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Comparison of GGMs based on one year GOCE observations with the EGM08 and terrestrial data over the area of Sudan

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ABSTRACT

Since 2010, a series of Global Geopotential Models (GGMs) based on Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite gravimetry mission have been released. In this study, the GGMs based on approximately 12 months of GOCE satellite gravity gradiometry (SGG) data have been compared over the area of Sudan with the EGM08 and terrestrial data. Geoid heights and free-air gravity anomalies from four GOCE/GRACE satellite-only GGMs, and one GOCE/GRACE GGM combined with terrestrial/altimetric gravity data were compared with the corresponding ones obtained from the EGM08, terrestrial free-air gravity anomalies and GNSS/levelling data.

The results reveal that geoid heights and free-air gravity anomalies obtained from the GOCE-based GGMs agree with the corresponding ones from the EGM08 truncated to d/o 200 with standard deviation of 18–20 cm, and 3.4–4.2 mGal, respectively. Their agreement with the terrestrial free-air gravity anomalies and the GNSS/levelling geoid heights, in terms of standard deviation is about 5.5 mGal, and about 50 cm, respectively.

1. Introduction

The GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) is the first European Space Agency (ESA) core explorer satellite mission that was launched on 17 March 2009, and it is expected to be in operation until the end of 2013. The main objectives of GOCE mission are to provide a high-accuracy, high-resolution global model of the Earth's static gravity field and thereby the geoid, i.e. gravity anomalies of 1 mGal accuracy and geoid of 1–2 cm accuracy, with spatial resolution of approximately 100 km half wavelength or degree and order (d/o) 200 of spherical harmonics (ESA, 1999; Drinkwater et al., 2003). To reach the mission objectives, GOCE satellite is equipped with on-board gravity gradiometer and GNSS receiver (ESA, 1999; Drinkwater et al., 2003). High-low satellite-to-satellite tracking (hl-SST) data obtained with the use of GNSS contributes mainly to the recovery of low frequency variations of the Earth's gravitational field while the medium- and high frequency variations are modelled with the use of satellite gravity gradiometer (SGG) data (e.g. Rummel, 2010).

Since June 2010, a number of GOCE-based Global Geopotential Models (GGMs) were developed. At present, four releases of GOCE-based GGMs have been made available for public by ESA; they are based on ∼2 months (1st release), ∼8 months (2nd release), ∼12 months (3rd release), and ∼27 months (4th release) of GOCE observation data. Three different approaches: direct approach, time-wise approach, and space-wise approach (in 1st and 2nd releases only) were implemented by ESA's High Level Processing Facility (HPF) to develop these models (e.g. Rummel et al., 2004; Pail et al., 2011). Besides the ESA's solutions, combined models based on the combination of GOCE data with the complementary gravity signal from other satellite and terrestrial data, such as the Gravity Observation Combination (GOCO) solutions (Pail et al., 2010), have also been developed with the use of similar GOCE observation data period. In addition, a few solutions such as the Delft Gravity Model (DGM) solution (Farahani et al., 2013), and the European Improved Gravity models of the Earth by New techniques (EIGEN) solution (Förste et al., 2011) were released recently by other teams.

In order to ensure the accuracy performance of GOCE-based GGMs for local gravity field modelling it is important to evaluate them with the use of independent gravity field functionals, e.g. ground truth data. Through the past years, several studies concerning the validation of GOCE-based GGMs have been performed; they were conducted for different parts of the world and by different research teams, e.g. Gruber et al. (2011) and Hirt et al. (2011) worldwide, Voigt et al. (2010) in Germany, Janák and Pitoňák (2011) in Central Europe, Godah and Krynski (2012) in Poland, Šprlák et al. (2012) in Norway, Guimarães et al. (2012) in Brazil, etc. The evaluation of GOCE-based GGMs over the area of Sudan is important

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for modelling the regional gravity field as well as for heighting using GNSS techniques. Abdalla et al. (2012) have recently validated the 1st and 2nd releases of GOCE/GRACE GGMs over a small part of Sudan (about 5% of its territory) using GNSS/levelling data, terrestrial free-air gravity anomalies and the recent geoid model. The standard deviations of differences between the geoid heights obtained from GOCE-based GGMs of 1st and 2nd releases and the corresponding ones obtained from geoid model and GNSS/levelling data are at the level of 36–49 cm. In the case of free-air gravity anomalies, the fit of truncated to d/o 210 GOCE-based GGMs of 1st and 2nd releases to the terrestrial gravity data is at the level of 11.4 mGal in terms of standard deviation of differences.

The aim of this contribution is to validate the GGMs based on approximately one year of GOCE data by comparing free-air gravity anomalies and geoid heights obtained from those models with the available terrestrial free-air gravity anomalies, GNSS/levelling data and the EGM08 (Pavlis et al., 2012) Global Geopotential Model over the whole area of Sudan ($3^\circ <$ latitude $< 24^\circ$; $21^\circ <$ longitude $< 39^\circ$). The latest release of GGMs based on ~27 months GOCE observation data became only available in the time of submission of the manuscript and thus has not been taken into consideration in the analysis.

2. Data used

The data used in this study consists of (1) the GOCE-based GGMs and the EGM08, and from the territory of Sudan, (2) terrestrial free-air gravity anomalies and (3) GNSS/levelling data.

2.1. Global Geopotential Models (GGMs)

Several GOCE-based GGMs have been developed over the past years. Major differences between them are the GOCE data that have been used for developing those models. In this study, four GGMs based on approximately 12 months of GOCE SGG observations data were used. These models were released by ESA and became available for public use via the International Centre for Global Earth Models (ICGEMs) http://icgem.gfzpotsdam.de/ICGEM/. The main characteristics of those models have been summarized in Table 1.

The DIR-R3 and TIM-R3 are official ESA's GOCE GGMs. The DIR-R3, TIM-R3 and GOCO-03s are the models of resolution up to d/o 240–250 developed with the use of Kaula regularization constraints over d/o 180–200. The TIM-R3 is distinguished as a GOCE-only solution in a rigorous sense; no external gravity field information was used, neither as reference model, nor for solution constrains (Pail et al., 2011). In the DIR-R3 solution GRACE GPS-SST&K-band range-rate data as well as LAGEOS data were used together with the GOCE data (Bruinsma et al., 2010). The GOCO-03s solution was based the same satellite-only data additionally augmented with CHAMP GPS-SST data (Mayer-Güerr et al., 2012). The EIGEN-06c2 (Förste et al., 2012) is a high resolution model up to d/o 1949, obtained as a combination of LAGEOS, GRACE and GOCE data, augmented with the DTU10 (Andersen, 2010) surface gravity data. It is worth to mention that the DTU10 global gravity anomaly grid has been obtained from altimetry data over the oceans and EGM08 data over the land.

The EGM08 Global Geopotential Model was used in this study as state-of-the art high resolution GGM for evaluating GOCE-based GGMs as well as for estimating higher gravity signals, e.g. gravity signals beyond the applied maximum resolution of GOCE-based GGMs. When developing the EGM08, over Africa, the values of $5' \times 5'$ mean gravity anomalies were synthesized using GGM02S spherical harmonic coefficients (Tapley et al., 2005) for degrees 2–60; coefficients for degrees 61–360 were augmented with those of EGM96 (Lemoine et al., 1998), and for further degrees, i.e. 361–2159 coefficients were augmented from the analysis of the residual terrain modelling (Pavlis et al., 2012). This model was extensively evaluated worldwide. Its fit to the African geoid measured with standard deviation of geoid height differences was evaluated as equal to 0.73 m (Merry, 2009).

The EGM08 has not been evaluated so far over the area of Sudan. An attempt of its evaluation with the use of available terrestrial gravity and GNSS/levelling data from the area of Sudan has been undertaken in this study (see Sections 5 and 6).

2.2. Free-air terrestrial gravity data

A regular $5' \times 5'$ grid of terrestrial free-air gravity anomalies covering the area of Sudan (Fig. 1a) was provided by the GETECH (Geophysical Exploration Technology), a division of the University of Leeds Industrial Services Ltd. This grid was developed by GETECH team using a simple interpolation of 28 156 point gravity data covering about 33% of the total area investigated (Fig. 1b), acquired in the framework of the African Gravity Project (Fairhead et al., 1988). Major error sources of free-air gravity anomalies determined are the station positions and heights. The stations were positioned in pre-GNSS days; some station heights were determined using barometric methods. For most surveys the error details are not available. Accuracy of terrestrial free-air gravity anomalies of the order of 1 mGal has been suggested by the provider of gravity data. It is expected, however, that the accuracy of the provided free-air gravity anomalies based on highly inhomogeneous data is substantially lower in majority of the country.

2.3. GNSS/levelling data

GNSS/levelling data at 19 stations distributed over the area of Sudan (Fig. 1b) have been used in this investigation. The heights of those stations have been determined by spirit levelling referred to 1st order, 2nd order and 3rd order vertical control which is defined in the normal orthometric height system based on normal gravity. They are referred in this study as orthometric heights. Ellipsoidal heights of those stations were obtained from GNSS survey conducted in 12 h observing sessions in the framework of several commercial geodetic projects between 2005 and 2008. The estimated accuracy of the GNSS/levelling geoid heights at these stations ranges from 0.1 to 0.5 m (for more details, see Abdalla, 2009).

3. Basics formulae

The Earth's gravitational potential V at any point on the Earth's surface and in the exterior space is commonly expressed using a series of fully normalized spherical harmonic coefficients C_{nm}. and S_{nm} (Torge and Müller, 2012, p. 71)

$$V_{(r,\varphi,\lambda)} = \frac{\mathrm{GM}}{r}\left(1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\varphi)\right) \quad (1)$$

where r, φ, λ are the geocentric radius, latitude and longitude of the computation point, respectively, GM is the geocentric gravitational constant, a is the semi-major axis of the reference ellipsoid, $\bar{P}_{nm}(\cos\varphi)$ is the fully normalized associated Legendre function, and n, m being degree and order of spherical harmonics, respectively.

Taking into account the boundary condition of the physical geodesy, the related quantities such as gravity anomalies Δg and geoid heights ζ were computed with the following formulae

Table 1
GOCE-based GGMs evaluated.

GGM	DIR-R3	TIM-R3	GOCO-03s	EIGEN-06c2
Name in ICGEM	GO_CONS_GCF_2_DIR_R3	GO_CONS_GCF_2_TIM_R3	GOCO-03s	EIGEN-06c2
Maximum d/o	240	250	250	1949
Semi-major axis *a* [m]	6378136.46	6378136.30	6378136.30	6378136.46
GOCE GPS-SST data	d/o 100 12 months	d/o 100 12 months	d/o 110 12 months	–
GOCE SGG data	d/o 240 12 months	d/o 250 12 months	d/o 250 12 months	d/o 240 12 months
GRACE K-band ranging	d/o 160 7 years	–	d/o 180 7.5 years	d/o 130 7.8 years
GRACE GPS-SST data	d/o 160 7 years	–	d/o 180 7.5 years	d/o 130 6.5 years
CHAMP GPS-SST data	–		d/o 120 5 years	–
LAGEOS-1/2 SLR data	d/o 3 7 years	–	d/o 5 5 years	d/o 30 25 years
Kaula regularization constraints beyond d/o	200	180	180	–
Terrestrial/altimetric gravity data	–	–	–	d/o 1949 DTU10
Time of releasing	November 2011	November 2011	July 2012	December 2012

(Torge and Müller, 2012, p. 272):

$$\Delta g_{(r,\varphi,\lambda)} = \frac{GM}{r^2}\sum_{n=2}^{N_{\max}}(n-1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta C_{nm}\cos m\lambda + \Delta S_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\varphi) \quad (2)$$

$$\zeta_{(r,\varphi,\lambda)} = \frac{GM}{r\gamma}\sum_{n=2}^{N_{\max}}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta C_{nm}\cos m\lambda + \Delta S_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\varphi) \quad (3)$$

where γ is the normal gravity at point (r,φ,λ), $N_{\max}$ is the maximum degree of geopotential model applied, ΔC_{nm} are differences between fully normalized spherical harmonic coefficients of actual and the normal gravity field and $\Delta S_{nm} = S_{nm}$.

4. Comparison of GOCE-based GGMs with the EGM08

With the use of Eqs. (2) and (3), free-air gravity anomalies and geoid heights from the GOCE-based GGMs (Table 1) and from the EGM08 were calculated over the area of Sudan. Since the significant contribution of GOCE to the development of GGMs is expected in the spectral band from d/o 100 to 250, these functionals were calculated from GGMs truncated to $N_{\max}$ = 100, 110, 120,. . ., 250 (with d/o 10 step), on the grids from $108' \times 108'$ to $43.2' \times 43.2'$, respectively. Figs. 2 and 3 show the standard deviations $\sigma_{\delta\Delta g}$ and $\sigma_{\delta\zeta}$ of the differences between corresponding geoid heights and corresponding free-air gravity anomalies, respectively, obtained from GOCE-based GGMs and the EGM08. Moreover, the statistics of those differences for the models truncated to d/o 180 and 200 are given in Tables 2 and 3.

Figs. 2 and 3 exhibit quite similar results up to d/o ~175 for all GOCE-based GGMs investigated. From d/o 180 onward, the

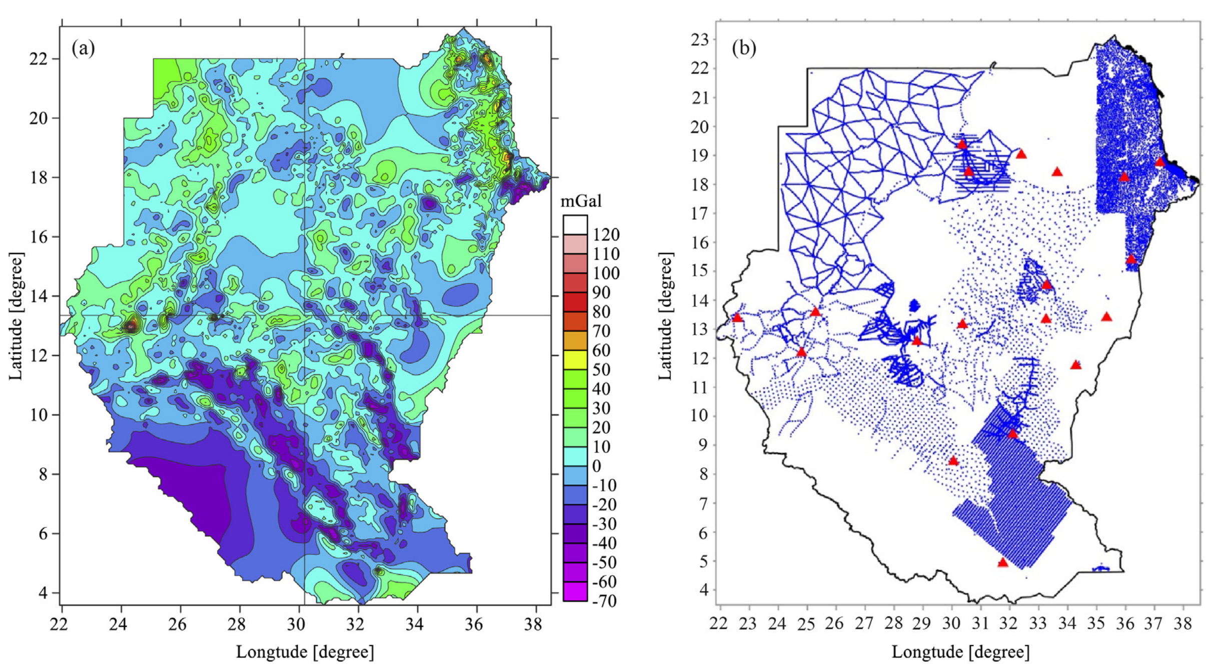

Fig. 1. (a) Terrestrial free-air gravity anomalies covering the area of Sudan provided by the GETECH on a regular $5' \times 5'$ grid, and (b) distribution of original GETECH point gravity data (blue dots), and GNSS/levelling stations (red triangles) in the tested area. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

Table 2
Statistics of the differences between geoid heights $\delta\zeta$ obtained from GOCE-based GGMs and the corresponding ones calculated from the EGM08 [m].

Model	$\delta\zeta$ (N_{max} = 180)				$\delta\zeta$ (N_{max} = 200)			
	Min	Max	Mean	Std.	Min	Max	Mean	Std.
DIR-3R	−0.855	0.776	0.003	0.189	−0.960	0.956	0.002	0.201
TIM-3R	−0.884	0.796	0.004	0.188	−0.961	0.948	0.004	0.199
GOCO-3R	−0.877	0.822	0.002	0.187	−0.935	0.956	0.003	0.197
EIGEN-06c2	−0.933	0.864	0.002	0.183	−0.851	0.818	0.002	0.184

Table 3
Statistics of the differences between free-air gravity anomalies $\delta\Delta g$ obtained from GOCE-based GGMs and the corresponding ones calculated from the EGM08 [mGal].

Model	$\delta\Delta g$ (N_{max} = 180)				$\delta\Delta g$ (N_{max} = 200)			
	Min	Max	Mean	Std.	Min	Max	Mean	Std.
DIR-3R	−18.83	15.22	0.030	3.73	−20.22	20.61	0.00	4.23
TIM-3R	−18.44	15.44	0.029	3.71	−20.17	20.78	0.02	4.13
GOCO-3R	−18.67	14.92	0.028	3.67	−19.60	20.76	0.03	4.08
EIGEN-06c2	−16.52	13.77	0.023	3.44	−17.24	16.75	0.02	3.46

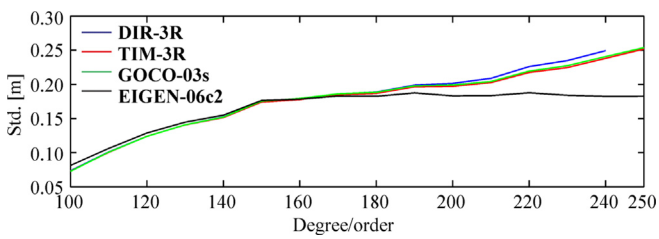

Fig. 2. Standard deviation $\sigma_{\delta\zeta}$ of the differences between geoid heights from GOCE-based GGMs and the respective ones from the truncated EGM08.

contribution of terrestrial/altimetry data such as in EIGEN-06c2 solution is clearly visible. Moreover, due to the contribution of EGM08 data to higher order harmonics of the EIGEN-06c2 model, the difference between the solutions in terms of the standard deviation becomes constant beyond d/o 180 over the area of Sudan (flattened curves for EIGEN-06c2 solution in Figs. 2 and 3). From d/o 200 onward, the standard deviations have increased rapidly for all the GOCE-based GGMs solutions. This might be since the coefficients beyond d/o 180–200 were estimated with the use of Kaula's rule (Bruinsma et al., 2010; Pail et al., 2011; Mayer-Güerr et al., 2012), and also because above d/o 200 noise starts to dominate signals (Rummel, 2010).

The statistics presented in Tables 2 and 3 are consistent for all truncated to d/o 180 GOCE-based GGMs investigated; the standard deviations of the differences for geoid heights and free-air gravity anomalies range from 18.3 to 18.9 cm, and from 3.4 to 3.7 mGal, respectively. The consistency of the statistics for truncated to d/o 200 GGMs concerns satellite-only models; the standard deviations of the differences for geoid heights and free-air gravity anomalies are at the level of 20 cm and 4 mGal, respectively. The combined model (EIGEN-06c2) exhibits uncertainty reduced by more than 1.3 cm and 0.6 mGal for geoid heights and free-air gravity anomalies, respectively, with regard to other GGMs investigated.

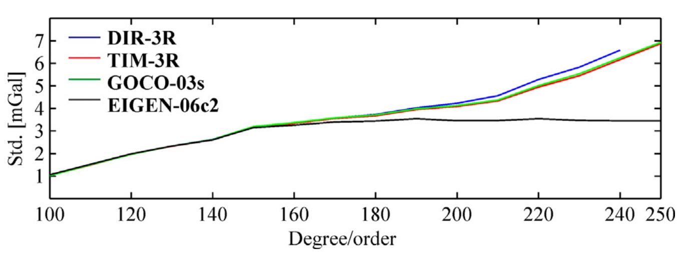

Fig. 3. Standard deviation $\sigma_{\delta\Delta g}$ of the differences between free-air gravity anomalies from GOCE-based GGMs and the respective ones from the truncated EGM08.

5. Comparison of GOCE-based GGMs with terrestrial gravity data

Theoretically, the free-air gravity anomalies obtained from terrestrial measurements contain the full spectral information of the gravity field. On the other hand, the GGMs are represented by finite spherical harmonic expansion. Thus, in order to evaluate GGMs with the use of terrestrial free-air gravity anomalies, the spectral inconsistency between them must be considered. Numerous evaluation procedures were developed, e.g. spectral enhancement method (Hirt et al., 2011), Gauss' filter (Voigt et al., 2010) and a method by means of orbit residuals and geoid comparisons (Gruber et al., 2011).

Two approaches to remove spectral inconsistency in the data being compared have been implemented in this study. To make the results of comparison more reliable, all GGMs used were truncated to d/o 200. Furthermore, to remove the effect of large gaps and inhomogeneous distribution of the available terrestrial gravity data over the test area, the evaluation was performed only at the locations of gravity stations (see Fig. 1b). Unfortunately, point gravity data were not available. The terrestrial free-air gravity anomalies Δg at the location of gravity stations (Fig. 1b) were thus interpolated from $5' \times 5'$ grid provided by GETECH.

In the first approach, free-air gravity anomalies Δg_{GGM} were calculated from the spherical harmonic coefficients up to d/o 200 from GOCE-based GGMs, while the remained gravity signals in the spectral range from 201 up to 2190 Δg_h were compensated from the EGM08. The differences $\delta\Delta g$ between terrestrial free-air gravity anomalies Δg and the corresponding ones Δg_{GGM} obtained from GGMs investigated were calculated as follows

$$\delta\Delta g = \Delta g - [\Delta g_{GGM} + \Delta g_h] \tag{4}$$

In the second approach, a simple low pass filter based on reciprocal distances from the computation point was applied to generate a grid of terrestrial free-air gravity anomalies, resolution of which corresponds to the one of the investigated GGMs. The weighting function W of this low pass filter is expressed as follows

$$W = \begin{cases} W_c & \text{if } \Delta x_i = 0 \text{ and } \Delta y_i = 0 \\ \left(\dfrac{1}{\Delta x_i^2 + \Delta y_i^2} \right)^{p/2} & \text{otherwise} \end{cases} \tag{5}$$

Table 4
Statistics of the differences between the terrestrial and computed from GOCE-based GGMs free-air gravity anomalies [mGal].

Statistics	≏Δg (1st approach)				≏Δg_{LPF} (2nd approach)			
	Min	Max	Mean	Std.	Min	Max	Mean	Std.
EGM08	−50.29	72.46	1.22	5.62	−23.60	31.86	1.69	5.56
DIR3	−50.56	70.64	0.81	5.61	−19.35	38.10	1.28	4.93
TIM3	−50.79	71.50	0.84	5.61	−19.45	38.66	1.31	4.90
GOCO3	−50.69	71.13	0.84	5.61	−20.05	38.80	1.32	4.94
EIGEN06c2	−51.57	70.80	0.72	5.41	−15.74	36.94	1.18	5.00

where Δx_i, Δy_i are the components of the distance from the computation point in x, and y directions, respectively. In this study, the central weight W_c and the power p were both fixed to 1; the maximum distance from the computation point was fixed to 100 km (corresponds to d/o ∼200 resolution). Fig. 4 illustrates the applied low pass filter.

The terrestrial free-air gravity anomalies in $5' \times 5'$ grid, specified in Section 2.2 (see Fig. 1a), have been filtered using the above filter. The higher frequency signals, i.e. from d/o 201 to 2190, were removed in the filtering process from the terrestrial data. The resulting terrestrial gravity signals after applying the low pass filter are presented in Fig. 5.

The terrestrial free-air gravity anomalies Δg_{LPF} at the location of gravity stations (Fig. 1b) were interpolated from the filtered grid shown in Fig. 5. The differences Δg_{LPF} between filtered terrestrial free-air gravity anomalies Δg_{LPF} and the corresponding ones Δg_{GGM} computed from GOCE-based GGMs were obtained as follows

$$\delta \Delta g_{LPF} = \Delta g_{LPF} - \Delta g_{GOCE} \tag{6}$$

Table 4 gives a summary of the statistics of 28 156 differences between terrestrial free-air gravity anomalies and the corresponding ones obtained from GOCE-based GGMs, calculated using the described above two approaches. As an example, Fig. 6 illustrates the distribution of the differences between the terrestrial free-air gravity anomalies and the corresponding ones obtained from DIR-R3.

The results presented in Table 4 show that the standard deviations of the differences between free-air gravity anomalies computed from GGMs and corresponding terrestrial gravity data are within the range of 4.9–5.6 mGal. It should be pointed out that the fit of all GOCE-based GGMs and EGM08 to the terrestrial gravity anomalies is similar when using 1st approach while in case of using 2nd approach (LPF) GOCE-based GGMs exhibit improvement with respect to the EGM08 by about 0.5 mGal in terms of the standard deviation. The use of low pass filter approach results in about 10% reduction in standard deviation of the differences for all investigated GOCE-based GGMs. In addition, it reduces significantly (by a factor >2) the dispersion of the differences for all investigated GGMs (Table 4, and Fig. 6 for DIR-R3). This may indicate that, the EGM08 is not accurate enough for modelling higher frequency gravity signals (e.g. d/o 201–2190) in the test area, since it is based on fill-in observations for that area. The means of the differences (Table 4) have slightly increased (by about 0.45 mGal) when applying low pass filter approach.

6. Comparison of GOCE-based GGMs and EGM08 with GNSS/levelling data

Comparison of geoid heights computed from GGMs with the corresponding ones obtained from the combination of ellipsoidal heights derived from GNSS data and orthometric heights determined from spirit levelling gives reasonable evaluation of the GGMs. In this study, the geoid heights obtained from GOCE-based GGMs and the EGM08 have been evaluated with the use of GNSS/levelling data at 19 stations distributed over the area of Sudan (see Fig. 1). First, the geoid models were developed over the area of Sudan at a grid of $54' \times 54'$ (i.e. approximately 100 km spatial resolution) using the investigated GGMs truncated to d/o 200, and afterwards these models were evaluated at the GNSS/levelling sites. The statistics of the differences $\Delta\zeta$ between geoid heights calculated from the GGMs and the corresponding ones from the GNSS/levelling data are given in Table 5.

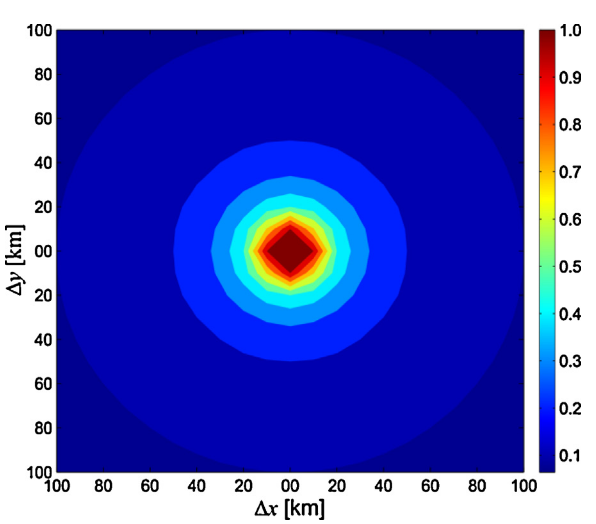

Fig. 4. The applied low pass filter.

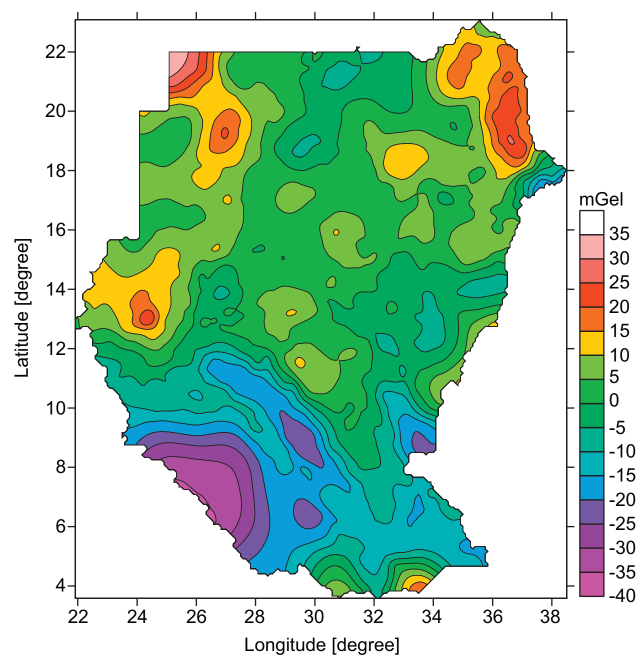

Fig. 5. The terrestrial gravity signals after applying the low pass filter.

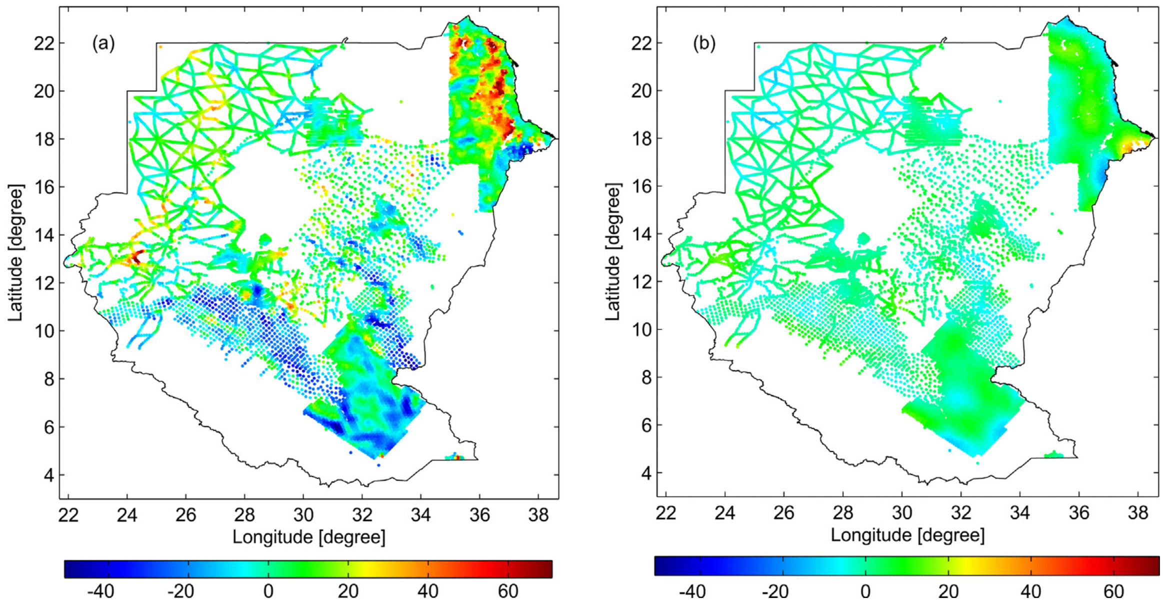

Fig. 6. Distribution of the differences between the terrestrial and computed from DIR-R3 free-air gravity anomalies (a) $\delta\Delta g$ (1st approach applied) and (b) $\delta\Delta g_{\text{LPF}}$ (low pass filter applied) [mGal].

Table 5
Statistics of the differences $\Delta\zeta$ between geoid heights calculated from the GGMs and the corresponding ones from the GNSS/levelling data at 19 stations [m].

GGM ($N_{\max}=200$)	Before fitting			
	Min	Max	Mean	Std.
DIR-3R	−0.348	1.874	0.797	0.722
TIM-3R	−0.340	1.871	0.798	0.711
GOCO-3R	−0.351	1.875	0.804	0.714
EIGEN-06c2	−0.397	1.889	0.805	0.720
EGM08	−0.456	2.004	0.804	0.730

The mean values of geoid height differences presented in Table 5 are at the level of 80 cm and they are close to each other within 1 cm. Such values are conventionally interpreted as a bias due to datum inconsistences when combining different vertical networks. Large dispersion of geoid height differences observed is additionally caused by the omission error in GNSS/levelling data, which has not been considered in this evaluation due to very limited number of available GNSS/levelling sites distributed over large area of Sudan, and insufficient accuracy of high resolution GGMs (e.g. EGM08) in the study area. However, in order to minimize this bias and to determine better fit of geoid models obtained from GGMs to GNSS/levelling data, 4-, 5-, and 7-parameter transformation models of corrector surface have been applied:

4-parameter transformation model (Heiskanen and Moritz, 1967)

$$\Delta\zeta = \cos\varphi_i \cos\lambda_i \cdot x_1 + \cos\varphi_i \sin\lambda_i \cdot x_2 + \sin\varphi_i \cdot x_3 + x_4 + v_i \tag{7}$$

5-parameter transformation model (Kotsakis and Sideris, 1999)

$$\Delta\zeta = \cos\varphi_i \cos\lambda_i \cdot x_1 + \cos\varphi_i \sin\lambda_i \cdot x_2 + \sin\varphi_i \cdot x_3 + \sin^2\varphi_i \cdot x_4 + x_5 + v_i \tag{8}$$

7-parameter transformation model (e.g. Fotopoulos, 2003)

$$\Delta\zeta = \cos\varphi_i \cos\lambda_i \cdot x_1 + \cos\varphi_i \sin\lambda_i \cdot x_2 + \sin\varphi_i \cdot x_3 + \frac{\cos\varphi_i \sin\varphi_i \cos\lambda_i}{w_i} \cdot x_4 + \frac{\cos\varphi_i \sin\varphi_i \sin\lambda_i}{w_i} \cdot x_5 + \frac{\sin^2\varphi_i}{w_i} \cdot x_6 + x_7 + v_i \tag{9}$$

where $x_{i=1,2,3,\ldots,7}$ are the transformation parameters between two datums, v_i is the residual of random noise term, (φ_i, λ_i) are the geodetic coordinates and $w_i = \sqrt{1 - e^2 \sin^2\varphi_i}$ where e is the ellipsoid eccentricity.

The statistics of the differences between geoid heights obtained from GGMs and GNSS/levelling after removing biases using Eqs. (7)–(9) are presented in Table 6.

The 4-parameter transformation model belongs to the family of models based on the general similarity datum shift transformation. Such transformation applied to a geoid model removes datum shift and provides the corrector surface that allows to evaluate actual discrepancy between the transformed data set and the ground truth, i.e. in this case geoid heights from GNSS/levelling

Table 6
Statistics of the differences between geoid heights obtained from GGMs investigated and the corresponding ones from GNSS/levelling data after applying the 4-, 5- and 7-parameter transformation [m].

GGM ($N_{\max}=200$)	4-Parameters model				5-Parameters model				7-Parameters model			
	Min	Max	Mean	Std.	Min	Max	Mean	Std.	Min	Max	Mean	Std.
DIR-3R	−0.888	1.256	0.000	0.644	−1.119	1.065	0.000	0.608	−0.886	1.089	0.000	0.508
TIM-3R	−0.885	1.226	0.000	0.639	−1.107	1.038	0.000	0.603	−0.873	1.089	0.000	0.506
GOCO-3R	−0.897	1.237	0.000	0.638	−1.100	1.049	0.000	0.602	−0.886	1.068	0.000	0.503
EIGEN-06c2	−0.935	1.208	0.000	0.636	−1.086	1.032	0.000	0.604	−0.920	1.084	0.000	0.507
EGM08	−0.964	1.190	0.000	0.639	−1.038	1.020	0.000	0.610	−0.930	1.097	0.000	0.495

referred to the national vertical datum. The results obtained for all GGMs investigated practically do not differ. They suggest the standard deviation of vertical datum in Sudan at the level of 64 cm, however, this estimate might not be fully representative due to very limited number of GNSS/levelling heights available. It should also be mentioned that the uniformity of vertical datum with orthometric heights to which the heights of all 19 GNSS/levelling sites are referred, is questionable. The other transformation models considered, i.e. 5-parameter transformation model and 7-parameter transformation model, are not any longer similarity transformations. They result in changing the original shape of corrector surface (the one spanned on geoid heights) in the process of transformation providing better fit to the ground truth. Such deformed corrector surface model is more suitable for being used for heighting with GNSS technique, since it allows to provide heights in the national vertical datum.

7. Summary and conclusions

This paper describes the validation of GOCE-based GGMs developed with the use of approximately 12 months of GOCE satellite gravity gradiometer (SGG) observations data, over the area of Sudan. Three different comparisons have been employed to evaluate GOCE-based GGMs.

First, the geoid heights and free-air gravity anomalies obtained from GOCE-based GGMs were compared with the corresponding ones calculated from the EGM08. All satellite-only GOCE-based GGMs investigated provide similar results within the spectral band from d/o 100 to 240/250, while combined model EIGEN-06c2 gives better fit to the EGM08 beyond d/o 175. This is because high frequency gravity signal of the EGM08 was used in developing that model. At spatial resolution of ~100 km ($N_{\max}$ = 200), the deviation of the geoid heights and the free-air gravity anomalies obtained from GOCE-based GGMs and the corresponding ones from the EGM08 are within the range of 18.4–20.1 cm and 3.4–4.2 mGal, respectively.

Second, the free-air gravity anomalies obtained from GGMs were compared with the corresponding ones from terrestrial database using two different approaches. In the 1st approach, the EGM08 was used for the recovery of high frequency gravity signals (from d/o 201 to 2190) in Δg from the GGMs, and in 2nd approach, low pass filtering was implemented to remove those high frequency signals from terrestrial free-air gravity anomalies. It has been shown that the use of low pass filtering over the test area provides better results for all investigated GOCE-based GGMs than the use of the 1st approach. Significant reduction (by a factor >2) of the dispersion of the differences has been observed for all investigated GGMs, including EGM08, when using low pass filtering approach. In addition, the use of gravity anomalies for the evaluation of the GGMs investigated indicates slightly better fit of GOCE-based GGMs to the terrestrial data than the EGM08 over the test area. The improvement is about 0.5 mGal in terms of standard deviation of differences which corresponds to 10% reduction. This, first of all results from poor coverage of gravity data in the area of Sudan and neighbouring countries applied for developing the EGM08. On the other hand it is due to the higher quality of gravity signals provided by GOCE-based GGMs. The free-air gravity anomalies obtained from GOCE-based GGMs agree with the corresponding terrestrial ones within the range from 4.9 to 5.6 mGal. Despite different methodologies and area size used for the evaluation of GOCE-based GGMs in Sudan in the previous study (cf. Abdalla et al., 2012) and in this one, the comparison of GGMs based on one year GOCE observations with terrestrial gravity anomalies has indicated substantial improvement (by 5.8–6.5 mGal) compared with 1st and 2nd releases of GOCE-based GGMs. It should be mentioned that the obtained results are also subject to the quality of the terrestrial data. The recent satellite-based GGMs may be used to validate terrestrial gravity data, in particular if it suffers from serious sampling and accuracy deficiencies.

Finally, the geoid heights obtained from EGM08 and GOCE-based GGMs were compared with GNSS/levelling geoid heights at 19 GNSS/levelling sites. As it was in case of using gravity anomalies, GOCE-based GGMs performed slightly better than the EGM08. The results obtained for all GGMs investigated suggest that the standard deviation of vertical datum in Sudan is at the level of 64 cm, however, this estimate might not be fully representative due to very limited number and inhomogeneous distribution of GNSS/levelling heights used. It should also be mentioned that the uniformity of vertical datum of orthometric heights of all 19 GNSS/levelling sites is questionable. Although the obtained results do not match the now a day's geoid heights accuracy worldwide, the use of geoid models computed from GOCE-based GGMs could be recommended for GNSS heighting in Sudan.

Discrete corrector surface models developed applying 7-parameter transformation model, exhibiting the fit of geoid heights obtained from GOCE-based GGMs and from the EGM08 to those from GNSS/levelling data within the range from 49.5 to 50.8 cm are more suitable for being used for heighting with GNSS technique; they provide heights in national vertical datum. Unfortunately, the low number, accuracy and inhomogeneous distribution of the available GNSS/levelling data do not allow a more reliable evaluation of GOCE-based GGMs. Future research will be required to investigate the better fit of GOCE-based GGMs to the observed GNSS/levelling geoid heights.

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