

## Lab 10

For this lab, you will implement a dynamic programming algorithm to estimate the state of the coin (Fair or Biased) for the fair bet casino. This will require two arrays (lists),  $VF_i$  and  $VB_i$ , where  $VF_i$  is the maximum probability of the path up to flip  $i$  (1-indexed) ends in state F. Likewise,  $VB_i$  is the maximum probability of the path up to flip  $i$  (1-indexed) ends in state B. The main recursion is:

$$VF_i = P_{x_i} \max \begin{cases} 0.9 VF_{i-1} \\ 0.1 VB_{i-1} \end{cases}$$

$$VB_i = P_{x_i} \max \begin{cases} 0.9 VB_{i-1} \\ 0.1 VF_{i-1} \end{cases}$$

where  $P_{x_i}$  is the probability of getting coin flip result  $x_i$ . Initialize  $VF_0 = VB_0 = 0.5$ .

Here is the pseudocode (where strings are 1-indexed):

Provide string  $x$ , the sequence of H and T for heads and tails and output string  $pi$ , the sequence of coins being used for the maximum probability path.

FairCasino( $x$ )

Make arrays  $VF$  and  $VB$  for  $\text{length}(x)+1$  states, 0-indexed

$VF_0 \leftarrow 0.5$

$VB_0 \leftarrow 0.5$

for  $i=0$  to  $\text{length}(x)$

if ( $x_i = H$ )

$P_x \leftarrow 0.75$

else

$P_x \leftarrow 0.25$

$VF_i \leftarrow 0.5 \times \max(TP \times VF_{i-1}, (1-TP) \times VB_{i-1})$

$VB_i \leftarrow P_x \times \max(TP \times VB_{i-1}, (1-TP) \times VF_{i-1})$

Traceback the path through  $pi$  (F or B) that gives the maximum probability.

One goal this week is for you to develop the traceback strategy on your own. However, the pseudocode with the traceback strategy will become available on blackboard at 4:15.

Run the algorithm with the sequence:

$x =$  HHHHTTTTHHHHHHTHTHTHTHHHHHTTTTTHHTHHT

Try also changing  $TP$  to see how the results change. (Try 0.5, for example.)