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Fast Combination of Satellite and Marine Gravity Data

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ABSTRACT

A new method has been developed that improves upon current methods of heterogeneous gravity data integration, with a much reduced computation time and memory requirement. Gridded datasets of satellite altimeter observations of the geoid height, and of shipboard measurements of the free-air gravity anomaly are combined through an iterated weighted superposition, using the fast Fourier transform for conversion between the datasets.

Keywords: geoid, gravity anomalies, Fourier transforms, satellite altimetry

INTRODUCTION

Although there exists an almost global coverage of satellite altimetry data, with the newly released Geosat/GM data and ERS-1 data greatly improving resolution, there is also an extensive global marine gravity dataset. The marine gravity dataset should not be ignored as it can easily be incorporated with satellite altimetry data to give a more accurate map of the geoid than presumably could ever be obtained from purely altimetric data. A method extensively used to combine heterogeneous gravity data is least squares collocation (LSC) (Moritz, 1980). However, LSC is notoriously costly in central processing unit (CPU) execution time. To combine M data points, a matrix of size M x M must be inverted: "inversions of this size obviously present time problems even on a supercomputer and results will suffer from round-off errors" (Schwarz et al., 1990, p. 507).

Other procedures to transform gravity anomalies to geoid heights are based directly upon the classical Stokes integral (Heiskanen and Moritz, 1967). These include direct numerical integration of the anomalies; ring integration methods (Kearsley, 1985); Fourier-domain approximations of the Stokes integral, a number of different techniques being summarised in Tziavos (1996); and a method by Sideris (1995) to construct a geoid from irregularly spaced gravity data using Fourier techniques.

The method of combination presented here makes use of the wavenumber-domain relationship between geoid height and free-air gravity, described below. This approach enables the use of the fast Fourier transform (FFT), which not only speeds up operations, but can be implemented on a computer with a relatively small memory.

The FFT, however, requires the data to exist on a complete and regular grid, necessitating the gridding of satellite and ship-point observations. This paper describes the combination of such satellite-geoid and ship free-air grids, using the FFT.

ANALYSIS OF DATA

The Fourier method

Taking the Fourier transform of the fundamental equation of physical geodesy, the geodetic boundary condition (Heiskanen and Moritz, 1967) in the flat-Earth approximation, yields a simple relation between geoid height (AO and free-air anomaly (Ag):

$$F\{\Delta g\} = \gamma \; k \; F\{N\} \tag{1}$$

where $F\{A\}$ indicates the 2-D forward Fourier transform of the function A, y is the normal gravity field, and k is the 2-D circular wavenumber

$$k = {}^{2\pi}\!/_{\lambda} \tag{2}$$

with A as wavelength.

The flat-Earth approximation of Laplace's equation is valid for areas of side length less than approximately 2000 km (Strang van Hees, 1990), and this is especially true if the data are manipulated on a map projection grid, with minimal coordinate scale distortion throughout the area of study. An example of such a projection is the Lambert conical conformai (Richardus and Adler, 1972). To quote from Dorman and Lewis (1970, p. 3364): "The difference between distances measured on the spherical Earth and those measured on a Lambert conical conformai map are slight; hence we expect little distortion due to using the plane approximation in transforming into the frequency domain".

Combining the altimeter and ship data

The algorithm combining heterogeneous gravity datasets makes use of equation (1) to carry out a rapid conversion between geoid height and free-air anomaly. The algorithm presented here has been given the name IFC, for Iterative Fourier Combination.

The IFC routine requires not only a grid of gravity values, but also a grid of weights for the dataset. This grid should reflect the relative influence which the corresponding grid node on the gravity grid has in the dataset combination. This depends upon the distance of the node from the actual observation points. The weighting grids can be generated by any means, but they should be smooth and continuous to reduce the ringing effect of the Gibbs phenomenon during wavenumber-domain operations.

In the combination procedure, a provisional gravity field model is created which is improved upon in successive iterations of a weighted superposition with grids of geoid or free-air anomaly measurements.

1. The starting grid is the geoid height generated solely from altimeter data. This is transformed to a free-air anomaly field using equation (1).

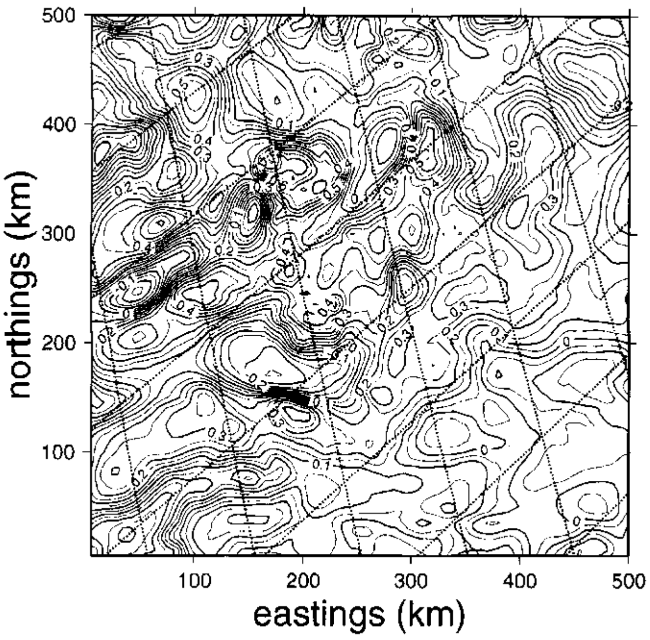

Figure 1. The test model geoid height showing simulated satellite ground tracks.

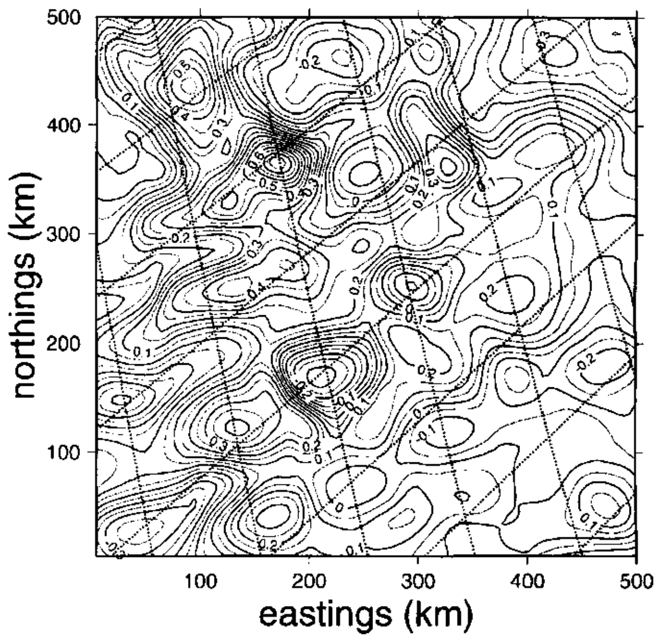

Figure 2. Geoid height from the simulated satellite tracks, showing location of simulated ground tracks.

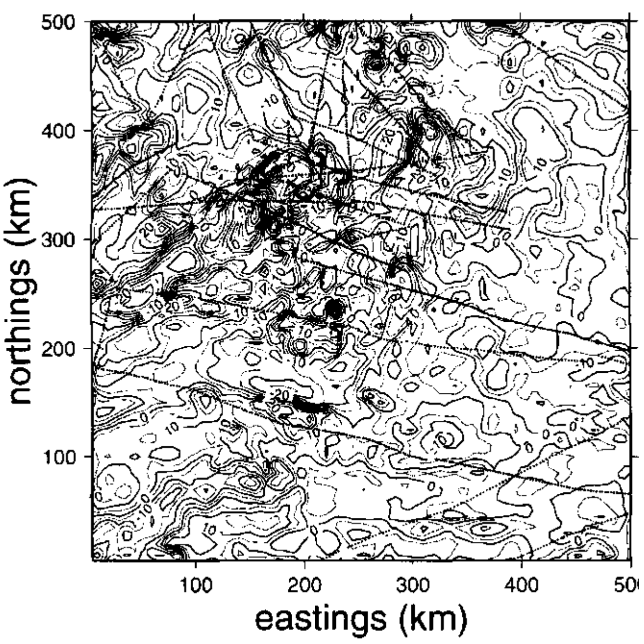

Figure 3. The test model free-air anomaly showing simulated ship tracks.

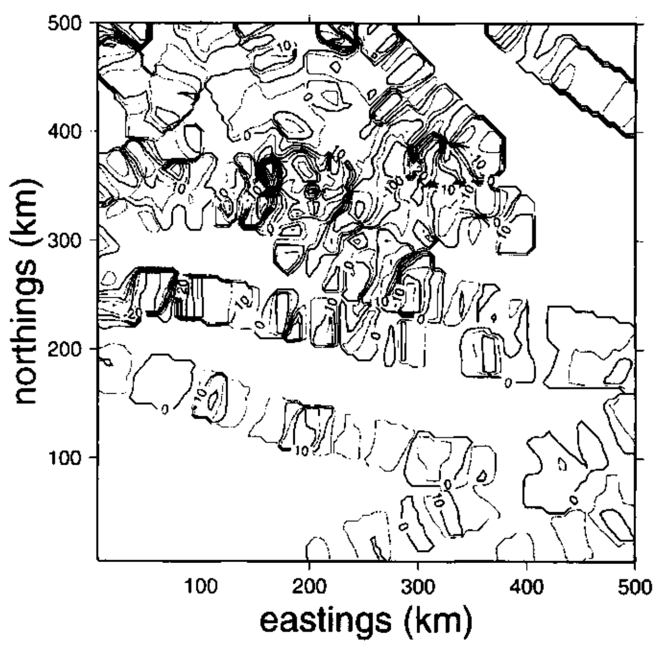

Figure 4. Free-air anomaly from the simulated ship tracks.

2. The grid of ship gravity measurements is added to this field, weighted by its weighting grid. Land gravity grids may also be included.
3. This provisional model is transformed to a grid of geoid heights using equation (1).
4. The new provisional model is combined with the satellite-derived geoid height grid.
5. This updated grid is transformed back to free-air anomaly by equation (1).
6. A weighted superposition is carried out with the original shipboard free-air gravity grid.

Steps (3) to (6) can be iterated any number of times until the provisional model stabilises. Stabilisation is considered to have been achieved when the RMS difference between successive 'like' provisional models (ie, geoid or free-air anomaly) reaches a previously specified value, indicating convergence.

The test dataset

A satellite geoid was created by gridding fictitious altimeter passes over the test area. These simulated passes were interpolated directly off the test model geoid along the tracks shown in Figure 1. The satellite geoid derived from these profiles is shown in Figure 2.

The gridded ship dataset was created using the gridding routine of Kirby and Hipkin (in prep.), and applied to fictitious ship track data derived from the free-air anomaly grid complementary to the test model geoid. The test model

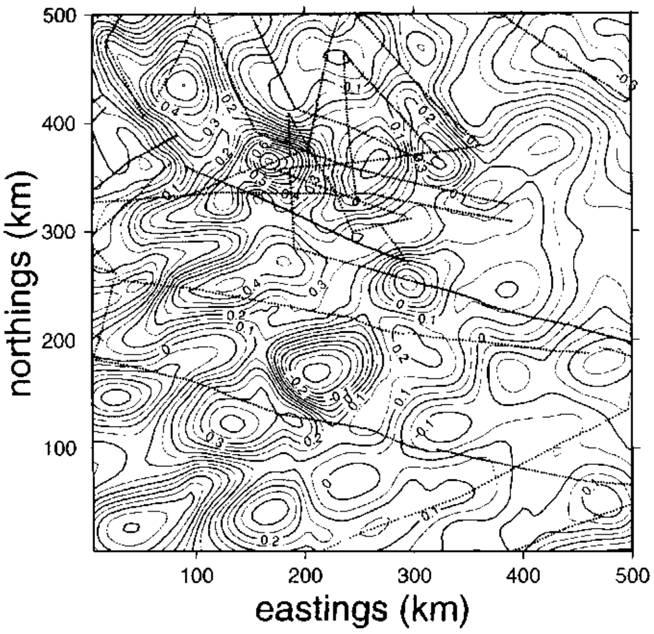

Figure 5. The geoid height from the simulated satellite and ship tracks, combined using the Iterative Fourier Combination algorithm.

free-air anomaly is shown in Figure 3, with the grid of ship data in Figure 4. As is evident, the routine has not attempted to interpolate data where no tracks exist.

As the test models in Figures 1 and 3 are accurately known everywhere, self-consistent, and fully interchangeable, the derived grids (Figures 2 and 4) are expected to contain accurate information about them, with the degree of recovery of the test model geoid (or free-air gravity) indicating the success of the presented algorithm.

RESULTS

The extent to which the inclusion of ship track data have improved the geoid model is evident from a comparison of the satellite geoid (Figure 2), the IFC geoid (Figure 5), and the test model geoid (Figure 1).

The satellite geoid contains the longer wavelength features of Figure 1, but not the smaller anomalies, as is expected from the distribution of the satellite tracks. Upon integration with the ship data, the shorter wavelength anomalies are resolved, especially in the area north of centre where the ship track distribution is most dense, where geoid recovery is greatest.

As expected, in areas with minimal ship data, the improvement has been slight but still noticeable.

CONCLUSION

A new method has been developed to combine grids of altimeter geoid heights and ship free-air anomalies using the fast Fourier transform.

This method has advantages over other methods in that the conversion between geoid height and free-air gravity is both straightforward and fast, enabling a rapid iteration of the combination procedure.

The Fourier combination also lends itself to integrating other types of gravity data, such as gravity gradients, or airborne data. This arises from simple wavenumber-domain relationships between the geoid height, gravity anomaly, and their spatial derivatives of any order.

The algorithm can be applied to both large and small areas, but is particularly suited to small-scale surveys where gravity measurements can be combined with one of the existing global satellite gravity databases over the oceans (eg, Sandwell et al., 1995).

It is suggested that, despite the global availability of high density altimeter datasets such as ERS-1 and Geosat/GM, they will never supersede the use of shipboard measurements of the gravity field. First, shipboard gravity meters can make point observations at very small track spacings, whereas altimeter measurements presently have an along-track resolution of at least 20 km, and a finite footprint diameter, over which the sea surface height is estimated.

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