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Configuring a Seismographic Network for Optimal Monitoring of Fault Lines and Multiple Sources

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Abstract The geometric configuration of a seismographic network has important consequences for the ability to determine hypocenters with high precision. We present a method for optimal configuration when the network must monitor a system of faults. Our optimality criterion is drawn from the statistical theory of experimental design and can be efficiently maximized using an extension of the DETMAX algorithm. Our work generalizes that of Rabinowitz and Steinberg (1990), which treated the problem of optimal network configuration for monitoring a point source.

Introduction

A primary function of a seismographic network is the determination of hypocenters from recorded arrival times. The ability to determine hypocenters with high precision depends to a large extent on the geometric configuration of the network. Given the high cost of installation and maintenance, it is essential that the network be designed to provide maximal precision.

Rabinowitz and Steinberg (1990) applied the statistical theory of optimal design of experiments to obtain a configuration with maximal precision for monitoring a point source. However, seismic sources are typically fault lines, not single point sources, and most networks must monitor several, often geographically diverse, systems of faults. In this work, we generalize the concepts from Rabinowitz and Steinberg (1990) and present a procedure for achieving an efficient network configuration in these more complex settings. The approach is general and can be applied to global, regional, and local network design.

Our focus in this work is on designing networks that reduce the statistical uncertainty that derives from random errors in the arrival times. Thus, when we refer to the precision of a network, or to its resolution power, we mean its ability to control statistical uncertainty. It should be remembered that other factors, such as the validity of the crustal velocity model and local velocity anomalies, will also affect the accuracy of hypocenter locations. See Pavlis (1987) for a complete analysis of this problem. We have not attempted to take explicit account of model-induced errors. However, we note that networks that are optimal with respect to statistical precision typically will "surround" potential sources (Rabinowitz and Steinberg, 1990) and thus should also effectively reduce errors related to deviations from the model.

Following a summary of prior research on network configuration, we present the methodology for the multiple source problem. We then illustrate the approach with several examples. We conclude with a brief discussion and summary of our proposal.

Optimal Configuration with a Single Source

The common approach to hypocenter determination is least-squares fitting of observed arrival times. Standard deviations for the estimated hypocenter parameters and the origin time are proportional to the square roots of the diagonal entries of the matrix $(A^TA)^{-1}$, where A is the matrix of partial derivatives of the travel times with respect to the parameters, evaluated at the estimated hypocenter. For details, see Buland (1976) and Lee and Stewart (1981). It follows that a network will efficiently monitor a point source if A^TA is "large" when the matrix A is constructed by evaluating the partial derivatives at the point source. A number of optimality functionals have been proposed in the statistical literature for measuring the size of A^TA. The most popular functional is the D criterion, $\det(A^TA)$ (Box and Lucas, 1959; Silvey, 1980; Atkinson, 1982; Pukelsheim, 1993). Kijko (1977) first proposed optimal configuration of a seismographic network by maximization of the D criterion. Rabinowitz and Steinberg (1990) described some theoretical properties of D-optimal seismographic networks.

Most algorithms for maximizing the D criterion require, as input, a list of candidate station sites, the desired number of stations in the network (N), and the location of the source. For small networks, modern computing power may make it possible to find the optimal network by conducting an exhaustive search, evaluating the criterion for all possible configurations. For larger problems, though, an algorithmic approach will be preferable.

Rabinowitz and Steinberg (1990) provided an efficient optimization scheme based on Mitchell's (1974) DETMAX algorithm. The DETMAX algorithm exploits special features of the criterion function and is much better suited to locating the global maximum than are standard optimization routines. The algorithm generates an initial network by selecting N sites at random from the list of candidates. The network is then modified sequentially by adding sites from the list and then removing sites, so as to improve the D criterion. When single additions and removals no longer improve the criterion, longer "excursions" are initiated in which several new sites are added and then an equal number removed. Mitchell (1974) recommended that several random starts be used to increase the probability of finding the global maximum.

Mitchell (1974) showed that a simple rule guides the choice of which station to add to, or remove from, a given network. Let **A** denote the matrix of partial derivatives of the arrival times for the given network, and let f_i denote the partial derivatives at the ith candidate site. Adding the ith candidate to the network increases the determinant to

$$\det(\boldsymbol{A}^t\boldsymbol{A})(1 + v_i), \tag{1}$$

where

$$v_i = f_i^T(\boldsymbol{A}^T\boldsymbol{A})^{-1} f_i. \tag{2}$$

Thus, the greatest improvement in the D criterion is achieved by adding the site that maximizes the quantity v_i. The best station to remove from an augmented network is the site at which v_i is minimal.

Optimal Network Configuration with Multiple Sources

We turn now to the important practical problem of optimal configuration when the network must monitor fault lines and multiple seismic sources. It is common seismological knowledge that a configuration that is optimal for one source may provide poor coverage for other sources. In terms of the D criterion, this problem reflects the fact that the matrix $\boldsymbol{A}$ depends on the source coordinates as well as the station sites. To monitor many sources, then, it is necessary to find a "compromise" network that may not be optimal for any individual source but provides effective coverage for all the sources.

The D criterion can be extended in several ways to handle multiple sources. For now, we assume that k isolated point sources must be monitored. We will discuss later how to apply our approach to line or area sources. The matrix of partial derivatives of the arrival times with respect to source j is denoted by $\boldsymbol{A}_j$. We propose maximization of the DMS criterion (D multiple source):

$$\sum_1^k a_j \ln[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)]. \tag{3}$$

Here a_j reflects the relative importance attached to source j by the designer. We recommend that large weights be assigned to those sources where effective monitoring is critical and to sources that have demonstrated a high level of seismicity.

Theoretical properties of the weighted log determinant criterion (3) are discussed by Lauter (1974, 1976), Chaloner and Larntz (1989), and Pukelsheim (1993, Ch. 11). A version of this criterion was first considered by Lindley (1956), who derived it in connection with Bayesian analysis of linear statistical models. In that context, the weights a_j are interpreted as representing a prior probability distribution on the set of possible hypocenters and so must sum to unity. The DMS criterion then arises as the expected increase in Shannon information provided by the experiment, with an optimal experiment maximizing the expected information gain. Lindley's result will be approximately true in nonlinear models, such as hypocenter determination, where the log likelihood functions are approximately quadratic. The criterion has been used for mixtures of linear models by Lauter (1974, 1976) and Cook and Nachtsheim (1982).

The Multi-Source Optimization Algorithm

A straightforward extension of the DETMAX algorithm can be used to maximize the DMS criterion. We generalize the formula used to compare stations when deciding which to add to, or subtract from, a given network. With the exception of this slight change, the DETMAX algorithm can be applied exactly as in the single-source problem.

The updating formula for a candidate site now involves the partial derivatives of the arrival times with respect to each of the sources. The vector of partial derivatives of arrival times at site i with respect to source j is denoted by $\boldsymbol{f}_{i,j}$. Let

$$v_{i,j} = \boldsymbol{f}_{i,j}^T(\boldsymbol{A}_j^T \boldsymbol{A}_j)^{-1} \boldsymbol{f}_{i,j}. \tag{4}$$

Thus, $v_{i,j}$ gives the quantity v_i defined in the previous section for source j, $j = 1, \ldots, k$. Combining equations (1), (3), and (4), we find that the criterion value, after adding a station at site i, is

$$\begin{aligned} \mathrm{DMS} &= \sum_1^k a_j \ln[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)(1 + v_{i,j})] \\ &= \sum_1^k a_j \ln[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)] + \sum_1^k a_j \ln(1 + v_{i,j}). \end{aligned} \tag{5}$$

Thus, for the DMS criterion, the best site to add to a given network is the site that maximizes

$$\sum_1^k a_j \ln(1 + v_{i,j}). \tag{6}$$

The best site to remove from an augmented network is the site for which equation (6) is minimal.

Efficient network configurations for monitoring line or area sources can be found with our approach by discretizing the sources. A rather coarse grid of locations should typically be sufficient to guarantee precise hypocenter determination throughout the full extent of line and area sources. To justify this claim, we note that optimal network configurations for a point source tend to surround the source, in accord with seismological experience of what constitutes "good coverage" (Rabinowitz and Steinberg, 1990). Thus, including a particular source location on a fault line will typically lead to a network that surrounds that location. Such a network will be efficient for nearby locations, since they will also be surrounded by stations. Thus, it should not be necessary to include proximate locations as separate sources in the optimization. We examine this issue empirically in one of the examples.

The OPTINET program (Shimshoni *et al.,* 1992) is a convenient software package for finding D-optimal and DMS-optimal seismographic networks.

Scaled Criteria for Network Comparison

A simple rescaling of the D and DMS criteria facilitates the comparison of different networks. Consider first the single-source case. For a particular network configuration, let f_i denote the partial derivatives of the arrival times to the ith station in the network, and let $\boldsymbol{A}$ denote the matrix whose ith row is $\boldsymbol{f}_i$. The information matrix $\boldsymbol{A}^T\boldsymbol{A}$ can be written as

$$\boldsymbol{A}^T\boldsymbol{A} = \sum_1^N f_i f_i^T. \tag{7}$$

Since the information matrix is a sum of N single-station information matrices, it will typically be roughly proportional to the number of stations in the network. For example, doubling the number of stations in a network will approximately double the information matrix and will thus increase its determinant by a factor of about 2^4. For comparing configurations, it is convenient to formulate the criterion in a way that preserves the proportionality to the number of stations. The common device in the statistical literature is to scale the D criterion by taking the $1/p$th root of the determinant, where p is the dimension of the information matrix (Pukelsheim, 1993, p. 135). In our case, the scaled D criterion is

$$\mathrm{D} = [\det(\boldsymbol{A}^T\boldsymbol{A})]^{1/4}. \tag{8}$$

The scaled D criterion provides a convenient interpretation in terms of network size: if the scaled D criterion for network A is 20% larger than that achieved by network B, then network A is effectively providing the resolution power that would be achieved by adding 20% more stations to network B. We will refer to the scaled value (equation 8) as the D criterion in our discussion of the examples in the next section. We will measure the relative efficiency of two networks for monitoring a particular source by computing ratios of their D-criterion values.

In the multiple-source case, the simplest generalization of the scaling procedure is to define the DMS criterion to be

$$\mathrm{DMS} = \exp\left(\sum_1^k a_j \ln([\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)]^{1/4}\right) = \prod_1^k [(\det \boldsymbol{A}_j^T \boldsymbol{A}_j)^{1/4}]^{a_j}. \tag{9}$$

If the a_j have been scaled to sum to unity, the scaled DMS criterion is a generalized geometric mean of the scaled D-criterion values for the individual sources; larger weight is given in computing the geometric mean to point sources with larger values of a_j. Again, the effect of the scaling is such that the modified DMS-criterion value should be roughly proportional to the number of stations in the network. The relative efficiency of two networks for monitoring the same fault zone can be assessed by computing the ratio of their DMS-criterion values.

Note that the scaling conventions do not alter the optimal network configuration. A network will be optimal with respect to the scaled DMS criterion if and only if it is optimal with respect to the original (unscaled) DMS criterion. The unscaled criterion facilitates description of the algorithm, but the scaled criterion is more useful for evaluating and comparing networks. The near proportionality of the scaled criterion to the number of stations also provides a good basis for comparing networks of different sizes.

Alternative Criteria

Other generalizations of the D criterion to the multiple-source case might also be considered. Kijko (1977) considered optimal network configuration with multiple sources and proposed maximization of $\sum_1^k a_j[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)]$. Kijko's criterion treats the determinants as additive, which we think is undesirable. An immediate consequence is that the optimal station to add to a given network will be that which maximizes $\sum_1^k a_j[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)]v_{i,j}$. This criterion is similar in form to our equation (6). In fact, a first-order approximation to equation (6) for $v_{i,j}$ close to 0 gives $\sum_1^k a_j \ln(1 + v_{i,j}) \approx \sum_1^k a_j v_{i,j}$. With Kijko's criterion, the weight assigned to source j is effectively modified from a_j to $a_j[\det(\boldsymbol{A}_j^T \boldsymbol{A}_j)]$. Thus, sources that are monitored effectively by the current network are given added weight in selecting the next station site. The end result is that this criterion will produce networks that tend to overmonitor sources for which good coverage is possible but ignore sources that are difficult to monitor. Good criterion values can be achieved even if the determinants of the information matrices for some sources are close to 0. Logging the determinants, as in the DMS criterion, guarantees that the network will provide minimal coverage for all the sources.

Illustrative Examples

In this section, we present several examples to illustrate use of our method for multiple-source network configuration. We first present two constructed examples to demonstrate the method and provide insight into the nature of the solutions obtained. Table 1 shows the crustal model used in the constructed examples. The final example applies the method to a realistic setting of seismic monitoring in Israel. All the computations were carried out with the program OPTINET (Shimshoni *et al.,* 1992), developed by the authors. In all the examples, we compute standard deviations of hypocenter coordinates under the assumption that the standard deviation of the arrival times is $\sigma = 0.1$ sec.

Case 1

We consider the problem of monitoring three seismic point sources, two of which lie outside the region where stations can be placed. All of the sources are at a depth of 5 km. For simplicity, we assume that the stations must be located in a square region with sides 100-km long. There are 64 candidate sites, equally spaced on an 8×8 grid. (Some initial experimentation showed that there was almost no advantage in using a finer grid.)

Figures 1, 2, 3, 4, and 5 show the DMS-optimal networks with 6, 7, 8, 9, and 10 stations, respectively. The sources are indicated by triangles, the candidate station sites by the grid of points, and the sites in the networks by circles. For each network size, we found the best configuration possible, rather than adding one station to the previous network (both options are available in the OPTINET program). Nonetheless, some of the networks can be seen as one station additions to the previous network.

Table 2 shows the overall DMS value for each network, the D value for each source, and the standard deviation for each hypocenter coordinate. The DMS values increase in a slightly sublinear fashion with the addition of new stations.

Table 1
Crustal Model for Case 1 and Case 2

Layer	Thickness	*P*-Wave Velocity*
1	10 km	4.0
2	10 km	5.5
3	10 km	6.5
4	Half-space	8.0

*Velocity in km/sec.

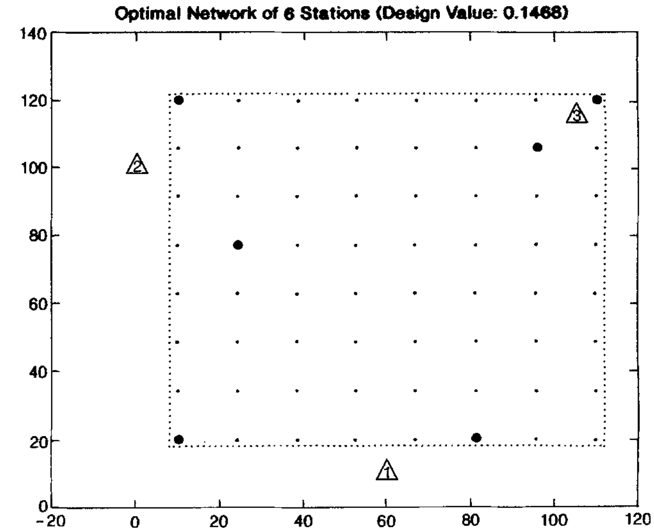

Figure 1. Optimal six-station network for monitoring three point sources. The candidate sites form an 8×8 grid. The sources are indicated by triangles and the sites included in the network by the solid circles. Distance units in km.

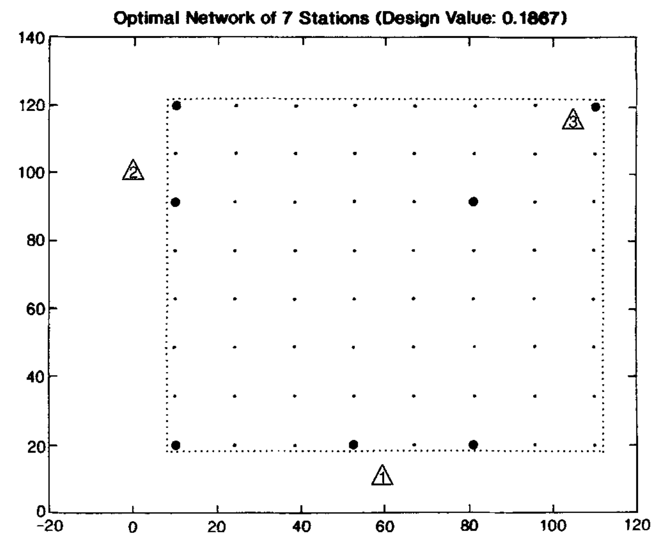

Figure 2. Optimal seven-station network for monitoring three point sources. The candidate sites form an 8×8 grid. The sources are indicated by triangles and the sites included in the network by the solid circles.

The D values for the individual sources increase much less smoothly, reflecting the fact that the network modifications may be much more beneficial to precise hypocenter determination at one source than another. This information can be of great value in assessing the marginal utility of adding a new station.

The major difference between the 6 and 7 station networks is the addition of a station on the southern border of the region, near Source 1. This is reflected in Table 2 by the

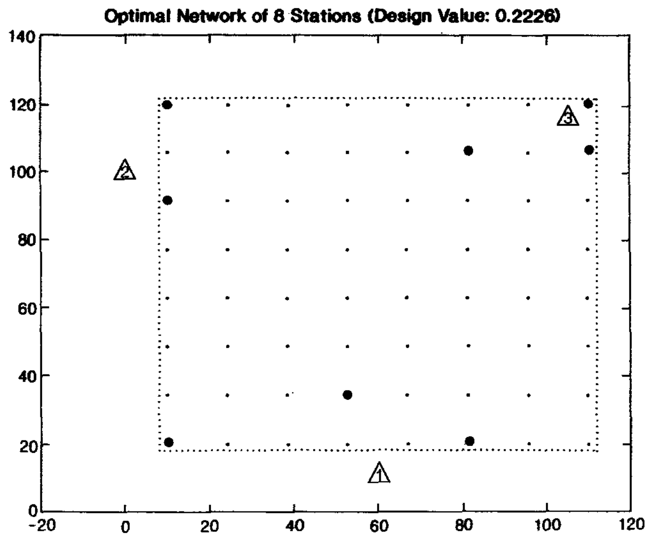

Figure 3. Optimal eight-station network for monitoring three point sources. The candidate sites form an 8 × 8 grid. The sources are indicated by triangles and the sites included in the network by the solid circles.

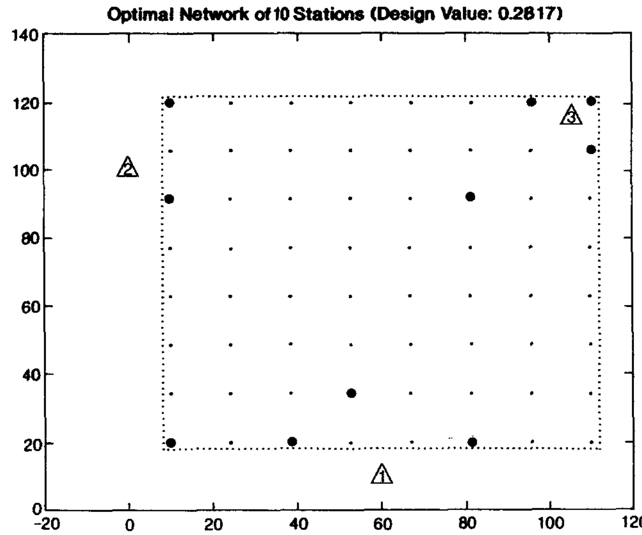

Figure 5. Optimal 10-station network for monitoring three point sources. The candidate sites form an 8 × 8 grid. The sources are indicated by triangles and the sites included in the network by the solid circles.

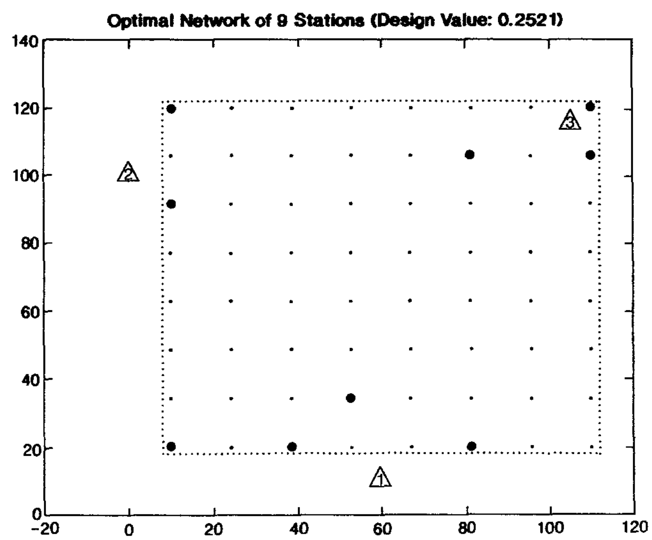

Figure 4. Optimal nine-station network for monitoring three point sources. The candidate sites form an 8 × 8 grid. The sources are indicated by triangles and the sites included in the network by the solid circles.

Table 2
Summary of Results for Optimal Network Configuration in Case 1

	Number of Stations				
	6	7	8	9	10
DMS criterion	0.147	0.186	0.222	0.252	0.281
Source 1					
D criterion	0.103	0.173	0.183	0.236	0.251
SD (X_0)	2.32	0.94	1.12	0.70	0.68
SD (Y_0)	15.51	8.91	6.34	5.48	5.14
SD (Z_0)	3.77	1.11	1.48	0.86	0.81
Source 2					
D criterion	0.160	0.183	0.197	0.210	0.223
SD (X_0)	7.71	6.54	6.52	6.16	5.96
SD (Y_0)	0.98	0.98	0.94	0.91	0.86
SD (Z_0)	1.48	1.21	1.03	1.03	0.95
Source 3					
D criterion	0.191	0.204	0.304	0.320	0.398
SD (X_0)	3.38	3.00	1.26	1.24	1.00
SD (Y_0)	3.41	2.82	1.36	1.35	0.94
SD (Z_0)	0.91	0.99	0.78	0.72	0.54

The standard deviations (SD) for the source coordinates are in kilometers.

large increase in the criterion value for Source 1. The standard deviations of the hypocenter parameters at Source 1 are roughly halved. Only slight improvements are found for the other sources.

The eight-station network adds a station in the northeast corner of the region, near Source 3. The major impact on the D values is improved estimation capability at that source, in particular with respect to the epicenter parameters, whose standard deviations decrease by a factor of more than 2. There is only a slight change in the SD for focal depth and for all parameters at the other sources.

The nine-station network modifies the configuration in the south central region, near Source 1. One station is added there and the locations are modified slightly. The major change in D value occurs at Source 1 and affects primarily the ability to precisely determine the X coordinate and the focal depth. The new station helps "surround" Source 1 along the X axis, hence, the large improvement there. But the practical constraints of the problem make it impossible to achieve the same effect on the Y axis, and SD (Y_0) decreases by just 14%.

The 10-station network adds a fourth station in the northeast corner, near Source 3. There is substantial improvement in precision for all the hypocenter parameters at Source 3, but little change occurs at the other sources.

Case 2

In this example, we show how the methodology can be used to select station sites for a network that must monitor a fault line. We assume that the fault is about 40-km long and runs from northwest to southeast, just northeast of the region where stations can be located. The candidate station sites form a 10 × 10 square grid with sides of 100 km. We wish to install an eight-station network that will efficiently monitor seismic activity along the entire length of the fault.

We compare several different ways to represent the fault by a sequence of point sources. We marked seven point sources along the fault and will refer to them as Source 1 (at the northwest end of the fault) to Source 7 (at the southeast end). Figure 6 shows the grid of candidate sites and Sources 1, 4, and 7, marked by triangles. We considered three possible sets of point sources to represent the fault: three point sources (1, 4, and 7) with equal weights, three point sources (1, 4, and 7) with double weight on source 4, and all seven point sources with equal weights.

All three schemes for representing the fault led to similar network configurations. Figure 6 shows the DMS-optimal configuration for the first scheme. Table 3 lists the D values for each of the seven sources under each of the three possibilities mentioned. The D values at each source are very similar for all three representations of the fault. Increasing the number of point sources in the representation has almost no effect on the ability to precisely locate events at each source. The practical implication of this finding is that a small set of sources will often be adequate to represent finite fault lines. We expect that more than three sources will be needed only if the fault is much longer, relative to the potential station site grid, than in our Figure 6.

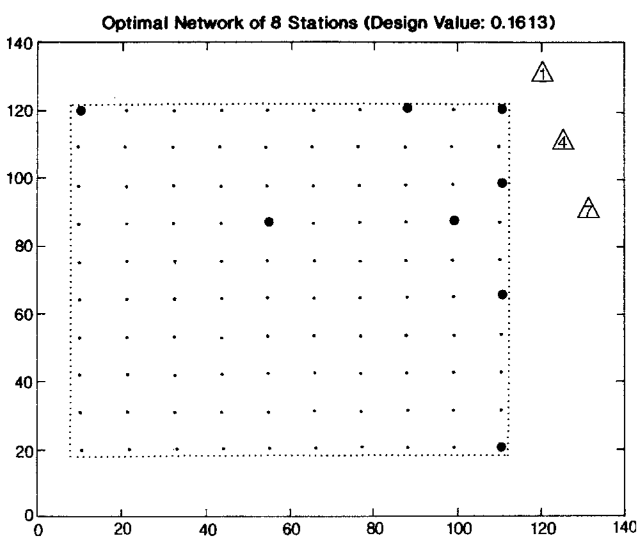

Figure 6. Optimal eight-station network for monitoring a fault line located northeast of the potential site area. The triangles mark the ends of the fault line and its midpoint and correspond to sources 1, 4, and 7, as indicated by the numbers. The candidate sites form a 10 × 10 grid, and the sites included in the network are indicated by the solid circles.

Table 3
Summary of Results for Optimal Network Configuration in Case 2

Source	Three Sources (Equal Wt.)	Three Sources (Unequal Wt.)	Seven Sources (Equal Wt.)	Single Source (D value)
1	0.116	0.120	0.116	0.123
2	0.133	0.135	0.130	0.140
3	0.153	0.159	0.170	0.178
4	0.176	0.183	0.183	0.188
5	0.183	0.192	0.193	0.202
6	0.187	0.197	0.199	0.207
7	0.203	0.189	0.192	0.213

The first three columns give the D value for each source under different schemes of weighting the fault for the DMS criterion. The final column lists the D value for each source for the D-optimal network that is devoted to that source alone.

The standard deviations for the hypocenter coordinates are also similar for all three networks. The source at the northwest end of the fault line is the most difficult to monitor. The epicentral coordinates at that source have standard deviations of about 14 km, and the focal depth has a standard deviation of about 6 km. Proceeding southeast along the fault line, the standard deviations steadily decrease. At the southeast end, the standard deviation for the X coordinate is about 6 km, that for the Y coordinate is about 1 km, and that for the focal depth is about 2 km. The poor precision at the northern end of the fault line arouses some concern. To enhance our comparison, we also computed D-optimal eight-station networks for each of the seven point sources along the fault. The respective D values are listed in Table 3. We find that the combined network achieves D values equal to at least 88% of the maximum for each of the point sources. Thus, the large standard deviations at the northern end of the fault line reflect a basic problem in monitoring (within the constraints we have placed on potential station sites), not a failure of the method to appropriately consider each of the sources. Relative to what can be achieved by an eight-station network, the DMS-optimal networks achieve near-maximal efficiency along the full length of the fault.

Case 3

We now turn from our constructed examples to a realistic problem in network configuration. Figure 7 shows the locations of 17 stations in the Israel Seismographic Seismic Network. These stations provide coverage of seismic events in northern Israel. Most of the seismic activity in that region is along the Bekaa valley (Jordan rift) and the Yagur-Carmel fault, which are also shown on the figure. We apply our methodology here to identify the most important stations for monitoring these two faults and examine whether some stations may be largely redundant, indicating that a smaller network might be able to provide nearly equal resolution power. We then test this assertion on recorded arrival times from an earthquake, comparing the solutions obtained with various subsets of the phase arrivals.

First consider what can be accomplished with just six stations, as might occur if there were a dramatic reduction in funds for seismic monitoring. The seismologist realizes that this task entails difficult trade-offs. Most of the seismic activity occurs along the Bekaa valley, and this must be a focus of interest. There is less activity on the Yagur fault, but its potential for catastrophic events cannot be ignored. The Yagur fault passes through the Haifa area, which is heavily populated and includes much heavy industry, and there was a strong earthquake ($M = 5.2$) on 24 August 1984 near the midpoint of the fault.

To help the seismologist select the sites for the six stations, we found optimal networks for monitoring each separate fault and a network that optimally monitors both faults. We represented each fault by three point sources, as indicated in Figure 7, and assigned equal weights to five of the sources and double weight to the source in the Mediterranean Sea, at the northwest end of the Yagur fault. The importance of this source is due to an observation of increased activity in this area during the last few years and because of its proximity to heavy industrial plants.

Figures 8 through 10 show the six stations selected for each individual fault and for monitoring both faults together. The most immediate observation is that the optimal network for monitoring both faults is identical to the network for the

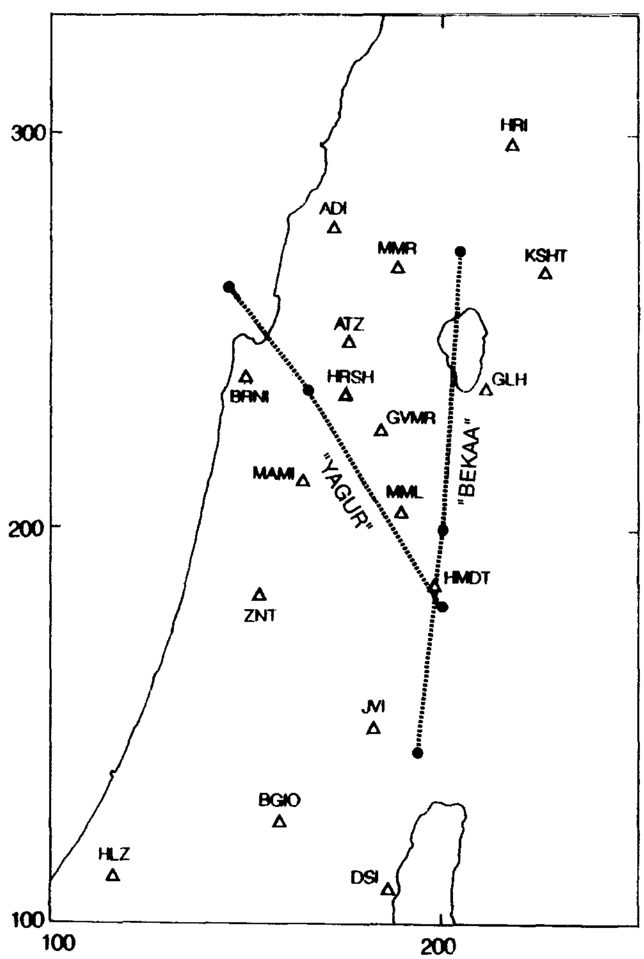

Figure 7. Seismographic stations in the Israel Seismographic Seismic Network (triangles) and the two major fault lines that threaten northern Israel, the Bekaa fault (which runs along the Jordan valley) and the Yagur-Carmel fault. For the purpose of optimal network design, we represent each fault by three point sources, which are marked as circles in the map. The source at the northwest end of the Yagur fault was given double weight in determining the network. Distance units in km.

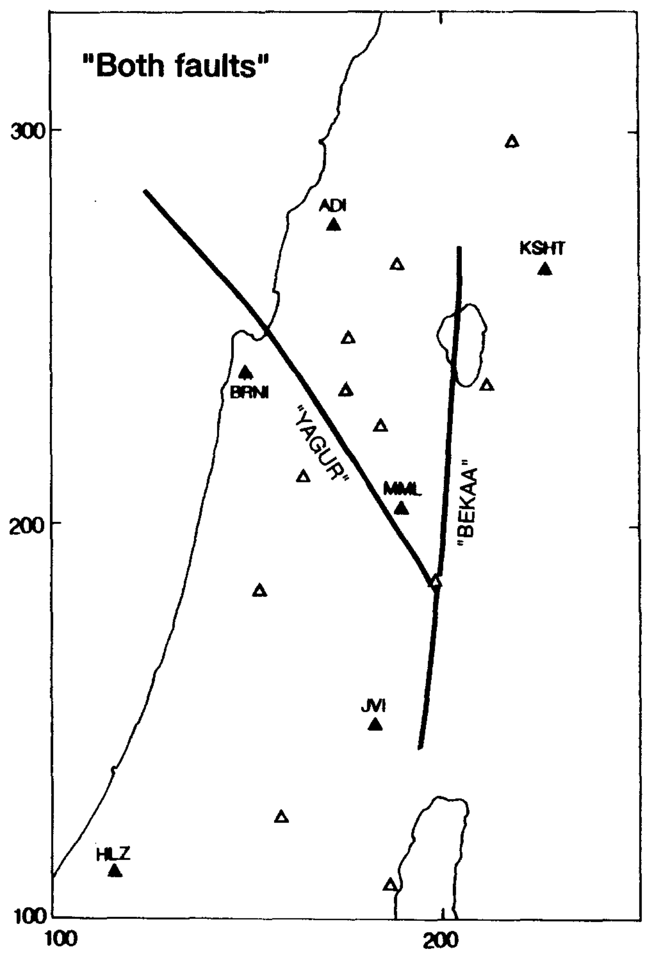

Figure 8. Optimal six-station network for monitoring both the Bekaa fault and the Yagur fault. The stations selected for the network are indicated by the solid triangles and the station labels. The remaining candidate sites are indicated by empty triangles.

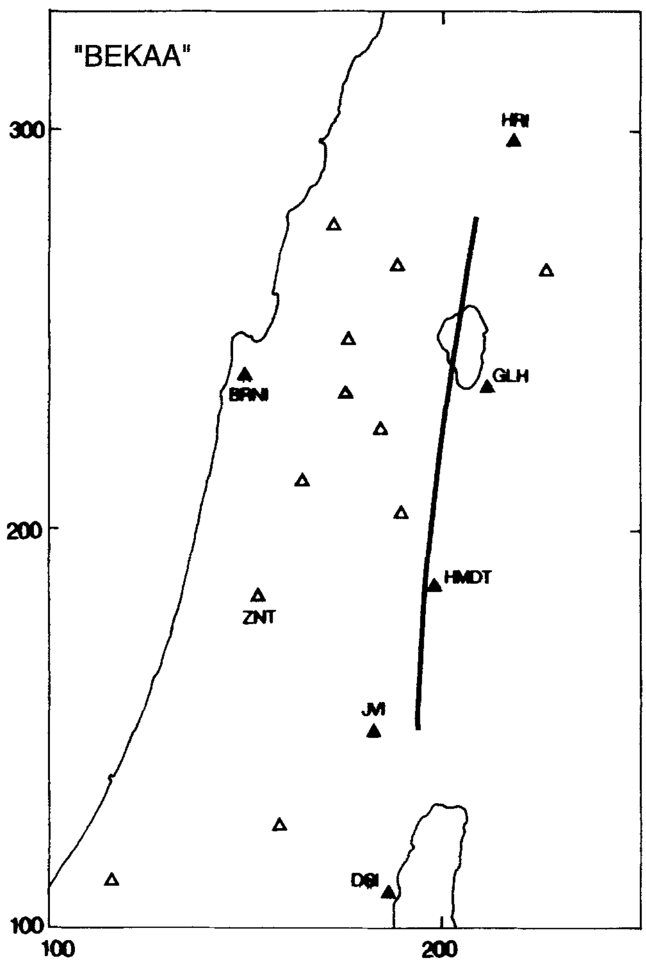

Figure 9. Optimal six-station network for monitoring only the Bekaa fault. The stations selected for the network are indicated by the solid triangles and the station labels. The remaining candidate sites are indicated by empty triangles.

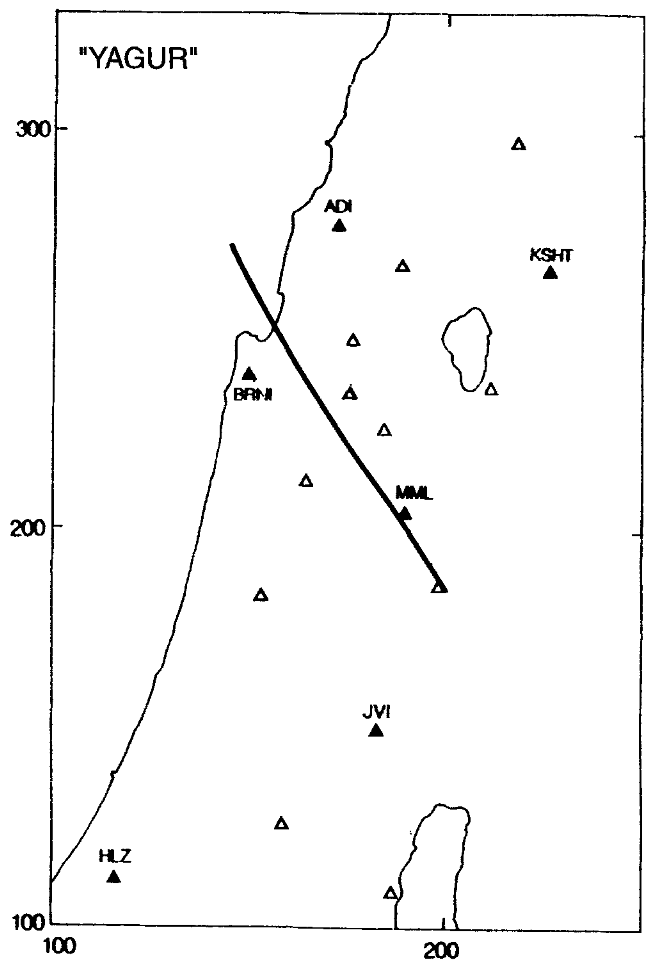

Figure 10. Optimal six-station network for monitoring only the Yagur fault. The stations selected for the network are indicated by the solid triangles and the station labels. The remaining candidate sites are indicated by empty triangles.

Yagur fault alone. Only one station, JVI, also appears in the optimal network for the Bekaa.

The D-criterion values achieved by the networks for each source are shown in Table 4. The Bekaa network provides poor coverage of the central and northern sources on the Yagur fault. Its efficiency relative to the Yagur network at these sources is 57% and 65%, respectively. The northern Yagur source, not surprisingly, is the most difficult to monitor. Even with the dedicated Yagur network, the standard deviations of the epicentral coordinates are 7.8 km (for X) and 3.2 km (for Y). The standard deviation of the focal depth is 1.6 km. With the Bekaa network, all these standard deviations are roughly twice as large. The central Yagur source is quite effectively monitored by the Yagur network, with all standard deviations less than 1.5 km. With the Bekaa network, all the standard deviations are substantially larger and that for focal depth increases from 1.4 to 8.9 km. The southern source on the Yagur fault is quite close to the Jordan rift and is actually monitored more efficiently by the Bekaa network. Both networks give small standard deviations for the Y coordinate and the focal depth but have some difficulty fixing the X coordinate ($SD(X_0) = 7$ km for the Yagur network and is 5.5 km for the Bekaa network).

Similar results hold for the Jordan rift sources. The networks provide almost equal coverage for the central source, which is close to both fault lines. Standard deviations for the hypocenter coordinates there range from 1.0 to 4.3 km. The Yagur network is not able to effectively monitor the northern and southern Bekaa sources. At the southern source, for example, the standard deviations are 28 km for the X coordinate and 10.7 km for the focal depth. However, even the dedicated Bekaa network has difficulty monitoring these sources. The corresponding standard deviations at the southern source are 16.1 and 10 km, respectively. In terms of the overall D value, the Yagur network is 74% efficient at the northern source and 70% efficient at the southern source. Comparison of the D-criterion values and the standard deviations does not indicate clear superiority for the Yagur network over the Bekaa network and suggests that the decision to double weight the northwest source in the Yagur

Table 4
Summary of Results for Network Configuration in Case 3 with Six Stations

	D values for Each Source When the Network is Configured to Monitor:		
	Bekaa Valley Only	Yagur Only	Both Faults
Bekaa Valley Sources			
North	0.178	0.131	0.131
Central	0.174	0.173	0.173
South	0.121	0.085	0.085
Yagur Sources			
North	0.097	0.150	0.150
Central	0.150	0.265	0.265
South	0.218	0.175	0.175

Table 5
Summary of Results for Network Configuration in Case 3 with Eight Stations

Stations Included When the Network Is Configured to Monitor:		
Bekaa Valley Only	Yagur Only	Both Faults
ADI	ADI	ADI
DSI	ATZ	BRNI
GLH	BRNI	DSI
HMDT	HLZ	GLH
HRI	HMDT	HLZ
JVI	JVI	HRI
MML	KSHT	JVI
ZNT	MML	MML

	D values for Each Source When the Network is Configured to Monitor:		
	Bekaa Valley Only	Yagur Only	Both Faults
Bekaa Valley Sources			
North	0.293	0.150	0.302
Central	0.297	0.281	0.212
South	0.168	0.100	0.175
Yagur Sources			
North	0.113	0.200	0.187
Central	0.191	0.358	0.320
South	0.305	0.259	0.226

fault plays an important role in determining the combined DMS-optimal network.

Suppose now that funds are made available for two additional stations. We repeated our analyses for eight-station networks. The results are shown in Table 5. We now see that the combined network includes stations from both of the individual fault networks. Three stations—ADI, JVI, and MML—appear in all the networks; the stations DSI, GLH, and HRI appear in the Bekaa and the combined networks; the stations BRNI and HLZ appear in the Yagur and the combined networks. Interestingly, station HMDT appears in both individual networks but is not in the combined network. This is quite plausible, since the resolution power of a station depends on the other stations in the network. The three stations in the combined network that are not in the Yagur network are spread out along the full length of the Bekaa fault, and two of the three are located to the east of the fault. These stations are so important to precise location of events on the Bekaa that they are included despite the extra weight given to the source in the sea, on the Yagur fault.

Comparison of the D values shows that the combined network is actually a bit better than the Bekaa network for monitoring the sources at either end of the rift. However, the combined network is 71% efficient for the central source due to the removal of stations HMDT and ZNT, which are located close to that source. The combined network is less efficient than the Yagur network for all three sources on that fault, with efficiencies, as measured by ratios of the D criterion, equal to 93.5%, 89%, and 87%, respectively. Thus, there is a moderate loss of precision all along the Yagur fault. Both single-fault networks are quite inefficient for two of the three sources on the second fault, just as we found with the six-station networks.

The major difference between the six-station and the eight-station combined networks is the inclusion of stations DSI and HRI. As already noted, these stations appear to be crucial for locating earthquakes on the Bekaa. This effect is also evident if we compare the D-criterion values and standard deviations. At the northern end of the Bekaa, the D criterion increases by 131% and at the southern end by 106%. All the standard deviations at the northern source are at most 1.2 km for the eight-station network. At the southern source, SD(X_0) is reduced from 28 to 6.6 km, SD(Y_0) from 13.4 to 1.2 km, and SD(Z_0) from 10.7 to 4.0 km. By contrast, the D-criterion values at all the remaining sources increase by 21% to 28%, slightly less than the 33% increase in the total number of stations. Most of the standard deviations decrease by about 20% relative to those for the six-station network. With the eight-station combined network, all of the standard deviations at the six indicated sources are less than 7 km and most are less than 1.5 km. Thus, the eight-station network is able to provide at least reasonable monitoring capability for the two faults.

As further stations are added, the DMS-criterion values continue to increase almost linearly with the number of stations. Thus, each station contributes to the overall precision of earthquake location. This finding does not agree with the accepted belief that a station will be largely redundant if it is located near an existing station.

We further illustrate our conclusions about the monitoring ability of a smaller network using data from a magnitude 3.6 earthquake that occurred on 24 April 1993 near the northern end of the Bekaa fault. Phase arrivals for this earthquake were recorded for all 17 seismographic stations shown in Figure 7, as well as 16 stations located further to the south in Israel. We located the event using several different subsets of stations: (a) all 17 stations in Figure 7; (b) the eight-station DMS-optimal network; (c) the six-station DMS-optimal network; (d) all 33 stations with recorded phase arrivals. The results are listed in Table 6. For each subset, the hypocenter was determined using Nelder and Mead's simplex algorithm (Rabinowitz, 1988).

Table 6
Hypocenter Locations for the Earthquake of 24 April 1993, Based on Different Subsets of Phase Arrivals

	X_0	Y_0	Z_0
17 stations in Figure 7	205.4	278.7	10.5
DMS-optimal eight-station network	206.0	278.6	8.1
DMS-optimal six-station network	203.0	269.7	0.0
All stations reporting	205.3	275.1	5.2

The estimated hypocenter location for the eight-station DMS-optimal network is quite similar to that for the full set of 17 stations. Both epicenter coordinates are within 0.6 km of the original solution and the focal depth differs by 2.4 km. By contrast, the solution for the six-station DMS-optimal network places the hypocenter on the surface, as opposed to the 10.5-km depth estimated by the set of 17 stations, and locates the epicenter 9 km farther south. Thus, the example reinforces our conclusion that the eight-station network can do an adequate job of monitoring both the Yagur and the Bekaa faults, while the six-station network provides poor coverage for the extreme ends of the Bekaa.

When all the phase arrivals are used to estimate the hypocenter, the results are surprisingly different from those obtained with just the 17 stations in north and central Israel. The estimated epicenter is 3.6 km farther south and, most noticeably, the focal depth changes from 10.5 to 5.2 km. What is responsible for these changes? Examination of residuals provides much insight. Large positive residuals (about 0.5 sec) are obtained at each of six stations in a geographically dense local array located in southwest Israel, about 70 km south of station HLZ. Most of the stations in central Israel have residuals of about -0.5 sec. Adjusting the hypocenter to fit the arrival times at the stations in the array results in a poorer fit at the stations in central Israel. The final hypocenter estimate is a compromise that does not fit well at any of these stations. Our interpretation is that local anomalies in the velocity structure in the region of the array bias the hypocenter estimates when all the phase arrivals are used. Thus, we believe that the 17-station solution is more accurate than the solution based on all the phase arrivals. This comparison emphasizes the fact that statistical uncertainty is not the only cause for inaccurate hypocenter locations; bias related to errors or anomalies in the crustal model coupled with an unfavorable station configuration also contribute to location errors. It is worth noting that the eight-station DMS-optimal network spreads the stations out and does not suffer from the same bias that occurs when all the phase arrivals are used.

Summary and Conclusions

Careful design of a seismographic network can guarantee precise estimates of hypocenters. In this article, we have proposed the DMS criterion for optimal configuration of a network that monitors a region with multiple sources and fault lines. Simple modifications to the DETMAX algorithm (Mitchell, 1974) allow for easy and efficient implementation of the multiple-source criterion. We have illustrated the usefulness of this approach on two constructed examples and on an example using the Israel Seismographic Station Network. Our work extends that of Rabinowitz and Steinberg (1990), who treated the problem of optimal configuration for a single source.

We think the DMS criterion is an important step forward in configuring efficient seismographic networks. However, further research is still needed in this area. In particular, one important practical consideration that we have not addressed here is detectability. We have assumed throughout that all stations in the network will record primary phase arrivals from all events. As a result, we have seen in the examples a tendency to favor stations that are rather distant from the sources. In practice, of course, the set of useful arrival times for microseismic events will typically be limited to stations that are close to the source. We are currently investigating several methods to reflect detectability in the network configuration criterion.

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