GEOPHYSICS, VOL. 74, NO. 5 (SEPTEMBER-OCTOBER 2009); P. L67–L73, 5 FIGS.
10.1190/1.3170690

# Reduction to the pole and transformations of scattered magnetic data using Newtonian equivalent sources

Fernando Guspí[1] and Iván Novara[1]

## ABSTRACT

We have developed an equivalent-source method for performing reduction to the pole and related transforms from magnetic data measured on unevenly spaced stations at different elevations. The equivalent source is composed of points located vertically beneath the measurement stations, and their magnetic properties are chosen in such a way that the reduced-to-the-pole magnetic field generated by them is represented by an inverse-distance Newtonian potential. This function, which attenuates slowly with distance, provides better coverage for discrete data points. The magnetization intensity is determined iteratively until the observed field is fitted within a certain tolerance related to the level of noise; thus, advantages in computer time are gained over the resolution of large systems of equations. In the case of induced magnetization, the iteration converges well for vertical or horizontal inclinations, and results are stable if noise is taken into account properly. However, for a range of intermediate inclinations near 35°, a factor tending to zero makes it necessary to perform the reduction through a two-stage procedure, using an auxiliary magnetization direction, without significantly affecting the speed and stability of the method. The performance of the procedure was tested on a synthetic example based on a field generated on randomly scattered stations by a random set of magnetic dipoles, contaminated with noise, which is reduced to the pole for three different magnetization directions. Results provide a good approximation to the theoretical reduced-to-the-pole field using a one- or a two-stage reduction, showing minor noise artifacts when the direction is nearly horizontal. In a geophysical example with real data, the reduction to the pole was used to correct the estimated magnetization direction that originates an isolated anomaly over Sierra de San Luis, Argentina.

## INTRODUCTION

Reduction to the pole is a well-known technique that transforms an observed total magnetic field (TMF) into a vertically induced field, assuming only that the magnetization direction of the sources is constant and known. A magnetic field reduced to the pole provides a better picture of source distribution by removing the pattern distortion caused by oblique directions of observation and magnetization. Even when other interpretation techniques, which do not rely on magnetic directions, are applied (i.e., Euler or Werner deconvolution) (Reid et al., 1990; Hansen, 2005; Salem et al., 2008), a comparison of the results with a reduced-to-the-pole map provides a more integrated view; so reduction to the pole has become a standard part of magnetic data processing. Moreover, when the magnetization direction of the sources is unknown, correlation between reduced-to-the-pole fields and total gradients is used as a criterion for estimating the unknown direction (Roest and Pilkington, 1993; Dannemiller and Li, 2006).

Reduction to the pole often is calculated in the frequency domain, which requires prior gridding of the observed field at a constant altitude and sometimes lateral data extension to avoid aliasing effects. This technique is fast to perform; but for induced fields with nearly horizontal direction, noise is greatly amplified and several filters have been proposed to overcome the problem (Hansen and Pawlowski, 1989; Mendonça and Silva, 1993; Keating and Zerbo, 1996; Li and Oldenburg, 2001). Li (2008) summarizes and compares different algorithms for reduction to the pole at low latitudes.

A different approach is based on the concept of equivalent sources (Emilia, 1973; Silva, 1986; Li and Oldenburg, 2000; Nakatsuka and Okuma, 2006). A layer of sources is found, usually at an appropriate constant depth, whose magnetic effect reproduces the observed field. By shifting the magnetization and observation directions to the

Manuscript received by the Editor 27 November 2008; revised manuscript received 11 March 2009; published online 21 August 2009.
[1]Universidad Nacional de Rosario, Instituto de Física Rosario, Grupo Geofísica, Rosario, Argentina. E-mail: guspi@fceia.unr.edu.ar; novara@ifir-conicet.gov.ar.

vertical, the magnetic effect of the sources yields the field reduced to the pole. The equivalent sources usually are represented by point dipoles, doublets, or parallelepipeds. This approach can be applied to horizontally and vertically scattered data, but a drawback is its need to solve a large system of equations, even with resources such as conjugate gradients or wavelet transforms (Li and Oldenburg, 2000).

Leão and Silva (1989) propose a method to perform any linear transformation of potential field data using the equivalent-layer principle by means of a moving data window that reduces the total processing time and memory requirements, yet data are assumed to be spaced regularly at a horizontal plane. When working with magnetic data, attention should be paid to the source pattern. The effect of a magnetic dipole is easy to calculate, but it produces a narrow peak with side lobes and limited lateral extension. The effect of a prism seems to be better, but it requires more complex computer operations.

Cordell (1992) devises a method for regularizing a scattered potential field of any nature by means of equivalent sources. Two features of Cordell's approach are remarkable. First, regardless the nature of the potential field processed, the Newtonian potential generated by the masses is always applied to fit the data. This holds true inasmuch as the calculated field satisfies Laplace's equation and vanishes as $z \rightarrow \infty$. Hansen and Miyazaki (1984) use the magnetic field of a set of magnetic sources to continue any potential field between arbitrary surfaces. However, Cordell (1992) prefers the Newtonian potential instead of its first or second derivatives (gravity and magnetic effects) because the former is more general, computes simpler and faster, and attenuates least with distance. Guspí (1991) also uses equivalent sources of potential type to calculate effects of cylindrically symmetric structures.

Second, it is unnecessary to solve a system of equations in the ordinary sense. The magnitudes of the sources, located one below each measurement point, are found by an iterative procedure that removes the maximum residual until all residuals become enclosed by a given envelope related to the noise level. For large data sets, this scheme is fast and advantageous; often, not all possible sources are used to attain convergence.

This paper aims to generalize Cordell's (1992) iterative scheme to perform reductions to the pole by incorporating observation and magnetization directions in such a way that when both directions are vertical, the magnetic field is represented by an inverse-distance Newtonian potential. As a by-product, quantities such as Hilbert transforms (Nabighian, 1984; Nabighian and Hansen, 2001), analytical signals and total gradients (Li, 2006), or tilt angle derivatives (Salem et al., 2008) can be computed directly from the equivalent sources without previously gridding, leveling, or extending the field. The upward continuation and correlation between vertical and total gradients of Dannemiller and Li's (2006) method for estimating magnetization directions, which we use in the geophysical example, are also computed directly from the equivalent sources.

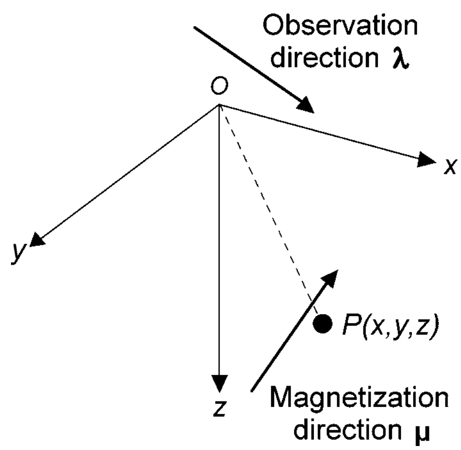


Figure 1. Observation and magnetization directions used to evaluate at the origin of a Cartesian coordinate system the magnetic effect of a point source located at $P(x,y,z)$.

## MODELING THE MAGNETIC EFFECT OF AN EQUIVALENT SOURCE

In a Cartesian system (Figure 1), consider a unit mass located at the point of coordinates $P(x,y,z)$. The Newtonian potential generated by this source at the origin of coordinates is

$$U = \frac{1}{(x^2+y^2+z^2)^{1/2}}. \tag{1}$$

The vertical magnetic effect from a vertically magnetized unit dipole at the same location is given by the second vertical derivative of this potential:

$$\frac{\partial^2 U}{\partial z^2} = -\frac{x^2+y^2-2z^2}{(x^2+y^2+z^2)^{5/2}}. \tag{2}$$

In our model, the equivalent sources are not magnetic dipoles; they are defined in such a way that their vertical magnetic effect, when vertically magnetized, is expressed by the function $U$, which is formally equal to the inverse-distance Newtonian potential of equation 1. Taking into account that $U$ can be obtained through a double integration of equation 2 with respect to $z$, the physical interpretation of the model is that the first integration yields the effect of a monopole; the second yields the effect of a vertical column of monopoles ranging from $P$ to infinity. This column of monopoles is the Newtonian equivalent source we use to perform the field transformations, and its location is defined only by point $P(x,y,z)$.

Because $U$ in equation 1 now represents a magnetic effect, it is in turn the second vertical derivative of a higher-order potential function $T$. Hence, $T$ can be found through a double vertical integration of $U$ with respect to $z$, which yields

$$T = z\ln(z+\sqrt{x^2+y^2+z^2}) - \sqrt{x^2+y^2+z^2}. \tag{3}$$

Therefore, $T$ is the potential of a Newtonian source whose vertical magnetic effect, when vertically magnetized, is $U$.

To calculate the magnetic effect of this type of source for arbitrary observation and magnetization directions, it is necessary to evaluate the tensor $\mathbf{T}$ of the second derivatives of the potential $T$. Calling $r = \sqrt{x^2+y^2+z^2}$, we have

$$t_{11} = \frac{\partial^2 T}{\partial x^2} = \frac{x^2}{r(z+r)^2} - \frac{1}{z+r},$$

$$t_{12} = t_{21} = \frac{\partial^2 T}{\partial x \partial y} = \frac{xy}{r(z+r)^2},$$

$$t_{13} = t_{31} = \frac{\partial^2 T}{\partial x \partial z} = \frac{x}{r(z+r)},$$

$$t_{22} = \frac{\partial^2 T}{\partial y^2} = \frac{y^2}{r(z+r)^2} - \frac{1}{z+r},$$

$$t_{23} = t_{32} = \frac{\partial^2 T}{\partial y \partial z} = \frac{y}{r(z+r)},$$

and

$$t_{33} = \frac{\partial^2 T}{\partial z^2} = \frac{1}{r}. \tag{4}$$

Let $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ be the unit vectors of magnetization and observation directions, respectively. We know (i.e., Baranov, 1975) that the magnetic field corresponding to those directions is given by the directional derivative of the potential:

$$m = \frac{\partial^2 T}{\partial \boldsymbol{\mu} \partial \boldsymbol{\lambda}}. \tag{5}$$

In matrix terms, considering that $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are column vectors, this can be expressed as

$$m = \boldsymbol{\mu}^T \mathbf{T} \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \mathbf{T} \boldsymbol{\mu}, \tag{6}$$

where the superscript $T$ stands for transposition.

Equation 6, with the tensor elements given by expressions 4, allows us to calculate the magnetic effect of a model source with unit strength at any point and arbitrary directions $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. Note that directions $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are interchangeable.

## DISTRIBUTION OF EQUIVALENT SOURCES AND ITERATIVE DETERMINATION OF THEIR MAGNITUDE

Following the criterion defined by Cordell (1992), we place a Newtonian equivalent source beneath each data point at a depth proportional to the horizontal distance between it and its nearest neighbor. The proportionality factor can be decided by experience, and values between 1.4 and 3 often produce good results. The choice of variable-depth equivalent sources, as pointed out by Cordell (1992), improves the resolution by accommodating the field generated by the sources to the texture and density of the scattered data. This criterion also is applied by Guspí et al. (2004) to the problem of combining geoid and gravity measurements by means of equivalent sources.

Let $m_i$, $i = 1,\ldots,n$, be a set of TMF observations on unevenly spaced data points. The field is measured in a direction given by the unit vector

$$\boldsymbol{\lambda} = [\lambda_x \quad \lambda_y \quad \lambda_z]^T. \tag{7}$$

We assume that the causative sources are magnetized in the constant direction of the unit vector

$$\boldsymbol{\mu} = [\mu_x \quad \mu_y \quad \mu_z]^T. \tag{8}$$

To find equivalent sources with these directions that fit the observed field and following Cordell's (1992) criterion, we first determine the maximum absolute value of the total magnetic field $m_i$, $i = 1,\ldots,n$, assumed to occur at station $i$, and fit the corresponding field observation $m_i$ considering only the effect of the Newtonian source lying vertically beneath the $i$th station at a depth $z_i$. From equations 4, noting that in this case $x = y = 0$ and $r = z = z_i$, the tensor becomes

$$\mathbf{T}_i = \begin{bmatrix} -\frac{1}{2z_i} & 0 & 0 \\ 0 & -\frac{1}{2z_i} & 0 \\ 0 & 0 & \frac{1}{z_i} \end{bmatrix}. \tag{9}$$

By applying equation 6, affected by a factor $s_i$ as the strength of the source, the magnetic effect is expressed as

$$m_i = \frac{s_i}{z_i}\alpha, \tag{10}$$

with

$$\alpha = -\frac{\lambda_x \mu_x}{2} - \frac{\lambda_y \mu_y}{2} + \lambda_z \mu_z. \tag{11}$$

Then the strength or intensity of the source is

$$s_i = \frac{m_i z_i}{\alpha}. \tag{12}$$

The effect of this source is calculated over all of the stations using equation 6 and subtracted from them. The next step places a new source under the maximum absolute value of the residuals, subtracts its effect from all the stations, and repeats until all of the residuals are bounded by an envelope of prefixed semiamplitude $\varepsilon$, chosen to account for the level of noise in the data because we do not attempt to fit all of the data exactly — only their expected values.

Once the sources have been determined, the reduced-to-the-pole field can be calculated from them at the measurement points or at any other location, such as a plane grid. This calculation is equivalent to computing the Newtonian potential generated by the sources.

### Alternative two-step solution

Divergence can occur when $\alpha$ in equation 12 approaches zero. To overcome this difficulty, the equivalent sources for reduction to the pole can be determined using a two-step procedure. The first step calculates the vertical component of the observed field without changing the direction of the source magnetization. The second step keeps the vertical observation direction and shifts the magnetization direction to the vertical.

The first step is to calculate, using equivalent sources, the vertical component $v$ of a total field $m$ observed in the direction $\boldsymbol{\lambda}$. It is unnecessary to attribute the true magnetization direction $\boldsymbol{\mu}$ to the equivalent sources. Instead, according to the equivalent-layer principle, any other magnetization direction $\boldsymbol{\eta}$ can be chosen (Emilia, 1973) as long as the combination of $\boldsymbol{\lambda}$ and $\boldsymbol{\eta}$ keeps factor $\alpha$ far from zero. The field generated from the result on the measurement stations, without modifying $\boldsymbol{\eta}$, and shifting only $\boldsymbol{\lambda}$ to the vertical, always represents the vertical component of the original field. The magnetization direction of the actual sources that give rise to $m$ in direction $\boldsymbol{\lambda}$ and to $v$ in the vertical direction is $\boldsymbol{\mu}$. Direction $\boldsymbol{\eta}$ is only an auxiliary direction assigned to the equivalent sources.

In step 2, taking into account the interchangeability of the observation and magnetization directions (equation 6), $v$ is exactly the same field as $w$, observed in direction $\boldsymbol{\mu}$ when the magnetization direction is vertical. To complete the reduction to the pole, it is only necessary to calculate the vertical component of $w$. Whatever the true magnetization direction of the sources (vertical in this case), the operation can be done as in the previous stage, using $\boldsymbol{\eta}$ or other direction as auxiliary direction if necessary.

In practice, the solution algorithm can combine steps 1 and 2 at each iteration without greatly increasing computer time. Errors can be kept within the prescribed envelope.

### Convergence pattern

The main difference between Cordell's (1992) method and our iterative scheme is the presence in equation 12 of the factor $\alpha$ defined by equation 11. This factor accounts for the obliquity of the directions, and its value is one when both observation and magnetization directions are vertical. When $\alpha$ is not close to zero, convergence follows Cordell's pattern: residuals decrease robustly at the first iterations and then tend to oscillate until they reach the prefixed envelope. An inspection of the equivalent sources obtained often shows that not all possible locations under the stations are used. This is because of the correlation or covariance between contiguous data values, which in turn is favored by the variable depth chosen to define the sources. This fact, for large data sets, represents an important advantage over methods that need to solve a system of equations involving all possible sources (i.e., Silva, 1986; Nakatsuka and Okuma, 2006), as illustrated in the examples.

Considering the case of induced magnetization, $\alpha$ is not only nonzero for directions tending to the vertical but also for nearly horizontal ones where, as shown in the examples when $\alpha$ is nonzero, the convergence is equally good. The case for $\alpha$ close to zero, for which convergence cannot be reached and even divergence occurs, corresponds to a range of intermediate inclinations centered at $\arctan(1/\sqrt{2})$, or 35°. As explained, the alternative is to arrive at the solution in two steps. Convergence also can be influenced by the equivalent-source depth, station distribution, anomaly pattern, and noise envelope; its behavior is illustrated further in the next section.

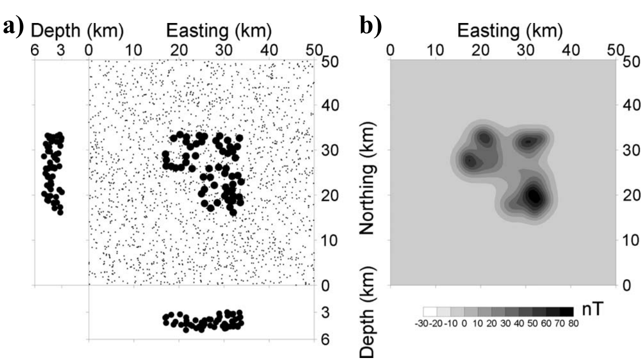


Figure 2. Synthetic example showing a distribution of 57 underground dipole sources (big dots) and 2000 scattered stations (small dots). Station heights range from 0 to 500 m. (b) Vertical magnetic field generated on the plane $z=0$ by the 57 dipoles with vertical magnetization, considered as a test result for reduction to the pole.

## SYNTHETIC EXAMPLE

We construct a source composed of 57 point dipoles at random locations within a volume, with depths varying randomly between 3 and 5 km, and assign to them random magnetization values (Figure 2a). Considering that the magnetization is induced, we calculate for different magnetization directions the effect of the sources on 2000 stations scattered over an area of $50 \times 50$ km, with elevations ranging from 0 to 500 m (Figure 2a). In all tests, the observations are contaminated with Gaussian noise of zero mean and standard deviation of 1 nT. The vertical field generated by the dipoles with vertical magnetization, at the measurement stations or on the plane $z=0$, is considered as the theoretical reduced-to-the-pole field (Figure 2b) to be tested by applying the method we describe in this paper.

First, as an illustration of the convergence pattern of the algorithm, declination is assumed to be 0°, and the effect of the dipoles is calculated at the stations for inclinations ranging from 0° to 90°. Then, equivalent sources are found with a depth-proportionality factor of two and tolerance of 3 nT. Figure 3 shows the number of iterations necessary to attain convergence, using a one- or a combined two-step solution. Negative inclinations (not shown) would produce an analogous pattern of convergence.

Three cases are analyzed in more detail.

### Case 1

Figure 4a shows the noise-corrupted total field at station level produced by the 57 dipoles in Figure 2a, assuming induced magnetization with 61° inclination and 27° declination. The Newtonian sources are defined with a depth-proportionality factor of two, and the error envelope is taken as $\varepsilon = 3$ nT, larger than the standard deviation of the noise (1 nT) because $\varepsilon$ accounts for the peak of the errors, whose mean value is less. Convergence is reached after 652 iterations, with 411 source locations used out of 2000 possible. The reduction to the pole, calculated from those sources at the measure-

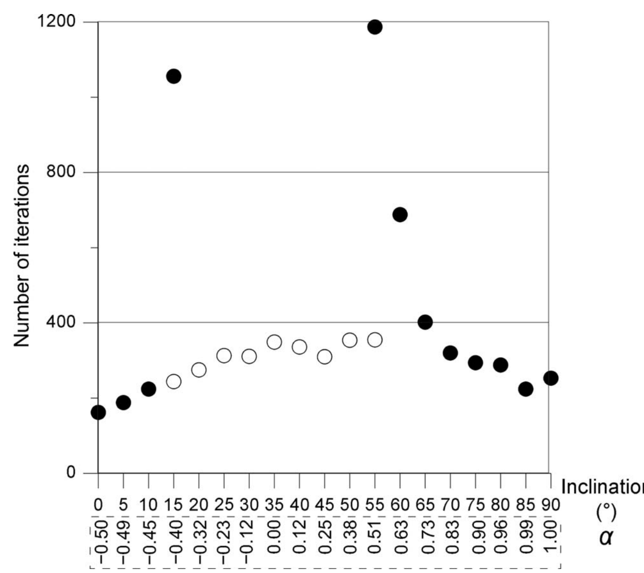


Figure 3. Total number of iterations needed to reach convergence as a function of magnetic inclination from data of the synthetic example (Figure 2a). The induced magnetization is assumed to have 0° declination; the corresponding values of coefficient $\alpha$ are indicated. Black and white dots are one- and two-step iterations, respectively.

ment stations, shows a mean square error of 1.42 nT compared with the true noise-free values, with maximum differences of $-6.88$ and 6.86 nT. Figure 4d shows the reduced-to-the pole map on the plane $z = 0$.

## Case 2

Here, we apply our method by assuming induced magnetization with inclination of 45° and declination of 35°. Figure 4b shows the total field anomaly. This is a case where $\alpha = 0$, and the reduction to the pole is performed using an intermediate magnetization direction. We choose 45° declination (the same of the observed field) and $-35°$ inclination (opposed sign to the actual direction). The depth-proportionality factor is three, and the tolerance is 3 nT. After 361 combined iterations, the equivalent source is defined over 219 locations. The reduced-to-the-pole field (Figure 4e) fits the theoretical field with an rms error of 1.77 nT.

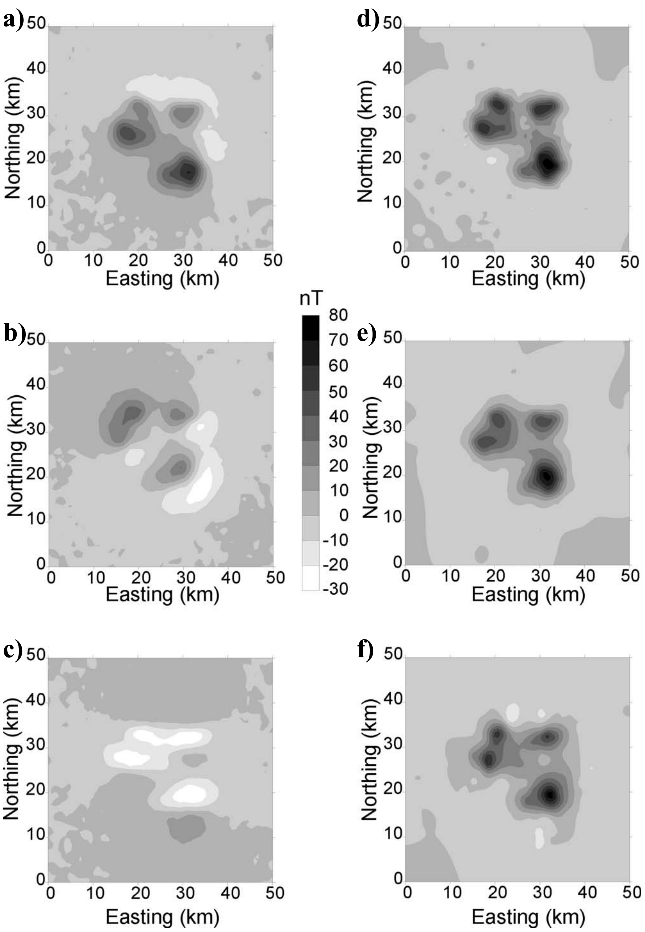


Figure 4. (a-c) Representation at station level of the induced magnetic field generated by the 57 dipole sources of Figure 2, contaminated with 1 nT Gaussian noise, with (a) 61° inclination, 27° declination; (b) 35° inclination, 45° declination; and (c) 5° inclination, 0° declination. (d-f) Reduction to the pole at level $z = 0$ of the fields represented in (a-c), respectively. Total theoretical field at the pole is shown in Figure 2b.

## Case 3

Figure 4c shows the total-field anomaly by assuming the inducing-field inclination of 5° and declination of 0°. For this nearly horizontal direction with depth factor of two and $\varepsilon = 3$ nT, the sources are found without needing an intermediate direction. The reduced-to-the-pole field fits the theoretical field with an rms difference of 3.32 nT. Nearly horizontal directions amplify noise, yet the inversion process is stable and the estimated reduced-to-the pole field gives a good picture of the theoretical field (Figure 4f).

## GEOPHYSICAL EXAMPLE

We analyze a magnetic anomaly at Sierra de San Luis, Argentina (Figure 5a), over a $30 \times 30$-km area, using data from a more extensive survey presented by Hungerford et al. (1996), performed between 66° 30′ W and 64° 30′ W and between 33° 30′ S and 32° 30′ S. The results of this survey are presented in the form of a digitized map of the magnetic anomaly reduced to the pole at a horizontal level, considering that magnetization is induced with an inclination of $-33°$ and a declination of 0°, without mentioning the technique used to regularize and reduce the field to the pole. Using Euler deconvolution over the entire area, Chernicoff and Ramos (2003) find that the magnetized basement is less than 3 km deep.

The isolated anomaly we consider is embedded in the background of the entire area. To obtain a better picture of it, we remove its mean value. Despite being an anomaly formally reduced to the pole, the map of Figure 5a shows a negative side lobe on the southern part with a peak of $-143$ nT. Compared to the positive peak (218 nT),

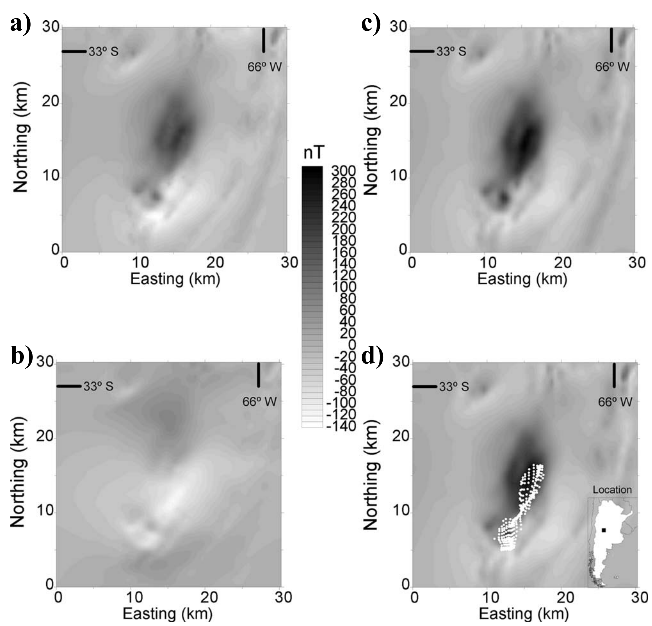


Figure 5. Geophysical example showing (a) magnetic anomaly over Sierra de San Luis, Argentina, taken from a digitized map (Hungerford et al., 1996), presented as the formal reduction to the pole of an induced field with $-33°$ inclination and 0° declination. (b) Reconstruction, using Newtonian sources, of the presumed original total field. (c) Reduction to the pole of the field in (b), considering the same observation direction and a magnetization direction of $-15°$ inclination and 0° declination. (d) Results of the Euler deconvolution (white dots) of the field in (c), where gradients and Hilbert transforms have been calculated from the sources that generate this field.

this suggests the possible presence of remanent magnetization. To test such a hypothesis, we try to find whether a different magnetization direction yields a better field reduced to the pole using the criterion of Dannemiller and Li (2006) of maximum correlation between vertical and total gradients.

First, we reconstruct the original total field by an inverse reduction to the pole. From the vertical field in Figure 5a, we determine a corresponding vertically magnetized Newtonian source (depth-proportionality factor $= 1.4$; envelope $\varepsilon = 3$ nT), and from this source we calculate the presumed original total field anomaly assuming induced magnetization with $-33°$ inclination and $0°$ declination (Figure 5b). At the same time, the gradients of the field in Figure 5a are calculated from the same vertical source, provided that if the field reduced to the pole is represented by a Newtonian potential, its gradients are the gravity components along the coordinate axes. Following Dannemiller and Li (2006) to get rid of short-frequency anomalies, the gradients are calculated on a plane 1 km above the reference surface; their correlation factor, obtained from the expression in that paper, is 0.45.

After some trials to obtain a better magnetization direction from the field of Figure 5b, we perform a reduction to the pole with a magnetization inclination of $-15°$ and declination of $0°$; the result is represented in Figure 5c. Compared with Figure 5a, the positive peak is enhanced and the side lobe is less negative. The Newtonian source that defines this field also is used to calculate the gradients 1 km above the reference plane. Their correlation factor is now 0.65, significantly better than 0.45 from the field of Figure 5a.

Finally, to complement the interpretation, the same Newtonian source provides a means to calculate the $x$ and $y$ Hilbert transforms of the field reduced to the pole because they are equivalent to the horizontal components of the field in both directions. Putting together gradients and Hilbert transforms, we perform a generalized Euler deconvolution (Nabighian and Hansen, 2001) using a structural index of one, estimated by upward continuation (Fedi, 2007). Figure 5d shows that the Euler solutions (white dots) are densely grouped in a northeast-southest elongated area at a depth of about 1.5 km (Figure 5d). The basement of this area (Las Aguilas Group) is composed of folded mafic and ultramafic rocks of Ordovician age, partly metamorphized into granulites, with the detected presence of platinum minerals (Sims et al., 1997; Chernicoff and Ramos, 2003). Because the estimated magnetization direction does not differ significantly from the external earth's field, we can interpret the magnetization of the mafic rocks as being mainly induced yet can retain a remanent component from their original magnetization.

## CONCLUSIONS

We have presented an equivalent-source method for reduction to the pole and other transforms on scattered noisy magnetic observations; it operates without the need for previous gridding or extension of data. Whereas the Newtonian potential functions associated with the point sources beneath the stations provide better coverage of the discrete data points, iterative determination of the source magnitudes, after setting a proper noise envelope, involves fewer operations than resolving complete systems of equations. As shown in the examples, direct convergence fails for certain magnetic inclinations, but the solution is found through a two-step iteration without affecting accuracy and computer effort significantly. Thus, as illustrated in the geophysical example, the equivalent-source approach can be incorporated into standard techniques for processing magnetic data because reduction to the pole and related transformations can be efficiently obtained from it.

## ACKNOWLEDGMENTS

Antonio Introcaso first suggested that we elaborate a method for reduction to the pole with equivalent sources. We are indebted to him for advice and support. We are also grateful to Servicio Geológico-Minero Argentino for access and permission to use their aeromagnetic data. This work was supported in part by PICTR2002-00166 of Agencia Nacional de Promoción Científica y Tecnológica, Argentina. Helpful comments and suggestions from Associate Editor Valéria Barbosa, and reviewers Marc A. Vallée and Fabio Caratori Tontini helped to improve the final presentation.

## REFERENCES

Baranov, V., 1975, Potential fields and their transformations in applied geophysics: Gebruder Borntraeger.
Chernicoff, C. J., and V. A. Ramos, 2003, El basamento de la Sierra de San Luis: Nuevas evidencias magnéticas y sus implicancias tectónicas: Revista de la Asociación Geológica Argentina, **58**, 511–524.
Cordell, L., 1992, A scattered equivalent-source method for interpolation and gridding of potential-field data in three dimensions: Geophysics, **57**, 629–636.
Dannemiller, N., and Y. Li, 2006, A new method for determination of magnetization direction: Geophysics, **71**, no. 6, L69–L73.
Emilia, D. A., 1973, Equivalent sources used as an analytic base for processing total magnetic field profiles: Geophysics, **38**, 339–348.
Fedi, M., 2007, DEXP: A fast method to determine the depth and the structural index of potential fields sources: Geophysics, **72**, no. 1, I1–I11.
Guspí, F., 1991, Rapid modeling and inversion of axisymmetric potential fields using equivalent sources: Revista de Geofísica, **47**, 19–26.
Guspí, F., A. Introcaso, and B. Introcaso, 2004, Gravity-enhanced representation of measured geoid undulations using equivalent sources: Geophysical Journal International, **159**, 1–8.
Hansen, R. O., 2005, 3D multiple source Werner deconvolution for magnetic data: Geophysics, **70**, no. 5, L45–L51.
Hansen, R. O., and Y. Miyazaki, 1984, Continuation of potential fields between arbitrary surfaces: Geophysics, **47**, 787–795.
Hansen, R. O., and R. S. Pawlowski, 1989, Reduction to the pole at low latitudes by Wiener filtering: Geophysics, **54**, 1607–1613.
Hungerford, N., J. P. Sims, and P. G. Stuart-Smith, 1996, Interpretación geofísica (magnética) de las Sierras de San Luis y Comechingones, Mapeo geocientífico de las Sierras Pampeanas, Proyecto Cooperativo Argentino-Australiano: Servicio Geológico-Minero Argentino.
Keating, P., and L. Zerbo, 1996, An improved technique for reduction to the pole at low latitudes: Geophysics, **61**, 131–137.
Leão, J. W. D., and J. B. C. Silva, 1989, Discrete linear transformations of potential-field data: Geophysics, **54**, 497–507.
Li, X., 2006, Understanding 3D analytic signal amplitude: Geophysics, **71**, no. 2, L13–L16.
———, 2008, Magnetic reduction-to-the-pole at low latitudes: Observations and considerations: The Leading Edge, **27**, 990–1002.
Li, Y., and D. W. Oldenburg, 2000, Reduction to the pole using equivalent sources: 60th Annual International Meeting, SEG, Expanded Abstracts, 386–389.
———, 2001, Stable reduction to the pole at the magnetic equator: Geophysics, **66**, 571–578.
Mendonça, C. A., and J. B. C. Silva, 1993, A stable truncated series approximation of the reduction-to-the-pole operator: Geophysics, **58**, 1084–1090.
Nabighian, M. N., 1984, Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms — Fundamental relations: Geophysics, **49**, 780–786.
Nabighian, M. N., and R. O. Hansen, 2001, Unification of Euler and Werner deconvolution in three dimensions via the generalized Hilbert transform: Geophysics, **66**, 1805–1810.
Nakatsuka, T., and S. Okuma, 2006, Reduction of magnetic anomaly observations from helicopter surveys at varying elevations: Exploration Geophysics, **37**, 121–128.
Reid, A. B., J. M. Allsop, H. Granser, A. J. Millett, and I. W. Somerton, 1990, Magnetic interpretation in three dimensions using Euler deconvolution: Geophysics, **55**, 80–91.
Roest, W. R., and M. Pilkington, 1993, Identifying remanent magnetization

effects in magnetic data: Geophysics, **58**, 653–659.
Salem, A., S. Williams, D. Fairhead, R. Smith, and D. Ravat, 2008, Interpretation of magnetic data using tilt-angle derivatives: Geophysics, **73**, no. 1, L1–L10.
Silva, J. B. C., 1986, Reduction to the pole as an inverse problem and its application to low-latitude anomalies: Geophysics, **51**, 369–382.
Sims, J., P. Stuart Smith, P. Lyons, and R. Skirrow, 1997, Informe geológico y metalogénico de las Sierras de San Luis y Comechingones, provincias de San Luis y Córdoba: Servicio Geológico Minero Argentino Anales, **28**, 1–148.