A two-step point mass method for regional gravity field modeling: a case study over the Auvergne test area

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Introduction

Regional gravity field modeling using Remove-Compute-Restore technique

- Numerical integral method
  - advantages: fast, straightforward
  - disadvantages: gridded gravity data, not well suited for data combination
- Least-squares collocation (LSC)
  - advantages: scatterred gravity data, optimal data combination
  - disadvantages: high computation burden
- **Parameter estimation method using radial basis functions (RBFs)**
  - advantages: scatterred gravity data, data combination, fewer unknown parameters
  - disadvantages: model construction, e.g., the number, the positions, and the type of the RBFs

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  - disadvantages: model construction, e.g., the number, the positions, and the type of the RBFs

The type of the radial basis function is fixed to be the point mass radial basis function

Goal of this presentation: develop a point mass method, which can determine the number and positions of the RBFs in a reasonable way

Point Mass Method – overview

Gravity field representation

$$T\left(\phi_i, \lambda_i, r_i\right) = \sum_{k=1}^{K} \beta_k B\left(\mathbf{r}_i, \mathbf{r}_k\right),$$

where $B\left(\mathbf{r}_i, \mathbf{r}_k\right) = \sum_{n=0}^{\infty} \left(\frac{r_k}{r_i}\right)^{n+1} (2n+1)\, b_n^{\text{PM}} P_n\left(\cos\psi_{ik}\right),$

$$\begin{cases} b_n^{\text{PM}} = \frac{1}{r_k(2n+1)} & \text{for } N_{\text{min}} \leq n \leq N_{\text{max}} \\ b_n^{\text{PM}} = 0 & \text{else} \end{cases}.$$

$N_{\text{min}} = 0$ & $N_{\text{max}} = \infty \rightarrow$ the full point mass RBFs

$N_{\text{min}} > 0$ & $N_{\text{max}} = \infty \rightarrow$ the reduced point mass RBFs

Point Mass Method – overview

Strategies to determine the positions of the point mass RBFs

- Point mass method with fixed positions
  - The number and positions of the RBFs are pre-defined, only the magnitudes are unknowns
  - Linear equation system: $\mathbf{l} - \mathbf{e} = \mathbf{A}\mathbf{x}$
- Point mass method with free positions
  - Not only the magnitudes, but also the number and positions of the RBFs are unknowns
  - Nonlinear equation system: $\mathbf{l} - \mathbf{e} = F(\mathbf{m})$

Point Mass Method — a two-step approach

The first step

- Goal: find a set of point mass RBFs at reasonable positions
- Realization: a search process by solving a series of small-scale bound-constrained nonlinear problems to minimize the misfit between the observations and predictions
  - Model factors: initial depth and depth limit, optimization direction, number of nearest point mass RBFs, and the iteration limit

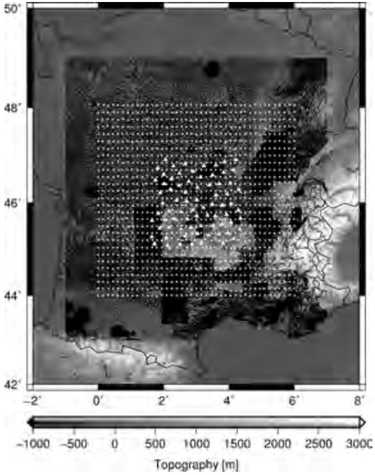
The second step

- Goal: provide a stable estimate of the magnitudes of all searched RBFs
- Realization: readjustment of the magnitudes by solving a linear system with known number and positions of the RBFs
  - Unconstrained solution: the reduced RBFs in the adjustment
  - Constrained solution: the full RBFs while considering the constraints in the adjustment

Numerical Experiment — data and preprocessing

Overview of the data in the Auvergne area

- Terrestrial data sets
  - 243954 (242809 + 1145) gravity anomaly measurements
  - DEM based on 3″ SRTM height data (version 3)
  - 75 GPS/leveling observations
- Global geopotential model
  - EGM2008 model up to d/o 360
- Residual terrain model (RTM)
  - Reference (30′ × 30′) & Coarse (30″ × 30″) & Dense (3″ × 3″)
  - Crustal density of 2.67 kg/m^3

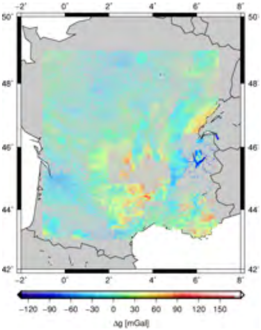


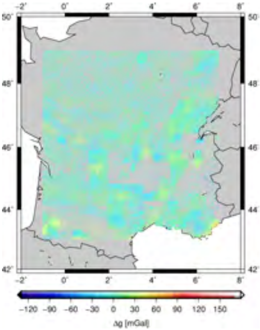
242809 $\triangle g$ observations (black dots), **1145** $\triangle g$ control points (white circles), and **75** GPS/leveling points (white triangles) in the Auvergne area.

Numerical Experiment – data and preprocessing

Original vs. reduced gravity measurements

	Mean	STD	RMS	Min	Max
Δg [mGal]	3.066	20.697	20.922	-127.459	177.841
Δg_{res} [mGal]	1.394	9.544	9.645	-49.426	72.054

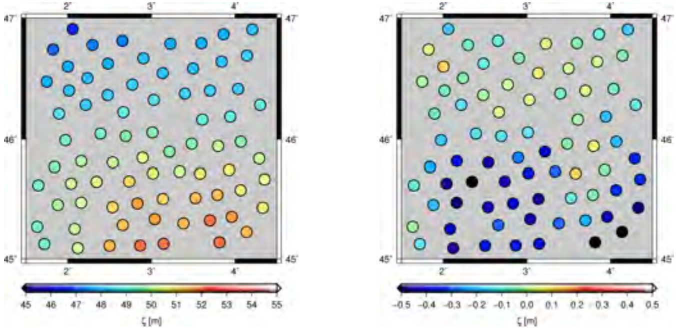




Numerical Experiment – data and preprocessing

Original vs. reduced GPS/leveling-derived height anomalies

	Mean	STD	RMS	Min	Max
ζ [m]	49.550	1.473	49.571	46.774	52.147
ζ_{res} [m]	-0.192	0.185	0.266	-0.605	0.108



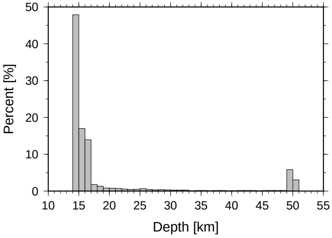
Numerical Experiment – model setup

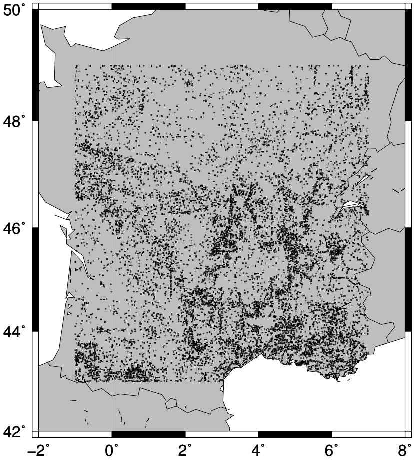
Model factors used for the regional modeling in Auvergne area

Spectral band of the RBFs	1) $N_{\text{min}} = 51$ (unconstrained solution) 2) $N_{\text{min}} = 0$ and $n' = 40$ (constrained solution)
Optimization direction	Radial-direction
Initial depth [km]	16
Depth limit [km]	14–50
Number N_ε of the nearest point mass RBFs	10
Total number K of point mass RBFs	10000
Iteration limit N_{it} for each new point mass RBF	20

Numerical Experiment – result of the first step

Depth histogram and horizontal distribution for 10000 searched point mass RBFs

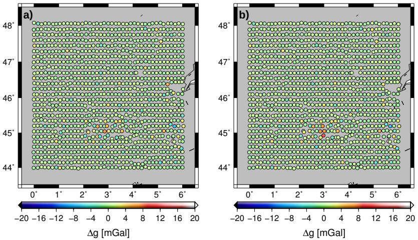


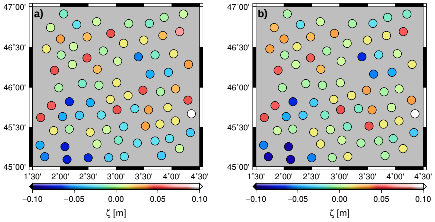


Numerical Experiment – validation of the gravimentric solutions

Predictions at 1145 Δg control points and 75 GPS/leveling points

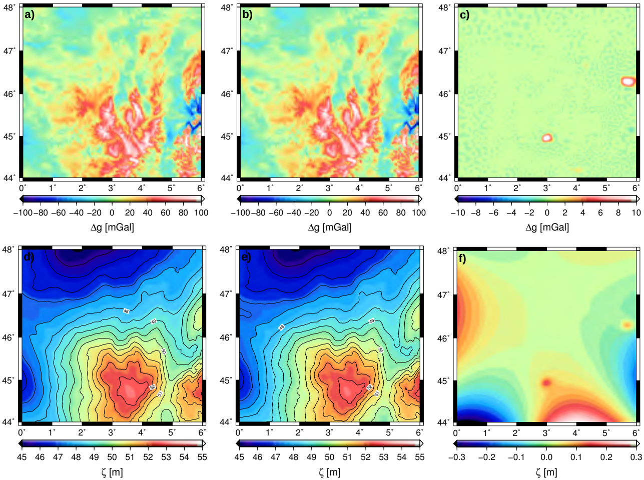
	Mean	STD	RMS	Min	Max
Δg [mGal]	0.048	1.190	1.191	-6.625	17.412
ζ [m]	-0.182	0.034	0.185	-0.255	-0.069
Δg [mGal]	0.078	1.239	1.241	-6.802	17.589
ζ [m]	-0.165	0.035	0.169	-0.247	-0.070





Numerical Experiment – comparison of the gravimentric models

$1' \times 1'$ gravimetric gravity anomaly and quasigeoid models and their differences



Numerical Experiment – combination of gravity and GPS/leveling data (1)

Computation details

- The first step for searching the point mass RBFs is based only on the gravity data
- Directly combination of gravity and GPS/leveling data by using the VCE technique in the second step

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Computation details

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Case 1

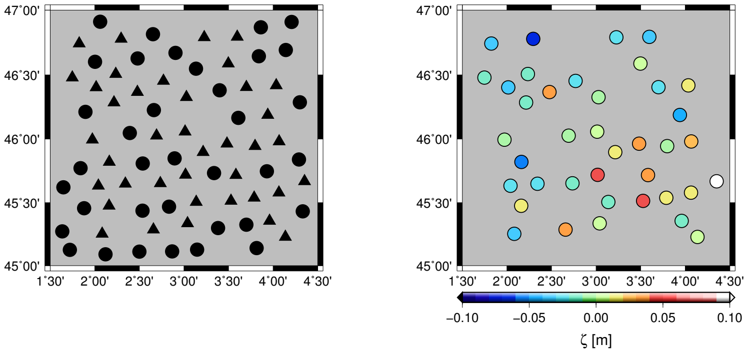
- 242809 Δg_{res} and 75 ζ_{res} are used as input
- Both full and reduced point mass RBFs are tested
- Predictions at 1145 Δg control points and 75 GPS/leveling points

	Mean	STD	RMS	Min	Max
Δg	0.051	1.190	1.190	-6.620	17.413
ζ	-0.175	0.034	0.179	-0.246	-0.058
Δg	0.047	1.191	1.191	-6.601	17.373
ζ	0.000	0.031	0.031	-0.066	-0.096

Numerical Experiment – combination of gravity and GPS/leveling data (2)

Case 2

- 242809 Δg_{res} and 36 ζ_{res} (the circles) are used as input
- Only full point mass RBFs are used for the combination
- **Validation at 39 GPS/leveling points (the triangles)**



Numerical Experiment – combination of gravity and GPS/leveling data (2)

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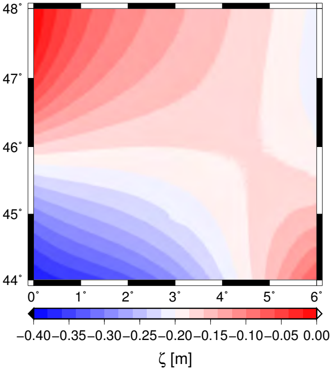
- Only full point mass RBFs are used for the combination
- **Validation at 39 GPS/leveling points (the triangles)**

	Mean	STD	RMS	Min	Max
(36)	0.001	0.032	0.031	-0.067	-0.064
(39)	-0.001	0.033	0.032	-0.062	-0.111

Numerical Experiment – combination of gravity and GPS/leveling data (2)

Case 2

- 242809 Δg_{res} and 36 ζ_{res} are used as input
- Only full point mass RBFs are used for the combination
- **Combined quasigeoid model – gravimetric quasigeoid model**



Summary

- The applicability of the proposed two-step point mass method is verified by the numerical experiment in the Auvergne area with moderate topography
- The computed gravity model is at the level of about 1.2 mGal, and the accuracy of the quasigeoid model is about $3 - 4$ cm
- The inconsistency between the gravimetric and GPS/leveling-derived height anomalies can be eliminated by combing the GPS/leveling data in the modeling
- The reduced point mass RBFs are suited for computing gravimetric solutions, but they fail to combine the GPS/leveling data. Instead, the full RBFs are better for computing combined solutions

Thank you for your attention!

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