Present Value

Given: - An interest rate r - Over m compounding periods per year

The following are equivalent: 1. Receive A after k compounding period 2. Receive $d_k A$ now

Where d_k denotes the ***discount factor***

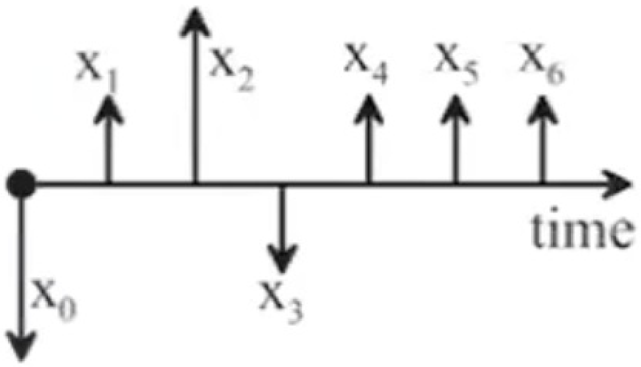
$$d_k = \frac{1}{(1 + r/m)^k}$$

Ideal Bank 1. An ***ideal bank***: - Gives the same interest for both *deposits* and *loans* - Has no service charge or fees - Gives the same interest no matter the size of the principal

2. If an ideal bank has an ***interest value*** which does not change over the period of time, it is called a ==***constant ideal bank***==

Ex: In practice a bank might give an interest of r for a 1-year *Certificate Deposit*, but would give a higher interest q, where $q > r$ for a 2-year period.

Cash Streams (Cash Flows)



Pasted image 20231019093238.png

Given a stream where $x_0, x_1, \cdots, x_n$ are the returns at some period and (for example) a present time step at x_0 and future time step at x_6

We can find a relation between present value PV and future value FV of the given stream.

$$ \begin{align} FV&=\sum\limits_{k=0}^nx_k\times(1+r/m)^{n-k} \tag{1. Compounding} \\\\ PV&=\sum\limits_{k=0}^nx_k\times(1+r/m)^{-k} \tag{2. Discounting} \end{align} $$ - Notice that in (1) the power is reversed ($n - k$). This is because each cash value occurs in a given period and then compounding is applied to it until the end period k. E.g. the value x_0 occurs in time period 0 and gets compounded at all n compounding periods. The value x_1 occurs 1 period later and receives interest only $n - 1$ times. - Conversely, in (2) the first

$2 * 1.1^3 + 2 * 1.1^2 + 2 * 1.1 = 7.282$