Noisy-Intermediate Scale Quantum Computers as Open Quantum Systems

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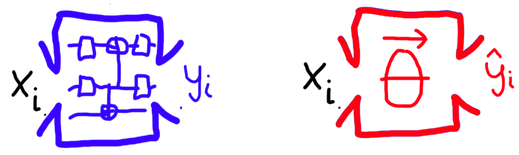
Parameterized Model Tuning - Basic Ingredients

Given a NISQ computer that implements some quantum circuit, how can we retrieve a model that approximates the action of the circuit in the best possible way?



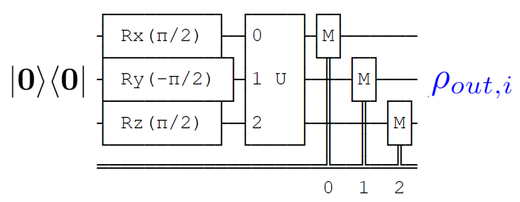
Probe the computer with a set of inputs x_i and measure outputs y_i. Replicate this procedure on the model and measure output $\hat{y}_i$.

$$C(\vec{\theta}) = \frac{1}{N}\sum_{i=1}^{N} C(y_i, \hat{y}_i) \qquad \vec{\theta}^* = \arg\min_{\vec{\theta}} C(\vec{\theta})$$

Introduce a cost function comparing the output of the NISQ computer and the model. Minimizing over the model parameters will then yield the best possible description.

$$\rho_{in,i} = P_i|\mathbf{0}\rangle\langle\mathbf{0}|P_i^\dagger$$

$$P_i = R_{i1} \otimes R_{i2} \otimes \cdots \otimes R_{in}$$



The input x_i is chosen to be a Pauli string, because they are easy to prepare in practice and introduce minimal state preparation error. After applying the target circuit **U**, the complete density matrix ρ is estimated as output.

$$C(\vec{\theta}) = -\frac{1}{N}\sum_{i=1}^{N} F(\rho_{out,i}, \hat{\rho}_{out,i}) \qquad \vec{\theta} \rightarrow \vec{\theta} - \mu\nabla_{\vec{\theta}} C(\vec{\theta})$$

A natural cost function is the negative Quantum State Fidelity averaged over all output pairs for the computer and the model. The parameters can then be optimized by minimizing cost with, for example, gradient decent.

Parameterization of Quantum Maps

What are good ways of parameterizing a quantum map in a differentiable way? A possible general description is the Kraus-form formulation, with a possible inclusion of a unitary prior **U**.

$$T_{\vec{\theta}}(\rho) = (1-c)U\rho U^\dagger + c\sum_{i=1}^{r} K_i(\theta)\rho K_i^\dagger(\theta) \qquad \sum_{i=1}^{r} K_i^\dagger K_i = I \qquad K_i^\dagger \in \mathbb{C}^{d\times d}$$

The Kraus operators $\mathbf{K_i}$ can be generated by performing QR-decomposition on a Ginibre matrix G. This yields a semi-unitary matrix Q, which can be blocked into Kraus operators with the appropriate properties. By changing the values of G, different maps can be obtained.

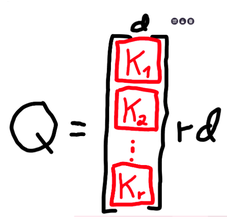
$$G = A + iB \in \mathbb{C}^{rd\times d}$$

$$\vec{\theta} = \{A, B, c\}$$

$$Q, R = \mathbf{QR}(G)$$

$$Q^\dagger Q = I_{d\times d}$$

$$QQ^\dagger \neq I_{rd\times rd}$$



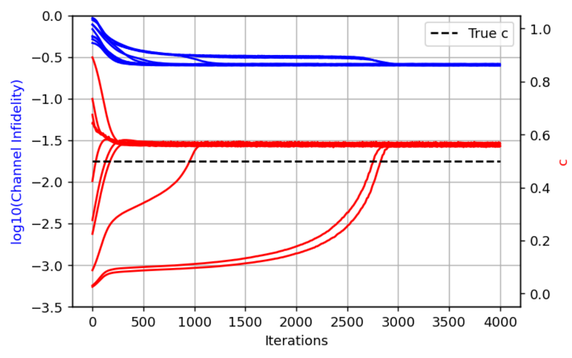
The parameter c controls how much of unitary part U is included in the model. For **c=1**, the map is completely unitary. For **c=0**, it is completely general. If the underlying map is well described by a known U, one would assume that less information is needed to recover the map

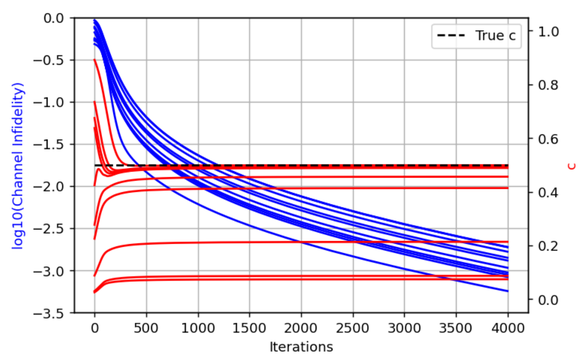
$$T_{True}(\rho) = (1-c)U\rho U^\dagger + c\sum_{i=1}^{r} K_i(\theta)\rho K_i^\dagger(\theta)$$

It possible to create synthetic data by instansiating a true model with random parameters. This model can then be applied to different inputs, and the resulting output can be interpreted as experimental data. Finally, another randomly initiated model can be trained on the synthetic data and compared to the true model.

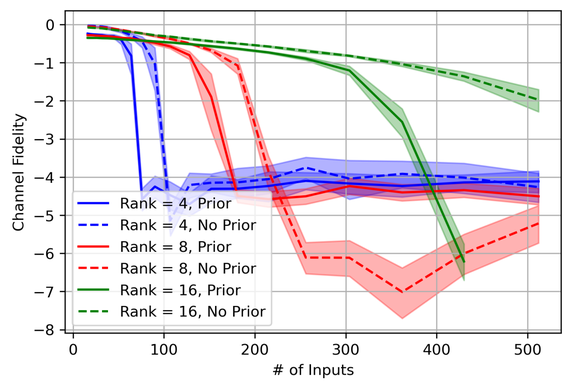
Numerical Benchmarks

We perform numerical benchmarks by generating synthetic data for a **n=3** qubit system. The True Model has Kraus-rank of **r=3**, with a random prior unitary **U** and c**=0.5**. We train several random models, both with rank **r=2** and **r=64**, to test under- and overparametrization





We observe that underparameterization causes poor model fit, and overestimation of the parameter c. Overparameterization, on the other hand, results in good fits, but general underestimation of c.

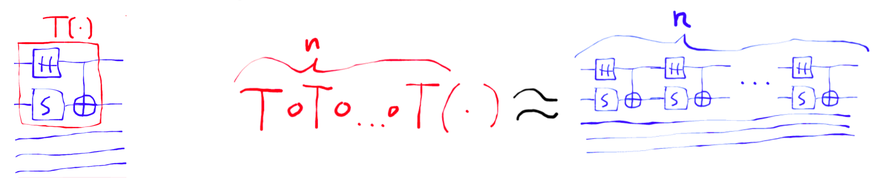


We also investigated how rank and use of prior unitary affects the number of inputs needed to recover the map.

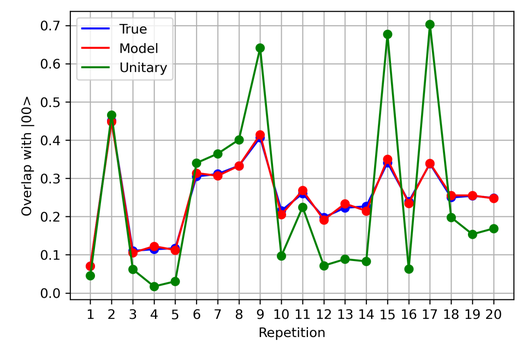
Higher rank generally needs more inputs, as there is more structure to be discovered. However, the use of a prior unitary is able to reduce the number of inputs by around 20%.

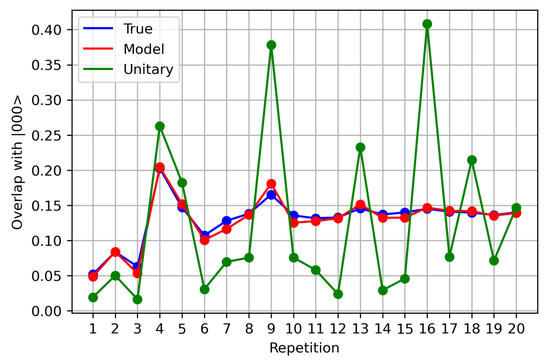
Experiment

The models are finally fitted to data produced by simulating mock noisy quantum computers using the Qiskit platform. As a benchmark, we study whether the repeated effect of the fitted model reproduces the action of the repeated circuit, as illustrated below.



The plots below show that models fitted to two and three qubit circuits capture the dynamics much better than the unitary description. This shows that the approach captures important details of the underlying quantum map from experimental data.





Outlook

With an accurate mathematical description of a quantum map, interesting aspects such as its spectrum, asymptotic state and degree of Markovianity can be studied.
To improve the efficiency of the procedure, estimation of density matrices should be changed to estimation of Pauli-string observables.