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IMPROVEMENT OF INERTIAL SURVEYS THROUGH POST–MISSION NETWORK ADJUSTMENT AND SELF–CALIBRATION *

Abstract

Present day inertial surveys are limited to single traverse runs in which the number of unknown system parameters to be determined are few, depending on the number of control points available along the traverse. Further, conventional inertial surveys are generally restricted to the determination of coordinates with no possibility for a rigorous post–mission adjustment of the observations. The consequence is the continued presence of systematic trends in the residuals, even after the use of error models such as those proposed by Ball, Gregerson or Kouba. Future work aiming at higher accuracies obviously requires more comprehensive models and rigorous adjustment procedures. These can be accomplished by the development of such error models and by the use of "area surveys", instead of the single traverses, together with rigorous adjustment procedures suitable for the network of criss–crossing lines inertially surveyed. In such a network the cross–over points serve as constraints for the geodetic parameters (latitude, longitude, height, gravity anomaly, deflection components) and allow the addition of hardware and software related error parameters. Thus an opportunity is provided to effectively self–calibrate the system– a concept successfully used, for example, in photogrammetry or in satellite tracking. The number and the strength of such parameters depend on the number of control and cross–over points. The adjustment, of course, also provides the necessary statistical information on the adjusted parameters, such as their precision and the correlation between them.

The presentation will describe current work at OSU in this area.

1. Introduction

Although inertial technology itself is not new (having first been developed in the 1940–1950 era), the concept of an Inertial Survey (Positioning) System is, having first made its appearance only as recently as 1975. It may safely be stated that it provides a revolutionary new concept in geodetic positioning and is certainly the first tool in this technological age which is able to produce relative positions and components of the anomalous gravity vector both simultaneously and quickly. Although these systems have been shown to be very much cost effective in producing geodetic control to a second–order standard (Babbage 1977, Kouba 1977), production experience with earlier models has clearly demonstrated the need to limit traverses to those of a "generally straight line"

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variety in order to avoid considerable deteriorations in accuracy (Harris 1977). In order to achieve this safeguard, various user organizations have applied their own operational restrictions (Ball 1978, Carriere et *al.* 1977). Clearly implicit in this restriction on traverse heading variations for purposes of maintaining highest accuracies is the implication of uncompensated systematic error.

Experience has shown that with generally straight line traverses and with some relatively simple smoothing routine, the major portion of any systematic errors affecting the raw inertial data (at least as they affect positions) is largely eliminated. In this context, the term "raw data" refers to data which has been processed by the real time Kalman filter alone but which has not been subjected to any further post–mission processing. It must be noted, however, that with one exception, post–mission methods of smoothing the raw data have essentially been carried out on the basis of the closure information available from single traverses alone. For example, when using only horizontal position information, the presence of, say, n control points has implied a maximum of $(n-1) \times 2$ system error parameters able to be determined in the smoothing process from any single traverse. This is certainly true for the methods proposed by Ball (1978) and Gregerson (*private communication,* 1979). The method used in the Litton software is one in which six error parameters are determined from the latitude, longitude and azimuth (if available) closures at the traverse end points. This is only achievable by using a priori variances for each of these error parameters (Lentz & Buchler 1975), and therefore means that the validity of the smoothing process is completely dependent upon these variances. There is some evidence that these variances are actually unsatisfactory and should, therefore, be changed (Schwarz 1980).

These types of procedures, while satisfactory for many purposes, do not, however, allow for the rigorous (in the least squares sense) post–mission adjustment of the observations. Indeed the only rigorous adjustment procedure presently used is that described by Kouba (1977). All present methods leave residuals which show clearly systematic trends. Thus it seems clear that if higher accuracies are to be obtained with the present hardware, then there is a need for an improved post–mission procedure which not only has more comprehensive models to remove systematic trends from the raw data but also enforces a strictly rigorous least squares adjustment. Such an adjustment, of course, would also provide the necessary statistical information on the adjusted parameters, including correlations between them.

It is obvious that as long as single traverses are used in which the smoothing process depends entirely on the closure at the end point, then little further progress can be expected over and above that which already exists. However, if an "area survey" technique is used in which direct and reverse traverses along the same line are intersected at common points by other criss–crossing traverses, then this provides an entirely new set of opportunities. Such surveys used in conjunction with the adjustment may not only provide optimum geodetic parameters (positions, gravity anomalies and deflections of the vertical) but also allow for more comprehensive error models. In such a network the crossover points serve as constraints since their geodetic parameters should be equal irrespective of which traverse they are determined from. These constraints thus increase the degrees of freedom in the adjustment thus allowing the extra system error parameters necessary in a more comprehensive error model. By this means an opportunity is provided to effectively self–calibrate the system— a concept which has been so successfully used in photogrammetry. It should be clear that the number and strength of such parameters depends on the number of control and cross–over points.

This paper then will first describe efforts in developing an error model for the IPS system together with the adjustment procedure utilized, and then present some preliminary results.

2. Error Model Development

Before a realistic error model could be developed it was first necessary to carry out a comprehensive review of the error sources affecting the Litton inertial system. In broad terms these may be regarded as being of either a hardware or non–hardware related origin. Rather than specifically detail the hardware related sources, such references as (Huddle & Maughmer 1972, Macomber & Fernandez 1962, Agard 1978) are recommended as good sources of information. The multiplicity of these effects may be gauged from the fact that Huddle and Maughmer (1972) note the existence of up to 30 principal sources of erroneous drift in the gyroscopes alone.

Non–hardware related sources, while less numerous, are by no means less complex. The easiest of these to deal with (provided, of course, that it is available) is the noise inherent in the control data itself. This is readily overcome by using a priori variances in the adjustment. Errors caused by the anomalous gravity field are more difficult to handle. The present Litton software uses a preprogrammed normal gravity field in its computation, variations from this field being the source of residual accelerations which in turn propagate into position errors. These residual accelerations are estimated by the Kalman filter and are output as accelerometer bias terms. Schwarz (1978) raises some pertinent questions as to the reliability of this process, at least as it relates to the IPS software. Recent studies have revealed the very strong possibility that the Kalman filter used by Litton is not in fact optimal but actually is of the fixed gain variety and can therefore be expected to produce additional systematic errors (Schwarz 1981). An adjustment process must be designed to model these additional trends as far as possible.

In order to derive the effects of these error sources on positions, the dynamics of the inertial system may be characterized by a single axis block diagram which can in turn be used to derive a closed loop transfer function. This function is then subjected to forcing terms corresponding to the type of input expected from a given error source and then solved for the resulting position error. As an example, the type of error propagation which would be expected from an incorrect scale factor, in the absence of platform releveling at ZUPTs is shown in *Fig. 1.*

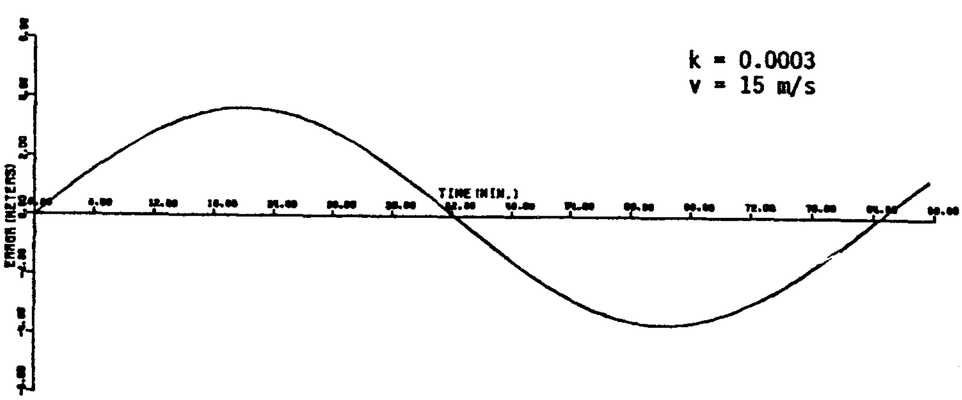

Fig. 1 – Accelerometer scale factor error.

This is clearly a sine function modulated at the Schuler frequency, the amplitude being dependent upon the scale factor constant, k multiplied by a velocity v, which occurs during initial acceleration from rest. If a time between ZUPTs of approximately four minutes is assumed, then for the IPS software in which the inertial platform is torqued back into the local vertical at each ZUPT point, errors will propagate in the form of the first four minutes of the above function (ignoring a deceleration when stopping). At the first ZUPT, the platform is torqued back into the local vertical and the error propagation process effectively begins again at the zero point of the above graph. Since this curve, at least for the first few minutes may be readily approximated by a straight line, the accumulating error effect will be in a piece–wise linear form, each additive portion being dependent on the magnitude of the initial acceleration from rest and its direction. If it is assumed that time of motion and distance traveled in a given cardinal direction are proportional, then, for example, error north (scale factor) $= S_\phi(\phi_2 - \phi_1)$.

In order for these piece–wise linear accumulations of error to have the same slope and therefore be reasonably approximated in accumulation during the mission by a simple linear relationship, it is *essential* that a homogeneous traversing technique be utilized, homogeneous in the sense of similar accelerations and decelerations after and before each ZUPT as well as intervals between ZUPTs being very much the same. This then explains the need for the "constant rate" type survey noted by Huddle (1976).

From such considerations, the following provisional error models have been established.

$$\phi_{obs_2} = S_\phi(\phi_2 - \phi_1) + \theta_n(\lambda_2 - \lambda_1) + \phi_2$$

$$\lambda_{obs_2} = S_\lambda(\lambda_2 - \lambda_1) + \theta_e(\phi_2 - \phi_1) + a \sin\omega_s(t_2 - t_1) + \lambda_2$$

In these equations, the subscript "obs" indicates the observed value of the geodetic quantity, ω_s = Schuler frequency, and $(t_2 - t_1)$ = time difference between a station in the traverse and the initial station. All other quantities on the right–hand side of these equations are considered to be unknowns with the θ terms being misalignments, the S being scale factors, and a being a constant.

It should be noted at this point that the inertial system actually observes differences (latitude differences, free air gravity anomaly differences, etc.) rather than coordinates or components of the anomalous gravity vector itself. Thus, the quantities referred to as "observed" in the previous error equations should more strictly be considered as "quasi–observables", although the distinction will not be drawn in the ensuing discussion.

3. Adjustment Procedure

In designing this procedure consider two uncorrelated sets of observations

$L_a^1 = F_1(X_a)$ and $L_a^2 = F_2(X_a)$, which when linearized, have the form

$$V_1 = A_1\hat{X} + L_1 \qquad V_2 = A_2\hat{X} + L_2$$

in which the V's are the residuals, the A's the design matrices relating to the parameters X, and the L's the misclosure vectors. It can readily be shown (Uotila, 1973) that the least squares solution for the adjusted parameters is given by the expression

$$X_a = X_0 + \hat{X}$$

in which X_0 is the vector of approximate values assumed for the parameters prior to adjustment, and

$$\hat{X} = -(A_1^T P_1 A_1 + A_2^T P_2 A_2)^{-1} (A_1^T P_1 L_1 + A_2^T P_2 L_2).$$

In this expression P_1 and P_2 are the weight matrices corresponding to the observations L_b^1 and L_b^2. In other words, $P_1 = \sigma_0^2 \Sigma_{L_b^1}^{-1}$ and $P_2 = \sigma_0^2 \Sigma_{L_b^2}^{-1}$ for some a priori variance of unit weight σ_0^2.

In the adjustment procedure, the first set of adjusted observations L_a^1 is from the inertial system itself and is a function of both the geodetic parameters for each point plus the system parameters for each traverse. The second set of adjusted observations L_a^2 is considered to come from an a priori knowledge of some of the geodetic parameters (e.g., control point coordinates) or a priori information concerning the system parameters. In other words, these correspond to "observations" on the parameters carried in the adjustment. In this instance the A_2 matrix is equal to the identity matrix and

$$\hat{X} = -(A_1^T P_1 A_1 + P_2)^{-1} (A_1^T P_1 L_1 + P_2 L_2)$$

It can readily be verified (Uotila 1973) that the variance–covariance matrix for the adjusted parameters is given by

$$\Sigma_{\hat{X}_a} = \hat{\sigma}_0^2 (A_1^T P_1 A_1 + P_2)^{-1}$$

and the a posteriori variance of unit weight by

$$\hat{\sigma}_0^2 = \frac{V_1^T P_1 V_1 + V_2^T P_2 V_2}{n_1 + n_2 - u}$$

in which n_1 equals the number of inertial observations, n_2 equals the number of a priori parameter observations, and u equals the number of parameters in the adjustment.

With the exception of the P_1 matrix, the matrices necessary for the adjustment are readily formed (Uotila 1967). The P_2 matrix, for example, comprises the a priori variances (and covariances if desired) of the underlying geodetic control. If it is desired to fix any given combination of control points, then this is easily achieved by assigning arbitrarily small variances to these points. The real difficulties, however, relate to the formation and the use of the P_1 matrix. Schwarz (1979) has demonstrated the high correlations between, for example, all the observed latitudes of points within a given traverse. If, as might be expected, correlations exist between traverses and also between the different observed quantities within a traverse, then in order to form the normal matrix in the adjustment, it becomes necessary to form and store large blocks of the P_1 matrix. Computer core storage problems can be dramatically decreased by storing only the non–zero elements occurring in the design matrix and by accepting correlations within a traverse and between the same type of observations (e.g., latitudes) but ignoring the cross–correlations between both different types of observations within the same traverse and between traverses. However, for the purposes of working within the capabilities of our present adjustment program and to provide a preliminary test of this self–calibrating area–survey concept, only a diagonal weight matrix has been used in the following test.

4. Test and Results

The above concept has been tested on horizontal position data provided by the Geodetic Survey of Canada and which was obtained as part of a production mapping operation in southern Manitoba in June, 1980. The traverses selected together with the points used are shown in *Fig. 2.* It should be noted that, in general, more points on each traverse and more traverses along the same routes were observed than actually used in this test. A single direct run followed by a reverse run along each route were selected, giving 24 runs to be used in the adjustment. Of the total southern Manitoba ISS scheme, only the well–controlled western portion has been used and even then some intermediate traverse points have been dropped.

Points to which a priori position constraints have been applied are all external to the net itself. There are two reasons for this. Firstly, the program itself requires that any point external to the net from which a traverse originates, or is considered to originate, must have some a priori constraint. But secondly, and most importantly, it was desired to use all the geodetic control inside the net for comparisons with the results of the adjustment subjected to the crossover constraints.

In order to assign weights to the observations a number of factors were considered. If the traverse originated from a control point, then an a priori σ of $0''.01$ for the particular quantity in question (e.g., latitude) at that point was used. If the traverse originated from a point which has been derived from system smoothed values in the field (e.g., traversing from 84166 to 84178), an a priori σ of $0''.02$ for the initial point was assumed. It should also be noted here that these a priori σ' s were changed to $0''.05$ for a control point and $0''.08$ for a system smoothed point in a later adjustment of the same data. This change produced no significant effect in the results to be mentioned shortly.

Traverse errors were considered to propagate in the simple form of $\sigma_i^2 + (\omega t)^2$ in which t is the time from the initial point of the traverse, and ω an error accumulation per hour. Due to the azimuth characteristic found in the raw inertial data, traverses in a northerly (southerly) direction had $\omega_\phi = 0''.15/\mathrm{hr}$ and $\omega_\lambda = 0''.3/\mathrm{hr}$. For traverses in an easterly (westerly) direction, these values were reversed. As a test on the differing values for ω used between latitudes and longitudes on the same run, equal values were also assigned. The results indicate that the adjustment itself is not strongly dependent upon these.

From the raw inertial data available, the a priori weights to be assigned to the system error parameters were estimated to be $\sigma_{s_\phi} = \sigma_{s_\lambda} = 0.00015$, $\sigma_{\theta_n} = \sigma_{\theta_e} = 0.00015$ and $\sigma_a = 0.00000015$. These a priori weights are necessary to ensure convergence of the solution.

With the adjustment completed, the position residual vectors *(Fig. 3)* were obtained by differencing the published geodetic coordinates of all unconstrained control points and their coordinates as obtained from the inertial adjustment. It is clear that systematic trends exist, trends which are quite different between the northern and southern portions of the network. With the exception of point 25411, all residuals in

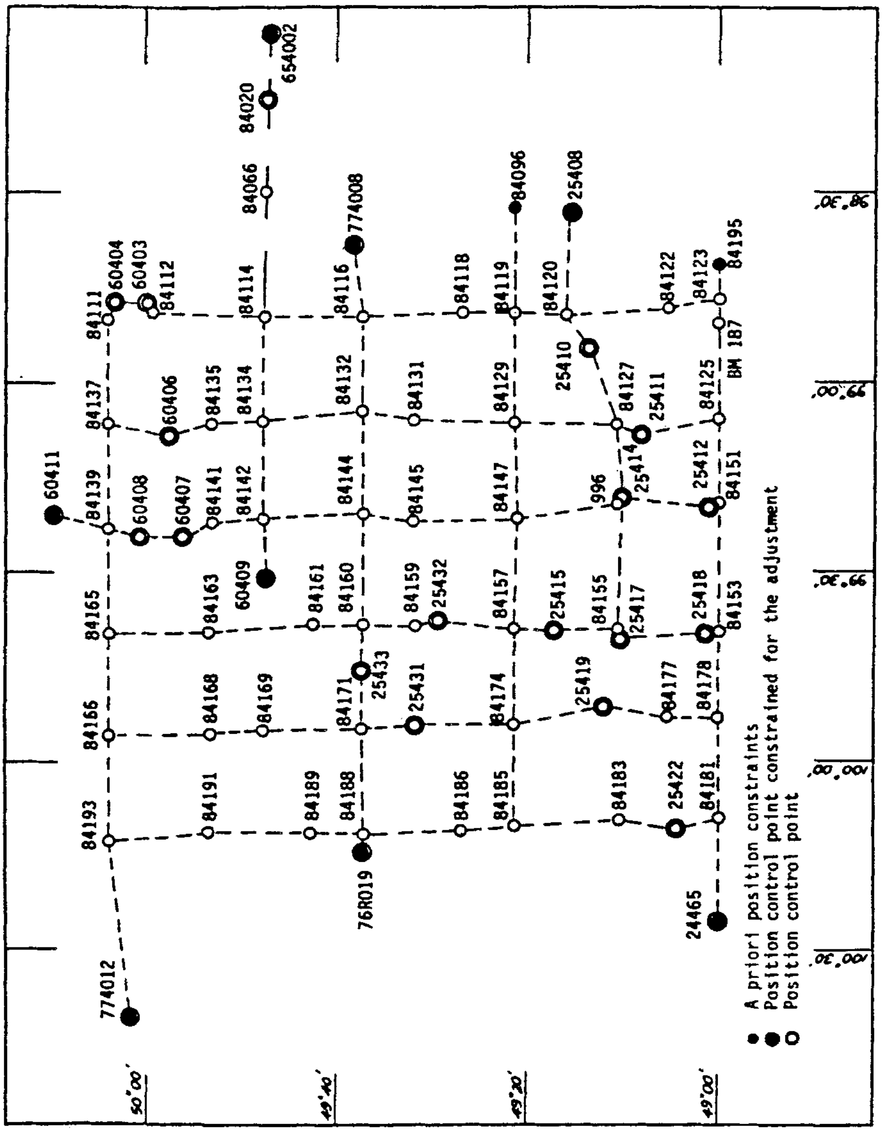

Fig. 2 – Traverse and control configuration.

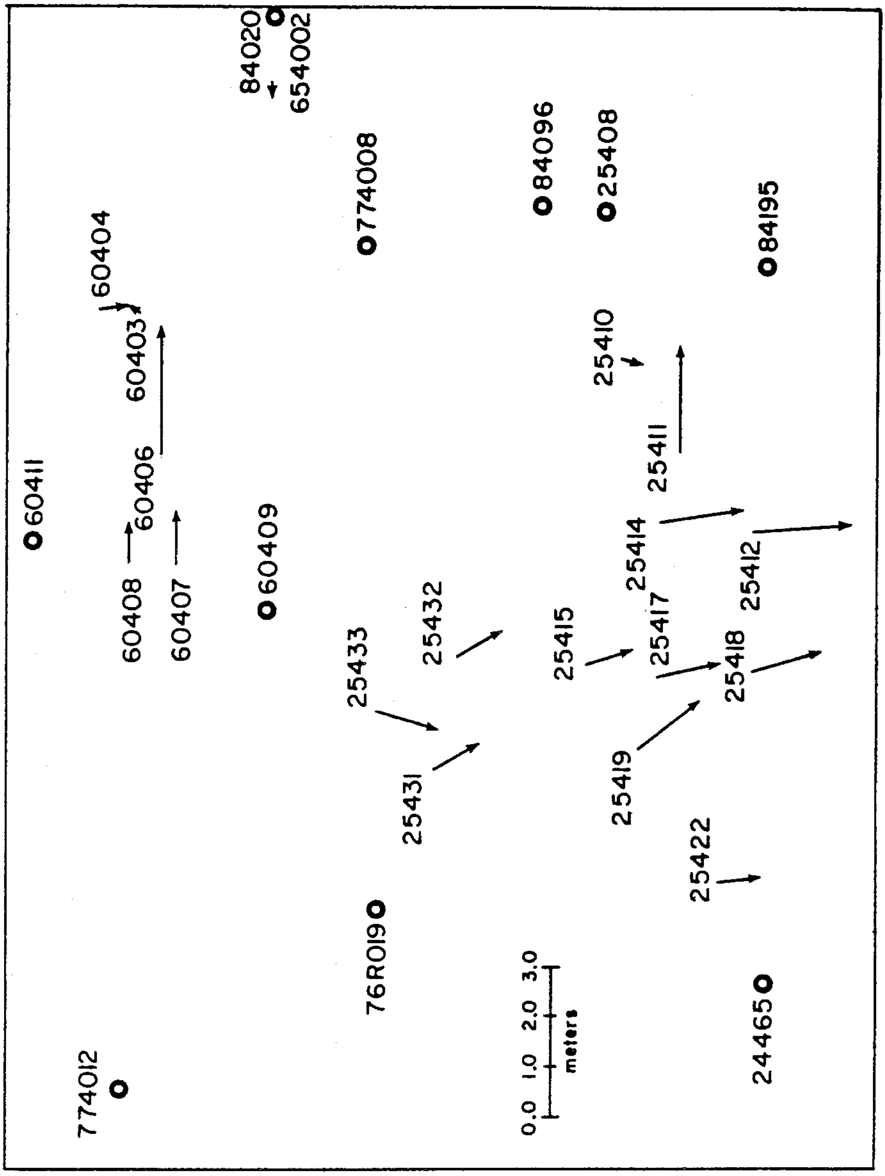

Fig. 3 — Position residual vectors.

the southern section show a pronounced southerly trend while those in the north, with the exception of points 60403 and 60404 show a clear easterly trend. These large residuals, together with their systematic trends, prompted a review of the underlying geodetic control with the thought that perhaps the strength of this procedure was such that the inertial positions may in fact reveal deficiencies in the control.

Detailed information concerning the establishment of this control and its quality was requested from the Geodetic Survey of Canada. The following details were readily provided. The geodetic coordinates of point 25411 are known to be in error by about 2 m in an easterly direction, exactly as indicated by the residual vector. The points in the 25000 series (the southern section of this scheme) were the earliest established and were connected by means of triangulation arcs along the 49^{th} parallel. These arcs had two northern spurs, one approximately at longitude $99^\circ 45' W$ and the other near Winnipeg at longitude $97^\circ W$. These two spurs were subsequently connected to each other by a chain of braced quadrilaterals more or less along the 50^{th} parallel (giving the 60000 series points). Scale in these quadrilaterals was controlled by means of tellurometer and geodimeter distances. Points 76R019 and 77408 have been established by Doppler translocation methods and transformed onto the MAY76 Datum. All coordinates provided refer to the MAY76 coordinate system and were derived from a simultaneous adjustment of the Canadian primary geodetic framework. All control is considered first order and expected to have errors of 20 ppm or under at a 95 % confidence interval.

A comparison of our residuals with this information reveals an almost perfect match between these systematic patterns and the historical sequence and direction of the triangulation arcs with which the control was established. This in turn offers evidence that the major contributor to these residuals may not be the inertial system, but rather the underlying geodetic control itself.

As a further simple test of the systematic nature of these residuals, the RMS's of the residuals at the 25000 series stations were computed as 1.30 m and 0.45 m in latitude and longitude respectively. After removing the means of the residuals (1.17 m and 0.27 m) in lieu of a more suitable datum transformation, the RMS's of the residuals were reduced to 0.40 m and 0.35 m respectively. The removal of these biases leaves both latitude and longitude differences well below the noise level of the control.

In the northern section of the scheme systematic patterns also exist although the magnitude of the residual at point 60406 is at this time unexplained.

One further aspect noted earlier is the fact that none of the points for which a priori position constraints have been applied form part of the net itself— they are all peripheral and are connected to the "mesh" by single traverse lines only. When the "internal" points 60404 and 25417 were added to the list of points constrained a priori (these being selected due to the fact that they are traverse update points) their residual vectors after adjustment were of course zero, but interestingly enough the only other points to show significant differences were 25432 and 25415 where the southerly component of their residual vectors decreased by approximately 30 cm. Clearly this demonstrates that the crossovers are exercising the major influence in the adjustment process rather than the control points which merely provide system definition and barely that according to the biases discussed above. It would also seem to indicate that the net is adjusting on the basis of its own geometric strength and deforming only at the ties which link it to the surrounding geodetic control.

5. Conclusions

These tests provide strong preliminary evidence of the value of using the inertial survey systems in an area mode of survey rather than in the single traverse mode as has generally been the practice until now. With the present hardware and a rigorous adjustment procedure which models the systematic errors known to occur in the raw inertial data, it appears that we may now be at the point of producing geodetic positions with inertial survey systems which in many cases will be well below the noise level of the underlying geodetic control. The strength inherent in using crossover points certainly provides a means of removing systematic errors from raw inertial data which is presently not available in single traverse smoothing techniques. It must be pointed out though that conclusive results, plus the necessary data upon which to base the observational variance–covariance matrices for the adjustment await the existence of a data set obtained under carefully controlled conditions from a test area of known high internal accuracy. With the results of such a data set available, together with the necessary software, considerable improvements in present results even with the current hardware packages seem now to be on the horizon.

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