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Analysis of the impact on the gravity field determination from the data with the ununiform noise distribution using block-diagonal least squares method

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ABSTRACT

The block-diagonal least squares method, which theoretically has specific requirements for the observation data and the spatial distribution of its precision, plays an important role in ultra-high degree gravity field determination. On the basis of block-diagonal least squares method, three data processing strategies are employed to determine the gravity field models using three kinds of simulated global grid data with different noise spatial distribution in this paper. The numerical results show that when we employed the weight matrix corresponding to the noise of the observation data, the model computed by the least squares using the full normal matrix has much higher precision than the one estimated only using the block part of the normal matrix. The model computed by the block-diagonal least squares method without the weight matrix has slightly lower precision than the model computed using the rigorous least squares with the weight matrix. The result offers valuable reference to the using of block-diagonal least squares method in ultra-high gravity model determination.

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1. Introduction

The Earth's gravity field is the basic physical field of the Earth, which has an important role in geodesy and geophysics. Surface grid data such as the surface grid gravity anomaly data are crucial to the determination of the Earth's gravity filed. Numerical quadrature technique and least squares method are main methods to compute the gravity field using surface grid data. Different from numerical quadrature technique, least squares method can evaluate the precision of the computed model coefficients, and the least squares method is widely used in gravity field determination [1–6]. However in ultra-high degree gravity field determination, the number of parameters and the dimension of the normal matrix are huge. Under current computation conditions it might be impossible to use the least squares method directly. Fortunately, spherical harmonic functions are orthogonal. When the observation data and precision distribution of the data satisfy the specific requirements and one arranges the parameters in a particular sequence, the normal matrix in the least squares method is block-diagonal matrix. Using block-diagonal least squares method to compute the model coefficients, may greatly reduce the calculation quantity. This issue has been studied in some papers [4,7,8]. Colombo [7] firstly described the structure of the normal matrix. Pavlis [8] compared the block-diagonal least squares method with the numerical quadrature method in gravity filed determination. Li Xinxing [4] focused on the detailed application of the block-diagonal method in gravity field modeling. Lemoine [9], Pavlis [1] and Förste [10–12] all used the block-diagonal least squares method in gravity field determination.

The necessary condition of the block-diagonal least squares method for gravity field modeling is that the weight of surface grid data must be independent of longitude which means the weight at the same latitude line must be equal. But the actual observation data can't satisfy this condition, so the normal matrix is not block-diagonal, which brings difficulty to the using of block-diagonal method. In ultra-high gravity model determination, the computation amount is so huge that one has to use the block-diagonal least squares method to simplify the computation amount. Therefore, we intend to research on the data processing strategy for the situations when data don't satisfy the requirements and give some valuable advice to the using of block-diagonal least squares method. This paper firstly studies the forms of the normal matrix that formed when using least squares method, and then analyzes the results of three data processing strategies in gravity field determination through numerical simulation.

2. Basic principle

The relation between surface grid gravity anomaly observations and gravity model coefficients is [2]:

$$\Delta g_{ij}^{c} = \frac{GM}{r_{ij}^{2}} \sum_{n=2}^{\infty} (n-1) \left(\frac{R}{r_{ij}}\right)^{n} \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda_j + \overline{S}_{nm} \sin m\lambda_j) \overline{P}_{nm}(\cos\theta_i) \tag{1}$$

where (*i*,*j*) represent the grid on the sphere or ellipsoid surface at the *i*-th row and *j*-th column. $\overline{P}_{nm}(\cos\theta)$[13] is the normalized associated legendre function. $\overline{C}_{nm}, \overline{S}_{nm}$ are gravity model coefficients. Δg_{ij}^{c} is the gravity anomaly in which the ellipsoid correction, atmosphere correction, second-order normal gravity gradient correction and analytical downward continuation correction are taken, which satisfies the basic equation of boundary value problem. The derivation of this formula can be found in the reference [2].

Based on equation (1), one can form the observation equation and the normal equation using the least squares method [14]. Then the model coefficients can be computed based on the normal equation. The observation equation is:

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{x} - \boldsymbol{L} \tag{2}$$

Every parameter in equation (1) has a corresponding coefficient function, and the element of the design matrix **A** in equation 2 is the value of the coefficient function $\frac{GM}{r_{ij}^2}(n-1)\left(\frac{a}{r}\right)^n \begin{pmatrix} \cos \\ \sin \end{pmatrix} (m\lambda_j)\overline{P}_{nm}(\cos\theta_i)$ corresponding to a certain parameter taking at the (*i*,*j*) grid on the surface. The elements of matrix **A** in the same column are the values that a same coefficient function $\frac{GM}{r_{ij}^2}(n-1)\left(\frac{a}{r}\right)^n \begin{pmatrix} \cos \\ \sin \end{pmatrix} (m\lambda_j)\overline{P}_{nm}(\cos\theta_i)$ takes at all the surface grids. The element of the normal matrix **N** is the inner product of two columns in the design matrix **A**, which means that the element of the normal matrix **N** is the inner product on the surface of two coefficient functions corresponding to two parameters. If an element of the normal matrix **N** is the inner product on the surface of two coefficient functions corresponding to the parameter C_{nm}^{α} and C_{rs}^{β}, then the element can be written as:

$$[\mathbf{N}]_{C_{nm}^{\alpha} C_{rs}^{\beta}} = GM^2 (n-1)(r-1) \sum_{i=0}^{N-1} \overline{P}_{nm}^{i} \overline{P}_{rs}^{i} \sum_{j=0}^{2N-1} \frac{1}{r_{ij}^4} \left(\frac{a}{r_{ij}}\right)^{n+r} \begin{Bmatrix} C_m^j \\ S_m^j \end{Bmatrix} \begin{Bmatrix} C_s^j \\ S_s^j \end{Bmatrix} P_{ij} \tag{3}$$

In ultra-high degree gravity field determination, the dimension of the normal matrix is huge, so it is impossible to use the least squares method directly under current computation condition. When the observation data satisfies the specific requirements and one arranges the parameters in a particular sequence, block-diagonal least squares method can be used in the computation. At first one can divide the normal equation into several "block" equations due to the form of the normal matrix and then solve each "block" equation. The set of the solutions of the "block" equations are the solution of the normal equation. In this way the gravity field coefficients can be obtained. The block-diagonal least squares method needs much less amount of computation than the rigorous least squares method. To differentiate the two kinds of least squares methods in this paper, the least squares represents the rigorous least squares method and the block-diagonal least squares represents the block-diagonal least squares method. The particular sequences of the parameters in the block-diagonal least squares method can be seen in reference [8] and the requirements for the data can be seen in reference [8]:

(a) The data reside on a surface of revolution (e.g., a rotational ellipsoid),
(b) The grid is complete and the longitude increment is constant,
(c) The data weights are longitude independent, which means the data located at the same latitude line has the same precision.

Even though the block-diagonal least squares method is very efficient, it has its own defect, that is, the method is confined to dealing with the observation data that meet the requirements (a), (b) and (c). The gridded surface grid data can meet the requirements (a) and (b), but the requirement (c) which is about data precision is hardly to meet. When the requirement (c) is not satisfied, the normal matrix is not block-diagonal. Then the adoption of block-diagonal least squares method in gravity field determination brings error to the coefficients. But to simplify the computation amount, the block-diagonal least squares method has to be used in ultra-high degree gravity field determination.

There are three kinds of data processing strategies that can be used in determining the gravity field. They are:

Strategy 1: compute the model coefficients using least squares method with consideration of data weight matrix;

Strategy 2: compute the model coefficients using block-diagonal least squares method with consideration of data weight matrix;

Strategy 3: compute the model coefficients using the block-diagonal least squares method without consideration of data weight matrix.

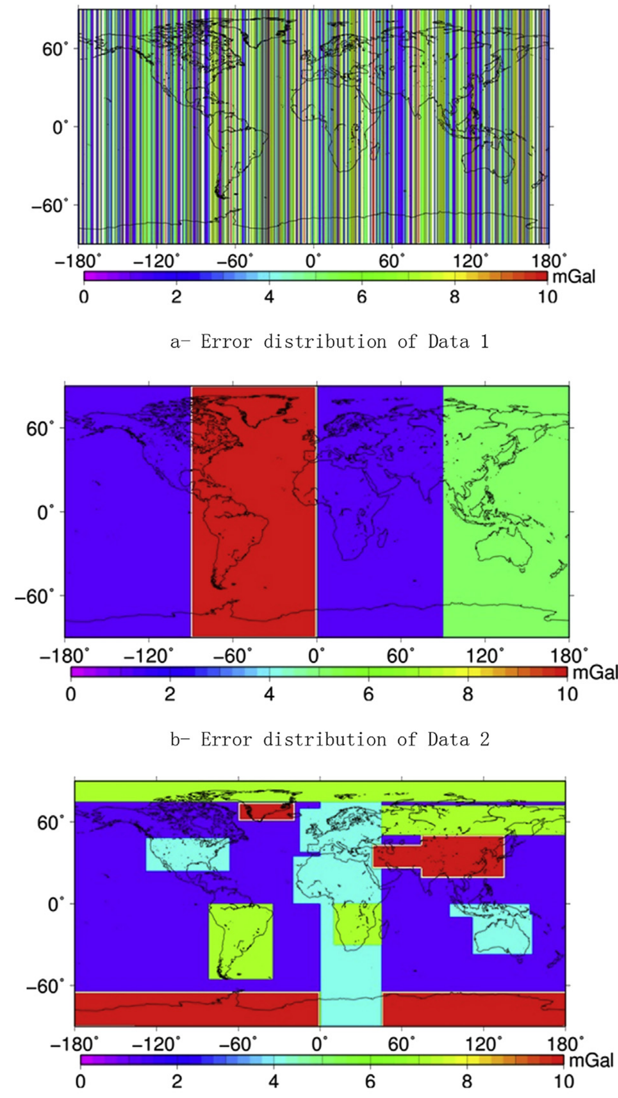
3. Numerical simulation analysis

To analyze the three data processing strategies mentioned above, 40' resolution surface grid gravity anomaly data on a 6378136.3 m radius sphere is simulated by the referenced gravity model EGM2008 which is truncated to degree 269. Three different kinds of observation data are formed by adding different white noises to the simulated data. They are:

Data 1: time sequences of noise with a fixed standard deviation are added to the grid data and the standard deviation corresponding to each longitude line is selected randomly from 1 mGal, 5 mGal and 10 mGal. The error distribution of Data 1 is shown in Fig. 1a;

Data 2: white noises with standard deviation of 1 mGal, 5 mGal, 1 mGal and 10 mGal are added to the grid data in the longitude interval [0°,90°], [91°,180°], [181°,270°] and [271°,360°], respectively. The error distribution of Data 2 is shown in Fig. 1b;

Data 3: referring to the error distribution of the surface gravity data in the determination of EGM2008, the sphere surface is divided into fourteen regions. White noises with



a– Error distribution of Data 1

b– Error distribution of Data 2

c– Error distribution of Data 3

Fig. 1 – The random error distribution of the data.

standard deviation of 1 mGal, 4 mGal, 7 mGal and 10 mGal are added to the simulated gravity anomaly data according to the location and the error distribution of Data 3 is shown in Fig. 1c.

Based on Data 1, Data 2 and Data 3, nine groups of gravity coefficients are determined employing three kinds of data processing strategies. Table 1 listed the data and strategy used in each gravity field model.

Table 1 – Coefficients computed using three kinds of data processing strategies and three kinds of data.

	Data 1	Data 2	Data 3
Strategy 1	CFR269	CFS269	CFC269
Strategy 2	CWGR269	CWGS269	CWGC269
Strategy 3	CBR269	CBS269	CBC269

The group of coefficients CFR269, CWGR269 and CBR269 in Table 1 are computed based on Data 1 using three different strategies. The degree root mean square (RMS) error and the spectral error of the coefficients are shown in Fig. 2 and Fig. 3, respectively. The value in the error spectral figure is the logarithm of the coefficient error's absolute value. Figs. 2 and 3 show that: the errors of coefficients computed using Strategy 1 are the smallest, the ones computed using Strategy 3 are slightly bigger. And the errors of coefficients computed using Strategy 2 are the biggest.

Even though the grid data in Data 1 have different errors at different longitude, the data errors are distributed randomly on the global scale due to the random standard deviation of the errors on a same longitude. Still, it can be inferred from the result that if considering the weight matrix of observation data, the error distribution of the data will completely destroy the block property of the normal matrix or the character of block dominating. That's why the coefficients computed employing Strategy 2 that only uses the block part of the normal matrix have relatively bigger error. Considering the optimal character of least squares method, one can get the best solution using Strategy 1, which is in accordance with the Figs. 2 and 3. It can be seen from the Figs. 2 and 3 that the coefficient model computed using Strategy 3 is very close to the one using Strategy 1, and the difference only grows bigger slowly from degree 170. The fact that difference mainly exists at high degree is due to the ignorance of the relative precision between the observations.

The groups of coefficients CFS269, CWGS269 and CBS269 in Table 1 are computed based on Data 2 using three different strategies, and the group of coefficients CFC269, CWGC269 and CBC269 in Table 1 are computed based on Data 3 using three different strategies. The degree error RMS of the coefficients is shown in Figs. 4 and 5. Just as Figs. 2 and 3, it can be drawn from Figs. 4 and 5 that the errors of coefficients computed using Strategy 1 are the smallest, the errors of coefficients computed using Strategy 3 are slightly bigger than the ones using Strategy 1, the errors of coefficients computed using Strategy 2 are the biggest. It can be inferred from the result that the error distribution of Data 2 and Data 3 also completely destroys the block property of the normal matrix or the character of block dominating, which is the same as Data 1, so the model computed using Strategy 2 have relative bigger error. The results of Strategy 1 and Strategy 3 are very close, but also due to the ignorance of the relative precision between the observations, the errors of the coefficients computed using Strategy 3 at degrees higher than 250 become bigger.

To analyze the error feature of the model computed employing Strategy 1 and Strategy 3, gravity anomaly data are computed based on the coefficients CFC269 and CBC269, respectively which are determined using Data 3. Then the differences of the computed gravity anomaly data with the gravity anomaly data simulated using EGM2008 are computed and the spatial distribution of the differences is shown in Fig. 6a and b. Fig. 6c shows the difference of the simulated gravity anomaly data based on CFC269 and CBC269 and Table 2 shows statistics of the differences. It can be inferred from Fig. 6a and b that the error of the gravity anomaly data computed employing Strategy 1 and Strategy 3 are very similar. The areas where the gravity anomaly observation errors are big have the big difference, which indicates that both the two models are closely fit to the observation data. Fig. 6c illustrates that the differences of gravity anomaly data computed using the two models are small and there is no obvious correlation between the differences and their locations.

One can get from Figs. 2–6 and Table 2 that the errors of coefficients computed using Strategy 2 are relatively big and this strategy can't be employed in gravity field determination. In some data error situations, the errors of coefficients computed using Strategy 3 are only little bigger than the ones using Strategy 1 and it's far smaller than the errors of the observation data. So taking account of the computation resource and observation data precision, Strategy 3 can be employed in the ultra-high degree gravity field determination.

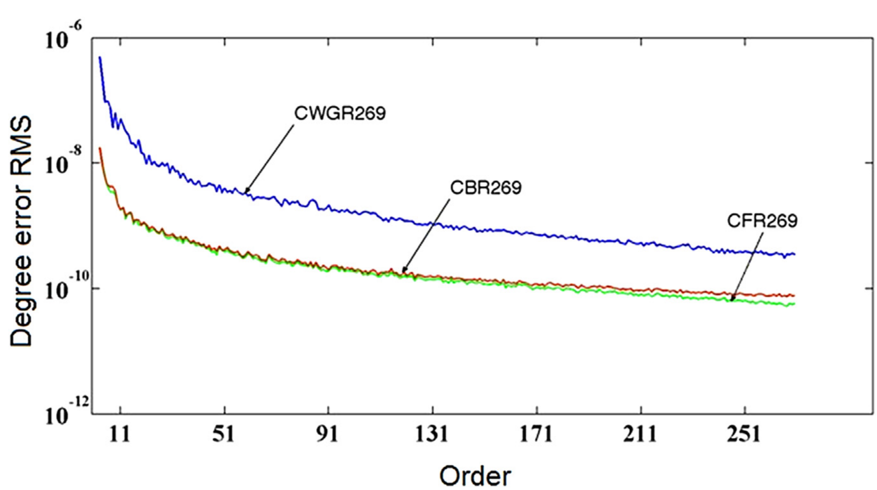
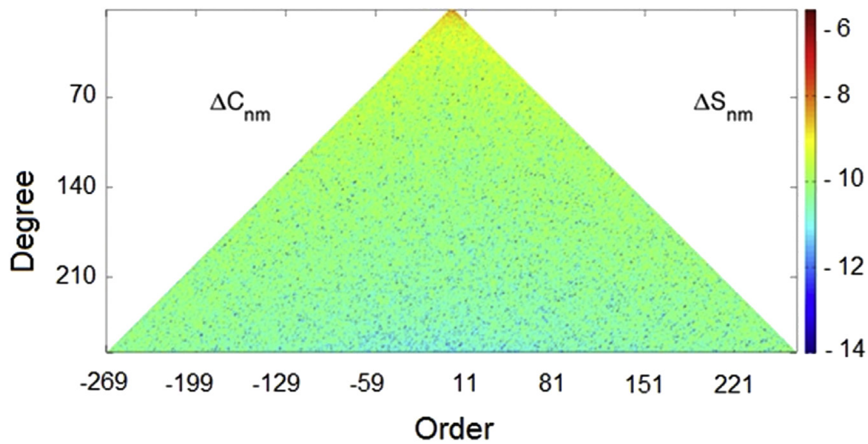
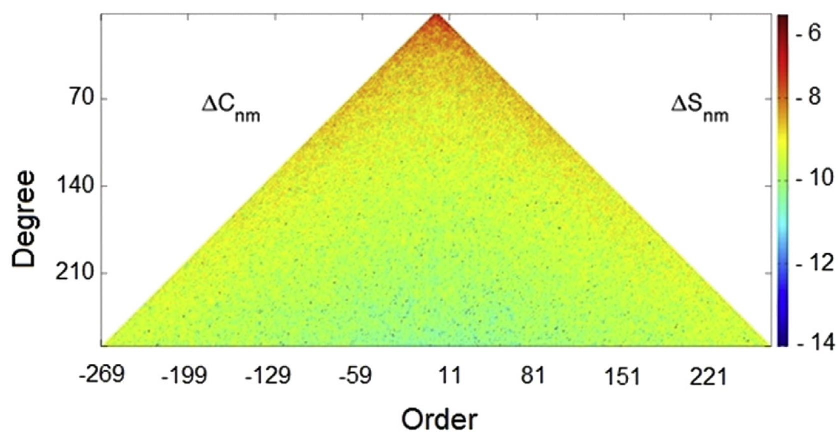


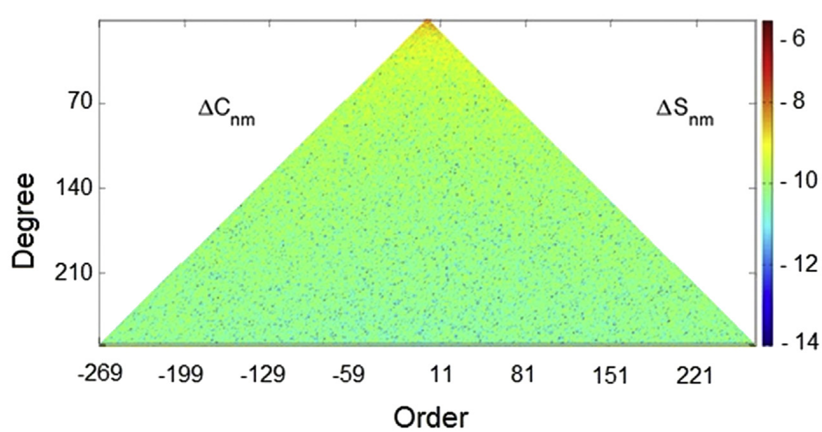
Fig. 2 – Degree error RMS of CFR269, CWGR269 and CBR269.



a– CFR269



b– CWGR269



c– CBR269

Fig. 3 – Spectral error of CFR269, CWGR269 and CBR269.

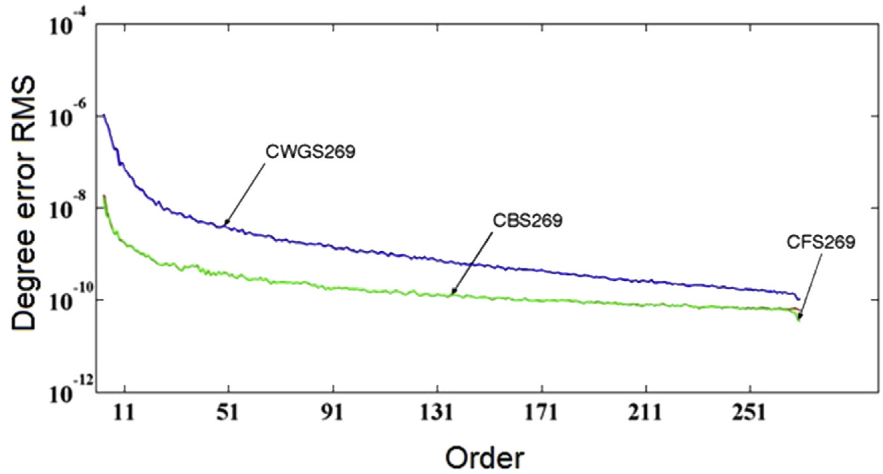


Fig. 4 – Degree error RMS of CFS269, CWGS269 and CBS269.

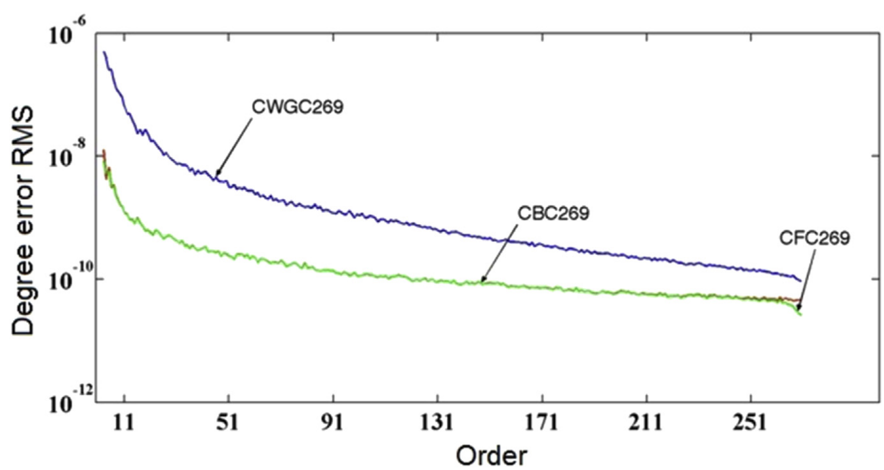


Fig. 5 – Degree error RMS of CFC269, CWGC269 and CBC269.

4. Conclusions

The data processing strategies in gravity field determination are analyzed in this paper by numerical simulation analysis under different kinds of data precision conditions. It shows that: (1) The coefficients computed using rigorous least squares method with consideration of data weight matrix have the highest precision; (2) The coefficient errors computed using block-diagonal least squares method with consideration of weight matrix which only use the block part of the normal matrix are relatively bigger, this kind of strategy can't be employed in gravity field determination; (3) The precision of the coefficients determined using block-diagonal least squares method without consideration of data weight matrix is slightly lower than that using rigorous least squares method with consideration of the weight matrix. So taking account of the computation resource and observation data precision, this strategy can be employed in the ultra-high degree gravity field determination.

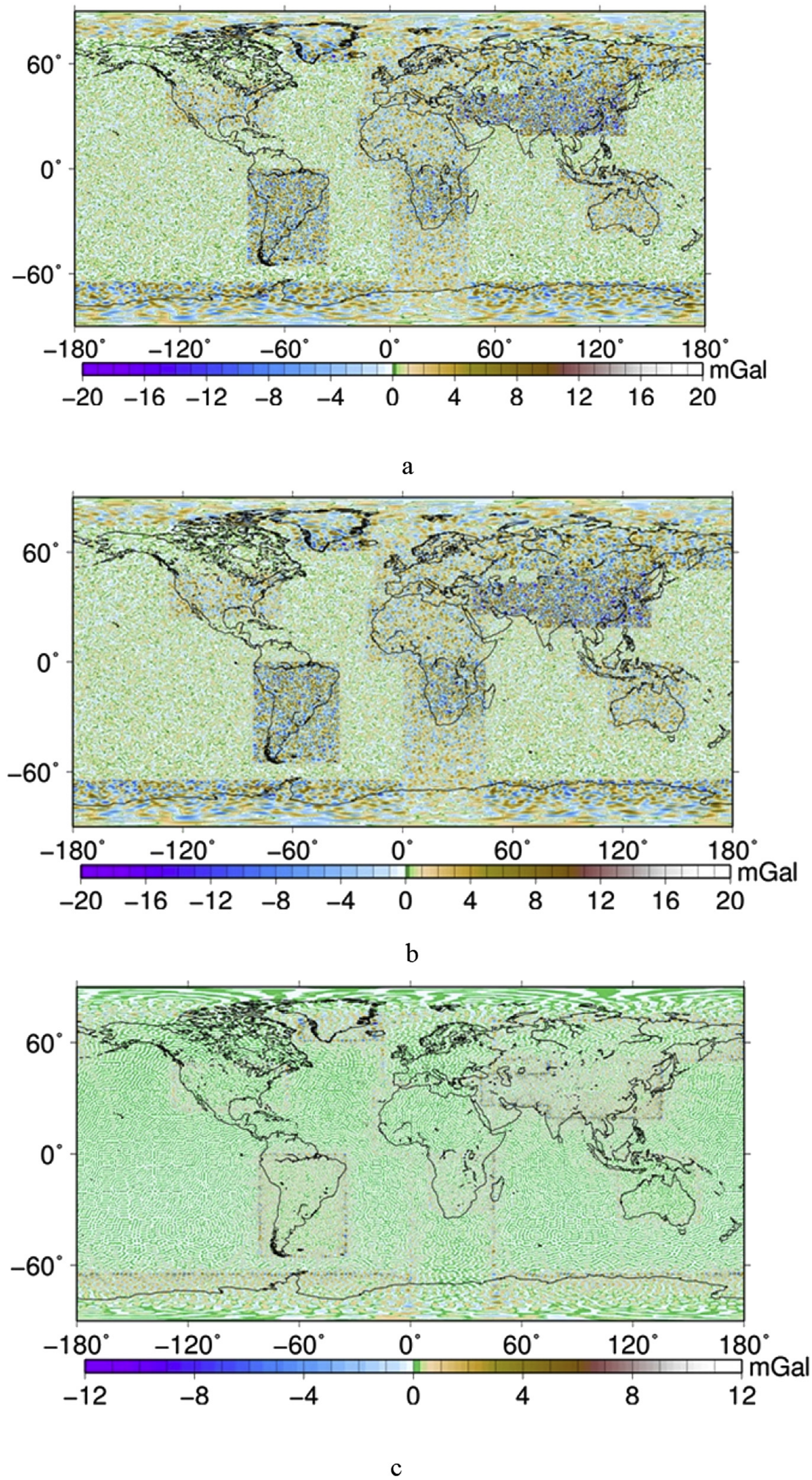


Fig. 6 – Gravity anomaly differences simulated from different gravity models (a – differences between CFC269 and EGM2008; b – differences between CBC269 and EGM2008; c – differences between CFC269 and CBC269).

Table 2 – The statistics of gravity anomaly differences (mGal).

Coefficients	Max	Min	Mean	RMS
CFC269 and EGM2008	28.4318	−30.0420	−0.00056	3.0858
CBC269 and EGM2008	29.5556	−30.8179	−0.0058	3.1688
CFC269 and CBC269	10.1723	−13.1377	0.0052	0.7213

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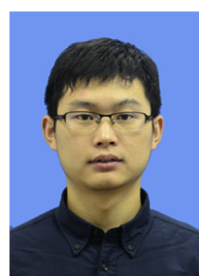
REFERENCES

[1] Pavlis NK, Holmes SA, Kenyon SC, Factor JK. The development and evaluation of the Earth gravitational model 2008 (EGM2008). J Geophys Res 2012;117(04406).
[2] Rapp RH, Pavlis NK. The development and analysis of geopotential coeffcient models to spherical harmonic degree 360. J Geophys Res 1990;95:21855–911.
[3] Li JC, Chen JY, Ning JS, Chao DB. Approximation theory of the earth gravity field and determination of the Chinese gravity geoid model 2000. Wuhan: Wuhan University Press; 2006.
[4] Li XX, Wu XP, Li SS, Liu XG, Cui ZW. The application of block-diagonal least squares methods in geopotential model determination. Acta Geod Cartogr Sin 2014;43(8):778–85.
[5] Ning JS. The progress in the Earth's gravity field in China. Northeast Surv Mapp 2002;25(4):6–9.
[6] Wang ZT, Dang YM, Chao DB. Theory and methodology of ultra-high degree geopotential model determination. Beijing: Surveying and Mapping Press; 2011.
[7] Colombo OL. Numerical methods for harmonic analysis on the sphere. Columbus: The Ohio State University; 1981.
[8] Pavlis NK. Modeling and estimation of a low degree geopotential model from terrestrial gravity data. Columbus: The Ohio State University; 1988.
[9] Lemoine FG, Kenyon SC, Factor JK, Trimmer RG, Pavlis NK, Chinn DS, et al. The development of the joint NASA GSFC and the national imagery and mapping agency (NIMA) geopotential model EGM96. Washington, D. C: NASA Goddard Space Flight Cent; 1998.
[10] Förste Ch, Schmidt R, Stubenvoll R, Flechtner F, Meyer U, König Rolf, et al. The GeoForschungsZentrum Potsdam/Groupe de Recherche de Gèodésie Spatiale satellite-only and combined gravity field models: EIGEN-GL04S1 and EIGEN-GL04C. J Geod 2008;82:331–46.
[11] Förste Ch, Bruinsma S, Marty JC, Flechtner F, Abrikosov O, Dahle C, et al. EIGEN-6C3 stat- the newest high resolution global combined gravity field model based on the 4th release of the GOCE direct approach. 2013. AGU 2013 Fall Meeting (San Francisco, USA).
[12] Förste Ch, Bruinsma SL, Abrikosov O, Lemoine JM, Schaller T, Götze HJ, et al. EIGEN-6C4 the latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse. Paris: 5th GOCE User Workshop; 2014. 11.25.– 28.11.
[13] Wellenhof BH, Moritz H. Physical geodesy. Wien New York: Springer; 2006.
[14] The Course Group of Surveying Adjustment in the School of Geodesy and Geomatics in Wuhan University. Error theory and foundation of surveying adjustment (the second edition). Wuhan: Wuhan University Press; 2009.



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