Geophysical Journal International

Geophys. J. Int. (2013) **195,** 870–882 doi: 10.1093/gji/ggt283
Advance Access publication 2013 August 16

Mutual evaluation of global gravity models (EGM2008 and GOCE) and terrestrial data in Amazon Basin, Brazil

E. P. Bomfim,[1,2] C. Braitenberg[2] and E. C. Molina[1]

[1] *Institute of Astronomy, Geophysics and Atmospheric Sciences—IAG, University of São Paulo, São Paulo, Brazil. E-mail: evertonbomfim@yahoo.com.br*
[2] *Department of Mathematics and Geosciences, University of Trieste, Trieste, Italy*

Accepted 2013 July 15. Received 2013 July 13; in original form 2012 October 13

SUMMARY

The gravity observations of the satellite GOCE have a global homogeneous coverage and precision. This data set constitutes an independent new tool to control the quality of terrestrial gravity data. Terrestrial data reach higher resolution and precision, but can be affected by errors due to factors such as different vertical geodetic datums, wrong position in latitude and longitude, geodynamic effects and gravimeter drift, which tends to accumulate over long distances. Terrestrial data recover gravity signals at shorter wavelengths compared to the GOCE satellite, but the average gravity anomaly values can be compared to the GOCE derived values which are bandlimited to lower frequencies.

We consider the area of the Amazon Craton, and in particular the Solimões, Amazon and Parnaiba Basins, and part of the Tocantins and São Francisco Provinces in Brazil, to estimate the systematic errors in terrestrial gravity data. We calculate the average terrestrial gravity anomaly by spatial averages applying Gaussian, inverse distance and simple averages, which allows to compare the long- and medium-wavelength part of the terrestrial gravity anomalies with the gravity field derived from GOCE. We also consider the combined satellite-terrestrial model EGM2008 up to degree and order 250 (i.e. maximum expansion from satellite GOCE). The results show that the systematic errors range from about −28.1 to 25.2 mGal with a standard deviation value of 6.4 mGal. The mean value over the study area is about zero, obtaining 0.27 mGal difference between the Gaussian average of the terrestrial gravity data and the gravity data from the GOCE satellite-only model and is smaller than the commission error associated to the geopotential model. Also, we verified that 64.8 per cent of the study area does not present systematic errors, as their difference is within the commission error of 5.1 mGal of the GOCE model in the harmonic expansion up to degree 250. The comparison of the terrestrial data with the model EGM2008 gives slightly smaller differences, which can be attributed to the fact that the EGM2008 contains terrestrial data. The results vary only slightly according to the type of averaging used, with improved values for the Gaussian average. The analysis also shows where the terrestrial data are scarce and require an improvement in data coverage in order to correctly represent the gravity field. The method we propose can be directly used to control other gravity databases and constitutes a tool for the quality assessment of terrestrial gravity observations.

Key words: Satellite geodesy; Satellite gravity; Gravity anomalies and Earth structure; Geopotential theory; South America.

INTRODUCTION

Global geopotential models (GGMs) make use of spatially averaged terrestrial data in order to include short-wavelength spherical harmonic coefficients and to improve estimates of the Earth's gravity field (Jekeli 1981; Wahr *et al*. 1998; Tapley *et al*. 2004; Chen *et al*. 2006). In this work we will use the Gaussian averaging function (Jekeli 1981), inverse distance weighting and a simple average to analyse the available terrestrial gravity data in the Amazon Basin area, Brazil, in order to compare them with the satellite-only and mixed satellite-terrestrial geopotential models. The terrestrial data allow to have detailed and accurate information about the short wavelengths of the gravity field, and we can estimate a mean gravity anomaly corresponding to the long wavelengths, comparable to the resolution of satellite observations.

The spherical harmonic gravity models obtained from geodetic satellite missions describe the low-degree components of the Earth's gravity field homogenously and accurately. The terrestrial gravity anomalies are accurate at short wavelengths, but are highly susceptible to the systematic errors at medium and long wavelengths, as pointed out by Heck (1990). Therefore, as described in Amos & Featherstone (2003), the terrestrial gravity anomalies do not form a flawless set of data to test GGMs, especially the satellite-only GGMs derived from the satellite gravity field missions. For the same reasons combined GGMs (e.g. EGM2008: Pavlis *et al.* 2012) can carry with them spurious errors from the terrestrial gravity data.

The main goal of this paper is to use the gravity field derived from the GOCE satellite-only model (Pail *et al.* 2011) to make a regional quality assessment of the terrestrial gravity data. Since the terrestrial gravity data are often used to validate the satellite data, the existence of systematic errors in the terrestrial gravity data may cause serious problems in the development of the GGMs. Furthermore, we analyse the differences between EGM2008 and GOCE, and GGMs and the terrestrial data to compare the performance of these free-air anomalies in the study area. In order to study this subject, we start from the accuracy estimates of the commission error associated to the GGMs in the form of expected degree error of the gravity anomaly (free-air anomaly) for GOCE and EGM2008.

The terrestrial gravity anomalies are relatively precise, nevertheless they are often contaminated by systematic errors such as geodetic datum errors, positioning errors (mainly elevation), reduction errors, geodynamic effects and instrumental errors (e.g. gravimeter drift), which tends to accumulate over long distances (Amos & Featherstone 2003). These and other systematic errors were studied by Heck (1990). Roland & Denker (2003) showed that the largest error components come from inconsistencies in the gravity observations and the horizontal and vertical positioning reference systems. It is almost impossible to estimate and correct these errors due to the lack of quantitative information about them (Huang *et al.* 2008).

Huang *et al*. (2008) described a method to assess the systematic errors of the low spherical harmonic degrees due to errors in the surface gravity anomalies. For this they used three types of techniques on averaging process of surface data, such as low-pass filtering, the inverse Stokes integral, and spherical harmonic analysis. Only a few authors estimated the systematic errors in the global terrestrial gravity data (Mainville & Rapp 1985; Pavlis 2000; Amos & Featherstone 2003), and Véronneau & Huang (2003) also used the averaging filter and the inverse Stokes integral techniques to estimate systematic errors in the GGMs.

As the Brazilian territory has continental dimensions, the gravity data were collected by many institutions and with different aims. Fig. 1 maps the distribution of the terrestrial data in Brazil, which contributed to the global 5 arcmin merged gravity anomaly database used in the formulation of EGM2008. Locations with missing data were incorporated with fill-in anomalies computed from Residual Terrain Model (RTM) effects through forward modelling (Pavlis *et al.* 2012).

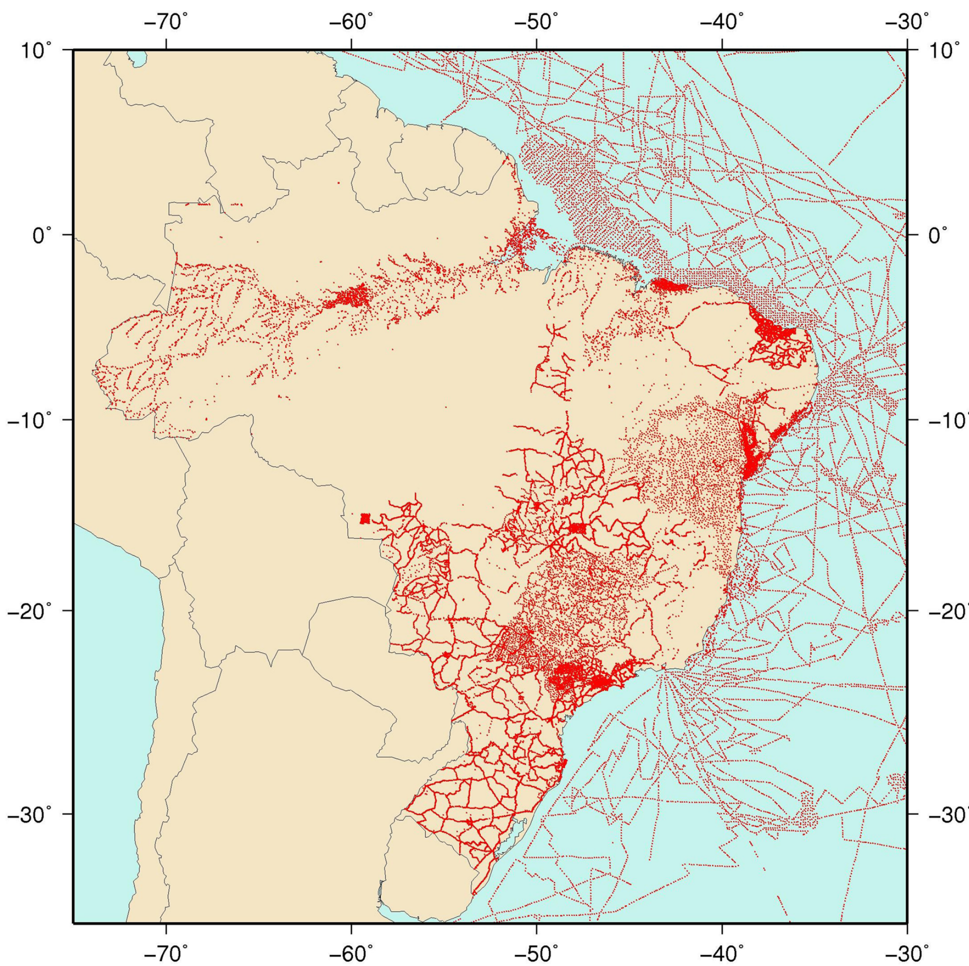

Figure 1. Distribution of terrestrial gravity anomalies available in Brazil (Courtesy: IAG/USP).

The final combination solution of the EGM2008 was developed and evaluated by a least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix and the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arcmin equiangular grid. As described in Pavlis *et al*. (2012), they formed the final grid by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, the higher-degree spectral content was supplemented with gravitational information implied by the topography.

The EGM2008 is complete to degree and order 2159 with additional coefficients extending up to degree 2190. In order to compare this model with the GOCE satellite-only model (Pail *et al.* 2011) we compute the EGM2008 values up to degree and order 250, the maximum available degree of the GOCE model.

STUDY AREA AND A GENERAL OUTLINE OF THE GEOLOGY

The study area embraces the region of the Amazon, Solimões and Parnaíba sedimentary basins and a small part of Tocantins, São Francisco and Borborema Provinces in Brazil (Fig. 2).

Geologically, the area comprises the Solimões, Amazon and Parnaíba Palaeozoic Basins. Amazon and Solimões basins are classified as intracratonic palaeozoic basins. Together they cover almost 1 000 000 km^2. The basins belong to an intracontinental rift system that straddles the border between the Brazil and Guyana Shields, covering some 4500 km^2 of the Brazilian territory. The Parnaíba Province is also of Palaeozoic age that covers some 600 000 km^2, filled with Ordovician to Early Triassic sediments, mostly of marine environment, but also with fluvio-deltaic and desertic contributions (Bizzi *et al.* 2004).

The terrestrial Bouguer anomaly map shows a chain of gravity highs that transect the Amazon Basin and are roughly coincident with the maximum thickness of the sedimentary rocks (Fig. 3). The structure of the Amazon Basin is presently not perfectly known, and several hypotheses exist to explain the positive and negative axial signals (Eiras *et al.* 1994; Nunn & Aires 1988). It is essential to have a reliable gravity field and have perfect knowledge on the precision of the terrestrial field.

METHODS FOR ERROR ESTIMATION

Our analysis requires the evaluation of the errors in the terrestrial data and the errors on the GOCE and EGM2008 spherical harmonic models. We also analyse the error of the difference between the terrestrial observations and the gravity derived from the GGMs. We proceed in three steps. The first describes the definition of the degree errors for the GGMs, the second the systematic errors in the low spherical harmonic degrees of the terrestrial data, and the third the procedure to correctly average the terrestrial data with an adequate spatial filter that corresponds to the maximum harmonic degree of the expansion of the GGM.

Errors for the GGMs: from degree amplitude to gravity anomaly errors

Wahr *et al*. (1998) defined the error of the geoid in terms of the GGM estimate as:

$$\delta N(\theta,\phi) = a \sum_{l=0}^{L} \sum_{m=0}^{l} P_{lm}(\cos\theta)\left(\delta C_{lm}\cos(m\phi) + \delta S_{lm}\sin(m\phi)\right), \tag{1}$$

where a, θ, ϕ are the equatorial radius of the Earth, co-latitude and longitude, respectively. δC_{lm} and δS_{lm} are the errors of the fully normalized geopotential model coefficients. L is the maximum degree of the expansion, l, m are the degree and order of the spherical harmonic expansion and $P_{lm}(\cos\theta)$ are the normalized associated Legendre functions. Then the spatial variance of the geoid error, taken over the entire globe, is

$$\delta\sigma_{sp}^2 = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \delta N^2(\theta,\phi) = a^2 \sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\delta C_{lm}^2 + \delta S_{lm}^2\right). \tag{2}$$

The degree amplitude or root mean square (rms) degree variances from the geopotential model error is defined as

$$\delta N_l = a\sqrt{\sum_{m=0}^{l}\left(\delta C_{lm}^2 + \delta S_{lm}^2\right)}. \tag{3}$$

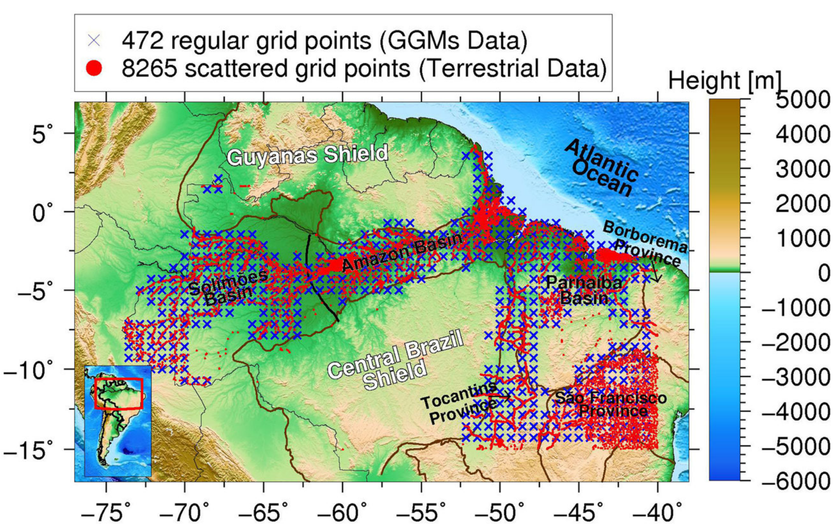

Figure 2. Availability of terrestrial gravity data (red) used to estimate the gravity field on a regular grid with approximately 80 km spacing (blue dots). These averaged values will be used in the comparison with the GGMs. In background is depicted the Digital Elevation Model from ETOPO1.

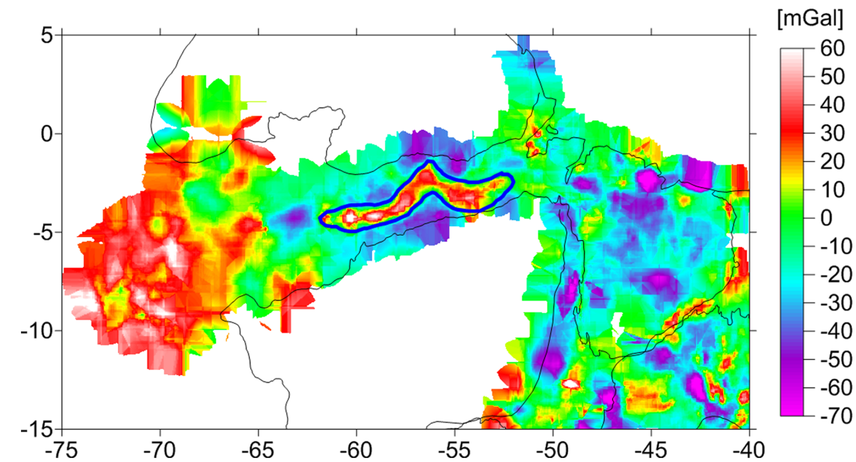

Figure 3. Bouguer anomaly map obtained from terrestrial data. The blue line delineates a chain of gravity highs that is discussed in the text.

The error estimate of the gravity anomaly is obtained similarly. Therefore the rms anomaly errors at degree l are computed by

$$\delta g_l = \frac{GM}{a^2} \sum_{m=0}^{l} (l-1) \sqrt{\left(\delta C_{lm}^2 + \delta S_{lm}^2\right)}. \tag{4}$$

δN_l^2 and δg_l^2 are the geoid and gravity anomaly degree variances that can be estimated by using the standard error of the disturbing potential coefficients δC_{lm}, δS_{lm}. GM is the gravitational constant multiplied by the Earth mass. The degree l is a measure of the spatial scale of the spherical harmonic, as the spatial resolution expressed as half-wavelength is given by 20 000 km/l. So δN_l^2 and δg_l^2 are a measure of the contribution to the variances at degree l, their sum up to maximum degree L the variance for all terms up to a given spatial scale.

The cumulative error in gravity anomaly is shown in Fig. 4 for spherical harmonic coefficients up to degree and order 250 and is about 1.6 mGal for EGM2008 and 5.1 mGal for GOCE. The GOCE model we use is the third TIM release GO_CONS_GCF_2_TIM_R3 of Pail *et al.* (2011). Fig. 4 shows that the cumulative error of GOCE exceeds the cumulative error of EGM2008 at about degree 200 and increases then very quickly. This is due to the fact that the error on the observed gradient components becomes increasingly predominant on the signal beyond degree 200. Thus, by using the full GOCE model up to maximum degree 250, the strong GOCE model error at high degrees contaminates the estimated systematic errors.

Systematic errors of low spherical harmonic degree in surface gravity anomalies

The systematic errors of low spherical harmonic degree in the terrestrial gravity anomalies were studied by Huang *et al*. (2008), and expressing the Earth gravity field in terms of GGM can be modelled by spherical harmonic expansion as follows:

$$\Delta g^{GGM} = \sum_{l=2}^{L} g_l + \varepsilon_L^{GGM}, \tag{5}$$

where Δg^{GGM} is the gravity anomaly; L is the maximum degree of the GGM; g_l is the spherical harmonic component of degree l and ε_L^{GGM} is the commission error associated with the GGM.

Similarly, the terrestrial gravity anomaly Δg^{TG} described in spherical harmonics can be expressed by

$$\Delta g^{TG} = \sum_{l=2}^{\infty} g_l + \varepsilon_L^{TG} + \varepsilon_H^{TG} + \varepsilon_l. \tag{6}$$

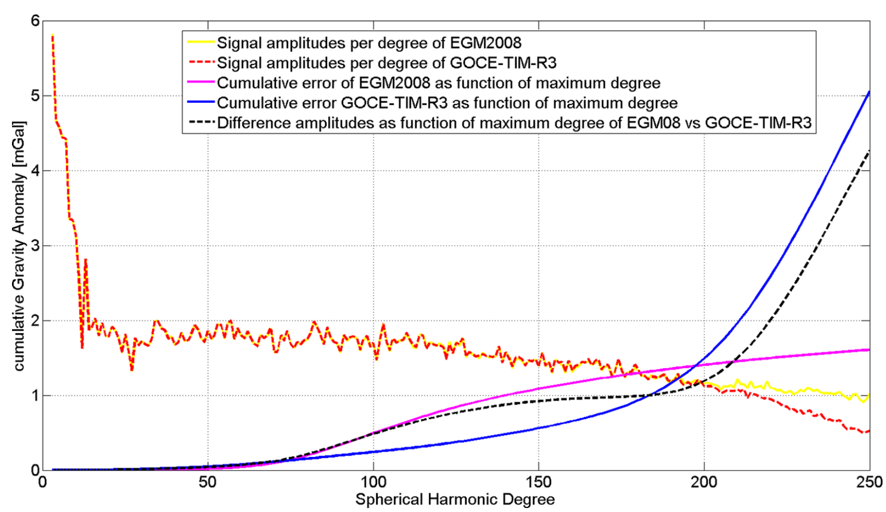

Figure 4. RMS errors in the gravity anomaly from the geopotential models GOCE and EGM2008 in the gravity anomaly (free-air), signal, error and difference between EGM2008 and GOCE.

The second and third terms of the summation are the low and high-degree systematic errors, respectively. The last term ε_l is the random error. By subtracting eq. (5) from eq. (6), we obtain

$$\Delta g^{TG} - \Delta g^{GGM} = \sum_{l=L+1}^{\infty} g_l + \delta\varepsilon_L + \varepsilon_H^{TG} + \varepsilon_l, \tag{7}$$

where

$$\delta\varepsilon_L = \varepsilon_L^{TG} - \varepsilon_L^{GGM}. \tag{8}$$

As discussed in Huang *et al.* (2008) for the satellite models, we assume that if the error of the GGMs is much smaller than the systematic error in the terrestrial gravity data, $\delta\varepsilon_L$ is approximately the low-degree systematic error in the terrestrial data below the maximum degree L. Thus, we can rewrite eq. (7) as

$$\delta\varepsilon_L = \Delta g^{TG} - \sum_{l=L+1}^{\infty} g_l - \varepsilon_H^{TG} - \varepsilon_l - \Delta g^{GGM} \tag{9}$$

obtaining the estimate of $\delta\varepsilon_L$. In order to remove the high-degree field components, and high-degree systematic and random errors, Huang *et al.* (2008) smoothed Δg^{TG}. Here, we smooth the observation through spatial averaging as shown in the section below.

Spatial averaging

Before comparing the terrestrial observations with the field derived from GOCE and EGM2008, the observations have to be smoothed so the averaged field corresponds only to the long-wavelength part of the field and spatial resolution is adequate for the GOCE geopotential model.

Here, we will use three averaging functions, the simple average, the inverse distance weighted average and the Gaussian averaging function (Jekeli 1981). The terrestrial data have more detailed and accurate information at short wavelengths, with respect to the satellite observations, and these will be used to obtain a mean gravity anomaly corresponding to the long wavelengths. The idea is to get the averaged gravity anomaly from the terrestrial data at the corresponding grid point of the gravity anomalies calculated from the geopotential global models (e.g. EGM2008 and GOCE up to degree and order 250) so that we can make comparisons between the gravity anomalies at long wavelengths obtained from the GGMs and the terrestrial data, and estimate the errors of the terrestrial data.

The simple average of the gravity anomalies is defined as follows:

$$\overline{gz} = \frac{\sum_1^n gz_i}{n}, \tag{10}$$

where gz_i, $i = 1,\ldots, n$ are the n gravity anomaly observation points. The simple average operator in eq. (10) is applied at regular grid points using the averaging radius r.

We define the weighting function for the weighted average as being the inverse distance $(1/d_i)^p$ commonly used in weighted average with a known scattered set of points, since this weight decreases as inverse of the distance of the point of the gravity anomaly observation around the computational point (knot) by the averaging radius r. Thus, the computational point obtained from weighted average is the weighted sum of each gravity anomaly observation within the computational cap. This filtering aims to get the long wavelengths from the gravitational field (e.g. ground data) which contain the full high frequency signal using inverse distance averaging. The p is a positive real number, called the power parameter. The smaller the parameter p, the more the filtering is dominated by points far away, and consequently greater values of p give greater influence to values closest to the computational point.

Here, we choose the value of $p = 1$ for inverse distance averaging (i.e. weights equal to $1/d_i$) because the degree of smoothing gives a consistent result to the long-wavelength part of the GGMs values obtained in the same computational point. Therefore, the weighted average value ($\overline{gz}$) at computational point (regular grid point) is based on gravity anomaly observations within the radius r using the inverse distance weighting function $(1/d_i)$

$$\overline{gz} = \frac{\sum_{i=1}^n gz_i.1/d_i}{\sum_{i=1}^n 1/d_i}, \tag{11}$$

where d_i is distance between the observation point and the regular grid point, r is the radius of the computational cap where the filtering is computed (Fig. 5), n is the total number of observation points used in filtering within this computational cap.

The Gaussian average is also a weighted average, where the weight is the Gaussian averaging function $W(\alpha)$ for the radius at r

$$\overline{gz} = \frac{\sum_{i=1}^n gz_i.W(\alpha_i)}{\sum_{i=1}^n W(\alpha_i)}, \tag{12}$$

where the Gaussian averaging function or operator $W(\alpha)$ is supposed to depend only on α (i.e. the spherical distance between the computation point and integration point), where the region of integration is a spherical cap centred at the computation point (Jekeli 1981):

$$W(\alpha) = \frac{2be^{-b(1-\cos\alpha)}}{1 - e^{-2b}}, \tag{13}$$

$$b = \frac{\ln 2}{(1 - \cos(r/a))}, \tag{14}$$

where r is the averaging radius (i.e. the distance on the Earth's surface at which W has dropped to half of its value at origin ($\alpha = 0$), where the distance on the Earth's surface is $a\alpha$, $0 \le \alpha \le \pi$, a is the Earth's mean radius (~6371 km) and b is a dimensionless parameter that defines the smoothing process of the Gaussian operator and $b > 0$. The (Gaussian) weighted function resembles for small α and was normalized so that the global integral of $W(\alpha)$ is equal to 1.

In terms of the spherical harmonic development, Jekeli (1981) showed that the coefficients W_l of the Gaussian weighting function can be obtained following the recursion formula (eq. 15)

$$\begin{aligned} W_0 &= 1 \\ W_1 &= \left[\frac{1 + e^{-2b}}{1 - e^{-2b}} - \frac{1}{b}\right] \\ W_{l+1} &= -\frac{2l+1}{b} W_l + W_{l-1}. \end{aligned} \tag{15}$$

Fig. 6 shows the Gaussian weighting function $W(\alpha)$ and the spherical harmonic coefficients W_l up to degree and order 300; the degree 250 corresponds to the averaging radius $r = 80$ km that was used for smoothing the terrestrial data.

DATA ANALYSIS

In order to avoid a significant portion of systematic errors related to positioning errors, or rather, the inconsistencies in the gravity and in the altitude due to measurement uncertainties of data acquired in the 1970s, at first we have checked the altitudes in the terrestrial database and compared them with the SRTM (Shuttle

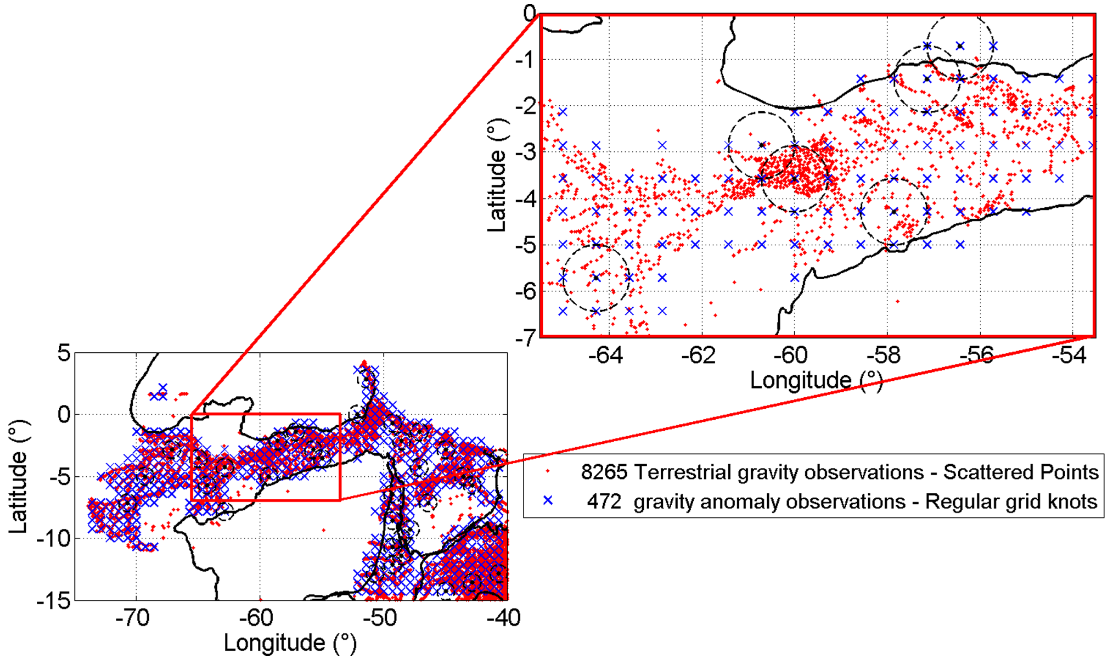

Figure 5. Schematic figure showing how the averages were estimated from terrestrial data within a radius of 80 km (black circle) to obtain the long-wavelength field. 8265 terrestrial data measurements are distributed sparsely (red dots) and 472 data from the geopotential models (blue cross) are distributed on a regularly spaced grid of 80 × 80 km grid spacing. The radius is centred in each of the 472 regular grid points.

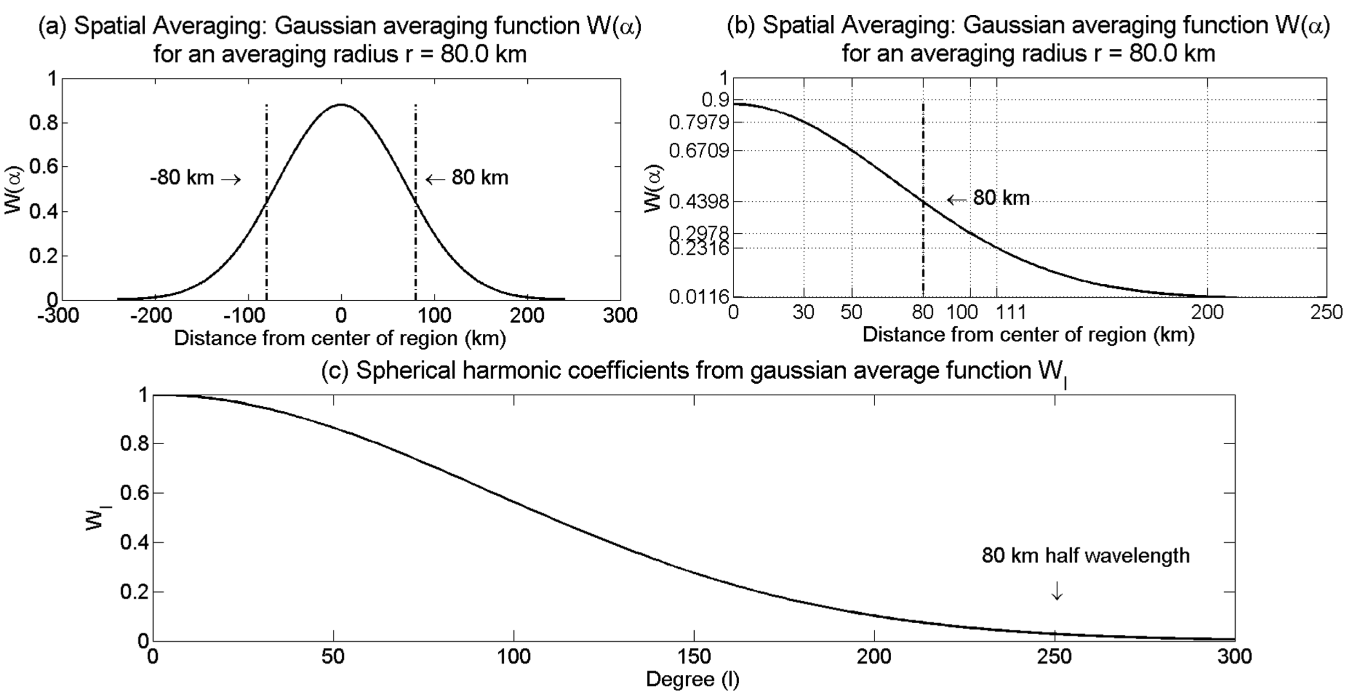

Figure 6. (a) Spatial averaging: Gaussian averaging function $W(\alpha)$ (continuous line) for the averaging radius $r = 80$ km (vertical stippled line); (b) zoom on (a) for positive distance from centre of region; and (c) spherical harmonic coefficients of Gaussian averaging function for the same averaging radius $r = 80$ km.

Radar Topographic Mission) altitude data interpolated at the same point. The histogram of the altitude differences was used to analyse the mean and the standard deviation (SD) of these differences, and then discard the gravity measurements with errors greater than the 99 per cent confidence interval. We consider acceptable only those gravity points in which the altitude difference with SRTM lies within three SDs of the altitude differences. The eliminated data could also be related to the interpolation errors. However, it was verified that the altitude differences higher than three SDs coincide or are very close, to the SRTM gridpoints of 3 arcsec (approximately 90 m) resolution (Jarvis *et al.* 2008) and therefore cannot be explained by interpolation errors.

Note that the SD, of about 28 m, is the SD of the differences and the standard error of the mean was estimated by the sample estimate of the SD of the height differences divided by the square root of the sample size.

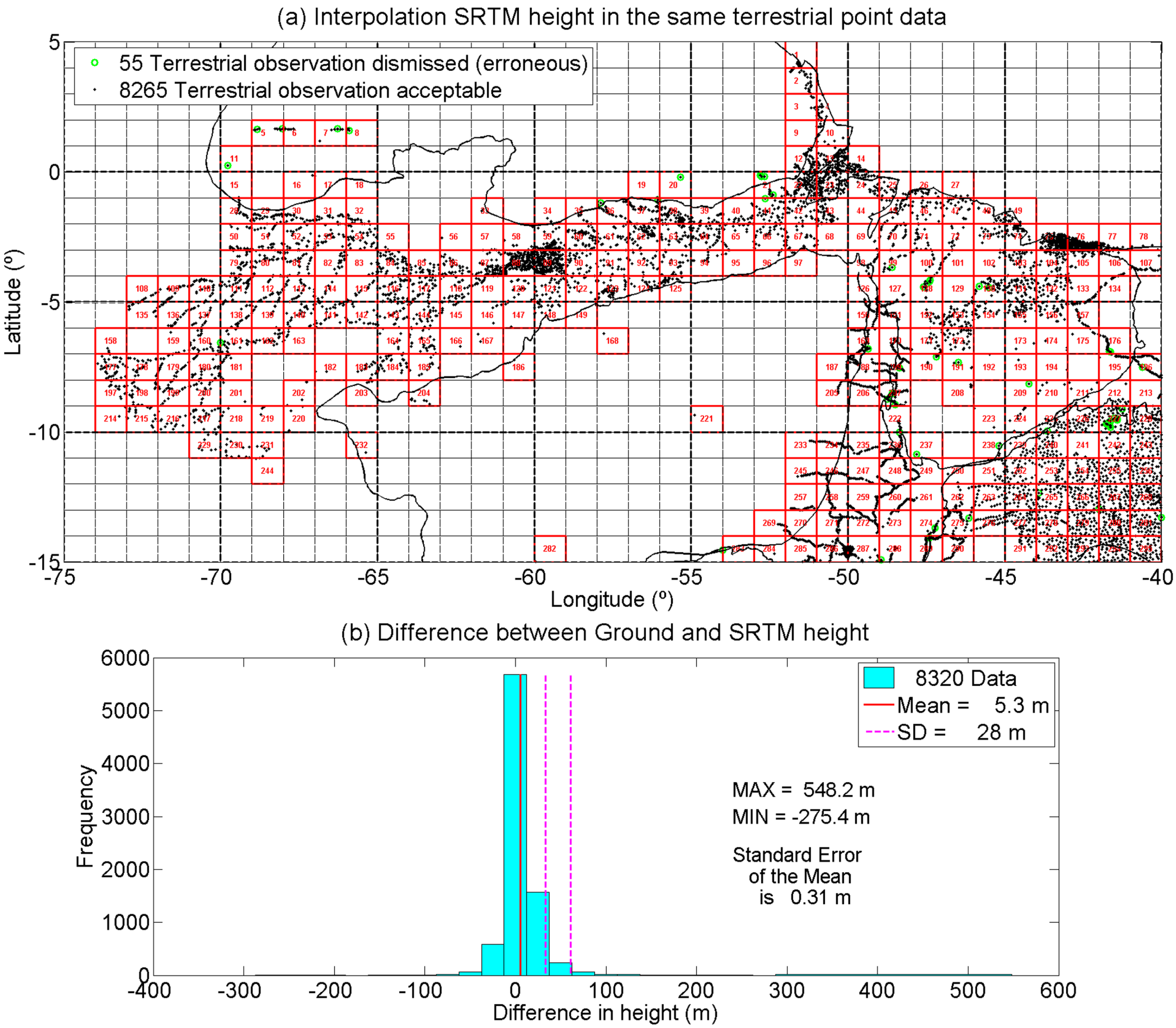

Figure 7. Comparison of data from terrestrial altitudes and interpolated SRTM altitudes at the same points: (a) subdivision of the study area in tiles of $1^{\circ} \times 1^{\circ}$ to overcome overload of the computer's memory in interpolating; and (b) histogram and statistic of the differences between the interpolated SRTM altitudes with the terrestrial altitude data.

Among the originally 8320 terrestrial altitude data compared with the SRTM altitudes, we have found only 33 points that could not be compared with SRTM data, as they had NaN (Not a Number) values in SRTM. These points were then interpolated with ETOPO1 altitude data (Amante & Eakins 2008) which has a resolution of 1 arcmin (spatial resolution ∼ 1850 m).

To facilitate the calculations and not overload the computer's memory, the study area was subdivided into tiles of $1^{\circ} \times 1^{\circ}$, resulting in 700 tiles in the region. Only 287 blocks had gravity data and were interpolated to obtain the SRTM altitude data at the same points of the terrestrial height data (Fig. 7a).

The differences between the interpolated SRTM heights and the terrestrial altitude data in the histogram (Fig. 7b) showed a standard Gaussian distribution with a mean value of 5.3 m having a standard error of the mean of 0.3 m and a SD of 28 m. Although we have no intention to assess the quality of the SRTM and the systematic errors due to the different height datums, it is worth noting that SRTM is not error-free (Rodriguez *et al.* 2005), that is there is a error of the order of some meters, about 5 m, in the Brazilian region. Probably, this error is responsible for the displacement of the mean of the height differences (between ground and SRTM) in 5.3 m seeing that the SRTM error at least should be more homogeneous and not affected by gross data errors. Furthermore, Ferreira & Freitas (2012) have estimated the systematic errors in the Brazilian height system when compared to the world height system with offset of 0.224 m.

Some anomalous points in the height differences were found greater than 90 m, and we assumed that these points probably have wrong altitudes in terrestrial data. Note that the SD found is far greater than the error on the SRTM data. Only 55 of 8320 values showed height differences higher than three SDs of the altitude differences. Fig. 7(a) depicts 55 points (bold-faced green fluorescent circles) that have been considered erroneous (gross errors) and so should be discarded. All the other 8265 remaining points (black dots) were considered acceptable within the confidence interval (three SDs). Finally, a total of 8265 reliable terrestrial data were used for our main study to obtain a regional assessment of the terrestrial gravity observation errors.

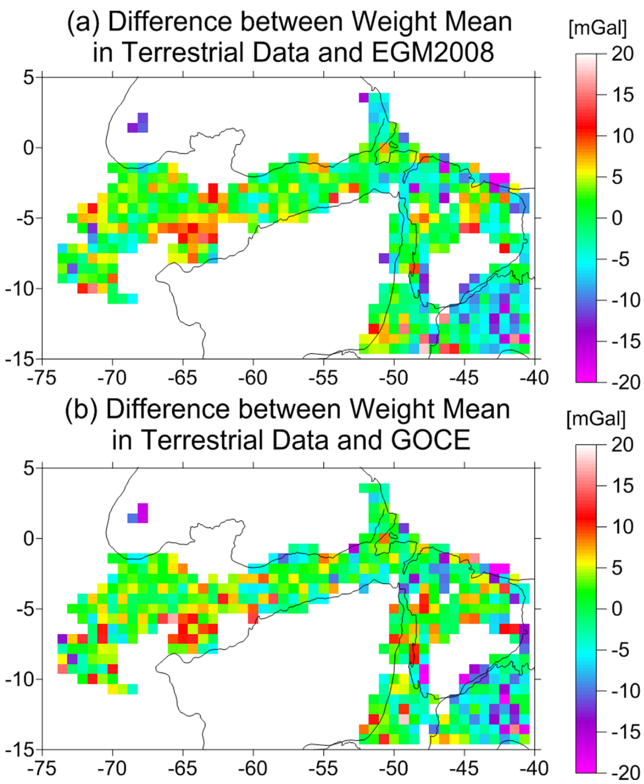

Figure 8. Differences between the gravity anomaly estimated from the weighted inverse distance average of terrestrial data and the gravity anomaly calculated from the (a) EGM2008 and (b) GOCE geopotential model up to degree and order 250. The spatial resolution is 80 km.

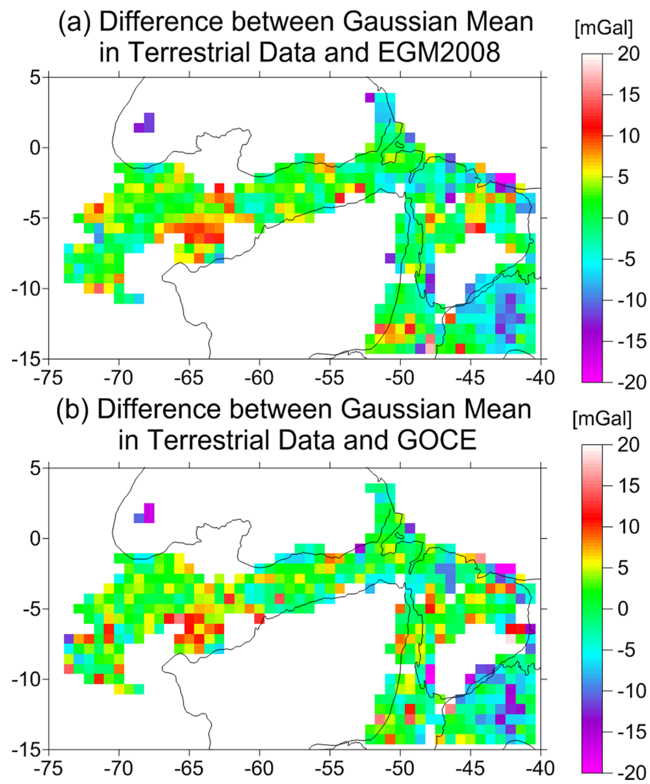

Figure 9. Differences between the gravity anomaly estimated from the Gaussian average of terrestrial data and the gravity anomaly calculated from the (a) EGM2008 and (b) GOCE geopotential models up to degree and order 250, respectively. The spatial resolution of 80 km.

We proceed now in applying the averaging process to the terrestrial data presented in Fig. 2. We calculate the gravity anomalies for the two GGMs EGM2008 and GOCE (Pail *et al.* 2011) up to degree and order 250, which is equivalent to a maximum spatial resolution of 80-km half-wavelength. The comparison between the obtained values (Fig. 5) allows us to estimate the errors of the terrestrial gravity data when compared to the gravity anomalies obtained from the geopotential models. The interest is to investigate whether this approach allows us to make a regional quality assessment and detect the systematic errors of the terrestrial observations (Figs 8–13).

The scattered values of the terrestrial gravity anomaly data points were averaged within a radius of 80 km from regularly spaced grid points using simple, inverse distance and Gaussian averages given by eqs (10), (11) and (12), respectively.

We estimated the gravity anomaly differences between the averaged terrestrial data and the GOCE model or the averaged terrestrial data and the EGM2008 model, and also between the GOCE and EGM2008 models, in order to determine the low-degree systematic errors in the terrestrial gravity measurements from eq. (9).

Fig. 10 shows the comparison between the GGMs, that is the difference between the gravity anomaly from the GOCE and EGM2008 models, which is due to commission errors of the finite spherical harmonic expansion of these models, respectively. The EGM2008 can be affected by errors in the terrestrial data, because only up to degree 70 does it contain purely satellite observations. We expect a smaller difference between the GGMs due to the low commission errors of the models (Fig. 4) when compared to the differences between the averaged terrestrial and GGMs measurements (Fig. 11), although for the GOCE model beyond degree 200 some error increase can be expected. Of course, the previous sentence also is not necessarily true depending on the quality of terrestrial gravity data. If the terrestrial gravity data have a good quality, we also would expect a small difference between the GGMs and the averaged terrestrial data.

In the calculations the filtering on the terrestrial data was done around regular grid knots where the regular grid spacing is equal to the spatial resolution of the maximum spherical harmonic degree. Therefore, for maximum degree 250 we define a regular grid spacing of 80 km, and consequently, this represents a data set of 472 computational points (knots) of regular grid within the study area. For maximum degree 200 we obtain a regular grid spacing of 100 km, resulting in 333 computational points (knots), and for maximum degree 180 we obtain a regular grid with grid spacing of 111.111 km or 1 arcdegree resulting in 279 computational points (knots).

As shown in Braitenberg *et al*. (2011) and Alvarez *et al*. (2012), this analysis permits controlling and testing the quality of terrestrial data, for example airborne campaigns. The EGM2008 global gravity field in some regions shows problems that can be traced back to missing data or poor quality of the terrestrial gravity data used as input. As the GOCE data are independent of terrestrial data, they can produce a new quality assessment tool for GGMs of higher resolution, and are also adequate to assess the quality of terrestrial observations even if the spatial resolution of the terrestrial data is higher than that of the GOCE model (Braitenberg *et al.* 2011; Reguzzoni *et al.* 2011). Geological structures can be studied with EGM2008 data, as far as the terrestrial data points used to calculate the model are available at the required resolution and of sufficient

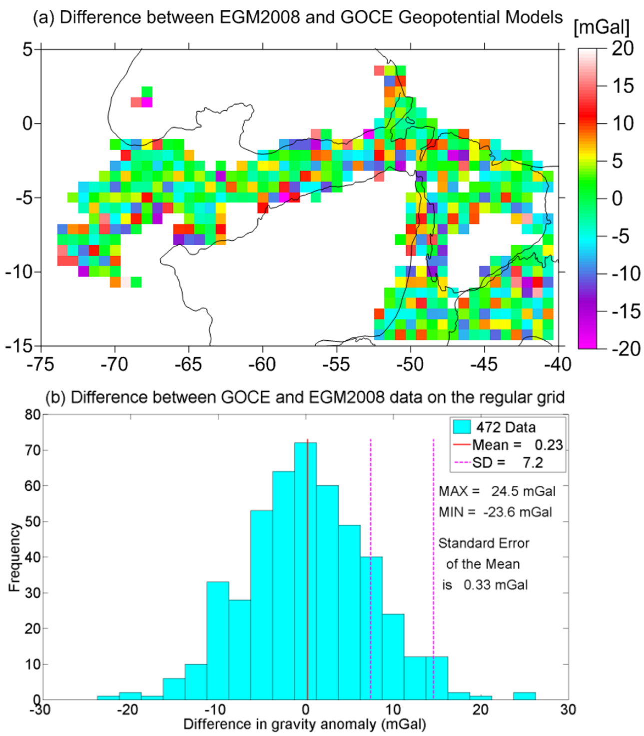

Figure 10. (a) Differences between the gravity anomaly of EGM2008 and GOCE geopotential models up to degree and order 250, respectively. The spatial resolution is 80 km; (b) histogram of the gravity anomalies differences between EGM2008 and GOCE.

precision. In this aspect the GOCE data, as an independent source with high-quality information, can be a valuable tool to work with, provided that the error analysis is successfully conducted.

The differences between the averaged terrestrial data points and the mean values calculated from the GGMs with $L = 250$ at the same regular grid are shown in Fig. 11(a). The mean values of the differences are near zero, with little variations according to the filtering scheme used in the terrestrial mean data calculation. These small values are within the commission error of about 1.61 and 5.06 mGal for gravity anomalies from EGM2008 and GOCE geopotential models up to degree 250, respectively. In Fig. 11(b) the analogous results are shown for the GGMs calculated with $L = 200$ where the GOCE commission error is smaller than for $L = 250$. The SDs of the gravity anomaly differences between terrestrial data and the GGMs are only slightly changed, which demonstrates that the errors are mainly due to the terrestrial data and not to errors in the GGMs. We assume that the points which have deviations in the same order of the commission errors at the respective maximum degree L are considered as errorless, and that points that lie within two SDs of the gravity anomaly differences can be considered acceptable.

The statistical values are also reported in Table 1, where the GGMs are used up to degree 180 (spatial resolution of about 111.111 km), 200 (spatial resolution of about 100 km) and 250 (spatial resolution 80 km). In case of $L = 180$ and $L = 200$ the GOCE errors are smaller than the model error to maximum degree 250. Consequently, the gravity anomaly differences obtained for averaged terrestrial data and averaged GOCE data are smaller than gravity anomaly differences obtained for averaged terrestrial data and averaged EGM2008 data. The averaged data refer to simple, weighted inverse distance and Gaussian gravity anomaly mean.

As the terrestrial gravity measurements do not cover homogeneously the computational cap defined by the radius r, the filtering is incomplete due to the errors caused by missing data within the cap. The greater the radius, the greater the probability of missing terrestrial data within the cap in the Amazon region. The filtering calculation is accepted only if it has at least 10 points within the radius (cap). It is worth noting that if we calculate the average position of the data points that contribute to the averaging made around the regular grid point, we find that, in fact, we are making comparisons at different points. The ideal would be to get the gravity anomalies (free-air) from GGMs in the same terrestrial points, that is obtaining scattered values of the GGMs. In this way, all (GGMs and terrestrial) gravity anomalies will suffer the same errors caused by missing data, and so, we can cancel out errors caused by missing data and biases.

In principle, the computational cap should be fully covered by gravity measurements to make filtering complete and so, the errors caused by missing data need to be considered. However, the GGMs are treated in the same way as the terrestrial data (scattered points) resulting in GGMs filtered around the same computational point (regular grid knot). This way the problem caused by missing data within the averaging cap is reduced because it affects the averaging of the terrestrial and the GGMs in the same way. Thus, the errors caused by missing data within the cap of radius r for spatial averaging is softened since, in this way, all (GGMs and terrestrial) gravity anomalies suffer the same errors caused by missing data in a part of the sector within the cap.

The maps allow us to identify also the areas (Fig. 12) with the greatest systematic errors (here considered greater than two SDs from the mean). Comparisons to the gravity maps obtained from terrestrial data and EGM2008 and GOCE show that the errors are in the order of the gravity values (Fig. 13).

The results of the quality analysis in Fig. 12 are identified as follows: green tiles are considered the most reliable areas, free of systematic errors, as they are within the commission error of the respective geopotential model. The blue tiles have systematic errors considered acceptable, and are within two SDs of the gravity anomaly differences, and the red tiles have large systematic errors, greater than two SDs, defining the areas where the terrestrial data cannot be used for further study.

We have also verified that the higher amounts of systematic error-free area were 66.3 and 64.8 per cent using the Simple and Gaussian average in Fig. 12(a) ($L = 250$), respectively, with the GOCE model, with lower percentages for the differences to EGM2008. These results stem from the fact that we extract a higher cumulative error from GOCE to degree and order 250 than from EGM2008 on the gravity anomaly differences. We note in Fig. 11, that the differences with respect to EGM2008 are slightly smaller than those obtained from GOCE- this could be due to the fact that the latter GGM exceeds the accumulative error of the EGM2008 and increases very quickly from maximum degree 198. But it is also ascribable to the fact that EGM2008 includes some of the same terrestrial gravity data, which carried eventual systematic errors, and so, are cancelled in the gravity anomaly differences.

If we compare the distribution of the observed scattered gravity data with the systematic error map, we find that the areas with smaller systematic errors correlate with areas of higher data density, likely due to more complete filtering on the terrestrial gravity data. The error estimates (Figs 11 and 12) are robust with respect to the type of spatial average filter used, as the results do not differ significantly.

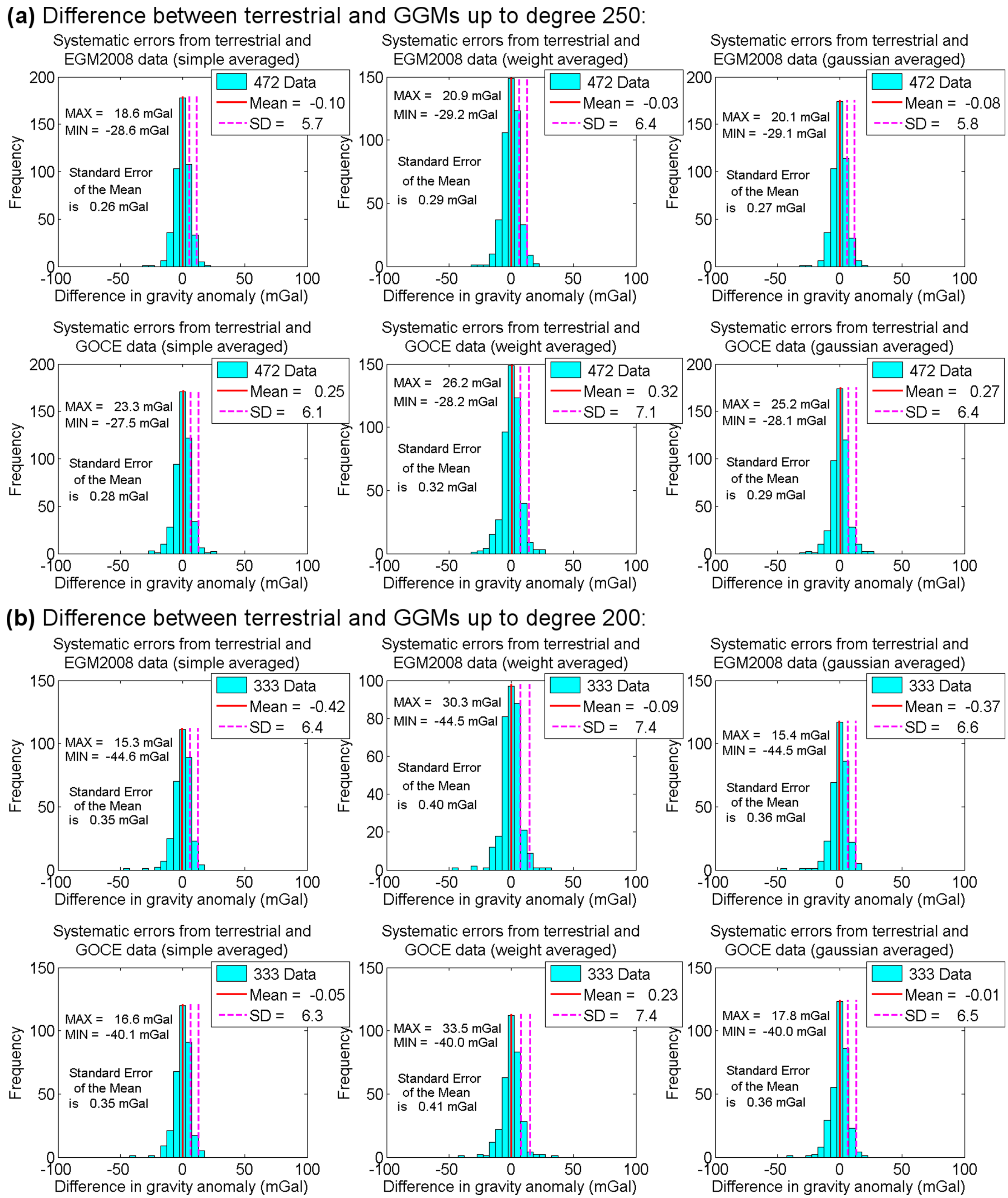

Figure 11. Systematic errors of gravity anomaly elements obtained for (averaged terrestrial data: EGM2008) (upper) and (averaged terrestrial data: GOCE) (lower) for spherical harmonic coefficients of the GGMs up to degree and order 250: the averaged terrestrial data refer to simple, weighted inverse distance and Gaussian gravity anomaly mean, respectively in the left, centre and right column.

Note that in Fig. 12(a) ($L = 250$) and Fig. 12(b) ($L = 200$) the pattern of the systematic errors is similar, but the distributions of data without systematic errors (green) or data with small errors (blue) is swapped between Figs 12(a) and (b) for GOCE. This is due to the fact that the cumulative error results to be bigger for $L = 250$ for GOCE, the average differences (Fig. 11) with the terrestrial values not changing much when considering $L = 200$ or $L = 250$. Since the error-free terrestrial data are defined as those having

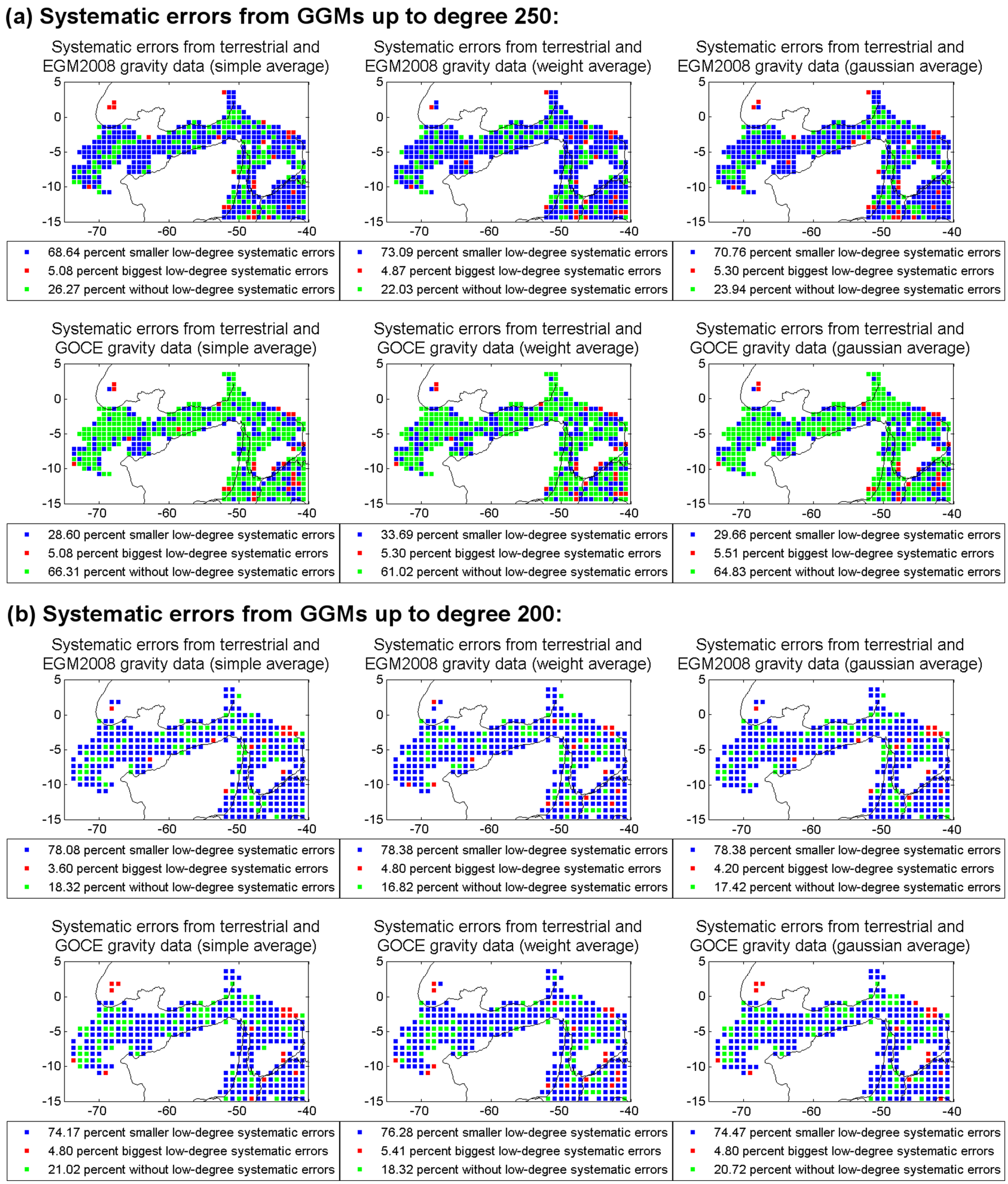

Figure 12. The low-degree systematic errors of the terrestrial gravity data indicate the locations where the terrestrial measurements should not be used for geological studies.

differences smaller than the cumulative error of the GGM, it results in the change from the error-free to small-error class when considering the two different maximum degrees for the GGMs. Fig. 13 shows the free-air anomaly for the models EGM2008, GOCE and terrestrial data. We describe the fields shortly in the next paragraph, setting them also in the broader geologic context.

Description of the gravity fields in relation to geology

The Bouguer gravity anomaly maps show approximately a chain of gravity highs of +15 to +60 mGal that transect the Amazon Basin, roughly coincident with the maximum thickness of sedimentary rocks (Nunn & Aires 1988). This gravity high is flanked by gravity

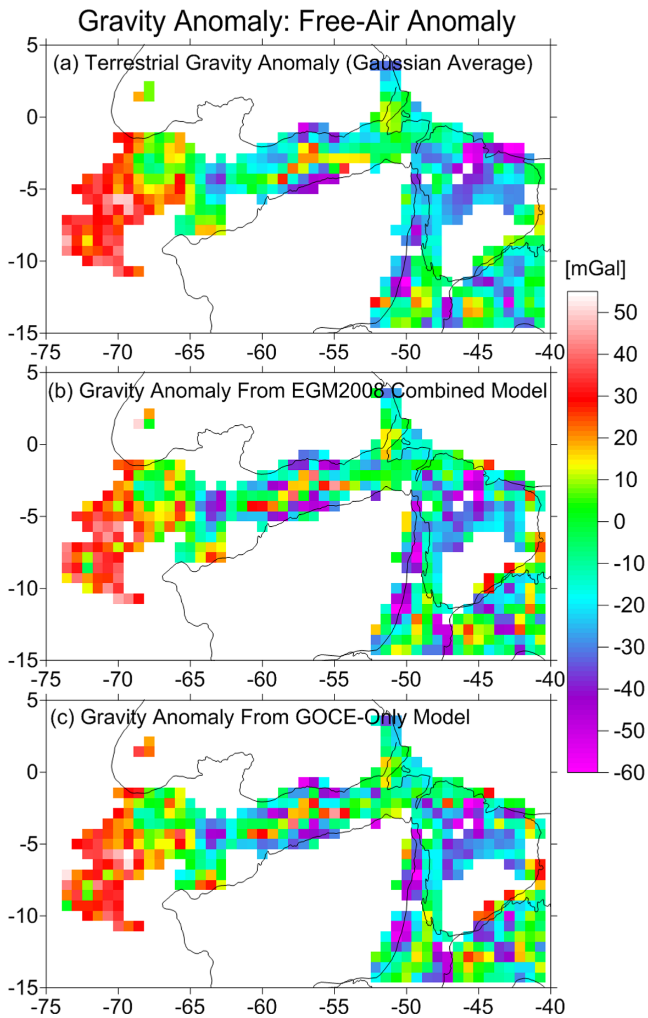

Figure 13. Free-air gravity anomaly from terrestrial, combined gravity model (EGM2008: Pavlis *et al.* 2008) and satellite-only geopotential model (GOCE: Pail *et al.* 2011) data, respectively.

lows to the north and south. Nunn & Aires (1988) interpreted the flanking gravity anomaly lows as being primarily due to the presumed downward deflection of the crust/mantle boundary beneath the basin.

The relatively high values of the gravity anomaly in correspondence to the basin axis point to a possible densification of the crust or upper mantle, or to the presence of intrusive rocks, because the sediments contribute to lower the gravity anomaly signal, as well as the Moho, which has a depth of about 42 km or more (Lloyd *et al.* 2010). In particular, the presence of possible diabase sills lying within or beneath the Amazon Basin has been postulated by some authors, and originated during late Triassic to early Jurassic, in an event known as the Penatecaua magmatism (Eiras *et al.* 1994). These igneous rocks may have migrated in a natural way following the sedimentary beds down dip causing the presence of a high density material that transects the Amazon Basin roughly coincident with the maximum thickness of the sedimentary rocks present in the basin. A more detailed analysis can be fulfilled using the terrestrial data, except where the areas with 'bad data' have been identified by our analysis.

CONCLUSIONS

The differences between the regional terrestrial gravity anomalies smoothed in order to remove the high-degree components and the satellite-only gravity anomalies recovered from satellite GOCE showed that the terrestrial gravity anomalies in the study area are affected by low-degree (up to $L = 250$) systematic errors. The statistical analysis in terms of histograms of the gravity differences reveal systematic errors in the terrestrial data, showing that the region in the study area presents about 5.5 per cent of data points with large systematic errors (considering large errors as values greater than two SDs from a mean which is close to zero) and 64.8 per cent data points with are high quality (i.e. without systematic errors). The most appropriate averaging of the terrestrial data is accomplished with the Gaussian smoothing procedure. The systematic errors have a mean value close to zero, which is smaller than the commission errors associated to the EGM2008 and GOCE models.

The terrestrial data are also compared to the EGM2008 global model that was built with both satellite and terrestrial data. The resulting systematic errors on terrestrial data are slightly lower than those found with the GOCE model, probably by the simple fact that the EGM2008 model contains the gravitational information derived from a global terrestrial data in its $5' \times 5'$ area-mean values set (Pavlis *et al.* 2012); as it is known that the terrestrial gravity data are often contaminated by systematic errors, such errors could be included in the geopotential model.

The satellite-only derived geopotential model of GOCE does not present such systematic errors. This analysis shows that the GOCE model is reliable to define the areas where the terrestrial data show greater systematic errors and should be used only with care or not be used at all for accurate geophysical modelling.

One final point is that we can see the effect of varying amount and irregular distribution of the available terrestrial data in the study region, which is common to most inaccessible areas of our planet. Thus, we can identify the existence of terrestrial data gaps that correlate with the large systematic errors.

The scheme we propose has been illustrated for the Amazon Basin, but is easily applied to any other area. An important application could be the systematic control of terrestrial databases by national agencies. We predict that cross-national datum shift which affect the gravity value could be detected in databases that include gravity anomalies from different nations.

ACKNOWLEDGEMENTS

We thank PETROBRAS, ANP and IAG-USP (Institute of Astronomy, Geophysics and Atmospheric Sciences—University of São Paulo) for the use of the terrestrial data, the Italian Space Agency (ASI) for supporting the GOCE-Italy project, the Ministero dell'Istruzione, dell'Universita' e della Ricerca (MIUR) under project PRIN, 2008CR4455_003 for financial support, and ESA for granting of AO_GOCE_proposal_4323_Braitenberg. This work was done in the frame of a Doctoral cotutela between the University of Trieste and the University of São Paulo, the latter of which is thanked for supporting part of this research next to scholarships from CAPES and CNPq, in Brazil. We acknowledge the use of EGM2008 (Pavlis *et al.* 2008) and GOCE (Pail *et al.* 2011) gravity models. We also gratefully acknowledged the two anonymous reviewers from the GJI were extremely insightful and useful.

Table 1. Statistics for gravity anomaly differences obtained between the averaged terrestrial data and the EGM2008 data filtered in the same way as the terrestrial data: simple, weighted inverse and Gaussian averaging and analogously for the differences between the averaged terrestrial data and the GOCE data. Spherical harmonic coefficients of the GGMs up to degree 250, 200 and 180 and regular grid spacing of 80, 100 and 111.111 km, respectively.

Maximum degree	GGM	Cumulative error [mGal]	Computational points for spatial average	Spatial average	Mean [mGal]	Max [mGal]	Min [mGal]	SD [mGal]	Standard error [mGal]
250	EGM2008	1.607	472	Simple	– 0.10	18.6	– 28.6	5.7	0.26
				Weighted	– 0.03	20.9	– 29.2	6.4	0.29
				Gaussian	– 0.08	20.1	– 29.1	5.8	0.27
	GOCE	5.056	472	Simple	0.25	23.3	– 27.5	6.1	0.28
				Weighted	0.32	26.2	– 28.2	7.1	0.32
				Gaussian	0.27	25.2	– 28.1	6.4	0.29
200	EGM2008	1.405	333	Simple	– 0.42	15.3	– 44.6	6.4	0.35
				Weighted	– 0.09	30.3	– 44.5	7.4	0.40
				Gaussian	– 0.37	15.4	– 44.5	6.6	0.36
	GOCE	1.491	333	Simple	– 0.05	16.6	– 40.1	6.3	0.35
				Weighted	0.23	33.5	– 40.0	7.4	0.41
				Gaussian	– 0.01	17.8	– 40.0	6.5	0.36
180	EGM2008	1.296	279	Simple	– 0.41	17.3	– 31.2	6.8	0.40
				Weighted	– 0.36	18.2	– 33.8	7.4	0.44
				Gaussian	– 0.41	17.1	– 33.2	6.8	0.41
	GOCE	0.929	279	Simple	– 0.12	15.9	– 30.4	6.7	0.40
				Weighted	– 0.11	18.5	– 31.5	7.4	0.44
				Gaussian	– 0.14	15.5	– 30.4	6.8	0.41

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