Homogén, szeparábilis diffegyenletek

$$y' = \frac{xy - y}{x^2 + 8x + 15}$$

$$y' = \frac{x - 1}{x^2 + 8x + 15} y$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x - 1}{x^2 + 8x + 15}$$

$$\int \frac{1}{y} dy = \int \frac{x - 1}{x^2 + 8x + 15} dx$$

Másodfokú megoldása "ránézésre":

$$a \cdot (x - x_1) \cdot (x - x_2) = 0$$

$$a\left[x^2 - (x_1 + x_2)x + (x_1 \cdot x_2)\right] = 0$$

$$b = -(x_1 + x_2) \qquad c = x_1 \cdot x_2$$

$$\ln(y) = \int \frac{x - 1}{x^2 + 8x + 15} dx$$

Parciális törtekre bontás:

$$\frac{x - 1}{(x + 3)(x + 5)} = \frac{A}{x + 3} + \frac{B}{x + 5}$$

$$x - 1 = Ax + 5A + Bx + 3B$$

$$1x - 1 = (A + B)x + (5A + 3B)$$

$$\begin{aligned} A + B &= 1 \\ 5A + 3B &= -1 \\ \\ A &= 1 - B \\ 5(1 - B) + 3B &= -1 \\ \\ 5 - 5B + 3B &= -1 \\ -2B &= -6 \\ \\ B &= 3 \\ A &= -2 \end{aligned}$$

$$\ln(y) = \int \frac{-2}{x+3} + \frac{3}{x+5} dx$$

$$\ln(|y|) = -2\ln(|x+3|) + 3\ln(|x+5|) + \ln(C)$$

$$y = (x+3)^{-2} \cdot (x+5)^{3} \cdot C$$