Cross-nested logit models (CNL)

Estimating complex network structures

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Research question

RQ:

- Show how the cross-nested logit model can extend the concepts of nested choices to a range of complex choice puzzles.
- Implement an estimator for the cross-nested logit on synthetic and real data (for the Danish unemployment benefits systems).

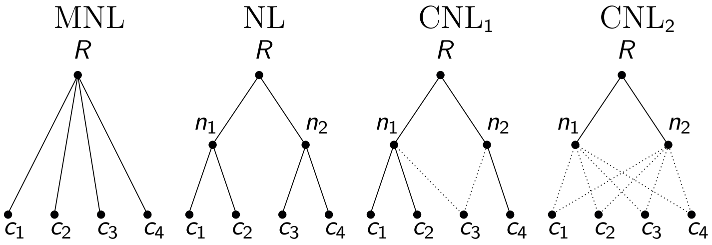
Status:

- We know how the estimator should be coded up (there are linear algebra related technicalities making this difficult).
- Optimizing such a complex likelihood function is probably too difficult for us (but we are looking into it).

Motivation

Independence of Irrelevant Alternatives (IIA):

- Assumes that the relative odds ratio between two alternatives $\frac{Pr(c_1)}{Pr(c_2)}$ is independent of the whether other alternatives exist.
  - $\rightarrow$ i.e. there is equal competition between all pairs of alternatives. This is violated if a pair of alternatives share unobserved attributes.
- Holds everywhere for the Multinomial Logit, only within nests for the Nested Logit, but never in nest where cross-nesting is allowed.



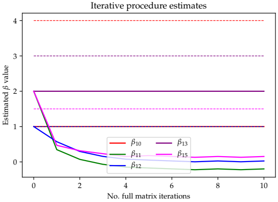
The mathematics

In general the model class has $p_i = \frac{e^{V_i + \ln G_i}}{\sum_j e^{V_j + \ln G_j}}$ We can derive a closed form likelihood, from which we can derive

$$\begin{aligned}\Pr(i|\mathcal{C}) &= \sum_m \frac{\left(\sum_j \alpha_{jm} z_j^{\mu_m}\right)^{\frac{\mu}{\mu_m}}}{\sum_n \left(\sum_j \alpha_{jn} z_j^{\mu_n}\right)^{\frac{\mu}{\mu_n}}} \times \frac{\alpha_{im} z_i^{\mu_m}}{\sum_j \alpha_{jm} z_j^{\mu_m}} \\ &= \sum_m \Pr(m|\mathcal{C}) \times \Pr(i|m)\end{aligned} \tag{1}$$

The log likelihood is then $\mathcal{L} = \sum_K d_{k \text{ chooses } i} \log \Pr(i)$

Estimation



- Clearly not correct estimates. Perhaps some kind of convergence? Depending on initial values this might look better or worse.

Current procedure:

- initiate with some parameters set to 0,1 or true value.
- for each parameter find a univariate optimum
- repeat n times over all parameters.

Out thoughts:

- The model is over-identified?
- The likelihood has a very weak global minimum $\Rightarrow$ we cannot feasibly find the correct minimum without advanced optimization?

Estimation

Alternatives

- Within-nest optimization - basically double counts cross nested options, but with a weighting due to sampling effects in each nest.

Usability

We haven't written a lot about this so far, but we want to emphasize how complex parameter interpretation is in this framework.

- There potentially are many local minima of the likelihood function.
- The math is complex, i.e. marginal effects are

$$\frac{\partial \mathrm{Pr}(i|\mathcal{C})}{\partial x} = \mathrm{Pr}(i|\mathcal{C}) \left(\beta_i - \sum_j \left[\beta_j \mathrm{Pr}(j|\mathcal{C}) + \frac{e^{x\beta_j} \frac{\partial^2 G}{\partial z_j \partial x}}{\sum_{j'} e^{x\beta_{j'}} \frac{\partial G}{\partial z_{j'}}} \right] \right) + \frac{e^{x\beta_i} \frac{\partial^2 G}{\partial z_i \partial x}}{\sum_j e^{x\beta_j} \frac{\partial G}{\partial z_j}} \tag{2}$$

So all in all you either

a) Need parameter estimates for interpreting and should therefore use a simpler model

b) or need a model general enough to fit weird correlations, in which case you should try out Machine Learning.