Hydrodynamical simulations of circumstellar discs
master thesis defense

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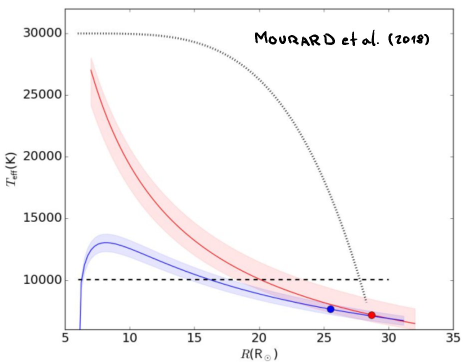


MOTIVATION: β LYRAE A

Mass transferring binary with an accretion disc around the gainer. Two B-type stars. Less massive donor.[1] [2]

Table. Adopted parameters of the system[3]

Parameter	Unit	Value
R_g	$R_\odot$	5.987
T_g	K	30000
M_g	$M_\odot$	13.048
M_d	$M_\odot$	2.910
q	1	0.223
$\dot{M}$	$M_\odot \mathrm{yr}^{-1}$	$2 \cdot 10^{-5}$
P	days	12.9440

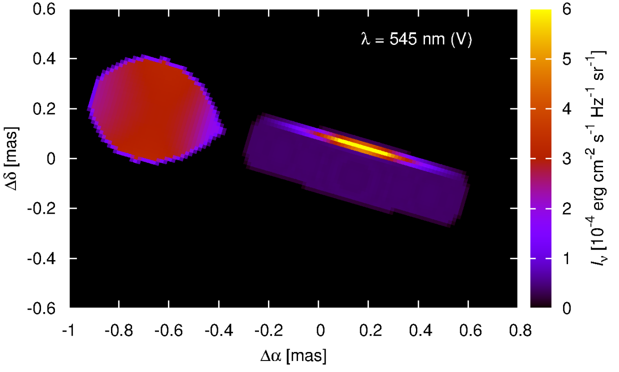


Figure. β Lyrae synthetic image in the V band [4]

[1] H. Ak et al. (Feb. 2007). "New findings supporting the presence of a thick disc and bipolar jets in the β Lyrae system". In: *A&A* 463.1, pp. 233–241. DOI: 10.1051/0004-6361:20065536

[2] R. E. Mennickent and G. Djurašević (Apr. 2013). "On the accretion disc and evolutionary stage of β Lyrae". In: *Monthly Notices of the Royal Astronomical Society* 432.1, pp. 799–809

[3] M. Brož et al. (Jan. 2021). "Optically thin circumstellar medium in the β Lyr A system". In: *A&A* 645, A51, A51. DOI: 10.1051/0004-6361/202039035. arXiv: 2010.05541 [astro-ph.SR]

[4] D. Mourard et al. (Oct. 2018). "Physical properties of β Lyrae A and its opaque accretion disk". In: *A&A* 618, A112, A112. DOI: 10.1051/0004-6361/201832952. arXiv: 1807.04789 [astro-ph.SR]

GENERAL OVERVIEW

Aim of the thesis: A dynamical study of the circumstellar accretion disc.

Two-step approach:

1. A modification of the analytical Shakura–Sunayev models.
2. Radiation-hydrodynamical numerical simulations with the FARGO_THORIN code.

Observational constraints

- **Kinematic model:** Mourard et al.(2018)[5], Brož et al.(2021)[6] fitted a kinematic model to spectroscopic, light-curve, spectral energy distribution, interferometric (VEGA/CHARA) and differential interferometric data.
  - The temperature profile best constrained in the outer part.
  - $T_{\text{innb}} \approx T_\star$
  - Converged values for the extent of the disc: $R_{\text{innb}} = 5.987 R_\odot$ and $R_{\text{outnb}} = 31.5 R_\odot$.
  - $\left(\frac{H}{r}\right)_{\text{kin}} \approx 0.08$ (3.8 times the hydrostatic eqilibrium value).
  - Σ_{kin} is a lower limit.
- **Main dynamical constraint:** Observed $\dot{P} = 19\ \frac{\text{s}}{\text{yr}} \implies \dot{M} = 2 \cdot 10^{-5}\ \text{M}_\odot \text{yr}^{-1}$

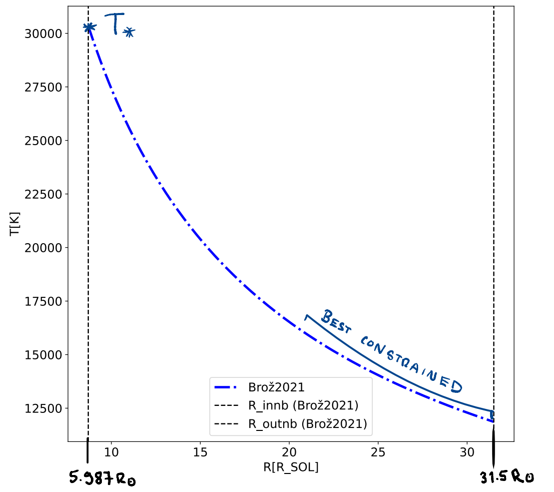


Figure. Temperature profile of the kinematic model.

[5] D. Mourard et al. (Oct. 2018). "Physical properties of β Lyrae A and its opaque accretion disk". In: *A&A* 618, A112, A112. DOI: 10.1051/0004-6361/201832952. arXiv: 1807.04789 [astro-ph.SR]

[6] M. Brož et al. (Jan. 2021). "Optically thin circumstellar medium in the β Lyr A system". In: *A&A* 645, A51, A51. DOI: 10.1051/0004-6361/202039035. arXiv: 2010.05541 [astro-ph.SR]

Section 1

Analytical models

Modified Shakura–Sunayev models

Main assumptions and equations:

- Steady-state Keplerian axially symmetric thin disc in a vertically integrated approximation.
- Parameterized kinematic viscosity[7] [8]:

$$\nu = \alpha c_s H \ . \tag{1}$$

- The optically thick approximation to the grey atmosphere[9].

$$\tau_{\rm eff} = \frac{3}{8}\tau_{\rm opt}, \qquad \tau_{\rm opt} = \frac{C_k}{2}\kappa\Sigma \ . \tag{2}$$

$C_k = 0.6$ accounts for a change in the opacity above the midplane.

- Energy balance: $Q_{\rm visc} = Q_{\rm vert}$

$$Q_{\rm vert} = \frac{2\sigma_{\rm B}T^4}{\tau_{\rm eff}}, \ Q_{\rm visc} = \frac{3GM\dot{M}}{8\pi R^3}\left(1 - \sqrt{\frac{R_\star}{R}}\right) . \tag{3}$$

- Eq. of state;

$$P = \rho\frac{k_B T}{\mu m_p} + \frac{4\sigma_B}{3c}T^4 . \tag{4}$$

- A general opacity approximation (one type of opacity dominates):

$$\kappa = \kappa_0 \rho^A T^B . \tag{5}$$

Three types of models:

1. $P_{\rm g} \gg P_{\rm r}$
2. $P_{\rm g} \approx P_{\rm r}$
3. $P_{\rm g} \ll P_{\rm r}$

For each assumption about the pressure we get for hydrodynamical quantities profiles:

$$q = q(R; \dot{M}, M_\star, \alpha, A, B, \kappa_0) \tag{6}$$

[7] N. I. Shakura and R. A. Sunyaev (Jan. 1973). "Black holes in binary systems. Observational appearance.". In: *A&A* 24, pp. 337–355

[8] J. E. Pringle (Jan. 1981). "Accretion discs in astrophysics". In: *ARA&A* 19, pp. 137–162. DOI: 10.1146/annurev.aa.19.090181.001033

[9] I. Hubeny (Mar. 1990). "Vertical Structure of Accretion Disks: A Simplified Analytical Model". In: *ApJ* 351, p. 632. DOI: 10.1086/168501

Rejection of the $P_g \ll P_r$ and $P_g \approx P_r$ models

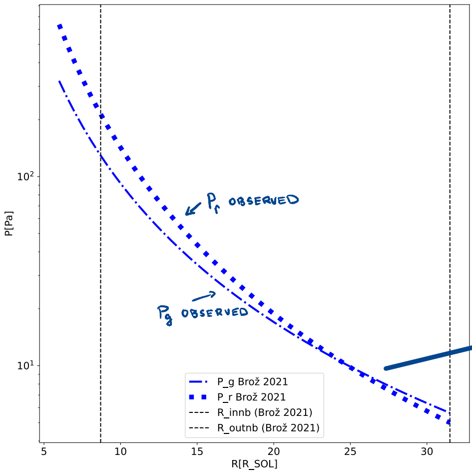


Figure. Top: Pressure profiles calculated from the kinematic model.

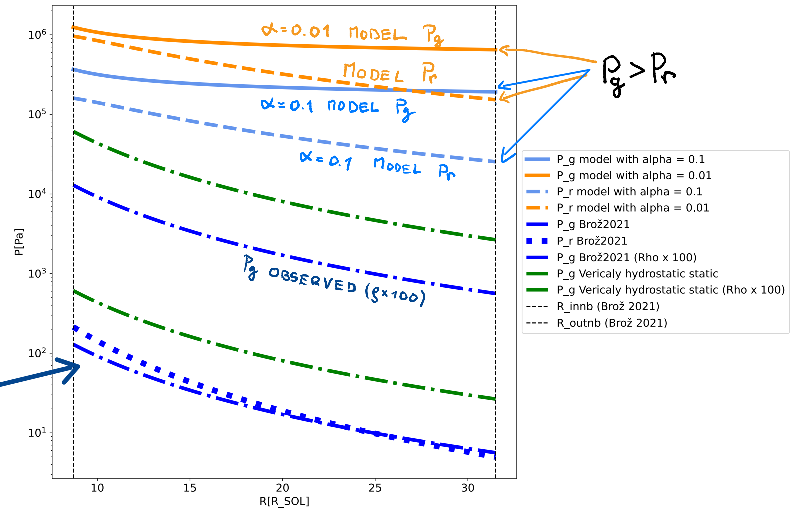


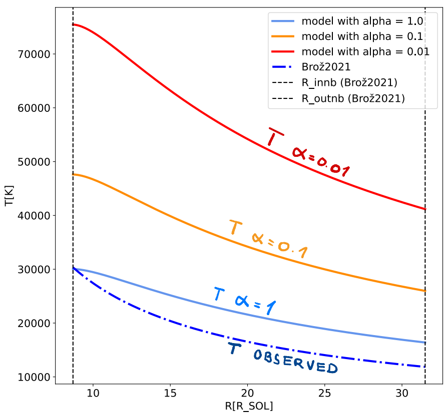
Figure. Pressure profiles computed for $P_g \ll P_r$; $\kappa = 10^{7.67} \rho^{0.72} T^{-0.1}$. The opacity was obtained as a fit to the 2D opacity function by Rogers and Iglesias(1992)[10]

[10] Forrest J. Rogers and Carlos A. Iglesias (Dec. 1992). "Rosseland Mean Opacities for Variable Compositions". In: *ApJ* 401, p. 361. DOI: 10.1086/172066

Gas pressure dominated models $P_g \gg P_r$

- Profiles are **consistent** with assumptions. Best results for **Krammer's opacity**: $\kappa = 10^{24} \cdot \rho T^{-\frac{7}{2}}$

$$T = T_\star \alpha^{-\frac{2A+2}{D}} \dot{M}^{\frac{2A+4}{D}} M^{\frac{2A+3}{D}} R^{-\frac{6A+9}{D}} \left(1 - \sqrt{\frac{R_\star}{R}}\right)^{\frac{2A+4}{D}} \quad \text{where } D = 3A - 2B + 10 \tag{7}$$



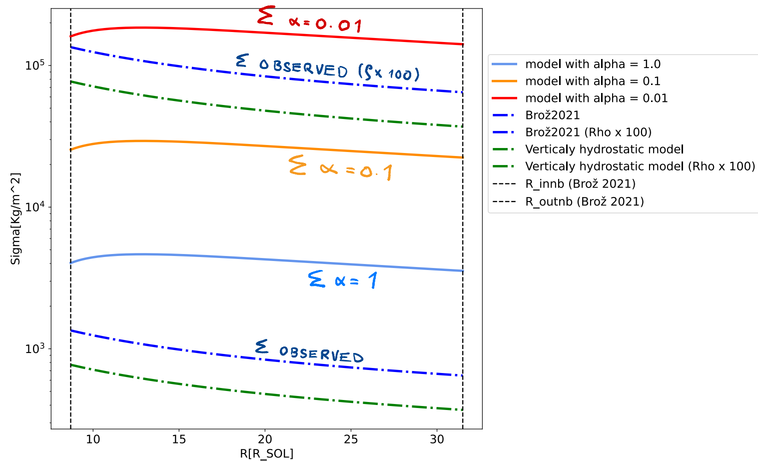


Figure. Left is the temperature profile and right is the Σ profile for the gas pressure dominated model with Krammer's opacity.

Section 2

Numerical 1D RHD modeling with the FARGO_THORIN code

Physics of the numerical model

- Vertically integrated hydrodynamical equations[11] [12]:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \boldsymbol{v}) = 0 \,, \tag{8}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\frac{1}{\Sigma}\nabla P + \frac{1}{\Sigma}\nabla \cdot \boldsymbol{\pi} - \frac{\int \rho \nabla \Phi \mathrm{d}z}{\Sigma} \,, \tag{9}$$

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\epsilon \boldsymbol{v}) = -P \nabla \cdot \boldsymbol{v} + Q_{\mathrm{visc}} + Q_{\mathrm{irr}} - Q_{\mathrm{rad}} \,, \tag{10}$$

  the set is closed by a vertically integrated ideal gas equation of state.
- Radiative losses now consider also optically thin matter: $\tau_{\mathrm{eff}} = \frac{3}{8}\tau_{\mathrm{opt}} + \frac{1}{2} + \frac{1}{4\tau_{\mathrm{opt}}}$.
- A more sophisticated viscous heating is implemented:

$$Q_{\mathrm{visc}} = \frac{1}{2\nu\Sigma}\left(\pi_{rr}^2 + 2\pi_{r\theta}^2 + \pi_{\theta\theta}^2\right) + \frac{2\nu\Sigma}{9}(\nabla \cdot \boldsymbol{v})^2 \,. \tag{11}$$

- Stellar irradiation is added:

$$Q_{\mathrm{irr}} = \frac{2\sigma_{\mathrm{R}} T_{\mathrm{irr}}^4}{\tau_{\mathrm{eff}}} \tag{12}$$

  where T_{irr} is the effective temperature of the star projected onto the surface of the disc reduced by the disc's albedo.

[11] O. Chrenko, M. Brož, and M. Lambrechts (Oct. 2017). "Eccentricity excitation and merging of planetary embryos heated by pebble accretion". In: *A&A* 606, A114, A114. DOI: 10.1051/0004-6361/201731033. arXiv: 1706.06329 [astro-ph.EP]

[12] F. Masset (Jan. 2000). "FARGO: A fast eulerian transport algorithm for differentially rotating disks". In: *A&AS* 141, pp. 165–173. DOI: 10.1051/aas:2000116. arXiv: astro-ph/9910390 [astro-ph]

Result 1.: The temperature problem.

Model: $\alpha = 0.1$, Zhu et al. (2012)[13] opacity table.
Temperature: We must reinterpret the results of the kinematic model.

- T is the midplane temperature.
- T_{eff} id the atmospheric temperature calculated as:

$$T_{\mathrm{eff}} = T \tau_{\mathrm{eff}}^{-\frac{1}{4}} \qquad (13)$$

- T_{irr} is the irradiation temperature, a possible thermodynamical limit.

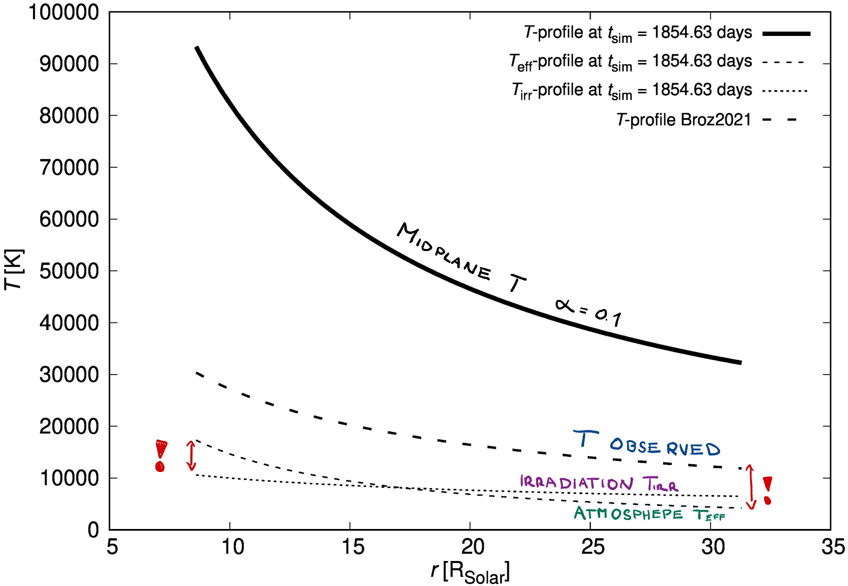


Figure. Temperature profile.

[13] Zhaohuan Zhu et al. (Feb. 2012). "Challenges in Forming Planets by Gravitational Instability: Disk Irradiation and Clump Migration, Accretion, and Tidal Destruction". In: *ApJ* 746.1, 110, p. 110. DOI: 10.1088/0004-637X/746/1/110. arXiv: 1111.6943 [astro-ph.SR]

Possible solutions to the temperature problem.

- **Vertical convection:** energy transfer by means of convection could imply a lower vertical temperature gradient $\implies$ Lower midplane temperatures.
- **Stronger irradiation:** the model underestimates irradiation by the central star (e.g. there is no edge on irradiation).
- **Temperature inversion:** non thermal radiation causes a temperature inversion (see fig.).

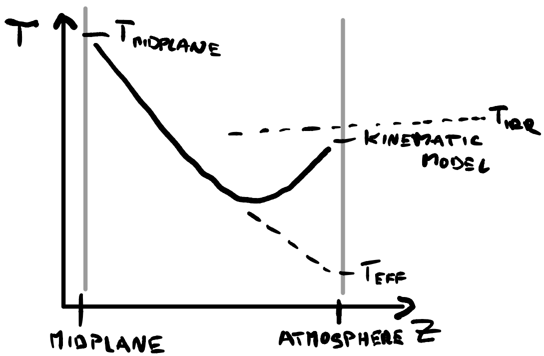


Figure. The relation of the three temperatures.

Result 2: Σ-profile

High densities are confirmed. $\Longleftarrow$ Necessary due to the fixed mass transfer rate.

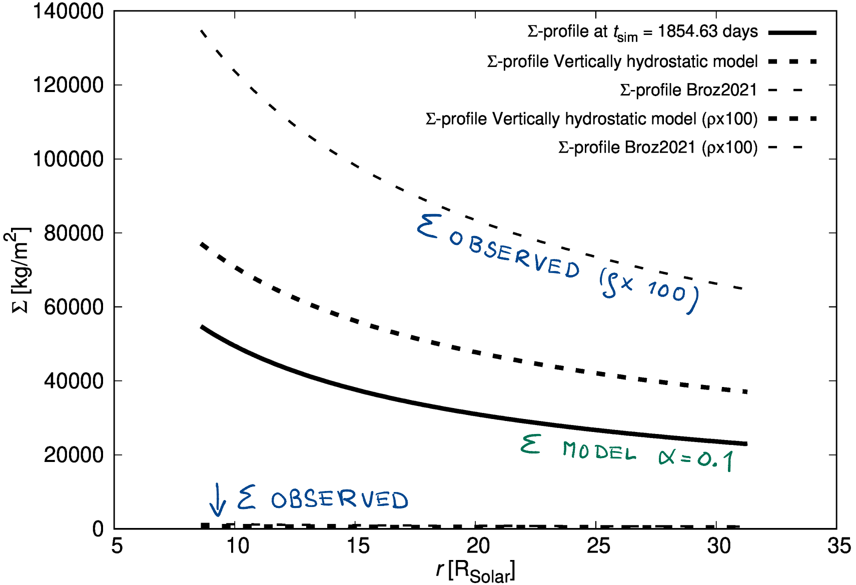


Figure. Σ-profile compared to observations

Result 3: Aspect ratio

The observed aspect ratio can be reached in **vertical hydrostatic equilibrium**. $\Longleftarrow$ High midplane temperatures.

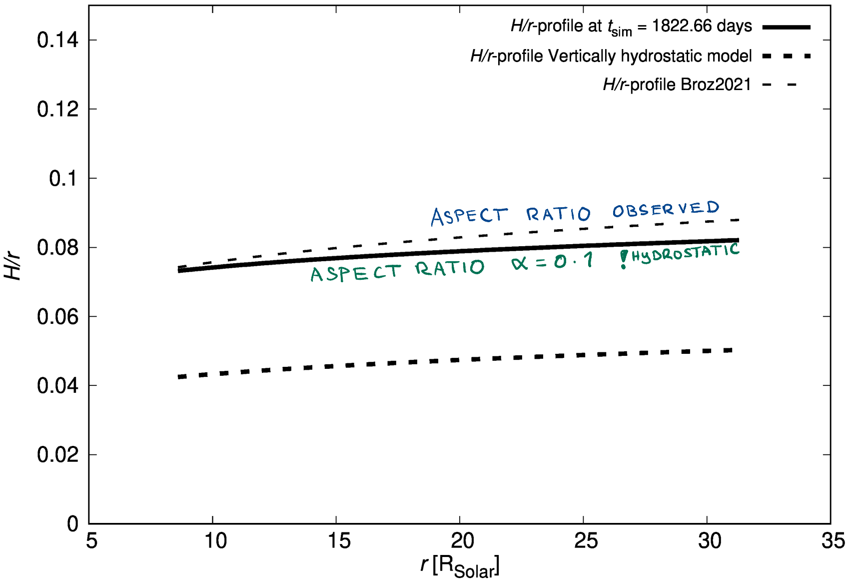


Figure. Aspect ratio compared to observations.

Other conclusions from the model

- The orbital velocity is almost Keplerian.
- The disc relaxed within a viscous timescale. Steady state for the rest of the simulation time.

$\implies$ Steady state Keplerian disc theory applicable.

- Radial velocities in the disc are of the order of 100 m s^{-1}.
- General opacity table produces an opacity close to Krammer's.
- $\alpha = 0.1$ is consistent with gathered evidence for fully ionized thin accretion discs[14] [15] [16].

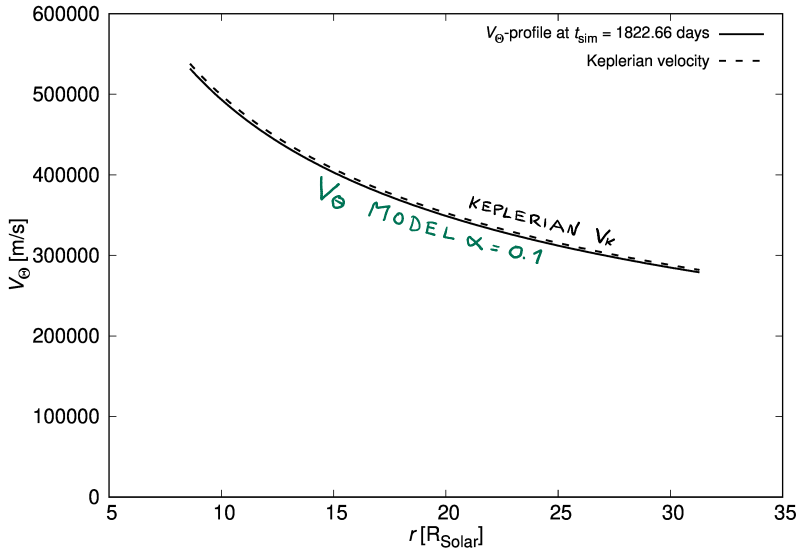


Figure. The rotational velocity profile.

[14] A. R. King, J. E. Pringle, and M. Livio (Mar. 2007). "Accretion disc viscosity: how big is alpha?" In: *Monthly Notices of the Royal Astronomical Society* 376.4, pp. 1740–1746

[15] A. Granada, C. E. Jones, and T. A. A. Sigut (Dec. 2021). "The Viscosity Parameter for Late-type Stable Be Stars". In: *ApJ* 922.2, 148, p. 148

[16] M. R. Ghoreyshi et al. (Sept. 2018). "The life cycles of Be viscous decretion discs: The case of ω CMa". In: *MNRAS* 479.2, pp. 2214–2228

OPACITY: ROGERS AND IGLESIAS(1992)[17]

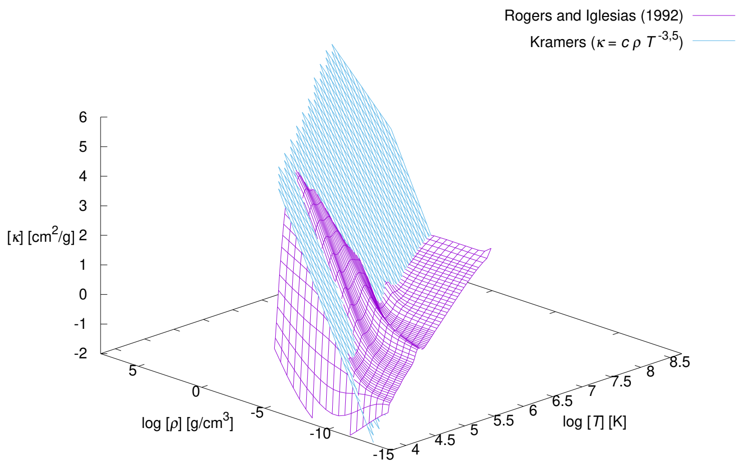


Figure. Fitted 2D opacity function - Krammer's opacity.

[17] Forrest J. Rogers and Carlos A. Iglesias (Dec. 1992). "Rosseland Mean Opacities for Variable Compositions". In: *ApJ* 401, p. 361. DOI: `10.1086/172066`

Optical depth

$$\tau_{\text{eff}} = \frac{T^4}{T_{\text{eff}}^4} \approx 300 \qquad (14)$$

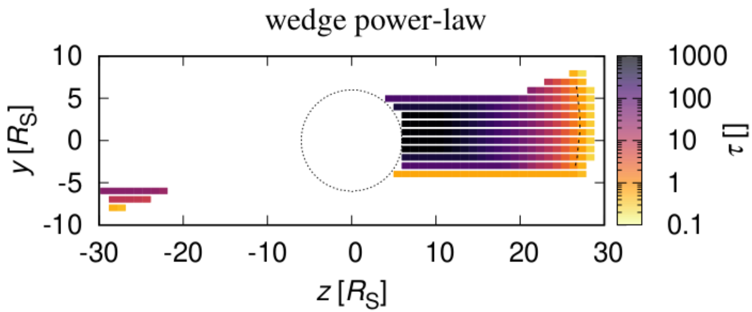


Fig. 11. Optical depth τ in the vertical (perpendicular) cross-section of the disk; other parameters are the same as in Fig. 10. The lines of sight are seemingly different from the wedge shape, but this only because they start either in the vacuum, or at the non-transparent object.

Figure. Optical depth in the disc [18]

[18] D. Mourard et al. (Oct. 2018). "Physical properties of β Lyrae A and its opaque accretion disk". In: *A&A* 618, A112, A112. DOI: 10.1051/0004-6361/201832952. arXiv: 1807.04789 [astro-ph.SR]

Radial dependency of α

Formula dependent on temperature[19]:

$$\alpha = (\alpha_{\mathrm{MRI}} - \alpha_{\mathrm{DEAD}}) \left[\frac{1 - \tanh(\frac{T_{\mathrm{MRI}} - T}{\Delta T})}{2} \right] + \alpha_{\mathrm{DEAD}}. \tag{15}$$

But we have no dead zones. Perhaps a weak power-law?

$$\alpha = C_\alpha R^E \tag{16}$$

$$T = T_\star C'_\alpha \dot{M}^{\frac{2A+4}{D}} M^{\frac{2A+3}{D}} R^{-\frac{6A+9+E(2A+2)}{D}} \left(1 - \sqrt{\frac{R_\star}{R}}\right)^{\frac{2A+4}{D}} \quad \text{where } \; D = 3A - 2B + 10 \tag{17}$$

[19] M. Flock et al. (Aug. 2016). "Radiation Hydrodynamics Models of the Inner Rim in Protoplanetary Disks". In: *ApJ* 827.2, 144, p. 144. DOI: 10.3847/0004-637X/827/2/144. arXiv: 1604.04601 [astro-ph.EP]

VERTICAL TEMPERATURE PROFILE IN A DECRETION DISC.

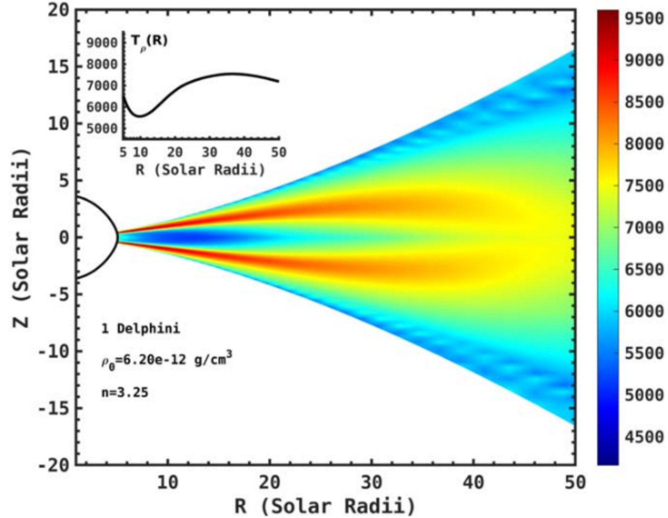


Figure 2. Disk temperatures, T(R, Z) in Kelvin, for the model with $\rho_0 = 6.2 \times 10^{-12}$ g cm^{-3} and $n = 3.25$. The stellar outline is indicated. The upper left insert shows the density weighted disk temperature as a function of disk radius.

Figure. Decretion disc [20]

[20] A. Granada, C. E. Jones, and T. A. A. Sigut (Dec. 2021). "The Viscosity Parameter for Late-type Stable Be Stars". In: *ApJ* 922.2, 148, p. 148.

THE LIMITATIONS OF 1D SIMULATIONS

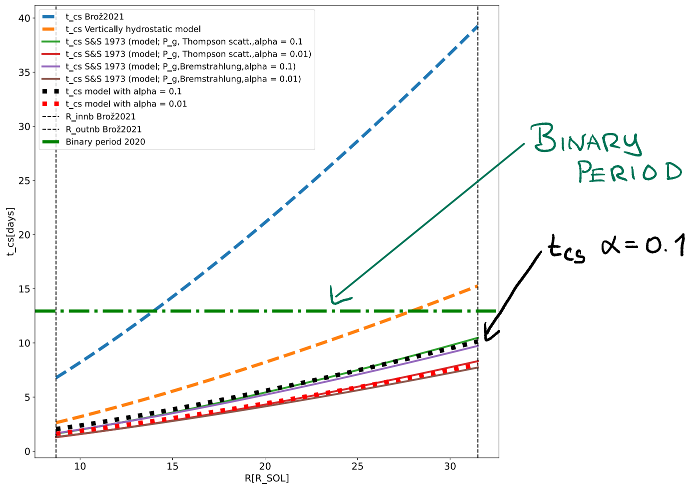


Figure. Sound speed timescale and binary period $\implies$ possibility of spiral waves from tidal interactions [21].

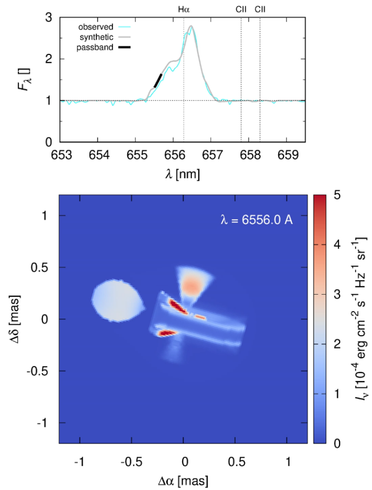


Figure. Scan of the H_α line supplementary to Broz et al. (2021) [22].

[21] Henri Boffin (Jan. 2008). "Spiral Waves in Accretion Discs— Theory". In: pp. 69–87. ISBN: 978-3-540-42213-6. DOI: 10.1007/3-540-45339-3_6

[22] M. Brož et al. (Jan. 2021). "Optically thin circumstellar medium in the β Lyr A system". In: *A&A* 645, A51, A51. DOI: 10.1051/0004-6361/202039035. arXiv: 2010.05541 [astro-ph.SR]