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The GeoForschungsZentrum Potsdam / Groupe de Recherche de Géodésie Spatiale satellite-only and combined gravity field models: EIGEN-GL04S1 and EIGEN-GL04C

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Abstract The recent improvements in the Gravity Recovery And Climate Experiment (GRACE) tracking processing data GeoForschungsZentrum Potsdam (GFZ) and Groupe de Recherche de Géodésie Spatiale (GRGS) Toulouse, the availability of newer surface gravity data sets in the Arctic, Antarctica and North-America, and the availability of a new mean sea surface height model from altimetry processing at GFZ gave rise to the generation of two new global gravity field models. The first, EIGEN-GL04S1, a satellite-only model complete to degree and order 150 in terms of spherical harmonics, was derived by combination of the latest GFZ Potsdam GRACE-only and Toulouse (EIGEN-GRACE04S) GRGS GRACE/LAGEOS (EIGEN-GL04S) mean field solutions. The second, EIGEN-GL04S1 combined with surface gravity data from altimetry over the oceans and gravimetry over the continents to derive a new high-resolution global gravity field model called EIGEN-GL04C. This model is complete to degree and order 360 and thus resolves geoid and gravity anomalies at half-wavelengths of 55 km at the equator. A degree-dependent combination method has been applied in order to preserve the high accuracy from the GRACE satellite data in the lower frequency band of the geopotential and to form a smooth transition to the high-frequency information coming from the surface data. Compared to pre-CHAMP global high-resolution models, the accuracy was improved at a spatial resolution of 200 km (half-wavelength) by one order of magnitude to 3 cm in terms of geoid heights. The accuracy of this model (i.e. the commission error) at its full spatial resolution is estimated to be 15 cm. The model shows a reduced artificial meridional striping and an increased correlation of EIGEN-GL04C-derived geostrophic meridional currents with World Ocean Atlas 2001 (WOA01) data. These improvements have led to select EIGEN-GL04C for JASON-1

Keywords: Earth gravity field model, global gravity field recovery, GRACE, LAGEOS, surface gravity data

satellite altimeter data reprocessing.

1 Introduction

High-resolution global gravity field models can be inferred from satellite tracking measurements and surface gravity data (e.g. Lemoine et al. 1998 or Gruber et al. 2000a). In the past, data from a large number of satellites, at different altitudes and inclinations, had to be processed in order to generate the so-called satellite-only solutions (e.g. Reigber et al. 2005), but the twin-satellite mission Gravity Recovery and Climate Experiment (GRACE; Tapley et al. 2004) with it's objective to map the global gravity field of the Earth on a monthly basis made a compilation from satellite ensembles superfluous.

The GRACE satellites, jointly managed by National Aeronautics and Space Administration (NASA) and Deutsches Zentrum für Luft- und Raumfahrt (DLR), were launched on March 17, 2002 in a near-circular orbit at about 500 km altitude. They are separated from each other by approximately 220 km along-track, and this distance and its rate of change are measured using a K-band microwave ranging system. Furthermore, the science payload of each satellite consists of a Global Positioning System (GPS) receiver, laser retro-reflector, star sensors, and a high precision three-axis accelerometer. Gravity field models derived from the GRACE mission and the precursory CHAMP mission are much more accurate than any other precursor satellite-only model derived from dozens of spacecraft and analyzing tracking data from more than 10 years (e.g. Reigber et al. 2003b), except for the very-low degrees, especially for C₂₀ (e.g. Eanes et al. 2005; Biancale et al. 2004). Thus, additional LAGEOS data should be used in order to estimate C_{20} accurately.

Since the launch of GRACE, a number of satellite-only gravity field models from different Analysis Centers have become available. They all claim to resolve the geoid with an accuracy of 1 cm for half-wavelengths down to approximately 270 km (e.g. EIGEN-GRACE02S, Reigber et al. 2005, GGM02S, Tapley et al. 2005, ITG-GRACE02s, Mayer-Gürr et al. 2006). This is an accuracy improvement of more than two orders of magnitude compared to the latest pre-CHAMP satellite-only model GRIM5-S1 (Biancale et al. 2000). The most recent European Improved Gravity model of the Earth by New techniques (EIGEN) models generated at the GeoForschungsZentrum Potsdam (GFZ) and the Groupe de Recherche de Géodésie Spatiale Toulouse (GRGS) are EIGEN-GRACE04S (an update of EIGEN-GRACE02S; Schmidt et al. 2007) and EIGEN-GL04S (Lemoine et al. 2007). The EIGEN as well as the preceding GRIM (GRGS and German geodetic research Institute Munich) gravity field solutions were produced jointly by GFZ Potsdam (resp. the earlier GRIM-models by DGFI Munich = Deutsches Geodätisches Forschungsinstitut) and GRGS Toulouse. Both groups operate equivalent and harmonized data reduction software packages, called EPOS (Earth Parameter and Orbit System, e.g. Schmidt 2007), and GINS (Géodésie par Intégrations Numériques Simultanées, Schwintzer et al. 1991), allowing a shared data processing at the level of normal equations. The dynamic approach based on the analysis of orbit

perturbations (e.g. Reigber 1989) is used in the processing of GRACE and LAGEOS observations. In this way, the long- to medium-wavelength features of the Earth's gravity field are derived from GRACE satellite data, whereasthe shorter wavelengths must be inferred from surface gravity data. These data, compiled from satellite altimetry, ship- and airborne gravimetry over the oceans, and airborne and terrestrial gravimetry over land, provide – except for Antarctica – an almost complete global coverage if condensed to block mean values of a regular equalangular 30'x 30' grid. Due to inconsistencies between the various data sets (e.g. in the vertical datum, see Heck 1990), and accuracies varying regionally, the surface data do not contain precise long- to medium-wavelength gravity information. However, if properly combined with satellite-only gravity field models (on the basis of normal equations), the resolution of the global model can be extended down to 55 km half-wavelength. Such a combination, based on the pre-CHAMP satellite-only model EGM96S, resulted in the broadly used model EGM96 (Lemoine et al. 1998).

In this paper, a new GRACE- and LAGEOS-based satellite-only model (EIGEN-GL04S1), and a new high-resolution combination gravity field model (EIGEN-GL04C) are presented. The latter model, when compared to EGM96 and GGM02C (Tapley et al. 2005), benefits in its long- to medium-wavelength part from the unprecedented performance and improved processing of the GRACE data, whereas at higher frequencies it is slightly more accurate thanks to the assimilation of a more complete and updated surface data compilation.

Section 2 depicts the processing of the satellite tracking data and the satellite-only model, whereas Sect. 3 describes the preparation and processing of the surface gravity data. The combination solution strategy and the resulting EIGEN-GL04C model are described in Sect. 4, followed by a Section concerning the model evaluation.

2 EIGEN-GL04S1: a satellite-only model derived from GRACE and LAGEOS tracking data

The satellite contribution to the gravity field combination model consists of a GRACE data processing at GFZ Potsdam and a GRACE/LAGEOS data processing at CNES/GRGS Toulouse.

At GFZ Potsdam, 30 months of GRACE Level 1B instrument data, covering the period February 2003 until July 2005, have been processed using the classical orbit perturbation analysis by a "two-step method" (e.g. Reigher et al. 2002, 2003a): (1)

adjustment of the orbit and clock parameters of the high-flying GPS spacecraft from ground-based GPS tracking data and (2) GRACE orbit determination and computation of observation equations for the GPS code and carrier phase measurements and for the K-band range-rate (KBRR) observations with fixed GPS spacecraft positions and clocks from step 1.

Monthly normal equation matrices have been computed from these observation equations, containing the gravitational spherical harmonic coefficients complete up to degree and order 150. Finally, these monthly normal matrices have been accumulated to one multi-month normal equation system. The inversion of this multi-month system provided a temporally averaged, satellite-only gravity field model. Since the underlying monthly normal equations have been used for the monthly EIGEN-GRACE04S gravity field time series (Schmidt et al. 2007), this label is used for the static gravity field also.

In the context of the dynamic orbit determination and gravity recovery method applied here we use the GRACE accelerometry as measurements of the nonconservative forces acting on the satellites replacing conventional force models (for drag and solar radiation pressure) used in the past. These measurements have to be corrected for instrument biases and scaling factors, which are included as arcdependent parameters in the adjustment process. Similar to EIGEN-GRACE02S (Reigher et al. 2004) we estimate daily biases for each of the instrument axes and a scaling factor for the along-track axis allowing for a linear drift of these parameters. Another set of arc-dependent parameters are K-bandrelated parameters, again in agreement with Reigber et al. 2004 (EIGEN-GRACE02S), i.e., we solve for a range-bias and drift every revolution and for periodic terms every 180 min (i.e. about two revolutions). These are introduced to compensate for potential systematic errors introduced by the K-Band instrument and the accelerometers, respectively, as described in Kim (2000).

The GRACE data processing for EIGEN-GRACE04S is the third release of GFZ Potsdam since the beginning of this satellite mission. While the satellite-only gravity field models of the previous GFZ releases 01 (EIGEN-GRACE02S; Reigber et al. 2005), and 02 (EIGEN-GRACE03S; Förste et al. 2005) have been calculated using 1.5-day arcs, the maximum arc length of the new release 03 has been shortened to 1 day. The processing in 1-day batches, for which the orbit and the gravity recovery results showed a quasi-optimum performance, was motivated by internal investigations on the influence of the arc length on processing results. In this context, a major achievement is the improvement of

the GPS satellites constellation ephemerides and clock accuracies which are generated in-house at GFZ, used in the mentioned two-step approach; thus improving the quality of the initial orbits for the GRACE satellites in the gravity recovery process. Furthermore, during the adjustment of the GPS satellite orbits and clocks (step 1) an improved ambiguity fixing method (integer ambiguities) has been applied for the determination of GPS carrier phase ambiguities between GPS satellites and ground receivers. This enhancement resulted in significantly improved GPS ephemerides and clocks, and led to an improved determination of the GRACE satellite orbits in the second step.

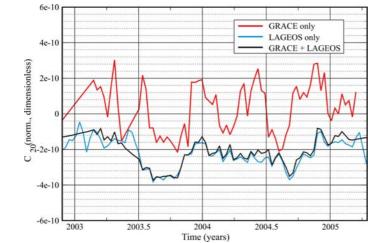


Fig. 1. C_{20} time-series derived from LAGEOS-only, GRACE-only and LAGEOS+GRACE gravity field solutions. An offset of $-0.484165137503\cdot10^{-03}$ and a slope of $+0.11628\cdot10^{-10}$ / year (reference epoch 1997.0) have been subtracted. The temporal resolution of the time series is 10 days

As before, GRACE high-low GPS code and carrier phase observations have been used undifferenced. Now, however, only data from an elevation of 10 degrees above the local horizon of the antennas were used. This gives an almost equal number of GPS observations for both GRACE satellites. Additionally, several background force models have been updated or added. For example, EIGEN-CG01C (Reigber et al. 2006) was used as apriori gravity force model, and the non-tidal atmosphere and ocean short-term mass variations were calculated using the AOD1B product release 03, which is based on a baroclinic instead of a simple barotropic ocean model (Flechtner et al. 2006). EIGEN-GRACE04S also takes into account a model for the self-consistent ocean pole tide (Desai 2002). Further details can be found in the GFZ Level-2 Processing Standards Document for release 03 (Flechtner 2005), which is also available as Electronic Supplementary Material (ESM) to this paper.

At GRGS Toulouse, exactly 2 years of GRACE and LAGEOS data (February 003 to February 2005, cf. Lemoine et al. 2007) have been processed and combined to a mean satellite-only gravity field model complete up to degree and order 150, which is called EIGEN-GL04S. The LAGEOS satellite laser ranging (SLR) data are included to increase the accuracy of the lower degree coefficients, especially for C₂₀. This is illustrated in Fig. 1. This picture shows a comparison of the C₂₀ values from three different satellite-only gravity field time series, derived at GRGS Toulouse from LAGEOS, GRACE and the combination of both satellites for a time span of

about 2 years. The GRACE-only time series contains a bias of about $+2 \times 10^{-10}$ compared to the LAGEOS-only solution. This bias in the GRACE C_{20} values is assumed to be unrealistic, and its cause is still an open topic for investigations. However, the bias is avoided in the combination with LAGEOS. Furthermore, the GRACE-only C_{20} values have more variability than those in the LAGEOS-only and combined solution.

The GRACE data processing strategy and background modeling at CNES/GRGS was identical to those of GFZ as mentioned above, but with two exceptions:

- The model for the non-tidal oceanic mass variations was based on the barotropic MOG2D model (Carrère and Lyard 2003), but the difference when using the baroclinic model OMCT (GFZ RL03) can be neglected (Flechtner et al. 2006).
- An ocean pole tide model was not taken into account.

The weighting of GPS and KBRR at GFZ and GRGS is based on the root mean square (RMS) of the data from orbital fits in which empirical acceleration parameters are adjusted to compensate for residual modeling errors. Based on such 'optimally' fitted orbits we obtain RMS values of ~ 0.3 /s for the KBRR data and ~ 0.5 cm for the GPS-Phase measurements. These values were then used for the weighting of the GPS and KBRR data in the computation of the initial orbits for the gravity recovery. No additional mutual weighting factor between the two data types has been applied. This weighting scheme is confirmed by the estimated apriori unit weighting factor after the adjustment of the monthly GRACE gravity models being close to the expected value of 1. The weighting of the SLR data of LAGEOS-1 and -2 at GRGS was done according to an obtained optimal orbital fit of ~ 1.5

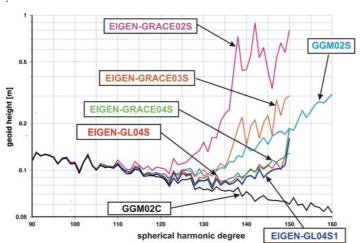


Fig. 2 Signal amplitudes per degree in terms of geoid heights (meter) for non-stabilized versions of the gravity field models as given in Table 1 in comparison with GGM02C

cm RMS. During the accumulation with the GRACE normal equation no additional weighting was applied.

In preparation for the final combined solution, two individual combined solutions were performed with the two satellite-only normal equations from both teams. The surface data and the combination procedure (overlapping range, weighting, etc.) were the same as later on applied for EIGEN-GL04C (to be described in Sects. 3, 4). With both solutions, the same orbit adjustment tests and GPS/leveling comparisons as mentioned in Sect. 5 were performed. The orbit adjustment tests gave slightly smaller residuals for the GRGS-based solution compared with the GFZ-based solution. Vice versa, the GFZbased solution performed better during the GPS/leveling comparisons. Hence, in order to benefit from the advantages of both solutions, it was decided to combine the normal equation systems of both teams. Thus, the two GRACE normal equation systems and the LAGEOS normal equation were combined and solved, resulting in a mean satelliteonly gravity field model complete to degree and order 150, which is called EIGEN-GL04S1. In order to take into account that the GRACE data are added twice, the two GRACE normal equations have been half-weighted before they were combined.

Figure 2 illustrates the progress of EIGEN-GL04S1 compared to some preceding GRACE data based satellite-only models (cf. Table 1). In Fig. 2, the geoid degree amplitudes (i.e. the square root of power per degree in terms of geoid heights) between degrees 90 and 160 are displayed. Additionally, the geoid degree amplitudes of GGM02C are shown as a reference for an independent realistic spectral behavior at higher degrees. For EIGEN-GRACE02S, EIGEN-GRACE03S and EIGEN-GL04S1, their GFZ-internal non-stabilized versions were taken.

EIGEN-GRACE04S, EIGEN-GL04S and GGM02S were intrinsically non-stabilized solutions, while the published EIGEN-GRACE02S, EIGEN-GRACE03S solutions had EIGEN-GL04S1 been computed using a stabilization of the underlying normal equations by stochastic a priori information for the short wavelength spherical harmonic coefficients bevond a certain degree (e.g. Reigber et al. 2005). The degree variances of the satellite-only models show the typical runoff at higher degrees, which is an indication for the limit of the sensitivity and therefore a measure for the stability at higher degrees of a satellite-only gravity field model.

The beginning and the intensity of the spectral run-off reflects the quantity of the included data. For the 110 days-only model, the run-off starts at about degree

110. This is in contrast to the models EIGEN-GRACE03S and GGM02S, containing 14 and 16 months of GRACE data, respectively, where the run-off is visible only beyond degree 120. The degree variance behavior of the latest three EIGEN-04 models is close, and their run-offs are again less developed compared with the others. Within these three last models, however, the combined model EIGEN-GL04S1 shows a further slight improvement,

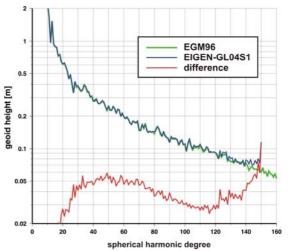


Fig. 3 Signal amplitudes per degree in terms of geoid heights (meter) for the gravity models EIGEN-GL04S1 (blue) and EGM96 (green) as well as the geoid degree amplitudes of the differences between both models (red)

because its degree variances are mostly close to those of GGM02C. This implies that EIGEN-GL04S1 has the highest stability at higher degrees compared with the other satellite-only models.

Table 1 GFZ/GRGS satellite-only models, based on GRACE data

Model	Originator	Included GRACE data	Max. degree	Reference	Remark
EIGEN-GRACE02S	GFZ	110 days	150	Reigber et al. 2005	
EIGEN-GRACE03S	GFZ	16 months	150	Förste et al. 005	GRACE contribution to EIGEN-CG03C, not separately published.
GGM02S	CSR	14 months	160	Tapley et al. 2005	GRACE contribution to GGM02C.
EIGEN-GRACE04S	GFZ	30 months	150	Schmidt et al. 2007	Mean gravity field from the EIGEN-GRACE04S time series (see Section 2).
EIGEN-GL04S	GRGS	24 months	150	Lemoine et al. 2007	
EIGEN-GL04S1	GFZ/GRGS	EIGEN-GRACE04S and EIGEN-GL04S	150		

Figure 3 shows the signal geoid degree amplitudes, again in terms of geoid heights, for the EIGEN-GL04S1 satellite-only gravity field solution and the pre-CHAMP EGM96 combination solution, as well as the difference between both solutions. Beyond degree 70, EGM96 mainly reflects the surface data information of altimetry and gravimetry, i.e. only these data contributed to the solution. Inspecting the difference, it can be deduced that for degrees higher than 115 the GRACE data contribution is inferior to that of the surface data, since there the difference geoid degree amplitudes reach their minimum (about 3 cm) at degree 115. This gives an indication for the choice of the overlapping spectral bands used in the combination of GRACE satellite-only with surface data, as described in Sect. 4.

3 Surface gravity data and processing

In order to enhance the spatial resolution of EIGEN-GL04S1 to degree and order 360, the following surface gravimetry data were used for the combination with the satellite-only GRACE and LAGEOS normal equation systems (see Fig. 4 for coverage):

- (1) Arctic Gravity Project (ArcGP) gravity anomalies (Forsberg and Kenyon 2004) for regions above 64° latitude,
- (2) National Ressources Canada (NRCan) gravity anomalies (Véronneau 2003, personnel communication), covering North America,
- (3) Alfred Wegener Institut Bremerhaven, (AWI; Studinger 1998) and Lamont Doherty Earth Observatory of Columbia University (LDEO; Bell et al. 1999) gravity anomalies over two small areas of Antarctica and, in the case of AWI, adjacent sea ice,

- (4) National Geospatial Intelligence Agency (NGA; formerly NIMA) altimetric gravity anomalies over the oceans, including standard deviations,
- (5) Geoid undulations over the oceans derived from GFZ mean sea surface heights (T. Schöne and S. Esselborn, 2005, GFZ Potsdam, personal communication, the herein included data are ERS-1 missions A-G, ERS-2 until 2003, Topex/Poseidon 1992-2004) minus Estimating the Circulation & Climate of the Ocean (ECCO) modeled sea surface topography (Stammer et al. 2002),
- (6) NGA terrestrial gravity anomalies (if not covered by data sets 1 to 3) including standard deviations, almost worldwide continental coverage, except for Antarctica and some smaller data gaps, and
- (7) NGA ship-borne gravity anomalies over water depths of less than 2,000 m.

These surface data sets are available on an equal angular 30' x 30' grid, either in their original form or after averaging to block mean values, except for data sets 5 and 7, which are originally provided with a 1° x 1° resolution. The NGA data sets (Kenyon and Pavlis 1997) are those already incorporated in the EGM96 solution (Lemoine et al. 1998). Due to limited computer resources, the normal equation system for the corrections to the spherical harmonic coefficients was generated from these data in two essentially different ways:

1) For the lower frequency part (up to degree and order 179) a rigorous normal equation system with individual data weighting (e.g. Gruber 2001) was set up using geoid undulations over the oceans (data set 5) and gravity anomalies (data sets 1 to 3 and 6) elsewhere. Data of set 4 were used to cover smaller seas like the Hudson Bay or the Black Sea and to fill the gaps in near-coastal areas (see Fig. 4). Ship gravimetry along coastlines (data set 7) overlaps with the

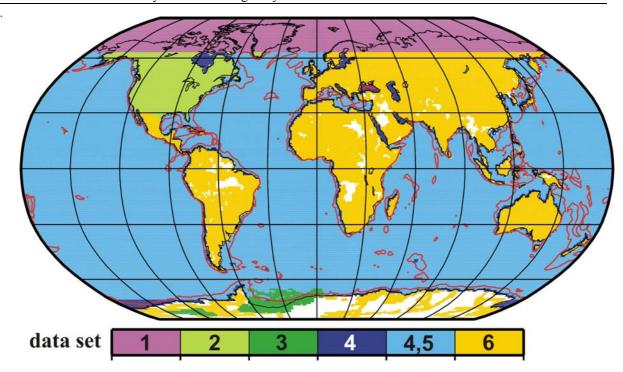


Fig. 4 Coverage of surface data sets 1 through 6 (cf. text). Red lines mark used ship gravimetry data (data set 7) over water depths of less than 2000 m. In white areas, no surface data are available (substituted by EIGEN-GRACE03S derived values).

altimeter-derived geoid undulations and NGA terrestrial gravity anomalies in order to strengthen the transition between geoid undulations and gravity anomalies. All data used for this normal matrix were evaluated (as given) on the Earth surface, i.e. no downward continuation due to topographic heights was necessary ("Molodensky approach", e.g. Pavlis 1988; note that this will result in quasi-geoid undulations). Prior to the setup of this full normal matrix the data were filtered to suppress the contribution from the spectral gravitational constituents higher than degree 179.

In the case of data sets with given standard deviations (data sets 4–7) these values have been used for the covariance matrix of observations. For the data sets without given standard deviations the following values were used: 5 mgal for the data sets 1 and 3 and 2.5 mgal for data set 2. The overlapping ship gravimetry data, which are given with a 1° x 1° resolution, were up-weighted by a factor 2. For all data a cosine-weighting as function of latitude was applied.

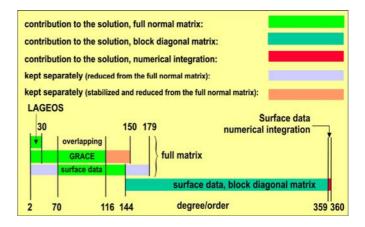
2) For the higher frequency part up to degree and order 359, a block-diagonal normal equation system was created as described in Gruber (2001) using gravity anomalies only (data sets 1 to 4 and 6). For this case the 30' x 30' block

mean values were continued downward to the ellipsoid and reduced for topographic masses, using Bosch's modified Helmert condensation method. (Gruber 2000b). Since a block diagonal structure requires a uniform weight per latitude, a measurement standard deviation of 2 mgal was assumed for all included data and a cosine-weighting as function of latitude has been applied.

In both cases, EIGEN-GRACE03S-derived gravity anomalies were used to fill areas that are not covered by any surface data (8.6%). EIGEN-GRACE03S was used up to degree and order 150. Additionally, prior to the data evaluation, all data sets were transformed to a common reference ellipsoid, and the correction for the quadratic terms of the normal gravity gradient and ellipsoidal corrections were applied to those gravity anomalies that were given in spherical approximation (Rapp and Pavlis 1990). Finally, the coefficients for degree 360, which cannot be estimated with a 30' x 30' data grid in the block-diagonal approach, are computed through numerical quadrature, using the same data as used in the block-diagonal normal equation system.

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Fig. 5 EIGEN-GL04C combination scheme of contributing satellite and terrestrial data sets.



4 EIGEN-GL04C combination and solution strategy

First, the EIGEN-GL04S1 satellite-only normal equation system [complete to degree and order 150 and stabilized onwards from degree 116 using Kaula's (1966) degree variance model] was added to the full surface data normal equation system (complete to degree and order 179) in such a way that the coefficients up to degree 70 and onwards from degree 116 were kept separate in the resulting normal equation system, i.e. only the coefficients of degree 70 through 115 were actually estimated from both normal matrices (see Fig. 5). In principle, this combination method is an enhancement of the degree-dependend normal equation combination technique used for the computation of JGM-1/-2 and -3 (Nerem et al. 1994; Tapley et al. 1996) and EGM96 (Lemoine et al. 1998), where the spherical harmonic coefficients up to degree/order 5 were adjusted separately for the surface gravity.

In the overlapping spectral band between degree 70 through 115, the surface data normal equation system, initially weighted roughly following the estimated data accuracy, was down-weighted relative to the satellite-only system by an empirically found optimal factor (0.05). This weight value gave the best results concerning the quality evaluation procedures as described below, like degree variance behavior, orbit adjustment tests and GPS/leveling comparisons. The resulting normal equation system, in which the long-wavelengths up to degree 70 are exclusively attributable to GRACE and LAGEOS, whereas the mid-wavelengths between degrees 116 and 179 are derived from the surface data only, was solved by Cholesky decomposition. This procedure prevents long-wavelength errors in the surface data affecting the solution, and assures a smooth transition within

the overlapping part of the individual normal equation systems, while maintaining the high quality of the satellite-only gravity field model up to the limit of its resolution of around degree 116 (see Fig. 3). Then, the block diagonal system was solved separately, and the resulting solution was used to extend the spherical harmonic coefficients based on the full combined normal matrix from degree 144 to degree 359, disregarding the shorter wavelength coefficients in the full normal equation derived solution. The substitution of the full normal matrixbased coefficients between degree 144 and 179 by the block diagonal-based coefficients was done to get a smooth transition from the full combined to the block diagonal solution and to avoid remaining truncation errors in the shortest wavelengths of the full matrix based solution.

Finally, the degree 360 coefficients (obtained through numerical quadrature) were added for completeness, although a drop in power is observed for this degree (see Fig. 6, this drop in power could be caused by a non-optimal weighting scheme for the numerical quadrature). The resulting model complete to degree and order 360 was called EIGEN-GL04C. The overall combination scheme of the contributing satellite and terrestrial data sets is summarized in Fig. 5

The decision of fixing the upper end of the overlapping range at degree 115 and the maximum degree for the coefficients from the full normal equation at degree 143 was made after varying these numbers between 110 and 120 and 130 and 160, respectively. It has to be mentioned that this variation had only marginal influence on the evaluation results as described in Sect. 5. In the end, the choice for 115 and 143 was taken in order to achieve the best possible smooth behavior of the degree amplitude differences to other gravity field models. As mentioned in Sect. 3, the ECCO model has been used to derive geoid undulations from altimetric sea

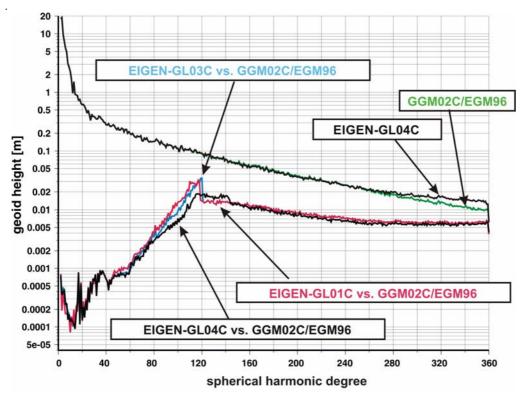


Fig. 6 Degree amplitudes for EIGEN-GL04C and GGM02C/EGM96 in terms of geoid heights (meter) to the maximum resolvable degree 360, as well as the degree amplitude differences between three EIGEN combination models and GGM02C/EGM96

surface heights. But one must have in mind, that the applied ECCO model is too smooth compared with zthe here included surface data wavelength part above degree 70 (concerning the ECCO model, see also: Pavlis and Holmes 2006). That means, we cannot expect a significant contribution of this ocean topography model to our combined solution. But anyway, the application of the ECCO model was needed for the computation of surface-only solutions, which were used for tests in the context of the preparation of the final combined solution.

5 EIGEN-GL04 quality validation

Figure 6 shows the degree signal amplitudes of EIGEN-GL04C in terms of geoid heights. Additionally, the degree-amplitude differences between several EIGEN combination models and the GGM02C/EGM96 model are shown for comparison. EIGEN-CG03C (Förste et al. 2005) is an update of the CHAMP and GRACE satellite data derived combination model EIGEN-CG01C (Reigher et al. 2006). GGM02C (Tapley et al. 2005) is a combination solution complete up to degree and

order 200 incorporating GRACE satellite data and EGM96-spherical harmonic coefficients obtained from NGA surface gravity data. GGM02C/EGM96 stands for the extension of the GGM02C model with the EGM96 coefficients up to degree and order 360 (in accordance with Tapley et al. 2005).

The degree amplitude difference to GGM02C shows the following improvements of EIGEN-GL04C compared with the precursor EIGEN-models: for EIGEN-CG01C and EIGEN-CG03C there are significant unrealistic peaks at degrees 118 and 120, respectively. These peaks coincide with the upper end of the full surface normal equations of these models. In contrast, EIGEN-GL04C is almost smooth at these degrees. This difference is mainly caused by the fact that the full normal equations of the surface data for the preceding models used 1°×1° block mean value grids, made from the original 30'× 30' grids, but without proper filtering; this was remedied for EIGEN-GL04C. For the former models, this compression of the surface data was done to save computing time, but obviously bred an omission error, resulting in these peaks at the upper end of the full surface normal equations around degrees 118 and 120.

Table 2 Orbit adjustment fits: Mean RMS values (cm) of SLR residuals

Satellite	maximum	Number of observations /	EGM96	GGM02C	EIGEN-	EIGEN-
	degree	Data period /			CG01C	GL04C
	used	Tested arcs: Number and lengths				
GFZ-1	120	2029 / October 1995 / 5 x 3 days	24.7	14.3	15.1	13.8
STELLA	120	1528 / October 1997 / 5 x 3 days	6.8	3.2	3.0	2.9
STARLETTE	120	1815 / October 1997 / 5 x 3 days	3.2	2.4	2.6	2.6
LAGEOS-1	120	3140 / October 1997 / 3 x 6 days	1.18	1.14	1.15	1.11
LAGEOS-2	120	2591 / October 1997 / 3 x 6 days	1.16	1.04	1.10	1.03
ENVISAT	120	10176 / July 2002 / 7 x 48 days	6.7	4.3	4.4	4.2
JASON	120	20003 / Nov Dec 2004 / 6 x 10 days	2.18	1.89	1.88	1.88
CHAMP	150	358 / October 2001 / 4 x 1.5	88.3	5.2	5.6	5.4
GRACE	150	592 / September 2002 / 4 x 1.5	69.9	5.5	5.2	5.2

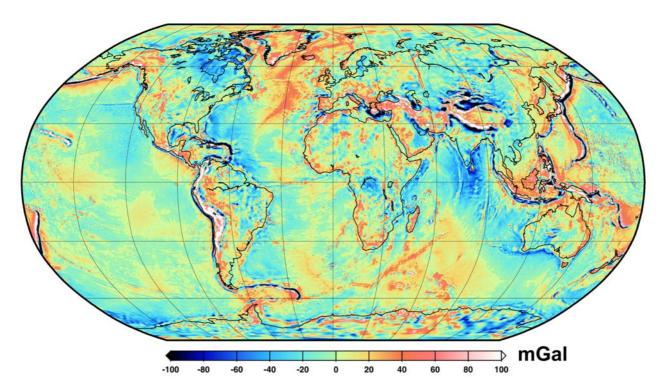


Fig. 7. Geographical distribution of gravity anomalies (mgal) derived from EIGEN-GL04C (maximum degree/order 360, reference system WGS84)

The formal standard deviations of the EIGEN-GL04C coefficients were calibrated a-posteriori in a degree-dependent way in order to produce realistic accuracy estimates. For the coefficients up to approximately degree and order 150, the calibration was based on internal subset solutions and on comparisons with external independent gravity field solutions (Reigber et al. 2002; Schmidt et al. 2007), such as GGM02C. The subset solutions were based on normal equations derived at GFZ only, not on twice-added GRGS/GFZ normal equations.

For the higher-degree coefficients, the standard deviations were fitted to the differences between EIGEN-GL04C and EGM96. This spectral range beyond degree 150 is based mainly on the same surface data (cf. Sect. 3) and the differences between both models, especially those beyond degree 280 as shown in Fig. 6, should be primarily caused by different combination algorithms and software tools. For instance, in the EGM96 solution a Kaularegularization has been applied particularly for the higher degrees (Lemoine et al. 1998). Thus it is realistic to assume that the errors of the coefficients

are at least of the same order of magnitude as the differences between both models. We are aware of the fact, that this is a very optimistic approach, since therewith other possible errors are implied to be neligible. But as discussed below in the course of this section, there are indications, that this error estimate is not unrealistic. The obtained calibrated error degree amplitudes (in terms of geoid heights) are shown in Fig. 10.

One traditional measure of the gravity field model's long-to medium wavelength accuracy is satellite orbit fits (e.g. Lemoine et al. 1998 or Gruber et al. 2000a). Table 2 lists mean RMS values of the SLR residuals after satellite adjustments using EIGEN-GL04C in comparison with results obtained from EGM96, GGM02C and EIGEN-CG01C. The orbit tests presented in this table were done for the satellites GFZ-1, STELLA, STARLETTE, LAGEOS-1 and 2, ENVISAT, JASON-1, CHAMP and GRACE. Table 2 shows that the new EIGEN-GL04C model gives the best orbit fits for most of the tested satellites except for CHAMP and STARLETTE, for which the smallest RMS values are obtained using GGM02C. Obviously, for the gravitational force modeling required satellite in orbit computations, the EGM96 is not state-of-the-art anymore.

The gravity anomaly plots of Figs. 7 and 8 illustrate the high spatial resolution of the EIGEN-GL04C model for the whole globe and for the region of Europe and western Asia. Additionally, Fig. 8 highlights the gain in the spatial resolution of the gravity field between the satellite-only model EIGEN-GL04S1 (Fig. 8, top) and the combined model EIGEN-GL04C (Fig. 8, bottom). For instance, the gravity signatures of mountainous regions like the Ural. Anatolia or the Atlas are much better resolved in the combined model. Furthermore, the satellite-only model contains some unrealistic meridional stripes in the western part of Siberia and in the

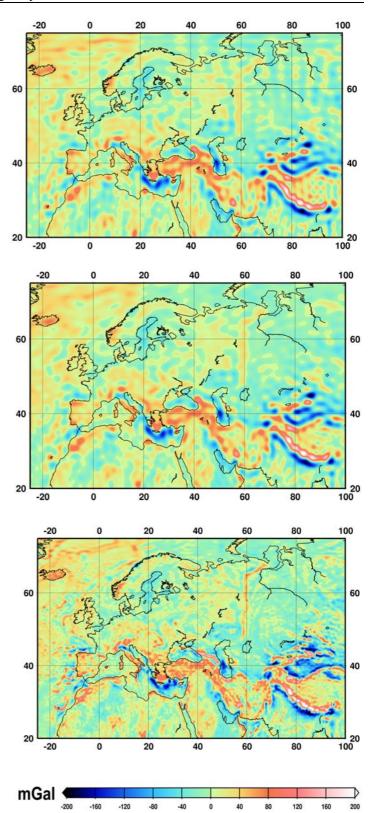


Figure 8. Gravity anomalies (in mgal) over Europe/western Asia derived from the GRACE/LAGEOS satellite-only model EIGEN-GL04S1 (top) and from the combined model EIGEN-GL04C for maximum degree/order 150 (middle) and 360 (bottom)

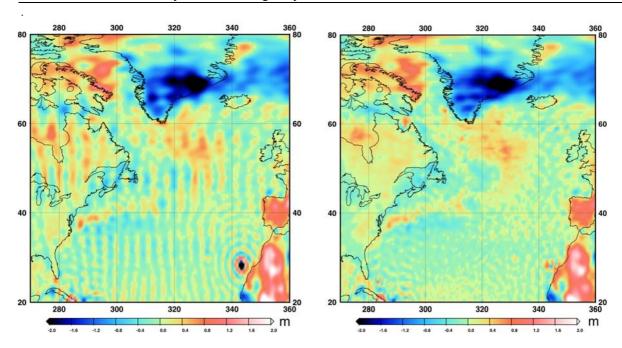


Fig. 9. Geoid height differences over the North Atlantic [m] between EIGEN-CG03C (left) and EIGEN-GL04C (right) and a global model based on ground data only.

highlands of Tibet, which are no longer present in the combined model (cf. Fig. 8, top and middle).

With EIGEN-GL04C, as just mentioned in the context of Fig. 8, a significant improvement in the reduction of the spurious meridional stripes has been achieved. Those stripes are typical for GRACE geopotential solutions (cf. Tapley et al. 2005). A second example of this improvement is shown for the North Atlantic in Fig. 9, where EIGEN-GL04C is compared with the quite "stripy" EIGEN-GC03C. For both models, the geoid difference to the corresponding geoid based on ground data is shown. These ground data based geoids were obtained simply by inversion of the full surface data normal matrix mentioned in Sect. 3, which was afterwards used for the combination with the satellite normal equation. Apart from the reduction of the meridional

stripes, Fig. 9 shows the correction of an artefact around

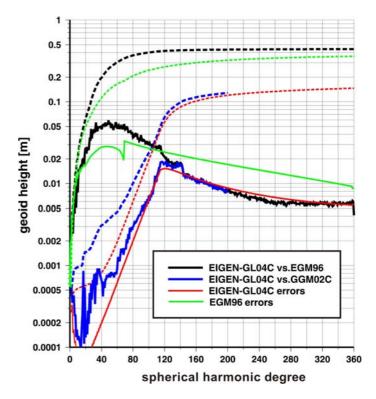


Fig. 10. Amplitudes per degree (lower, wiggly curves) and degree-wise accumulated (upper, dashed curves) in terms of geoid heights (meter) of the differences EIGEN-GL04C vs. EGM96 and GGM02C, respectively, and of the EIGEN-GL04C and EGM96 calibrated errors.

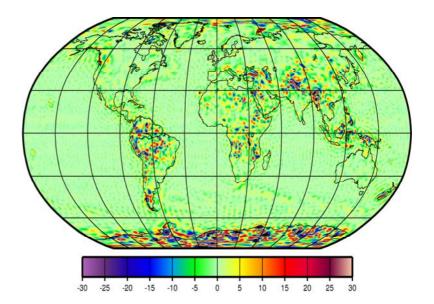
the Canary Islands. This was accomplished by substituting the obviously wrong Canary Island surface gravity data with EIGEN-GRACE03S values. Furthermore, one can notice the Gulf Stream signature, suggesting an imperfect removal of the ocean dynamic topography from the ground data.

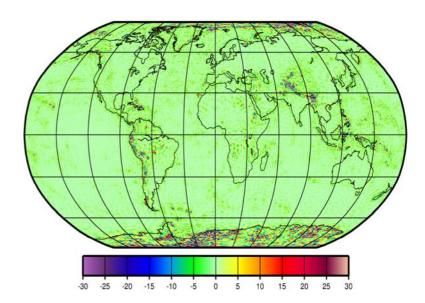
Apart from satellite orbit computations, EGM96 is still one of the most used global gravity field models and for that reason it is of interest to compare it with the new EIGEN model. Figure 10 presents a comparison with EGM96 in the spectral domain. It shows that, up to degree 360, the cumulated differences between EIGEN-GL04C and EGM96 add

up to 45 cm (in terms of geoid heights). In contrast, the EIGEN-GL04C and GGM02C difference up to degree 200 is 15 cm. The difference geoid degree amplitudes of EIGEN-GL04C and EGM96, and those of EIGEN-GL04C and GGM02C, respectively, almost coincide for degrees higher than 100 because similar surface data contributed to all three models.

Assuming that the differences reflect the realistic order of magnitude for the standard deviations of EIGEN-GL04C's spherical harmonic coefficients up to degree 100, the cumulated differences illustrate a one-to-two order of magnitude accuracy improvement from the pre-CHAMP model EGM96

Figure 11. Gravity anomaly differences between EIGEN-GL04C and EGM96 for half-wavelengths larger than 200 km (spherical harmonic degrees 2 to 100, top) and shorter than 200 km (spherical harmonic degrees 101 to 360, bottom) in units of mgal.





to the new GRACE-based models for this wavelength range.

The comparison in the spectral domain does not provide a regional discrimination of the model characteristics. For this purpose, the geographical distribution of the gravity anomaly differences between EIGEN-GL04C and EGM96 are shown in Fig. 11, representing only the long- to mediumwavelength part (half-wavelengths larger than 200 km, or spherical harmonic degrees 2 to 100), and the remaining higher frequency part (half-wavelengths from 200 to 55 km, or spherical harmonic degrees 101 to 360). The frame on the top in Fig. 11 shows the areas where the main contribution to the improvement of the global gravity field model comes from the GRACE/LAGEOS satellite data: the polar caps, previously not very accurately resolved by satellite-only models due to the relatively small inclinations of the analyzed satellites, and the continents of Africa, Asia and South America, which are only inhomogeneously covered by satellite tracking stations and surface gravimetric data. The frame at the bottom of Fig. 11 mainly reveals the impact of the new gravity anomaly data from the Arctic Gravity Project, and over two Antarctic regions. Especially the usage of the Arctic Gravity Project data is a major advance, because for EGM96 only airborne gravity data from Greenland and the Canadian Arctic were available. Larger discrepancies

Table 3 Weighted (cosine of latitude) root mean square (rms) of gravity anomaly differences between EIGEN-GL04C and EGM96 as a function of spherical harmonic degree range

Degree	Global	Oceans	Continents
2 - 360	6.8 mgal	4.1 mgal	10.9 mgal
2 - 100	4.4 mgal	2.4 mgal	7.3 mgal
101 - 360	3.6 mgal	2.6 mgal	5.4 mgal

Table 4 Root mean square (cm) about mean of GPS/leveling minus model derived geoid heights (number of points in parentheses).

Gravity Model		GPS/	Leveli	ing Ge	eoid H	eight	Data	Set	
		SA 69)		nada (30)		ope 86)		tralia 01)	_
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	
EIGEN-GL04C EIGEN-CG01C GGM02C/EGM96 EGM96	36 37 36 40	36 37 36	26 28 27 37	27 28 27	33 41 31 49	31 34 31	26 28 27 31	27 29 27	

The spherical harmonic coefficients of the tested models are (1) used up to degree 360 and (2) replaced by the EGM96 coefficients beyond degree 120

with EGM96 also appear in surface data gap areas, because these were filled with different data in both models

Table 3 gives the statistics of the differences in terms of gravity anomalies between the two high-resolution global gravity field models for the lower (degree 2 to 100) and higher (degree 101 to 360) frequency part as well as for the entire spectrum. The mean differences over the oceans are lower than those over the continents due to the homogeneous coverage with satellite altimeter data in both models.

An independent comparison with external data can be made using geoid heights determined pointby **GPS** positioning and leveling wise ("GPS/leveling"). Table 4 shows the results for EIGEN-GL04C, EIGEN-CG01C, GGM02C (filled up to degree and order 360 with EGM96 coefficients) and EGM96 using GPS/leveling points of the USA (Milbert, 1998), Canada (M. Véronneau, personal communication 2003, Natural Resources Canada), Europe (Ihde et al. 2002) and Australia (G. Johnston, Geoscience Australia and W. Featherstone, Curtin University of Technology, personal communication 2007). For this comparison, height anomalies were calculated from the spherical harmonic coefficient data sets and reduced to geoid heights (c.f. Rapp 1997). The topographic reduction was done by using the ETOPO2 model (National Geographic Data Center NGCD, ETOPO2: Bathymetry/Topography Data, US Dep. of Com., Washington DC 2001), which has been transformed into spherical harmonic coefficients until degree/order of 360 before. The improvement in the new EIGEN model with respect to EGM96 and the older EIGEN-CG01C is evident. Only in the case of the European GPS/leveling data set GGM02C/EGM96 still performs better.

As pointed out in the previous section, the short wavelength part of the EIGEN-GL04C and EGM96 models was made from more or less the same terrestrial data sets. In this context, it should be of interest to probe the contribution of the higher degree wavelength portions of these models to the results of the GPS/leveling tests. To this purpose, the GPS/leveling tests were carried out with composite models in which the higher degree portions of the spherical harmonic coefficients beyond degree 120 have been replaced by the corresponding EGM96 coefficients. The results are given in Table 4 in the columns denoted by (2). Only in the case of Europe does the higher degree portion of EGM96 perform better than EIGEN-GL04C, which corresponds to the finding of the tests with the complete models [see Table 4, columns denoted by (1)], where GGM02C/EGM96 performs best. But in the cases of the other GPS/leveling data sets, the EIGEN-GL04C portion performs just as well or better than the EGM96 portion. The results of the GPS/leveling

Table 5 Residuals of the geostrophic currents obtained from different combined gravity field models vs. World Ocean Atlas 2001 (WOA01) data, (courtesy of John Ries and Don Chambers, Center for Space Research, The University of Texas at Austin)

Gravity Model	Standard Deviation (cm/s)		Cori	elation
	zonal	meridional	zonal	meridional
EIGEN-GL04C	3.0	3.0	0.915	0.542
EIGEN-CG03C	2.9	3.2	0.921	0.494
GGM02C	3.0	3.2	0.914	0.481
EIGEN-CG01C	3.2	3.8	0.905	0.398
EGM96	8.2	7.0	0.352	0.288

comparisons can also be taken for the verification of the error estimates. The estimated cumulated error of the combined model at degree 360 is 15 cm in terms of geoid heights (= comission error, cf. Fig. 10). Considering an omission error of 18 cm based on Kaula's degree variance model beyond degree 360 (e.g. Lemoine et al. 1998), the total error of EIGEN-GL04C is of about 23 cm ($\approx \sqrt{15^2 + 18^2}$). On the other hand, our GPS/leveling comparison results for both Canada and Australia are 26 cm both (see Table 4). Uncertainties in the topographic correction alone could be of the order of some centimetres in mountainous regions (Milbert 1998). And leveling errors could be up to 3 mm per kilometer (Milbert 1998), which results in a couple of centimeters in the error over ranges of 1,000 km. Keeping in mind these uncertainties of the GPS/leveling, at least the fits for Canada and Australia are consistent with the estimated total 23 cm error level. An oceanographic validation of a global gravity field model can be done by the comparison of the dynamic ocean topography derived by subtracting from an altimetry-derived mean sea surface with an ocean topography model obtained from other sources. One variant of this comparison is the computation of residuals between geostrophic current maps obtained from the ocean topography surfaces to be compared. Table 5 presents results of such a validation, courtesy of John Ries and Don Chambers from the Center for Space Research of the University of Texas at Austin. The numbers given in Table 5 are obtained from comparisons of geostrophic currents computed from the various global gravity field models with an ocean circulation map derived from the World Ocean Atlas 2001 (WOA01, Stephens et al. 2002) relative to 4,000 m ocean depth, and after 400 km smoothing. A description of the test algorithm can be found in Tapley et al. (2003). Concerning the meridional component, the results in Table 4 show a smaller standard deviation and a higher correlation for EIGEN-GL04C than for any other precursor model,

which indicates a more accurate marine geoid for this model. The values for the zonal component are comparable for all tested GRACE-based models, and they show a significant improvement compared to EGM96.

6 Conclusions

A new satellite-only gravity field model, EIGEN-GL04S1, complete up to degree and order 150 has been inferred from GRACE and LAGEOS data. Using this satellite-only model as a starting point, a new combined global gravity field model EIGEN-GL04C, complete

up to degree and order 360, has been developed, incorporating surface gravity data including newly available or improved data sets of the Arctic, Antarctica and North-America and improved mean sea surface heights from altimetry processing at GFZ. Compared to the pre-CHAMP high-resolution model EGM96, the long- to medium-wavelength ($\lambda/2 > 200$ km) gravity and geoid accuracy was improved by about one order of magnitude to 3 cm due to the EIGEN-GL04S1 contribution. Up to degree and order 360 ($\lambda/2 = 55$ km), the EIGEN-GL04C mean accuracy is estimated to be 15 cm. Especially at high latitudes, EIGEN-GL04C benefits from the better coverage of the newly released gravity anomaly compilations. Additionally, the observed artificial meridional striping of earlier EIGEN models could be reduced thanks to improved GRACE data processing algorithms. Lastly, former "hot spots", e.g. the Canary Islands, could be corrected by substituting obviously erroneous surface gravity data with GRACE-derived values. The new model can be used in precise orbit determination, as background model in regional Geoid modeling, or for geodynamic interpretation over a wide range of topics, such as the study of the Earth's crust and mantle mass distribution. The notable improvements, especially in the reduction of artificial meridional striping and the increased correlation of EIGEN-GL04C-derived geostrophic meridional currents with WOA01 data, convinced the JASON-1 project to use EIGEN-GL04C for JASON-1 data reprocessing.

Remark. The EIGEN-GL04S1 and EIGEN-GL04C models can be downloaded from the ICGEM database of the IAG (http://icgem.gfz-potsdam.de). Additionally, both models are included to this paper as ESM.

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