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## Problem Statement:

The function `longest_consecutive_sequence` takes a list of integers as input and returns the length of the longest consecutive sequence found within that list. For example, if the input is `[100, 4, 200, 1, 3, 2]`, the function identifies the consecutive sequence `[1, 2, 3, 4]` and returns 4.

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## Solution 1

The provided solution works by first converting the list into a set to enable  $O(1)$  lookups. It then iterates over each number in the original list. For each number, if the preceding number (`num - 1`) is not in the set, it marks the start of a new sequence. A while loop is then used to count how many consecutive numbers follow, updating the longest sequence length accordingly.

```
def longest_consecutive_sequence(nums: List[int]) -> int:
    if not nums: return 0
    longest = 0
    num_set = set(nums)

    for num in nums:
        if num - 1 not in num_set: # no prior number means start of sequence
            streak = 1

            while num + 1 in num_set:
                streak += 1
                num += 1

            longest = max(longest, streak)

    return longest
```

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## Step-by-Step Breakdown

### 1. Input:

A list of integers (e.g., `[100, 4, 200, 1, 3, 2]`).

### 2. Initial Check & Set Creation:

**Step:** If the list is empty, the function returns 0 immediately. Otherwise, convert the list to a set for fast lookups.

```
1. if not nums: return 0
2.
3. num_set = {100, 4, 200, 1, 3, 2} → {1, 2, 3, 4, 100, 200}
4.
```

### Explanation:

Converting to a set ensures that checking if a number exists (`num in num_set`) is done in constant time.

### 3. Iteration Over Each Number:

- **Step:** The function loops over every number in the original list.

```
1. For each number in [100, 4, 200, 1, 3, 2]:
2.   └─ Process the number
```

### 4. Identifying the Start of a Sequence:

- **Step:**

For each number, check if  $\text{num} - 1$  is not in the set. This indicates the number is the beginning of a consecutive sequence.

- For 100: Check if 99 is in the set. It is not, so start a new sequence.
- For 4: Check if 3 is in the set. Since 3 is present, skip starting a sequence here.

```
1. Number = 100
2.   └─ 100 - 1 = 99 → 99 not in set → Start sequence [100]
3.
4. Number = 4
5.   └─ 4 - 1 = 3 → 3 in set → Skip (Not the start)
6.
```

### 5. Counting the Consecutive Sequence:

- **Step:**

If the number is the start of a sequence, initialize a counter ( $\text{streak} = 1$ ) and then use a while loop to check for consecutive numbers.

- $\text{num} = 1 \rightarrow \text{Check } 1 - 1 = 0 \rightarrow \text{Not in set} \rightarrow \text{Start a new sequence.}$
- Set  $\text{streak} = 1$ .

```
1. While (num + 1) is in num_set:
2.   1. num = 1: Check if 2 is in set? Yes → streak becomes 2, update num to 2.
3.   2. num = 2: Check if 3 is in set? Yes → streak becomes 3, update num to 3.
4.   3. num = 3: Check if 4 is in set? Yes → streak becomes 4, update num to 4.
5.   4. num = 4: Check if 5 is in set? No → End loop.
6.
7. Sequence starting at 1: [1 → 2 → 3 → 4]
8. Final streak for this sequence: 4
9.
```

- **Other Numbers:**

- For 200:
  - $200 - 1 = 199$  is not in the set, so sequence starts at 200.
  - Check 201 in set  $\rightarrow$  Not found  $\rightarrow$  Sequence length = 1.
- For 100 (as seen earlier):
  - Sequence is just [100]  $\rightarrow$  Length = 1.

### 6. Updating the Longest Sequence:

- **Step:**  
After processing each starting number, update the variable longest with the maximum streak found so far.

```
1. longest = max(longest, streak)
```

- **Flow Example:**
  - After processing 100:  $\text{longest} = \max(0, 1) = 1$
  - After processing 1:  $\text{longest} = \max(1, 4) = 4$
  - Other numbers do not change the longest value.

## 7. Output:

- **Final Step:**  
Return the longest streak found.

```
1. return longest → returns 4
```

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## Efficiency Details for Solution 1:

- **Time Complexity:**
    - **Step 1:** Converting the input list to a set takes  $O(n)$  time.
    - **Step 2:** Iterating over each number in the list is  $O(n)$ .
    - **Step 3:** For each number that starts a sequence, the while loop only processes each number once across all iterations.
    - **Overall:** The algorithm runs in  $O(n)$  time on average.
  - **Space Complexity:**
    - The set created from the input list uses  $O(n)$  extra space.
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## Visual Flow Diagram

```
1. Input: [100, 4, 200, 1, 3, 2]
2.      |
3.      v
4. Convert List → Set: {1, 2, 3, 4, 100, 200} [O(n) time, O(n) space]
```

```
5.      |
6.      ▼
7. For each number in input:
8.
9.      [Check: is (num - 1) in set?]      (O(1) per check)
10.
11.     |
12.     ▼
13. If not, start counting consecutive sequence:
14.
15.     [while (num + 1) in set:             (each element processed once)
16.         count++
17.         num++]
18.
19.     |
20.     ▼
21. Update longest streak found (O(1) per update)
22.     |
23.     ▼
24. Return longest streak      (O(1))
```