Journal of Geodesy (2003) 76: 641–652
DOI 10.1007/s00190-002-0302-5

Journal of Geodesy

A Monte-Carlo technique for weight estimation in satellite geodesy

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Received: 11 April 2002 / Accepted: 1 October 2002

Abstract. Precise orbit determination, and satellite-geodetic applications such as gravity field modelling or satellite altimetry, rely on different observation types and groups that have to be processed in a common parameter estimation scheme. Naturally, the choice of the relative weights for these data sets as well as for added prior information is of importance for obtaining reliable estimates of the unknown parameters and their associated covariance matrices. If the observations are predominantly affected by random errors and systematic errors play a minor role, variance component models can be applied. However, most of the methods proposed so far for variance component estimation involve repeated inversion of large matrices, resulting in intensive computations and large storage requirements if more than a few hundred unknowns are to be determined. In addition, these matrices are not necessarily provided as standard output from common geodetic least-squares estimation software. Therefore, a method is proposed which is based on Monte-Carlo estimation of the redundancy contributions of disjunctive observation groups. The method can handle unknown variance components without the need for repeated inversion of matrices. It is computationally simple, numerically stable and easy to implement. Its application is demonstrated in an experiment concerning low-medium-degree gravity field recovery from simulated orbit perturbations of the GOCE mission, and compared in performance with Lerch's method of subset solutions.

Keywords: Weight estimation – Variance component estimation – Satellite geodesy – Gravity field modelling – Monte-Carlo method

1 Introduction

Precise orbit determination, and satellite-geodetic applications and techniques such as global and regional gravity field modelling or sea surface mapping by satellite altimetry, rely on many different observation types or groups that have to be processed in a common parameter estimation scheme. Global gravity field models, for example, are computed nowadays from multi-arc, multi-satellite, multi-station laser tracking data, altimetry, terrestrial gravimetry, GPS-based satellite-to-satellite tracking, and specific techniques such as intersatellite ranging with the Gravity Recovery And Climate Experiment (GRACE) spacecraft or satellite gravity gradiometry with the Gravity and Steady-State Ocean Circulation Explorer Mission (GOCE). Needless to say, the choice of the relative weights for these different observation groups as well as for those models which introduce additional prior information, e.g. on the gravity field, is of importance for obtaining reliable estimates of the unknown parameters. The reliability of estimated and/or calibrated covariance matrices of the unknown parameters also depends on the assumptions about the observation weights.

Therefore, a weight optimization process normally forms a part of the activities. This involves assessing the formal instrumental errors as well as carrying out empirical considerations and intercomparison with independent models and data sets. Optimization with respect to the weights given by formal errors is necessary due to the limited knowledge of the instrument noise characteristics, and due to the imperfections in the physical models involved in the estimation process. Down- or overweighting in global gravity field modelling or orbit determination can easily reach a factor of 10 or more (Marsh et al. 1988; Lerch 1991; Schwintzer et al. 1997; Kizilsu and Sahin 2000). The question as to whether re-weightings are appropriate is likely to appear in the course of the European Space Agency's (ESA's) GOCE satellite mission, although a lot of effort is currently put into the investigation of error models for the gradiometer instrument and the precise orbit determination (POD) process.

A method proposed by Lerch (1991) establishes the weighting scheme based on the behaviour of the common parameters – the spherical harmonic coefficients of the Earth's gravity field in his paper – and their error estimates when we derive subset solutions where data groups have been excluded. This method is approximative in the sense that it assumes that each data subset in question has only small influence on the overall solution in comparison to the full data set. Consequently, in a variance component model, this method gives unbiased estimates to the same degree of approximation. Lerch's method requires repeated evaluation of the trace of the solution covariance matrix and the subset solution covariance matrices, which certainly will pose a problem for application to GOCE data analysis given the number of unknowns and the size of these covariance matrices. Lerch's method is, however, known to be quite robust with respect to unmodelled systematic errors.

It should be added that an assessment of the proper weight of prior information can be viewed as Tikhonov regularization with a posteriori parameter choice. Within this approach the regularization or weighting parameter is to be derived from minimization of some statistically or mathematically motivated target function, which usually involves the least-squares (LS) residuals of the data. Triggered by the new gravity field missions, quite recently different techniques have been studied in this field [see Kusche and Klees (2002) and the references therein]. If more than one regularization or weighting parameter is to be determined, however, these methods lead to multi-dimensional optimization problems for which numerical solutions lack efficiency in computation or may not be feasible at all.

The weighting method presented in this paper is based on the estimation of variance components (VCE), which goes back to Helmert (1924). Rao (1973) developed the MINQUE technique, and Grafarend and d'Hone (1978) investigated estimation methods for analysing geodetic data. For further references, see e.g. Schaffrin (1983), Rao and Kleffe (1988), Koch (1990), or Grafarend and Schaffrin (1993). A recent survey on exact and simplified VCE approaches when observation groups are disjunctive is provided by Crocetto et al. (2000). In global gravity modelling, VCE has been proposed by Schwintzer (1990) for assessing the proper weight of prior information on the Earth's low-degree gravity field. Sahin et al. (1992) and Kizilsu and Sahin (2000) apply Helmert's VCE for precision analysis in satellite laser ranging. Recently, VCE has been applied by Zhang et al. (2001) for improving orbit determination for the ERS-2 satellite. Lucas and Dillinger (1998) applied MINQUE to very-long-baseline interferometry (VLBI) data analysis and found good agreement with the conventional weighting methods in this field. Apart from in their problem, only relatively few unknown parameters were involved in the applications as reported. Looking at the GOCE data analysis concepts, the main limitation of the VCE techniques proposed so far seems the costly and repeated computation of the redundancy contributions of the observation groups, involving the normal matrix contribution of the particular data set as well as the inverse of the weighted combined normal matrix. For large systems, like those encountered when solving for a high-resolution gravity field model, this is prohibitive. The inverse normal matrix will not necessarily be computed if iterative solvers are employed. Moreover, the matrices involved are generally not provided as standard output by common geodetic parameter adjustment software. Even the technique worked out by Lucas and Dillinger (1998), which reduces the size of the problem to the number of common parameters present in the adjustment problem, cannot be expected to be of great improvement if the number of these global parameters dominates the problem. Therefore we apply a variant described by Koch and Kusche (2002), which makes use of a stochastic trace estimation technique invented by Girard (1989) and Hutchinson (1990).

The method is re-structured and developed further in a Monte-Carlo sense, that is, on input for an arbitrary LS estimation software we use cyclically randomized versions of the original data set where for each individual observation group an artificial noise sequence has to be added in turn. On output, from a comparison of the residuals obtained with the original data and the randomized data, the new weights are estimated in an iterative sense. The method will be called Monte-Carlo VCE (MCVCE) in the remainder of this paper. Three sub-variants will be described which differ slightly in the artificial noise characteristics and the necessary matrix–vector operations. A Monte-Carlo approach appears particularly attractive since it seeks to extract statistical information on the inversion scheme by passing random input through the given algorithm, without requiring modifications of the code. This means that an existing software package for solving the data inversion problem can be used as a black box. A similar method has been proposed by Purser and Parrish (2000) in the context of variational assimilation of meteorological data.

The material is organized as follows. First, we briefly review the model setup for the estimation problem in satellite geodesy. Then, VCE and the particular algorithm we intend to use will be introduced. We will show how this algorithm can indeed be recast in a Monte-Carlo fashion when using a randomized trace estimation technique. Geodetic parameter estimation software can be imbedded in an iterative MCVCE process without internal modifications or computing non-standard output. Finally, a simulation of low-/ medium-degree gravity field recovery from the GPS orbit determination for GOCE is considered. For comparison, we include results obtained with a randomized version of Lerch's method for weight estimation. A discussion closes the article.

2 Data combination in satellite geodesy

The linear observation model which will be adopted throughout this contribution reads

$$\mathbf{X}\boldsymbol{\beta} = \mathbf{y} + \mathbf{e} \qquad E(\mathbf{e}) = \mathbf{0} \qquad D(\mathbf{e}) = \boldsymbol{\Sigma} \tag{1}$$

with $n \times u$ design matrix $\mathbf{X}$, $u \times 1$ vector $\boldsymbol{\beta}$ of unknowns, $n \times 1$ vector $\mathbf{y}$ of observations, and $n \times 1$ vector $\mathbf{e}$ of