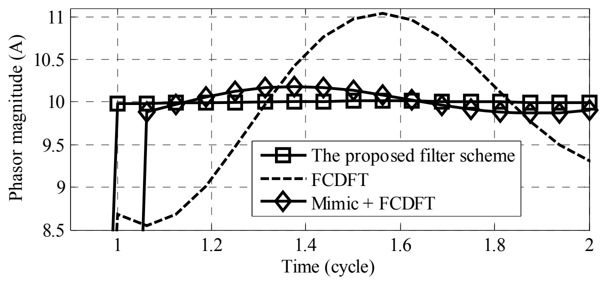


Fig. 4 Fundamental phasor magnitude responses of different filters with pure DC component measurement

In this test, the signal $x_2(t)$ containing a pure DC component demonstrates the abilities of the proposed filter scheme to deal with such components. Fig.4 depicts the fundamental phasor magnitude responses of different filter algorithms. However, we can note that the conventional FCDFT still cannot obtain accurate results, and the mimic filter + FCDFT can only generate results with less error. On the other hand, since the measurements contain pure DC components, the results of the proposed filter scheme only have little oscillation. In this case, the maximum percentage error of the proposed filter scheme is 0.43%. Thus, when handling measurements containing pure DC components, the proposed iterative computations can fit the obtained decaying constant Γ_2 to deal with the pure DC components and the results are supported with enough accuracy.

c) Statistical performance analysis – with Decaying DC

In this test, the same signal $x_1(t)$ is used, but a different decaying DC time constant is considered. Random noise with different Signal-to-Noise ratio (SNR) is then added to test filter performances. Also, the decaying DC time constant used for simulations are 1/40sec, 1/80sec, 1/120sec, and 1/160sec. Random noise is generated by the **rand** function of MATLAB [10], and its magnitude is matched with different SNRs (40dB, 60dB, 80dB, and 100dB). Since the results of the conventional FCDFT always contain many errors, it is replaced by the Gu's algorithm [4] in comparing the filter performances. Meanwhile, the two indices [7], PRMSE and

PPE, are defined to demonstrate the filter performances. These two indices are defined as follows:

(1) The percentage root-mean-square error (PRMSE):

$$\mathrm{PRMSE} = \frac{\sqrt{\dfrac{\sum_{k=n}^{n+N-1} (\text{Filter output} - \text{Steady state value})^2}{N}}}{\text{Steady state value}} \times 100\% \quad (10)$$

This performance index is used to evaluate the averaged performance in one cycle. The data used for analysis are N samples (one cycle), and the beginning index n depends on the filter algorithm. For example, n = N for the proposed filter scheme, n = N+1 for mimic filter + FCDFT, and n = N+2 for Gu's algorithm.

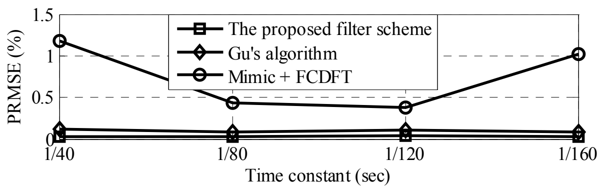
(2) The percentage peak error (PPE):

$$\mathrm{PPE} = \frac{\mathrm{Max}\left|\text{Filter output} - \text{Steady state value}\right|}{\text{Steady state value}} \times 100\% \quad (11)$$

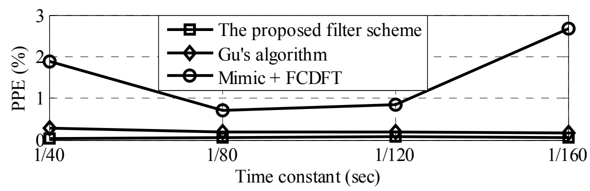
This performance index is used to evaluate the maximum error. The data used for this index are similar to those used in PRMSE.

Fig.5 shows the simulation results of the various decaying DC time constant. Obviously, the errors of mimic filter + FCDFT results decrease when time constants are near to its preset value. However, it remains having worst results. Meanwhile, the proposed filter scheme and Gu's algorithm are not affected by the decaying DC time constant. Notably, the results of the proposed filter scheme are better compared with Gu's algorithm. This is because the proposed filter scheme is less sensitive to the noise contained in the signal.

Fig.6 shows the simulation results of various SNR ratios. Notably, the effects of these ratios on mimic filter + FCDFT is not obvious. In addition, its ordinary errors are much higher than the errors caused by noise. On the other hand, both the proposed filter scheme and Gu's algorithm are obviously affected by different SNR ratios. However, the proposed filter scheme is less sensitive to noise.

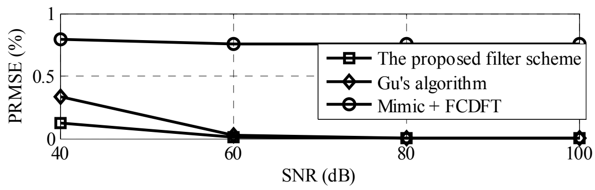


(a) PRMSE %

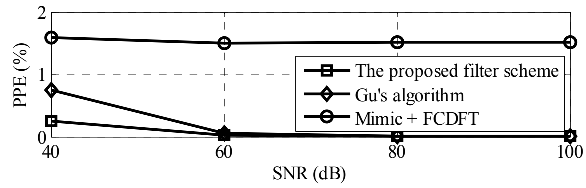


(b) PPE %

Fig. 5 The statistical results of PRMSE% and PPE% under various decaying DC time constant



(a) PRMSE %



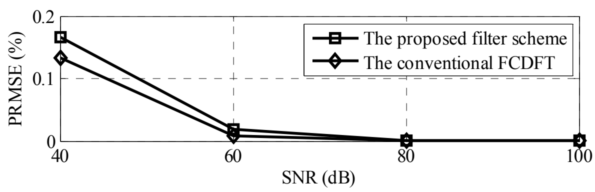
(b) PPE %

Fig. 6 The statistical results of PRMSE% and PPE% under various SNR ratios

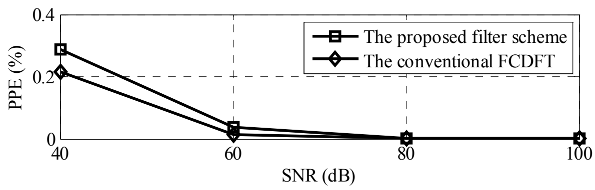
d) Statistical performance analysis – without Decaying DC

With previous simulations, it is concluded that the proposed filter scheme is affected by noise. In this test, the decaying DC containing in $x_1(t)$ is removed and the only considered effects are those from various SNR ratios. Hence, the considered SNR ratios are still 40dB, 60dB, 80dB, and 100dB. In this simulation, the results of the proposed filter scheme are exclusively compared with those of the conventional FCDFT.

Fig.7 shows the simulation results of various SNR ratios. Note that the effects of SNR ratio on the proposed filter scheme and the conventional FCDFT are similar. Thus, it is concluded that the proposed filter scheme can process the signal with or without decaying DC components. Meanwhile, when the signal does not contain the said components, the proposed filter scheme has the same response time and has similar errors of the conventional FCDFT.



(a) PRMSE %



(b) PPE %

Fig. 7 The statistical results of PRMSE% and PPE% of the signal without decaying DC components under various SNR ratios

B. Power System Case

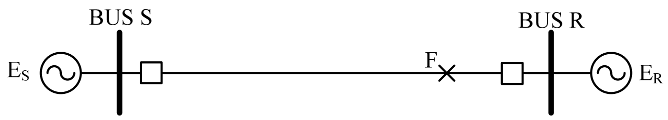


Fig. 8 The single-line-diagram of the power system for testing

TABLE I PARAMETERS OF THE TRANSMISSION LINE

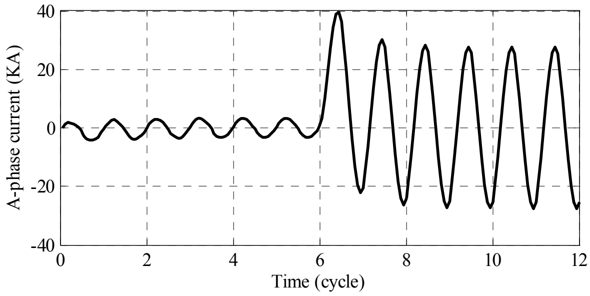
System voltage 345kV	System frequency 60Hz
Generator parameters:	
$E_S=1.0\angle 0^{\circ}$pu	$E_R=1.0\angle -10^{\circ}$pu
$Z_{S1}=0.238+j5.7132(\Omega)$	$Z_{R1}=0.238+j6.19(\Omega)$
$Z_{S0}=2.738+j10\ (\Omega)$	$Z_{R0}=0.833+j5.118(\Omega)$
Transmission line parameters : Length L=100km	
Positive sequence: R1=0.0321(Ω/km) L1=0.473(mH/km) C1=0.038(uF/km)	
Zero sequence: R0=0.3479(Ω/km) L0=1.370(mH/km) C0=0.038(uF/km)	
Data Acquisition:	
Sampling frequency : 960Hz,	
2nd order Butterworth with 360Hz cut-off frequency	

In Fig.8, a simple power system is used to evaluate the performances of the proposed filter scheme. The system parameters are listed in Table I. The fault currents are also measured at BUS S. The second order low-pass Butterworth anti-aliasing filters with 360-Hz cut-off frequency are applied to current measurements. Afterwards, these measurements are sampled by 960 Hz for digital filter processing.

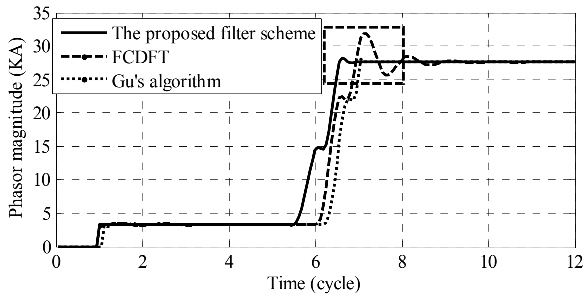
For comparison, Gu's algorithm and the conventional FCDFT are calculated. Assuming an A-phase to ground fault occurs at 10km from BUS S, the fault resistance is $0.1\,\Omega$ and the fault is incepted at the end of cycle 6. The total simulation time is 12 cycles. Fig.9a shows the A-phase current waveform while its fundamental phasor magnitudes through different algorithms are in Fig.9b. Meanwhile, Fig.9c is the expanded view of the dashed rectangle in Fig.9b. Obviously, the proposed filter scheme can accurately remove the decaying DC components without requiring any extra sample. Table II shows the simulation results analyzed by the performance indices PRMSE and PPE. Therefore, the proposed filter scheme has the fastest response and the lowest error.

IV. DESIGN CONSIDERATIONS

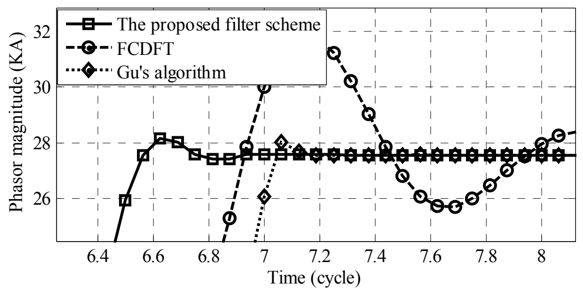
In previous section, the initial decaying constants are all set as $\Gamma_1 = 0.7$ and $\Gamma_2 = 0.75$ for all the computations of secant method. The reason is due to the considerations of practical transmission line parameters. $\Gamma_1 = 0.7$ denotes the decaying DC time constant $\tau_1 = -\Delta t/\ln(0.7)$ = 0.0028sec (=0.17 cycle), and $\Gamma_2 = 0.75$ denotes τ_2 = 0.0035sec (=0.2 cycle). Obviously the both initial time constants are shorter than the normal transmission line time constant. Meanwhile, they are also shorter than the response time of FCDFT. Thus, these initial values will lead the proposed iterative computations to a convergent result. However, if the sampling time Δt is changed, the initial values must be redesigned.



(a) A-phase fault current waveform



(b) The fundamental phasor magnitude obtained by different algorithms



(c) The expanded view of (b)

Fig.9 A-phase fault current waveform and fundamental phasor of an A-phase to ground fault

TABLE II ERROR ANALYSIS OF DIFFERENT FILTER ALGORITHM

	PRMSE (%)	PPE (%)
The proposed filter scheme	0.024	0.048
FCDFT	8.50	15.72
Gu's algorithm	0.109	0.431

V. CONCLUSIONS

In this paper, a new filter scheme is proposed to obtain the accurate fundamental phasor of the fault currents. The proposed scheme combining an iterative computation and the FCDFT to achieve the filtering computations, which only needs one-cycle samples. In contrast with previous research, the response time of the proposed filter scheme is faster. Meanwhile, since this scheme is not sensitive to noise, it has the potentials for practical environment application. Furthermore, simulations results illustrate that the proposed filter scheme is effective in various fault conditions

ACKNOWLEDGMENT

The research was supported by the National Science Council of the Republic of China under the grant NSC 96-2628-E-027-115-MY2

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