Steps 1 & 2: $cont_t = \beta_0 + \beta_1 dpi_t + \beta_2 aaa_t + \varepsilon_1$

Where:

$cont_{t =}$ Real personal consumption expenditures in year t, in billions of 2009 dollars

$dpi_{t =}$ Real disposable personal income in year t, in billions of 2009 dollars (estimated sign = +)

$aaa_{t=}$ Real interest rate on Aaa corporate bonds in year t (estimated sign = -)

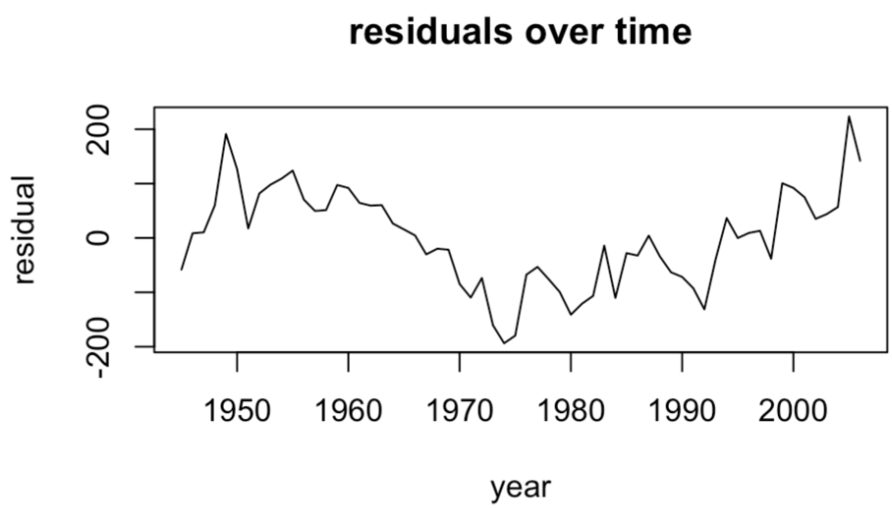
```
Call:
lm(formula = con ~ dpi + aaa, data = df)

Residuals:
     Min       1Q   Median       3Q      Max 
-193.670  -66.484    6.578   60.291  223.573 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.654e+02  2.309e+01  -7.165 1.42e-09 ***
dpi          9.320e-01  4.861e-03 191.726  < 2e-16 ***
aaa         -1.254e+01  3.804e+00  -3.297  0.00166 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 91.18 on 59 degrees of freedom
Multiple R-squared:  0.9987,    Adjusted R-squared:  0.9987 
F-statistic: 2.28e+04 on 2 and 59 DF,  p-value: < 2.2e-16
```

Step 3: The residuals do not appear random. There is a predictable pattern where residuals are more likely to be positive when surrounding residuals are positive and conversely negative when surrounding residuals are negative. This is positive auto-correlation.



Step 4: Durbin-Watson test for positive serial correlation

$H_{0:}$ p<=0 (no positive serial correlation)

H_1: p>0 (positive serial correlation)

```
        Durbin-Watson test

data:  reg1
DW = 0.38577, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0
```

The Durbin-Watson test statistic is 0.38577. I compare this with the two critical values for a sample size of 62 and 2 variables. d_L=1.522 $d_{U=}$1.654. According to my decision rule, I reject the null hypothesis that there is no positive serial correlation because the test statistic is less than d_L which puts us in the rejection region with likely positive serial correlation. This is verified by the R generated p-value of < 2.2e-16 meaning the lowest level of significance that we can reject the null hypothesis is less than 0.01.

Step 5: Estimate the model with GLS (Prais-Winsten)

```
prais_winsten(formula = con ~ dpi + aaa, data = df, index = "year")

Residuals:
    Min      1Q  Median      3Q     Max
-164.73  -77.16  -18.85   66.65  210.73

AR(1) coefficient rho after 5 iterations: 0.8589

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -186.34412   74.40963  -2.504   0.0151 *
dpi            0.93393    0.01274  73.301   <2e-16 ***
aaa           -4.78633    2.72331  -1.758   0.0840 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 51.02 on 59 degrees of freedom
Multiple R-squared:  0.9858,    Adjusted R-squared:  0.9853
F-statistic:  2051 on 2 and 59 DF,  p-value: < 2.2e-16

Durbin-Watson statistic (original): 0.3858
Durbin-Watson statistic (transformed): 2.187
```

My coefficients are different from the OLS regression, particularly the coefficient of aaa. Additionally, the t-statistics are different. The standard error for dpi increased leading to a lower t-value; however it is still statistically significant even at the 0.01 level of significance. The standard error for aaa decreased; however, with the change in coefficient the t-value went down leading to it being statistically significant at only the 0.1 level (versus 0.05 for OLS). The changed slopes are a known problem of GLS; however, we can expect different estimates to have different values even though their expected values are the same. We could try Newey-West standard errors to correct serial correlation without changing the slopes.

After the GLS transformation, it seems autocorrelation no longer exists. The new Durbin-Watson statistic is 2.187. Given the same critical values of d_L=1.522 $d_{U=}$1.654, we can apply our decision rule. We fail to reject the null hypothesis that there is no positive serial correlation because 2.187 > 1.654 (d_U).