

## Present Value

Given: - An interest rate  $r$  - Over  $m$  compounding periods per year

The following are equivalent: 1. Receive  $A$  after  $k$  compounding period 2. Receive  $d_k A$  now

Where  $d_k$  denotes the **discount factor**

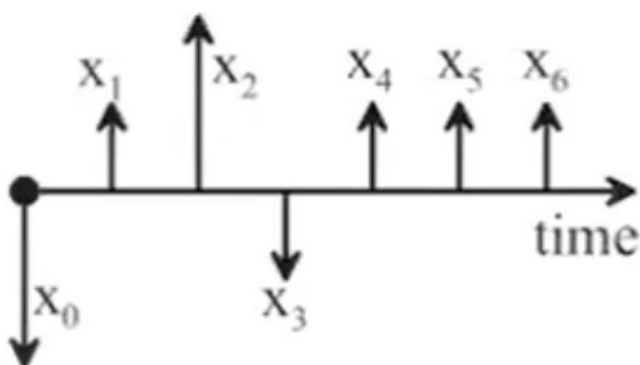
$$d_k = \frac{1}{(1 + r/m)^k}$$

### Ideal Bank 1. An **ideal bank**: - Gives the same interest for both *deposits* and *loans* - Has no service charge or fees  
- Gives the same interest no matter the size of the principal

2. If an ideal bank has an **interest value** which does not change over the period of time, it is called a **constant ideal bank**

Ex: In practice a bank might give an interest of  $r$  for a 1-year *Certificate Deposit*, but would give a higher interest  $q$ , where  $q > r$  for a 2-year period.

## Cash Streams (Cash Flows)



Pasted image 20231019093238.png

Given a stream where  $x_0, x_1, \dots, x_n$  are the returns at some period and (for example) a present time step at  $x_0$  and future time step at  $x_6$

We can find a relation between present value  $PV$  and future value  $FV$  of the given stream.

$$FV = \sum_{k=0}^n x_k (1 + r/m)^{n-k} \quad \text{\tag{1. Compounding}}$$

$$PV = \sum_{k=0}^n x_k (1 + r/m)^{-k} \quad \text{\tag{2. Discounting}}$$
 - Notice that in (1) the power is reversed ( $n - k$ ). This is because each cash value occurs in a given period and then compounding is applied to it until the end period  $k$ . E.g. the value  $x_0$  occurs in time period 0 and gets compounded at all  $n$  compounding periods. The value  $x_1$  occurs 1 period later and receives interest only  $n - 1$  times. - Conversely, in (2) the first

$$2 * 1.1^3 + 2 * 1.1^2 + 2 * 1.1 = 7.282$$