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Absolute airborne gravimetry: a feasibility study

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ABSTRACT

We report here the results obtained during a feasibility study that was pursued in order to evaluate the performances of absolute airborne gravimetry. In contrast to relative systems, which use spring-type gravimeters, each measurement acquired by absolute systems is independent from the others and the instrument is not suffering from problems like instrumental drift, frequency response of the spring and variation of the calibration factor. After a validation of the dynamic performance of the experimental setup in a moving truck, a comparison between the experimental airborne data retrieved over the Swiss Alps and those obtained by ground upward continuation at flight altitude allow us to state that airborne absolute gravimetry is feasible. The first test flight shows a spatial resolution comparable to those obtained by relative airborne gravimetry. For a wavelength on the order of 12 km the absolute value of gravity can be evaluated with an uncertainty of 6.9 mGal.

Key words: Airborne, Absolute gravity, GPS, Gravimetry.

INTRODUCTION

In recent years, global change has become of concern for mankind. In this context precise knowledge of the Earth's gravity field is of major importance in order to establish the physical reference surface needed, for instance, for monitoring global sea level changes. There is a large uncovered wavelength domain, which can only be filled by airborne or satellite borne techniques. Until now, airborne gravity surveys were carried out with relative techniques using spring-type gravimeters. These suffer, however, from correlated measurements, instrument drift and frequency response of the spring. Because absolute gravimeters are not suffering from any one of these problems, absolute airborne gravimetry may be a possible tool for determining the geoid where ground measurements are very difficult or even impossible. The aim of the project was to develop a new airborne gravity measurement system, which should provide absolute values of the Earth's gravitational acceleration and its anomalies. These would not be influenced by any instrumental disturbing effects like drifts and tares because the absolute system does not need any experimental calibration and only measures two basic physical units: length and time.

In this experiment, we used a small version of a land-based absolute gravimeter FGL modified for use on dynamic platforms, global positioning system (GPS) receivers to remove the non-gravitational accelerations and an inertial measuring unit to determine the attitude of the platform. The absolute gravimeter employs a laser interferometer that measures the free fall trajectory of an optical element within an evacuated chamber. Standards for length and time are provided respectively by an iodine stabilized primary reference laser and by a rubidium frequency reference, which provide high accuracy and stability. Such a system is therefore drift-free and quasi insensitive to horizontal accelerations. In principle it is able to perform measurements at a rate of one measurement every two seconds. The advantage of an absolute gravimetry is that the measurements are uncorrelated, which means that a sporadic perturbation, like a shock or a very strong vertical

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Figure 1 Map showing the location of the test flight in Switzerland.

acceleration, will affect only the actual measurement and not the following ones as it is the case with spring-type gravimeters measuring in a relative mode.

The complete experimental setup was installed in a De Havilland Twin-Otter aircraft belonging to the Swiss Federal Geographic Survey and operated by the same office. The line Martigny-Leuk in the Swiss Alps was flown four times at a nominal speed of approximately 70 m/s at a flight altitude of about 2500 m a.s.l.

THEORETICAL BACKGROUND

Gravity measurements on a moving platform are a classical problem where a physical phenomenon, defined in an accelerated reference system, has to be described in a fixed one with data that where acquired in different reference systems.

The principal reference systems

In the following a short description of the principal reference systems, used in terrestrial mechanics, will be given. These systems are illustrated in Figs 2 and 3.

The Copernican reference system

The Copernican reference system (R_I) is a quasi-inertial system with its origin at the centre of the mass of the solar system. It is a Cartesian system with its axis oriented in the direction of fixed stars.

Terrestrial reference system

This is a Cartesian reference system rigidly tied to the Earth (R_T) with its origin at the centre of mass of the Earth-Atmosphere system. The z-axis corresponds to the mean rotation axis of the Earth whereas the x-axis and y-axis are located in the equatorial plane with x crossing the meridian of Greenwich. Since the rotation axis of the Earth is the subject of precession and nutation this system has to be materialized by ground reference stations, which are referenced with respect to celestial bodies. The most widely known terrestrial systems are the ITRF (International Reference System) and the WGS-84 (World Geodetic System).

Local reference systems

A local reference system (R_L) is a system that can be defined in any point P at the surface of the Earth. Its z-axis corresponds to the direction of g at the point P and the x and y form a tangent plane to the equipotential surface at the same point. The x-axis points towards geographic north and the y-axis points towards east. In order to be able to define such a system it is necessary to know the direction of g at each point at the surface of the Earth. In practice one constructs a model of the Earth's gravity field, which allows a mathematical description of the Earth's ellipsoid. In this manner the z-axis corresponds to the normal of the ellipsoid.

In the frame of this project we will use, for defining the position of a point P, the terrestrial reference system WGS-84 in which P is fully defined by its geographical coordinates: latitude, longitude and ellipsoidal height (λ, φ, h). In this manner

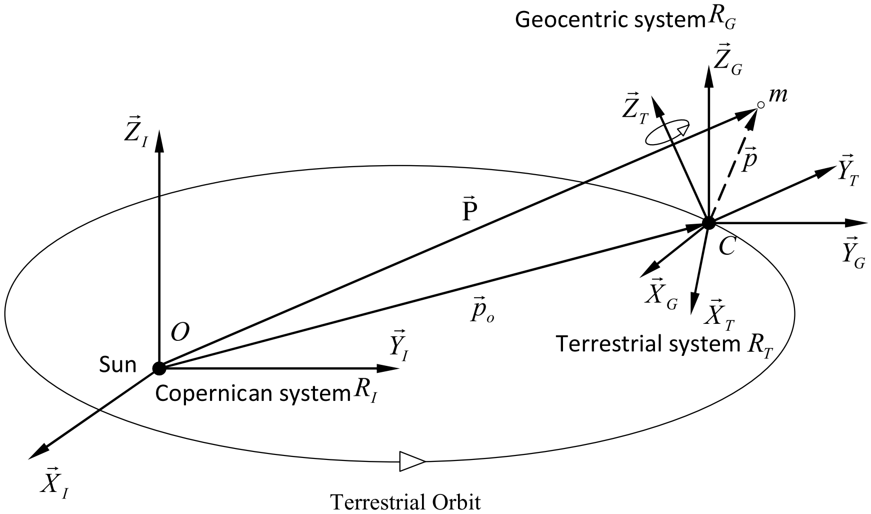

Figure 2 The principal reference systems used in celestial and terrestrial mechanics. R_C or R_I is the Copernican system, R_G is the geocentric system with its translation with respect to R_C, R_T is the terrestrial system with its origin at the same point as R_C and rotating around Z_T. m is the moving mass with geocentric vector $\vec{p}$.

it is possible to form a base ($\vec{u}_E$, $\vec{v}_N$, $\vec{n}$) in P (λ, φ, h), which is commonly called the local geographic reference system.

Geocentric reference system

This system (R_G) is a Cartesian reference system with its origin at the centre of mass of the Earth. This system has a translation movement with respect to the Copernican reference system. It is materialized by the observation of celestial bodies with invariant positions.

Measurement reference system

This system, (R_M), corresponds to the reference system into which each measuring instrument located inside the moving platform works.

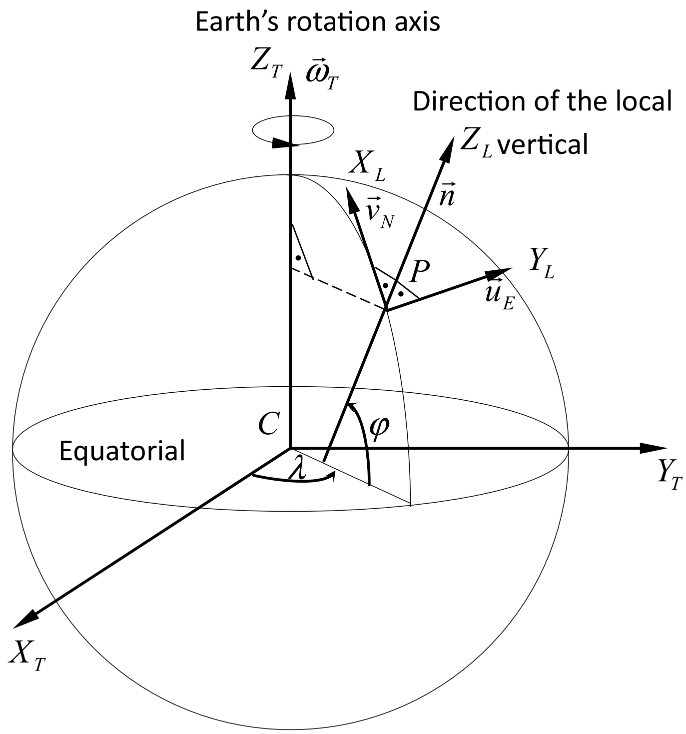

Figure 3 Definition of the reference system of the local gravity field R_L. The three unit vectors $\vec{u}_E$, $\vec{v}_N$, $\vec{n}$ are defining the local base at the point P (λ, φ, h) referred to the ellipsoid.

Figure 4 View of the interior of the aircraft during the test survey. 1) Gravimeter FGL, 2) data acquisition system of the EpiSensor, 3) rack containing the GPS receivers and the PCs, 4) navigation system for photogrammetry, 5) inertial navigation system (INS), 6) steering and data acquisition system of the inertial navigation system.

Dynamics on a moving platform, gravity measurements

The basic expression for the measurement of g on board a moving platform (aircraft) in the Earth's reference frame is given by:

$$\vec{P}(\vec{p})_{apparent} = \vec{g}(\vec{p}) - [\dot{\vec{V}}(\vec{p})_{Platfom} + (2\vec{\omega}_T + \vec{\omega}_L) \times \vec{V}(\vec{p})_{Platform}], \tag{1}$$

whereby

$\vec{P}(\vec{p})_{apparent}$: sum of the accelerations measured by the used sensor (apparent acceleration)
$\vec{g}(\vec{p})$: value of the gravity
$\vec{V}(\vec{p})_{Platform}$: velocity of the platform
$\dot{\vec{V}}(\vec{p})_{Platfom}$: acceleration of the platform
$\vec{\omega}_T$: angular velocity of the Earth's rotation with respect to the geocentric reference system.
$\vec{\omega}_L$: angular velocity of the local reference system with respect to the terrestrial system.

This expression contains two parts: the real gravity $\vec{g}(\vec{p})$on one side and the sum of all non-gravitational accelerations $\vec{A}_{cin} = \dot{\vec{V}}(\vec{p})_{Platform} + (2\vec{\omega}_T + \vec{\omega}_L) \times \vec{V}(\vec{p})_{Platform}$, which are tied to the movement of the platform on the other side.

The main difficulty of dynamic gravimetry is to separate the two terms in order to obtain the value of g. This separation can be done either by mechanically or numerically filtering or by modelling the non-gravitational acceleration before subtracting them from the total acceleration.

In a simplified manner equation (1) can be written:

$$\vec{P}(\vec{p})_{apparent} = \vec{g}(\vec{p}) - \vec{A}(\vec{p})_{cin}. \tag{2}$$

In the frame of this project we focused our attention on the intensity of the gravity field $g(p)$ at a point p. $g(p)$ is a scalar value defined by $\vec{g}(p) = -g(p)\vec{n}_{true}$, where $\vec{n}_{true}$ corresponds to the unit zenith vector pointing toward the local vertical. In order to be able to determine the kinematics accelerations we will make the approximation that the local vertical is parallel to the normal of the ellipsoid ($\vec{n} \approx \vec{n}_{true}$). This is justified, because, if we assume that the deflection of the vertical does not exceed $\varepsilon = 30''$ on the surface of the Earth, the relative error on the static measurement $g_{measured}$ is given by:

$$g_{measured} = g\cos(\varepsilon) \approx g\left(1 - \tfrac{\varepsilon^2}{2}\right)$$
$$\Rightarrow \frac{g - g_{measured}}{g} = \frac{\varepsilon^2}{2} = 10^{-8}.$$

Then for $g_{measured} = 9.80\ \mathrm{ms}^{-2}$, the error is $\delta g = 9.80 \cdot 10^{-8}\ \mathrm{ms}^{-2}$, which is much smaller than the accelerations produced by the movements of the aircraft. Consequently this approximation can be used without restriction.

For equation (2), using our approximation we can write:

$$\vec{P}_{apparent} = (-g(p) - A_{cin_n}(p))\vec{n} = -P_{apparent}\vec{n} \Rightarrow P_{apparent} = g(p) + A_{cin_n}(p).$$

Projecting $\vec{A}_{cin}$ on $\vec{n}$, one obtains A_{cin_n}

$$(\dot{\vec{V}}_{Flight}(p) + (2\vec{\omega}_T + \vec{\omega}_L) \times \vec{V}_{Flight}(p))\vec{n}.$$

In the local base system $(\vec{u}_E, \vec{v}_n, \vec{n})$ the global rotation's vector can be decomposed in the following manner:

$$2\vec{\omega}_T + \vec{\omega}_L = \begin{bmatrix} -\dot{\varphi} \\ (2\omega_T + \dot{\lambda})\cos(\varphi) \\ (2\omega_T + \dot{\lambda})\sin(\varphi) \end{bmatrix}.$$

We then obtain for A_{cin_n}:

$$A_{cin_n}(p) = \ddot{h} - (V_N\dot{\varphi} + V_E(2\omega_T + \dot{\lambda})\cos(\varphi) = \ddot{h} - E(p),$$

where E is the Eötvös correction, $\ddot{h}$ the vertical acceleration of the platform (aircraft) and V_E, V_N the flight parameters given in the reference system of the local gravity field R_L.

The basic relationship that gives the intensity of gravity in airborne gravimetry can then be written as:

$$g(p) = P_{apparent} + E(p) - \ddot{h}.$$

The Etövös corrections can be written as a function of the flight parameters V_E, V_N and h as follows:

$$V_E = (N(\varphi) + h)\cos(\varphi)\dot{\lambda}$$
$$V_N = (M(\varphi) + h)\dot{\varphi},$$

where $N(\varphi)$ and $M(\varphi)$ are the principal radii of the curvature normal and meridian at the point p, respectively.

If we call $f = \frac{a-b}{a}$ the ellipsoidal flattening, with a and b major respectively the minor axis of the ellipsoid, the Eötvös correction can be written (Torge 1991)

$$E(p) = \frac{V^2}{a}\left[1 - \frac{h}{a} - f(1 - \cos^2(\varphi)(3 - 2\sin^2(\alpha))\right] + 2V\omega_T\cos(\varphi)\sin(\alpha),$$

where $V^2 = V_E^2 + V_N^2$ and where α represents the flight azimuth so that $V_E = V\sin\alpha$ and $V_N = V\cos\alpha$

Using the speed of the aircraft projected to the Earth's surface and writing $V_{Earth}^2 = V_{E_{Earth}}^2 + V_{N_{Earth}}^2$ the relationship becomes (Harlan 1968):

$$E(p) = \frac{V_{Earth}^2}{a}\left[1 + \frac{h}{a} - f(1 - \cos^2(\varphi)(3 - 2\sin^2(\alpha)))\right] + 2V_{Earth}^2\omega_T\cos(\varphi)\sin(\alpha)\left(1 + \frac{h}{a}\right). \quad (3)$$

Consequently all cinematic accelerations can be determined from the values of the coordinates, (λ, φ, h), the first derivatives with respect to the time of the longitude and the latitude, $\dot{\lambda}$ and $\dot{\varphi}$ and the vertical acceleration $\ddot{h}$ of the aircraft.

In conclusion, in order to be able to determine g from a moving platform (aircraft) it is necessary to have a very accurate positioning system in addition to the gravimeter allowing the computation of the disturbing accelerations.

EXPERIMENTAL SETUP

The experimental setup was formed by two main parts: the absolute gravimeter for the measurement of the apparent acceleration and a combined inertial measuring unit IMU – GPS receivers system for the determination of the attitude of the platform as well as the non-gravitational accelerations.

Absolute gravimeter FGL

The core of the system is the absolute gravimeter FGL from Micro-g LaCoste that allows the determination of the apparent acceleration $P_{apparent}$.

The FGL measures the position of a free-falling prism by means of a Michelson's interferometer, one arm of which is fixed and the second one is able to move vertically (Niebauer 1995). The incoming laser beam is separated into a reference beam and a measuring beam. These beams are reflected by the prisms parallel to the incident beams and recombined at the splitter, creating the interferences that are converted into an electrical signal by means of a photo-detector. By counting and timing the occurrences of the fringes, it is possible to determine the position of the falling prism, relative to the reference prism and the associated time. The equation of movement of the free falling prism relative to the reference can be described by the following equation,

$$z = z_0 + v_0 t + \frac{1}{2}P_{apparent}t^2 + \frac{1}{6}\gamma v_0 t^3 + \frac{1}{24}\gamma P_{apparent}t^4 \quad (4)$$

$$P_{apparent} = g + A_{cin},$$

where

g : gravity
A_{cin} : non-gravitational accelerations acting on the reference prism
v_0, z_0 : initial values of the velocity and position of the falling prism relative to the reference prism
γ : local vertical gravity gradient (free air gradient)

Inertial measuring unit IMU – GPS receivers system

In order to determine the attitude of the platform and the non-gravitational accelerations the system was equipped with an inertial measuring unit IMULN-200 from Litton combined with four NAVSTAR GPS receivers. A fifth receiver was permanently located at a ground station and served as a reference for the positioning. Two of the GPS receivers were installed on the wings, just above the engines and two on the roof of the cabin. These four receivers allowed computing pitch, roll and yaw of the aircraft and these results were used as redundant information of the IMU system.

THE DATA ACQUISITION

The data were acquired along a line of around 30 km, at an altitude of 2500 m a.s.l, located in the middle of the Swiss Alps (more or less vertical of the Rhone river). The mean velocity of the aircraft was 70 m/s (252 km/h). The line was flown four times, twice in each direction with a mean measurement rate of 2 seconds. The azimuth of the measurement line was 65 degrees east.

The aircraft was equipped with an automatic pilot and the pitch and roll angles were maintained below 2 degrees according to the results of the airborne survey of the French Alps (Verdun and Klingelé 2005).

The most difficult problem in absolute airborne gravimetry is the compensation of the perturbing vertical accelerations. To solve this problem it is absolutely necessary to have a system able to measure both the accelerations induced by the movements of the aircraft and those generated by the engines and the propellers.

Before the airborne gravity measurements were performed many experiments were performed in the laboratory in order to design the best possible vibrations absorbing system. For designing such a system a special two-hour flight performed with only a vibrations measurements system aboard was accomplished. This special flight gave very valuable information about the spectrum of the vibrations induced by the engines and the propellers. This spectrum shows two peaks at 83 Hz and 26.7 Hz (Fig. 5) with amplitudes of around 17 000 mGal and 7000 mGal respectively due to a chock wave produced by the propellers when passing in front of the wings, a low-frequency peak of amplitude of around 5000 mGal of unknown origin and finally a background noise centred at 130 Hz with a maximum amplitude of 2500 mGal. When seeing such a spectrum any one immediately feels the huge difficulty to which we were confronted in order to design a system able to measure g, in absolute value, with an accuracy and at a wavelength comparable of that obtained in relative airborne measurements.

In order to compensate these accelerations two different methods were studied. The first consisted of decoupling the gravimeter from its support by means of a mechanical filter while the second used a mathematical model for subtracting numerically the perturbing accelerations.

Three mechanical filters were studied in the laboratory: a commercial optical table, a multi-layer table made of three layers of honeycomb aluminium plates and one consisting in a suspending system (Figs 6 and 7).

The application of the transfer functions of these filters to the amplitude spectrum of the vibrations of the aircraft has shown that the hanging table horizontally constrained presented the best characteristic in flying conditions. Unfortunately this system like any other mechanical filter has the disadvantage of having a resonance frequency close to the frequency of some typical movements of an aircraft, consequently inducing broad oscillations of the table. The only possibility for controlling these oscillations would be to introduce a damper but that would also decrease the quality of the transfer function.

The numerical compensation and the general behaviour of the FGL were studied in detail also in the laboratory. From this study we could clearly see that the principal components of the FGL (laser, GT 650 board) allow measuring g with a resolution of 2.5 mGal for each single fall without the use of a super-spring. It was also possible to confirm that with the numerical compensation of the movements of the reference's mirror the residuals are reduced by a factor of 8.

In a secondary phase and in order to test the anti-vibration system that we developed, a ground test was performed using a small truck as a moving platform. The test consisted of performing measurements along a line of around 150 m long with known g values each 3 m at a velocity of 1.5 m/s. These ground reference data were obtained by measurements performed with a relative gravity meter Lacoste and Romberg model G # 319 and tied to the absolute gravity station of

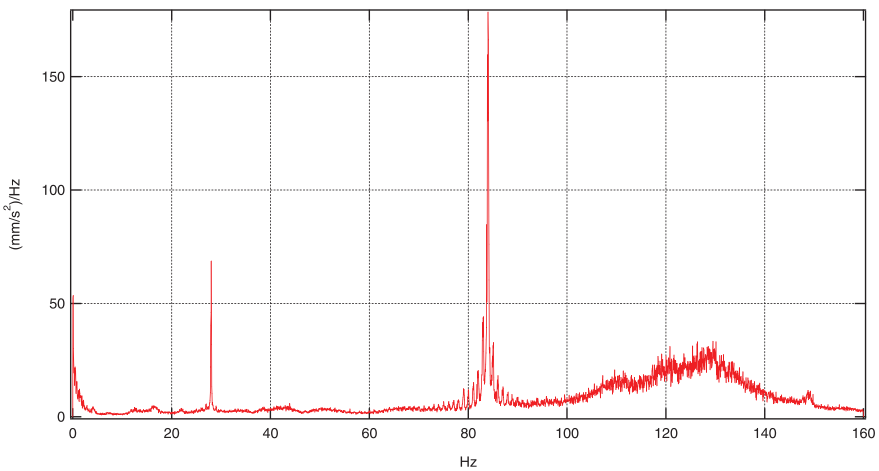

Figure 5 Power spectrum of the vibration recorded inside the aircraft during the vibration's test flight.

Switzerland at the ETH Zurich. The differences between the values obtained with the absolute moving system and those obtained by the static measurements were of few tens of mGal (Baumann 2004).

This test also served for choosing the best shock-absorbing base for the FGL to be installed inside the aircraft. According to the results obtained during these tests the base made of three layers honeycomb aluminium plates was chosen because of its good absorbing properties and of its lightness.

EVALUATION OF THE UNCERTAINTY CONTRIBUTIONS

In the following paragraphs the uncertainty contributions associated to the measurement and the evaluation of the absolute airborne gravity are estimated. The standard uncertainty, or uncertainty u, of a result of measurement reflects the lack of exact knowledge of the value of the measured. The expanded uncertainty, denoted U, is obtained by multiplying the uncertainty by a coverage factor k (ISO GUM 1995).

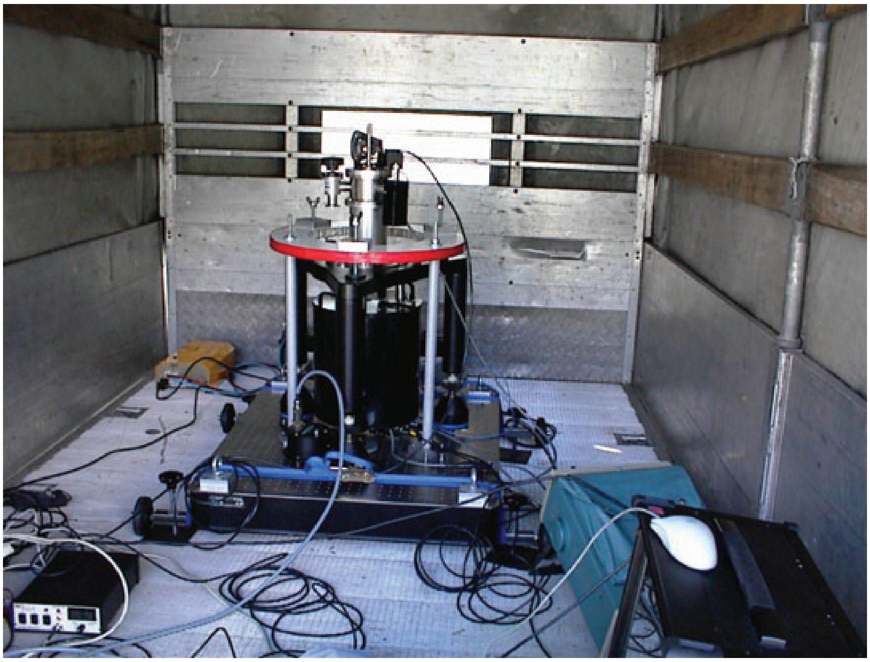

Figure 6 The absolute gravimeter mounted on the optical table inside the small truck during the dynamic ground experiment.

Figure 7 The absolute gravimeter mounted in the suspended anti vibration system in a small truck during the dynamic ground experiment.

Single drop uncertainty $\mathbf{u}_d$

The main difference between a traditional FG5 gravimeter and the gravimeter FGL used in this project is the absence of a super-spring (Rinker 1983). The super-spring is a long period electromechanical low-pass filter that allows decoupling the reference prism from the seismic environmental noise. The use of this kind of filter in a dynamic configuration is not possible because of the long time decay needed to reach stable working conditions.

To reduce the seismic noise, we used an external accelerometer, in our case the inertial measuring unit accelerometers, from which the double integrated measurement was introduced as observable in the equation system for the numerical determination of *g* (Brown, Niebauer and Klingelé 2001). With this approach, an uncertainty of 2.5 mGal can be reached for a single drop.

Verticality uncertainty u_θ

The metrological performances of the FG5 gravimeters in a laboratory condition have been described in detail by Niebauer (1995). In these conditions the non-gravitational accelerations that can generate a systematic error are essentially due to geophysical phenomena like tides, polar motions or pressure loads. These phenomena are inducing systematic errors smaller than one mGal. For this reason these contributions were not taken into consideration.

From all the contributions that have been considered by Niebauer (1995) only the influence of the vertical orientation has to be considered. The influence of the deviation of the reference laser beam from the verticality can be described by,

$$\frac{\delta g}{g} = -\frac{\theta^2}{2}, \tag{5}$$

where θ is the angle between the vertical and the reference laser beam. The deviation angle θ between the reference angle and the reference laser beam was defined by the mean of the inertial measuring unit – GPS system. The uncertainty associated to this correction is given by

$$u_{dg} = g\theta u_\theta.$$

The mean value of the uncertainties associated to the verticality correction has been evaluated to be in the order of 2 mGal.

Vertical acceleration uncertainty $u_{\ddot{h}}$

The correction of the apparent acceleration measured by the FGL during the flight can only be corrected by the non-gravitational accelerations with the help of a measuring system that is not affected by the geographical variation of gravity. Because the accelerometers of the inertial measuring unit are at least relative gravimeters it is not possible to use this information for the correction of the apparent acceleration to obtain *g*.

Because GPS can be considered independent of the geographical variation of g in the considered wavelengths the four GPS arrays fixed to the aircraft were used for the determination of the vertical acceleration. The uncertainty associated to the vertical acceleration has been estimated at 2 mGal for a wavelength longer than 10 km. This value is absolutely in the same order of magnitude with the values estimated by Klingelé, Cocard and Kahle (1997) and Verdun *et al.* (2003) for relative measurement.

Eötvös acceleration uncertainty u_E

The estimation of the uncertainty associated to the Eötvös acceleration is based on the estimation that was done during the ASFAG project (Verdun 2000) and that was evaluated to be 0.5 mGal.

Positioning uncertainty u_{GEO}

The uncertainty associated to the positioning of the gravity measurement is directly dependant to the horizontal and vertical gradient. The horizontal gradient in the area of our test survey (Alps) is around 2 μGal/m and a normal vertical gradient of 300 μGal/m. An uncertainty of half a metre in the horizontal position produces an uncertainty of 1 μGal and an uncertainty of 1 m in the vertical position produces an error of 300 μGal on the measurement of g. Because the GPS positioning has an uncertainty of the order of some cm the uncertainty associated to the mis-positioning can be neglected.

DATA PROCESSING AND ESTIMATION OF g

The main difficulty in processing the acquired data is to separate the non-gravitational accelerations from the apparent accelerations measured by the absolute gravimeter. The non-gravitational accelerations can be divided into two groups. The high-frequency accelerations are mainly generated by the engines and propellers and the low-frequency acceleration is due to the movements of the aircraft. The compensation of these accelerations can be done in two distinguished steps.

In the first step, the apparent acceleration for each drop is approximated by least-squares adjustment of a physical model taking into account the movement of the reference prism during the drop. To estimate this movement the vertical acceleration measured by the inertial measuring unit is high-pass filtered with a cut off frequency defined by the drop time. The filtered signal is then integrated twice to obtain an approximation of the movement of the reference prism during the drop. This function is introduced into the physical model (equation (7)) used for the first evaluation of g by least-squares adjustment.

$$z_i = \frac{1}{2} g t_i^2 + \dot{z}_0(t_i) + z_0 + \alpha_s F_s(t_i). \tag{7}$$

z_i : position of the prism at a time i
z_0 : initial position of the prism
$\dot{z}_0$: initial velocity of the prism
g : gravity
t_i : time
α_s : scaling factor
F_s : measurements of the external sensor (inertial measuring unit)

Before the reduction of the low-frequency perturbations, an anti spike procedure was applied to the high-pass filtered data. This procedure consists in a consistency check between the measured values and a mathematical model. The used model has been calculated with following formula (Torge 1991)

$$g_0(\varphi, h) = \frac{a g_E \cos^2\varphi + b g_P \sin^2\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} \times \left[1 - \frac{2}{a}\left(1 + f + m - 2f\sin^2\varphi\right)h + \frac{3}{a^2}h^2\right], \tag{8}$$

with

$$m = \frac{a^2 b \Omega_E^2}{GM_E},$$

and

$g_E = 9.7803253359$ m/s^2	the normal gravity at the equator
$g_P = 9.8321849378$ m/s^2	the normal gravity at the pole
$GM_E = 3.986004418 \cdot 10^{14}$	the gravitational constant times the mass of the Earth
$\Omega_E = 3.986004418 \cdot 10^{-5}$ rad/s	the Earth's rotation rate
$a = 6378137$ m	the semi-major axis of the WGS84 ellipsoid
$b = (1-f)a$	the semi-major axis of the WGS84 ellipsoid
$f = 1/298.257223563$	the flattening factor of the WGS84 ellipsoid

All measured values that presented a deviation from the mathematical model larger than 20 000 mGal were eliminated. The criterion has been determined in such a way that at least 70% of all acquired measurement were satisfying it.

Finally, a band-pass filter, taking in account the length of the flown lines, with cut-off frequencies of $Fc_H = 3$ mHz (wavelength: 25 km) and $Fc_L = 6$ mHz (wavelength: 12 km) was applied to the remaining apparent accelerations as well as to the cinematic acceleration before subtracting them from each other. Because the flown lines were very short, an artificial line of 120 km, composed by the four flown straight lines was constructed and processed.

VALIDATION WITH GROUND DATA

The validity of the absolute airborne measurements can be done by comparison with data obtained by independent methods of measurement, ground data, for example.

For this purpose the ground data have to be upward continued at the flight altitude or the airborne data have to be downward continued to the altitude of the ground measurements. Because the prolongation operator works like an amplifier for the short wavelengths of the anomaly, we adopted the first approach of upward continuing the ground data.

Instead of computing a Bouguer anomaly (equation (9)) for each airborne measurement we computed the theoretical value of g from the upward continued ground anomaly following the definition of the anomaly.

$$g_{Bouguer} = g_{measure} - g_{norm} + g_{Top} - g_{BPl} + g_{free-air}. \tag{9}$$

$g_{Bouguer}$: full Bouguer anomaly
$g_{measure}$: measured g
g_{norm} : normal gravity
g_{Top} : topographic corrections
g_{BPl} : Bouguer layer effect
$g_{free-air}$: free-air effect

Re-ordering relationship (9) one obtains:

$$g_{measure} = g_{Bouguer} + g_{norm} - g_{Top} + g_{BPl} - g_{free\text{-}air}. \tag{10}$$

It is, therefore, quite simple to obtain the theoretical measurements.

From the data bank of the European Geo Traverse (EGT), a grid with a mesh size of 1 km covering the whole of Switzerland and a part of Italy, Germany and France was produced by interpolation (Klingelé *et al.* 1996). In order to obtain a support surface having no parts higher than the flight altitude the stations located higher than 2000 m were removed from the data bank before interpolation. Using the FFT technique the anomaly was upward continued from a level of 500 m to 2500 m according to equation (11)

$$g(x, y, z_o + h) = F^{-1}\{e^{-fh} F\{g(x, y, z_o)\}\}. \tag{11}$$

The hypothesis of a horizontal support surface of the Bouguer anomaly at 500 m altitude is not completely true and could introduce distortions on the upward continued anomaly. In principle the continuation should be done from the irregular support surface to the horizontal plane at 2500 m by an equivalent source technique, for example. However, because most of the ground measurement points are between 400–1000 m altitude and spread across a large surface ($> 50\,000$ km^2), the distortion effect is relatively small, as the following evaluation has shown.

The upward continued anomaly was compared to measurement points lying between 2400–2600 m, where the Bouguer anomaly is known to be better than 0.5 mGal. This comparison shows that the difference is at most around ± 3.5 mGal. It is worth noting that the nodes of the grid do not correspond exactly to the location of the measurement points in the horizontal coordinates and altitude and, therefore, a linear three-dimensional interpolation was necessary for the comparison.

Results

The results obtained by the described processing procedure are represented in Fig. 8. The comparison between the upward continued gravity values and those obtained from the measured data after processing, shows quite good agreement.

The uncertainties associated to the processed absolute data are summarized in Table 1. Additional to the uncertainty contributions described above, the average and the standard deviations of the difference between the upward continued and the measured gravity have been introduced.

For a coverage factor $k = 1$ the uncertainty given in Table 1 is evaluated to 6.9 mGal. This corresponds to a confidence level of 68%. With a coverage factor $k = 2$ the expanded uncertainty is then 13.7 mGal, which corresponds to a confidence level of 95%.

Compatibility between g_{theo} and g_{mes}

The evaluation of the compatibility factor E_n that defines the ratio between the difference of two estimated values and the uncertainty of the difference is given by:

$$E_n = \frac{x_i - x_j}{\sqrt{u^2(x_i) + u^2(x_j)}}.$$

An E_n factor greater than one signifies that the difference cannot be covered by the uncertainty. This would imply that either at least one of the two values is corrupted, or the claimed

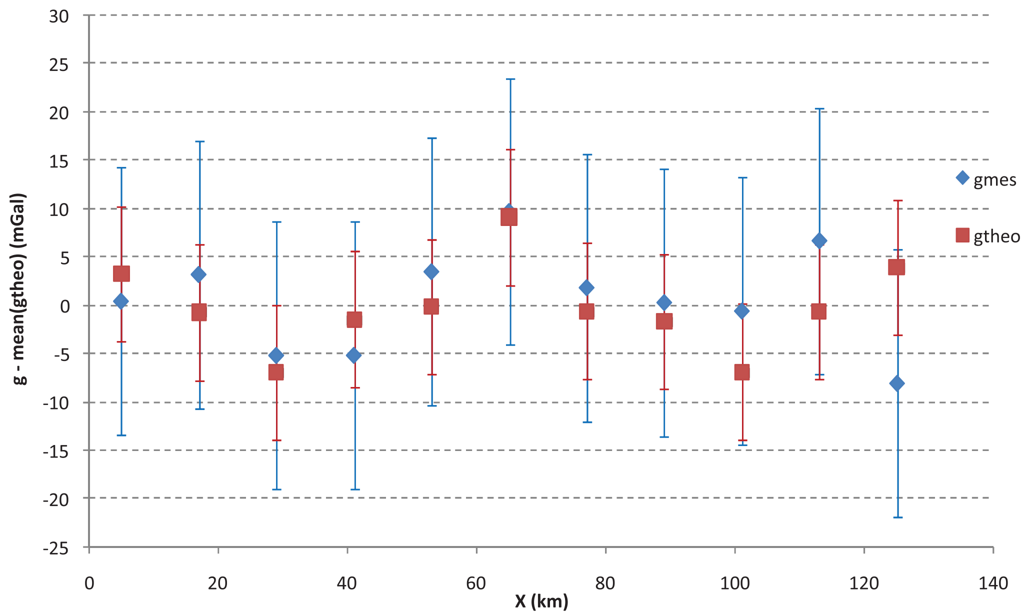

Figure 8 Comparison between the theoretical gravity value (gtheo, red squares) evaluated by upward continuation and the measured values after processing (gmes, blue lozenges). From both data series, the mean value of the theoretical value has been subtracted (mean (gtheo) = 979889.8 mGal). The error bars represent the expanded uncertainties ($k = 2$) that are respectively estimated at 7 mGal for the theoretical values and at 13.7 for the measured values.

uncertainties are too small. The evaluation of E_n is given in Table 2.

The evaluation of the compatibility between the airborne gravity data and the upward continued presented in Table 2 shows that even if one E_n value is larger than one the compatibility between both measurements is statistically satisfied. Effectively for $k = 1$ a global agreement larger than 68% must be reached while for $k = 2$ the agreement must be better than 95%. For both coverage factors the conditions are satisfied.

Based on the obtained results after the proceeding procedure and the evaluation of the uncertainty it can be stated that for a wavelength of 12 km the absolute value of the gravity at 2500 m can be determined by absolute airborne garvimetry with an uncertainty ($k = 1$) of 6.9 mGal and with an expanded uncertainty ($k = 2$) of 13.7 mGal.

Table 1 Summary of the contributions to the global uncertainty of the measured and processed gravity

Single drop uncertainty	2.0	mGal
Verticality correction	2.0	mGal
Vertical acceleration	3.0	mGal
Eötvös	0.5	mGal
Mean difference	0.8	mGal
Std of the difference	5.4	mGal
Uncertainty ($k = 1$)	6.9	mGal
Expanded uncertainty ($k = 2$)	13.7	mGal

CONCLUSIONS

In order to overcome the limits and deficiencies of relative airborne gravimetry the potential of the absolute one was studied

Table 2 Evaluation of the compatibility between the airborne and the upward continued gravity values. (For $k = 1$, $u_{gtheo} = 3.5$ mGal and $u_{gmes} = 6.9$ mGal. For $k = 2$, $u_{gtheo} = 7$ mGal and $u_{gmes} = 13.7$ mGal)

x (km)	g_{mes} (mGal)	g_{theo} (mGal)	E_n ($k = 1$)	E_n ($k = 2$)
5	979889.7	979892.50	0.4	0.2
17	979892.4	979888.50	0.5	0.3
29	979884.1	979882.29	0.2	0.1
41	979884.1	979887.77	0.5	0.2
53	979892.7	979889.07	0.5	0.2
65	979898.9	979898.35	0.1	0.0
77	979891.1	979888.62	0.3	0.2
89	979889.5	979887.58	0.3	0.1
101	979888.6	979882.34	0.8	0.4
113	979895.9	979888.59	0.9	0.5
125	979881.2	979893.12	1.5	0.8

in great detail. The absolute technique measures directly g by tracking a freely falling object in a vacuum chamber. The problem is to separate the non-gravitational aircraft accelerations and vibrations from the real gravity signal. The work presented here is, as far as we know, the first attempt worldwide to construct such a system for application in airborne mode.

The goal of this experiment was to find out whether absolute airborne gravimetry is feasible at all. From the results summarized in Table 1 we can clearly say that absolute airborne gravimetry is feasible.

For a wavelength of 12 km the absolute value of the gravity can be evaluated with an uncertainty of 6.9 mGal, which corresponds to the one obtained in relative airborne surveys (Klingelé *et al*. 1996; Verdun *et al*. 2003). The experimental setup we used during this first attempt can be further optimized. In particular, the levelling of the table must be improved to reach stability on the order of a few tenths of degrees. The uncertainty strongly depends on the flight conditions. In our experiments we did not have a choice of the flight area or of the flight altitude. These were imposed to us by air traffic control and by the meteorological conditions. A flight in a region with a very rugged topography like in the Swiss Alps with a flight altitude of 2500 m is far from ideal. It would have been better to fly higher than the highest mountain in order to reduce the influence of the turbulences. This first step of absolute airborne gravimetry opens the way for new methods, like airborne gradiometry and will contribute helping the measurement for geodynamic studies in general and to geophysical prospecting, in particular.

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