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WORLD STANDARDS FOR GRAVITY MEASUREMENTS

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Although gravity is one of the most important factors affecting our existence on this planet our understanding of the subject is limited to laws deduced from physical measurements. By his experiments with objects on inclined planes and by refuting the idea that heavy objects fall faster than light ones Galileo laid the groundwork for the science of mechanics which led to Newton's laws of motion and his law of universal gravitation. According to this law the gravitational force between any two bodies of mass M and m, separated by distance d is given by

$$F_G = G(Mm)/d^2, \tag{1}$$

in which G is the gravitational constant. If the masses are measured in grams and the distance in centimetres, the numerical value of G is approximately 0.000000066 cm.^{3}/gr. sec.^{2} If the object, m, falls subject *only* to this force, F_G, it will, according to Newton's second law of motion, acquire an acceleration, a_G, in the direction of the force, given by

$$a_G = F_G/m = GM/d^2 \text{ (toward the centre).} \tag{2}$$

To rotate with the earth the mass, m, requires a centripetal force, F_c. This centripetal force is given by

$$F_c = m\omega^2 r \text{ (toward the axis of rotation),} \tag{3}$$

in which ω is the angular velocity and r is the distance from the axis of rotation; the corresponding centripetal acceleration is given by

$$a_c = F_c/m = \omega^2 r \text{ (toward the axis of rotation).} \tag{4}$$

It should be noted that both a_G and a_c are independent of the mass, m; on the other hand if the mass is doubled the force is also doubled. For this reason it is more convenient to use the term *acceleration of gravity* instead of *force of gravity* to describe the gravity field of the earth.

The observed acceleration, known as the acceleration of gravity and denoted by g, is the difference (strictly the vector difference) between a_G and a_c. As neither d in (2) nor r in (4) are simple functions of latitude, ϕ, it follows that g cannot be expressed as a simple function* of latitude

*The expression for g is further complicated by the fact that (1) must be modified by a mass-shape factor due to the oblateness of the earth spheroid. The gravitational attraction of the sun and the moon cause small periodicities, known as earth tides, about the mean value of gravity at any point. Neither of these factors invalidates the general argument presented here.

either. In fact the function defining g at sea level over a smooth, i.e. theoretical, earth has been found empirically from measurements; it is

$$g = 978.0490\ (1+0.0052884 \sin^2 \phi - 0.0000059 \sin^2 2\phi)\ \text{cm./sec.}^2 \quad (5)$$

This is known as the international gravity formula. It was adopted at the General Assembly of the International Union of Geodesy and Geophysics (I.U.G.G.) meeting at Stockholm in 1930.

At any point the acceleration of gravity is the acceleration caused by the resultant of all the forces acting at that point. In tribute to Galileo the unit *gal* has been adopted as the unit of acceleration of gravity and this has been subdivided to *milligal* for convenience, thus

$$1 \text{ milligal (mgal.)} = 0.001 \text{ gal} = 0.001 \text{ cm./sec}^2. \quad (6)$$

Although the term gal was coined by a German physicist named von Oettingen in 1896 (Bartels 1962) the use of it and its obvious derivative, milligal, have achieved general acceptance only in the last two decades; even within this period the term *dyne*, *centimetres per second per second*, and *c.g.s. units* were still being used. In common parlance the terms *observed gravity*, *force of gravity* (per unit mass implied) and even just *gravity* are frequently used synonymously for the *acceleration of gravity*.

The acceleration of gravity at sea level increases from approximately 978 gals at the equator to approximately 983 gals at the poles. This increase is partly due to the flattening of the earth—in equation (2) d is smaller at the poles hence a_G is larger—and partly due to the centripetal force—in equation (4) r decreases to zero at the poles hence a_c decreases to zero. From data on the shape and rate of rotation of the earth the centripetal acceleration can be computed, but there is no known method by which it can be measured separately from the gravitational acceleration.

The Physical Standard for a Milligal. In the realm of physical standards, acceleration is a derived rather than a defined standard. If you adopt a standard of length,* and a standard of time,† you cannot adopt an independent unit for acceleration and still maintain a consistent system.

There is no fundamental reason why distance and time should be defined and acceleration derived; an equally valid system would result by defining a standard of acceleration and of, say, time and deriving from these a standard of distance. However there is no reason to revise a system which, historically, is well established and for which no better alternative is proposed. We have inherited standards of length and time and in order to be consistent and compatible with the world-wide system

*Formerly was represented by a platinum-iridium bar kept at Sèvres, France, and now is a particular wave-length of orange light emitted by krypton 86 (Howlett 1961).

†One second is defined as 1/31556925.9747 of the tropical year 1900 (Smith and Thomson 1958).

of physical standards, gravity must be expressed in terms of these internationally adopted standards. A measurement of g directly in terms of the fundamental standards of length and time is called an *absolute* measurement or absolute determination of gravity whereas measurements of the difference in gravity are referred to as *relative* measurements and have the symbolic notation Δg.

From this discussion of standards it follows that, in terms of our adopted systems of standards, there is one true value of gravity for any point. Because of limitations in the accuracy with which absolute measurements of gravity can be made it has been necessary to adopt provisional standards, knowing full well that they were not true values and would subsequently be revised.

Absolute Measurements. The acceleration of gravity, like the velocity of light, is one of the most important fundamental quantities in physics. Measurements accurate to one or two gals can easily be made with the equipment in a high school physics laboratory, but to attain accuracy of a milligal many years of careful preparation are necessary. The objective of a determination accurate to a tenth of a milligal, that is to one part in ten million, is a challenge still facing physical scientists. Several methods have been tried but the methods of free fall and of the reversible pendulum have been used most frequently. Regardless of the method employed the accuracy of the final result is dependent on the accuracy to which lengths and intervals of time can be measured. At the National Research Council in Ottawa Dr. Preston-Thomas and colleagues worked for several years on a variant of the free fall method before obtaining results with a possible error of ± 1.5 mgals. (Preston-Thomas *et al.* 1960).

A summary of absolute measurements completed and in progress in 1962, listed in Table I, has been abstracted from an unpublished report by Morelli (1960), and from Bulletin d'Information, no. 4, Jan. 1963, an unpublished report of the Bureau Gravimétrique International. The value of 981.274 gals obtained by Kühnen and Furtwängler (1906) at Potsdam was adopted as an international reference value at the 16th General Conference of the International Association of Gravity (I.A.G.) in 1909 and since that time all gravity measurements and gravity formulas have been reduced to this reference value, now known as the Potsdam System. It is interesting to note that adoption of this value for Potsdam made it necessary to apply a correction of -16 mgals. (Borrass 1911) to the Wien system which had been in use until that time. As is apparent from the tabulation the choice was not a particularly happy one, nevertheless it has been retained because, as illustrated by the dispersion of the modern measurements, the mean value may still be significantly

TABLE I*a*

ABSOLUTE MEASUREMENTS OF GRAVITY COMPLETED

Station	Author	Year*	Method	g_{obs} (mgals.)	Indicated correction to Potsdam system (mgals.)
Madrid	Barrageur	1882	R.P.	980,981	0
Paris	Defforges	1883	"	,943	− 1
Wien	v. Oppolzer	1884	"	,859	+ 6
Padova	Lorenzoni	1886	"	,648	−10
Roma	Pisati and Pucci	1887	L.P.	,343	−16
München	v. Orff	1887	R.P.	,736	+ 2
Potsdam	Kühnen and Furtwängler	1906	R.P.	981,274.0±3.0	0
	Revis.: Heyl	(1935)		,260	−14.0
	Dryden	(1942)		,262.3	−11.7
	Jeffreys	(1948)		,263.3±2.2	−10.7
	Berroth	(1949)		,262.3	−11.7
Leningrad	Ivanoff and Boscov	1936	L.P.	980,929	− 4
Washington, N.B.S.	Heyl and Cook	1936	R.P.	,080.4±3.0	
	Revis.: Jeffreys	(1948)		,082.0±3.0	−17.5
Teddington, N.P.L.	Clark	1939	R.P.	981,181.7±1.6	
	Revis.: Jeffreys	(1948)		,183.4±0.6	−13.0
Leningrad, V.N.I.I.M.	Agaletzki and Egorov		R.P.	981,918.7±0.4	−12.1
	Agaletzki	1956	F.F.	,922.4±2.0	− 7.5
	Martsinyak		F. & N.F.F.	,921.5±1.6	− 9.3
Sèvres, A.	Thulin	1960	F.F.	980,928.0±1.0	−12.9
Ottawa, N.R.C.	Preston-Thomas	1960	F.F.	,613.2±1.5	−13.8
Buenos Aires, I.G.M.	Baglietto	1959	R.P.	(980,696)	− 8.7
		Weighted mean correction to the Potsdam system			−12.8±0.4

*In most cases year of publication is given, although in some cases the year refers to the final year of research.
R.P. = reversible pendulum; F.F. = free fall; L.P. = long pendulum; F. & N.F.F. = free and non-free falling.

TABLE I*b*

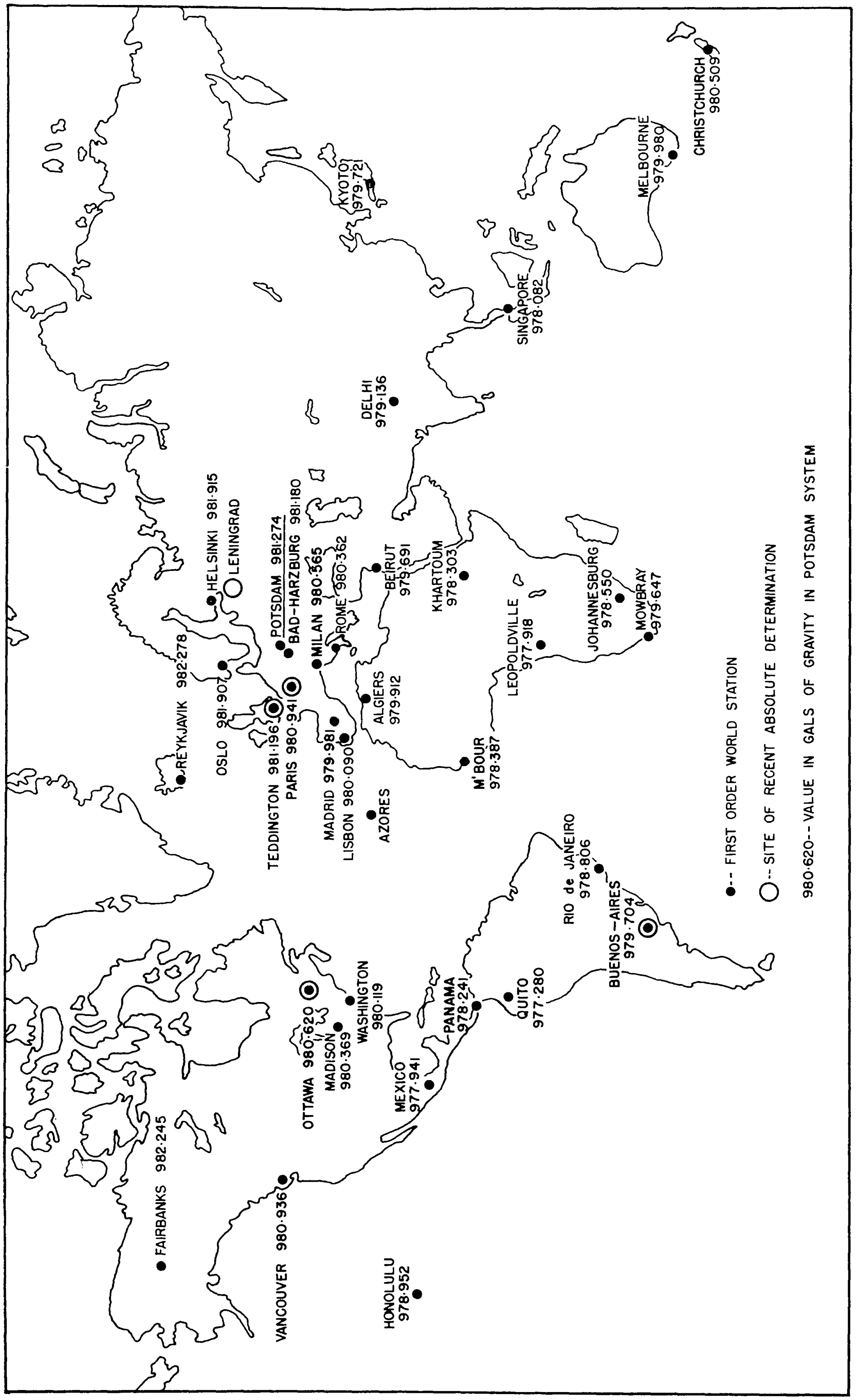
ABSOLUTE MEASUREMENTS OF GRAVITY IN PROGRESS, 1962

Laboratory and Location	Method
Physikalisch Technische Bundesanstalt, Braunschweig	Free fall of emulsion coated rod
Geodätischen Institut, Potsdam, East Germany	Two reversible pendulums in opposite phase
University of Buenos Aires, Argentina	Reversible pendulum
National Physical Laboratory, Teddington, England	Photoelectric timing of free rise and fall of body
National Bureau of Standards, Washington, U.S.A.	Photoelectric timing of free fall
The Finnish Geodetic Institute, Helsinki	200-metre pendulum
National Research Laboratory of Metrology, Tokyo	Photoelectric timing of free falling scale
University of Wisconsin, Madison, Wisconsin	Timing of freely falling mirror relative to velocity of light
University of Princeton, N.J.	Free fall of interferometer mirror
International Bureau of Weights and Measures, Sèvres, Paris	Free rise and fall of interferometer mirror
Air Force Cambridge Geophysics Research Directorate, U.S.A.	Reversible pendulum
Deutsches Amt für Messwesen, Berlin	Free fall of a body having a small aperture

wide of the true value and there is no point in change for its own sake. At recent triennial assemblies of the I.U.G.G. the question of adopting a new value has been deferred "until the next meeting when those measurements now in progress have been completed."

First Order World Gravity Network. To co-ordinate the efforts of the many institutions interested in establishing a consistent gravity network throughout the world a Special Study Group, No. 5, has been established by the I.A.G. This group has designated a network of key points as First Order World Stations; these include all the points of absolute determinations, all the National Reference Stations, and a few others strategically located for convenience in making international connections. In the figure the location of these stations is shown and a preliminary value of gravity in the Potsdam system is given for each station. These values, found from a few pendulum connections and many gravimeter measurements, have closure errors indicating that many of them are inconsistent by as much as one milligal and some may be in error by two milligals or more.

National Reference Station for Canada. Many years before the formation of Special Study Group 5 and the selection of the First Order World stations it was necessary to adopt a value of gravity at one reference



FIRST ORDER WORLD GRAVITY NETWORK 1962

point in each major geographical area. In Canada a value of 980.622 gals in the Postdam system for a pier in the basement of the Dominion Observatory was adopted in 1929 based on a pendulum connection to Potsdam by A. H. Miller (1931). This continues to be the National Reference Station for Canada and all gravity measurements in Canada are quoted in relation to this reference even though the pier itself has been removed and the value of g for it may be high by some 2.3 mgals., in relation to the Potsdam standard (Winter *et al.* 1961).

The adopted and the most recent values for both Potsdam and Ottawa are summarized in Table II. It must be stressed that the recent values are provisional and much more work is needed before new values are adopted.

TABLE II

ADOPTED AND RECENT GRAVITY VALUES FOR POTSDAM AND OTTAWA

Date of measurement or adjustment	Potsdam	Ottawa	Δg
(1) Adopted value 1909	981,274.0	—	—
(2) Adopted values after pendulum comparison 1929	981,274.0	981,622.0	−652.0
(3) Results of recent pendulum comparisons 1960	981,274.0	980,619.7	−654.3
(4) Best present values after a correction of −12.8 made to the Potsdam system	981,261.2	980,606.9	−654.3

Although the gravity values of (4) are now believed to be more accurate, those of (2) are being retained as international and national standards until international agreement is reached on the adoption of a revised system.

Scientists in Canada, such as those involved in establishing standards for electrical units, who require the best possible absolute value of gravity, may apply a correction of −15.1 mgals. to the value of g as given in the national reference system in Canada. Geodesists and geophysicists generally will continue to use existing reference systems until a complete revision of the Potsdam system and the international gravity formula have been officially adopted.

Why is a precise standard for relative measurements necessary? Each year relative measurements for an estimated 100,000 stations are made throughout the world. To be of maximum value to geodesists and geophysicists these observations should be in one reference system and in a consistent system of units, or at the very least in systems for which the inter-relationship is known. Virtually all of these stations are observed by a gravimeter, an instrument consisting essentially of a very sensitive spring with an equally fine gear and dial system so that the slightest change in the force acting on the spring alters its length. In most gravimeters the reading is obtained by rotating the dial so as to bring the spring back to a null position. In all gravimeters the response is directly *proportional* to the change in gravity but the proportionality factor *cannot* be determined from the gravimeter itself. The gravimeter must be calibrated against a known standard which, like any other standard, should be more accurate by one order of magnitude than the gravimeters which are to be calibrated against it. Thus, if a gravimeter is capable of measuring a gravity interval of 500 mgals. to ± 0.2 mgal. it should be calibrated against a standard accurate to ± 0.02 mgal.

In some respects gravity measurements are comparable to precise levelling. It is relatively easy to measure differences in gravity just as it is to measure differences in elevation but it is virtually impossible to measure absolute height directly from the centre of the earth and, as shown above, it is quite difficult to measure absolute gravity. If the standards for either gravity or elevation were dependent on our ability to find the absolute values at disparate points and take the difference, the quality of standards for relative measurements in both cases would be extremely low. Fortunately this is not necessary. Elevation differences are expressed in terms of our adopted but arbitrary standard of length which bears no *à priori* relationship to the radius of the earth. Gravity differences are expressed in the same units and to the same standards as absolute values and there is, fortunately, a method by which gravity differences can be measured directly in terms of fundamental standards without recourse to absolute measurements. This is the method of pendulum determinations.

Standards by Pendulum Apparatus for Relative Measurements. There is a direct relationship between the acceleration of gravity, the period, and the length of a simple pendulum that can be derived from fundamental laws of physics (e.g. Shortley and Williams 1950). At point A

$$P_A = 2\pi\sqrt{(l/g_A)} \tag{7}$$

where g_A is the acceleration of gravity at A, and P_A is the period of a pendulum of length l. From this it would appear that by measuring P_A and l it would be a straightforward matter to obtain g_A. Actually no pendulum is completely simple, so that, in practice, the accurate determination of the effective length of a real pendulum is extremely difficult to measure, but as indicated in Table I, it has been done for a few absolute experiments. However, if the same pendulum is taken to point B, then

$$P_B = 2\pi\sqrt{(l/g_B)} \tag{8}$$

and the two equations can be combined to give

$$\Delta g = g_B - g_A = g_A\{(P_A{}^2/P_B{}^2)-1\} \tag{9}$$

The length of the pendulum does not need to be known explicitly but the value of it at B must be *exactly* the same as it was at A. If this condition is met then the accuracy of Δg depends directly on the accuracy with which P_A and P_B can be measured. It should also be noted that Δg cannot be found unless the value of gravity at A is known. In practice, relative measurements originated at Potsdam and a worldwide network of gravity stations has been built up by successive measurements of gravity intervals in this manner.

The behaviour of pendulums has attracted the attention of scientists ever since Galileo discovered that the period of a chandelier in a cathedral in Pisa depended on the length of the cord but was independent of the amplitude of the swings. This principle was applied to timekeeping by Huygens in 1656 and continued to be the basis of time standards until the introduction of quartz crystal oscillators two decades ago. The effect of gravity on the period of a pendulum was used by Bouguer and de Maupertius in 1735–40 to assist in establishing that the earth was a flattened rather than an elongated spheroid. Until the development of the spring gravimeter three decades ago there was no other method of making relative gravity measurements and countless thousands of pendulum observations were made for geodetic and geophysical investigations. In that era the accuracy of the best pendulum apparatus was of the order of two or three milligals.

In the early stages of gravimeter development it was sufficient to accept any two pendulum values as an arbitrary standard for relative gravity measurements. Subsequently networks of pendulum values were adjusted by least squares to give an improved standard and in recent years research on pendulums has been renewed in the hope of being able to measure gravity intervals to very high accuracy to meet the needs of the extremely sensitive modern gravimeters. To achieve accuracy of the order of ± 0.1 mgal. every possible error whether due to temperature, pressure, magnetism, sway of the supports, rounding of knife edges, or stress in the metal, must be either eliminated or monitored so that adequate corrections can be applied. Two sets of apparatus, the Gulf-Wisconsin quartz and the Cambridge invar pendulums, have been used extensively in the last decade. Germany, Italy, Russia, Japan and Canada (Thompson 1959) have pendulum apparatus in various stages of development.

At the Ninth General Assembly of the I.U.G.G. in Brussels, 1951, a resolution was passed recommending the establishment of gravimeter calibration ranges by pendulum observations in various continents. It was suggested that intercomparisons of as many instruments as possible be made over these ranges. In Europe, a range between Catania, Italy and Hammerfest, Norway has been measured many times and in North America a range between Mexico City and Fairbanks, Alaska has been measured once with the Cambridge invar pendulums (Garland 1953 and 1955) and several times with the Gulf-Wisconsin quartz apparatus (Woollard *et al.* 1956; Woollard *et al.* 1962). The standard deviation for a gravity interval measured with modern pendulum apparatus is of the order of ± 0.3 mgal. To ensure that no systematic errors remain in the calibration ranges more measurements with different types of pendulum apparatus are needed.

As a provisional standard the gravity difference between the National Reference Station in Ottawa and the National Reference Station of the United States in the basement of the Commerce Building in Washington, was adopted in 1956 as 501.44 mgals. (Innes 1958). The calibration factor for all gravimeters used for control networks in Canada is found by comparing this adopted value with the observed readings over this line or a portion of it. Subsequent investigations have shown that this adopted difference may be small by some 0.3 mgal. in terms of fundamental standards. This is equivalent to a factor of 1:1500. If this is so then our measurement of gravity at, say, Frobisher Bay where g is some 1500 mgals. greater than at Ottawa, will be low by approximately one milligal.

All national datums and standards are continually being re-evaluated in terms of fundamental standards. In the meantime there is a great reluctance to abandon existing adopted standards because it is much better to have all the observations in an area in one consistent system, even though the system may be slightly in error, than it is to have three, four or more systems intermingled.

Summary and Conclusions. The Potsdam standard used throughout the world is known to be high by some 12.8 mgals.; the standard deviation of this correction is given as ± 0.4 but it would not be surprising if the correction finally adopted differed from this provisional correction by a milligal or more. The adopted value for the National Reference Station for Canada is known to be high relative to the existing Potsdam standard by some 2.3 mgals.; the standard deviation is estimated at ± 0.5 but this may also be an optimistic appraisal. The standard interval for relative measurements in Canada is suspected to be small by a factor of the order of 1:1500 but this ratio also may be in error by as much as 50 per cent.

It is concluded that, even though our present gravity standards are not entirely consistent with the fundamental standards of length and time, the necessary corrections are not yet known with sufficient accuracy to warrant adopting new standards.

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