Duration of a Portfolio

It is simply an average of the durations of each investment weighted by the price of the investment.

$$\begin{aligned}\mathbf{P} &= P_1 + P_2 + P_3 + \cdots + P_n \\ \mathbf{D} &= \frac{P_1D_1 + P_2D_2 + P_3D_3 + \cdots + P_nD_n}{\mathbf{P}}\end{aligned}$$

Investment Risks

- You have an obligation to pay £1,000 in 2 years, b
  you can only buy bonds of maturities 1 or 5 years.
  - Buying 1 year bonds: you face reinvestment risk
    as you do not know the bond prices in 1 year.
  - Buying 5 year bonds: you may fail to meet your
    obligation when interest rates change, i.e., you
    may not be able to sell the bond after 1 year at t
    desired price.

The two bonds impose 2 types of risks respectively: 1. Reinvestment Risk (***Interest rates go down***) - Reinvest the principal at a lower interest rate, which means the investment will now generate less income than before. This is a concern if a certain amount of income is needed to match future liabilities. 2. Market Risk (***Bond prices go down***) - This is the risk of bond prices falling due to rising interest rates. If you need to sell a bond before it matures (as you might in this scenario after 1 year if you initially purchase a 5-year bond), you might receive less than the bond's face value if interest rates have risen, because newer issues offer higher yields and make your lower-yielding bond less attractive.

Interest rates go up

PVs of bonds go down

1. The 1-year bond will mature, and the principal will be reinvested at lower price and a higher yield which means more profit (↑)
2. The 5-year bond will go down in price therefore a premature sell will result in less profit (↓)

Interest rates go down

PVs of bonds go up (both because present values are discounted less and because bonds are more desirable)

1. The 1-year bond will mature, and the principal will be reinvested at a higher price with a lower yield ↓
2. The 5-year bond will go up in price so selling prematurely results in more profit ↑

Immunization

- You have an obligation to pay £1,000 in 2 years, b
  you can only buy bonds of maturities 1 or 5 years.
  - Buying 1 year bonds: you face reinvestment risk
    as you do not know the bond prices in 1 year.
  - Buying 5 year bonds: you may fail to meet your
    obligation when interest rates change, i.e., you
    may not be able to sell the bond after 1 year at t
    desired price.

Striking a balance between the two bonds would ensure that fluctuations in the interest would not affect the final price of the portfolio. I.e. the portfolio is ***immunised*** against investment risk.

Construct a portfolio that matches the liability price of 1000 and duration of 2 years

$$\begin{aligned}\mathbf{P} &= x_1P_1 + x_2P_2 \\ \mathbf{D} &= \frac{x_1P_1D_1}{\mathbf{P}} + \frac{x_2P_2D_2}{\mathbf{P}}\end{aligned}$$

And just solve for x_1 and x_2

In order to match the liability even better convexity or even more terms can be accounted for. #### Duration Notice how the duration is used and not the actual maturities. This is because duration takes into account the time-value of money and creates a more accurate model.