

# Quantum Machine Learning

by

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for the degree of

**Master of Science**



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# Abstract

# Acknowledgements

• **Kristian Wold**  
Oslo, August 2021

# Abbreviations

<b>AURKA</b>	Aurora Kinase A
<b>AURKB</b>	Aurora Kinase B
<b>AURKC</b>	Aurora Kinase C
<b>CDK</b>	Cyclin-Dependent Kinase
<b>CHARMM</b>	Chemistry at HARvard Macromolecular Mechanics
<b>CML</b>	Chronic Myelogenous Leukemia
<b>CPC</b>	Chromosomal Passenger Complex
<b>DOF</b>	Degrees of Freedom
<b>EGFR</b>	Epidermal Growth Factor Receptor
<b>GROMACS</b>	GRoningen MACHine for Chemical Simulations
<b>HDX</b>	Hydrogen-Deuterium Exchange
<b>INCENP</b>	Inner Centromere Protein
<b>MD</b>	Molecular Dynamics
<b>MS</b>	Mass Spectrometry
<b>NMR</b>	Nuclear Magnetic Resonance
<b>PBC</b>	Periodic Boundary Conditions
<b>PCA</b>	Principal Component Analysis
<b>PK</b>	Protein Kinase
<b>PKA</b>	Protein Kinase A
<b>RMSD</b>	Root-Mean-Square Deviation
<b>RMSF</b>	Root-Mean-Square Fluctuation
<b>VMD</b>	Visual Molecular Dynamics

# Glossary

Dipole Blockade	Phenomenon in which the simultaneous excitation of two atoms is inhibited by their dipolar interaction.
Cavity Induced Transparency	Phenomenon in which a cavity containing two atoms excited with light at a frequency halfway between the atomic frequencies contains the number of photons an empty cavity would contain.

# Nomenclature

$c$	Speed of light ( $2.997\,924\,58 \times 10^8 \text{ ms}^{-1}$ )
$\hbar$	Planck constant ( $1.054\,572\,66 \times 10^{-34} \text{ Js}$ )
$k_B$	Boltzmann constant ( $1.380\,658 \times 10^{-23} \text{ JK}^{-1}$ )
$Z_0$	Impedance of free space ( $376.730\,313\,461 \, \Omega$ )
$\mu_0$	Permeability of free-space ( $4\pi \times 10^{-7} \text{ Hm}^{-1}$ )

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# 1

## Introduction and Objective of the Study

### 1.1 Introduction

#### 1.1.1 Parameter Identification

### 1.2 Objective of the Study

### 1.3 How This Thesis is Organized

How This Book is Organized (ISL)

Our view is that one must understand simple methods before trying to grasp more complex ones. Hence, after giving an overview of the supervising learning problem in Chapter 2, we discuss linear methods for regression and classification in Chapters 3 and 4. In Chapter 5 we describe splines, wavelets and regularization/penalization methods for a single predictor, while Chapter 6 covers kernel methods and local regression. Both of these sets of methods are important building blocks for high-dimensional learning techniques. Model assessment and selection is the topic of Chapter 7, covering the concepts of bias and variance, overfitting and methods such as cross-validation for choosing models. Chapter 8 discusses model inference and averaging, including an overview of maximum likelihood, Bayesian inference and the bootstrap, the EM algorithm, Gibbs sampling and bagging. A related procedure called boosting is the focus of Chapter 10. In Chapters 9–13 we describe a series of structured methods for supervised learning, with Chapters 9 and 11 covering

regression and Chapters 12 and 13 focusing on classification. Chapter 14 describes methods for unsupervised learning. Two recently proposed techniques, random forests and ensemble learning, are discussed in Chapters 15 and 16. We describe undirected graphical models in Chapter 17 and finally we study high-dimensional problems in Chapter 18. At the end of each chapter we discuss computational considerations important for data mining applications, including how the computations scale with the number of observations and predictors. Each chapter ends with Bibliographic Notes giving background references for the material.

# Part I

## Theoretical Background

# 2

## Theoretical Background

### 2.1 Theory

#### 2.1.1 Project Theory 1

This is [subsection 2.1.1](#).

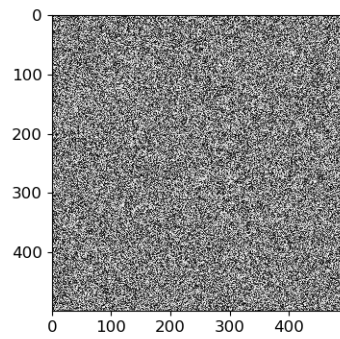
Citation is done with BibTeX [**Sakurai**].

Cross-reference equations such as

$$E = mc^2 \tag{2.1}$$

with [Equation \(2.1\)](#).

[Figure 2.1.1](#) brings the noise from the **figures folder**.



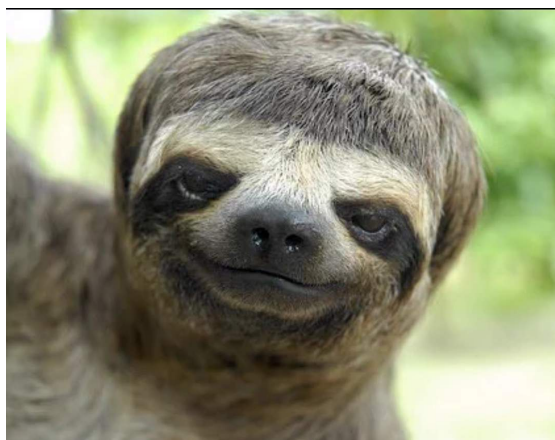
**Figure 2.1.1:** *Make some noise.*

Figure 2.1.2 shows a happy animal found in the **Images** folder.



**Figure 2.1.2:** *Sloths are arboreal mammals noted for slowness of movement and for spending most of their lives hanging upside down in the trees.*

Source: *Insert image source here*



**Figure 2.1.3:** *Sloths are arboreal mammals noted for slowness of movement and for spending most of their lives hanging upside down in the trees.*

Source: *Insert image source here*

Table 2.1.1 is from the `tables` folder.

**Table 2.1.1:** *From pandas to latex.*

$x$	$x^2$	$x^3$
0.250	0.062	0.016
0.500	0.250	0.125
0.750	0.562	0.422

Table 2.1.2 tabulates some values with alternating row colors.

**Table 2.1.2:** *Alternating background color for rows.*

$\alpha$	$\beta$	$\gamma$
0.1	0.2	0.3
0.4	0.5	0.6
0.7	0.8	0.9

Table with nice rulers

**Table 2.1.3:** *Generic table with different sized rulers.*

header1	header2	header3	header4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4

Table with nice rulers and alternating rows

**Table 2.1.4:** *Generic table with alternating rows and different sized rulers.*

header1	header2	header3	header4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4

Table with nice rulers and alternating rows 2

Given

$$f: \mathbb{R} \rightarrow \mathbb{R},$$

**Table 2.1.5:** *Generic table with alternating rows and different sized rulers.*

header1	header2	header3	header4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4
col1	col2	col3	col4

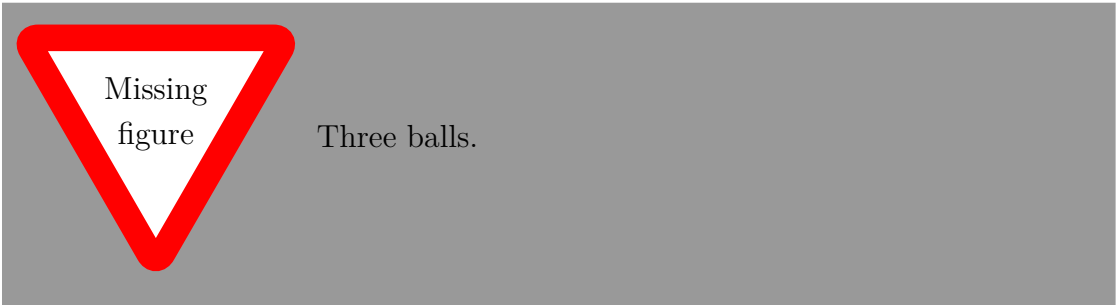
magic happens

$$\int_0^\infty e^{-x} \, dx$$

This is

Rewrite this!

## 2.2 Figures and Tables



**Figure 2.2.1:** *Three balls.*

Correct	Incorrect
$\varphi\colon X \rightarrow Y$	$\varphi : X \rightarrow Y$
$\varphi(x) \coloneqq x^2$	$\varphi(x) := x^2$

**Table 2.2.1:** *Proper colon usage.*

It is now easy to tell that Birch and Swinnerton-Dyer are two people.



Correct	Incorrect
$A \implies B$	$A \Rightarrow B$
$A \impliedby B$	$A \Leftarrow B$
$A \iff B$	$A \Leftrightarrow B$

**Table 2.2.2:** *Proper arrow usage.*

Correct	Incorrect
−1	-1
1–10	1-10
Birch–Swinerton-Dyer conjecture	Birch-Swinerton-Dyer conjecture
The ball — which is blue — is round.	The ball - which is blue - is round.
The ball—which is blue—is round.	

**Table 2.2.3:** *Proper dash usage.*

Correct	Incorrect
“This is an ‘inner quote’ inside an outer quote”	"This is an 'inner quote' inside an outer quote"

**Table 2.2.4:** *Proper quotation mark usage. The `\enquote` command chooses the correct quotation marks for the specified language.*

# 3

## Bayesian Inference

### **3.1 The Bayesian Paradigm**

## Part II

# Methodology & Computational Approach

# 4

## Computational Approach

### 4.1 Method

#### 4.1.1 Project Method 1

## Part III

### Results & Discussion

# 5

## Results and Discussion

### 5.1 Results

#### 5.1.1 Project Results 1

## Part IV

### Conclusion & Future Research

# 6

## Conclusion & Future Research

### **6.1 Conclusion**

### **6.2 Future Research**



# Appendices



## Appendix A

### **A.1   `Appendix.append(stuff)`**

Appendix A

# B

## Abbreviations and Units

### B.1 List of Abbreviations

<b>AURKA</b>	Aurora Kinase A
<b>AURKB</b>	Aurora Kinase B
<b>AURKC</b>	Aurora Kinase C
<b>CDK</b>	Cyclin-Dependent Kinase
<b>CHARMM</b>	Chemistry at HARvard Macromolecular Mechanics
<b>CML</b>	Chronic Myelogenous Leukemia
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<b>PK</b>	Protein Kinase
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<b>RMSD</b>	Root-Mean-Square Deviation
<b>RMSF</b>	Root-Mean-Square Fluctuation
<b>VMD</b>	Visual Molecular Dynamics

## B.2 Units

M	molar (1 mol/L)
$\mu$ s	microsecond ( $10^{-6}$ s)
ns	nanosecond ( $10^{-9}$ s)
ps	picosecond ( $10^{-12}$ s)
Å	Ångström ( $10^{-10}$ m)