Modelling Dissipation in NISQ Hardware from Experimental Data

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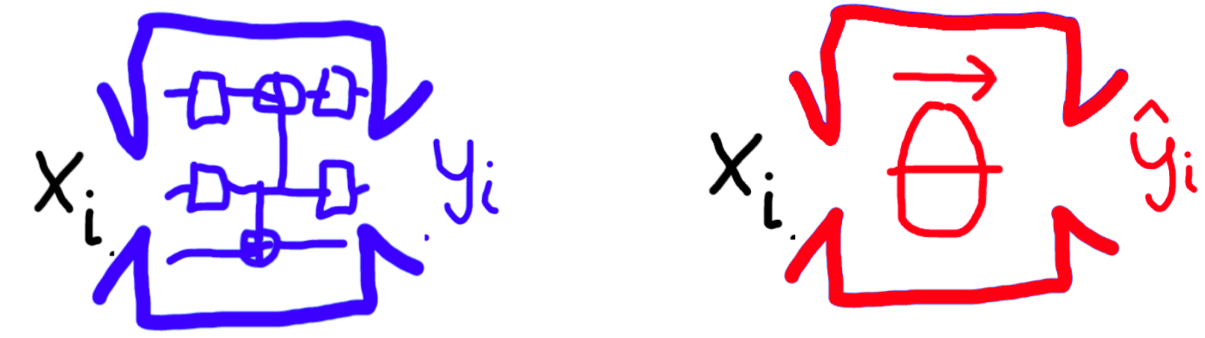3. NordSTAR, Nordic Center for Sustainable and Trustworthy AI Research, osloMet, Norway

Parameterized Model Tuning - The Basic Ingredients

Given a noisy quantum computer that implements some quantum circuit, how can we retrieve a mathematical model that describes the underlying dissipative quantum map?

Probe the hardware with a set of inputs x_i and measure outputs y_i. Replicate this procedure on the mathematical model and measure output $\hat{y}_i$.

$$C(\vec{\theta}) = \frac{1}{N} \sum_{i=1}^{N} C(y_i, \hat{y}_i) \qquad \vec{\theta}^* = \arg\min_{\vec{\theta}} C(\vec{\theta})$$

Introduce a cost function comparing the output of the real hardware and the mathematical model. Minimizing over the model parameters will then yield the best possible description.

$$\rho_{in,i} = P_i |\mathbf{0}\rangle\langle\mathbf{0}| P_i^\dagger$$

$$P_i = R_{i1} \otimes R_{i2} \otimes \cdots \otimes R_{in}$$

$$|\mathbf{0}\rangle\langle\mathbf{0}| \qquad \rho_{out,i}$$
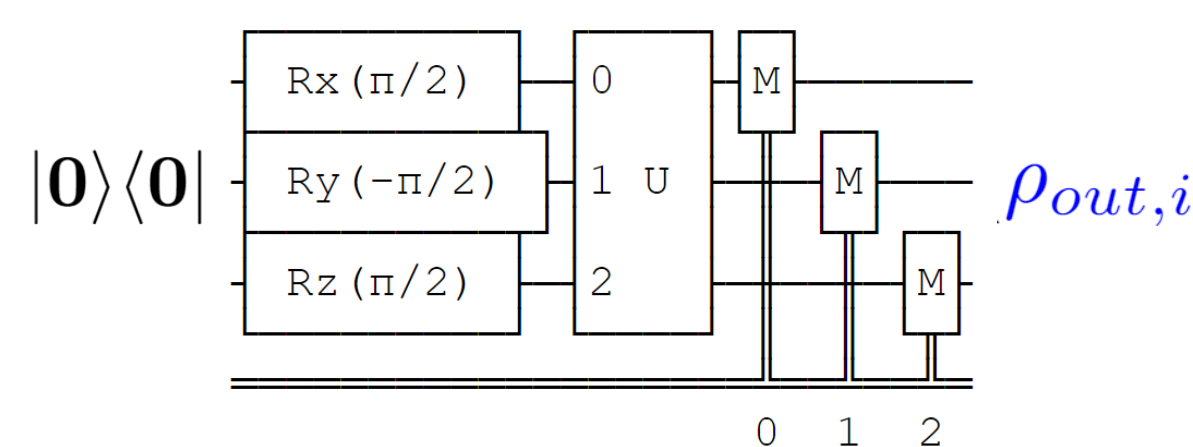
The input x_i is chosen to be Pauli-string, because they are easy to prepare in practice and probe the hardware well. After applying the target circuit U, the complete density matrix ρ is estimated as output.

$$C(\vec{\theta}) = -\frac{1}{N} \sum_{i=1}^{N} F(\rho_{out,i}, \hat{\rho}_{out,i}) \qquad \vec{\theta} \rightarrow \vec{\theta} - \mu \nabla_{\vec{\theta}} C(\vec{\theta})$$

A natural cost function is the negative Quantum State Fidelity averaged over all output pairs for the real hardware and the model. The parameters can then be optimized by minimizing cost with gradient decent.

Parameterized Quantum Maps

What are good ways of parameterizing a quantum map in a differentiable way? A possible general description is the Kraus-form formulation, with a possible inclusion of a unitary prior U.

$$T_{\vec{\theta}}(\rho) = (1-c)U\rho U^\dagger + c \sum_{i=1}^{r} K_i(\theta)\rho K_i^\dagger(\theta) \qquad \sum_{i=1}^{r} K_i^\dagger K_i = I \qquad K_i^\dagger \in \mathbb{C}^{d \times d}$$

The Kraus operators K_i can be generated in a parametric way by performing QR-decomposition on a Ginibre matrix G. This yields a semi-unitary matrix Q, which can be blocked into Kraus operators with the appropriate properties. By changing the values of G, different maps can be obtained.

$$G = A + iB \in \mathbb{C}^{rd \times d} \qquad Q, R = \mathbf{QR}(G)$$
$$\vec{\theta} = \{A, B, c\} \qquad Q^\dagger Q = I_{d \times d}$$
$$QQ^\dagger \neq I_{rd \times rd}$$

The parameter c controls how much of a unitary part U is included in the mathematical model. For $c=1$, the map is completely unitary. For $c=0$, it is completely general. If the underlying map is well described by a known U, one would assume that less information is needed to recover the map

$$T_{True}(\rho) = (1-c)U\rho U^\dagger + c \sum_{i=1}^{r} K_i(\hat{\theta})\rho K_i^\dagger(\hat{\theta})$$

It possible to create synthetic data by instansiating a true model with random parameters. This model can then be applied to different inputs, and the resulting output can be interpreted as experimental data. Another model can be trained and compared to the true model.

Numerical Benchmarks

We perform numerical benchmarks by generating synthetic data for a **n=3** qubit system. The True Model has Kraus-rank of **r=3**, with a random prior unitary **U** and **c=0.5**. We train several random models, both with rank **r=2** and **r=64**, to test under- and overparametrization
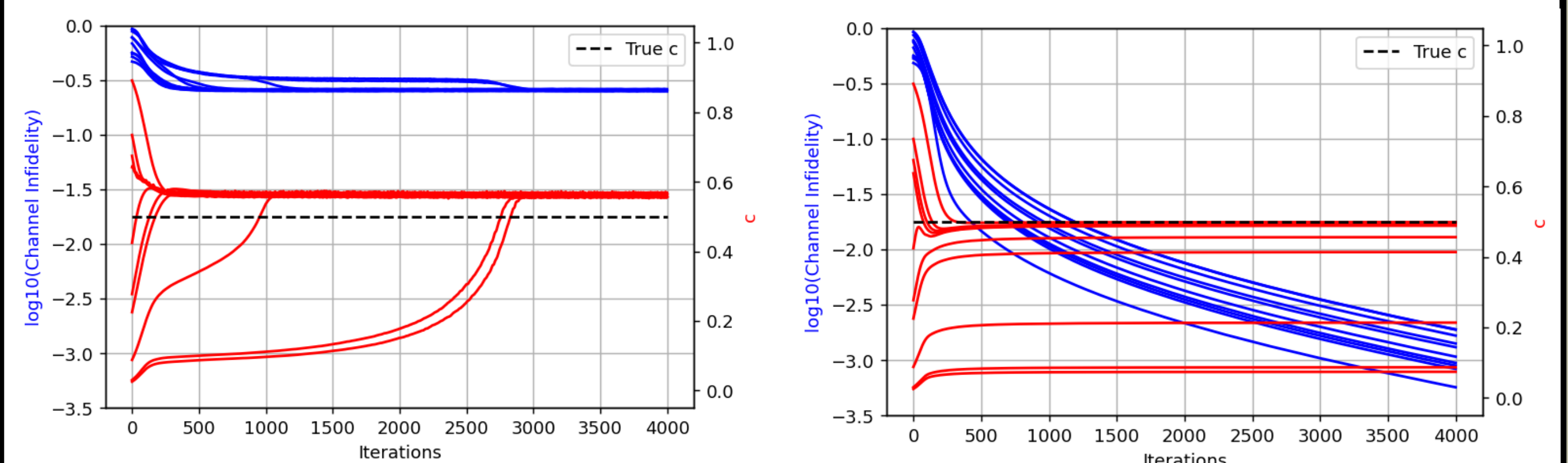
From the above plot, we see that underparameterization causes poor model fit, and overestimation of the parameter c. Overparameterization, on the other hand, results in good fits, but general underestimation of c.
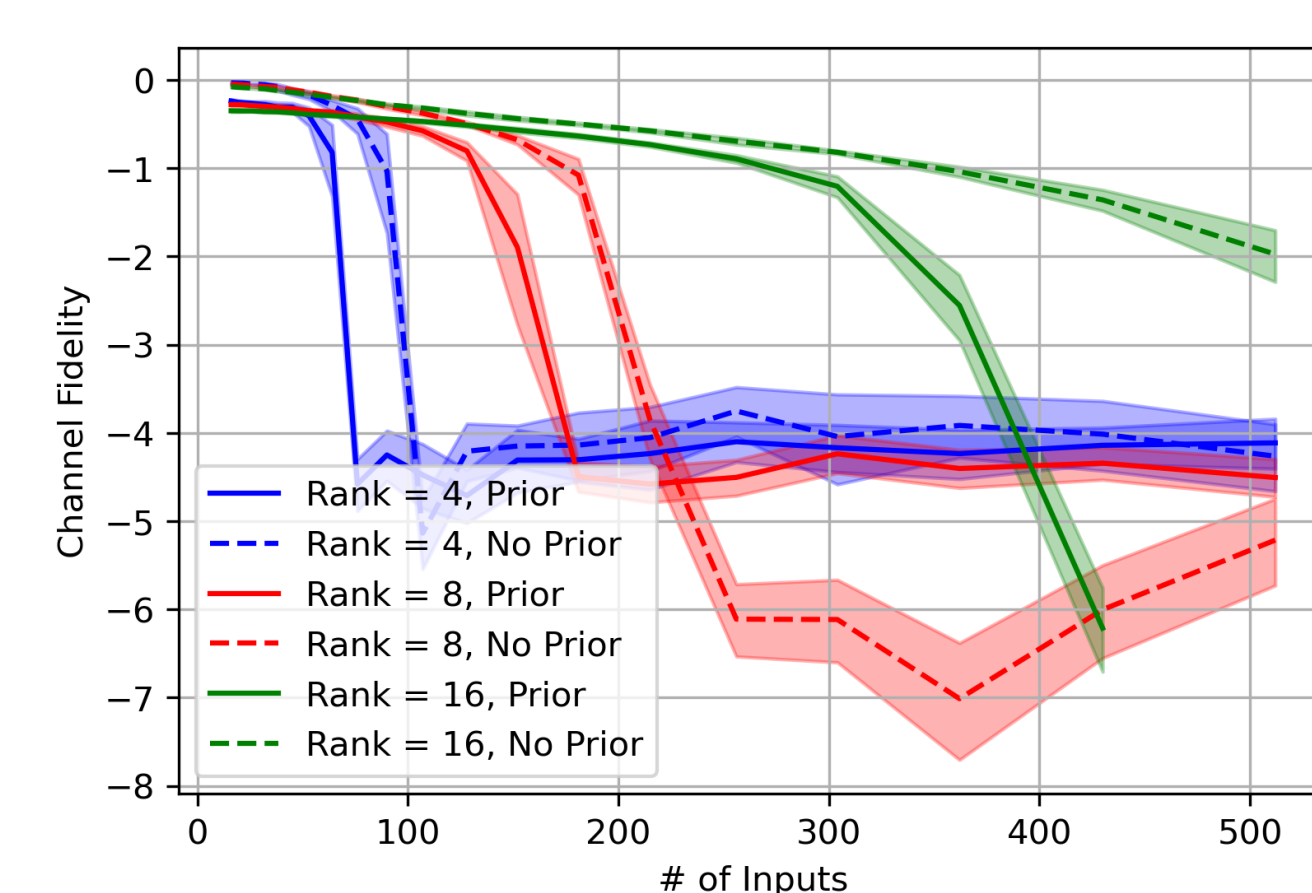

In the plot to the left, we see how the rank and use of prior unitary affects the number of inputs needed to recover the map.

Higher rank generally need more inputs, as there is more structure to be discovered. However, the use of a prior unitary is able to reduce the number of inputs by around 20%.

Experimental Data

The models are finally fitted to data produced by simulating mock noisy quantum computers using the Qiskit platform. As a benchmark, we study whether the repeated effect of the fitted model reproduces the action of the repeated circuit, as illustrated below.
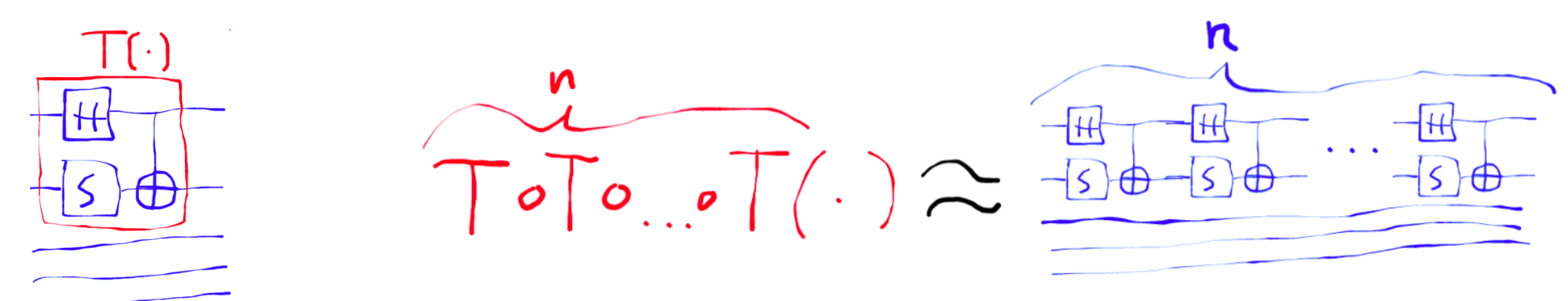
The plots below show that models fitted to two and three qubit circuits capture the dynamics much better than the purly unitary description. This shows that the approach captures important details of the underlying quantum map from experiental data.
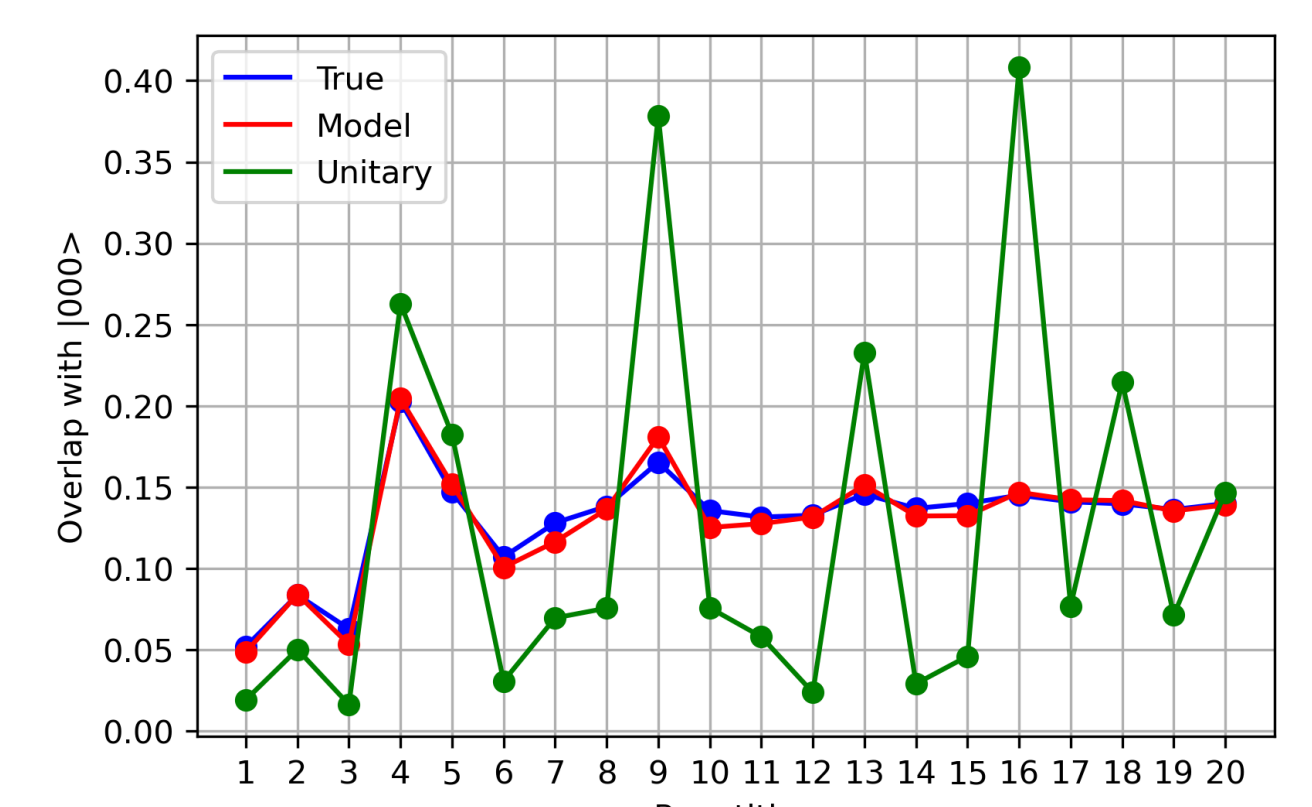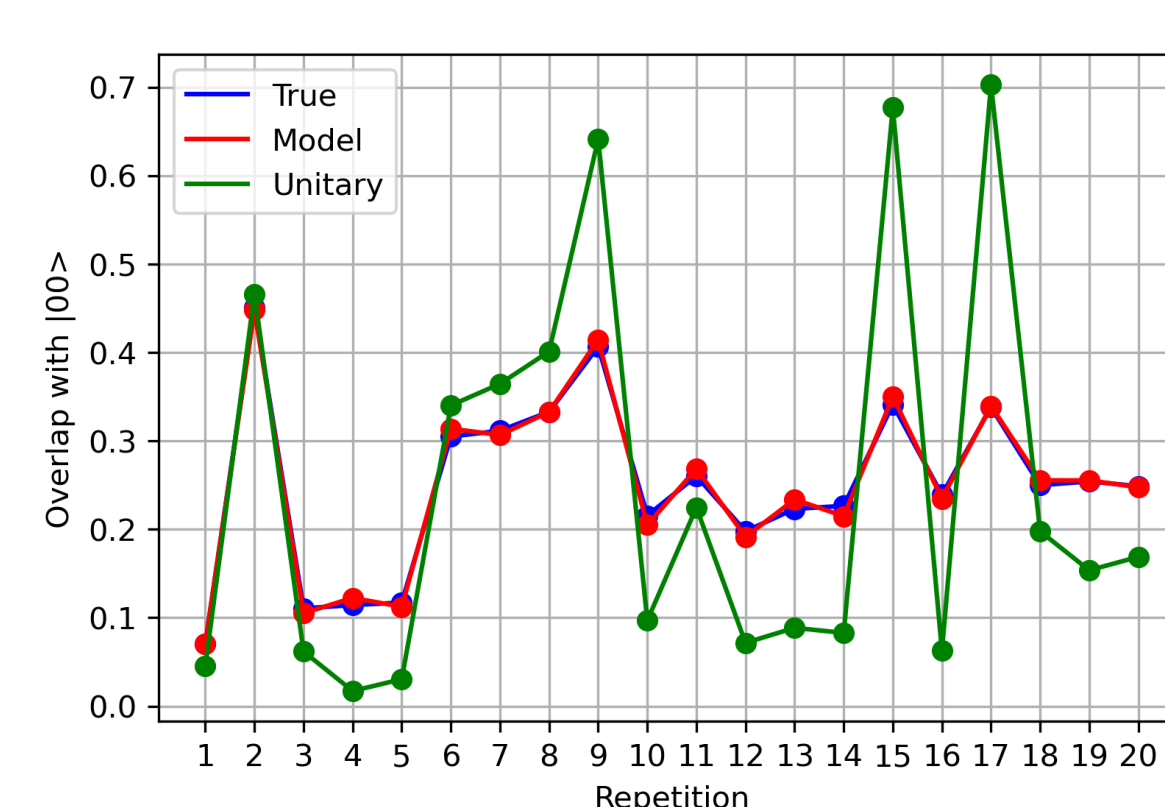
Outlook

With an accurate mathematical description of a quantum map, interesting aspects such as eigenvalue spectra, asymptotic states and degree of Markovianity can be studied.
To improve the efficiency of the framework, estimation of density matrices should be changed to estimation of Pauli-string observables.