CPTS 453 **Name:** *Kristofer Koehn*
Homework 6
10/28/22

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1. Consider rooted trees with exactly 10^{12} non-parent vertices. Recall that we denote by H the height of such a tree.

   (a) Find a tight lower bound for H among such trees.
   tight lower bound is $H = 1$

   (b) Find a tight upper bound for H among such trees.
   tight upper bound is $H = \infty$

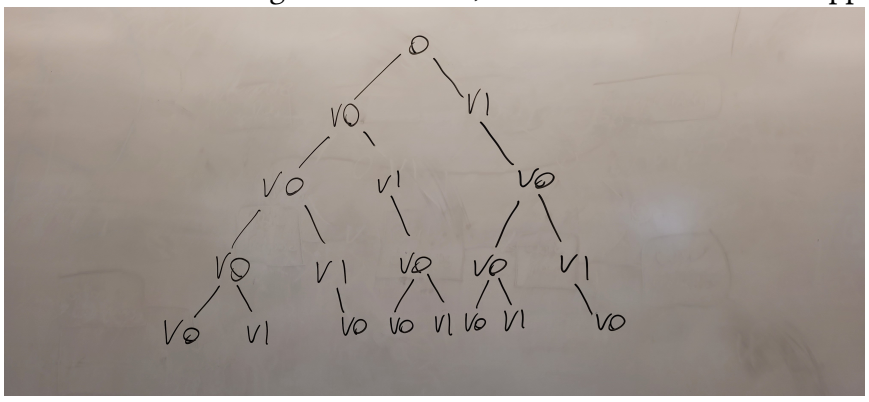
   (c) Find a tight lower bound for H among binary rooted trees with exactly 10^{12} non-parent vertices.
   $H = 39$

   (d) Find a tight lower bound for H among n-ary rooted trees with exactly 10^{12} non-parent vertices. your formula should involve n.
   $Log_n 10^{12}$

2. Consider the binary rooted tree (T, r) each of whose vertices is a finite string over 0,1. We construct our tree by the following rules:

   - The root is vertex 0
   - if v is a string that ends in a 0, then the left child of v is "v0" and the right child is "v1".
   - If v is a string that ends in a 1, then the only child of v is "v0".

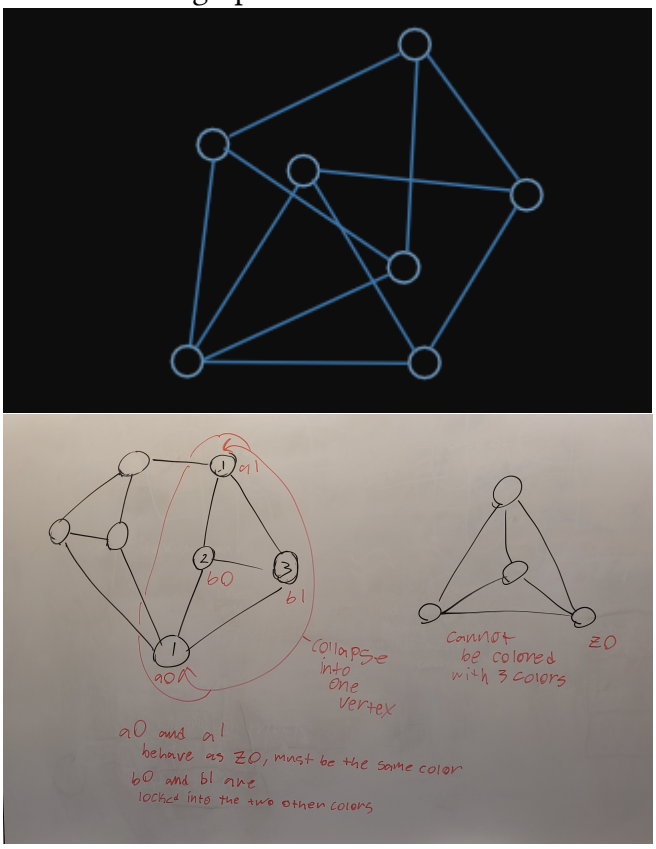
   (a) Draw Levels 0 through 4 of this tree, with the vertices labeled appropriately.



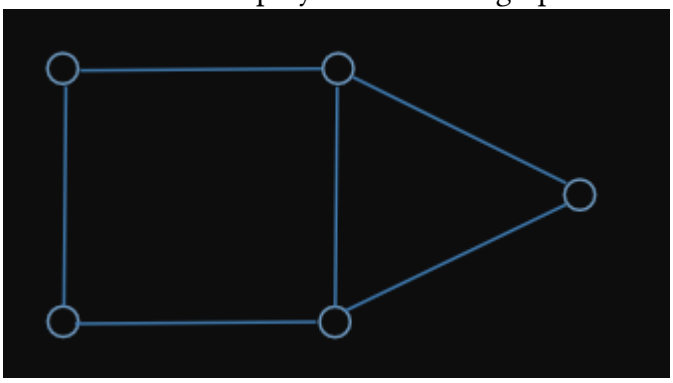
   (b) Make a conjecture of how to compute the number of vertices at a given level.

   To find the amount of vertices for a given level, you take the level number of the previous level $L - 1$ and add the number of vertices to it.

3. Show that the graph below cannot be 3-colored:



4. Find the chromatic polynomial of the graph below:



$x^5 - 6x^4 + 14x^3 - 15x^2 + 6x$