Surface Density Values for the Earth from Satellite and Gravity Observations

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Summary

The representation of the Earth's gravity field as the potential of a simple layer distributed over the surface of the Earth is determined by combining satellite observations and gravity anomalies. Density values of the simple layer for 192 surface elements are computed and converted into harmonic coefficients up to the 15th degree and order. These coefficients are used to determine surface density values referred to a reference ellipsoid with the flattening of an earth in hydrostatic equilibrium. The geophysical implications of these values are outlined.

Introduction

Recent determinations of the gravity field of the Earth by means of satellites (Kaula 1966; Smithsonian Institution 1966; Anderle 1967) have been used to interpret the sources of the geopotential (Kaula 1969; Moberly & Khan 1969). In these studies the gravity field was expressed by gravity anomalies. However, for geophysical interpretation a representation of the gravity field by means of the potential of a simple layer distributed over the surface of the Earth seems to be more suitable than one by gravity anomalies, since the computed density values allow both quantitative and qualitative interpretation of mass surplus or deficiency within the Earth. Density anomalies have already been computed from satellite results by Schwiderski (1968) under the assumption of a fluid mantle. However, the density values of a simple layer can be obtained without any hypotheses.

The representation of the Earth's gravity field by means of the potential of a simple layer has been proposed for satellite geodesy by Koch (1968) and applied to optical satellite observations by Koch & Morrison (1970). In this application 48 density values of the simple layer for 48 surface elements of about 30° × 30° were determined from satellite observations. In addition gravity anomalies were combined with the satellite observations to make a second determination of the 48 density values. For this computation only gravity anomalies found from a combination of satellite results and terrestrial data (Rapp 1968a) were available. In the meantime, however, gravity anomalies and their standard deviations collected by the Department of Geodetic Science of the Ohio State University have been provided by the Air Force Cambridge Research Laboratories. These gravity anomalies are given as mean values for 2592 5° × 5° surface elements. One thousand four hundred

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and seventy values are obtained from gravity measurements and the rest are model anomalies (Rapp 1968b). The standard deviations of the measured anomalies vary between ± 2 and ± 19 mgal, the standard deviations of the model anomalies are ± 20 mgal.

To incorporate these gravity anomalies, the combination of satellite observations and gravity data mentioned above is repeated, to determine the gravity field of the Earth represented by density values of a simple layer potential. However, instead of 48 density values for $30° \times 30°$ blocks 192 density values for elements of an approximate size of $15° \times 15°$ are determined here, in order to take more advantage of the information about the gravity field coming from the gravity measurements. Since the gravity anomalies are given for $5° \times 5°$ surface elements, density values for smaller than $15° \times 15°$ blocks could have been computed. However, as shown by Koch (1970), the results of the combined solution would not be improved with smaller surface elements, since the significant contribution to the determination of the gravity field comes from the satellite solution which uses $30° \times 30°$ surface elements.

Computational method

In the satellite solution mentioned above the optical satellite observations were a function of the unknown density values and the unknown orbital elements of the satellites being observed. Since approximate values for the density and the orbital elements were known, Taylor's series was applied to obtain the observation equations for the least squares adjustment (Koch & Morrison 1970, equation (4))

$$[\mathbf{A}, \mathbf{B}] \begin{vmatrix} \Delta\chi \\ \Delta\mathbf{e} \end{vmatrix} = \mathbf{l} + \mathbf{v}$$

where $\mathbf{A}$ and $\mathbf{B}$ are matrices of coefficients, $\Delta\chi$ the vector of the 48 corrections $\Delta\chi_i$ to the approximate density values for $30° \times 30°$ surface elements, $\Delta\mathbf{e}$ the vector of corrections to the orbital elements of the observed satellites, $\mathbf{l}$ the observation vector, and $\mathbf{v}$ the vector of residuals. The normal equations of the least squares adjustment for $\Delta\chi$ and $\Delta\mathbf{e}$ are obtained by means of the covariance matrix Σ_l of the observations $\mathbf{l}$

$$\begin{bmatrix} \mathbf{A}^T \Sigma_l^{-1} \mathbf{A} & \mathbf{A}^T \Sigma_l^{-1} \mathbf{B} \\ \mathbf{B}^T \Sigma_l^{-1} \mathbf{A} & \mathbf{B}^T \Sigma_l^{-1} \mathbf{B} \end{bmatrix} \begin{bmatrix} \Delta\chi \\ \Delta\mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T \Sigma_l^{-1} \mathbf{l} \\ \mathbf{B}^T \Sigma_l^{-1} \mathbf{l} \end{bmatrix}. \tag{1}$$

The values $\Delta\chi_i$ ($i = 1, ..., 48$) may be represented as mean values of four corrections $\Delta\bar{\chi}_{ij}$ ($j = 1, ..., 4$) of density values for surface elements of the approximate size of $15° \times 15°$ into which the $30° \times 30°$ blocks are divided

$$\Delta\chi_i = c_{i_1} \Delta\bar{\chi}_{i_1} + c_{i_2} \Delta\bar{\chi}_{i_2} + c_{i_3} \Delta\bar{\chi}_{i_3} + c_{i_4} \Delta\bar{\chi}_{i_4}. \tag{2}$$

In matrix notation we write

$$\Delta\chi = \mathbf{C}\Delta\bar{\chi} \tag{3}$$

with

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 & 0 & 0 & ... \\ 0 & 0 & 0 & 0 & c_{21} & c_{22} & c_{23} & c_{24} & ... \\ ... \end{bmatrix}$$

and

$$\Delta\bar{\chi}^T = |\Delta\bar{\chi}_{11}, \Delta\bar{\chi}_{12}, \Delta\bar{\chi}_{13}, \Delta\bar{\chi}_{14}, \Delta\bar{\chi}_{21}, \Delta\bar{\chi}_{22}, \Delta\bar{\chi}_{23}, \Delta\bar{\chi}_{24}, ...|$$

The coefficients c_{ij} are obtained by dividing the surface element associated with $\Delta\bar{\chi}_{ij}$ by the surface element for $\Delta\chi_i$, so that the mean value $\Delta\chi_i$ is formed by applying weights proportional to the size of the surface element for $\Delta\bar{\chi}_{ij}$ (Koch 1970).

With (3) we obtain instead of (1)

$$
\begin{bmatrix} \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{AC} & \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{B} \\ \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{AC} & \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{B} \end{bmatrix} \begin{bmatrix} \Delta\bar{\chi} \\ \Delta\mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{l} \\ \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{l} \end{bmatrix}.
\tag{4}
$$

This system of normal equations is singular, since not $\Delta\bar{\chi}$ but $\Delta\chi$ has been determined by satellite observations. However, by adding the contribution of the gravity anomalies to the density values we obtain a non-singular system.

Density values for $5° \times 5°$ surface elements are computed from the given gravity anomalies Δg for $5° \times 5°$ blocks by (Koch & Morrison 1970, equation (16))

$$
\chi_{5 \times 5} = \frac{\Delta g - G}{2\pi} + \frac{3}{(4\pi)^2} \int \int (\Delta g - G) \, S(\psi) \cos\phi \, d\phi \, d\lambda
\tag{5}
$$

where

$$
G = \frac{1}{4\pi} \int \int \Delta g \cos\phi \, d\phi \, d\lambda.
$$

$S(\psi)$ is Stokes' function, ψ the spherical distance between the fixed point and the variable point, and ϕ and λ the geographic latitude and longitude. Mean values $\bar{\chi}$ for the density of $15° \times 15°$ surface elements are formed again by applying weights proportional to the size of the surface elements

$$
\bar{\chi} = \mathbf{DF} \, \Delta\mathbf{g}.
\tag{6}
$$

$\mathbf{F}$ denotes the matrix by which according to (5) the anomalies Δg have to be multiplied to obtain the density values for $5° \times 5°$ blocks and $\mathbf{D}$ the matrix by which the mean values for the $15° \times 15°$ elements are computed from the $5° \times 5°$ blocks. From the covariance matrix $\boldsymbol{\Sigma}_{\Delta g}$ which is diagonal and has the standard deviations of the anomalies Δg on its diagonal, we find the covariance matrix $\boldsymbol{\Sigma}_\chi$ for the density values $\bar{\chi}$

$$
\boldsymbol{\Sigma}_\chi = \mathbf{DF} \, \boldsymbol{\Sigma}_{\Delta g} \, \mathbf{F}^T \mathbf{D}^T.
\tag{7}
$$

When analysing the satellite observations according to (1), the anomalies published by Rapp (1968a) were used to compute approximate density values. The gravity anomalies now being used lead to different density values for the $15° \times 15°$ surface elements. If the difference between these values is denoted by $\bar{\chi}_0$, we finally obtain the 192 corrections $\Delta\bar{\chi}$ to the density values from the combination of satellite observations and gravity measurements by least squares adjustment

$$
\begin{vmatrix} \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{AC} + \boldsymbol{\Sigma}_\chi^{-1} & \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{B} \\ \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{AC} & \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{B} \end{vmatrix} \begin{vmatrix} \Delta\bar{\chi} \\ \Delta\mathbf{e} \end{vmatrix} = \begin{vmatrix} \mathbf{C}^T \mathbf{A}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{l} + \boldsymbol{\Sigma}_\chi^{-1} \bar{\chi}_0 \\ \mathbf{B}^T \boldsymbol{\Sigma}_l^{-1} \mathbf{l} \end{vmatrix}.
\tag{8}
$$

Results

The variance of unit weight of the least squares adjustment (8) equals 1.44, and the number of degrees of freedom is equal to 8932. The standard deviations for the density values lie between ± 1.23 mgal and ± 0.14 mgal. The absolute values for the correlation coefficients for $\Delta\bar{\chi}_{ij}$ obtained from the inverted system (8) are smaller than 0.6. The standard deviations computed by the error propagation (7) vary between ± 1.45 mgal and ± 0.24 mgal. Hence, the accuracy of the density

Table 1

Normalized, harmonic coefficients to (15, 15)

n	m	$\bar{C}_{nm} \times 10^6$	$\bar{S}_{nm} \times 10^6$	n	m	$\bar{C}_{nm} \times 10^6$	$\bar{S}_{nm} \times 10^6$
2	0	-484.0664		11	3	.0192	.0127
2	1	-.0368	.1267	11	4	-.0856	-.0177
2	2	2.5984	-1.1877	11	5	.0399	-.0247
3	0	.6924		11	6	-.0586	-.0624
3	1	1.3537	.0112	11	7	.0107	-.0234
3	2	.7431	-1.1494	11	8	-.0046	-.0660
3	3	.4069	1.8872	11	9	.0167	.0644
4	0	.5893		11	10	.0182	.0005
4	1	-.4494	-.4338	11	11	.0494	.0426
4	2	.2918	.6287	12	0	.0377	
4	3	.9194	-.6375	12	1	.0354	-.0709
4	4	-.4078	.8734	12	2	-.0140	-.0126
5	0	-.0342		12	3	.0535	.0259
5	1	-.3367	-.0691	12	4	.0063	-.0352
5	2	.5450	-.5116	12	5	.0172	-.0169
5	3	-.2088	-.1431	12	6	.0254	.0322
5	4	.2697	-.8084	12	7	-.0295	-.0189
5	5	.0294	-.3942	12	8	-.0044	-.0107
6	0	.1233		12	9	-.0431	.0052
6	1	-.3514	.2825	12	10	.0022	-.0025
6	2	.0790	-.2447	12	11	.0138	.0267
6	3	.2014	-.8119	12	12	.0038	.0418
6	4	-.1517	-.3009	13	0	.0622	
6	5	-.2234	-.4729	13	1	.0168	-.0115
6	6	-.0169	-.4156	13	2	.0244	.0120
7	0	-.2293		13	3	.0153	.0067
7	1	-.1136	.1826	13	4	-.0068	-.0101
7	2	.1157	.0920	13	5	.0298	-.0045
7	3	.1859	-.0367	13	6	-.0025	.0020
7	4	-.3577	.2642	13	7	-.0103	.0428
7	5	.0401	.0933	13	8	.0279	-.0244
7	6	-.3000	-.2109	13	9	-.0128	.0813
7	7	.0862	-.0024	13	10	-.0480	-.0232
8	0	.1272		13	11	-.0033	-.0027
8	1	-.0909	.0286	13	12	.0142	.0205
8	2	.0768	-.0221	13	13	-.0377	.0334
8	3	-.0515	.1723	14	0	-.0073	
8	4	-.0168	.0801	14	1	.0204	-.0347
8	5	-.0030	.0713	14	2	.0030	.0226
8	6	-.1302	.2038	14	3	-.0275	-.0083
8	7	-.0261	-.1867	14	4	.0361	-.0323
8	8	.0093	.1035	14	5	.0098	.0357
9	0	-.1344		14	6	.0131	.0105
9	1	.0550	-.0452	14	7	.0337	.0082
9	2	.0887	.0172	14	8	-.0189	-.0051
9	3	-.0871	.0290	14	9	.0166	.0648
9	4	-.0388	-.1471	14	10	-.0166	-.0060
9	5	-.0963	.0492	14	11	.0158	-.0230
9	6	-.0867	.0572	14	12	.0199	-.0036
9	7	.0311	.0441	14	13	-.0015	.0212
9	8	.0960	-.2775	14	14	-.0260	-.0148
9	9	.0602	.0370	15	0	.0275	
10	0	.0933		15	1	.0434	-.0371
10	1	-.0089	-.0144	15	2	-.0050	-.0107
10	2	-.0297	-.0633	15	3	-.0342	.0066
10	3	-.0545	-.0173	15	4	-.0409	.0188
10	4	-.0960	-.0321	15	5	.0063	.0160
10	5	-.0451	-.0147	15	6	-.0084	-.0174
10	6	-.0329	.0400	15	7	-.0162	.0356
10	7	-.0148	.0234	15	8	.0023	.0171
10	8	.0298	-.0227	15	9	-.0015	.0157
10	9	.1002	.0867	15	10	.0015	-.0118
10	10	.0471	-.0264	15	11	-.0191	-.0073
11	0	-.2565		15	12	.0387	.0120
11	1	.0206	-.0146	15	13	-.0050	.0081
11	2	.0169	-.0304	15	14	-.0183	.0030
				15	15	-.0223	.0080

values could not be considerably improved by combining satellite observations and gravity anomalies. This indicates that the $15° \times 15°$ surface elements are sufficiently small for the combination solution. To investigate the influence of the model anomalies on the solution, a combined solution was completed with standard deviations of ± 40 mgal instead of ± 20 mgal assigned to the model anomalies. The results were not considerably changed.

The density values $\Delta\bar{\chi}_{ij}$ were then used to compute the normalized harmonic coefficients $\bar{C}_{nm}$ and $\bar{S}_{nm}$ of the expansion of the geopotential in spherical harmonics,
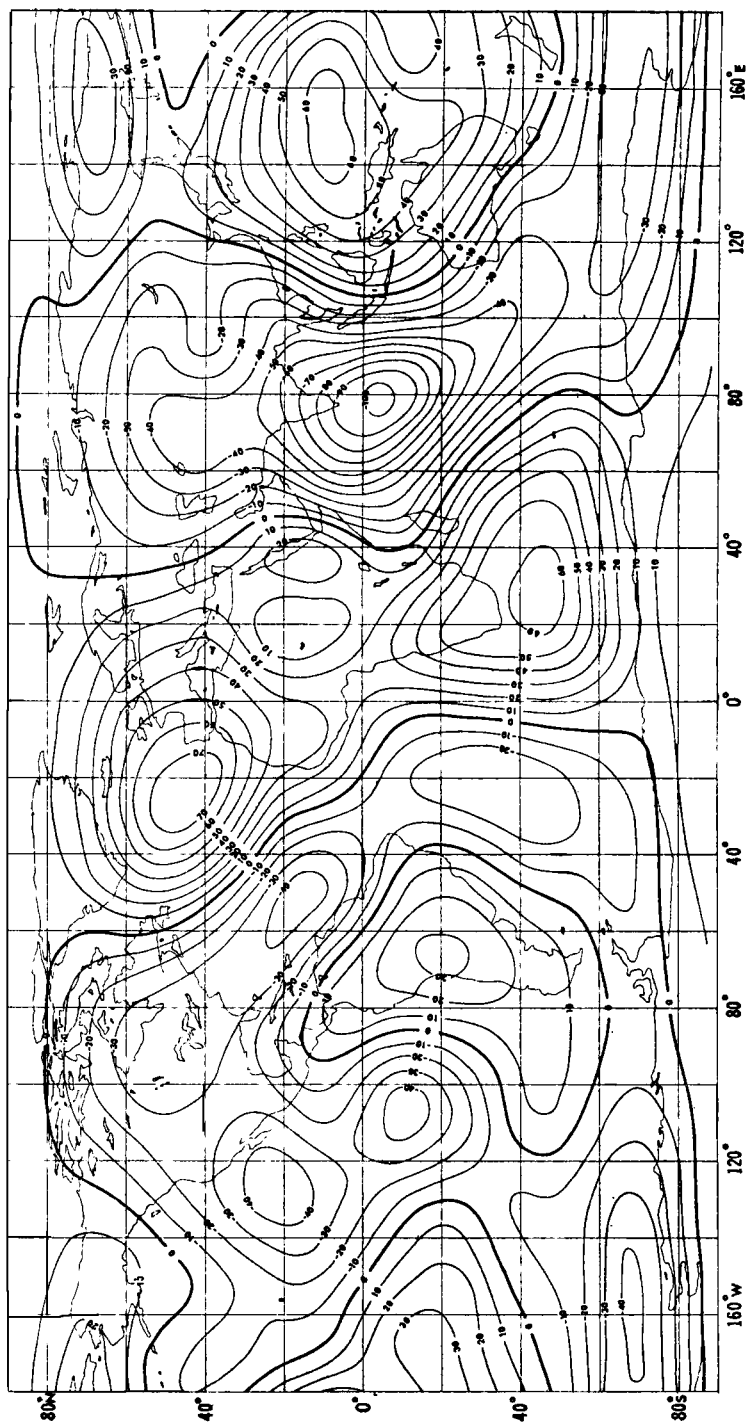
FIG. 1. Geoid undulations in metres referred to the best-fitting ellipsoid.

where n is the degree and m the order of the expansion. Although an arbitrary number of harmonic coefficients may be computed with a given set of density values (Koch 1968), it is obvious that the number of harmonic coefficients should not be much greater than the number of the computed density values. A development up to the 15th degree and order is chosen here. The coefficients are given in Table 1. For this expansion the values of the Smithsonian Institution (1966)

$$kM = 3 \cdot 986\,013 \times 10^{14}\,\mathrm{m^3\,s^{-2}}; \quad a = 6\,378\,155\,\mathrm{m} \tag{9}$$

have been assumed, where kM is the product of the gravitational constant and the mass of the Earth, and a the equatorial radius of the Earth. Using the harmonic coefficients in Table 1, the geoid shown in Fig. 1 has been computed. It refers to the best fitting ellipsoid, which is defined by the values for kM and a in equation (9), $\bar{C}_{20} = -484 \cdot 0664 \times 10^{-6}$ in Table 1, and the rotational velocity of the Earth. Its flattening equals $1 : 298 \cdot 286$ which is slightly too small in comparison with other satellite results. The reason for this is the incomplete coverage of the Earth with gravity anomalies, which gives insufficient results for the low order harmonics.

The harmonic coefficients and the geoid are compared with the solution of the Smithsonian Institution (1966) and the one of Anderle (1967) which were obtained from satellite observations, and with the results of Rapp (1968a, Table 1) and Koch & Morrison (1970, Table 3) which are found by combining satellite observations and gravity measurements. The rms discrepancy between the common coefficients of these solutions and the ones of Table 1 is per coefficient $\pm 0 \cdot 21 \times 10^{-6}$, $\pm 0 \cdot 29 \times 10^{-6}$, $\pm 0 \cdot 15 \times 10^{-6}$, $\pm 0 \cdot 13 \times 10^{-6}$. The rms discrepancy between the geoid heights computed at $10°$ intervals is $\pm 16 \cdot 1\,\mathrm{m}$, $\pm 17 \cdot 1\,\mathrm{m}$, $\pm 15 \cdot 4\,\mathrm{m}$, $\pm 10 \cdot 0\,\mathrm{m}$. Table 2 shows the anomaly degree variances

$$\sigma^2_{\Delta g,\,n} = \gamma^2 (n-1)^2 \sum_{m=1}^{n} (\bar{C}_{nm}{}^2 + \bar{S}_{nm}{}^2) \tag{10}$$

for the solutions mentioned above. γ denotes the normal gravity. The comparison shows that the results of the combination of satellite data and gravity measurements agree very well with previous solutions.

Table 2

Anomaly degree variances $\sigma^2_{\Delta g,n}$ in $mgal^2$

n	Smithsonian Institution (1966)	Anderle (1967)	Rapp (1968a)	Koch et al. (1970)	Coefficients in Table 1
2	7·2	8·0	7·1	7·5	7·9
3	32·7	40·1	30·4	34·4	30·1
4	17·0	22·4	16·2	26·4	26·7
5	18·3	21·9	12·3	20·0	25·0
6	18·8	32·2	14·5	27·1	36·8
7	13·6	33·3	9·4	11·0	17·5
8	11·9		6·7	7·2	8·5
9			5·3		11·0
10			6·7		4·3
11			3·3		9·5
12			4·5		2·5
13			4·4		3·1
14			5·6		2·7
15					2·5

Density values

The density values determined here do not refer to a reference ellipsoid but to a reference surface obtained by an expansion into spherical harmonics up to the 4th degree and order (Koch & Morrison 1970, equation (8)). Instead of computing directly the density values with respect to a reference ellipsoid by means of integral equations and adding them to the values determined here, the harmonic coefficients of Table 1 are used to compute the density values by assuming a spherical surface of the Earth.

Let U be the potential of a level ellipsoid, which is defined for instance by the values for kM, for a, for the second zonal harmonic $\bar{C}_{20}$, and for the rotational velocity ω of the Earth. Then the geopotential W is given by

$$W = U + T. \tag{11}$$

The disturbing potential T is represented by the potential of a simple layer with the density χ distributed over the surface Σ of the Earth

$$T = k \iint_{\Sigma} \frac{\chi}{r} d\Sigma \tag{12}$$

r is the distance between the fixed point and the variable point, $d\Sigma$ the surface element of the Earth and k as before the gravitational constant. If we assume a spherical surface of the Earth, we obtain the well-known integral equation for the density χ, if there are no zero-order harmonics in the gravity anomalies Δg

$$2\pi k \chi - \tfrac{3}{4} k \int \int \frac{\chi}{\sin(\psi/2)} \cos \phi \, d\phi \, d\lambda = \Delta g. \tag{13}$$

By substituting (12) we find with $r = 2R \sin(\psi/2)$ where R is the mean radius of the Earth

$$k\chi = \frac{\Delta g}{2\pi} + \frac{3}{4\pi} \frac{T}{R}. \tag{14}$$

The density χ of the simple layer can now be computed by using the expansion into spherical harmonics

$$T = \frac{kM}{R} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \bar{P}_{nm}(\sin \phi)(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \tag{15}$$

and

$$\Delta g = \frac{kM}{R^2} \sum_{n=2}^{\infty} \sum_{m=0}^{n} (n-1) \bar{P}_{nm}(\sin \phi)(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \tag{16}$$

where $\bar{P}_{nm}(\sin \phi)$ is the normalized associated Legendre function and where $\bar{C}_{20}$ and $\bar{C}_{40}$ have to be referred to the chosen reference ellipsoid.

Geophysical implication

The density values given here are referred to an equipotential ellipsoid whose values for kM, a, and ω are identical with the values obtained for the Earth. If we also take $\bar{C}_{20}$ as determined for the Earth, we refer the density values to the best fitting ellipsoid. The density values in Fig. 2 are referred to this ellipsoid, that means $\bar{C}_{20}$ equals the value given in Table 1.
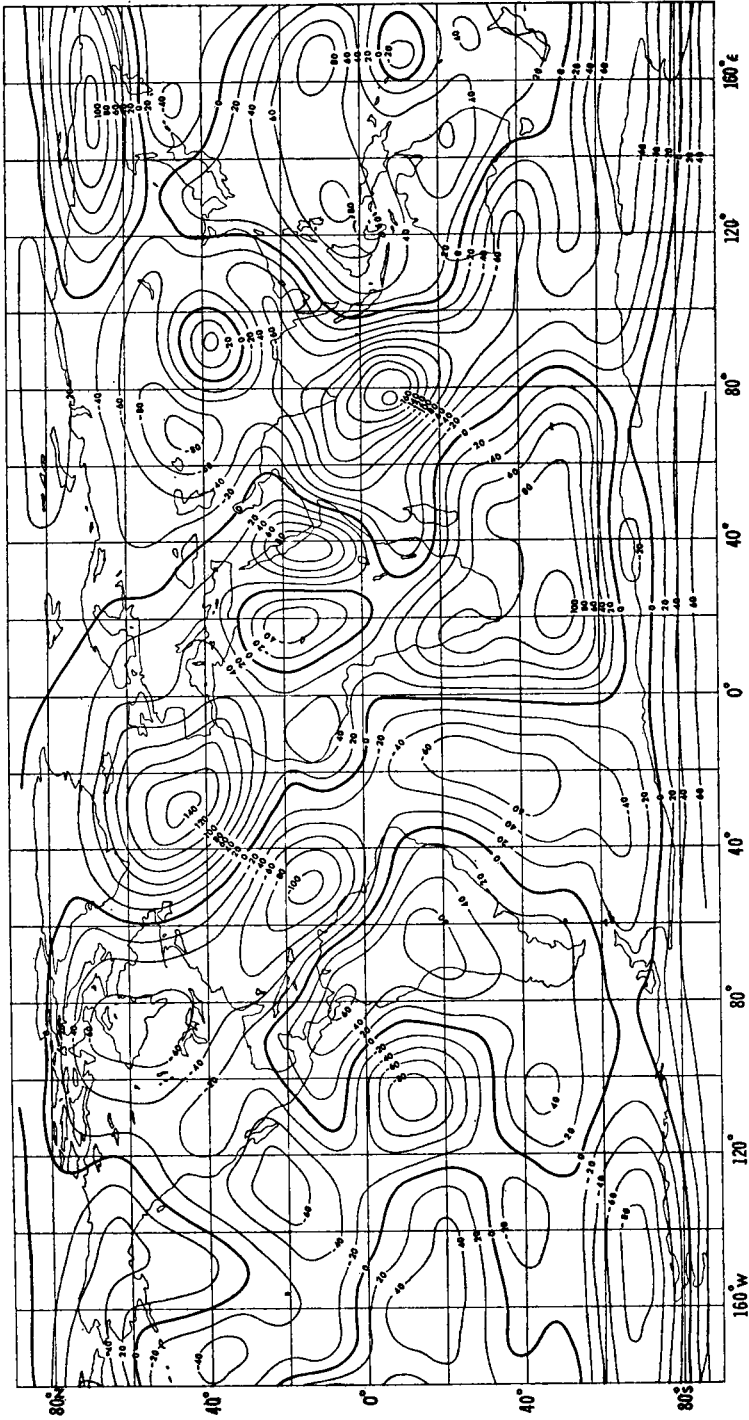
FIG. 2. Density values, referred to the best-fitting ellipsoid, in g cm^{-3} for a surface layer of 10 m thickness.
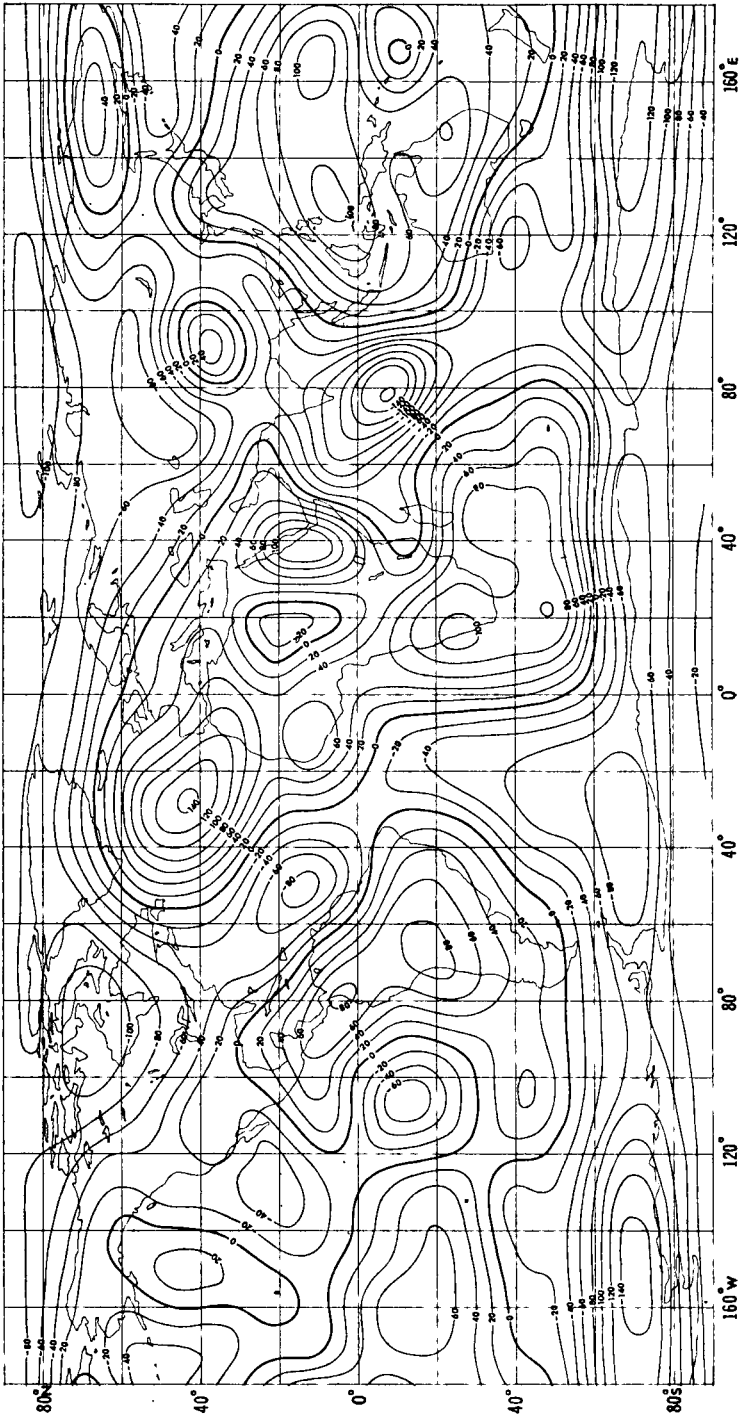
FIG. 3. Density values, referred to an equilibrium figure of the Earth, in g cm^{-3} for a surface layer of 10 m thickness.

For geophysical interpretation, however, the reference ellipsoid should have the shape of the Earth in hydrostatic equilibrium, since stress differences in the Earth's mantle arise from the difference between the actual shape of the Earth and the theoretical one for hydrostatic equilibrium (O'Keefe & Kaula 1963). This argument has been recently supported by Goldreich & Toomre (1969) who argue that the Earth seeks out a polar axis such that the products of inertia are dampened. The rotational bulge follows the polar wandering and therefore the present deviation of the Earth's flattening from the one of a hydrostatic equilibrium figure cannot be connected with a faster spin rate of the Earth in the past. In Fig. 3 the density values are referred to an ellipsoid with the second zonal harmonic for a fluid earth (Jeffreys 1964)

$$\bar{C}_{20} = -479 \cdot 458 \times 10^{-6}.$$

The flattening of this ellipsoid equals 1 : 299·67.

The units of the dimension of the density of a simple layer are by definition $g\,cm^{-2}$. To come to any meaningful interpretation of the computed density values we have to assume that the simple layer is spread out over a layer of finite thickness, in order to obtain the units $g\,cm^{-3}$ for the density. In Figs 2 and 3 a layer of 10 m thickness is chosen.

The comparison between the density values of Figs 2 and 3 shows that the absolute amount of the density values is considerably changed close to the poles while it is only slightly altered at the equator. However, the areas of positive and negative density values and the difference in the density values of adjacent areas have almost not changed at all. Thus, the flattening of the reference figure is not too important for the geophysical interpretation. For the following discussion we consider only the values of Fig. 3. These values represent density anomalies since the mass distribution of the equipotential ellipsoid, whose mass equals the mass of the Earth, has been subtracted. One possible mass distribution of a level ellipsoid is, for instance, a homogeneous core and a nearly homogeneous mantle (Moritz 1968).

Just from looking at Fig. 3 it becomes evident that there is no correlation between the density anomalies and the topography. The variation of the density has no connection with the change of the continents and oceans over the Earth's surface. The sources of the density variations have to lie at a considerable depth which becomes obvious from the following considerations. The largest differences between density values of adjacent areas appear between East Africa and the Indian Ocean, and between the Indian Ocean and the area north of New Guinea. These differences equal $260\,g\,cm^{-3}$. They are unrealistically large so that the simple layer must be assumed to be spread over a layer thicker than 10 m. With a layer 10 km thick the above mentioned variations in the density equal $0 \cdot 26\,g\,cm^{-3}$; they are still too large. Assuming a thickness of 100 km, we obtain the maximal density variations of $0 \cdot 026\,g\,cm^{-3}$ which are very reasonable. Hence, density variations, the sources of the variations of the gravity field, go as deep as the lithosphere which is assumed to be about 100 km thick.

Toksöz, Chinnery & Anderson (1967) concluded by means of seismic data that the sources for the satellite-determined gravity field lie even deeper than 100 km, and Moberly & Khan (1969) correlated the map of the gravity anomalies with the map of the asthenosphere, thus finding evidence for the new global tectonics founded on the hypotheses of continental drift, sea-floor spreading, and underthrusting of the lithosphere at island arc (Isacks, Oliver & Sykes 1968). Under such an assumption positive density values have to be found in areas of trenches and island arcs where the colder, and therefore denser crust is shoved into the mantle. Negative density values have to be expected in the areas of ocean ridges where material convecting out of the asthenosphere causes the sea-floor spreading.

The density anomalies of Fig. 3 are positive in the area of deep trenches east of the Philippines and north of New Guinea and there are negative density values at the Mid-Indian Ocean Ridge. Furthermore, a low can be found in the area of the ridges in the Pacific Ocean and a high along the trench off the west coast of South America. Positive density anomalies are encountered in the area of the Himalayas where the Indian shield underthrusts the Himalayas (Le Pichon 1968), and in the area of the Tonga and Kermadec Trench in the South Pacific Ocean. There is another high south of Alaska; however it does not follow the Aleutian Island arc.

In the South Atlantic Ocean negative density values are situated in the area of the Mid-Atlantic Ridge, however in the North Atlantic Ocean the Mid-Atlantic Ridge goes through the area with the maximal amount of positive density values. Positive gravity anomalies in the area of the Mid-Atlantic Ridge of the North Atlantic Ocean have been found not only by satellites but also by sea-borne gravity measurements (Talwani & Le Pichon, 1969). Another feature of the density values which cannot be explained by means of the new global tectonics is the high over the Red Sea where negative values are to be expected, since the bottom of the Red Sea is assumed to be spreading. The negative density anomalies over the Canadian shield are another feature unexplainable by continental drift. These values, however, can be interpreted by the uplift of the Canadian shield due to the isostatic compensation after the icecap over the shield had melted.

Although many features of the map of density anomalies of Fig. 3 support the theory of continental drift and sea-floor spreading, there are other features which cannot be explained by this hypothesis. Hence it must be concluded that not only density variations in the asthenosphere but also density anomalies in the lithosphere cause the variations of the gravity field although at the present time only the long-wave components of the gravity field are determined by satellite observations and their combination with gravity measurements.

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