The theory of the Bouguer gravity anomaly: A tutorial

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Although the corrections which bring about a Bouguer gravity anomaly are well established, the reasons for doing them are not well understood. One cause of this common misunderstanding is that the subject has been poorly presented in many of the basic texts. This claim was quantified by T.R. LaFehr, a major figure in gravity exploration for 30+ years, in 199 1 when he wrote in GEOPHYSICS that nine of 15 English-language textbooks contain erroneous statements on this subject.

I believe the major reason for these errors is the routinely used term "gravity reduction." This phrase implies that the gravity value is somehow moved or "reduced" to a location that is different from where it was measured. This is not correct - station values are *never* moved from the observation points.

In this article, I will attempt to explain in a simple way what the Bouguer gravity anomaly truly represents.

The gravity correction process. The whole thing really boils down to

Bouguer anomaly = observed gravity - earth model.

But that simple formula is not as easy to handle as it appears at first glance. The term "observed gravity" represents a measured gravity value at any point on the earth's surface. The value, however, doesn't come solely from an instrument measurement. The measurement has been subjected to several corrections related to surveying (e.g., drift and tide corrections, base station ties) and others related to instrumentation (cross-coupling, counter units to mGals, etc.). These corrections will not be discussed here. It will be assumed that all were performed correctly and that a reliable value for absolute gravity is available at each survey point.

Absolute gravity means the exact vertical acceleration due to gravity. This can be measured in many different ways and is another area that will not concern us. We will assume the data are of good quality. The earth's gravitational acceleration is often approximated as 9.8 m/s^2 or 980 000 mGa1. Anomalies in the range of *one part per million,* or one mGa1, of the Earth's field often have significance in exploration. This is equivalent in difficulty to weighing yourself in order to determine if you have lost a button off your shirt!

The goal is to remove from the observed gravity data any components that would be present if we were dealing with a simple and virtually homogenous earth (the "ideal earth model" in the formula). If we do this correctly, then whatever remains will be *anomalous* and perhaps of local exploration interest. LaFehr summed it up this way: "It is our intent that the Bouguer anomaly be free of all nongeologic effects that are unavoidable components of the basic measurement."

Gravity reference field. The largest contribution to the earth model comes from the Gravity Reference Field, which is a mathematical model of the earth's worldwide gravity field. This formula is also called the *theoretical gravity.* In differential form, it is known as the *latitude correction.* Both of these terms are somewhat inaccurate because they don't describe the whole story behind the model. The most recent version of this formula, derived in 1987, is:

$$g = \frac{978032.68\,(1 + 0.00193185138639\,\sin^2\Phi)}{(1 - 0.00669437999013\,\sin^2\Phi)^{0.5}}$$

where Φ is the latitude in degrees. There are many variations and earlier, less precise, versions. For exploration purposes, any post- 1967 version is usually adequate.

This formula accounts for three major phenomena that impact gravity measurements: (1) That the earth spins at different angular velocities at different latitudes and they produce different outward accelerations (resulting in a gravity reading different than that produced by a nonspinning body); (2) The earth's ellipsoidal shape (some locations are different than they would be if a spherical model were used); and (3) The ellipsoidal bulges contain rock (Figure 1). Because of these effects, gravity measurements can vary considerably. The range goes from about 978 000 mGa1 at the equator to about 983 000 mGa1 at the poles.

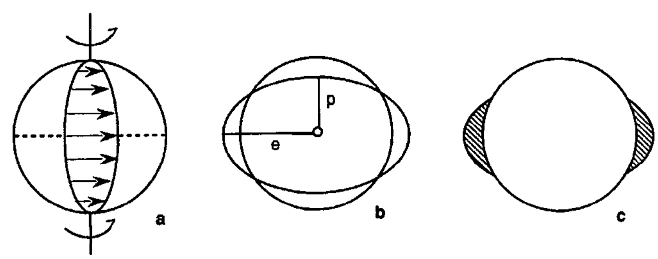

Figure 1. Components of the Gravity Reference Field model. (a) Angular velocities differ, depending upon latitude, because the earth spins faster at the equator. Therefore, outward acceleration is greatest there. (b) The earth's ellipsoidal shape means all points on the surface are not equally distant from the center of mass (e = ellipsoidal radius at equation and p = ellipsoidal radius at poles). The poles are closest to the center of mass and, therefore, gravitational acceleration is highest there. (c) Uneven mass distributions, due to ellipsoidal shape as compared with a sphere, need to be accounted for.

Obviously, the formula for the Gravity Reference Field involves some simplifying assumptions. Key ones include: (1) the earth is homogenous in lateral density distribution; (2) the observation point is static (not moving with respect to the earth); and (3) the observation is made at sea level. The first assumption is, of course, wrong in the local sense. Indeed, this local nonhomogeneity is what we want to exploit when analyzing an area's geology. In a sense, we are solving for the

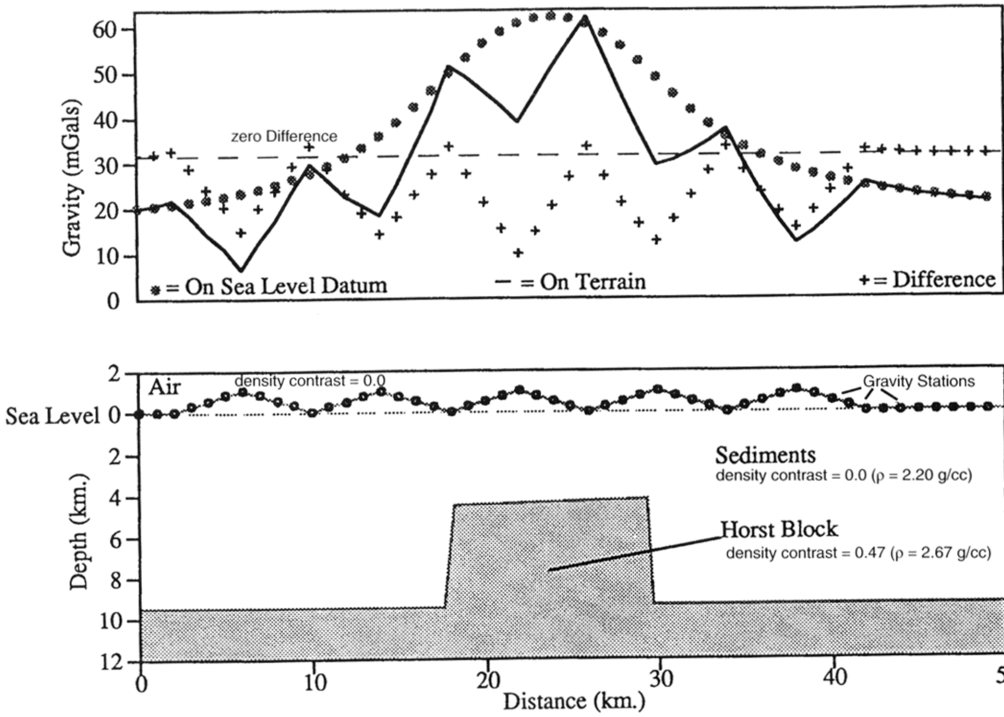

Figure 2. Comparison of observations on terrain with those on a constant datum over a synthetic model. Because observations made on varying terrain will differ in their distance to the anomalous body, short wavelenth undulations (not seen in the constant datum observations) will be imposed on the gravity data. These short wavelength components mimic the topography despite the elevation and terrain corrections already applied. Notice the density contrast at the land/air interface is zero. Land gravity surveys will always contain this effect. Notice that the "On Sea Level" and "On Terrain" values match only where the gravity stations are at sea level.

failures of this assumption. The second and third assumptions are the reasons we have to make Eötvös (for marine and airborne surveys) and elevation corrections.

rections: *They attempt to make up for the incorrect assumptions made in the original earth model.*
circle in that some corrections are made to correct for failures made by earlier corrections. We'll now consider some of the most important ones individually.

Free-air correction. This adjustment, when appropriate (nearly always for land surveys), accounts for the fact that the measurement was not made at sea level. The earth model assumes a theoretical gravity at sea level that is based upon Newton's very familiar inverse square law

$$g = GM/R^2$$

where g is the acceleration due to gravity, G is the universal gravity constant, M is the mass of the earth, and R is the distance between the observation and the center of the earth. However, this formula assumes the earth is a sphere and not an ellipsoid. We account for the earth's shape by applying the .*free-air graviry gradient,* the derivative of g with respect to h, which is expressed as

$$dg/dh = (2gh)/R — (3gh^2)/R^2$$

In practice, this usually is performed with a linear term, 0.3086 mGal/m. This correction is dependent upon latitude because R will vary due to the shape of the ellipsoid and g (the Gravity Reference Field) is different at each latitude. The 0.3086 value is based upon an inaccurately determined earth radius and the 1930 gravity formula calculated at latitude 45°.

A better, more valid approach would be to use a formula like that provided by Robbins (GEOPHYSICS, 1981). This approach takes into account worldwide variations in the shape of the earth and the Gravity Reference Field. It further provides advantages of more. robust free-air corrections at higher elevations.

The free-air correction itself includes an assumption that needs correcting, namely that there is nothing (except air) between the observation point and sea level. This, of course, is not true; the difference is more than a simple change in elevation from where the model said the point should be and where it actually is. That brings up the next logical step in the process.

Bouguer correction. To be as simple as possible, the purpose of this step is to replace the "air" in the previous correction with rock. The Bouguer correction formula is

$$B = -0.04193 \rho h$$

where p is density (in g/cm^3) and h is elevation in meters.

Two major assumptions are made-that we can fill in the elevation difference with a simple infinite slab, and that the "till" has a reasonable mass (density) distribution. The literature contains numerous techniques for calculating a reasonable slab density. If the estimate is wrong, we will either over- or undercorrect the data and thus accentuate the topography. This is not necessarily a serious matter; it depends upon how the data are subsequently used in interpretation. (For example, if you use the wrong Bouguer density, you can subsequently correct for this error and remove it in 2-D models during interpretation.)

Terrain correction and curvature correction. The widely-used terrain correction and the rarely-used curvature correction are, simply, attempts to make the infinite slab assumption more realistic. The terrain correction, not surprisingly, tries to account for the local topography-the bumps and pits on top of the infinite slab. Since these effects are close to the observation point and anomalies vary by the square of the distance to their source, these corrections can be important in high relief areas.

The curvature correction essentially bends the slab to conform to the shape of the earth in a more reasonable way. Since this particular correction effects the slab at a considerable distance from the observation point, these values are generally small.

Both of these procedures are too involved to be presented in this paper. LaFehr published a recent approach toward curvature correction in "An exact solution for the Gravity Curvature (Bullard B) Correction" (GEOPHYSICS 1991). Many terrain correction methods, both manual and automated, exist.

Eötvös corrections. The earth model, used in the previous corrections, assumes the observation point is fixed at a particular point on the earth and, thereby, rotates with it. However, this assumption will be violated in marine and airborne operations because the observation platform will have an angular velocity different than that predicted by the earth model for that particular latitude. This produces large errors only if the observation platform has a velocity component in the east-west direction. If the platform is moving north-south, it will be moving along with the rotation of the earth. (However, it will still produce a small outward acceleration not accounted for by the earth model.)

The Eotvos correction is

$$E = 7.508\ V \cos\Phi \sin\Theta + V^2/R$$

where V is the platform velocity in knots, Φ is the latitude in degrees, Θ is the platform heading in degrees from north, and R is the radius of the earth (in meters) at that latitude.

Final Bouguer gravity anomaly. In essence, all the above has only made a minor change in the original formula, namely:

Bouguer anomaly = observed gravity - corrected earth model

As you have seen, no attempt was made to "move" the observation point to any other location. All changes were made to correct for incorrect assumptions made in earlier corrections. In other words, we were attempting to force the com*putation to conform to the local conditions of the observation point*, rather than the other way around. The anomaly values would be significantly different if observed at a different datum, as implied in much of the literature which refers to gravity "reductions" (Figure 2).

In that case, we would have to attenuate or deattenuate the observed values appropriately. This is never done in any of the above corrections. The process simply subtracts a multiply-corrected earth model from our observation to yield a local anomaly. This local anomaly occurs at the varying elevation surface upon which the measurements were made. To "reduce it to a common datum" would require a totally different process - differentially continuing the data to a flat datum plane. (Differential continuations can be done but they generally involve complicated processing.) There are several exceptions to the above that I need to mention in passing. First, the commonly called 3-O *Bouguer correction* is simply a three-dimensional gravity model of topography; thus, no slab assumption is made and terrain/curvature corrections don't have to be made separately because they are integral parts of the process. Second, airborne gravity and marine gravity are, by design, observed at a constant datum and, consequently, these surveys are corrected/interpreted somewhat differently.

What is a Bouguer anomaly? We should think about it as the residual left over after a process of elimination has removed all possible components of the earth model. Ideally, anything that is left over will be the result of density inhomogenities due to local geology.

How do we determine if the "left over" is of exploration interest? Well that's another story.

Table 1: Conceptual flow chart for the gravity correction process

Input	Correction	Major assumptions		Failure leads to . . .
Observed gravity	Grav. ref. field (GRF)	Homogenous earth	→	*Desired outcome*
		Static measurement	→	Eötvös correction
		Sea level datum	→	Free-air correction
GRF corrected	Eötvös	None	→	*Desired outcome:* GRF
			→	& Eötvös corrected data
GRF (& Eötvös) corrected	Free-air	No rock above sea level	→	Bouguer correction
		Gradient computed at sea level	→	Higher-order correction terms
		Only valid at 45° lat.	→	Generally not corrected
Free-air corrected	Bouguer	Infinite slab	→	Terrain & curv. corrected ("Simple" Bouguer anomaly)
		Uniform slab density	→	*Desired outcome*
		"Correct" slab density	→	Interpretation issue
"Simple" Bouguer anomaly	Terrain (& curve.)	None	→	*Desired outcome:* ("complete Bouguer anomaly)