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OPTIMAL CONFIGURATION OF A SEISMOGRAPHIC NETWORK: A STATISTICAL APPROACH

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Abstract

We consider the problem of selecting sites for a seismographic network so that the network resolution will be optimal. Configurations are compared via the D-optimality criterion proposed in the statistical literature on optimal design of experiments. Optimal configurations are found using the efficient DETMAX algorithm. We also show how to implement the criterion and the algorithm in the presence of spatially correlated error terms.

Introduction

In recent years, there has been considerable interest among seismologists in assessing the potential resolution capability of global and local seismographic networks (Peters and Crosson, 1972; Minster *et al.*, 1974; Lilwall and Francis, 1978; Dziewonski and Woodhouse, 1983; Romanowicz *et al.*, 1984; Satake, 1985; Souriau and Woodhouse, 1985; Uhrhammer, 1980). These works discuss methods for studying the potential of a given network with respect to variations in hypocenter location.

Our purpose is to turn the assessment question around: given a possible hypocenter location, how should one locate the stations in a network so as to obtain precise location estimates? We suggest a general approach to the station siting problem based on recent developments from the statistical theory of optimal experimental design. An important precursor is the work of Kijko (1977), who first suggested applying ideas from optimal experimental design to the problem of seismographic network configuration. We extend Kijko's ideas, describe theoretical properties of optimal networks, describe efficient numerical algorithms (DETMAX; Mitchell, 1974), and consider some extensions and modifications.

Method

Optimal experimental design

In a typical geophysical inversion procedure, the unknown parameter vector $\boldsymbol{\theta}$ is estimated by fitting a set of observed data to their predicted values (with respect to a given assumed model) at the observation points. The estimation criterion is typically least squares, so that the parameter estimate minimizes the Sum of Squared Residuals (SSR) function:

$$\min_{\theta} \mathrm{SSR} = \sum_{i=1}^{N} (t_i^{\mathrm{obs}} - t_i^{\mathrm{cal}}(\boldsymbol{\theta}))^2 \tag{1}$$

where t_i^{obs} denotes the observed quantity t at station i, t_i^{cal} the corresponding predicted value of t (as a function of the unknown $\boldsymbol{\theta}$) and N the number of stations. To compute the estimate $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$, one usually starts with an initial guess $\boldsymbol{\theta}^{(0)}$ and updates iteratively, so that $\boldsymbol{\theta}^{(n+1)} = \boldsymbol{\theta}^{(n)} + \delta\boldsymbol{\theta}^{(n)}$, with $\delta\boldsymbol{\theta}^{(n)}$ chosen to reduce the value of the SSR function. At each iteration, $\delta\boldsymbol{\theta}$ is evaluated by fitting the linearized model

$$\delta \mathbf{r}^{(n)} = \mathbf{A}\delta\boldsymbol{\theta}^{(n)} \tag{2}$$

where $\boldsymbol{\delta}\mathbf{r}^{(n)}$ denotes the residual vector $(\mathbf{t}^{\text{obs}} - \mathbf{t}^{\text{cal}})$ for $\boldsymbol{\theta}^{(n)}$ and $\mathbf{A}$ is the $\mathbf{N} * \mathbf{p}$ matrix of partial derivatives of $\mathbf{t}$ with respect to the p components of $\boldsymbol{\theta}$, evaluated at $\boldsymbol{\theta}^{(n)}$.

The problem of optimal configuration can be formulated as follows: find a configuration of stations that optimizes the resolution of $\boldsymbol{\theta}$. The problem of achieving maximal resolution for a parameter estimate has been studied at length in the statistical literature under the heading optimal experimental design (Box and Lucas, 1959; Silvey, 1980; Atkinson, 1982). Of several optimality functionals that have been advanced there, the most popular is maximization of the quantity $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{A})$, known as the D-criterion. The D-criterion is based on the following argument: assuming normality and independence of the errors, an approximate confidence ellipsoid for $\boldsymbol{\theta}$ has the form:

$$\{\boldsymbol{\theta} : (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \leqq \text{constant}\} \tag{3}$$

where $\hat{\boldsymbol{\theta}}$ is the least-squares estimate of $\boldsymbol{\theta}$, $\mathbf{A}$ is evaluated at $\hat{\boldsymbol{\theta}}$, and the constant is an appropriate quantile from the χ^2_{N-p} distribution. The content of this ellipsoid is proportional to $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1/2}$. An obvious optimality criterion is therefore to make this ellipsoid as small as possible by maximizing $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{A})$. A configuration that maximizes $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{A})$ will be called D-optimal.

For nonlinear least-squares problems, such as that here, the criterion $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{A})$ depends on $\hat{\boldsymbol{\theta}}$ as well as the station configuration. For design purposes, we will replace $\hat{\boldsymbol{\theta}}$ by a particular parameter value for which high resolution is desired. There are several ways one might extend the D-criterion to design networks that must monitor several faults or fault zones. Let $\mathbf{A}_{\mathrm{i}}$ denote the matrix of partial derivatives for the ith hypocenter. Kijko (1977) suggested maximizing $\sum \alpha_i \mathbf{det}(\mathbf{A}_{\mathrm{i}}{}^{\mathrm{T}}\mathbf{A}_{\mathrm{i}})$, where α_i can be interpreted as the prior probability of the ith hypocenter, or simply as a weight expressing the relative importance of the ith hypocenter. Chaloner and Larntz (1990), following work of Lindley (1956), suggest that a better criterion is to maximize $\sum \alpha_i \ln[\mathbf{det}(\mathbf{A}_{\mathrm{i}}{}^{\mathrm{T}}\mathbf{A}_{\mathrm{i}})]$, which corresponds to the entropy of the posterior distribution of $\boldsymbol{\theta}$. We are currently investigating use of this criterion.

Naive application of D-optimality often leads to designs that concentrate observations at a small number of sites (Atkinson and Hunter, 1968). This phenomenon was also characteristic of our first example, described in a later section. As practical seismologists are quick to point out, however, two stations located at the same site will give identical arrival times so that the second station provides no additional information. This seeming contradiction arises from the assumption that the errors at all stations are statistically independent; the practical argument implies, to the contrary, that errors for two stations at the same site will be perfectly correlated. The idea of correlated errors can be justified physically by arguing that the error is composed of model inaccuracies and local anomalies, as well as observational error (Pavlis, 1987). The effect of model inaccuracies will be similar at proximate stations and identical if two stations are located at the same site. These similarities can be reflected by assuming that the errors are spatially correlated. The corresponding design criterion will be to maximize $\mathbf{det}(\mathbf{A}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{A})$, where $\mathbf{W}$ is the error correlation matrix for the sites. This criterion assures that all stations in an optimal network will be located at unique sites if one assumes perfect correlation between stations at the same site.

Theory of D-Optimal Seismographic Networks

Steinberg and Rabinowitz (unpublished manuscript) derive some properties of the D-criterion for seismographic networks and characterize D-optimal configurations. The theoretical results require somewhat restrictive assumptions, so they are of limited practical value. Nonetheless, they provide useful insight into the type of configurations that will be favored by the D-criterion. We present below a brief summary of the main results.

The assumptions are as follows: (i) stations can be placed at any location; (ii) stations will be located on the earth's surface at the same height; (iii) a standard layers above a half-space model is the basis for computing travel times; (iv) the error terms are statistically independent. The following results characterize D-optimal networks for a particular hypocenter.

1. Given any network configuration, the D-criterion will be improved if the stations in the network are redistributed so that they are symmetric about the epicenter. In particular, a D-optimal configuration must place all stations on concentric circles about the epicenter, with the stations on each circle equidistant from one another. Thus the D-optimality criterion agrees fully with seismological intuition that a good configuration must "surround" the epicenter.

2. For a hypocenter in the half-space, the triangular quadripartite network recommended by Lilwall and Francis (1978) and by Uhrhammer (1980) is D-optimal. This configuration places one station directly above the hypocenter and three stations at equidistant points on a circle about the epicenter. The D-criterion is a monotone increasing function of the sine of the take-off angle of the ray path from the hypocenter to the stations on the circle. The maximal value is thus obtained in the limit, as the radius tends to infinity. For typical crustal models, a moderate radius will yield a value near the maximum.

3. For a hypocenter in a layer above the half-space, the D-optimal configuration places a station above the hypocenter and at least three stations at equidistant locations on each of two concentric circles about the epicenter. The radii of the two circles must be chosen so that waves arrive at stations on the near circle via direct ray paths and reach stations on the far circle via refraction paths. The derivatives of travel time with respect to focal depth will then have different signs for stations on the two circles and the network can be designed so that the focal depth estimate will be uncorrelated with the estimate of origin time; high correlation between these parameter estimates typically leads to ill-conditioning, problems in convergence, and large uncertainty in the estimated hypocenter. The triangular quadripartite network, which uses only one circle, can be quite inefficient for a hypocenter above the half-space. The superior resolution power of a network with mixed arrivals, in particular for depth resolution, was noted by Peters and Crosson (1972). However, they considered only the problem of assessing a given seismographic network and did not extend this idea to network design.

4. For the D-optimal configurations described above, the D-criterion, when both P- and S-wave arrivals are available, is proportional to the criterion with P-wave arrivals only. Thus these configurations will be D-optimal in either situation. The marginal utility to the D-criterion of adding the S arrivals is substantial. This corroborates the observation by Buland (1976) that adding even a single S arrival can substantially improve location estimates. We conjecture that the benefit of adding S arrivals is even greater for sub-optimal configurations.

5. The D-criterion is proportional to the determinant of the information matrix for the hypocenter parameters; thus a configuration that is D-optimal is also optimal in the sense that it maximizes information about the hypocenter parameters. The D-criterion is thus equivalent to a criterion proposed by Kijko (1977) for optimal network configuration. Two alternative criteria are to maximize the determinant of the information matrix for the epicenter (also considered by Kijko) and to maximize

the information for focal depth. For all of these criteria, result 1 is still valid: redistributing the stations in a network so that they are symmetric about the epicenter always improves the criterion. Thus optimal configurations will place stations at equidistant points on concentric circles about the epicenter. The proportion of stations allotted to each circle will differ from the D-optimal proportions.

THE DETMAX ALGORITHM

The design of a seismographic network always involves practical constraints related to geography, communications, transmission, remoteness, transportation, etc. Thus the theoretical results described in the previous section will typically be impossible to implement. A more practical approach is to apply a computer algorithm that is able to search for D-optimal configurations taking account of the constraints. In this work we applied the DETMAX algorithm, proposed by Mitchell (1974) for the specific purpose of constructing D-optimal experimental designs. An important practical feature of the DETMAX algorithm is the ability to augment existing networks by adding stations, in an optimal manner, from a pre-defined set of candidate sites.

Numerical maximization of $\mathbf{det(A^TA)}$ is difficult, mainly because of the large number of variables ($\mathbf{N} * \mathbf{p}$) that must be handled and also because of the complicated nature of the function $\mathbf{det(A^TA)}$, which often has many local maxima. The DETMAX algorithm is much more efficient than standard optimization routines such as those used by Kijko in applying the D-criterion to seismographic networks. For example, Box and Draper (1971) reported that they were able to find D-optimal designs using a direct search method only for small linear problems.

The DETMAX algorithm for maximizing $\mathbf{det(A^TA)}$ exploits a fundamental result from the statistical theory of experimental design known as the "Duality principle between D- and G-optimal design" (Kiefer and Wolfowitz, 1960). In our context the duality principle can be described as follows. Let H be the set of all possible candidate sites and consider estimating the value of an observation at a site x_i, $i \in H$. The variance of the estimate, based on the linearized model about $\hat{\boldsymbol{\theta}}$, is given by

$$v_i = f_i^{\mathbf{T}}(\mathbf{A^TA})^{-1} f_i \sigma^2 \qquad (4)$$

where f_i gives the partial derivatives of an observation at x_i with respect to $\boldsymbol{\theta}$ (evaluated at $\hat{\boldsymbol{\theta}}$), and σ^2 is the data error variance (which without loss of generality is set here to 1). A configuration is called G-optimal if it minimizes, over all possible configurations, $\max_{i \in H} v_i$. The first consequence of the duality principle involves adding one station to an existing N-station network and states that the maximum possible increase in $\mathbf{det(A^TA)}$ is achieved by adding a station at the site where v_i attains its maximal value:

$$v_0 = \max_{i \in H} v_i. \qquad (5)$$

Further, the duality principle states that a configuration is D-optimal if and only if it is G-optimal and, for such a configuration, $v_i = p/N$ for each site in the configuration. This principle holds exactly only for determining the fraction of stations to be placed at each site, without requiring that the fractions be multiples of $1/N$; it is approximately true for the actual discrete problem of assigning stations. The importance of the final condition is that it gives a simple criterion for judging whether a configuration is D-optimal. This condition was essential to deriving the theoretical results in the previous section and it is useful in setting convergence criteria for iterative search algorithms. We have derived related criteria for problems with correlated errors (Koren, 1989) and have made extensive modifications to the DETMAX algorithm to apply these criteria.

We now describe the DETMAX algorithm in the context of the optimal configuration of a seismographic network, closely following Mitchell (1974). The algorithm starts by choosing at random an N-station network and improves this initial configuration by: 1. Adding one station to the network. This additional station is chosen from a set (H) of possible sites so that the maximum possible increase in $\mathbf{det(A^TA)}$ is achieved, i.e., adding that candidate station for which v_i (equation 5) is maximized, then 2. Subtracting from the (N + 1)-station network that station for which v_i is minimum, which guarantees the minimum possible decrease in $\mathbf{det(A^TA)}$. Here another rule is now introduced: the requirement that an (N + 1)-station network return immediately to an N-station network is relaxed. Instead the algorithm is permitted to make "excursions" in which networks of various (finite) sizes are constructed (always by adding or subtracting a single station to the previous network in the excursion), and eventually returning to an N-station network. If no improvement in $\mathbf{det(A^TA)}$ is made during an excursion, all networks constructed there are ostracized by placing them in a set of "failure networks" called F. The set F is then used to guide the course of the next excursion, which always starts with the best N-station network found this far. The rules are such that excursions always proceed in the direction of an N-station network unless they come to a network in F which has already led to a failure. The goal of the excursions is to enable the algorithm to escape from local maxima. To further increase the probability of finding global maxima, Mitchell (1974) recommended that a number of different random initial configurations be used.

Illustrative Examples

The following examples have been chosen for the sake of clarity and to give some insight into the notion of optimal resolution and optimal siting. They also serve as interesting cases where intuition may not agree with mathematical reasoning.

Case 1: Independent Errors

Consider the following "ideal" seismological experiment: We want to locate, with a six-station seismographic network, an earthquake which occurred in the right lower edge of a 30 * 60 km rectangular area and at a focal depth of about 20 km (Fig. 1a). A complete knowledge of the crustal structure is assumed, with a three layered velocity model as shown in Table 1. We assume that only P-wave arrivals will be recorded. We are able to deploy the network inside (or on the boundaries of) the rectangular area, and seek to find the optimal configuration of the network (with respect to that task only).

In order to apply the DETMAX algorithm the area was divided into a 7 * 13 grid where each grid point represents a candidate site. Input to the DETMAX algorithm was thus a 91 * 4 matrix whose ith row had the form:

$$f_i = \left(1, \frac{\partial t_i}{\partial y_0}, \frac{\partial t_i}{\partial x_0}, \frac{\partial t_i}{\partial z_0}\right) \tag{6}$$

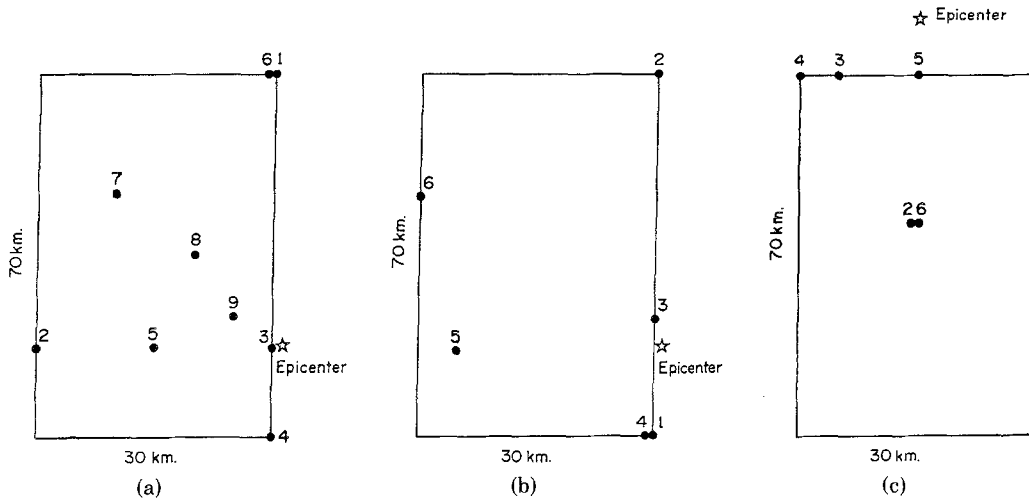

FIG. 1. Configurations obtained by DETMAX algorithm in case 1, for source (star) (a) at focal depth of 20 km. Points 7, 8, and 9 denote successive replacements of station 6; (b) at a focal depth of 5 km; (c) outside the network, with focal depth 5 km.

TABLE 1

VELOCITY MODEL USED IN THE EXAMPLE (SEE TEXT)

Layer	P-wave velocity (km/sec)	Thickness (km)
1	4.0	10
2	5.5	10
3	6.5	10
4	8.0	half-space

where t_i denotes the P travel time from the source to station i, and y_0, x_0, z_0 are the expected source coordinates. (It can be shown that for a given configuration, the function $\det(\mathbf{A}^T\mathbf{A})$ is insensitive to a small perturbation in the source position, so that a point source can represent a small area.) Figure 1a shows the optimal configuration to which DETMAX converged, and which was achieved for 95 out of 100 random starts. Upon observing this result, it is worth noting that such a configuration clashes with our "professional judgment." The DETMAX siting of stations 5, 4, and especially 3 (the station above the anticipated hypocenter) would be accepted by an experienced seismologist but the siting of station 2 and especially stations 1 and 6 (at the same site) would probably be graded as "unreasonable." In order to examine this result more carefully, station 6 was moved and relocated successively at points 7, 8, and 9 (shown in Fig. 1a). For each such new six-station configuration (denoted configuration +7, +8, and +9) the value of $\det(\mathbf{A}^T\mathbf{A})$ was calculated (see Table 2). As observed from Table 2, the DETMAX selection is indeed better than any of these alternatives.

Figure 1b shows the results of the same experiment, but for an event that is expected to occur at a shallow focal depth (5 km). For this shallower event, better resolution is achieved when the station on top of the hypocenter is moved about 5 km northward. The recommendation to place two stations at the same site is repeated here. This configuration was achieved in 13 out of 100 random starts.

TABLE 2

DETERMINANT VALUES OF FOUR CONFIGURATIONS

Configuration	$\det(A^T A)$
6+	$2.0\ 10^{-4}$
7+	$1.7\ 10^{-4}$
8+	$1.8\ 10^{-4}$
9+	$1.6\ 10^{-4}$

Values of $\mathbf{det}(\mathbf{A}^T\mathbf{A})$ for different configurations in Figure 1a.

A configuration for an event that is expected to occur significantly outside the network is shown in Figure 1c. Here the algorithm tries to encircle the hypocenter, in accord with seismological intuition, but again two stations are placed at the same site (stations 3 and 6).

We do not, in general, recommend replicate station sites and it should be understood that two "extreme" features of the above examples are responsible for the phenomenon. First is the strong assumption that errors from the model are statistically independent. In case 2 below, we show the effect of assuming correlated errors, which seems a much better representation of physical reality. Second, the examples are based on a single source, but seismographic networks are usually designed to simultaneously cover several expected sources in different areas. We believe that incorporating several sources will also alleviate the clustering of stations observed above.

Case 2: Correlated Errors

In this example, we consider selection of stations for an event located just outside the northeast corner of a 30 * 70 km rectangular region at a depth of 15 km. We assume the same velocity model as in case 1 and again seek a six-station network. Input to the modified DETMAX algorithm includes a 105 * 4 matrix of partial derivatives (with respect to each candidate point on a 7 * 15 grid) and a subroutine that defines the correlation between the errors at any pair of stations.

When errors are assumed to be independent, the DETMAX solution (Fig. 2a) places two stations in the northeast corner of the region. One station is located in the southeast corner, and another 5 km west of that corner. The station in the corner receives a direct wave, but the station to its west receives a refracted wave. The final two stations are located on the western border of the region. Again one station (the northern station) receives a direct wave and the other receives a refracted wave. As noted in the theoretical discussion, the presence of both direct and refracted waves can markedly improve the resolution ability of a network, especially with regard to determination of focal depth. The importance of this phenomenon, which is automatically recognized by the DETMAX algorithm, is likely to be overlooked by a seismologist working on professional intuition alone.

We then analyzed network design for this problem assuming that the errors at each pair of stations are correlated by

$$R(i, j) = \begin{cases} \left(1 - \dfrac{\alpha}{\pi}\right)\exp[-\beta\,|\,d_i - d_j\,|\,] & \text{for} \quad 0 < \alpha < \pi \\ 0 & \text{for} \quad \alpha > \pi \end{cases} \tag{7}$$

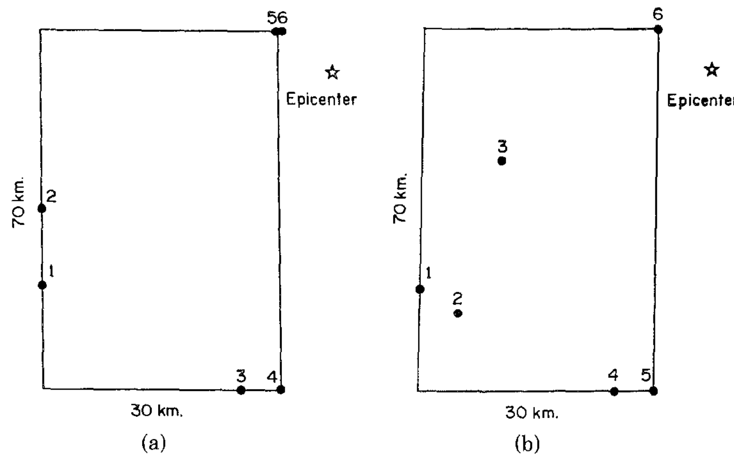

FIG. 2. Configurations obtained by DETMAX algorithm in case 2, (a) errors are assumed to be uncorrelated; (b) errors are correlated using equation 7 (see text).

where α is the difference in azimuths for the two stations and d_i is the distance of Station i from the epicenter. This correlation function assumes that errors will be highly correlated at two stations if they are approximately the same distance from the epicenter and have similar azimuth. The errors at stations on opposite sides of the epicenter ($\alpha > \pi$) are assumed to be independent. The free parameter β controls how rapidly the correlations drop off as a function of distance from the epicenter; it can be chosen subjectively or estimated from data. Our experience is that choices between 0.01 and 0.5 provide a useful range of correlation patterns.

We used the modified DETMAX algorithm to derive optimal configurations for several values of β. Replicate station sites were never included in the networks, since they cannot improve the D-criterion for a current network. The optimal configuration when $\beta = 0.05$ is shown in Figure 2b. This configuration is similar to that found with independent errors but spreads the stations out to a greater extent. The sites in the northeast and southeast corners provide substantial resolution power and are once again included. Similarly, the two stations that receive refracted waves are again included. A fifth station is located near the western border, 45 km north. The final station is located near the southwest corner, but in the region that receives direct waves. Moving the station slightly closer to the corner would result in a third refracted wave arrival and would adversely affect resolution potential. The D-criterion value for this network is $0.395\ 10^{-4}$. The criterion value for the original network (found assuming independent errors) is $0.175\ 10^{-4}$.

Summary and Conclusion

In this paper, we advocate the use of a statistical method for the purpose of optimal deployment of seismographic networks. The proposed D-optimality criterion is general in nature and independent of the context of the specific study, so it is potentially applicable to global, regional, and local networks. We have shown that, in general, the D-criterion will generate networks that "surround" the epicenter, in accord with seismologists' practical experience. Although naive application of the criterion may lead to networks that include duplicate sites, we have shown that this problem can be solved by adopting a more realistic model in which the error terms are correlated. The DETMAX algorithm has been found to be efficient in generating D-optimal networks, both for the case of independent errors (in its original form) and for the case of correlated errors (following our modifications).

Apart from the design of a complete network, the statistical approach also suggests natural solutions to other relevant problems. For example, the criterion, and the DETMAX algorithm, can be used to find the optimal site for an individual new station to be added to an existing network. One can also define a rather general measure of "station efficiency" by the relative increase in the determinant of the network due to the inclusion of each individual station. These problems are related to the work of Souriau and Woodhouse (1985), who compared candidate sites for addition to a given network with respect to final resolution. The criterion can also be used to decide how many stations are needed for a particular seismological study by comparing networks of different sizes and checking when the increase in the criterion is insignificant.

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