

2/3 K 9-ay yrony

① $U = 3 - 8x + 6y$, eam $x^2 + y^2 = 36$

$$L(x, y, \lambda) = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$$

$$L'_x = -8 + 2x\lambda$$

$$L'_y = 6 + 2y\lambda$$

$$L'_\lambda = x^2 + y^2 - 36$$

$$-8 + 2x\lambda$$

$$x = \frac{4}{\lambda}$$

$$6 + 2y\lambda$$

$$y = -\frac{3}{\lambda}$$

$$\left(\frac{4}{\lambda}\right)^2 + \left(-\frac{3}{\lambda}\right)^2 = 36$$

$$\lambda = \pm \frac{5}{6}$$

$$\lambda = \frac{5}{6}, \quad x = \frac{24}{5}, \quad y = -\frac{18}{5}$$

$$M_1\left(\frac{24}{5}, -\frac{18}{5}\right)$$

$$\lambda = -\frac{5}{6} : \quad x = -\frac{24}{5}, \quad y = \frac{18}{5}$$

$$M_2\left(-\frac{24}{5}, \frac{18}{5}\right)$$

$$\Delta = \begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{vmatrix} = -8\lambda(x^2 + y^2)$$

$$M_1 = -8 \cdot \frac{5}{6} \left(\left(\frac{24}{5} \right)^2 + \left(-\frac{18}{5} \right)^2 \right) = -240 < 0 \Rightarrow \max$$

$$M_2 = -8 \cdot \left(-\frac{5}{6} \right) \cdot \left(\left(-\frac{24}{5} \right)^2 + \left(\frac{18}{5} \right)^2 \right) = 240 > 0 \Rightarrow \min$$

$$M_1 - \max; M_2 = \min.$$

$$\textcircled{3} \quad U = x^2 + y^2 + z^2, \quad \vec{c} = (-9, 8, -12), \quad \nabla \cdot M(8; -12; 9)$$

$$|\vec{c}| = \sqrt{81 + 64 + 144} = 17$$

$$\vec{c}_0 = \frac{\vec{c}}{|\vec{c}|} = \left(-\frac{9}{17}; \frac{8}{17}; -\frac{12}{17} \right)$$

$$U'_x = 2x$$

$$U'_y = 2y$$

$$U'_z = 2z$$

$$\text{grad } U|_{(8; -12; 9)} = (16; -24; 18)$$

$$U'_z|_{(8; -12; 9)} = \left(-\frac{9}{17} \right) \cdot 16 - \frac{24 \cdot 8}{17} - \frac{12 \cdot 18}{17} = -\frac{552}{17}$$

$$\textcircled{4} \quad U = e^{x^2 + y^2 + z^2}, \quad \vec{J} = (4, -13, -16), \quad \nabla \cdot L(-16; 4; -13)$$

$$|\vec{J}| = \sqrt{16 + 169 + 256} = 21$$

$$\vec{D}_0 = \frac{\vec{D}}{|\vec{D}|} = \left(\frac{4}{21}; -\frac{13}{21}; -\frac{16}{21} \right)$$

$$U'_x = 2xe^{x^2+y^2+z^2}$$

$$U'_y = 2ye^{x^2+y^2+z^2}$$

$$U'_z = 2ze^{x^2+y^2+z^2}$$

$$\text{grad } U|_{(-16, 4, -13)} = (-32e^{441}; 8e^{441}; -26e^{441})$$

$$U_{\vec{b}}|_{(-16, 4, -13)} = \frac{4}{21} \cdot (-32e^{441}) + \left(-\frac{13}{21}\right) \cdot 8e^{441} + \left(-\frac{16}{21}\right) \cdot (-26e^{441}) =$$

$$= \frac{184}{21} \cdot e^{441}$$