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Inner and minimum constraint adjustment of marine gravity data ☆

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Abstract

A programme for marine gravity data validation and adjustment is presented—VALDAMA. The aim of the programme is to provide a complete user-friendly system for marine gravity data validation and adjustment that enables the user to define all intervening parameters. The programme is written in standard C and has the possibility of solving for three different types of adjustment: inner constraint, constraint and minimum constraint and allows the user to define the stochastic model. It also has the ability to detect and adjust individual sub-networks, which may occur in regional applications, where track network connectivity fails. VALDAMA uses a parameter file in which the validation parameters, adjustment method and stochastic model are defined. The programme calculates the main statistical parameters and presents the results in spreadsheet format compatible with most common computer office tools. A test case is presented for marine gravity data in the North-East Atlantic Ocean. The complete validation and adjustment of more than 190,000 gravity observations are presented and from an analysis of several adjustment possibilities, it was concluded that inner constraint adjustment may give unpredictable results for global gravity data and that minimum constraint would be the preferred solution.

Keywords: Gravity; Geoid; Constraint adjustment; Validation

1. Introduction

Marine gravity data is one of the fundamental observations for geodetic, geophysical and geological marine applications. Marine gravity data has been used extensively in crustal and mantle structure studies, in gravimetric geoid computation and in providing ground truth for satellite and airborne data. However, its use has been limited in some oceanic areas due to uncertainties in the precision of older data stored in archives. The main source of errors associated with marine gravity observations may be summarized following Wessel and Watts (1988) as instrumental errors of the gravimeter and external influences, positional and navigational errors, and incorrect tie-in to the base station. Gravimeter instrumental errors are mainly characterized by the introduction of a small amplitude noise in the signal (relative gravity value). This type of error is associated with the gravimeter type (cross-coupling), its position in the platform (off-levelling), or changeable weather conditions during observation. These errors, with a random behaviour, are mostly impossible to model and hence not considered in

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our analysis. Positional errors affect marine missions in two ways: first by giving incorrect geodetic coordinates, affecting all subsequent calculations and, second, by causing an error in azimuth and velocity that affects computation of the Eötvös correction. To obtain a 1 mGal precision in the Eötvös correction the velocity and the azimuth precision must be better than 0.1 m/s and 1°, respectively (Torge, 1989). Marine gravity measurements are relative to some known base station and therefore any incorrect tie-in to that station may lead to a constant offset in all the gravity data of that cruise. Moreover, most marine gravimeters tend to drift with time, introducing systematic errors in the data. The standard way to account for the latter error is to assure correct tie-in in the starting base station and in the ending base station, assuming that the precision of the g-value in these stations is better than the drift of the gravimeter.

The removal of systematic errors (navigation, positioning and tie-in to base station) and the associated random error minimization in marine gravimetric observations has been the subject of several studies (Wessel and Watts, 1988; Motao, 1995; Wenzel, 1992; Adjaout and Sarrailh, 1997; Sevilla and Catalão, 1993). It is believed by most authors that the major source of error is related to navigation and positioning. Even after the removal of systematic errors and the minimization of random errors, a residual error reconnected in the track intersections remains. The residual errors due to positional uncertainty and instrumental precision are inherent and are not removed from the gravimetric database, and therefore it is only possible to keep this error component to a minimum.

This paper presents methods and a standard C language programme for marine gravity data validation and adjustment. The adopted strategy is based on the detection of intersections between tracks, computing the differences between the anomaly estimated at the intersection point from both tracks (cross-over errors (COE)) and finally adjusting these residuals, estimating the best bias for each track. In regional applications or in poorly surveyed areas there is the possibility of losing overall network connectivity. In this programme, an algorithm was implemented with a pivot in which connectivity is ensured through the automatic identification of individual sub-networks and the adjustment is performed individually for each sub-network.

A test case was applied in the North-East Atlantic Ocean between the Azores archipelago and the Iberian Peninsula. In this area the programme was tested and the results of its potentialities are presented. Several tests were made covering almost all possibilities for marine gravity data validation and adjustment.

2. Theory

2.1. Pre-processing

A marine gravity survey is composed of a set of tracks. Gravity measurements belonging to the same track share common properties related to the ship, the gravimeter, sea conditions and the positioning system. A sequence of observations within a track is statistically correlated, allowing the detection of possible gross errors by a simple spatial filtering process. This procedure, called pre-processing or track internal validation, detects outliers in the data and flags them for deletion, assuming their spatial correlation. The method should minimize the error of rejecting valid observations and/or accepting wrong observations.

The detection of outliers within a track was preformed by least-squares collocation (LSC). Each gravity observation of a track is compared with the LSC estimate for that point and its prediction error variance. The LSC function is constructed with a subset of observations within a predefined distance from the interpolation point. At a given point P with known coordinates (geodetic coordinates, latitude and longitude (ϕ, λ)), the gravity anomaly is estimated through the following expression (Moritz, 1980):

$$\Delta g_P = C_{iP}(C_{ij} + D_{ij})^{-1}\Delta g_i, \tag{1}$$

where C_{iP} is the vector of covariances between observed gravity anomalies (Δg_i) and predicted gravity anomalies (Δg_P), C_{ij} is the auto-covariance matrix of the observations (Δg_i) and D_{ij} is the variance–covariance matrix of the associated noise (in practice this is a diagonal matrix). This method assumes that the set of observations is homogeneous and isotropic, which can be obtained by reducing the free air gravity anomalies to a global geopotential model (EGM96, Lemoine et al., 1997), or through a polynomial function. The selected covariance function is (Barzaghi and Sansò, 1983):

$$C(\psi) = C_0(1 + \psi/\alpha)e^{-\psi/\alpha}, \tag{2}$$

where C_0 is the local data variance, ψ is the spherical distance between the observation point and the prediction point and α is the correlation distance multiplied by 0.595 (a fixed parameter corresponding to a correlation distance of 15–25 km). The local variance value C_0 is determined with the 20 closest observations to the prediction point with a maximum distance of 15 km. The 20 observations used for interpolation are distributed symmetrically with respect to the prediction point, a maximum of 10 points before and after the prediction point (the prediction point is not considered a data point).

An observation is flagged for deletion, or considered suspicious, if the difference between the estimated value and the observed value is simultaneously greater than a given threshold and greater than $k\sigma_p$, where k is the critical value and σ_p is the estimated prediction error. The critical value will determine what percentage of good observations will be incorrectly rejected and for a probability of 99% or 99.9% (1% or 0.1% of good observations rejected) the critical value (k) is 2.58 or 3.29, respectively. This pre-processing step is essential for the track adjustment procedure to avoid the introduction of errors in the track intersection gravity value computation.

2.2. COE adjustment

In global marine gravity databanks there are millions of observations from the entire world spanning decades of acquisition. For a specific area (for example the North Atlantic) there are gravity data from different institutions with different periods of acquisition, but with some spatial correlation. The spatial correlation between different surveys separated in time by tens of years is achieved by their vicinity and particularly by the spatial intersection of both surveys at one or more points. The intersection point between two tracks is called the cross-over point and the associated difference in the gravity anomaly is called the COE. The basic principle underlying the validation and adjustment of marine gravity data from different surveys separated in time is based on the existence of intersections between tracks of two or more surveys. Besides the existing intersections between different surveys, each mission is designed to take into account the track intersection within the mission, constructing a connected network. This means that redundant observations exist and it may be assumed that COEs are due to a bias incorrectly applied to the gravity data. Hence, the determination of these biases for each track and the correction of the gravity data with these biases should reduce COEs.

The COE were divided into internal and external COEs. An internal COE occurs when a cruise crosses itself one or more times during the cruise. An internal COE has several common parameters, like the gravimeter, the positional system or the ship, and therefore it is expected that this COE will have a small standard deviation for each cruise. External COEs are those computed from two distinct tracks from two different cruises. An external COE may have larger values than internal ones, since it involves different time acquisition, different positional systems and different gravimeters. The basic principle underlying the systematic error adjustment of a gravimetric oceanic mission is to assume that the errors are almost constant for each mission, but with a random behaviour in the complete data set (from track-to-track). Let b_i be the bias to be applied to the gravity anomaly of track i and b_j the bias to be applied to the gravity anomaly of track j. At the crossing point P_0 (cross-over point):

$$v_{ij} = (\Delta g_i + b_i) - (\Delta g_j + b_j) \tag{3}$$

and the discrepancy d_{ij} is given by

$$d_{ij} + v_{ij} = b_j - b_i. \tag{4}$$

For each two tracks an observation equation is written which includes the COE d_{ij} and the two parameters to be determined, b_i and b_j (the bias). With n crossings and m tracks, the mathematical model is defined by the following vectorial equation:

$$\mathbf{d} + \mathbf{v} = \mathbf{Ax}, \tag{5}$$

where $\mathbf{d}$ is the discrepancy vector with n elements, $\mathbf{v}$ is the residual vector with n elements, $\mathbf{A}$ is the configuration matrix with $n \times m$ elements and $\mathbf{x}$ is the parameter vector with m elements. The weight matrix $\mathbf{P}$ is defined by the stochastic model of the COE, in which each equation is associated with a weight "p" computed as the inverse of a previously determined error variance. Only the diagonal elements of this matrix are set with non-zero elements assuming that COEs are stochastically independent. The least-squares solution of Eq. (5) is calculated from the normal equation:

$$\mathbf{A}^{\mathrm{T}}\mathbf{PAx} = \mathbf{A}^{\mathrm{T}}\mathbf{Pd} \quad \text{or} \quad \mathbf{Nx} = \mathbf{n}, \tag{6}$$

where $\mathbf{N}$ is the normal equation matrix and $\mathbf{n}$ is the right-hand side of the normal equation. In this system, the full rank of $\mathbf{N}$ and a solution for each track is only obtained if four conditions are fulfilled: there is at least one crossing for each track, the number of crossings is greater than the number of tracks, the set of tracks is a complete connected network and the adjustment datum is defined. The two first conditions are easily found in almost all regions of the world. It may happen that the subdivision of a mission into several tracks may lead to the existence of small tracks without any cross-over. In this particular case the track is not compensated, and difficulties arise when the track network is not fully connected and the adjustment datum is not defined. Assuming at a first step the connectivity of the complete track network, the singularity of the system is avoided by the addition of a constraint equation. There are two possibilities for this constraint equation: one is to fix one track assuming a zero bias (minimum constraint adjustment) and the other one is to constrain all track bias to a zero sum (inner constrain adjustment), Wenzel (1992). In both cases the following equation must be added to the system (Eq. (6)):

$$\mathbf{C}^{\mathrm{T}}\mathbf{X} = \mathbf{0}. \tag{7}$$

$\mathbf{C}$ is an identity vector in the last case ($\mathbf{C}^{\mathrm{T}} = \{1, 1, 1, \ldots, 1\}$) and equal to $\mathbf{C}^{\mathrm{T}} = \{0, 0, 1, 0, \ldots, 0\}$ in the first case, in which the identity element corresponds to the track with zero bias. In between these two cases there is one particular case in which a set of tracks with zero bias is fixed, called constrained adjustment. The system solution is computed by the Cholesky algorithm with a full pivot in order to obtain a minimum profile matrix (Hofmann-Wellenhof, 1982).

The stochastic model is defined by the weight matrix **P** calculated as the inverse of the error variances associated with each track. The error variance of a COE is computed by the sum of the error variances associated with each track. The a posteriori variance of unit of weight computed after least-squares adjustment is

$$\sigma_0^2 = \frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{n-q+1}, \tag{8}$$

where n is the number of COEs and q is the number of unknowns (tracks). The a posteriori variance is a measure of the misfit between the stochastic model and the residuals and a global statistical test must be performed. This test involves comparing the computed a posteriori variance factor against the chi-squared distribution

$$\frac{\chi^2_{1-\alpha/2,n-q}}{n-q+1} < \sigma_0^2 < \frac{\chi^2_{\alpha/2,n-q}}{n-q+1}. \tag{9}$$

If a unit value is assumed for the a priori variance and, if the observation residuals are consistent with their accuracy estimates (the **P** matrix) and the residuals are normally distributed, then the estimated variance factor would be expected to take a value close to unity for a number of degrees of freedom ($n-q$) greater than 100, which is a realistic value for marine cross-over adjustment. If the test fails, it may indicate significant gross errors in one or more of the observations, or a poor estimate of standard deviations of observations, and the observation residuals are significantly larger (or smaller) than those implied by the a priori standard deviations. In the latter case, the weight model must be redefined iteratively until the chi-square test is valid.

3. The programme

Specific software, VALiDAtion of MArine gravity data (VALDAMA), was written for the validation and adjustment of marine gravity data following the above-described mathematic and stochastic model. The programme was developed within a project of geoid determination in the North-Atlantic Ocean at the Faculty of Sciences, University of Lisbon. The programme is modular and written in standard C, allowing compilation and linkage in any operating system, accepting command-line arguments suitable for any operating system. The workflow of marine gravity data validation/adjustment using the VALDAMA programme is sequential and can be accomplished in four steps: filtering observations, detection of intersections, track statistics and track adjustment. The structure of the command-line arguments is: the name of the programme (VALDAMA), name of the input files and a code indicating the action to perform. The programme is driven by the command-line instructions and also by a parameter file in which all filtering, intersection and adjustment parameters are specified by the user. In this file the name of the output files in each step are also specified.

At the end of this programme three files are produced: a data (observation) file with all compensated gravity anomaly values, a report file with track adjustment results, and a statistics file with all statistics of the tracks and the adjustment. Analysis of the statistical parameters of the latter file will enable the user to eliminate or temporally suppress any track and proceed with the track adjustment of the last step. The programme can be run once or iteratively in the adjustment phase where the weight matrix must be defined or redefined, or when a track or set of tracks must be suppressed.

A detailed description of the programme, the workflow and the structure of all relevant files is presented in a text file supplied with the source code.

3.1. Input files

There are four mandatory files: the parameter file, the source file, the header file and the data file. All data files, parameter files and output files are in ASCII format with records separated by spaces. The output files are record formatted and can be opened in any spreadsheet programme.

In the parameter file (parameter.dat) the user must specify all output file names and all parameters for the filtering and adjustment process. In the filtering process, the user must specify the correlation distance, the noise and signal rms and the data rejection limit. For the adjustment, the user must specify the weight code, defining the stochastic model applied to each observation, and the fixed track code, defining the adjustment datum. The weight code determines the type of weight that is to be applied to each track. There are three possibilities: a weight system based on the date of the track, in which the newest tracks have the greatest weight; another weight scheme based on the error variance of each track written on the statistic file; and a third, unit weight for all tracks. The fixed track code indicates the adjustment datum with four possibilities: the first indicating the fixed track number, the second in which it is possible to fix a set of tracks belonging to the same source, the third in which it is possible to fix all tracks surveyed after a specific date, and the fourth indicating that the fixed track should be the longest track in the database.

In the source file the user must indicate the number of missions appearing in the data file, the source code of each mission (for example the BGI source code), a 10-digit integer number, and the respective survey year. The information in this file is crucial for the adjustment of the tracks both for the stochastic model definition and for the adjustment datum definition.

Most global gravity data banks use complex data formats preferably organized by mission. The sequential files supplied by these organizations are ASCII formatted files with a header area containing all information concerning a mission, followed by the data. This format, although very well structured and self-contained, is not suitable for validation or for plotting. The useful information contained in these files should be condensed in a more suitable data format that will enable the user to validate and adjust the data, and plot and analyse the data with commercial software, without loss of information. Data files from Bureau Gravimmetrique International, France (BGI) and National Geophysical Data Centre, USA (NGDC) must be split into two data files: the header file and the data file. The header file contains the minimum information on each track: the track number, the mission code (the code specified in the source file), the coordinates of the bounded rectangle of that track, a pointer to the start position of the first observation of that track in the data file, and the number of observations of this track. Each row corresponds to one track. The data file is also a sequential file containing all marine gravity data, with one observation in each row. The structure of this file is: a point number (must be an integer), latitude, longitude, gravity anomaly and validation flag. This structure is convenient and useful for statistics data analysis and graphic representation with commercial software; however, the user must be aware of the conceptual relation between the source file, the header file and the data file.

3.2. Output files

The output file names are specified in the parameter file. The output files from one step are the input files of the next sequential step with the format described above. However, there are four more files produced in the four main steps of the programme: the filtering file, the intersection file, the statistics file and the adjustment file. Their format is intuitive and is explained in the documentation file. The result of the filtering process is presented in the filtering file in which all information relevant to the flagged observation is presented, namely its complete identification (the number of the track, the sequential number of the observation in the track, the number of the observation), the gravity anomaly observation, the estimated gravity anomaly, the error difference and the estimated standard deviation (Eq. (3)). In the intersection file for each detected intersection there is one row with a sequential number of the intersection, the coordinates of the intersection point, the gravity anomaly intersection value computed from tracks i and j, and the number of tracks i and j. The statistics file is a complementary file of the header file. For each track a battery of statistical parameters are computed, including the length of the track, the minimum and maximum anomaly value, the number of intersections of this track, and a flag code indicating whether it is a valid track (with more than one intersection), a duplicate track (with more than 30 intersections) or a forbidden track (without intersections). The adjustment file is the final report file giving the user a compete description of the adjustment results. The estimated bias for each track and the computed residuals are listed. The a posteriori variance of unit weight and residual mean and standard deviation are also listed for error analysis.

Below, a case study of the application of this software in the North Atlantic Ocean is presented. Each step in the validation/adjustment process will be explained and documented in terms of the programme usage.

4. A case study in the North Atlantic

4.1. Gravity data set

The original data bank used in this study was the result of a compilation from BGI, NGDC and the Defense Map Agency of the USA (DMA) data banks, and covers an area with the following limits: $35°\text{N} < \cap\ \varphi < 45°\text{N}$, $32°\text{W} < \lambda < 6°\text{W}$. Most of the data were acquired from American, British and French institutions in the period from 1970 to 1990. This data bank was recently improved with a recent gravimetric campaign held in 1997 under the scope of the PDIC/C/Mar project (Fernandes et al., 1998) and the AGMASCO project (Timmen et al., 1999). The complete data set, obtained from a simple merging of data files, was cleaned from repeated missions recorded in different data banks, resulting in a data set with 197,702 data points. All data were transferred to the IGSN71 system and the anomalies converted to GRS80. The distribution of gravity data is depicted in Fig. 1. In our study area marine gravity measurements obtained before the 1980s represent more than 66% of the total available gravity observations.

Most BGI and DMA gravity data has no information concerning the mission that it belongs to, nor the leg number. An algorithm was developed that joins gravimetric data points which belong to the same track, based on their geographic positioning and some empirical criteria involving the length between observations and the angle between each three consecutive observations. The NGDC gravimetric data has a detailed description of all missions (MGD77 Marine Geophysical Data Exchange format, NGDC, 1993). Following

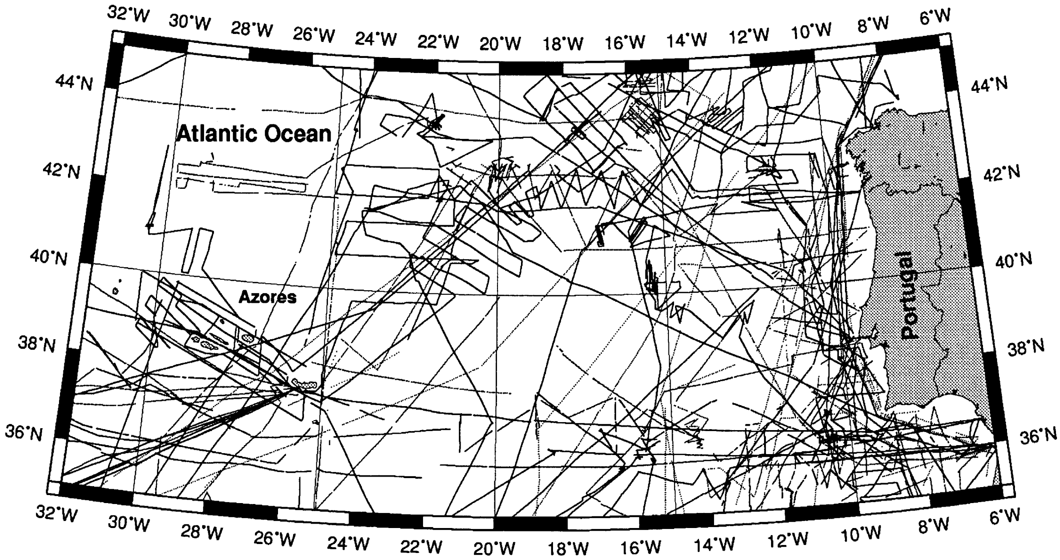

Fig. 1. Geographic distribution of marine gravity data in North-East Atlantic Ocean between Azores archipelago and Iberian peninsula.

this procedure, 776 tracks were detected, with a mean length of 199 km and a maximum length of 4264 km.

4.2. Pre-processing

Once the header, data, source and parameter files had been constructed, each track was analysed in order to detect internal gross errors. The pre-defined rejection limit was 15 mGal, the correlation distance 15 km and the critical value, k, was 2.58, meaning that there is a 1% probability of rejecting a valid observation. Applying this procedure to all 776 tracks, 203 observations were considered erroneous and eliminated, corresponding to less than 1% of the total data set. It was found that only 29 tracks (less than 4%) had outliers that had to be removed, most of which corresponded to missions in the period before 1978. The resulting data set was 196,229 observations with a mean value of 10.36 mGal and a standard deviation of 46.40 mGal.

4.3. Detection of intersections

In this step, each track is compared with all the others, in order to search for possible intersections. The programme follows the track order specified in the header file and checks to the minimum and maximum limits of each track written in this file to avoid searching all data points of all cruises. The cross-over point was determined by a simple line intersection algorithm and was computed only for those tracks in which the distance between two consecutive observations were less than 2 km. Linear interpolation was used to determine the anomaly gravity value at the cross-over point. Once an intersection is found, its geographic position and the anomaly gravity value in the intersection point for both legs is written in the intersection file.

Internal COEs were detected for 38 tracks with a maximum value of 20 mGal and a standard deviation of 5.3 mGal. Wessel and Watts (1988) showed that tracks with a small number of internal crossings tend to yield arbitrary drift rates. For this reason they suggest that the drift rate should not be computed for tracks with less than 100 internal crossings. If this recommendation were strictly applied to this data set, there would be only three cruises satisfying this criterion. This recommendation was applied to this data set and the drift rate was iteratively adjusted for two cruises with a drift rate of −0.11 and 0.23 mGal/day, and no drift rate was applied for the AGMASCO cruise.

When the entire data set of 776 tracks had been processed, 6017 intersections were detected involving 721 tracks. There are tracks without any intersection (55 tracks) and 41 tracks with only one intersection. The reason for this large number of tracks without any intersection was due to the algorithm used in the detection of tracks (from the BGI data file), which may divide a cruise into a large number of tracks. Therefore, data registered in the BGI data bank with minimal information concerning each cruise will not be properly handled and will introduce unavoidable errors in the data validation procedure. Nevertheless, it was decided to keep these gravity data, as long as each track has at least two intersections, in the validation procedure, providing better data coverage. The maximum number of intersections was 405 for cruise 8 806 311 (from France) with a total length of 3976 km.

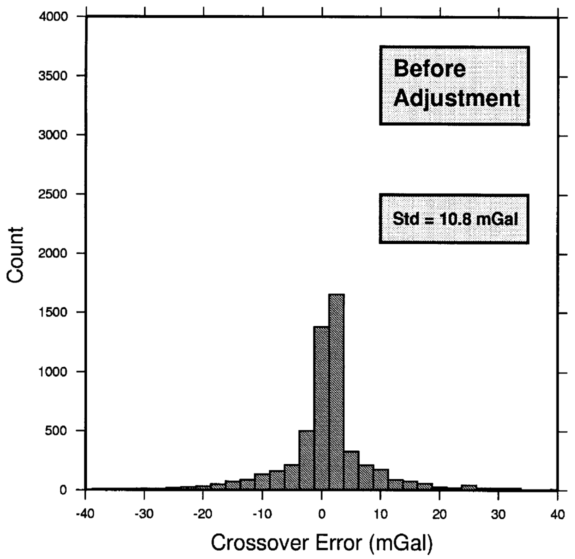

Fig. 2. Histogram of external COEs before adjustment (in mGal).

A histogram showing the frequency distribution of external COEs is presented in Fig. 2. For the 721 tracks with intersections, the external COE had a standard deviation of 10.81 mGal and a minimum and maximum value of −94.87 and 103.47 mGal, respectively. The amplitude of the COE is almost half of the gravity signal in this area, meaning that there are large discrepancies in the gravity observations belonging to different missions.

4.4. Adjustment

The bias for each track was determined through a global adjustment of the external COE and different weights were given to each observation equation. The adjustment is not a straightforward procedure and there are several ways of doing it. Three different adjustment schemes were tested and the a posteriori variance of unit of weight after the adjustment was used as a decision marker.

The first solution, solution A, was obtained from a one-step global-free network adjustment with inner constraints, and two weight models based on the cruise date (A.1) and on the COE standard deviation associated with each track (A.2). In model A.1, the standard deviation was attributed for each cruise, or leg, based on its surveying date, in which a 2.5 mGal was taken for the most recent tracks (after 1980), a value of 7.5 mGal was taken for the tracks between 1970 and 1980, and a value of 12.5 mGal was taken for tracks before that date (1970). In model A.2, the weight model is based on the standard deviation associated with a track and was computed as the standard deviation of all COEs belonging to that track.

A second solution, solution B, was also computed in a one-step global adjustment constraining to a zero bias all tracks surveyed after 1980 (constraint adjustment). We have assumed that since 1980 all cruises have been equipped with precise navigation systems and the derived gravity data quality has improved considerably. The same two weight models used in solution A were also tested: based on the cruise date (B.1) in which a 2.5 mGal was taken for the most recent tracks (after 1980), a value of 7.5 mGal was taken for the tracks between 1970 and 1980, and a value of 15.0 mGal was taken for tracks before that date (1970), and on the COE standard deviation (B.2).

The third solution, solution C, was again obtained as a one-step global adjustment constraining to a zero bias the AGMASCO cruise, surveyed in 1997 (minimum constraint adjustment). The weight models are the same as in the first two solutions: based on the cruise date (C.1), in which a 2.5 mGal was taken for the most recent tracks (after 1980), a value of 5.0 mGal was taken for the tracks between 1970 and 1980, and a value of 10.0 mGal was taken for tracks before that date (1970), and on the COE standard deviation (C.2).

The weight model for each solution was achieved after a small number of adjustments through the analysis of the variance of unit of weight, computed with the VALDAMA programme.

In the chosen area of the Atlantic most of the tracks (92%) were connected through a network, but there were small groups that were not connected to the main net. In this situation, it is also necessary to define the reference track for these independent adjustments. In these small independent networks, if there is at least one cruise that meets the imposed conditions the adjustment datum is already defined, but if this is not the case it is necessary to define the adjustment datum. In these particular cases the longest track belonging to that network was chosen as the fixed track. These small networks may introduce some inhomogeneities in the final data bank in cases when another adjustment datum was defined. Throughout, it was decided to analyse each net individually before including it in the final data bank in order to detect unfitted data.

The results of the external COE after adjustment of the three solutions are presented in Table 1. In Fig. 3 the histogram of COE after adjustment is depicted. After adjustment the standard deviation of the COEs falls to 3.0 mGal in the two last solutions. The adjustment method has a much larger effect on the standard deviation of the residuals than the stochastic model (weight method). From method A (inner constraint) to C (minimum constraint) we observe a decrease in the standard deviation of the adjusted COE from 7.22 to 3.0 mGal. This means that inner constraint adjustment may produce less precise results than minimum constraint adjustment. This is true if the minimum

Table 1
Statistics of COEs before and after adjustment (values in mGal) and computed variance of unit of weight after adjustment ($\hat{\sigma}_0$)

Type of adjustment/weight method	External [COEs]		
	$\hat{\sigma}_0$	Mean	Std
Inner Constraint			
A.1—weight as function of acquisition date	1.08	0.94	7.22
A.2—weight as function of intersection variance	1.89	0.36	6.77
Constraint (all tracks after 1980)			
B.1—weight as function of acquisition date	1.00	−0.03	3.13
B.2—weight as function of intersection variance	1.05	0.03	3.37
Minimum constraint (agmasco survey)			
C.1—weight as function of acquisition date	1.00	−0.03	3.00
C.2—weight as function of intersection variance	0.95	0.01	3.03
Original data (not validated)		1.12	10.81

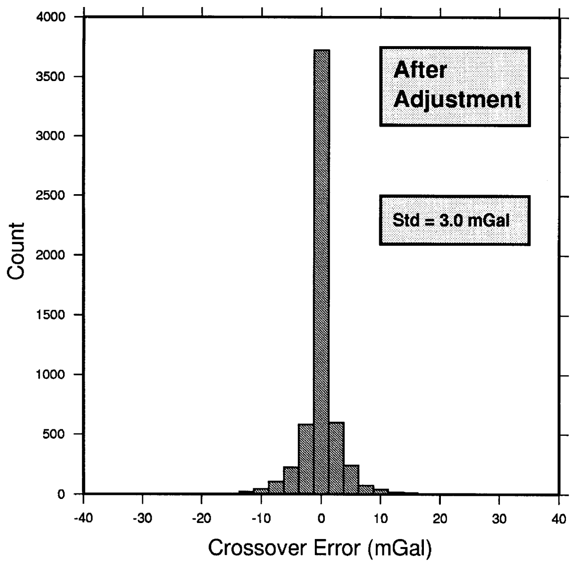

Fig. 3. Histogram of external COEs after adjustment, solution C.1 (in mGal).

Table 2
Results of comparison between gravity anomaly-adjusted values and EGM96 geopotential model gravity anomalies, KMS99 gravity anomalies and EGM96 geopotential gravity anomalies plus residual terrain model effects computed with AZDTM bathymetric model ($\Delta g_{residual}$), values in mGal.

Type of adjustment	$\Delta g - \Delta g_{EGM96}$		$\Delta g - \Delta g_{KMS99}$		$\Delta g_{residual}$	
	Mean	Std	Mean	Std	Mean	Std
A.1	−3.3	21.1	−2.7	8.4	−5.2	18.0
A.2	−3.1	21.0	−2.6	8.2	−5.0	17.9
B.1	−2.8	20.6	−2.1	7.5	−4.7	17.6
B.2	−3.2	20.6	−2.6	7.7	−5.2	17.7
C.1	4.1	20.5	4.9	7.2	2.2	17.4
C.2	4.1	20.5	4.8	7.1	2.2	17.5
Original data	−4.1	24.6	−2.8	12.6	−5.8	21.3

constraint is applied to a recent mission, the accuracy of which is beyond doubt. Considering the stochastic model (weight method), we see that there are no significant changes in the standard deviation of the adjusted COE, but the a posteriori variance of unit of weight after adjustment is not acceptable by the chi-square test. In fact, we do not control the weight applied to each observation, as it is defined by the standard deviation of the track it belongs to. Considering the adjustment method and the stochastic model, we may conclude that the preferred solution would be to adjust with a minimum constraint in the conditions described above and with a weight scheme based on the acquisition date such that the a posteriori variance of unit of weight after the adjustment would give a value of unity.

The adjusted gravity anomalies for each solution were also compared with the EGM96 geopotential model, with satellite-derived gravity anomalies global data set KMS99 (Andersen and Knudsen 1998) and a set of residual anomalies were also computed (Table 2). These residual anomalies were computed by removing the long wavelength and short wavelength of the gravity field, determined by the EGM96 geopotential model and the AZDTM (Catalão and Sevilla, 1999) digital terrain model, respectively. The result of this comparison shows that three different solutions give similar results in the standard deviation of the differences, but with a high mean value. When the main purpose of this validation and adjustment is geoid determination it is important that the residual anomalies (after the removal of the global geopotential model contribution and terrain effects) should be centred (with a zero mean value). In this test case the solution that is closest to this ideal assumption is again the third one. We may conclude again that the third solution, based on minimum constraint, fulfils the ideal conditions for geoid determination better.

5. Discussion and conclusions

There exist several programmes for marine gravity data validation, but most of them were designed for specific institutional purposes and are not in the public domain. The main advantages of VALDAMA over the formal programmes are the possibility of defining all adjustment and validation parameters and its import-export facilities.

This paper has presented the mathematical and stochastic formulation related to the validation and adjustment of marine gravity track data. A programme was presented for validation and adjustment of marine data followed by a case study in the North Atlantic Ocean. All adjustment possibilities were analysed in this area and the adjustment solutions are compared and analysed. It was shown that step-by-step and interactivity of the programme allowed full control of the validation and adjustment procedure. The adjustment statistical parameters are easily accessed by ASCII files written in fixed formats readable by MS Office tolls.

The case study presented in this paper enables us to conclude that the adjustment method has a much greater effect on the standard deviation of the residuals than the stochastic model (weight method). From the inner constraint adjustment an adjusted COE standard deviation of 7.22 mGal was obtained, as opposed to a value of 3.0 mGal from minimum constraint adjustment. This means that inner constraint network adjustment may produce less precise results than minimum constraint adjustment. Considering the adjustment method and the stochastic model (weight method), we may conclude that the preferred solution would be to adjust with minimum constraint in the conditions described above, and with a weight scheme based on the acquisition date such that the a posteriori variance of unit of weight after adjustment would give a value of unity. The results show that marine gravity data precision was significantly improved with this methodology, in a proportion of 3 to 1.

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