Combination in Gravity Modelling

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Gravity Modelling

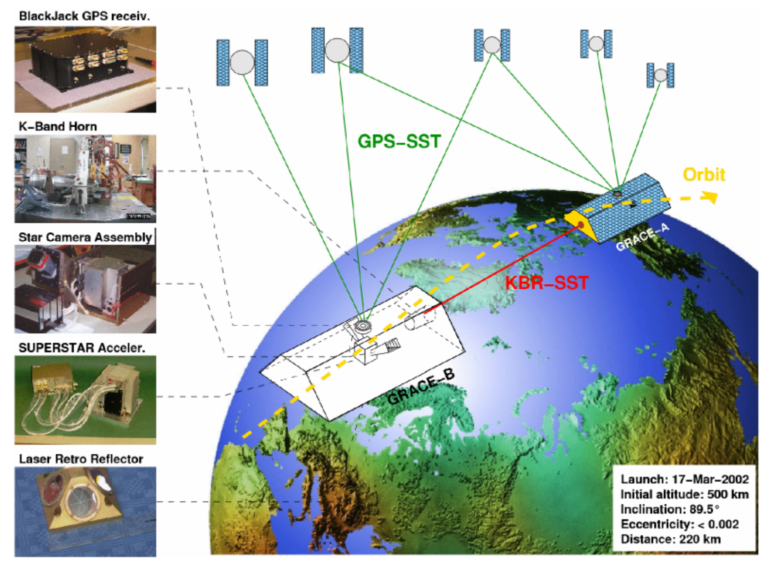
Gravitation is a 3D Force field

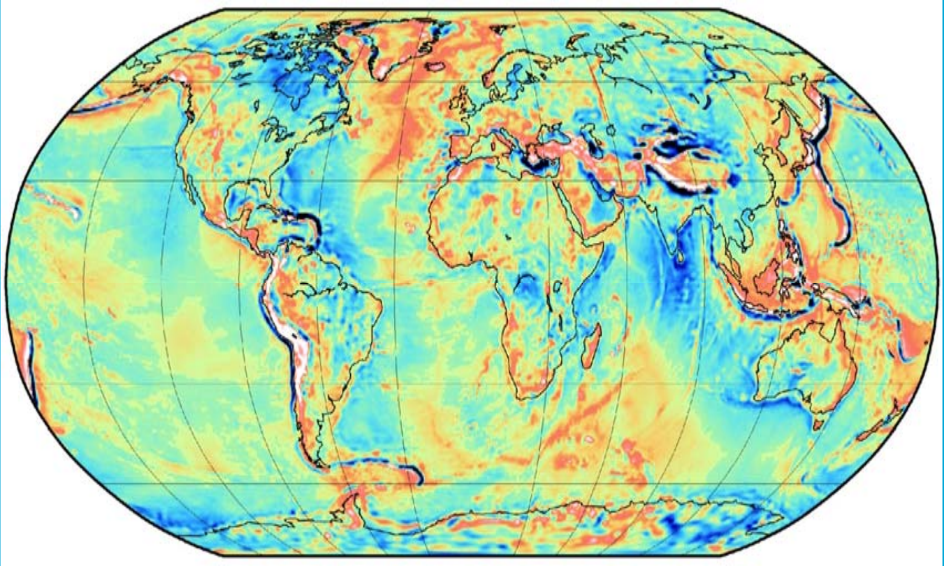
- Magnitude at point P depends on distance to source masses
- Variety of functionals is measurable (ΔV, grad V, grad grad V)
- Conventional gauge of force field: V(infinity)=0
- Conventionally described by spherical harmonic model for V
- “d.o.f. = infinite”, Commission/ommission errors
- Inversion of Gravity to Mass is always non-unique
- Computation of “the Geoid” is non-unique
- Satellite gravity (GRACE) employs a reference system which is “geo-centric” for the analysis interval

Gravity Modelling

Gravity Field “products” for GGOS: globally relevant quantities

- Global gravity models (satellite L...200, combination L...360...?)





- Precise regional geoid models around key stations
- Global vertical datum
- Global networks of superconducting gravimeters (SCG) and absolute gravimeters (SG)

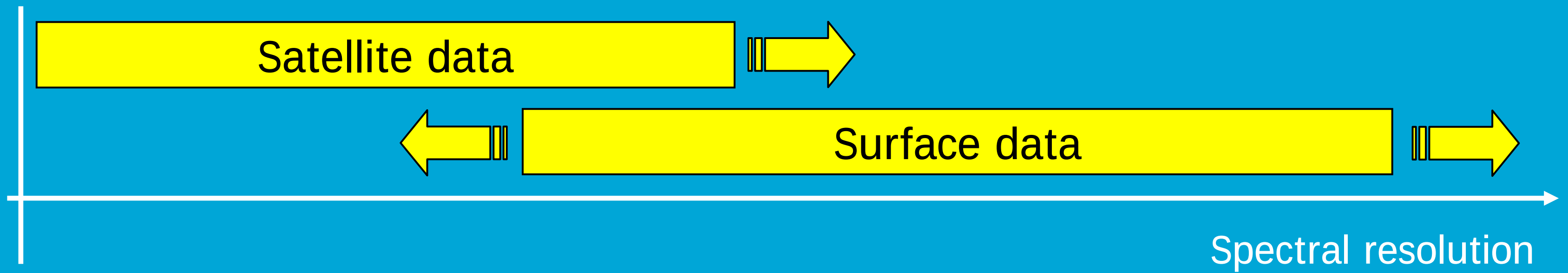
Combinations involving Gravity

Gravity functional <u>and</u> Gravity functional

- Satellite Gravity + Satellite Gravity: LAGEOS, GRACE, GOCE
- Satellite Gravity + Surface Gravity: terrestrial anomalies, airborne gravity, altimetry-derived gravity anomalies

Combination Product is again a gravity functional

- Logical limit in spatial and temporal resolution ?
- How to deal with the spectral overlap ?



Combinations involving Gravity

Gravity functional and Geometry functional

Examples

- Low-degree TV gravity + geometry change → seasonal surface mass change (Wu et al.)

  Needs a geophysical theory/loading (Q: Standard?)
- Degree-1 from geometry change or from models + GRACE → surface mass change
- AG + SCG + vertical motion → local mass change
- Satellite geoid + altimetry + ocean hydrography → dynamic topography (Rio et al. 03)
- Satellite geoid + local geoids + altimetry/tide gauge + GPS + levelling → global height reference

Combinations and Consistency, Issues

1. type of G. product / approximations of physical geodesy
   a. geopotential differences (free of hypothesis)
   b. gravity anomalies/disturbances (not free of hypothesis, height datum, gravity reference system, several definitions, approximations)
   c. geoid/height anomalies (not free of hypothesis, approximations, downward continuation through masses)
   d. mean sea surface (not free of hypothesis, mean topography)

2. different mathematical representations and spectral/spatial/temporal information content
   a. truncated spherical harmonics vs block-mean values (Global combination G.)
   b. truncated spherical harmonic models vs. point values (GRACE vs. AG, SCG)
   c. some “non-standard” global G. models are not expressed in SH anymore, different base functions

Combinations and Consistency, Issues

3. time (G. products always refer to a “time mean”)
    a. validity interval (e.g. 1 Month)
    b. data interval (not necessarily equal to validity interval)
    c. treatment of temporal effects in “mean” models

4. tide system

6. geometrical reference system (only in approximation this affects only degree-0,1,2 spherical harmonic coefficients)

7. computation/inclusion/removal/re-substitution after processing of
    a. atmosphere
    b. ocean, tides
    c. other (e.g. local hydrology)

Combination Methodologies

1. Common for global satellite models is combination of normal equations level (either different data or the same original data)
2. Global combination models have to use regional data sets, these are often proprietary and not well described
3. Common for high-resolution global modelling (e.g. EIGEN) is combination based on a mixture of normal equations and integration methods. This involves different theories (Molodensky, Stokes).
4. Common for regional models is a combination of a global model on the solution level with regional data and an integration method. Remove-restore using terrain information. No use of full covariance info.
5. Weighting often degree-wise (except normal equation combination)
6. TV Gravity: Degree-1/Geo-center motion is simply "added"

Combination Methodologies

Rank deficiencies in geometrical techniques – ill-posedness in gravity (e.g. satellite-only model from GOCE, polar gaps)

- In contrast to constraining (fixing a rank defect) in geometrical techniques, regularization applies to an infinite number of singular values of the problem.
- There is no agreed standard procedure for regularization of G. fields. Kaula regularization used to be a quasi-standard, but modern methods have proven to supersede.
- There is no agreed procedure to quantify biases due to regularization

Weighting of different data sets - VCE

Some recent non-standard G. solutions use Variance component estimation (VCE) for combining satellite data sets, will likely be applied for deriving the GOCE solution

No standard procedure (MINQUE, Bayesian, REML, Helmert,...)

1. several provide actually identical estimates
2. some provide different estimates in an initial stage of the algorithm (often terminated) but converge to the same estimate in the end
3. some provide actual different estimates

- Applicable for combination on observation and normal equations level. But can it be of use when integration methods are applied?
- LS-residuals in G. always contain truncation error VCE: bias-corrected methods (Koch & Kusche 2007, Xu et al 2007)?
- None of the common VCE methods is robust against outliers

How it is done for EIGEN-Models

		EIGEN-CG01C	EIGEN-CG03C	EIGEN-GL04C	EIGEN-GL05Cp.18	EIGEN-GL05Cp.19
	released:	May 2004	March 2005	March 2006	preliminary Sept. 2006	preliminary April 2007
	Resolution:	360 x 360	360 x 360	360 x 360	360 x 360	360 x 360
<u>Main differences:</u>						
Satellite data:	CHAMP	**33 months**: 10/2000 - 06/2003		----	---	---
	GRACE	**200 days**: 04/2002 - 11/2003	**16 months**: 02/2003 - 07/2004	**30 months**: 02/2003 - 07/2005	**34 months**: 02/2003 - 02/2006	**34 months**: 02/2003 - 02/2006
	LAGEOS	----	----	24 months: 02/2003 - 02/2005	24 months: 02/2003 - 02/2005	24 months: 02/2003 - 02/2005
Ocean data (direct altimetry)		**CLS01** sea surface heights		**GFZ** mean sea surface heights		
Max. degree of the the full normal matrix		**140**	140	**179**	**200**	**260**
Overlapping range [deg] between satellite and terrestrial data:		70 ... 109	70 ... 120	70 .. 115	70 ... 150 **using constraints**	
Terrestrial data: Grid size for the full normal equations		1° x 1°	1° x 1°	30‘ x 30‘	30‘ x 30‘	
Remarks:					including the latest ArcGP data (Forsberg 2006)	

Förste et al 07

How it is done for EIGEN-Models

GRACE and LAGEOS satellite data for EIGEN-GL04C and EIGEN-GL05Cp

Data Period:

CNES (for GRACE and LAGEOS): February 2003 – February 2005
GFZ (for GRACE): February 2003 - July 2005 (without January 2004) [for EIGEN-GL04C]
GFZ (for GRACE): February 2003 - Febr. 2006 (without January 2004) [for EIGEN-GL05Cp]

Arc Length:

LAGEOS: 7d GRACE: 1d

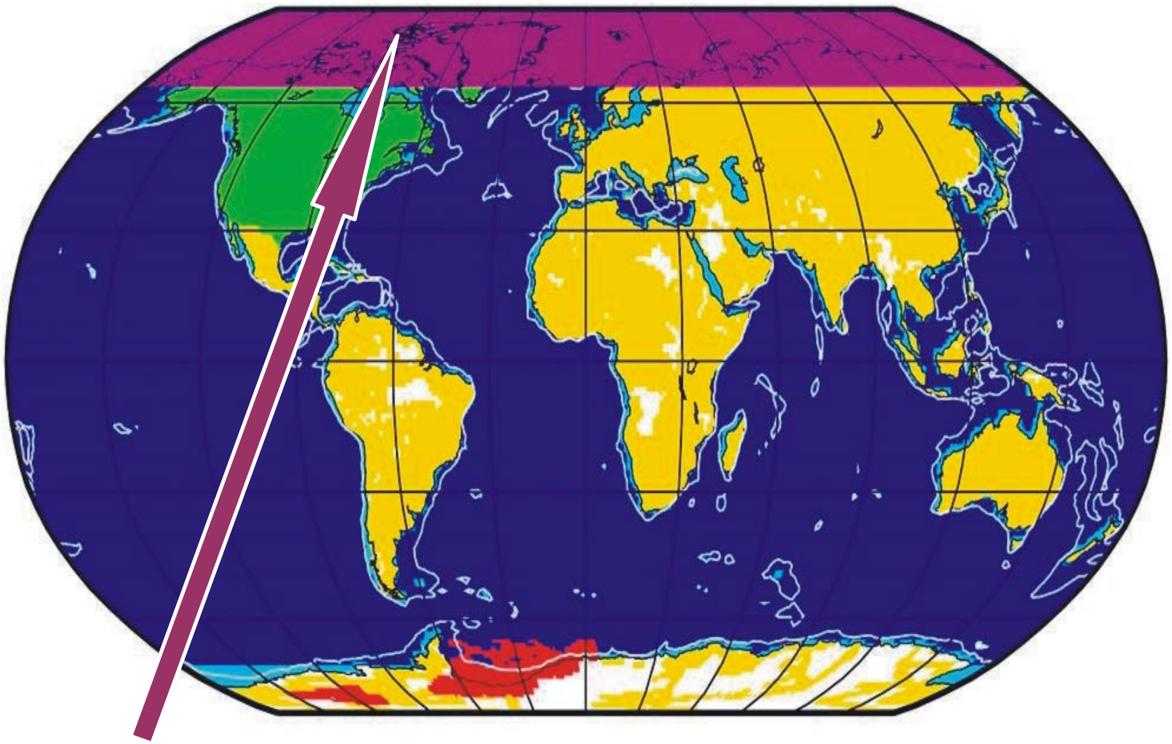
Dynamical Parametrization:

LAGEOS: emp. Coefficients, along-track polygon with 4d spacing, along-track and cross-track emp. 1/rev coefficients changing every 4 d
GRACE: K-band empirical coefficients, accelerometer 3D scaling factors and biases at begin and end of the arc

Processing Standards:

Gravity Field:	EIGEN-CG03C (150x150)	Ocean Tides:	FES2004 (80x80)
Earth Tides:	IERS2003	Solid Earth Pole Tide:	IERS2003
De-aliasing:	AOD1B RL03 (ECMWF, OMCT)	Ocean Pole Tide:	Desai 2002 (80x80)

Surface data sets for EIGEN-GL05Cp

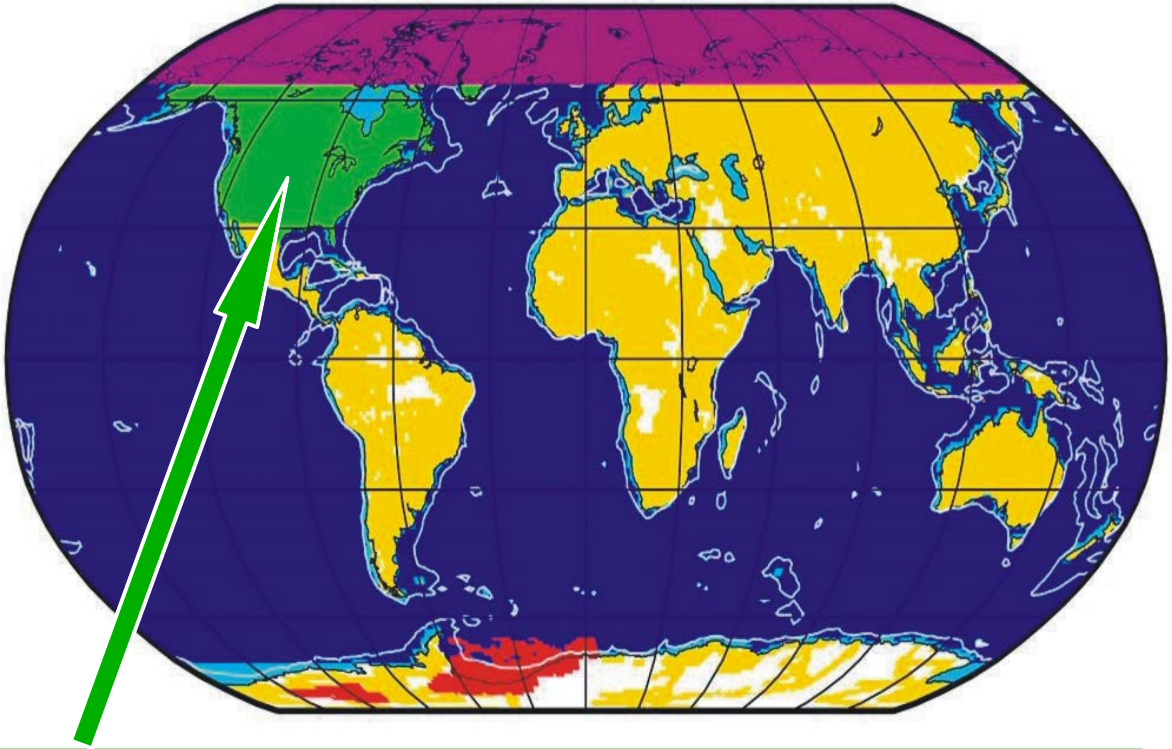


Arctic Gravity Project (ArcGP).

Gravity anomalies for regions of latitude > 64°

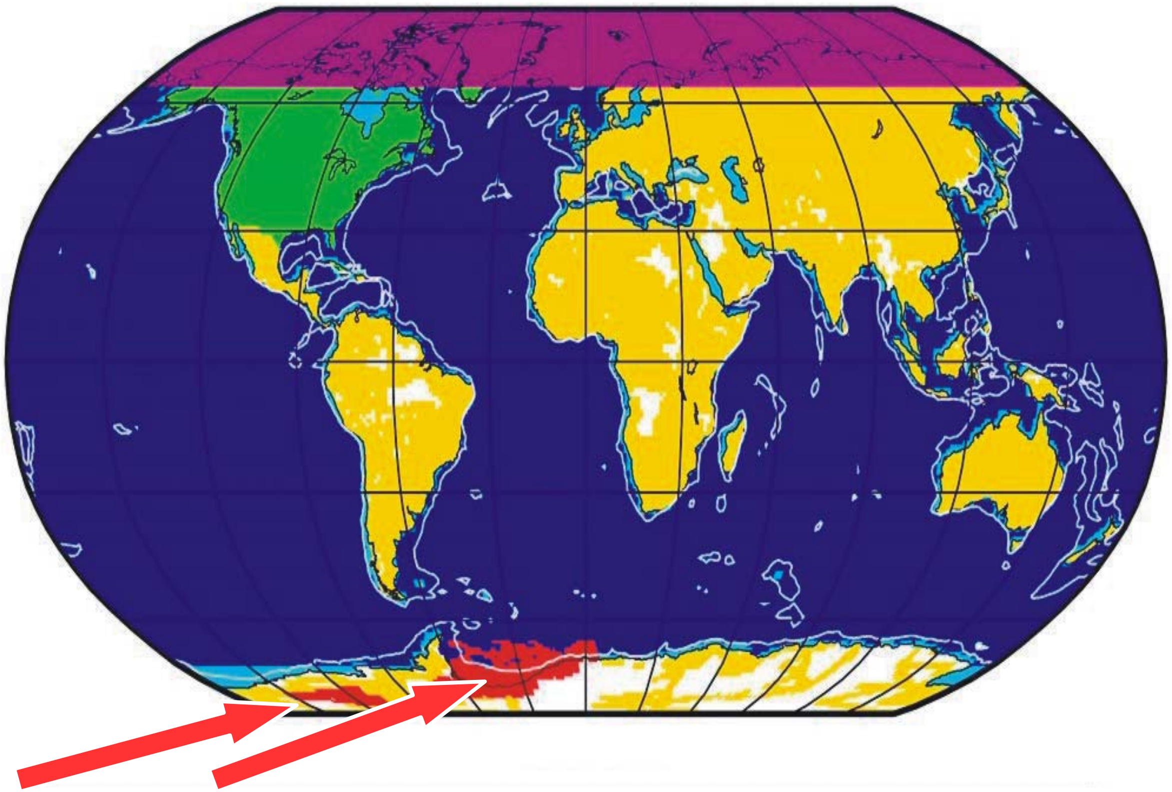
(Forsberg, Kenyon 2004 & 2006)

Surface data sets for EIGEN-GL05Cp



NRCan gravity anomalies covering North America.
(Véronneau 2003, personal communication)

Surface data sets for EIGEN-GL05Cp

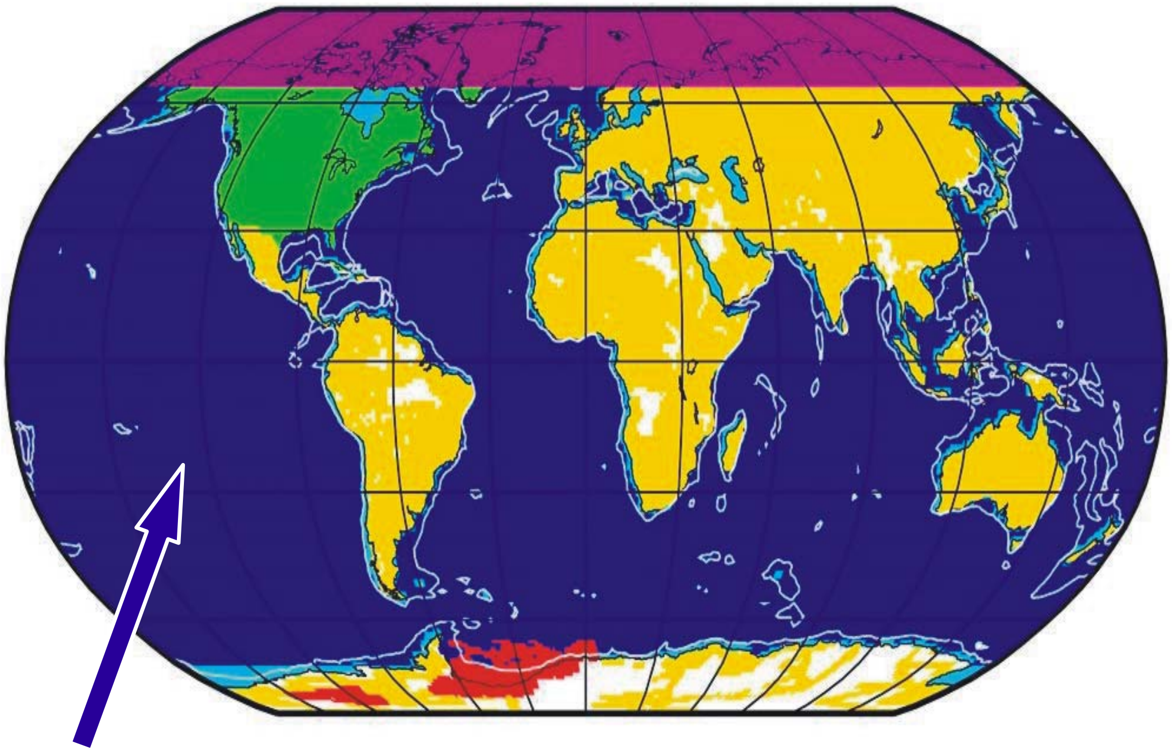


LDO and AWI gravity anomalies.

Small areas over Antarctica and adjacent sea ice.

(LDO: Bell et al., 1999; AWI:Studinger 1988)

Surface data sets for EIGEN-GL05Cp



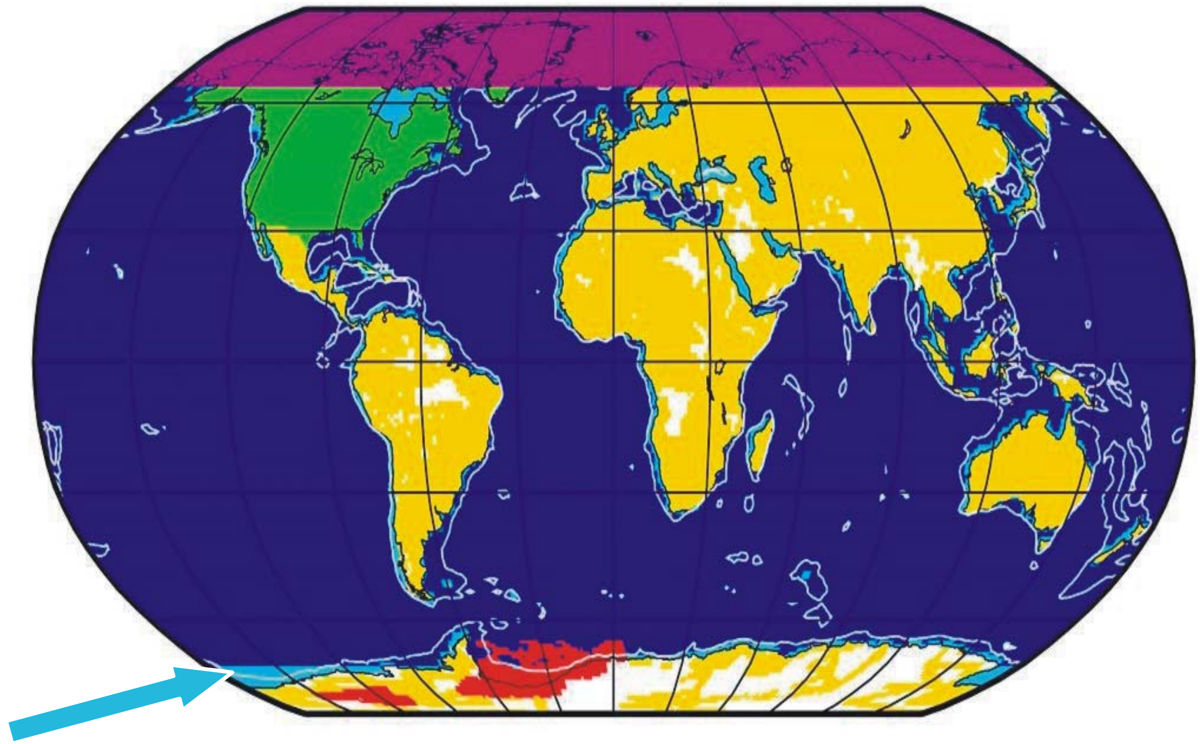
Geoid undulations over the oceans:

GFZ-MSSH minus ECCO-SSTop.

(GFZ: T. Schöne and S. Esselborn 2005, personal communication);

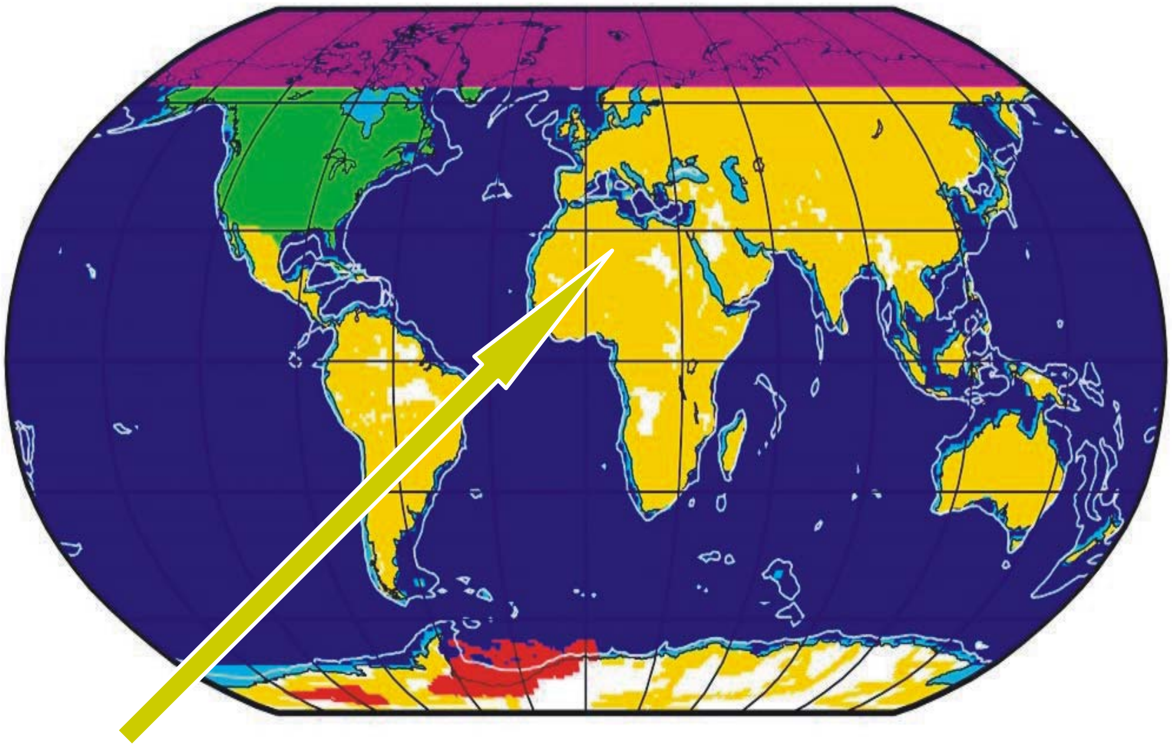
ECCO: Stammer et al., 2002),

Surface data sets for EIGEN-GL05Cp



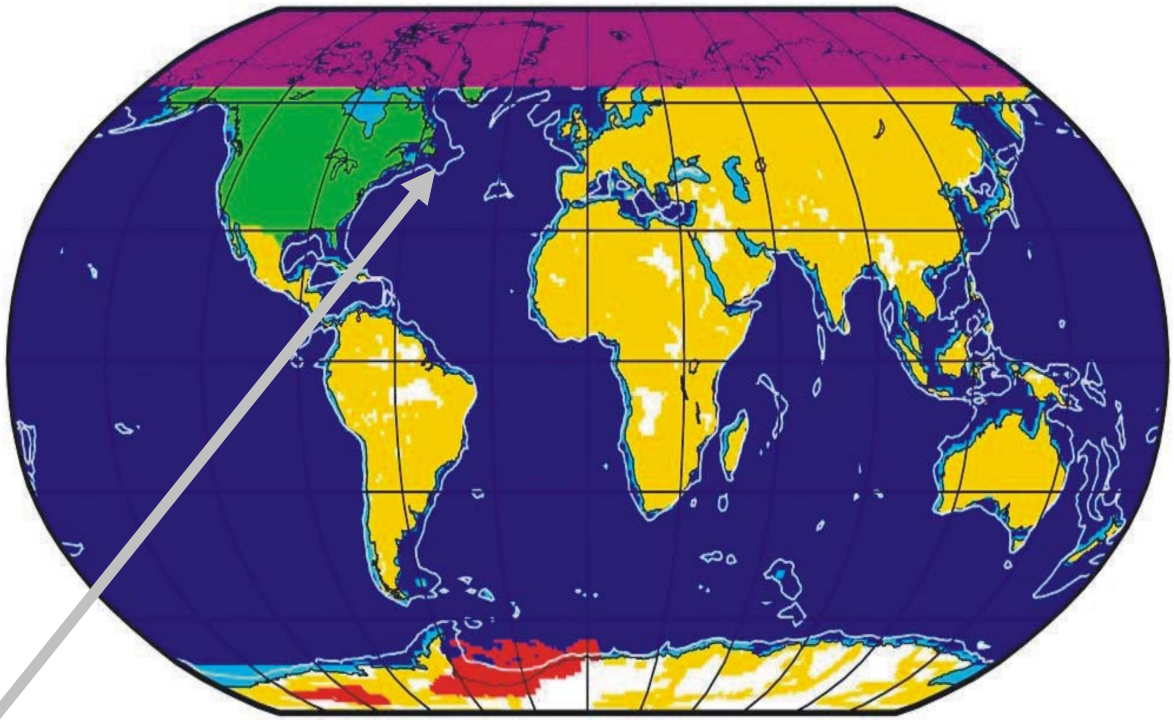
NIMA altimetric gravity anomalies
over the ocean if not covered by GFZ-geoid
(including standard deviations)

Surface data sets for EIGEN-GL05Cp



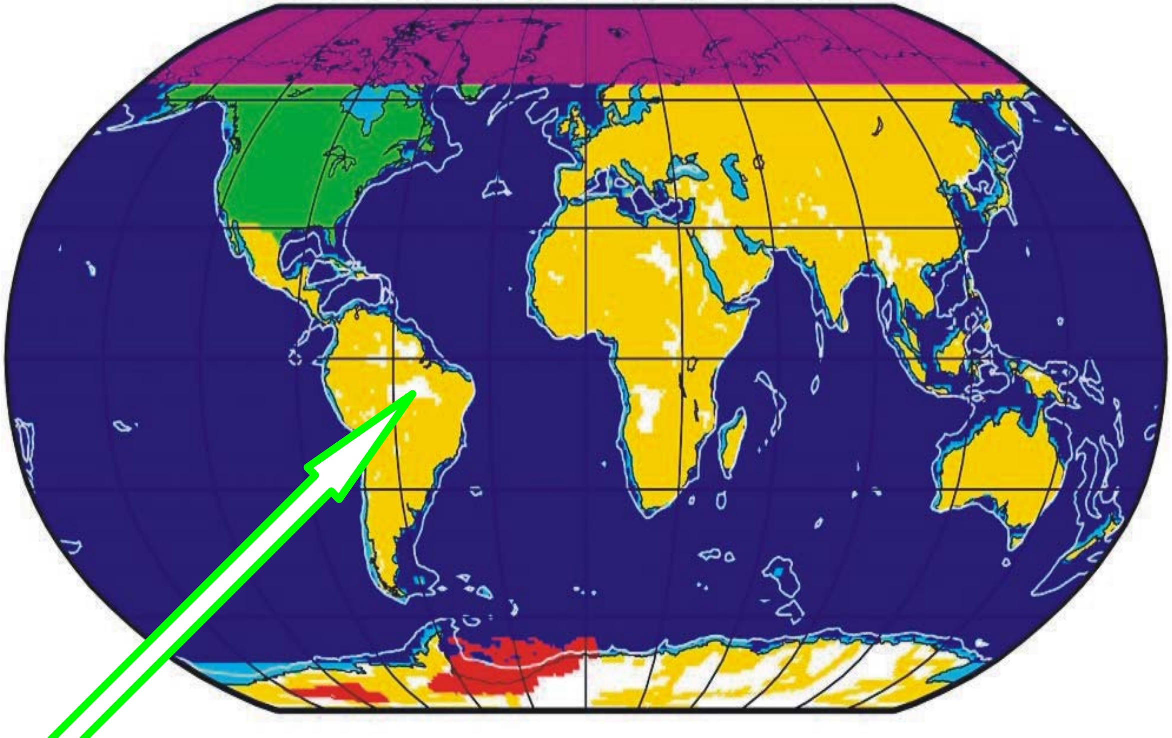
NIMA terrestrial gravity anomalies (including standard deviations),
almost worldwide continental coverage, except for Antarctica and some smaller data gaps.

Surface data sets for EIGEN-GL05Cp



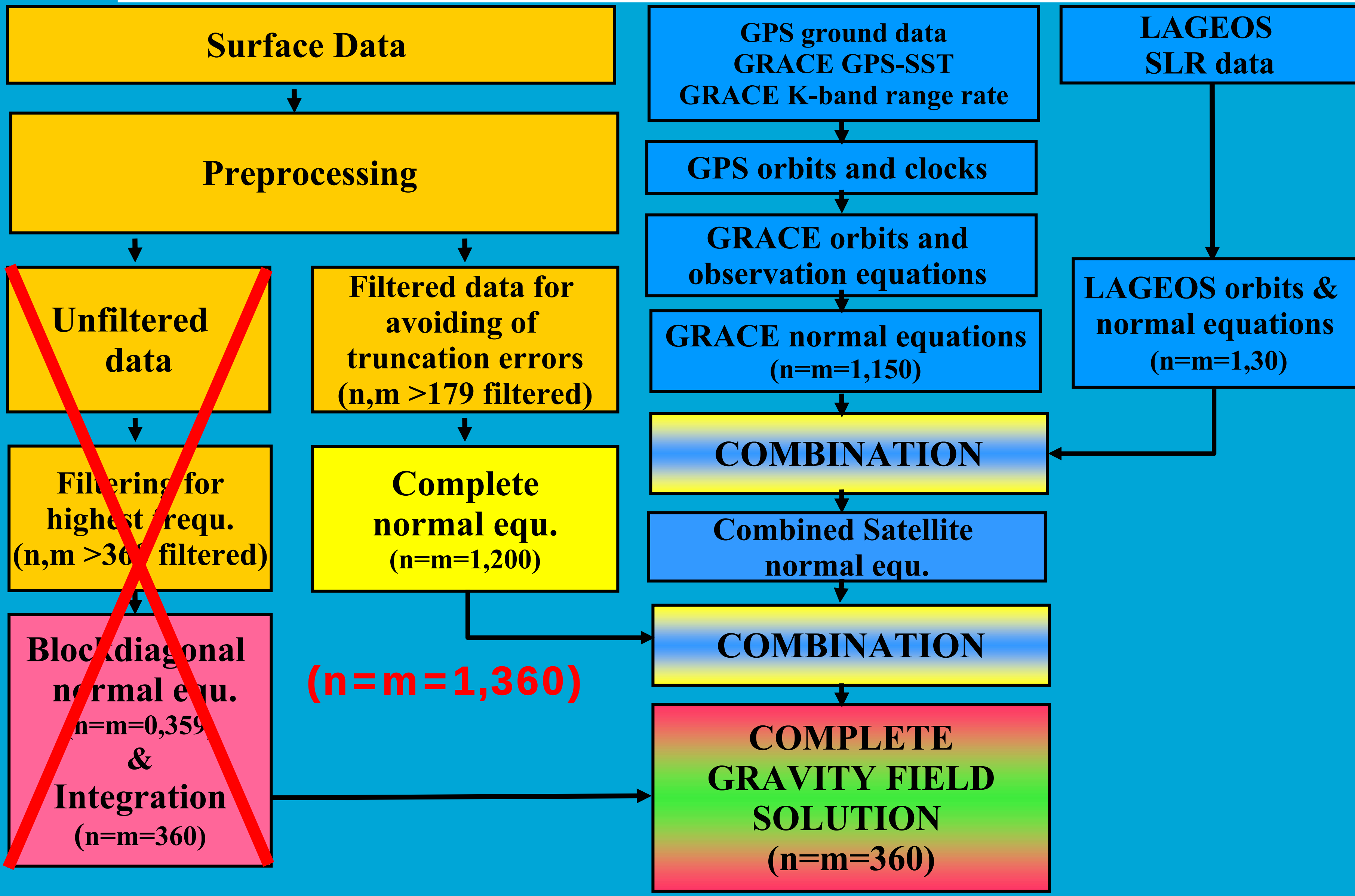
NIMA ship-borne gravity anomalies over water depths less than 2000 m.
(white contourline)

Surface data sets for EIGEN-GL05Cp

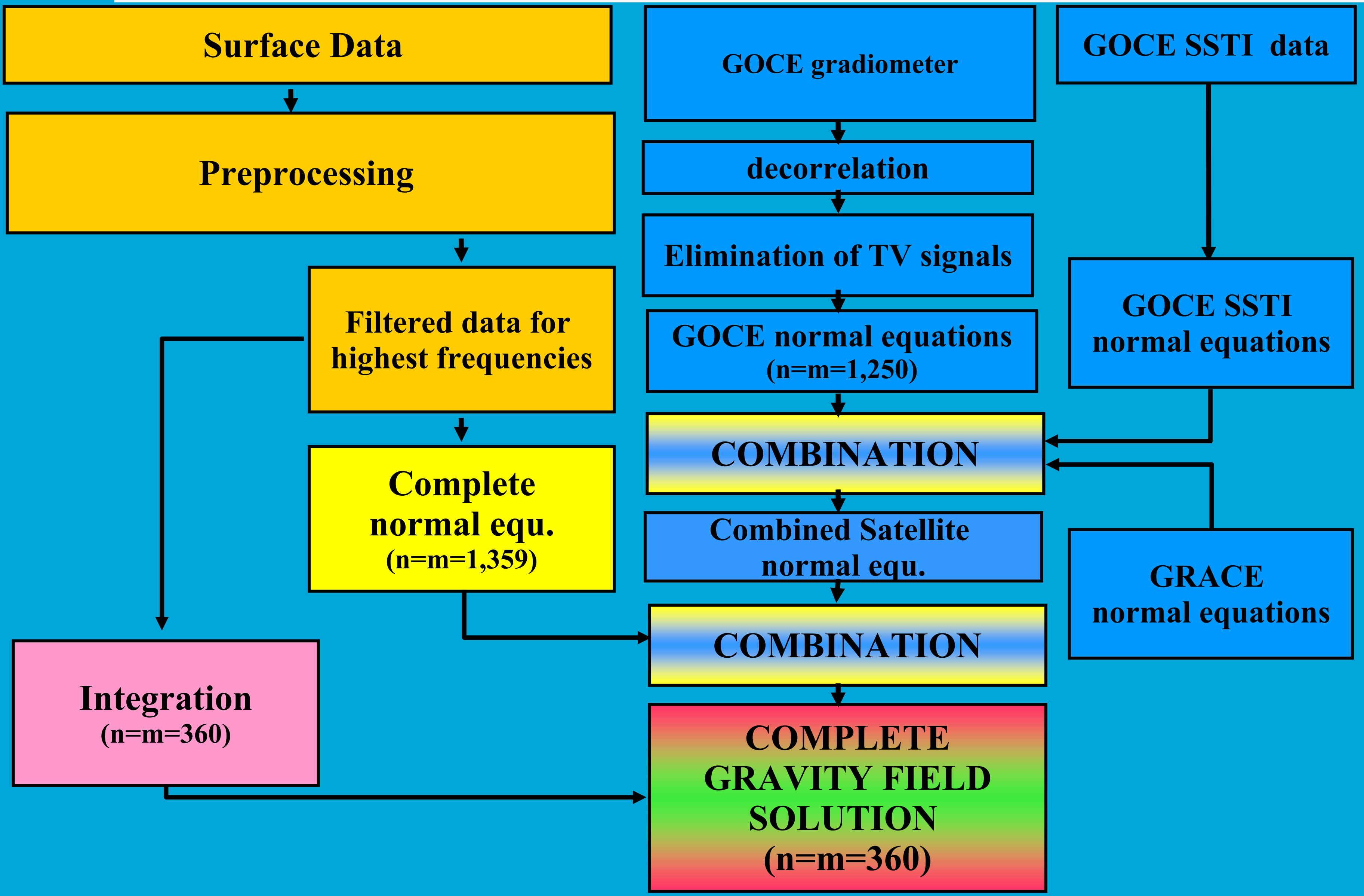


Data gaps filled with GRACE03S-solution.

Strategy for Combination in the EIGEN models



How it will be done for GOCE



Combination scheme of EIGEN-GL05Cp.19

contribution to the solution, full normal matrix:

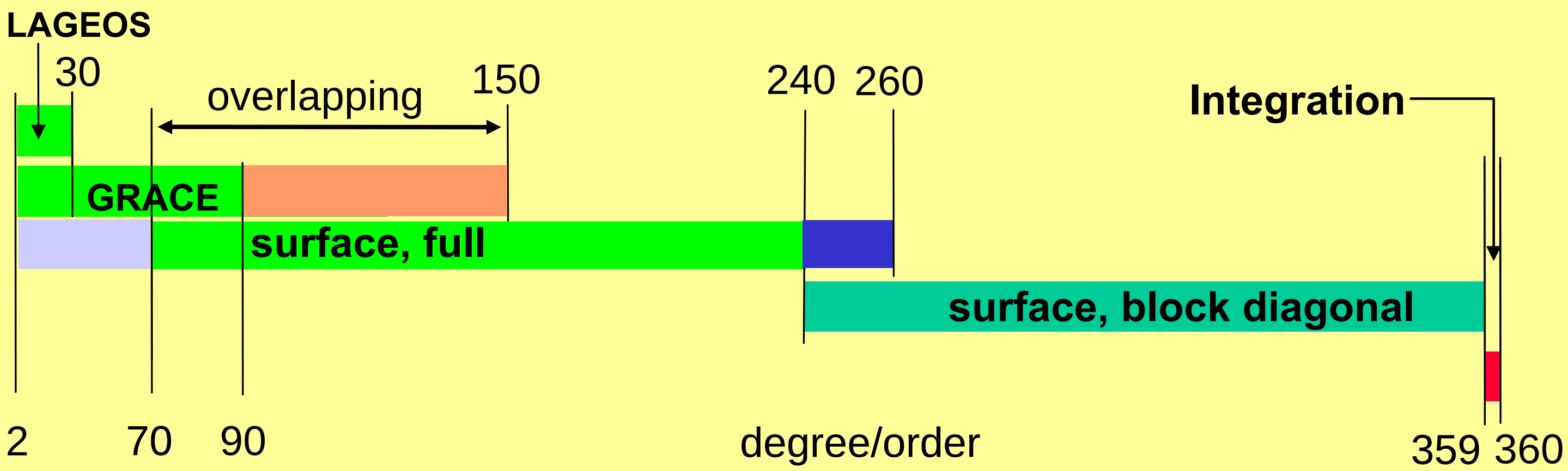
kept separately and bound together with the surface data using constraints):

kept separately (reduced from the full normal matrix):

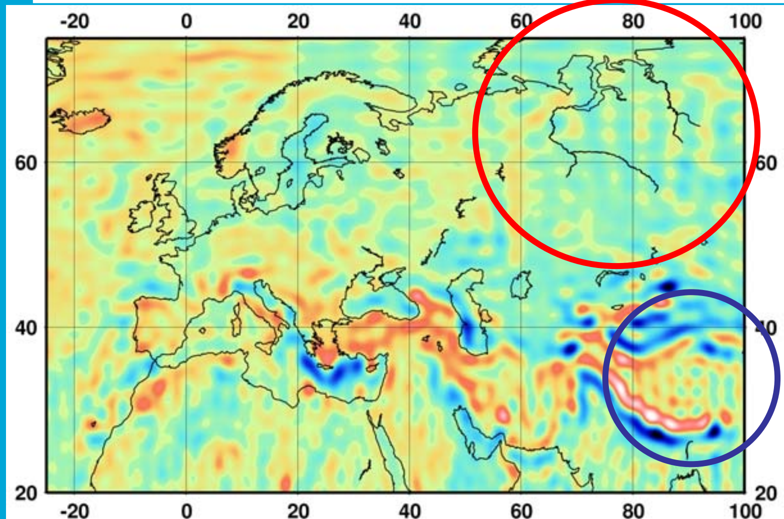
not used (low-pass filtered in order to avoid truncation errors) :

contribution to the solution, block diagonal matrix:

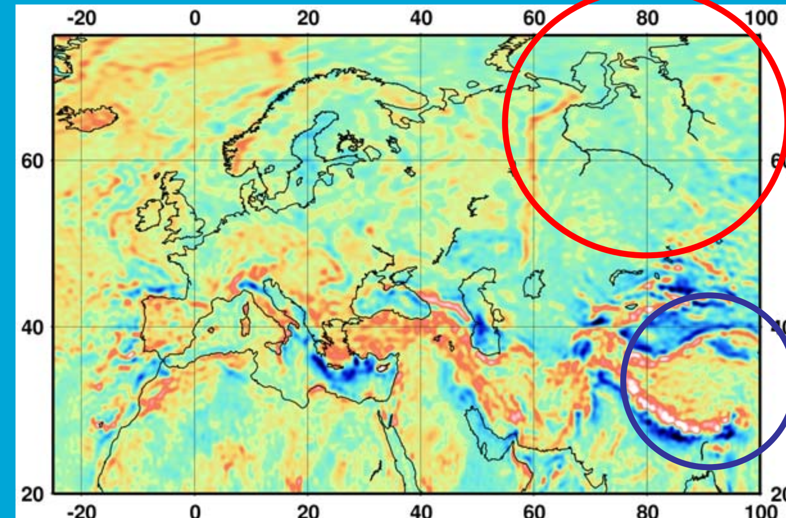
contribution to the solution, numerical integration:



Gravity field models: Quality enhancement (reduction of stripes) and increase of the spatial resolution by combination of satellite and surface data:

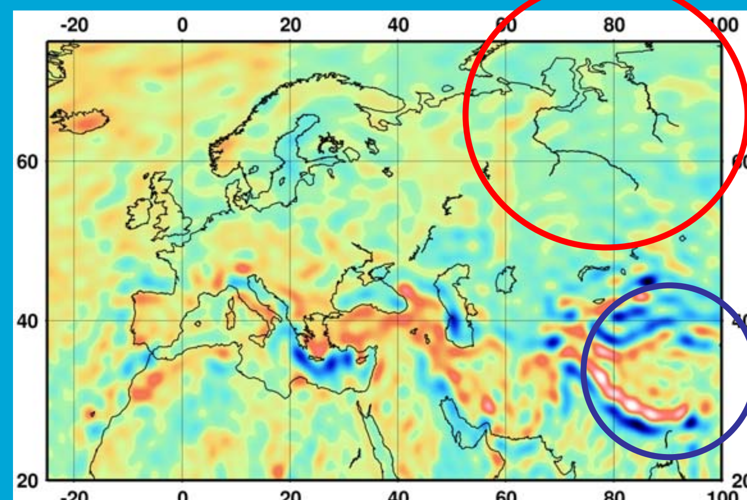


1) GRACE-only until d/o 150 (EIGEN-GL04S1)



3) GRACE + surface until d/o 360 (EIGEN-GL04C)

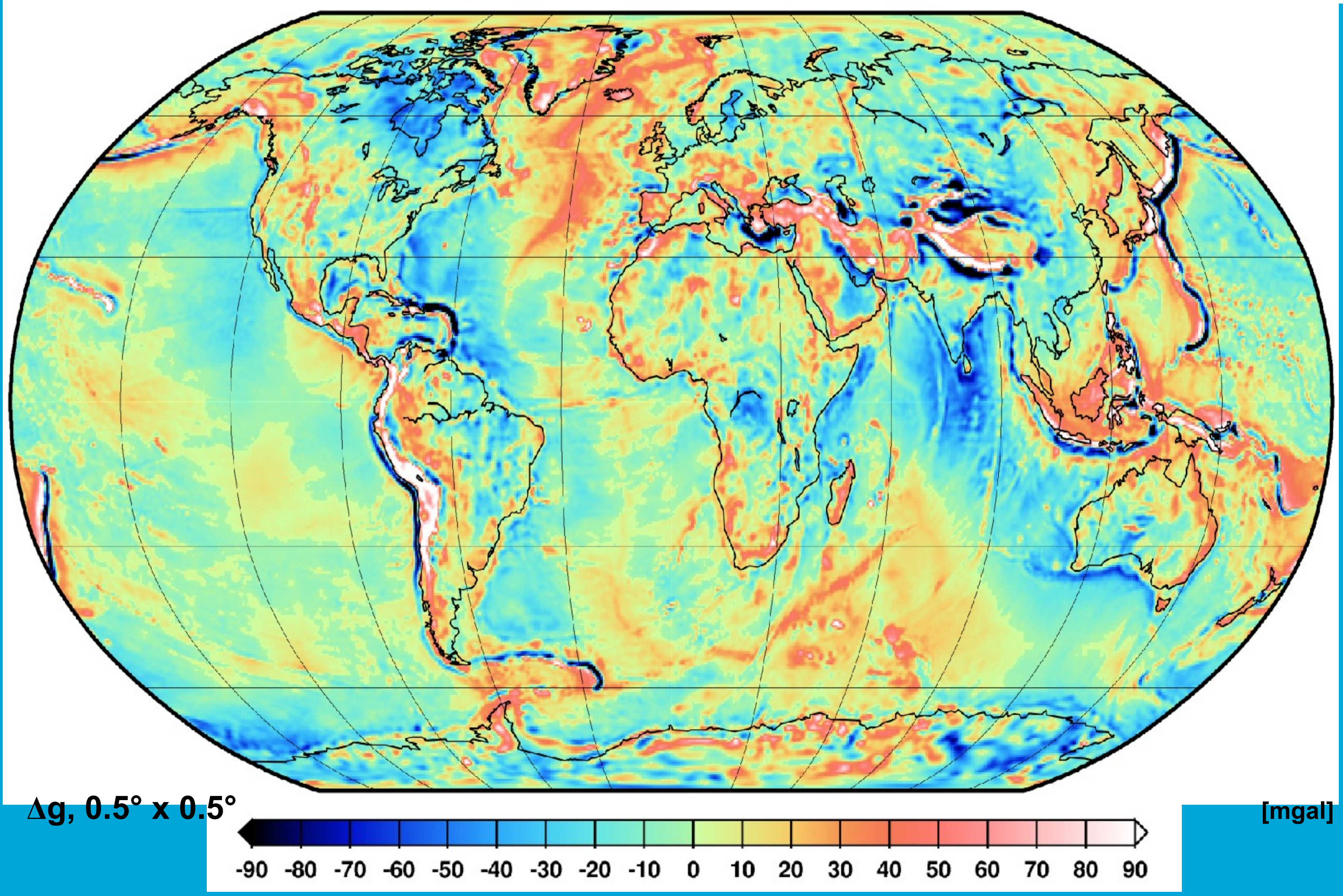
2) GRACE + surface
until d/o 150
EIGEN-GL04C)



Gravity anomaly

GFZ POTSDAM HELMHOLTZ GEMEINSCHAFT ICGEM

EIGEN-GL05Cp gravity anomaly

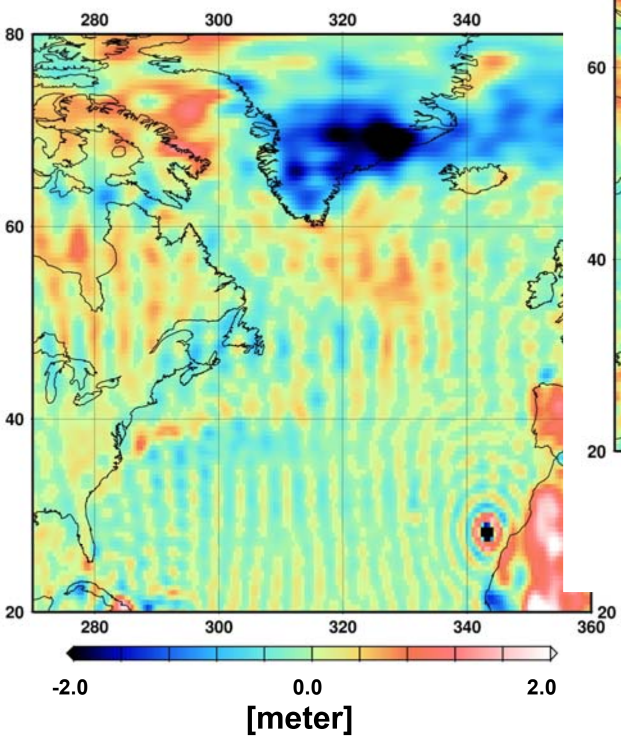


Improvement of EIGEN-models

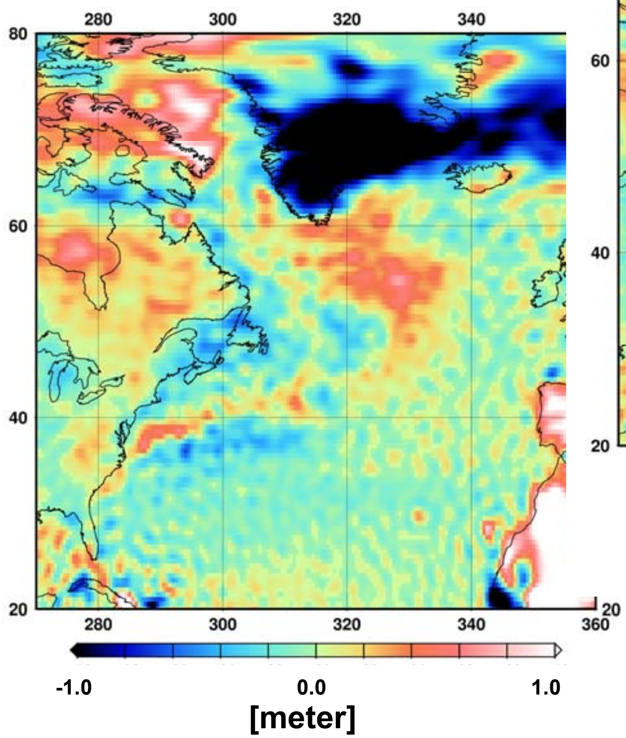
Geoid height differences vs. a global ground data only solution

$\Delta\zeta$, 0.5° x 0.5°

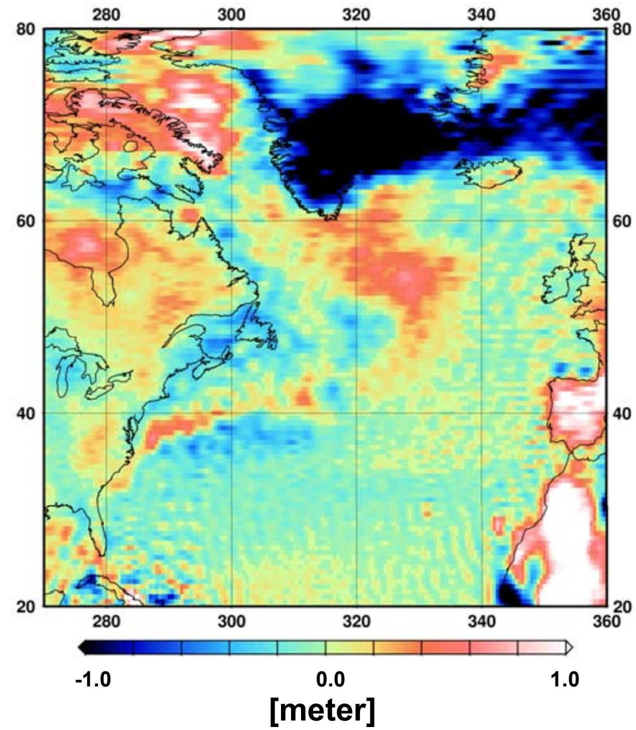
EIGEN-CG03C



EIGEN-GL04C



EIGEN-GL05Cp.18



$\longrightarrow$ Reduction of the meridional stripes !!

Improvement of EIGEN-models

Geoid height differences vs. a global ground data only solution

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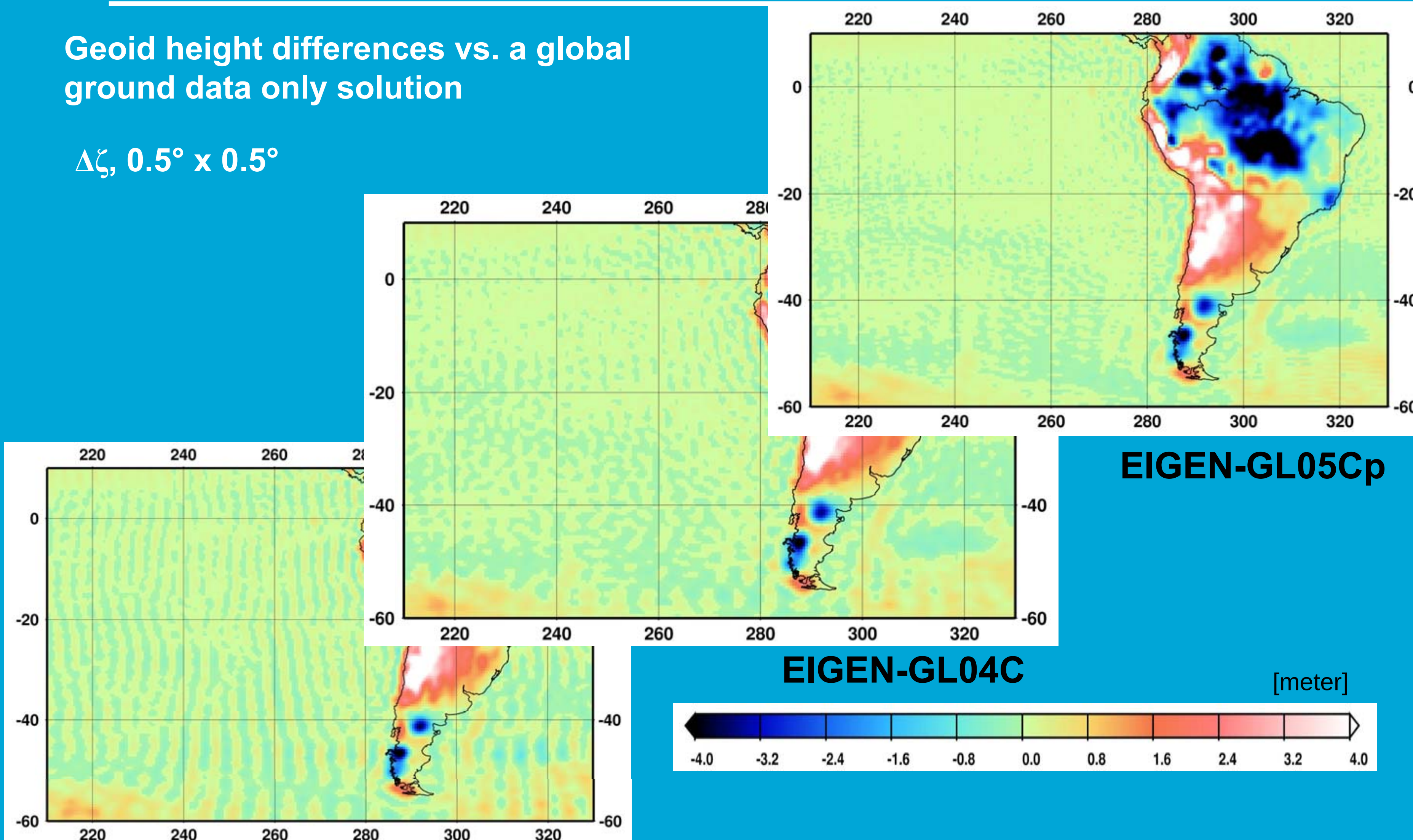
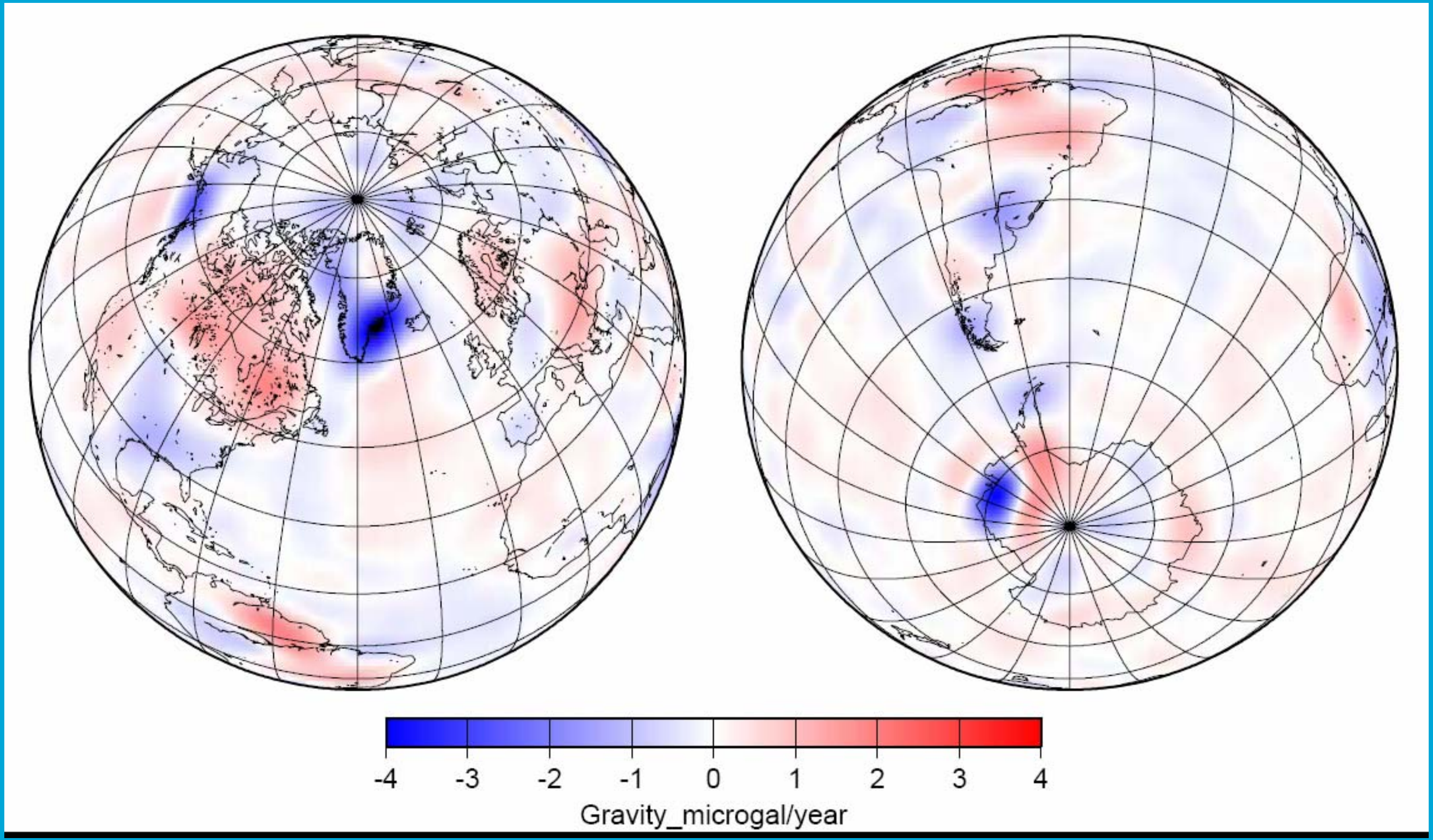


EIGEN-CG03C

$\longrightarrow$ Reduction of the meridional stripes !!

Temporal Gravity

Secular trend GRACE RL04 products GFZ GSM fields, de-striped



Max 4…6μGal/year, wrms 0.4…0.6 μGal/year)

Formal accuracy depends on averaging radius/de-striping method

Here ~ 0.03μgal/year, **degree-1 missing**