Geometric Series up to n

$$\begin{aligned}
S_n &= \sum_{i=0}^{n-1} a^i x \\
S_n &= a^0 x + a^1 x + a^2 x + \cdots + a^{n-1} x \\
aS_n &= a^1 x + a^2 x + a^3 x + \cdots + a^n x \\
S_n - aS_n &= x - a^n x \\
S_n - aS_n &= x(1 - a^n) \\
S_n(1 - a) &= x(1 - a^n) \\
S_n &= x\frac{(1 - a^n)}{1 - a}
\end{aligned}$$

Geometric series up to ∞ Using the proof for S_n we can calculate the limit of S when n approaches ∞

If $a < 1$ then the limit converges and we can find a definite value. Otherwise the limit diverges to negative infinity

$$\begin{aligned}
\lim_{n \to \infty} x\frac{1 - a^n}{1 - a} &= \frac{x}{1 - a}, \quad & a < 1 \\
\lim_{n \to \infty} x\frac{1 - a^n}{1 - a} &= 0 & a = 1 \\
\lim_{n \to \infty} x\frac{1 - a^n}{1 - a} &= \infty, & a > 1
\end{aligned}$$