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The comparison of L_1 and L_2-norm minimization methods

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L_2-norm, also known as the least squares method was widely used in the adjustment calculus. The weaknesses of the least squares method were the effect of gross measurements error on the solution and the disturbance and absorbance of gross error on the solution. L_1–norm, also known as the least absolute values method, was affected by almost none or very little from gross error. Therefore L_1-norm method, used for parameters estimation in some special case, has been successfully used for outlier measurements detection. In this study, the L_1 and L_2-norm adjustment methods have been taken relatively to each other's advantages and disadvantages and the numerical application of the two-dimensional similarity coordinate transformation were made and the results of both methods are discussed.

Key words: L_1 and L_2-norm, gross errors, outlier detection, similarity transformation.

INTRODUCTION

The subject of adjustment calculus is to determine the highest probability of certain values of unknown parameters and adjusted measurements with the measurement that is more than the required number. Also, unknown parameters and its functions sensitivity and reliability are determined with adjustment calculus.

In the problem, the measurements, which is much than the required number cause discrepancy between measurements and in this case, the solution is not unique. An objective function is made for the unique solution. It is seen that usually the objective functions are formed by minimization of corrections or a function of corrections and the two methods come forward. The most used methods are shown below:

(i) [pvv] = min. (L_2-norm) Least Squares Method (LSM)
(ii) [p|v|] = min. (L_1-norm) Least Absolute Values Method (LAVM).

It is known that L_1-norm method, first, is used for astronomical orbit calculation by Gauss in 1632.

Historically, L_1-norm method is a robust alternative to least squares (Huber, 1964; Fuchs, 1981; Kampmann, 1989), but it has been criticized by some researches for reasons of indirect solution, redundant measurements which are virtually ignored and difficult calculation (Rousseeuw et al., 1986; Dodge, 1987).

L_2-NORM MINIMIZATION METHOD

L_2-norm method, also known as the LSM, is widely used in geodesy for parameters estimation. There is no knowledge about distribution measurements as in the maximum likelihood methods for using LSM (Koch, 1999). If the inverse of variance-covariance matrix of measurements is taken as weight matrix of measurements, LSM estimation is unbiased and minimally variant. Also, if the measurements have normal distribution, the solution of LSM is identical to the maximum likelihood method (Simkooei, 2003).

LSM solution of a problem can be performed easily and the result is always unique. If v, p and p_i are taken respectively as correction of measurements, weight matrix of measurements and diagonal elements of weight matrix, the objective function of LSM can be written as follows:

$$v^T P v = [Pvv] = \sum p_i v_i^2 = min \tag{1}$$

As it can be understood from equation (1), the sum of the weighted squares of corrections is minimal.

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The most important issue about L_2–norm method is the assumption that the measurements, subjected to adjustment calculus, have the normal distribution; but in practice, the measurements may not always have the normal distribution due to gross and systematic errors. Therefore, before gross and systematic errors are cleaned out in the measurements, the adjustment calculus must be made with L_2-norm method; otherwise the result of adjustment would be adversely affected by these errors. The disadvantages of L_2-norm method is affected more from gross and systematic errors. In the case of the measurements that contain more gross and systematic errors, these errors are not easily determined with the outlier detection methods based on LSM. Different methods used by outlier detection give different results and the measurement, determined as outlier, is removed from the measurements group. As a result of this, there is a reduction in statistical confidence. If the consistent measurements are obtained inadvertently as outlier, then it has a sinking effect. Similarly, if the outlier measurements are not obtained as outlier, it has a hiding effect. These two effects are described with a distribution effect of LSM (Hekimoğlu, 1997). LSM solution of the problem as a classical formulation is as follows (Bektaş, 1991). In the problem, n, u $(n > u)$ and d are number of measurements, unknown parameters and rang defect, respectively. For the solution of the problem, the linear or linearized relationship, written between measurements and unknown parameters (one per measurement), consists of n equations, including u unknown parameters.

$$A_{n,u}\delta X_u = \ell_n \tag{2}$$

Here, A is for design matrix, δX is shifted unknown parameters, ℓ is measurements vector, respectively. Subsequently, the system of linear equations must be solved. Therefore, this system must be consistent with the rang of A design matrix and A design matrix, extended with constant terms, must be equal, so that $rang\,(A) = rang\,(A:\ell)$; whereas, the system of equation (2) is inconsistent because $(n > u)$. In this case, $rang\,(A) \leq u$. The extended matrix with ℓ measurements $rang\,(A:\ell)$ is generally more than $rang\,(A)$. There is no solution of inconsistent equations because δX *shifted* unknown parameters, not provided in equation (2), are not calculated. In this case, only the approximate solution of the system can be derived. The equation system with approximate solution is calculated by adding v corrections at the right side of equation (2).

$$A\,\delta X = \ell + v \tag{3}$$

Depending on the choice of v vector corrections, infinite solutions can be obtained. The solution should be made according to the objective function of the unique solution. For example, the LSM always give a unique solution.

The mathematical model, which consists of functional and stochastic model, is established in solving the adjustment problems. The relation between measurements and unknown parameters is reflected in the solution with functional model. However, the stochastic model shows a relation of precision and correlation between measurements.

Functional Model   Stochastic Model

$$A\,\delta X = \ell + v \qquad P_{\ell\ell} = \sigma_0^2 C_{\ell\ell}^{-1} \tag{4}$$

Here, $P_{\ell\ell}$ is for the weighted matrix, $C_{\ell\ell}$ is for the variance-

covariance matrix σ_0^2 the a-priori variance of measurements respectively. It should be noted that this mathematical model solves with an objective function of LSM

$$v^T P v = min.$$

and initially, normal equations were established.

$$(A^T P_{\ell\ell} A)\delta X = (A^T P_{\ell\ell}\,\ell) \tag{5}$$

Then, the unknown parameters and the corrections of measurements are obtained as follows:

$$\delta X = (A^T P_{\ell\ell} A)^{-1}(A^T P_{\ell\ell}\ell) \tag{6}$$

$$v = A\delta X - \ell \tag{7}$$

If the equation is not sufficient in identifying the unknown parameters, the mathematical model would have a rank defect, or if there are new conditions, they must be provided by the unknown parameters (d). However, the number of condition equations is written as follows and the functional model is extended (Bektaş, 2005):

$$G^T \delta X = 0 \tag{8}$$

Here, this is the $G_{u,d}$ is design matrix of condition equations between unknown parameters. The extension is made for P weight matrix. The number of equations, $m = n + d$, is in the extended model. In this case, the mathematical model is shown as follows:

Functional Model   Stochastic Model

$$\begin{bmatrix} A \\ G^T \end{bmatrix}_{m,u}[\delta X]_{u,1} = \begin{bmatrix} l \\ 0 \end{bmatrix}_{m,1} \qquad P_{m,m} = \begin{bmatrix} P_{\ell\ell} & 0_{n,d} \\ 0_{d,n} & I_{d,d} \end{bmatrix} \tag{9}$$

Here, $I_{d,d}$ is unit matrix, $0_{n,d}$ and $0_{d,n}$ is zeros matrix, $d = u - rang\,(A)$ is rank defect, respectively. This mathematical model solves equations according to $v^T P v = min.$ which is the objective function of LSM.

$$(A^T P_{\ell\ell} A + GG^T)\delta X = (A^T P_{\ell\ell}\ell) \tag{10}$$

After establishing normal equations, the unknown parameters and the corrections of measurements are obtained as follows:

$$\delta X = (A^T P_{\ell\ell} A + GG^T)^{-1}(A^T P_{\ell\ell}\ell) \tag{11}$$
$$v = A\delta X - \ell$$

If the mean of the measurements group is taken as the certain value for a special case of one unknown parameter determinate, the solution of LSM are obtained. If the corrections are calculated from the measurement group, they have a normal distribution that does not include gross and systematic errors, by L_2-norm methods, both the squared sum of correction and the absolute value sum of correction are minimum.

L_1-NORM MINIMIZATION METHOD

L1–norm minimization method, also known as the least absolute

values method (LAVM), is a robust method used for outlier detection in geodetic networks. This method is different from the re-weighted LSM. The objective function of L_1–norm minimization method and the sum of the absolute value weighted correction, is minimum, are shown as:

$$p^T|v| = [p|v|] = \sum p|v| = min. \tag{12}$$

The use of measurements weight is different from the L_2-norm method in the L_1-norm method. The weight of measurements directly reflects solution in L_2-norm method, whereas, the weight of measurements are not used in the non-redundant solutions, obtained by L_1-norm method. Once the solutions of $\binom{n}{u}$ combination are obtained, then these weights are used in determining the optimal solution.

The mathematical model is established by the L_2-norm. The relation between measurements and the unknown parameters according to equation (3) and the conditions for the unknown parameters that are required according to equation (8) are written respectively as:

$$A\,\delta X = \ell + v$$
$$G^T \delta X = 0 \tag{13}$$

This mathematical model is solved with principle of $\lfloor p|v| \rfloor = min.$ but the direct solution of the problem, given according to this principle, is not possible except in a special case. The solution is made by either the searching algorithm or linear programming. If n and u are respectively number of equations and unknown parameters $(n > u)$ in the linear equation system, then the number of solution is equal to the combination of $\binom{n}{u}$. The solution is obtained, excluding each time, with a different $(n > u)$ number of measurements. One of these solutions is the solution of desired L_1-norm. In the case where the number of freedom is small, the solution is obtained by L_1-norm method.

Generally, if the solution of a problem is obtained according to L_1-norm method, the problem is converted by linear programming (Simplex method) and then the iterative solution are made by searching algorithm (Barrodale et al., 1974). Moreover, optimal corner point is searched, when corner points of polygonal solution regions are formed by condition equations in the linear programming. As a simple example, if investigations are made graphically in the region formed by intersection of three line equation with L_1 and L_2-norm methods, the solution of problem with L_1-norm methods is K point in the triangular region formed by A, B and C points (Figure 1). However, the solution of the problem with L_1-norm methods is one or several corners of triangle ABC. In some cases, several corners can give an optimal solution. In this situation, if points A and B give an optimal solution, all points of this line are therefore an optimal solution.

For the solution of linear programming equations system, all unknown parameters are positive and an objective function must be seen as follows:

$$CX = d$$
$$f = b^T X = min.$$

In this case, the functional model, written in equation (13), is converted to this form. After unknown parameters are obtained, the corrections of measurements are calculated and used by these unknown parameters in the solution of L_2-norm. The unknown
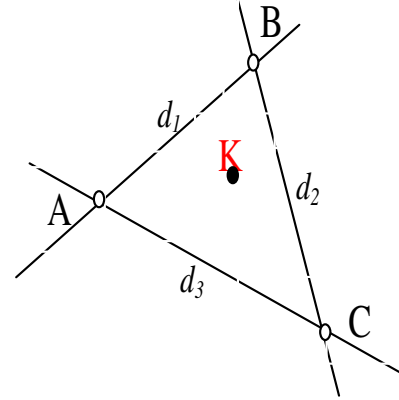

Figure 1. The triangular region is formed by A, B and C points.

parameters and corrections of measurements are obtained together in the solution of L_1-norm with linear programming. In other words, the unknown vector of L_1-norm includes both the unknown parameters and correction of measurements. In this case, the following equation system must be solved with the principle of $[p|v|] = min.$

$$A\,\delta X - v = \ell$$
$$G^T \delta X = 0$$

If this equation system can be written as the matrix representation, the following equation is obtained

$$\begin{bmatrix} A_{n,u} & -I_{n,n} \\ G^T_{d,u} & 0_{d,d} \end{bmatrix} \begin{bmatrix} \delta x \\ v \end{bmatrix} = \begin{bmatrix} \ell \\ 0 \end{bmatrix} \tag{14}$$

It should be noted that equation (14) is not solved with the objective function of $[p|v|] = min.$ All unknowns (unknown parameters and correction of measurements) must be positive to solve this equation system with the principle of $[p|v|] = min.$ Therefore, δX unknown parameters and correction of measurements are re-arranged to be differences of new parameters as positive and negative.

$$\delta X = \delta X^+ - \delta X^- \qquad \delta X^+, \delta X^- \geq 0$$
$$v = v^+ - v^- \qquad v^+ - v^- \geq 0$$

With this arrangement, equation (14) is obtained by the following equation system

$$\underbrace{\begin{bmatrix} A_{n,u} & -A_{n,u} & -I_{n,n} & +I_{n,n} \\ G^T_{d,u} & -G^T_{d,u} & 0_{d,d} & 0_{d,d} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \delta x^+ \\ \delta x^- \\ v^+ \\ v^- \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} \ell \\ 0 \end{bmatrix}}_{d} \tag{15}$$

$$CX = d$$

In this case, the number of unknown parameters and correction of measurements is doubled.

Also, the principle $[p|v|] = min.$ of L_1-norm method can be converted to an objective function $f = b^T X$ that is necessary for linear programming. In this situation, an objective function of L_1-norm in equation (12) can be written as follows:

$$[p|v|] = p^T v = p^T| v^+ - v^-| = \sum p| v^+ - v^-| = \sum p| v^+ + v^-| = min.$$

Table 1. The solutions of L_1-norm and L_2-norm for sample 1.

i	1	2	3	4	5	$[\lvert v \rvert]$	$[vv]$
ℓ_i	5	6	7	9	16		
The solution of L_1-norm (the solution of LAVM)							
$X = median\{\ell\} = 7$							
$v_i = X - \ell_i$	2	1	0	-2	-9	14	90
The solution of L_2-norm (the solution of LSM)							
$X = [\ell]/n = 43/5 = 8.6$							
$v_i = X - \ell_i$	3.6	2.6	1.6	-0.4	-7.4	15.6	77.2

Here, because v^+ or v^- is equal to zero, $\sum pl\ v^+ + v^-l = \sum pl\ v^+ - v^-l$. If the objective function system, with matrix representation, can be written as a function of unknowns, the following equation is obtained:

$$f = b^T X [p \lvert v \rvert] = min. \qquad (16)$$

$$b^T = [0^I \quad 0^I \quad P^I \quad P^I]$$
$$X^T = [\delta X^+ \quad \delta X^- \quad v^+ \quad v^-]^T$$

$0^T = [0 \quad 0 \quad \cdots \quad 0]^T$ zeros vector with u element
$P^T = [p_1 \quad p_2 \quad \cdots \quad p_n]^T$ weight of measurement vector with n element.

More so, the condition equations and an objective function of linear programming are obtained respectively from equations (15) and (16). The solution is made with simplex algorithms. For the solution, the software that is designed for this purpose can be used. For example, *Linprog* function in MATLAB can be used easily.

The solution of the problem, having a rank defect, can be done without the need for extra processing (such as pseudo inverse) by L_1-method (Harvey, 1993). This situation is one of the advantages of L_1-norm method. The other advantage of the solution with L_1-norm method is that the results are not or slightly impacted from measurements of gross errors. Therefore, L_1-norm method is generally used for the measurements, including gross error and cleaned out measurements group.

The solution is made with the measurement number equal to the unknown parameters in the L_1-norm method and the other measurements are excluded. This is a disadvantage of L_1-norm method.

The solution of the L_1-norm method is not always unique. So, in several solutions, the sums of absolute value corrections are minimal and are obtained for the problem. In the case of only one solution, the unknown is measured if the median of measurements group is taken as a certain value, in which the solution of L_1-norm is obtained. The median is the middle element of the group, arranged in order of the size. According to the number of elements, double or single in the group, the median of the group is determined as follows:

If number of element is single, (2n+1)   median = x_{n+1}
If number of element is double, (2n)   median = $(x_n + x_{n+1}) / 2$

In the case where only one unknown is measured, the following two examples show how the solution of L_1-norm and L_2-norm can be

obtained. However, the solution of L_1-norm is not unique (Bektaş, 2005).

Sample 1

The value of X unknown is measured 5 times and the value $\ell = \{5, 6, 7, 9$ and $16\}$ is obtained. A certain value of X unknown parameter is determined with L_1 and L_2-norm method. The solution is seen in Table 1.

The size of l_5 measurement did not change the result of the solution with L_1-norm method.

Sample 2

The size of l_5 measurement is the gross of the other measurements in sample 1 and the measurement has the largest correction. In this case, this measurement seems to be outlier. l_5 measurement was removed from the measurement group and the problem was resolved with L_1 and L_2-norm method in sample 1. The solution is seen in Table 2.

$l = \{5, 6, 7, 9\}$

The median of the measurements group is the averages of the two middle values, so the number of element is doubled. Although this value is 6.5, all real values between two values (including 6 and 7) are taken as a certain value. All x value in this range give the same least absolute value $[\lvert v \rvert] = 5$. Thus, it is seen that L1-norm method is not unique. All of the following values are taken as x value.

$x = \{6,\dots 6.2,.. 6.5,.. 6.887,.....7\}$

If x is taken as 7, results are obtained in the previous sample. It is obviously seen that L_1-norm method is not affected from the measurements, including gross errors.

THE COMPARISON OF L_1 AND L_2-NORM METHODS

(i) The solution of the L_2-norm method is always unique and this solution is easily calculated.
(ii) The L_2-norm method is used for the weighted and correlated measurement.
(iii) It is possible that measurements, unknown parameters and its functions sensitivity and reliability are determined in L_2-norm

Table 2. The solutions of L_1-norm and L_2-norm for sample 2.

i	1	2	3	4	$[\|v\|]$	$[vv]$
ℓ_i	5	6	7	9		

The solution of L_1-norm (the solution of LAVM)

$X = median\{\ell\}$ = {6,…,6.2,..,6.5,..,6.887,.....,7}

	1	2	3	4	$[\|v\|]$	$[vv]$
$v_i = X - \ell_i$ (x = 7)	2	1	0	-2	5	9
$v_i = X - \ell_i$ (x = 6)	1	0	-1	-3	5	11
$v_i = X - \ell_i$ (x = 6.5)	1.5	0.5	-0.5	-2.5	5	9
$v_i = X - \ell_i$ (x = 6.887)	1.887	0.887	-0.113	-2.113	5	8.825

The solution of L_2-norm (the solution of LSM)

$X = [\ell]/n$ = 27/4 = 6.75

	1	2	3	4	$[\|v\|]$	$[vv]$
$v_i = X - \ell_i$	1.75	0.75	-0.25	-2.25	5	8.75

Table 3. The coordinate of common and new points.

Point	Y_1	X_1	Y_2	X_2
248	9043.7400	5208.7900	4618.7200	4068.8300
257	9218.4200	4833.4900	5579.4100	1115.6000
253	9000.0000	5000.0000	4103.9800	2553.3800
124	9220.0200	5166.9100	5893.3800	3597.0300
125	9242.7000	5039.3800	5946.7000	2626.7000
New point				
251	9106.1700	5050.7100	?	?

method.

(iv) The disadvantages of L_2-norm method are distributed by errors effect and the sensitivity against measurements, to include gross error.

(v) The solution of the L_1-norm method is not always unique and there may be several solutions. Also, the solution of L_1-norm method is not generally obtained directly, but iteratively, calculations are made. Therefore, the solution is not easily calculated like L_2-norm method. Notwithstanding, computational tools, computer capacity and speed are considered and the difficulty of calculations are eliminated.

(vi) The correction of at least u measurements is always equal to zero per the solution in the L_1-norm method. Only the corrections are calculated for the number of $(n - u)$ measurements. It is clear that these corrections are not the measurement corrections. In the case of errorless corrections, some measurements are contrary to the errors theory.

(vii) The advantages of L_1-norm method are non-sensitivity against measurements, including gross error and the solution is not or is little affected by these measurements.

NUMERICAL APPLICATION

The coordinates of 5 common points was used as numerical application. These coordinates are in two coordinate systems, known as systems I and II. 2D similarity transformation was made with 5 common points (248, 257, 253, 124 and 125) used for L1-norm and L_2-norm methods and the solutions were compared. The coordinates of common and new points were given in Table 3. All coordinates were equal in precision and were uncorrelated. Firstly, the transformation parameters were calculated.

The similarity transformation was made between two coordinate systems by L_2-norm method and it is observed that the coordinates of the two systems were consistent. Then, 2D similarity transformation was made with the same data by L_1-norm method and the transformation parameters were calculated. The coordinates of 251 point were calculated and used for the transformation parameters of these methods. The solution is shown in Table 4.

It is seen that in Table 4, the solution of L_1-norm was obtained and used for Y_{248}, Y_{257}, X_{257} and X_{253} coordinates. The coordinate of 253 point was tainted by adding $\delta Y_{253}=+2m$. and $\delta X_{253}= -1m$. from the coordinates given in Table 3 and the second numerical application was made.

The new coordinates were given in Table 5a. The solutions were made using these coordinates and the solutions were given in Table 5b.

It is seen that in Table 5b, the solution of L1-norm was obtained and used for Y_{257}, X_{257}, Y_{124} and X_{125} coordinates.

Table 4. The solutions of 2D similarity coordinate transformation by L_1-norm and L_2-norm methods.

Point			L_2-norm				L_1-norm	
	v_y	v_x	Y_2	X_2	v_y	v_x	Y_2	X_2
248	-0.0016	-0.2020	4618.7184	4068.6280	-0.0000	-0.3639	4618.7200	4068.4661
257	0.0047	0.0110	5579.4147	1115.6110	-0.0000	0.0000	5579.4100	1115.6000
253	-0.1767	0.0977	4103.8033	2553.4777	-0.1311	0.0000	4103.8489	2553.3800
124	0.0835	-0.0068	5893.4635	3597.0232	0.0316	-0.1300	5893.4116	3596.9000
125	0.0901	0.1001	5946.7901	2626.8001	0.0484	0.0230	5946.7484	2626.7230
			New point					
251			4940.3658	2834.8896			4940.3686	2834.7896
$[vv]$	0.107				0.170			
$[\|v\|]$	0.774				0.728			

Table 5a. The tainted coordinate used for 2D coordinate transformation.

Point	Y_1	X_1	Y_2	X_2
253	9000.0000	5000.0000	4105.9800	2552.3800

Table 5b. The solutions of 2D similarity coordinate transformation by L1-norm and L2-norm methods with tainted coordinates.

Point			L_2-norm				L_1-norm	
	v_y	v_x	Y_2	X_2	v_y	v_x	Y_2	X_2
248	0.2945	-0.8510	619.0145	4067.9790	-0.0139	-0.4266	4618.7061	4068.4034
257	0.6558	0.3095	5580.0658	1115.9095	-0.0000	-0.0000	5579.4100	1115.6000
253	-0.4417	0.7302	4104.5383	2553.1102	-2.1199	0.9589	4103.8601	2553.3389
124	0.1507	-0.2768	5893.5307	3596.7532	0.0000	-0.1708	5893.3800	3596.8592
125	0.3407	0.0881	5947.0407	2626.7881	0.0260	0.0000	5946.7260	2626.7000
New point								
251			4940.8306	2834.6202			4940.3620	2834.7520
$[vv]$	4.171				5.626			
$[\|v\|]$	5.767				3.716			

DISCUSSION AND CONCLUSION

If there is an investigation between the first and second application used for the tainted coordinates of the L_2-norm method, the change of 251 point coordinates were determined. These changes were X-side -27 cm. and Y-side +46 cm., whereas these changes were X-side -4 cm. and Y-side +1 cm. in the application made by L_1-norm method. Hence, it was seen that L_1-norm method was not affected from gross errors. In other words, the gross errors reflected its correction, while the other corrections were not affected.

In the good parameters estimation method, the measurement including gross error must be corrected with its correction and as a result, these gross errors are not affected by the other measurements corrections. Therefore, the corrections of 253 point coordinates should be the opposite sign of errors in the second application. If the result of L_1-norm is examined, it is seen that the corrections of 253 point coordinates are v_y = -212 cm v_x = +96 cm. However, these corrections are nearly equal to the opposite sign of errors (δY_{253}=+2 m δX_{253}= -1 m). The result shows that in the L_1-norm method, the corrected gross error is very good; whereas, If the result of the L_2-norm is examined, it is seen that the corrections of 253 point coordinates are v_y = -44 cm v_x = +73 cm. Besides, the corrections of points, not including gross error, were bigger than the corrections of 253 point in this method (the corrections of 248 point v_x = -85 cm and the corrections of 257 point v_y = +66 cm). From here, it is seen in the L_2-norm method solution that the gross errors distributed the corrections of other measurements and the hidden measurement, including

the gross errors.

RESULTS

In this study, a comparison was made between L_1-norm (LAVM) and L_2-norm (LSM). L_1-norm method is not the alternative of L_2-norm method in terms of adjustment. It is used to clean-out gross errors and calculate approximate values of unknown parameters successfully. The solution of problem, including gross errors that have rank defect, is easily made with L_1-norm method despite the convergence problem in the L_2-norm method. The authors of this study proposed and used the L_2-norm method in the solution of adjustment calculus, after the measurement group cleaned up gross and systematic errors using L_1-norm method.

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