ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/238187424

Simplified method for evaluation of the lateral dynamic behaviour of a heavy vehicle

Article *in* International Journal of Heavy Vehicle Systems · January 2004

DOI: 10.1504/IJHVS.2004.004039

CITATIONS
4

READS
175

1 author:

Johan Wideberg
Universidad de Sevilla

30 PUBLICATIONS **343** CITATIONS

SEE PROFILE

All content following this page was uploaded by Johan Wideberg on 12 December 2013.

The user has requested enhancement of the downloaded file.

Simplified method for evaluation of the lateral dynamic behaviour of a heavy vehicle

J. P. Wideberg

Transportation Engineering, School of Engineering, University of Seville, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain

Email wideberg@us.es

Abstract: In this paper a method to add elastic effects to the classic 'bicycle model' for the simulation of the dynamic behaviour of vehicles is presented. The results obtained show that in the simulations the dynamic effects deteriorate when flexible vehicle frames are considered. This work demonstrates that the modified bicycle model is a useful instrument to predict the response of a vehicle. The method used is a numerical model of the dynamics coupled with a finite element model of the frame. MATLAB is used as a platform for evaluation of the dynamics and CALFEM, which is a MATLAB plug-in, is used for the finite element model. The proposed method offers an easy and quick way to evaluate the dynamic effects in models with flexible frames. The elasticity of the frame has an important impact on the directional response (jaw gain, lateral velocity and ultimately the trajectory) of the vehicles. The change of rigidity of the frame could give rise to considerable effects in the directional stability and the handling.

Keywords: bicycle model, flexible chassis, simulation, transient, vehicle dynamics.

Reference to this paper should be made as follows: Wideberg, J. P. (2004) 'Simplified method for evaluation of the lateral dynamic behaviour of a heavy vehicle', *Heavy Vehicle Systems,* A Series of the *Int. J. of Vehicle Design*, Vol. 11, No. 2, pp. 00–00.

Nomenclature

b	distance from the centre of gravity to the front axle (m)
c	distance from the centre of gravity to the rear axle (m)
L	distance between axles (m)
m	total mass of the vehicle (kg)
v_x	velocity in forward direction (m/s)
v_y	velocity in lateral direction (m/s)
Ω	yaw speed (rad/s)
E	Young's modulus (MPa)

Copyright © 2004 Inderscience Enterprises Ltd.

α_f	slip angle of the front tire (rad)
α_r	slip angle of the rear tire (rad)
θ_f	angle due to deformation of the frame, front axle (rad)
θ_r	angle due to deformation of the frame, rear axle (rad)
$C_{\alpha f}$	cornering stiffness, front tire (kN/rad)
$C_{\alpha r}$	cornering stiffness, rear tire (kN/rad)
I_z	moment of inertia (mm^4)
F_c	centrifugal force (N)
F_{yf}	reaction force at the front tire (N)
F_{yr}	reaction force at the rear tire (N)
K	understeer gradient
a_y	lateral acceleration (m/s^2)
v_{crit}	critical speed (m/s)
DOF	degree of freedom
FEA	finite element analysis
MBS	multi body system
SUV	Sport Utility Vehicle

1 Introduction

A very well known dynamic model for simulating the cornering of a vehicle is the 'bicycle model'. This model is discussed in many articles and text books (e.g. Milliken (1997) or Genta (1997)). In this model a vehicle with four wheels and two axles is modelled as a one track vehicle with two wheels, front and rear, with double lateral stiffness. The bicycle model is a very valuable tool to use and there is a wealth of information about the dynamics of a vehicle that can be derived from this relatively simple model.

One of the drawbacks of the model is that it is considered to be rigid. In this paper the 'bicycle model' is modified to take into consideration the elasticity of the frame, i.e. how the bending of the vehicle along the vertical z-axis (according to the SAE system) changes its transient response. It's important to mention that the elasticity of the frame is of particular importance in trucks with independent frame (ladder design). Also trucks are suitable because of the relatively small lateral acceleration. The bicycle model is less and less applicable with increasing lateral force. Segel (1956) says it should be used if the lateral acceleration is no greater than approximately 0.5 g. A car, as opposed to a truck, has a shorter distance between front and rear axles relative to the track width and also integrates the chassis with the frame; therefore it is much stiffer and has a very small influence on cornering . Ambrósio and Gonçalves (2001) studied the elastic effects in a car frame simulating two different cases: vehicle handling and a road bump. For the case of vehicle handling no definite conclusion is reached. In the paper written by Kuti (2001) a inite element model is used to evaluate the effect of the elastic deformations of the chassis of a truck. Wideberg (2002) presented an extension to the bicycle model where the elasticity was included for the quasi-static case. Some of the results and conclusions are similar to this paper. Finally it is worth mentioning is that the SUVs do have dual frame, and that this architecture might be adopted for future passenger cars.

In 1956 William F. Milliken David W. Whitcomb, and Leonard Segel, of the Cornell Aeronautical Laboratory, published the first major quantitative and theoretical analysis of vehicle handling in a series of papers (Milliken and Whitcomb, 1956; Segal, 1956a, 1956b). The final paper in the series, written by D. W. Whitcomb, draws a series of conclusions on automobile stability and control using a two degree of freedom model (yaw and side-slip) with experimentally determined parameters (Whitcomb and Milliken, 1956). Due to the lack of a roll degree of freedom, Whitcomb was able to assume that the car has no width and that the tyres lay on the centreline of the vehicle. This model is commonly referred to as the 'bicycle model'.

Since the early 1980s a shift in the vehicle modelling process has taken place. The demand for accurate vehicle dynamics models combined with the difficulty in deriving the equations of motion for large multibody systems led to the use of general multibody simulation codes. A wide range of capabilities are present in modern MBS codes, including the ability to handle non-inertial reference frames, to incorporate flexible elements in the model, to utilise generalised coordinates, and to symbolically generate the equations of motion. Also it is possible to couple calculations for instance between a MBS and FEA code so the output from one program is used as the input or boundary condition for the other. These type of calculations are often very time consuming due to there iterative nature.

To use MBS and FEA codes it is necessary to invest substantial amounts in computers, codes and engineer training. This is possible for large corporations, consulting houses and universities, but not for small firms and individual independent engineers.

It is very common that older trucks are modified for uses other than those originally intended. For instance a new owner might want to modify the structure by adding a crane. In this process it is common for the frame to be modified in a way that it is makes it more or less rigid. This is usually done by an individual or smaller consulting firm that has limited time, knowledge and resources to use MBS or FEA codes. For the reasons mentioned above, it would be useful to have a tool available that any engineer could use which is based on the 'bicycle model' but which adds a way to interpret the effect of the stiffness of the frame. This model, which is presented in this paper, is simple to use and both the dynamic model and the FEA model is coded using MATLAB.

It is important to stress that the objective of this work is to find a simple tool to calculate the dynamic behaviour of a truck. What is important is the tendencies i.e. will the behaviour be better or worse. Therefore there are effects that are not considered, for instance all inertia effects of the deflection are being ignored and that the deflection is limited to in-plane bending. Nevertheless, if all these effects would to be included then the simplicity and therefore the purpose of this study would be gone astray.

2 Problem statement

Steady-state cornering will be derived for the case of transient response, i.e. a sudden change in direction. A model will combine the classical ‘bicycle model’ and a FEA model of the truck frame subjected to a point load in the location for the centre of gravity (see Figure 1). The boundary conditions permit free rotation of the beam but do not permit displacements. The contact between the tyre and the road will prevent rotation slightly. The amount of rotation is very hard to quantify and therefore using a totally free rotation will give more conservative results. The load on the frame is equivalent to the lateral force and moment for the response of a transient change of the steer angle as explain below.

3 Dynamic equilibrium

In this paper the equations are expressed as scalar equations. It is desirable to express these without introducing the heading angle. ‘Vehicle fix coordinates’ are therefore used, otherwise an extra integration would be needed when solving (to keep track of heading angle).

The road is considered to be flat hence the motion will be planar. Equilibrium of forces gives (see Figure 1).

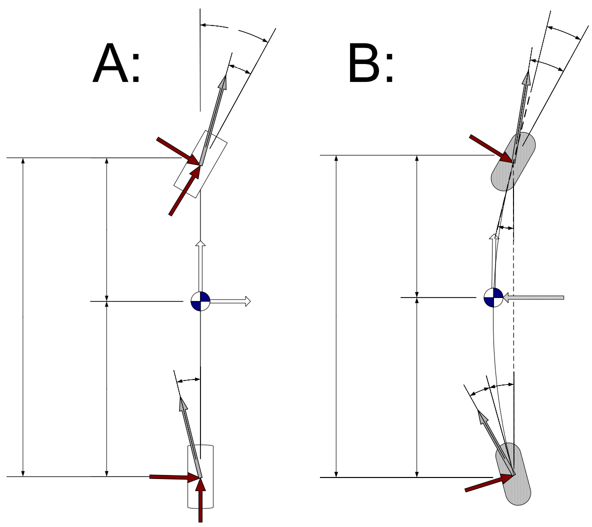

Figure 1 *To the left- the bicycle model, and to the right the elastic effect of the lateral bending of the frame.*

$$\begin{aligned} m \cdot a_x &= m \cdot (\frac{dV_x}{dt} - V_y \cdot \Omega) = F_{xr} + F_{xf} \cdot \cos(\delta) - F_{yf} \cdot \sin(\delta) \\ m \cdot a_y &= m \cdot (\frac{dV_y}{dt} + V_x \cdot \Omega) = F_{yr} + F_{xf} \cdot \sin(\delta) + F_{yf} \cdot \cos(\delta) \end{aligned} \tag{1}$$

and the equilibrium of the moments

$$I \cdot \frac{d\Omega}{dt} = -F_{yr}c + F_{xf} \cdot \sin(\delta) \cdot b + F_{yf} \cdot \cos(\delta) \cdot b \tag{2}$$

4 Nonlinear tyre model

The tyre model plays an important role in the performance of the vehicle model. With the exception of aerodynamic forces which are only relevant at higher speeds, all of the interactions of the vehicle with the external environment occur through the tire. Accurate modelling of the tyre is crucial to an accurate vehicle simulation. Normal modelling practices decouple the horizontal force generation from the model for vertical force generation, with the exception of the dependence of the horizontal force on normal load.

A common approach to calculating the lateral force generated by the tyre is to use either a linearisation (as in Equation (3)) which is the classic approach when using the bicycle model. Another method is to use the Magic Formula Tyre Model (Bakker *et al.*, 1987) which provides more accurate results for larger slip angles. The formula is based on a function whose behaviour approximates the shape of the curves obtained from experimental measurements on tyres. Its parameters are determined so as to fit the curve to a particular set of experimental results. A problem in many empirical/numerical approaches is that it requires a large amount of test data which can be difficult and expensive to obtain by testing, or often difficult to find by a literature search. Some authors Tönök and Ünlüsoy (2001) derived the necessary parameters using a finite element model. Due to the nature of this study neither of the methods mentioned above are acceptable.

In this article, a method will be used which has a simpler mathematical expression that mimics the most important, overall features of the Magic Formulae with fewer parameters. The method, often called the brush model (Dixon, 1991; Wennerström, 1999), will be used in this paper. The advantage is that it is still nonlinear, but does not require experiments and/or analysis. A graphic presentation can be seen in Figure 2.

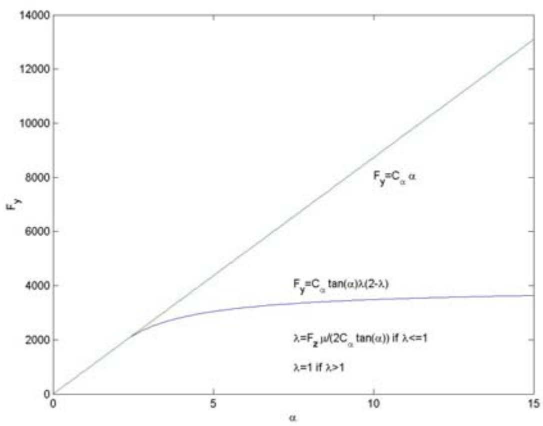

Figure 2 *Brush model tyre model.*

The formulation of a linear tyre model is defined in Equation (4). The slip angles for the rear and the front tyres can be expressed as:

$$\begin{aligned} F_{yf} &= C_{\alpha f}\alpha_f \\ F_{yr} &= C_{\alpha r}\alpha_r \end{aligned} \tag{4}$$

5 The brush model

As mentioned here in this section the brush model will be used due to its easy implementation. It is defined as:

$$\begin{cases} F_{\mathrm{yf}} = -\mathrm{C}_{\alpha}\tan\left(\alpha_{\mathrm{f}}\right)\lambda_{\mathrm{f}}(2\text{-}\lambda_{\mathrm{f}}) \\ F_{\mathrm{yr}} = -\mathrm{C}_{\alpha r}\tan\left(\alpha_{\mathrm{r}}\right)\lambda_{\mathrm{r}}(2\text{-}\lambda_{\mathrm{r}}) \\ \lambda_{\mathrm{f}} = \dfrac{F_{zf}\mu}{2C_{af}\left|\tan(\alpha_{\mathrm{f}})\right|} \quad ; \quad \lambda_{\mathrm{f}} \le 1 \\ \lambda_{\mathrm{r}} = \dfrac{F_{zr}\mu}{2C_{af}\left|\tan(\alpha_{\mathrm{r}})\right|} \quad ; \quad \lambda_{\mathrm{r}} \le 1 \\ \lambda_{\mathrm{f}} = 1 \quad ; \quad \lambda_{\mathrm{f}} > 1 \\ \lambda_{\mathrm{r}} = 1 \quad ; \quad \lambda_{\mathrm{r}} > 1 \end{cases} \tag{5}$$

The slip angles are defined relative to the tangent of the frame and this is rotated θ_f and θ_f, respectively. For the case of a rigid frame these angles are zero. The compatibility will then give the following relationship between angles:

$$\begin{aligned} \tan(\delta\text{-}\alpha_{\mathrm{f}} + \theta_f) &= \frac{b\cdot\Omega + V_y}{V_x} \\ \tan(\alpha_{\mathrm{r}} + \theta_r) &= \frac{c\cdot\Omega\text{-}V_y}{V_x} \end{aligned} \tag{6}$$

6 Finite element model

The calculation of the truck frame is integrated directly into the main MATLAB program of the bicycle model. In Figure 4 the location of the routine inside the main vehicle dynamics program can be seen. The advantages of this approach are that everything is integrated into one program, the time of execution is very short and changes and reruns are also easily executed in the batch oriented mode.

CALFEM is a MATLAB toolbox for finite element applications. The finite element analysis can be carried out either interactively or in a batch oriented fashion. In the interactive mode the functions are evaluated one by one in the MATLAB command window. In the batch oriented mode sequences of functions are written in a MATLAB file, and evaluated by writing the file name in the command window or in this case called by from the main MATLAB command file.

The group of element functions contains functions for the computation of element matrices and element forces for different element types. For each element type there is a function for the computation of an element stiffness matrix and an element load vector. Then the element stiffness matrices and element load vectors are assembled into a global stiffness matrix and a load vector. Unknown nodal values of temperatures or displacements are computed by solving a linear system of equations.

Beam elements are available for two-, and three-dimensional linear static analysis. Two-dimensional beam elements are also available for nonlinear geometric and dynamic analysis. In this particular case a linear two-dimensional static element is used. The frame of a real commercial truck has been modelled using a finite element technique. It is depicted in Figure 3. The physical parameters of the truck are listed in Table 1. The model corresponds to a smaller European truck. It is made up of linear beam elements. There are a total of 64 elements and 48 nodes with 3 DOF each. The model will also be used to validate the influence of the elastic frame, i.e. the angles θ_f and θ_r .

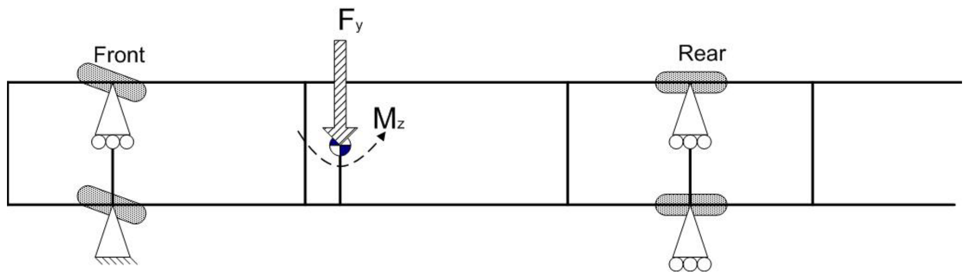

Figure 3 *Finite element model.*

Table 1 Vehicle parameters.

Mass	Total mass	7490 kg
Geometry	Distance CG to front axle, *b*	1.7 m
	Distance CG to rear axle, *c*	2.55 m
	Width of the frame	0.849 m
	Total length of vehicle	7.59 m
Cornering stiffness	Vehicle front axle	80 kN/rad
	Vehicle rear axle	130 kN/rad
Moment of inertia	I_z	4700 kgm^2
Forward velocity	V_x	100 km/h
Steering angle	δ	5 degrees

7 Numerical model

The results from the dynamic equilibrium, the tyre model and the FEA model of the frame are combined into a numerical model of truck behaviour.

The governing dynamics equations can be expressed as:

$$\begin{aligned}\frac{dV_y}{dt} &= \frac{1}{m}\left(C_{\alpha r} a\tan\left(\frac{c\Omega - V_y}{V_x} + \theta_r\right)\lambda_r(2-\lambda_r) + \right. \\ &\left. + C_{\alpha f}\left(\delta - a\tan\left(\frac{b\Omega + V_y}{V_x}\right)\lambda_f(2-\lambda_f) + \theta_f\right)\cos(\delta) - V_x\Omega\right)\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{d\Omega}{dt} &= \frac{1}{I}\left(-C_{\alpha r} a\tan\left(\frac{c\Omega - V_y}{V_x} + \theta_r\right)\lambda_r(2-\lambda_r)c + \right. \\ &\left. + C_{\alpha f}\left(\delta - a\tan\left(\frac{b\Omega + V_y}{V_x}\right)\lambda_f(2-\lambda_f) + \theta_f\right)\cos(\delta)b\right)\end{aligned} \tag{8}$$

$$\frac{d\psi}{dt} = \Omega \tag{9}$$

$$\frac{dX}{dt} = V_x \cdot \cos\ \psi\ -\ V_y \cdot \sin\ \psi \tag{10}$$

$$\frac{dY}{dt} = V_y \cdot \cos\ \psi\ +\ V_x \cdot \sin\ \psi \tag{11}$$

where Equations (7) and (8) are the governing equations of the vehicle dynamics with the tyre model and the influence of the elasticity included The standard simplification of small angles is not considered because a closed solution is not necessary and the equations are easy to solve numerically. The three following Equations (9)-(11) are needed to find out the global heading angle ψ and the global coordinates X and Y.

The equations are used to develop a numerical model in MATLAB. A flow chart representing the code developed in included as Figure 4. The differential equations where solved using MATLAB's ODE45 solver with a fixed time step of 0.006 s.

The method of evaluation is straightforward. The model consists of various scripts that are related to each other as is indicated in Figure 4. Starting from left there is the input file with all the physical parameters of the car. The second file is the main program that reads the input script and starts different scripts and sub routines. For instance the boundary conditions to solve Equations (7)-(11), and the duration of the simulation is defined here. Also the MATLAB routine ODE45 is called from the main program, to that routine a function defined in the third script is sent. This needs some further explanation: first the slip angles are calculated as it indicated in Figure 4. This is first done with the influence of the elastic frame from the last time step calculated. Then these angles are inserted in the equations for the lateral force and for the jaw moment and used as input for the CALFEM FEA calculation. The outputs from the structural model, the angles θ_f and θ_r, are then used as input in the differential equation (again Equations (7)-(11)).

Finally the output from the ODE for all the time steps is post processed so that the results that are presented in the next section can be obtained.

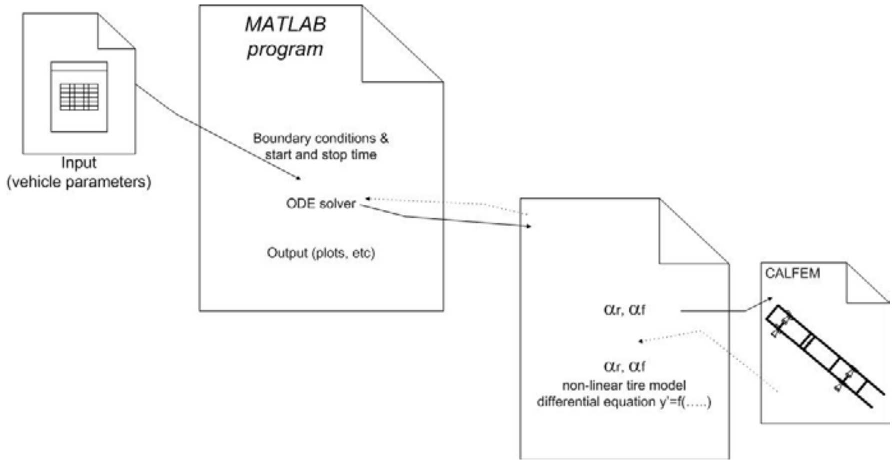

Figure 4 *Data flow of the vehicle dynamics routine in MATLAB*.

9 Results and comparison

The parameters used in this study are presented in Table 1. These values come from commercial specifications from a standard European truck with two axles. In this section several examples will be presented. The first case is for the standard truck as mentioned above. For each graph two versions are presented. As an example in Figure 5, the left plot is for the standard frame just as it is manufactured. In the second plot, the right hand side is an example of the same frame but without one of the traversal beams. That is a demonstration of what could happen if, by a modification for instance, one beam is removed. Also each plot has two results inside each graph. In all the figures there are two curves. The solid line stands for a model without the effect of elasticity and the dotted line is from results that include the effect of elasticity.

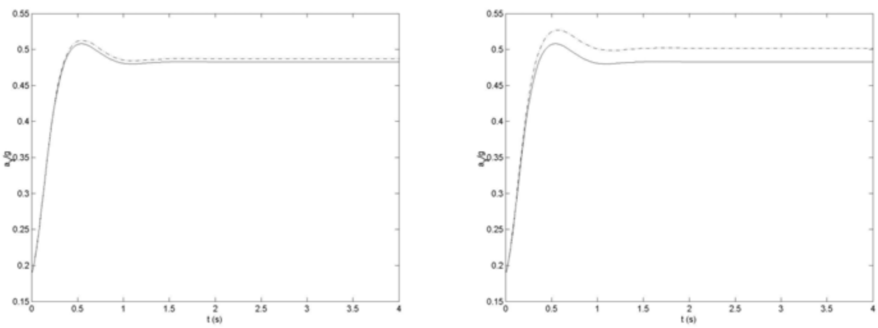

Figure 5 *Comparison between vehicles with different stiffness of the frame: lateral acceleration*.

In the paper by Kuti (2001) it is concluded that a truck with a rigid chassis has a lower peak lateral acceleration than a truck with an elastic chassis. This behaviour can also be observed in the numerical example calculated with the method proposed. In Figure 5 it can be seen that for the standard frame (the left graph) the acceleration is slightly higher, but the results for the weaker frame gives a result where the peak acceleration is noticeably higher. Similar behaviour is observed for the lateral velocity and for the yaw in Figure 6. The elasticity gives a bigger impact for the lateral velocity than for the yaw as can be seen in the plot. Figure 7 has been included as an illustration of the impact the elasticity has on the trajectory of the truck. The impact doesn't seem to be very large, but one has to keep in mind that the time span of the calculation is only three seconds. Also the truck chosen is by nature understeered. The elasticity of the frame makes it less understeered, but it doesn't make the vehicle oversteered, which would be the worst scenario. Therefore a second calculation has been made to show the impact the elasticity would have on a vehicle which is oversteered by nature (the critical speed is 130 km/h.) Here the effect of elasticity gives a very large influence. In Figures 8, 9 and 10 the elastic effect can be observed. In this case and because the vehicle is oversteered the effects are much more severe. This is due to the fact that the elasticity makes the oversteered vehicle even more oversteered. Or, to put it another way, the critical speed is lower.

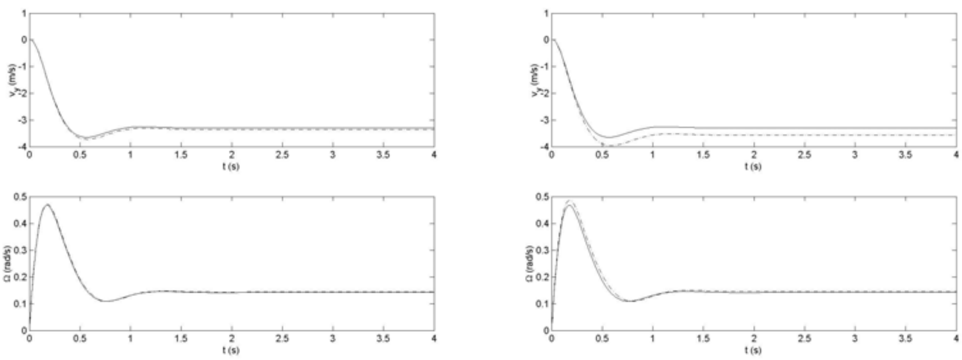

Figure 6 *Comparison between vehicles with different stiffness of the frame: Lateral velocity and yaw.*

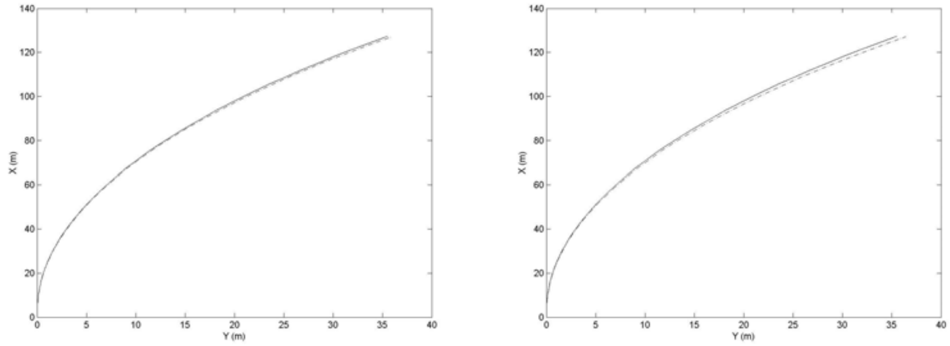

Figure 7 *Comparison between vehicles with different stiffness of the frame: trajectory of the truck.*

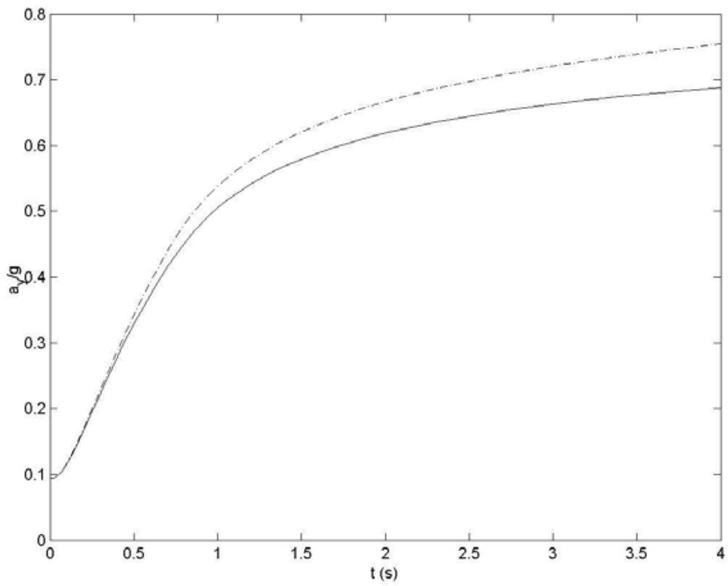

Figure 8 *Oversteered vehicle, lateral acceleration*.

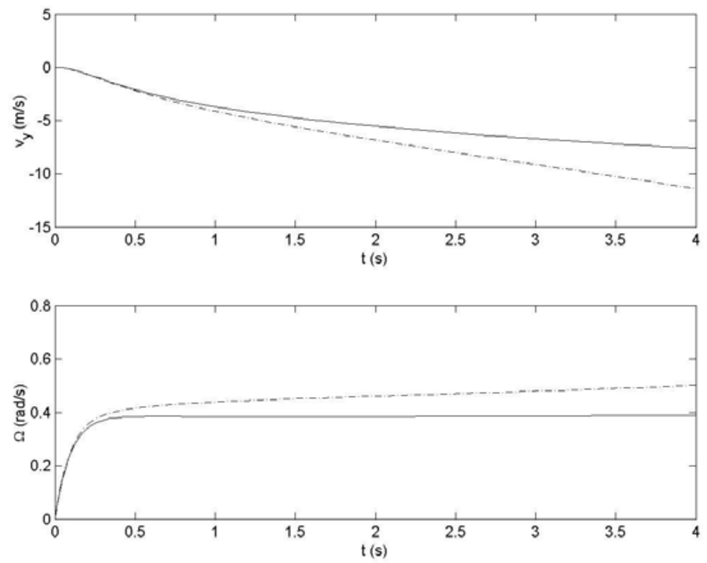

Figure 9 *Oversteered vehicle, lateral velocity and yaw*.

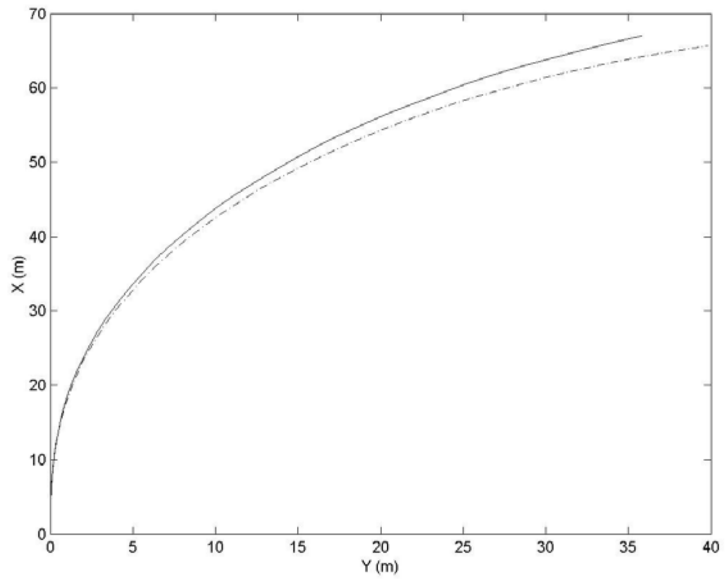

Figure 10 *Oversteered vehicle, trajectory*.

10 Summary

In this paper it has been shown that the bicycle model coupled with a FE analysis is a useful tool to predict the cornering dynamics of a vehicle. Nowadays much effort is made with complicated rigid body simulation programs, such as ADAMS, although the method proposed herein offers an easy way to evaluate the dynamic effects that occur due to handling in the early stages of the design process. It is worth mentioning that the elasticity has an important impact on the steering response of a vehicle, especially in heavy industrial vehicles, such as trucks, due to their design with independent frame. This method is aimed principally at the engineer who modifies second-hand trucks. Supposedly the truck manufacturer has tools and experience to avoid designing too-weak frames in the first place. Nonetheless, on the second-hand market it is very common that the truck and its structure are modified. Changing the bending stiffness of the frame (by removing one of the traverse beams for instance) could then give considerable effects in the directional stability and handling as it is shown in this paper.

References

Ambrósio, J. A. C. and Gonçalves, J. P. C. (2001) 'Complex flexible multibody system with application to vehicle dynamics', *Multibody System Dynamics,* Vol. 6, pp. 163-182.

Bakker, E., Nyborg, L. and Pacejka, H. B. (1987) 'Tyre modeling for use in vehicle dynamics studies', *SAE Paper #870421*.

Dixon, J. C. (1991) *Tyres, Suspension and Handling*. Cambridge University Press, Cambridge.

Kuti, I. (2001) 'Simulation of vehicle motions on the basis of the finite element method', *Vehicle System Dynamics*, Vol. 36, No. 6, pp. 445-469.

Pacejka, H. B. and Besselink, I. J. M (1997) 'Magic formulate tyre model with transient properties', *Vehicle System Dynamics Supplement*, Vol. 27, pp. 234-249.

Genta, G. (1997) *Motor Vehicle Dynamics: Modeling and Simulation*, World Scientific Publishing.

Milliken, W. F. and Milliken, D. L. (1997) *Race Car Vehicle Dynamics*, Society of Automotive Engineers.

Milliken, W. F. and Whitcomb, D. W. (1956) 'General introduction to a programme of dynamic research', *Proceedings of the Automobile Division,* The Institution of Mechanical Engineers, Vol. 7, pp. 287-309.

Segel, L. (1956a) 'Research in the fundamentals of automobile control and stability', SAE National Summer Meeting, Atlantic City, June 5.

Segel, L. (1956b) 'Theoretical prediction and experimental substantiation of the response of the automobile to steering control', *Proceedings of the Automobile Division; The Institution of Mechanical Engineers*, Vol. 7, pp. 310-330.

Tönük, E. and Ünlüsoy, Y. S. (2001) 'Prediction of automobile tire cornering force characteristics by finite element modeling and analysis', *Computers and Structures*, Vol. 79, pp. 1219-1232.

Wennerström, E., Nordmark, S. and Thorvald, B. (1999) *Fordonsdynamik*, KTH, Stockholm, Sweden.

Wideberg, J. P. (2002) 'Dynamic effect of the non-rigid modified bicycle model', *Proc Instn Mech Engrs Part D: J Automobile Engineering,* Vol. 216, pp. 717-722.

Whitcomb, D. W. and Milliken, W. F. (1956) 'Design implications of a general theory of automobile stability and control', *Proceeding of the Automobile Division*, The Institution of Mechanical Engineers, Vol. 7, pp. 367-391.

View publication stats