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Mathematical model of the vehicle in MATLAB Simulink

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Abstract

The article presents the creation mathematical model of the vehicle. Authors used the software MATLAB Simulink for building model. The article also discloses calculation of forces action on the car. Authors considering of the car as a plane-parallel motion solid body. The block diagram of the mathematical model of the vehicle are presented in the article.

Keywords: Mathematical model, vehicle, MATLAB Simulink, plane-prallel motion,block diagram

1. Introduction

Currently designers when they design any complex technical devices should be create mathematical model this devise. Mathematical model is needed for verification and research it. Mathematical model allows reducing costs of design this devise. The vehicle is no exception.

2. Mathematical modeling

Moving of the vehicle considering as plane-parallel motion of the solid body in this article. (fig.1.). This assumption is made to simplify the system of equations describing the motion. The motion of the vehicle can be describe by the following system differential equations:

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$$\begin{cases} \vec{a} = \dfrac{1}{m} \left(\sum_{i=1}^{4} \overrightarrow{P_{fi}} + \sum_{i=1}^{4} \overrightarrow{R_i} + \overrightarrow{P_w} \right) \\ J_z \dfrac{d\omega_z}{dt} = \sum_{i=1}^{4} M_{nki} + \sum_{i=1}^{4} M(\overrightarrow{R_i}) + \sum_{i=1}^{4} M(\overrightarrow{P_{fi}}) \end{cases}, \tag{1}$$

where $\vec{a}$ - the acceleration vector of the mass center of the car; m – vehicle weight; $\overline{P}_{fi}$ - the vector of the force of resistance of straight-ahead of wheel; $\overline{R}_i$ - the vector of force of confusion wheel with the ground; $\overline{P}_w$ - the vector of force of air resistance; J_z – moment of inertia of the vehicle; M_{nki} – the moment of resistance to rotation.
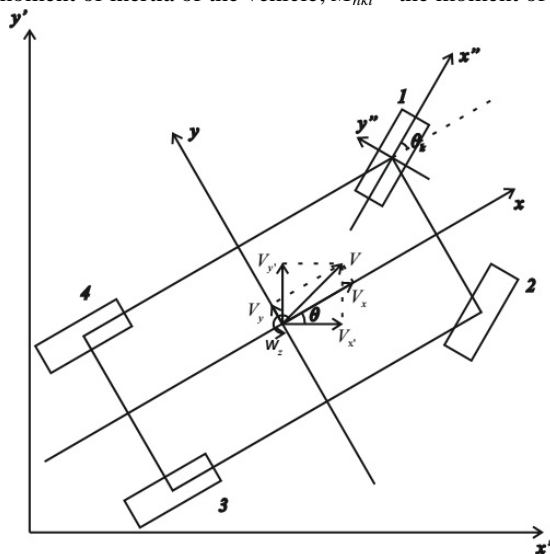

Fig. 1. The design scheme of the car:
x,y,z –. axis moving coordinate system; x',y',z' – axis of stationary coordinate system; θ - the angle of rotation of the vehicle around the axis of the vehicle; V - the velocity vector of the center of the vehicle; ω_z - angular velocity of turning of a vehicle.

The acceleration in plane-parallel moving is

$$\vec{a} = \frac{d\vec{V}}{dt} + \omega_z \vec{V} , \tag{2}$$

where dV/dt - the relative velocity of the center of mass derivative of the vehicle.

For fig. 1 projections of the speeds in coordinate system x',y',z':

$$\begin{cases} V_{x'} = \dfrac{dy'}{dt} = V_x \cos\theta - V_y \sin\theta \\ V_{y'} = \dfrac{dx'}{dt} = V_x \sin\theta + V_y \cos\theta \end{cases}. \tag{3}$$

Considering that

$$\omega_z = d\theta / dt , \tag{4}$$

then combining (1-4) we can write the following system of equations

$$\begin{cases} a_x = \dfrac{dV_x}{dt} - \omega_z V_y = \dfrac{1}{m}(\sum_{i=1}^{4} P_{fxi} + \sum_{i=1}^{4} R_{xi} + P_{wx}) \\[2mm] a_y = \dfrac{dV_y}{dt} + \omega_z V_x = \dfrac{1}{m}(\sum_{i=1}^{4} P_{fyi} + \sum_{i=1}^{4} R_{yi} + P_{wy}) \\[2mm] J_z \dfrac{d\omega_z}{dt} = \sum_{i=1}^{4} M_{nki} + \sum_{i=1}^{4} M(\overrightarrow{R_i}) + \sum_{i=1}^{4} M(\overrightarrow{P_{fi}}) \\[2mm] V_{x'} = \dfrac{dy'}{dt} = V_x \cos\theta - V_y \sin\theta \\[2mm] V_{y'} = \dfrac{dx'}{dt} = V_x \sin\theta + V_y \cos\theta \\[2mm] \omega_z = \dfrac{d\theta}{dt} \end{cases} , \qquad (5)$$

where a_x, a_y – the projection of the acceleration of the center of mass of the vehicle in coordinate axes x,y,z; P_{fxi}, P_{fyi}, R_{xi}, R_{yi}, P_{wx}, P_{wy} – the projection of the forces auctioning on the vehicle in coordinate axes x,y,z.

This system of equations allows determines the position, velocities and acceleration of the vehicle. This system of equations was creating in MATLAB Simulink. The inputs are the calculated values of the projections of the forces. The outputs are position, projections of the vehicle, acceleration, angular velocity, angle of rotation of the vehicle. Then this outputs value are used for calculation of the forces.

One of the wheels are considered for calculation of forces (fig. 2). For calculation forces, we used [2].
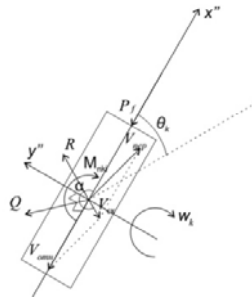

Fig. 2. The design scheme of the car:

The moment and the force of the rolling resistance depend of the properties of the tire and surface of a road. And it proportional to value of normal reactions Q_i. Normal reactions can be find of the system of equations:

$$\begin{cases} Q_1 - Q_2 + Q_3 - Q_4 = 0 \\ Q_1 + Q_2 + Q_3 + Q_4 = mg \\ Q_1 x_1 + Q_2 x_2 + Q_3 x_3 + Q_4 x_4 = -ma_x H_z \\ Q_1 y_1 + Q_2 y_2 + Q_3 y_3 + Q_4 y_4 = -ma_y H_z \end{cases} , \qquad (6)$$

where x_i, y_i – position of wheel in x-y; H_z – height, which is the center of mass of the vehicle.

The first equation is based on the assertion that the ends of the vectors normal reactions lie in one plane, and the second is derived from the condition that the sum of the normal reactions and weight of the car, the third and fourth equating moments.

The vector of slipping velocity $\overrightarrow{V_{sl}}''$ of the bottom point of wheel in the coordinate system x '' - y '' is determined by:

$$\vec{V_{sl}}\,'' = \vec{V_c}\,'' + \vec{V_r}\,'' ,\tag{7}$$

where $\vec{V_c}\,''$ - the vector of the carrying velocity in x''-y''; $\vec{V_r}\,''$ - the vector of relating velocity in x''-y''.

The vector of the carrying velocity:

$$\vec{V_c} = \vec{V} + \vec{\omega} \times \vec{\rho} ,\tag{8}$$

where $\vec{\omega}$ - the vector angular velocity; $\vec{\rho}$ - the radius vector defining the position of the moving coordinate system.

The projection of $\vec{V_c}\,''$, V_{cx} and V_{cy} in x-y:

$$\begin{cases} V_{cx} = V_x - \omega_z y_k \\ V_{cy} = V_y + \omega_z x_k \end{cases}.\tag{9}$$

The projection of $\vec{V_c}\,''$ in x''-y'':

$$\begin{cases} V_{cx}\,'' = (V_x - \omega_z y_k)\cos\theta_k + (V_y + \omega_z x_k)\sin\theta_k \\ V_{cy}\,'' = -(V_x - \omega_z y_k)\sin\theta_k + (V_y + \omega_z x_k)\cos\theta_k \end{cases}.\tag{10}$$

The projection of $\vec{V_r}\,''$ in x''-y'':

$$\begin{cases} V_{rx}\,'' = \omega_k r_k \\ V_{ry}\,'' = 0 \end{cases},\tag{11}$$

where ω_k - angular velocity of wheel; r_k - dynamic radius.

Then projection of $\vec{V_{sl}}\,''$ in x''-y'':

$$\begin{cases} V_{slx}\,'' = (V_x - \omega_z y_k)\cos\theta_k + (V_y + \omega_z x_k)\sin\theta_k - \omega_k r_k \\ V_{sly}\,'' = -(V_x - \omega_z y_k)\sin\theta_k + (V_y + \omega_z x_k)\cos\theta_k \\ V_{sl} = \sqrt{V_{slx}\,''^2 + V_{sly}\,''^2} \end{cases}.\tag{12}$$

The coefficient of sliding of the model "with a rectangular imprint" [3] for calculating the interaction with the wheel bearing surface:

$$S = \frac{V_{sl}}{\omega_k r_k} .\tag{13}$$

The force of interaction of the wheels to the road:

$$R = \mu_s Q ,\tag{14}$$

where μ_s - tire – terrain interaction coefficient

$$\mu_s = \mu_{s\alpha\max}(1 - e^{-\frac{S}{S_0}})(1 + e^{-\frac{S}{S_1}}) ,\tag{15}$$

where S_0 and S_1 - constant parameters of the curve shape; $\mu_{s\alpha max}$ – coefficient of the tire – terrain interaction at complete slip:

$$\mu_{s\alpha\,max} = \frac{\mu_{sx\,max}\,\mu_{sy\,max}}{\sqrt{\mu_{sx\,max}{}^2 \sin^2\alpha + \mu_{sy\,max}{}^2 \cos^2\alpha}}\,, \tag{16}$$

where here μ_{sxmax} and μ_{symax} – friction ellipse parameters (it determined empirically)

$$\begin{cases} \sin\alpha = \dfrac{V_{sly}{}''}{V_{sl}} \\ \cos\alpha = \dfrac{V_{slx}{}''}{V_{sl}} \end{cases}, \tag{17}$$

The projections of the tire – terrain interaction force in the road plane are calculated in the following way:

$$\begin{cases} R_x{}'' = -R\cos\alpha \\ R_y{}'' = -R\sin\alpha \end{cases}, \tag{18}$$

Then it in *x-y:*

$$\begin{cases} R_x = R_x{}''\cos\theta_k - R_y{}''\sin\theta_k \\ R_x = R_x{}''\sin\theta_k + R_y{}''\cos\theta_k \end{cases}, \tag{19}$$

The force of resistance rectilinear motion:

$$P_f = fQ\,, \tag{20}$$

where here f – tire rolling resistance coefficient:
Vector P_f is oppositely to projection $\overline{V_c}{}''$ in *x''*, then P_f in *x-y* definition by:

$$\begin{cases} P_{fx} = -P_f\,\dfrac{V_{cx}}{\left|V_{cx}{}''\right|}\cos\theta_k \\ P_{fy} = -P_f\,\dfrac{V_{cx}}{\left|V_{cx}{}''\right|}\sin\theta_k \end{cases}. \tag{21}$$

Assume that $\vec{P}_w$ is opposite to $\vec{V}$, then:

$$P_w = c_x F q_v\,, \tag{22}$$

where c_x - the coefficient of aerodynamics; F – the frontal area of the vehicle;

$$F = k_F B H\,, \tag{23}$$

where $k_F = 0,25\text{-}0,45$ – the coefficient of frontal shape of a vehicle; B and H – track and height of a vehicle;

$$q_v = \rho_a V^2 / 2\,, \tag{24}$$

where ρ_a - density of air.

Slipping of the footprint is causes of the steering force. With acceptable accuracy for find a moment of resistance to rotation, the following system of equations may be used:

$$
\begin{cases}
M_{nki} = \dfrac{M_{nк\max}}{1+0,15\dfrac{R_{nki}}{b_k}} \\
M_{nк\max} = 0,375\mu_{sa\max}Q\sqrt{\dfrac{\pi l_k b_k}{4}}
\end{cases},
\tag{25}
$$

where $M_{nкmax}$ – moment of resistance for turn wheel while standing still, l_k и b_k – the length and the width of the contact, R_{nki} – the turning circle.

A block diagram of a mathematical calculation of forces and moments acting on the wheel of the car and the car itself based on (6-25) are creating in MATLAB Simulink. Inputs are values of velocity, the angular velocity of rotation of the car, wheel speed, angle of rotation of the wheels. Outputs are values of projection of $\bar{P}_w$, P_f, R, M_{nki}.

Individual drive each wheel allows you to fully realize the advantages of the electric drive. To power the motors of electric transmission converters can be used as described in [6, 7]. This transmission allows individually controlled for each wheel traction characteristics. It allows form the optimal mode of operation, to minimize the movement of the wheels to skid and the slip and create quick to implement active safety systems. This transmission definition by:

$$
\begin{cases}
J_{k1}\dfrac{d\omega_{k1}}{dt} = M_{d1} - M_{r1} \\
J_{k2}\dfrac{d\omega_{k2}}{dt} = M_{d2} - M_{r2} \\
J_{k3}\dfrac{d\omega_{k3}}{dt} = M_{d3} - M_{r3} \\
J_{k4}\dfrac{d\omega_{k4}}{dt} = M_{d4} - M_{r4}
\end{cases},
\tag{26}
$$

where J_{ki} - moment of inertia of the i-th wheel-motor; M_{di} – traction moment of i-th wheel-motor, M_{ri} – the moment of resistance i-th wheel.

On the fig. 3 we presented block diagram of the mathematical model of vehicle.
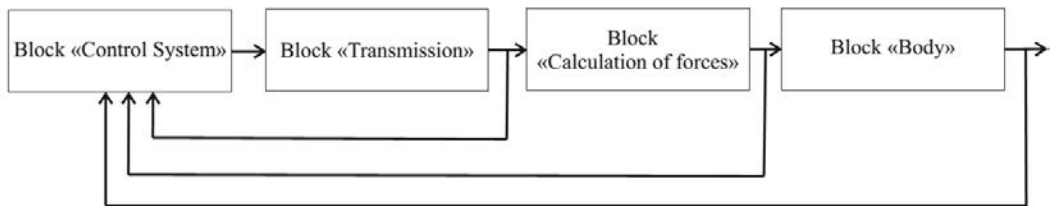

Fig. 3. Block diagram of the mathematical model of vehicle

The block «Control System» is the control system. It form control of traction moment of each wheel and form control of active safety system. The block «Transmission» is model of electric transmission with individual electric drive of each wheel (26). The block «Calculation of forces» make calculation of forces (6-25). And block «Body» is model of moving vehicle as plane-parallel moving solid body (5).

3. Conclusion

The developed mathematical model can synthesize the system of direct torque control on each wheel, depending on the strength of the interaction of wheel support surface, active safety systems and test their performance.

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