THE1 practical - probability : L16-L18.

1. Two fair dice are rolled in sequence and their upfacing numbers are recorded. Determine the probabilities of the following events and use random variable (rv) notation to describe the relevant events. Let the outcome of the first roll be the rv X and the outcome of the second roll be the rv Y.

   (a) The first shows a two and the second shows a six.

   (b) A three and a four are recorded.

   (c) The sum on the dice is greater than five.

   (d) One of the dice shows a number greater than 3.

   (e) The first die rolled shows greater than three or the second die shows lower than five.

   (f) The sum on the dice is greater than five, given that the first die shows the number three.

   (g) The sum on the dice is greater than eight, given that at least one die shows the number four or above.

2. In a school, 40% of the students study both Maths and Physics. 60% of the students study Maths. What is the probability of a student studying Physics given she/he is known to be studying Maths?

3. Two equally matched tennis players, player A and player B, play twelve games in a tennis match. At the end of each game, the player that is serving changes. Suppose initially that serving does not affect the outcome of each game.

   (a) What is the most probable score (number of games each) at the end of the match?

   (b) What is the probability of the most probable score?

   (c) What is the probability of player A winning the match?

   (d) (*) Now suppose that, when serving, a player has double the probability of winning the game compared to their opponent. Repeat parts (a) to (c) above.

   (e) In the limiting case, suppose that a server always wins the game. Repeat parts (a) to (c) above.

   (*) This is challenging and may be skipped if you are short of time.)

4. A fair coin is flipped ten times.

   (a) What is the probability of getting six heads?

   (b) Suppose that someone has learnt to flip the coin in a biased way such that they can attain a head with 80% success. If they try and achieve a head on each of the ten flips, what is the probability of getting six heads?

   (c) For each of questions (a) and (b) above, what is the most likely outcome of the ten flips? (Note: the mode of a binomial distribution is given by $\lfloor (n+1)p \rfloor$ if $(n+1)p$ is a not an integer, where n is the number of Bernouilli trials and p is the probability of success.)

5. A robbery is committed using a getaway car at dusk. A witness states that he is quite sure that he saw a green car driving rapidly away from the crime scene. Suppose that it has been shown that people can distinguish green and blue with an 80% reliability at dusk at the given observation distance. Assume that there are only blue and green cars in the city and 90% of them are blue. Is the witness statement most likely to be strong ecvidence in any court case? Explain your reasoning by using the technical terms for the individual component terms used in Bayes' theorem (i.e. prior probability, posteriori probability and liklihood).

6. Three companies A, B and C supply 15%, 35% and 50% of the pens to a University respectively. Past experience shows that 5%, 4% and 3% of the pens produced by these companies are defective, respectively for companies A, B and C. If a pen was found to be defective, determine the probabilities that it was supplied by each of the companies A, B and C. What is a good check for the correctness your three answers that is a necessary condition? Perform this check on your answers.

7. X is a random variable that can take the values -6, -3, 0, 3, 6 with probabilities 0.2, 0.1, 0.1, 0.3, and 0.3 respectively. Calculate the expected value and the variance of this random variable.

8. Let X and Y be discrete random variables with a joint distribution defined by the probabilities $\{\frac{1}{4}, \frac{1}{8}, \frac{1}{2}.\frac{1}{8}\}$ at the corners of the unit square $(0,0), (0,1), (1,1), (1,0)$ respectively. (a) Calculate the marginal distributions of each random variable and their expected values. (b) Calculate the variance of each variable and the covariance of their joint variablity. (c) Calculate the correlation between the two variables. (d) Comment on your answers for the two rvs and on their correlation.