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Regularization of gravity field estimation from satellite gravity gradients

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Abstract. The performance of the L-curve criterion and of the generalized cross-validation (GCV) method for the Tikhonov regularization of the ill-conditioned normal equations associated with the determination of the gravity field from satellite gravity gradiometry is investigated. Special attention is devoted to the computation of the corner point of the L-curve, to the numerically efficient computation of the trace term in the GCV target function, and to the choice of the norm of the residuals, which is important for the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) in the presence of colored observation noise. The trace term in the GCV target function is estimated using an unbiased minimum-variance stochastic estimator. The performance analysis is based on a simulation of gravity gradients along a 60-day repeat circular orbit and a gravity field recovery complete up to degree and order 300. Randomized GCV yields the optimal regularization parameter in all the simulations if the colored noise is properly taken into account. Moreover, it seems to be quite robust against the choice of the norm of the residuals. It performs much better than the L-curve criterion, which always yields over-smooth solutions. The numerical costs for randomized GCV are limited provided that a reasonable first guess of the regularization parameter can be found.

Keywords: GOCE – Satellite gravity gradiometry – Regularization – Generalized cross-validation – L-curve criterion

1 Introduction

The Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) mission is the first core mission of the Earth Explorer Programme of the European Space Agency (ESA). It will be launched in 2005. The main objective of the GOCE mission is to provide a high-accuracy, high-resolution global model of the Earth's static gravity field from a combination of spaceborne gravity gradiometry (SGG) and high–low satellite-to-satellite tracking (hl-SST). A homogeneous accuracy of about 1 cm in terms of geoid heights, and of better than $1 \times 10^{-5}\mathrm{m/s^2}$ in terms of gravity anomalies, will be feasible for half wavelengths down to approximately 100 km. It is expected that such an advance in the knowledge of the Earth's gravity field will provide new and fundamental insight into the areas of solid–earth physics, oceanography, and geodesy (ESA 1999).

Scientific data collection during the mission will last about 12 months, divided into two operational phases. During that time the prime sensors on board GOCE, the GPS receiver and the gradiometer, will collect hundreds of millions of phase, pseudorange, and gravity gradient observations. From these observations about 90 000 potential coefficients will be estimated. Even when limited to SGG-only solutions, the dimension of the problem is still impressive: assuming a measurement rate of 1 Hz and four tensor components to be measured simultaneously, about 10^8 observations will be collected over the 12-month period. A standard least-squares (LS) approach based on the functional relationship between measured gravity gradients and unknown potential coefficients generates a design matrix with about 10^{13} entries, and a normal matrix with about 10^{10} entries. The associated variance–covariance matrix of the measured gravity gradients will be dense, as end-to-end closed-loop simulations have shown (see e.g. SID 2000). In particular, consecutive SGG observations of the same type taken along the orbit are highly correlated, and the correlations are expected to damp out after two to three revolutions (ESA 1999). This implies that the variance–covariance matrix cannot be generated by a spatially isotropic covariance function, which, together with the huge number of entries, will make a proper LS approach even more elaborate numerically. Whether this matrix can be approximated by a sparse one (see e.g. Moreaux

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2000) or whether suitable linear filters can be defined that decorrelate the observations (see e.g. Schuh 2000) is the subject of current investigations. Anyway, the number of entries of the design matrix and the normal matrix implies that these matrices cannot be formed explicitly, thus making iterative solution methods, such as conjugate gradient methods, mandatory. Then, the normal matrix is only assessed indirectly by matrix–vector multiplications.

Another complication is illustrated in Fig. 1, which shows the error degree variances of a gravity field solution up to degree and order 300 from measured gravity gradients. Second radial derivatives of the disturbing potential are used, observed along a circular 60-day repeat orbit with an inclination of 96.6° and corrupted by a colored noise sequence generated according to the error budget as defined in ESA (1999). The uppermost curve shows the degree-order root-mean-square (RMS) error of the solution error if (almost) no regularization is applied. Obviously, they are orders of magnitude larger than the signal to be recovered. This reflects the ill-conditioning of the normal equation matrix. There are two reasons for that: first of all, the determination of gravity field functionals on the Earth's surface or any surface close to it from satellite sensors lacks stability. This is usually referred to as the downward continuation problem. It amplifies not only the data noise but also unmodeled signal. The second reason is the polar gap caused by the non-polar orbit, which in particular affects the low-order coefficients, independently of the degree. How downward continuation and polar gap degrade the gravity field solution depends on (1) the mission design, in particular the choice of orbit and instrument noise; (2) the type of gravity field functional to be determined at the Earth's surface (i.e. geoid heights or gravity anomalies); (3) the spatial resolution (in terms of the maximum degree and order to be estimated); and (4) the quality of the a priori gravity field model (i.e. how well it fits the true gravity field) and the gravity field recovery strategy. For instance, it is well known (e.g. Koop et al. 2000) that the gradiometer cannot determine the long-wavelength features of the Earth's gravity field well due to the $1/f^2$ increase of the error power spectral density function according to ESA (1999) for frequencies $f < 5$ mHz. Although this is adjusted for when combining SST and SGG observations, the problem is still worth investigating, if SGG-only solutions are combined with long-wavelength global gravity field models from the CHAMP and GRACE missions. Simulations conducted at Delft University of Technology in the framework of the ESA's E2mGal study (cf. Klees et al. 2000a) have shown that LS solutions up to degree and order 300 with realistic mission parameters and spherical harmonics as base functions lose about eight digits due to the downward continuation and about 16 digits due to the polar gap.

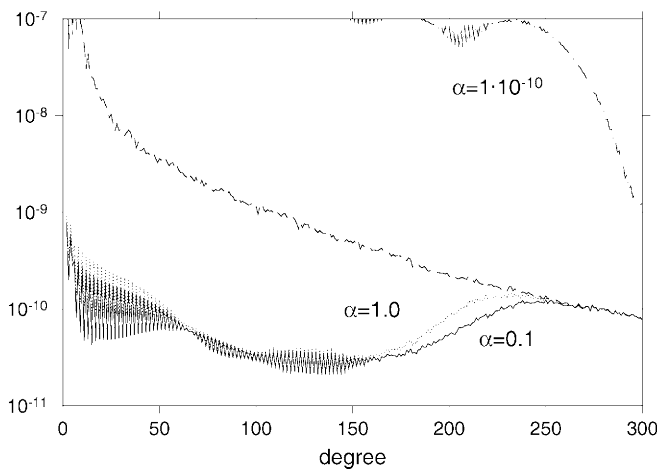

Fig. 1. Degree-error RMS for solutions stabilized by Tikhonov regularization with signal constraint. The three solutions are computed with almost no regularization (the tiniest α that allowed Cholesky decomposition, *dash–dotted line*), 'nominal' stabilization (*dotted line*), and optimal regularization (*solid line*). The *dashed line* shows the 'true' OSU91a input signal

For that reason, the LS solution shown in Fig. 1 is corrupted by observation noise to such an extent that it does not resemble any physically meaningful gravity field model. Although a first stabilization effort has already been made by modelling the gravity field in finite-dimensional space, regularization, or from a Bayesian point of view the use of prior information, is indispensable. On the other hand, ill-conditioned normal equations are not new in geodesy, in particular when satellite observations are used for gravity field estimation. Usually Kaula stabilization is used; that is, we add a diagonal matrix to the normals matrix, whose entries follow a degree variance model of the potential coefficients. The corresponding solution is also shown in Fig. 1. Obviously, Kaula stabilization provides a solution much better than the corresponding solution without any regularization. However, when compared with the optimal solution, we observe significant differences at medium and high degrees and orders. Therefore, one objective should be to improve the solution in that part of the spectrum by applying another regularization method in combination with a suitable parameter choice rule.

So far, various algorithms for computing a regularized solution and various methods for choosing the regularization parameter have been proposed in connection with the determination of gravity field functionals (gravity anomalies or geoid heights) on the Earth's surface from satellite gravity gradiometry or satellite-to-satellite tracking, e.g. by Xu (1992), Ilk (1993), Bouman (2000), and Lonkhuyzen et al. (2001). Among the regularization methods are Tikhonov regularization (Tikhonov 1963), biased estimation (Hoerl and Kennard 1970), unbiased LS estimation with stochastic prior information, and LS collocation (e.g. Moritz 1976). In particular, in Kaula stabilization, the diagonal part of the covariance matrix of the stochastic parameters is identified with the signal variances from a degree variance model or an existing satellite-derived geopotential model. Formally, these regularization methods lead to the same regularized normal equations; they only differ in how the results are interpreted.