Noslēguma darbs

Kristaps Rubuls

04.03.1019.

Paraugs:

spectrum of the system is continuous. This system has discrete negative and continuous positive eigenvalues for the operator corresponding to the total energy (the Hamiltonian).

▶*Using the Dirac notation, show that the eigenvalues of an Hermitian operator are real.*

Let $|a\rangle$ be an eigenstate of Hermitian operator A corresponding to eigenvalue a, then

$$\begin{aligned} A|a\rangle &= a|a\rangle, \\ \Rightarrow \quad \langle a|A|a\rangle &= \langle a|a|a\rangle = a\langle a|a\rangle, \\ &\text{and} \\ \langle a|A^\dagger &= a^*\langle a|, \\ \Rightarrow \quad \langle a|A^\dagger|a\rangle &= a^*\langle a|a\rangle, \\ \langle a|A|a\rangle &= a^*\langle a|a\rangle, \quad \text{since } A \text{ is Hermitian.} \end{aligned}$$

Hence,

$$\begin{aligned} (a - a^*)\langle a|a\rangle &= 0, \\ \Rightarrow \quad a &= a^*, \text{ since } \langle a|a\rangle \neq 0. \end{aligned}$$

Thus a is real. ◀

It is not our intention to describe the complete axiomatic basis of quantum mechanics, but rather to show what can be learned about linear operators, and in particular about their eigenvalues, without recourse to explicit wavefunctions on which the operators act.

Before we proceed to do that, we close this subsection with a number of results, expressed in Dirac notation, that the reader should verify by inspection or by following the lines of argument sketched in the statements. Where a sum over a complete set of eigenvalues is shown, it should be replaced by an integral for those parts of the eigenvalue spectrum that are continuous. With the notation that $|a_n\rangle$ is an eigenstate of Hermitian operator A with non-degenerate eigenvalue a_n (or, if a_n is k-fold degenerate, then a set of k mutually orthogonal eigenstates has been constructed and the states relabelled), we have the following results.

$$A|a_n\rangle = a_n|a_n\rangle,$$

$$\langle a_m|a_n\rangle = \delta_{mn} \quad \text{(orthonormality of eigenstates)}, \tag{19.5}$$

$$A(c_n|a_n\rangle + c_m|a_m\rangle) = c_n a_n|a_n\rangle + c_m a_m|a_m\rangle \quad \text{(linearity)}. \tag{19.6}$$

The definitions of the sum and product of two operators are

$$(A + B)|\psi\rangle \equiv A|\psi\rangle + B|\psi\rangle, \tag{19.7}$$

$$AB|\psi\rangle \equiv A(B|\psi\rangle) \quad (\neq BA|\psi\rangle \text{ in general}), \tag{19.8}$$

$$\Rightarrow \quad A^p|a_n\rangle = a_n^p|a_n\rangle. \tag{19.9}$$

spectrum of the system is continuous. This system has discrete negative and continuous positive eigenvalues for the operator corresponding to the total energy (the Hamiltonian).

► *Using the Dirac notation, show that the eigenvalues of an Hermitian operator are real.*

Let $|a\rangle$ be an eigenstate of Hermitian operator $\boldsymbol{A}$ corresponding to eigenvalue a, then

$$\boldsymbol{A}|a\rangle = a|a\rangle,$$

$$\Rightarrow \langle a|\boldsymbol{A}|a\rangle = \langle a|a|a\rangle = a\langle a|a\rangle,$$

and

$$\langle a|\boldsymbol{A}^\dagger = a^*\langle a|,$$

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Hence,

$$(a - a^*)\langle a|a\rangle = 0,$$

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Thus a is real. ◄

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Kods:

```
\documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\usepackage{amsmath,amssymb,latexsym}
\usepackage{verbatimbox}
\usepackage[table]{xcolor}
\graphicspath{ {/home/user/images/} }

\usepackage[a4paper, total={5.5in, 8.818898in}]{geometry}

\title{Noslēguma darbs}
\author{Kristaps Rubuls}
\date{17.02.1019.}

\begin{document}
\maketitle


\includegraphics[scale=0.12]{IMG_0690.jpg}

\clearpage
\setcounter{page}{651}
\begin{center}
\textbf{19.1 OPERATOR FORMALISM}\vspace{0.1cm}
\end{center}
\hline
\vspace{0.7cm}
{\baselineskip = 2.0\baselineskip
spectrum of the system is continuous.
This system has discrete negative and
continuous positive eigenvalues for
the operator corresponding to the total energy (the Hamiltonian).}\hfill\break
\vspace{0.2cm}\hfill\break
\begin{tabular}{ |l| }
\hline
    \cellcolor{gray!25}$\blacktriangleright$
    \textit{Using the Dirac notation,
    show that the eigenvalues of an Hermitian operator are real.}\\
\hline
\end{tabular}\vspace{0.1cm}\hfill\break
Let $|a\rangle$ be an eigenstate of
Hermitian operator $\textbf{\textit{A}}$ corresponding
to eigenvalue $a$, then\vspace{0.2cm}
\begin{center}
    $\textbf{\textit{A}}|a\rangle = a|a\rangle,$
\end{center}
\begin{center}
    $\Rightarrow \langle a|\textbf
    {\textit{A}}|a\rangle = \langle a|a|a\rangle =
    a\langle a|a\rangle,$
\end{center}
```

```
\begin{center}
    and
\end{center}
\begin{center}
    $\langle a|\textbf{\textit{A}}^
    \dagger = a^* \langle a|,$
\end{center}
\begin{center}
   $\Rightarrow \langle a|\textbf{\textit{A}}^
   \dagger|a\rangle = a^*\langle a|a\rangle,$
\end{center}
\begin{equation*}
\langle a|\textbf{\textit{A}}
|a\rangle = a^*\langle a|a\rangle,
\quad\textnormal{since \textbf{\textit{A}} is Hermitian}
\end{equation*}
Hence,
\begin{center}
    $(a-a^*)\langle a|a\rangle = 0,$
\end{center}
\begin{equation*}
\Rightarrow  a = a^*,\quad\
textnormal{since $\langle a|a\rangle \neq 0$}
\end{equation*}
Thus $a$ is real. $\blacktriangleleft$\hfill\break

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Where a sum over a complete set
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relabelled), we have the following results.
\vspace{0.5cm}
\begin{center}
    $\textbf{\textit{A}}|a_n\rangle = a_n|a_n\rangle,$
\end{center}
\begin{equation}
    \langle a_m|a_n\rangle = |
    \delta_{mn}\quad\textnormal
    {(orthonormality of eigenstates),}
    \tag{19.5}\label{eq:19.5}
\end{equation}
\begin{equation}
    \textbf{\textit{A}}(c_n|a_n\rangle +
    c_m|a_m\rangle)=c_n a_n|a_n\rangle
    + c_m a_m|a_m\rangle\textnormal\quad
    {(linearity).}\tag{19.6}\label
    {eq:19.6}\vspace{0.4cm}
\end{equation}\vspace{0.2cm}
The definitions of the sum and product of two operators are
\begin{equation}
    \textbf{\textit{(A+B)}}|\varphi)\equiv \textbf{\textit{A}}|\varphi)+\textbf
    {\textit{B}}|\varphi),\tag{19.7}\label{eq:19.7}
\end{equation}
\begin{equation}
    \textbf{\textit{AB}}|
    \varphi\rangle\equiv \textbf{\textit{A}}
    (\textbf{\textit{B}}|
    \varphi\rangle)\quad
    (\neq \textbf{\textit{BA}}|
    \varphi\rangle\textnormal { in general)},
    \tag{19.8}\label{eq:19.8}\vspace{0.1cm}
\end{equation}
\begin{equation}
    \Rightarrow \textbf{\textit{A}}^p|a_n\rangle = a_{n}^p|a_n\rangle
    \tag{19.9}\label{eq:19.9}
\end{equation}
\clearpage
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