Principal and Interest

- Principal: amount invested (W)
- Interest: rent paid on the principal (I)
- Interest rate: interest per unit currency invested (r)

$$I = Wr$$

Given a yearly interest r the account holdings in a year will be:

$$\begin{aligned} W_0 &= W \\ W_1 &= W_0 + I = W_0 + W_0 r \\ W_1 &= W_0(1 + r) \end{aligned}$$

Compound Interest

Given a varying interest r_k, for $k = 1, 2, \cdots, n$, after n years the account holdings will be

$$W_n = W \prod_{i=1}^{n} (1 + r_i)$$

For a fixed interest r the account holdings after n years are:

$$W_n = W(1 + r)^n$$

Compounding at different periods

Usually, interest is quoted in an yearly basis, but in reality it is added on a smaller ***compounding period***.

They year is divided in m compounding periods, and interest needs to be calculated based on them.

Nominal rate: r (yearly) Length of a compounding period: $\frac{1}{m}$ Interest rate for each of the compounding period: $\frac{r}{m}$ Growth of the account over k periods: $(1 + r/m)^k$ Growth of the account over 1 year: $(1 + r/m)^m$

In reality the effective interest rate r_{eff} is higher than just applying the interest rate once at the end of the year as compounding would take effect m times:

$$r_{eff} = (1 + r/m)^m$$

Continuous Compounding

Increasing the compounding periods to infinity, gives the notion for a continuous compounding. In fact the limit converges like so:

$$\lim_{m \to \infty} (1 + r^{1/k})^m = e^r$$

Introducing t to denote the number of years of compounding:

$$\lim_{m \to \infty} (1 + r^{1/k})^{mt} = e^{rt}$$