Gravity adjustment

by

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1. Absolute and relative gravity measurements

Two main principles for measuring gravity

Absolute

Principle: Free-fall

Instrument: FG-5

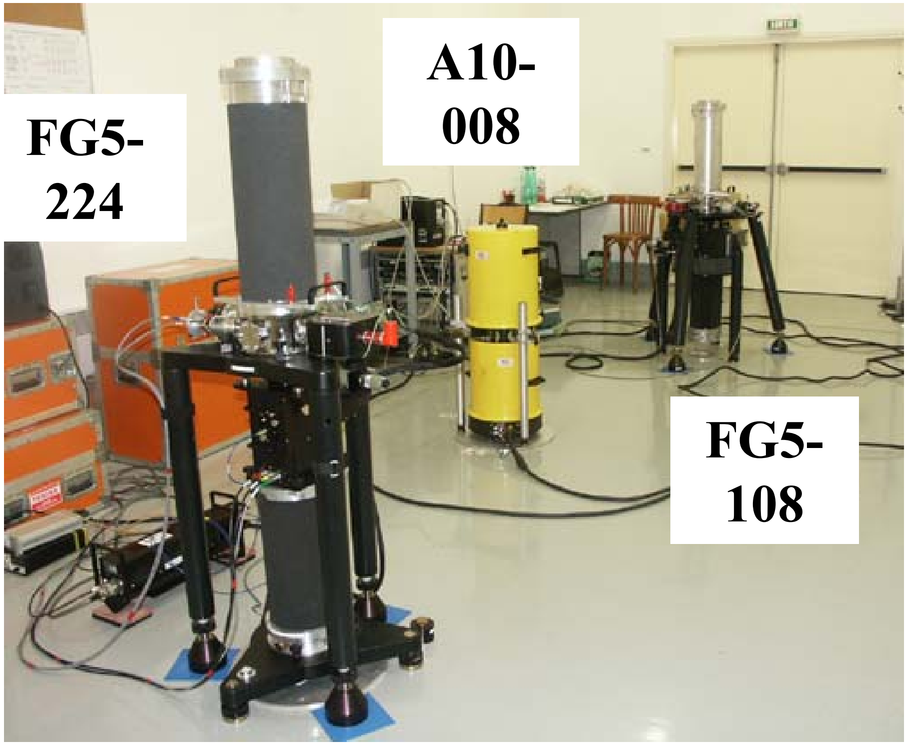
Relative

Principle: Spring force to counterbalance the force of gravity

Instrument: LaCoste & Romberg

1. Absolute and relative gravity measurements

Absolute gravity meters - instrumentation



FG-5

accuracy: ~2 μGal (2 days; 1500-3000 drops)

operation: can be operated in a tent but is more suited for a laboratory.

A10

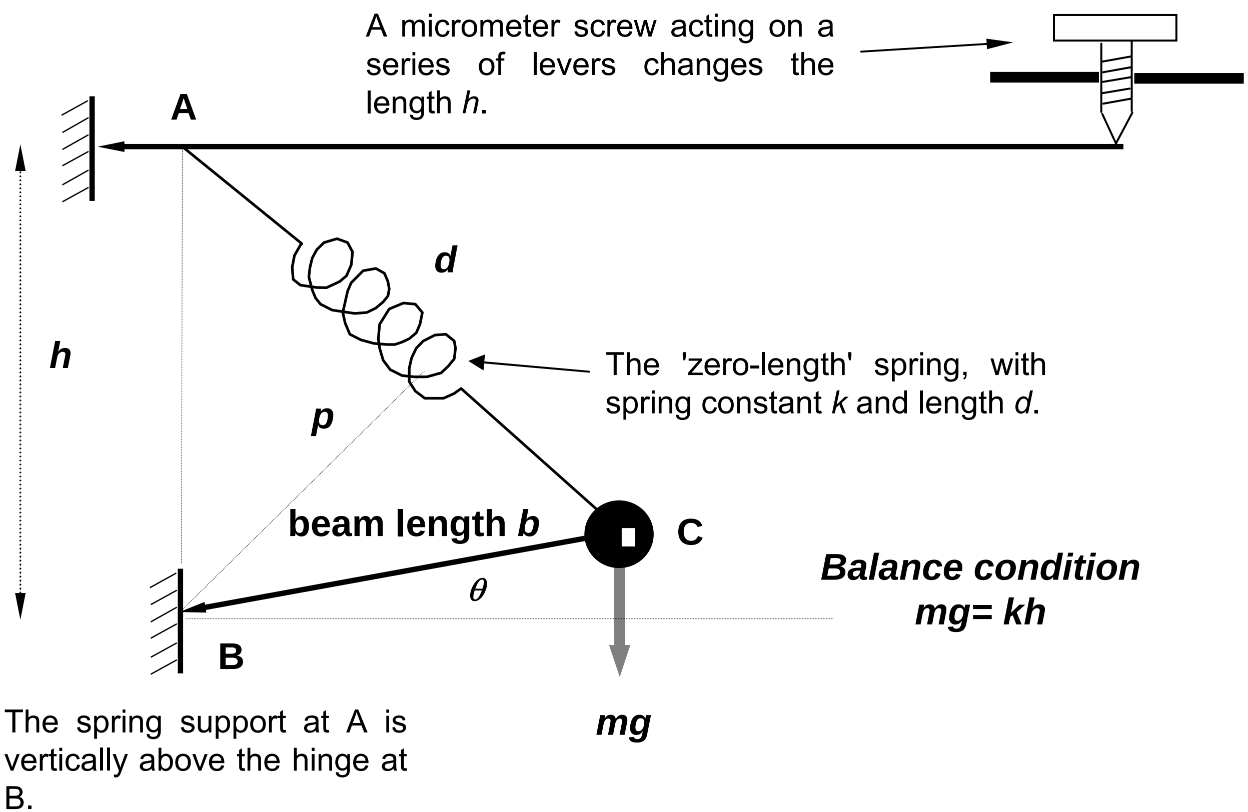
accuracy: ~5 μGal (1-1½ hr; 400-600 drops)

operation: outdoor instrument.

A10-019 purchased by DTU Space (July 2008)

1. Absolute and relative gravity measurements

LaCoste and Romberg relative gravity meters – “zero length” spring



Secret alloy in the super-elvinar group with ultra-low expansion coefficient and thermo-elastic coefficient

Thermo-stated oven to 0.01 K

Displacement of spring support point reproducibly measurable to ½ diameter of a hydrogen atom

Scale depends on uniformity of the screw thread.

Research accuracy ~ 1 μGal

1. Absolute and relative gravity measurements

A state-of-the-art relative gravity meter Scintrex CG-5

Sensor type: Fused Quartz using electrostatic nulling

Standard field repeatibility: <5 μGal

Range without resetting: 8000 mGal

Automatic tilt compensation. Range: +/- 200 arc/sec

Weight: 8 kg

Dimensions: 30 cm (H) x 22 cm x 21 cm

Residual long term drift: < 0.02 mGal/day

Other features: tidal correction; automatic data storage;

GPS antenna can be mounted.

1. Absolute and relative gravity measurements

Relative measurements, calibration and absolute gravity reference

Relative gravity meters like the LaCoste & Romberg instrument only measure a gravity difference.

They do so in arbitrary units e.g. micrometer dial turns (counter units, CU).

$$g_1 - g_0 = k\,(\Theta_1 - \Theta_0)$$

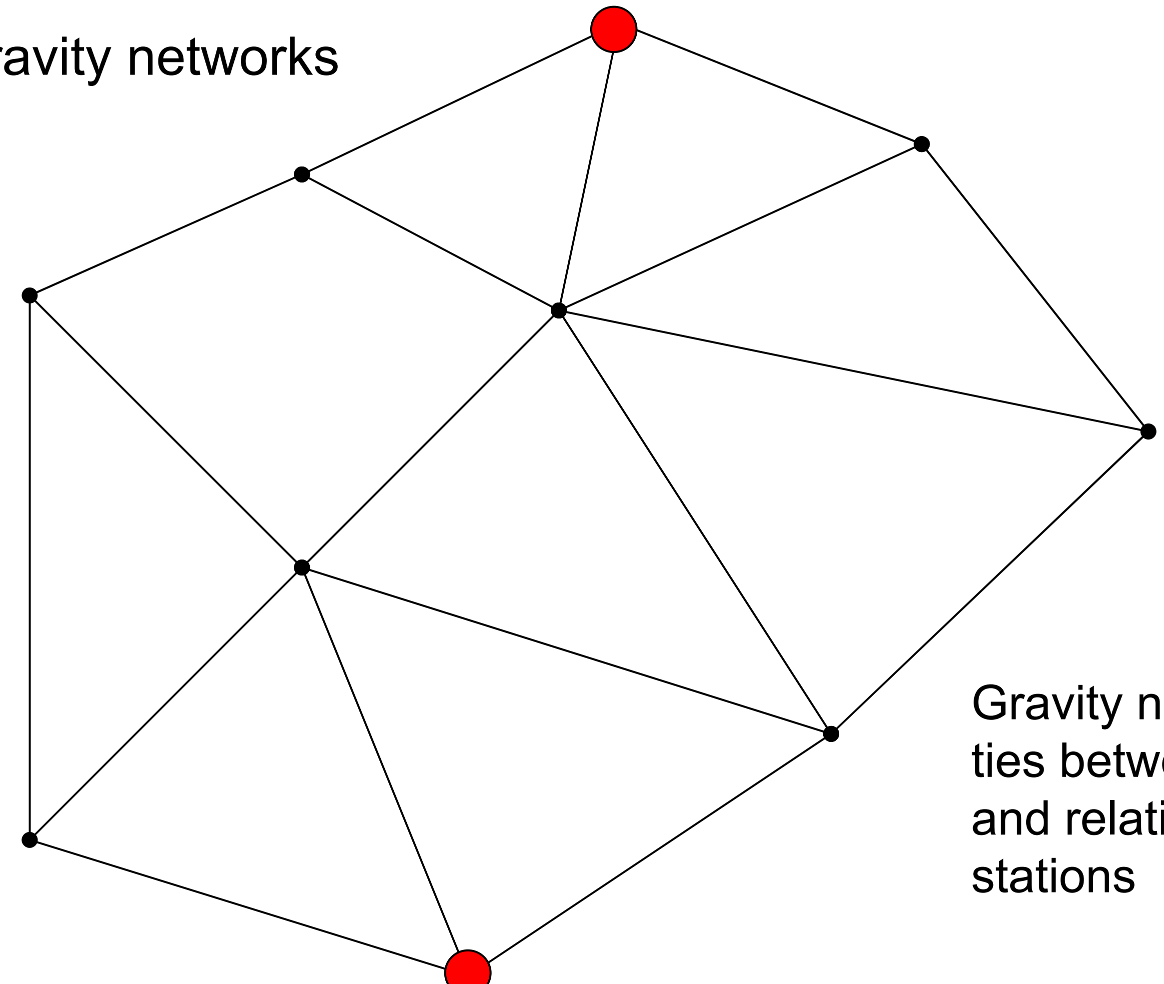
They need calibration – finding *k* – and a reference point where gravity, g_0, is known. For research instrument, *k* can be known to better than 1 in 10^5

Gravity reference sites have been established globally e.g. the 1971 International Gravity Reference Net has an accuracy generally better than 50 μGal.

The previous 1958 Potsdam reference system was offset by 15 mGal
IGSN71 is being replaced by stand-alone absolute gravity sites.

1. Absolute and relative gravity measurements

Gravity networks



Gravity network constructed with ties between absolute (red dots) and relative (black dots) base stations

Minimise closure errors around polygons

1. Absolute and relative gravity measurements

Gravity networks-DK

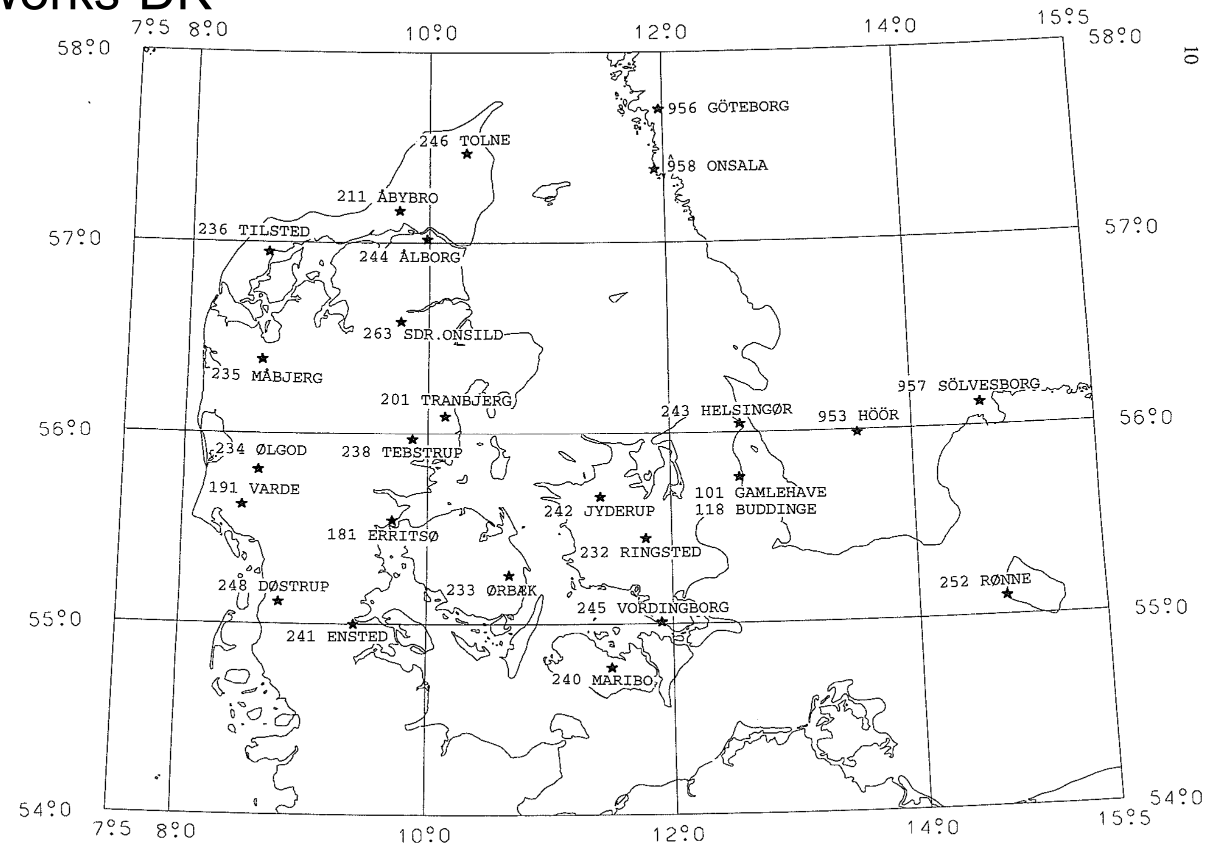


Fig. 1. Danish precise gravity reference network stations.

10

Andersen, O.B & R. Forsberg: Danish precision gravity reference network, KMS Skrifter 4 række bind 4, 1996.

1. Absolute and relative gravity measurements

Gravity networks-DK: repeated relative measurements yield better accuracy

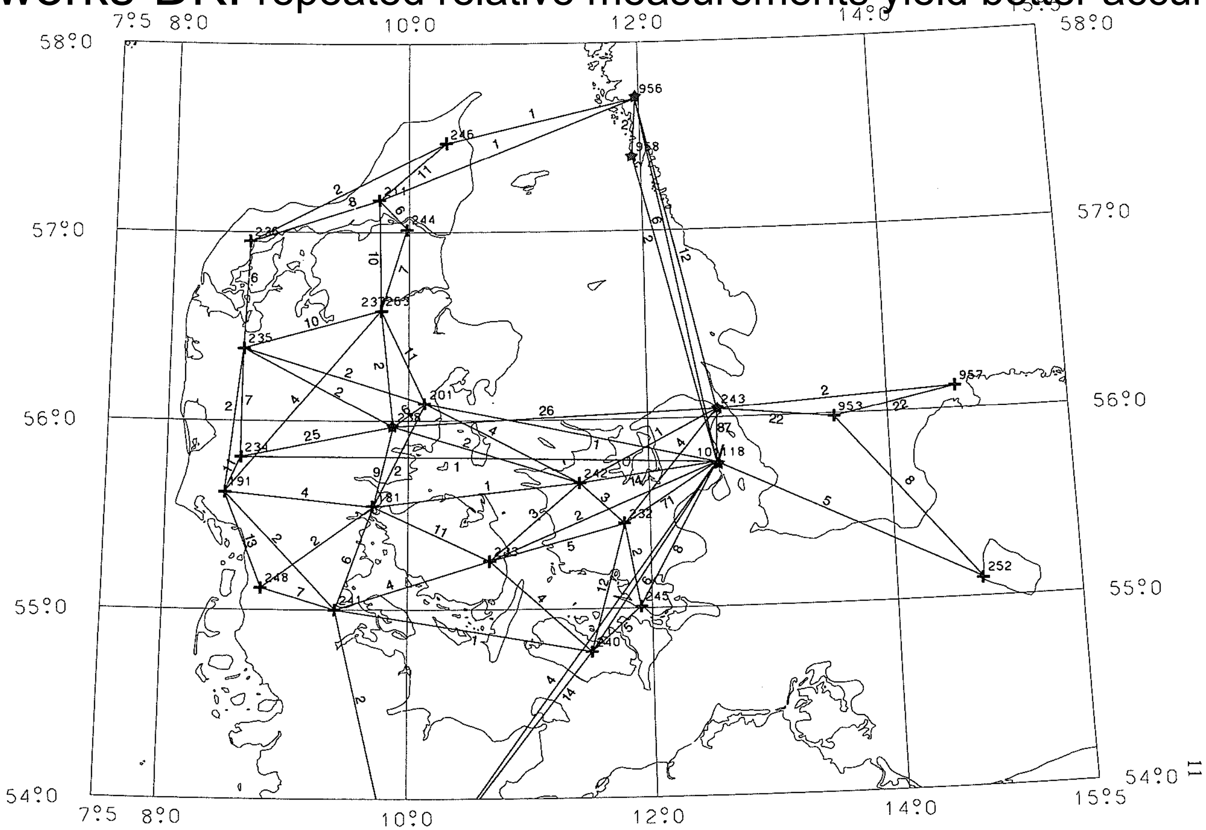
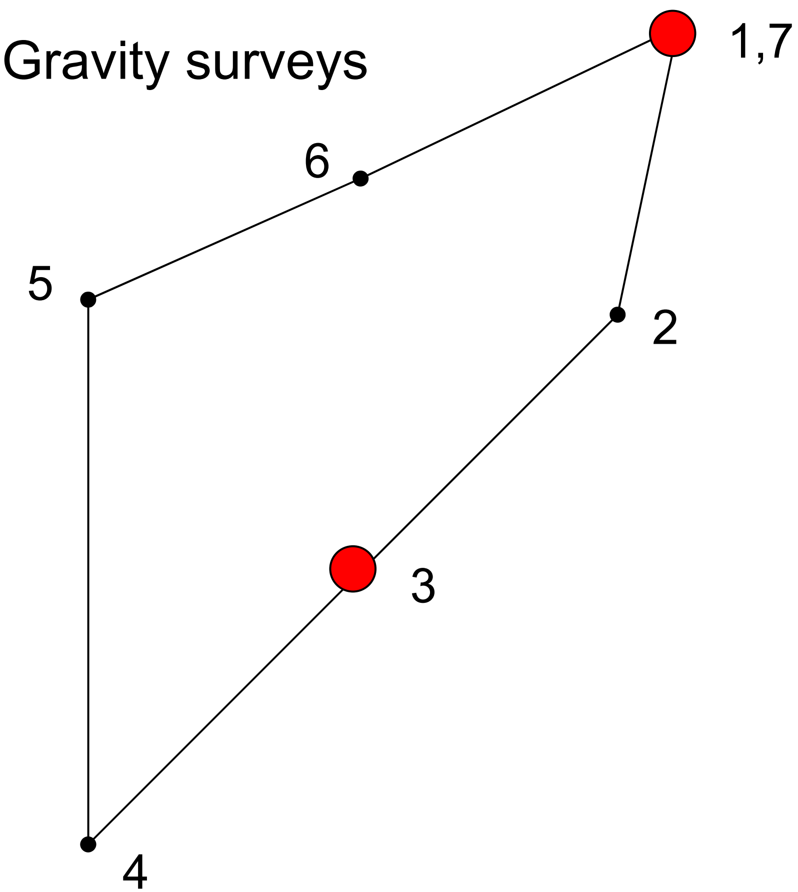


Fig. 2. Number of gravity ties in the gravity network.

Andersen, O.B & R. Forsberg: Danish precision gravity reference network, KMS Skrifter 4 række bind 4, 1996.

1. Absolute and relative gravity measurements

Gravity surveys



In principle, a relative gravity survey includes at least one station (red dot) with known absolute gravity value and a number of new stations (black dots)

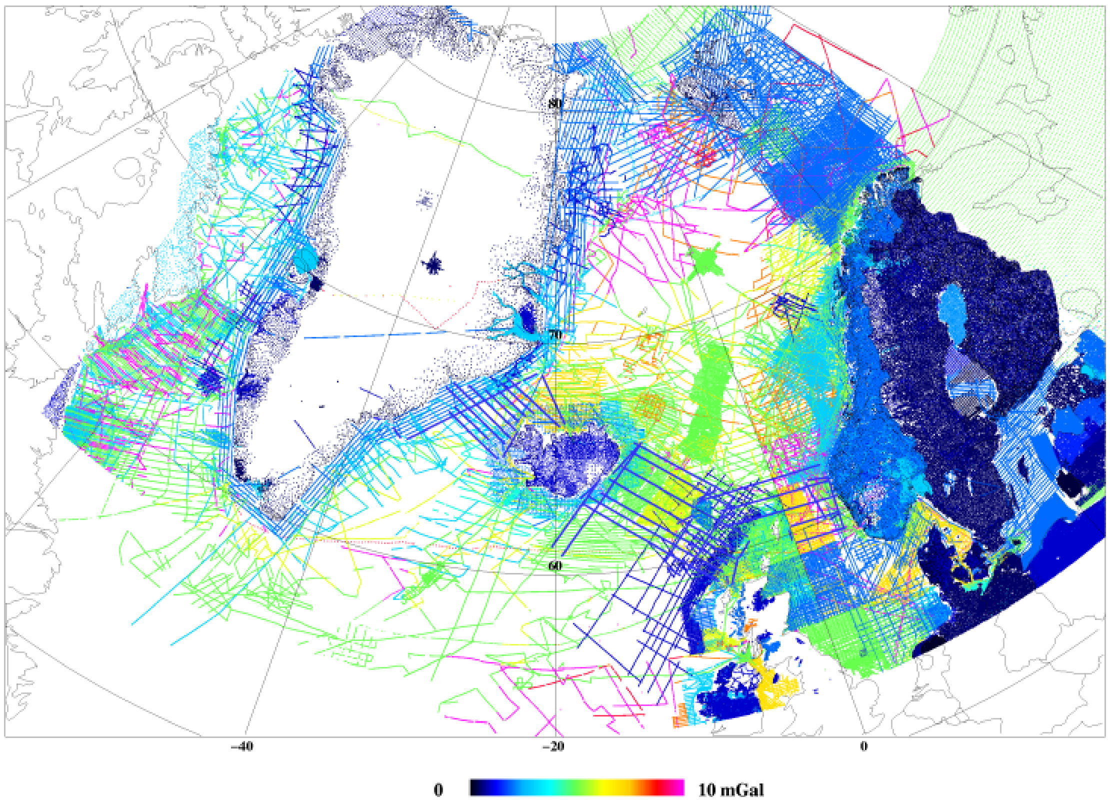
Additional stations with known gravity (red dot) can be included to control the survey

The survey is further strengthen by repeating the measurements for some new stations at different times.

The numbers denote the sequence of the measurement in the survey

1. Absolute and relative gravity measurements

Gravity surveys and regional gravity data base



More than 1.5 mill gravity points

NKG (Nordic Geodetic Commission) gravity DB including national DB for DK

Administrated by

DTU-Space, Copenhagen, DK
Statens Kartverk, Hønefoss, N

DK: land: approx. 1 gravity station pr. km^2; sea: approx. 1 gravity station pr 25 km^2

2. Adjustment software for relative gravity surveys

Adjustment software - overview

GRREDU: corrects for the tidal effects (attraction of Sun & Moon)
converts from counter units (CU) to mGals
input: raw gravity observations in CU, time, lat/long
output: (tidal attraction) reduced gravity observations

GRADJ: adjustment of reduced relative gravity observations
using one or more absolute gravity value
input: (tidal attraction) reduced gravity observations
output: absolute gravity values for all stations

GRANO: compute gravity anomalies
input: absolute gravity values g for all stations, H
output: gravity anomaly

2. Adjustment software for relative gravity surveys

Basic concepts – correction for the tidal forces

Gravity at the Earth's surface **changes with time** because of the attraction of the Sun and the Moon. Regarding both Sun and Moon as point masses, and for the fixed location P(φ, λ, H) on the Earth's surface, the relative distances of P to the CM of the Sun, $r_{sun}(t)$, and the CM of the Moon, $r_{moon}(t)$, changes with time at different frequencies.

This externally generated and time varying part of the gravity field is measured by the relative gravity meters, but not regarded as a part of the Earth's gravity vector:

$$\mathbf{g} = \nabla W \equiv \left(\frac{\partial W}{\partial X}, \frac{\partial W}{\partial Y}, \frac{\partial W}{\partial Z}\right) = \left(\frac{\partial (V+\Phi)}{\partial X}, \frac{\partial (V+\Phi)}{\partial Y}, \frac{\partial (V+\Phi)}{\partial Z}\right)$$

where V is the gravitational potential and Φ the rotational potential

GRREDU includes a subroutine that can compute the correction for the tidal forces for a given location P(φ, λ, H) on the Earth's surface, a given date, and the time of the day (converted to UTC-time).

2. Adjustment software for relative gravity surveys

Basic concepts – correction for the tidal forces

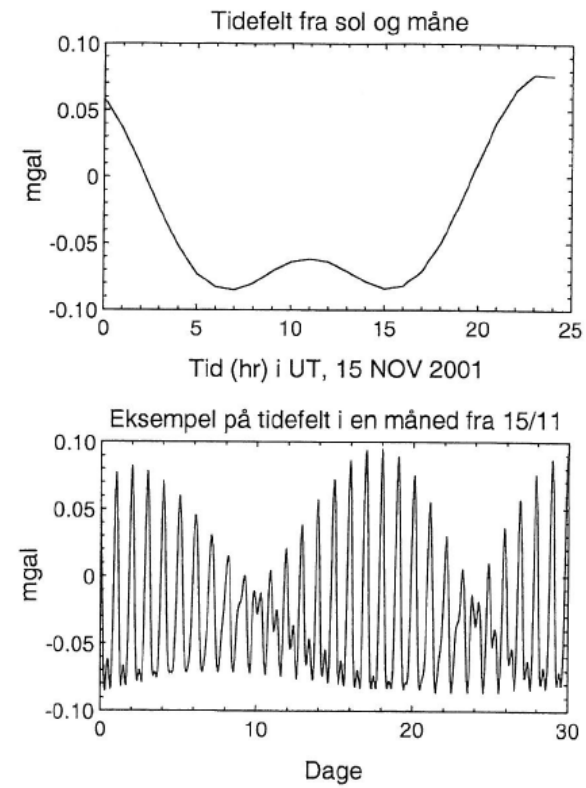
Furthermore, the Earth as a whole does not behave like a rigid body.
It is **deformed** by the tidal forces. The effect of the Earth's deformation caused by the tidal forces (through Love's numbers h and k) is included in the tidal correction, but not the so called permanent tide effect (the constant deformation of the Earth caused by the presence of Sun and the Moon).

The gravity data thus reduced are in the so called **zero-tide** system (which theoretically is the most convenient for the geoid computations).

2. Adjustment software for relative gravity surveys

Basic concepts – correction for the tidal forces

Example of the tidal correction for Møns Klint,DK



Notice the magnitude of the tidal signal and its variability during the day.

2. Adjustment software for relative gravity surveys

Basic concepts – other time varying signals and noise

The relative gravimeter registers both **the geophysical signals** affecting the gravity station at the time of measurement and the sensor signals related to the gravity meter. These sensor signals are viewed as a “**sensor noise”** that must be modeled and removed from measurements before the gravity differences can be modeled.
Two most important types of the ”sensor noise” (included in GRADJ) are:

gravimeter drift – a constant (in time) linear change of the gravimeter properties during a relative survey (LCR: 0.5-1 mGal/month, CG-5 <0.02 mGal/day).

tare – a sudden jump in the gravimeter level for the same station related to the transportation (bumps and jerks). The drift is assumed to be unaffected.

Other types of “sensor noise” (only relevant for microgravity surveys) are caused by pressure- & temperature changes and the mechanical periodic nulling dial errors.

2. Adjustment software for relative gravity surveys

Basic concepts – other time varying signals and noise

The importance of the **geophysical signals** affecting the gravity station is closely related to the measurement noise.

The more accurate you measure the more you see!!! (Consequently, geophysical signals which can be neglected in relative gravimetry can be of some importance in the systematics of the absolute gravimetry)

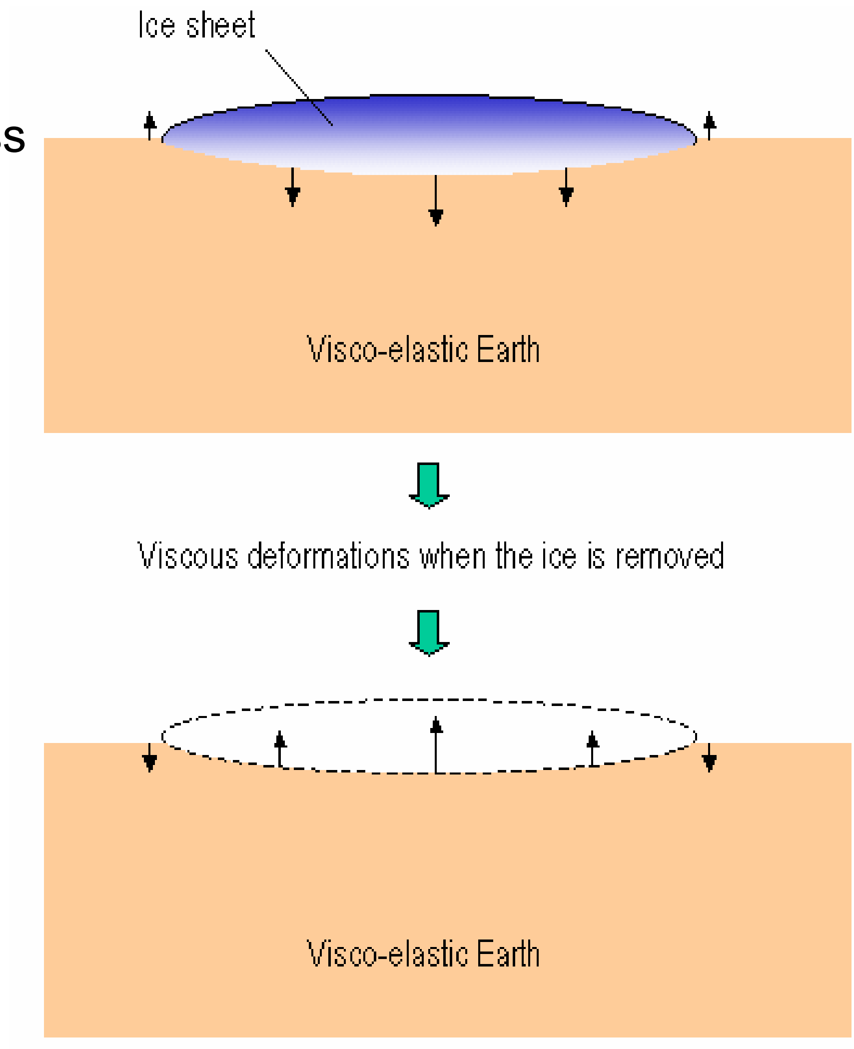
Both **the magnitude** of these signals and their **time duration** is relevant.

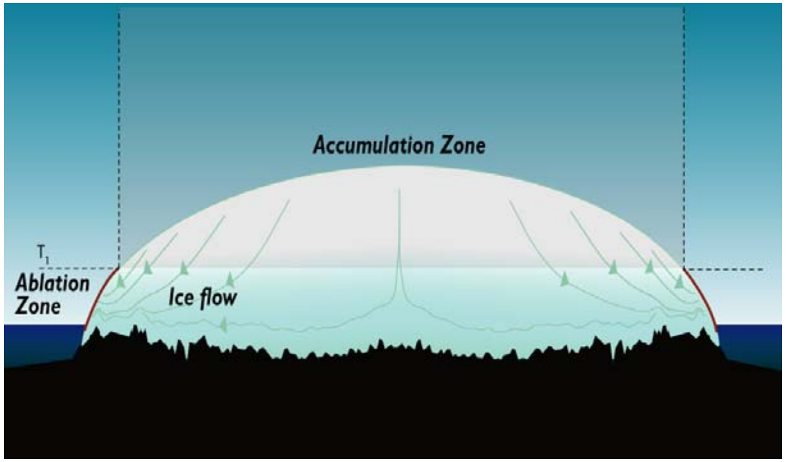
Seismological signals (surface waves)
Microseism and ocean loading. (Traffic/road construction)
Geodynamic signals: Land uplift (Scandinavia, Greenland), ice melting (Greenland).
Hydrology (snow, rain, ground water).
Air pressure loading (both direct mass attraction and surface deformation)
Deviation of the instantaneous Earth's axis of rotation to the mean axis of rotation.

2. Adjustment software for relative gravity surveys
Geophysical signals – land uplift

Measuring the ice by GPS – ”loading”

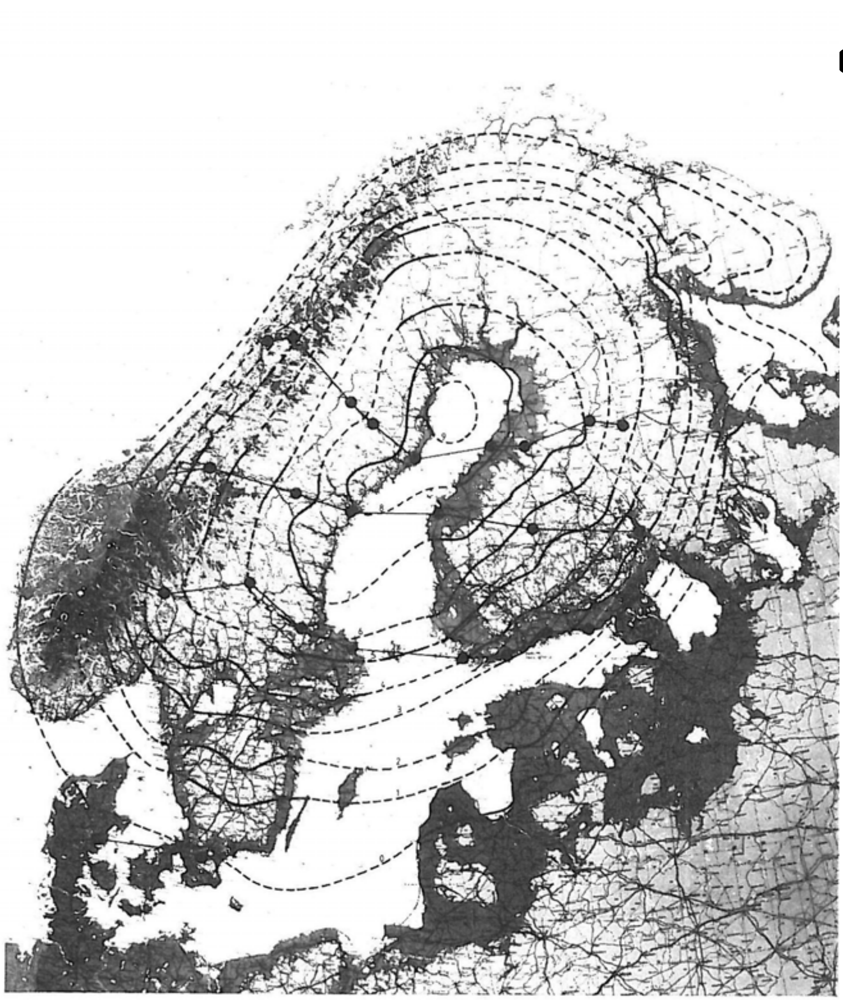
- Melting of glaciers in Greenland: mass loss
- Less mass => land uplift
  Earth's response is visco-elastic:
  - Short term elastic effect
  - Long term ”viscous” effect (10.000 år+)
- Elastic effect is up to 10 mm/year
- Measured land uplift is a function of the historical changes of ”ice load”



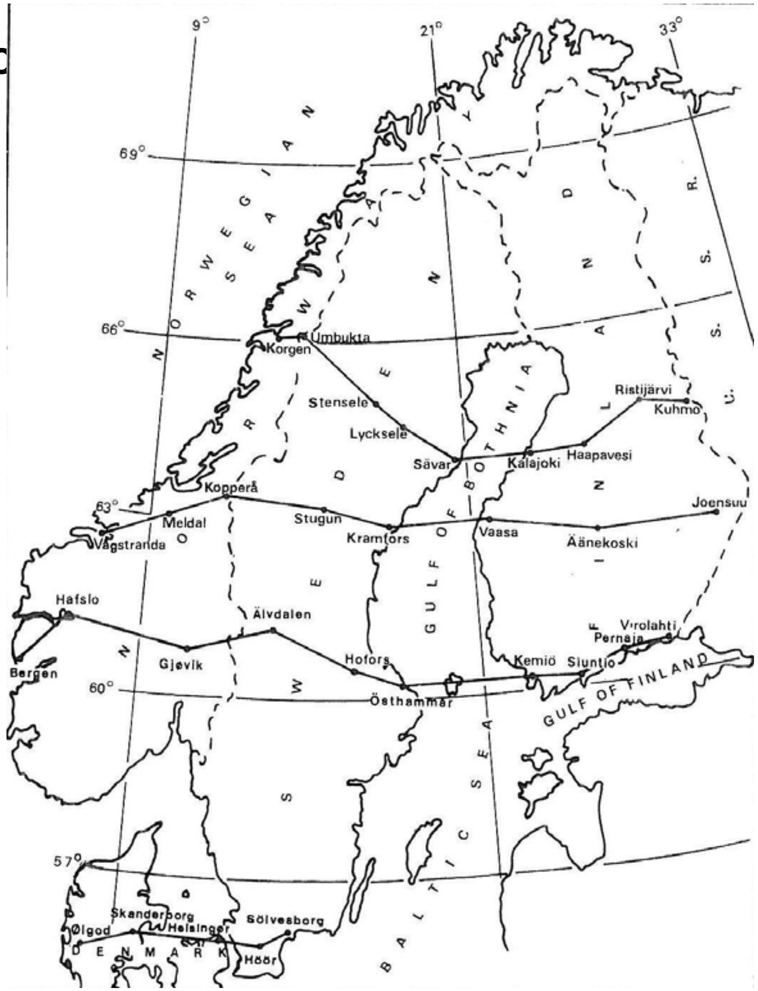


2. Adjustment software for relative gravity surveys

Scandinavia: land uplift – Nordic relative gravity lines



land uplift mm/year



land uplift gravity lines

2. Adjustment software for relative gravity surveys

Observational equations

The main idea: to describe mathematically all relevant (i.e. within noise limits) factors that affect the relative gravity measurements. (Not relevant factors can be neglected).

GRREDU: convert raw gravimeter measurements from CU to mGals and reduce them for the effect of (external) tidal forces.

But first:
Order the gravity stations in the OBSERVATION file in a time-wise sequence.

Index i=1,2,…,N, refers to $t_1 < t_2 < \ldots < t_N$ where N is the number of relative measurements in a survey.

$$G_i^{red} = s_i \times G_i^{raw} - g_{tide}(\varphi_i, \lambda_i, H_i, t_i)$$

where G_i^{red} is the reduced observation in mGal (not the absolute gravity!!!)
G_i^{raw} is the raw gravimeter reading, s_i is the scale factor (from CU to mGal)
and $g_{tide}(\varphi_i, \lambda_i, H_i, t_i)$ is the tidal correction

2. Adjustment software for relative gravity surveys

Basic concepts – use of LaCoste & Romberg calibration tables

lodel G&D Meter — Primary Information

Converting the Counter Reading to Milligals

Let us illustrate conversion of meter readings to milliGals with some examples.

Model G

If the counter reading is *2654.32*, look at the calibration table for your meter. Remember that each meter has its own unique table.

Portion of calibration table

Counter Reading	Interval Factor	Cumulative Value
2500	1.00794	2519.42
2600	1.00799	2620.21
2700	1.00805	2721.01
2800	1.00811	2821.82

Divide the reading into two parts.

2600.00	2620.21
+ *54.32*	+ 54.75
2654.32	2674.96

Interval factor x reading within interval

1.00799 x 54.32 = 54.75

Each LC&R gravimeter has its own unique table with interval factors s_i

These unique tables must be inserted into GRREDU

The tables are provided (and stored) by the factory.

Notice that the conversion from CU to mGal is non-linear.

2. Adjustment software for relative gravity surveys

Instrument height correction (optional)

In practice, it is advisable (each time) to measure the instrument height ΔH e.g. to the top of the housing of a LaCoste & Romberg gravity meter.

In principle, the instrument sensor is located in some fixed height with respect to the gravimeter housing (e.g. it is some 5 cm above the base of the housing for a LC&R gravimeter), i.e. the measurement is done in a height $\Delta H^* = \Delta H - \Delta H_s$, where ΔH_s is the fixed sensor height to the top of the gravimeter housing. With the relative instruments we measure:

$$\Delta g = g(H_2 + \Delta H_2^*) - g(H_1 + \Delta H_1^*) \approx g(H_2) - g(H_1) - 3.086\ \mu\text{Gal/cm}\ (\Delta H_2^* - \Delta H_1^*)$$

Notice that: $\Delta H_2^* - \Delta H_1^* = \Delta H_2 - \Delta H_1$, i.e. independent of sensor height

For $\Delta H_2 = \Delta H_1$, $\Delta g = g(H_2 + \Delta H_2^*) - g(H_1 + \Delta H_1^*) \approx g(H_2) - g(H_1)$

G_i^{red} can be further corrected after GRREDU for the relative differences in the station heights. This is particularly relevant for microgravity surveys.

2. Adjustment software for relative gravity surveys

Observational equations

G_i^{red} is not the approximation to the absolute gravity value $g(P(t_i))$ for the gravity station $P(t_i)$ on the Earth's surface, …. but (for i>1)

$$G_i^{red} - G_{i-1}^{red} = \left(g\left(P\left(t_i\right)\right) - g\left(P\left(t_{i-1}\right)\right)\right) + d_{LC\&R} \times (t_i - t_{i-1}) + T_{i,i-1} + \varepsilon_{i,i-1}$$

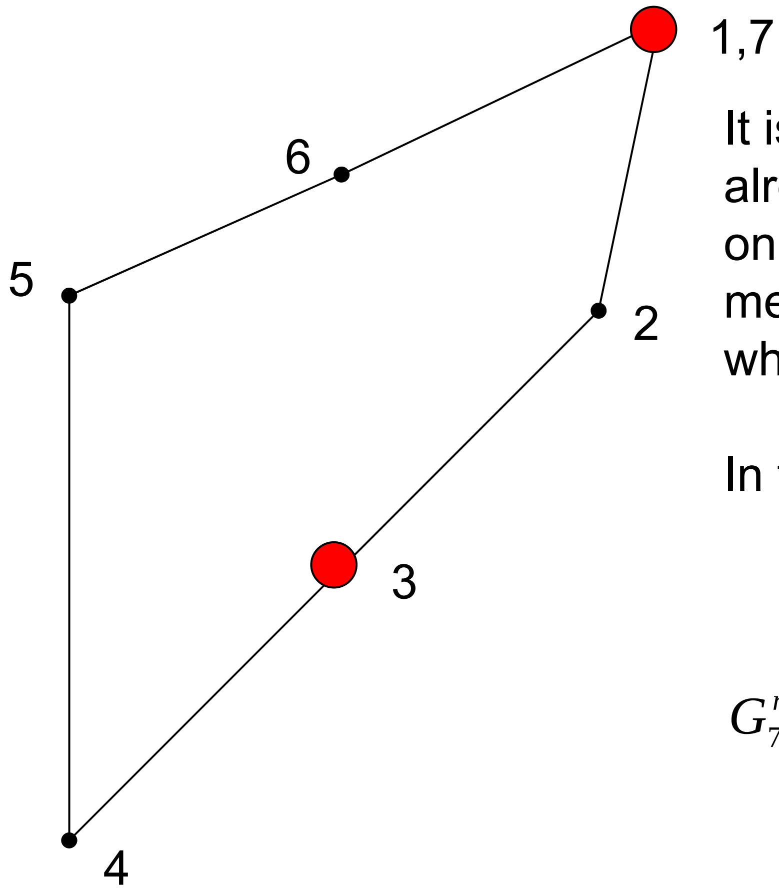
where

$d_{LC\&R}$ is the constant drift of the particular gravimeter for that survey

$T_{i,i-1}$ is the gravimeter tare between t_{i-1} and t_i (in most cases it is 0)

$\varepsilon_{i,i-1}$ is the uncorrelated measurement error reflecting the accuracy of a gravimeter reading

2. Adjustment software for relative gravity surveys



It is important to repeat measurements on already visited gravity station or to measure on known gravity points (red dots). Each such measurement yields an additional equation which strengthen the adjustment result!!!

In the example we have $P(t_1) = P(t_7)$ so that

$$g\left(P\left(t_7\right)\right) - g\left(P\left(t_1\right)\right) = 0$$

$$G_7^{red} - G_i^{red} = d_{LC\&R} \times (t_7 - t_1) + \sum_{i=1}^{7} T_{i,i-1} + \varepsilon_{7,1}$$

Notice! The number of stations is less than the number of measurements

2. Adjustment software for relative gravity surveys

A setup with M gravity stations, one gravimeter and one tare

$$G_i^{red} - G_{i-1}^{red} = \left(g\left(P\left(t_i\right)\right) - g\left(P\left(t_{i-1}\right)\right)\right) + d_{LC\&R} \times (t_i - t_{i-1}) + T_{i,i-1} + \varepsilon_{i,i-1}$$

$$\begin{bmatrix} \vdots \\ G_{k+1}^{red} - G_k^{red} \\ \\ \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 & \left(t_{k+1} - t_k\right) & 1 \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_k \\ g_{k+1} \\ \vdots \\ g_M \\ d_{LC\&R} \\ T_{k+1,k} \end{bmatrix}$$

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

2. Adjustment software for relative gravity surveys

A setup with M gravity stations and two gravimeters

$$
\begin{bmatrix} \vdots \\ G^{red}_{k+1,1} - G^{red}_{k,1} \\ \\ G^{red}_{k+1,2} - G^{red}_{k,2} \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 & \left(t_{k+1,1} - t_{k,1}\right) & 0 \\
0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 & 0 & \left(t_{k+1,2} - t_{k,2}\right)
\end{bmatrix}
\begin{bmatrix} g_1 \\ \vdots \\ g_k \\ g_{k+1} \\ \vdots \\ g_M \\ d_{LC\&R,1} \\ d_{LC\&R,2} \end{bmatrix}
$$

$$
\mathbf{y} = \mathbf{A}\mathbf{x}
$$

2. Adjustment software for relative gravity surveys

A setup with M gravity stations and fixed $g_k = g_{known}$

$$
\begin{bmatrix} \vdots \\ g_{known} \\ \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_k \\ g_{k+1} \\ \vdots \\ g_M \\ d_{LC\&R} \\ T_{k+1,k} \end{bmatrix}
$$

$$
\mathbf{y} = \mathbf{A}\mathbf{x}
$$

2. Adjustment software for relative gravity surveys

GRADJ: Solving for parameter vector **x**

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{T}\mathbf{y}$$

Using a-priori error covariance matrix

$$x = \left(\mathbf{A}^{\mathrm{T}}\mathbf{C}_{\varepsilon}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{T}\mathbf{C}_{\varepsilon}^{-1}\Delta\mathbf{y}$$

2. Adjustment software for relative gravity surveys

GRADJ: Further “complication”. The correction to the scale factors (the calibration tables from factory) can also be estimated! Inclusion of these additional parameters makes the system of equation non-linear, which however can be linearized around initial model $\mathbf{x_0}$. In each iteration step k we have

$$\Delta \mathbf{y}_k \equiv \mathbf{y}_k - \mathbf{y}_{\mathbf{k-1}} \approx \mathbf{A}_k \left(\mathbf{x}_k - \mathbf{x}_{\mathbf{k-1}} \right) \equiv \mathbf{A}_k \Delta \mathbf{x}_{\mathbf{k}}$$

For each iteration step the system is solved like for the linear case

$$\Delta \mathbf{x}_k = \left(\mathbf{A}_k{}^{\mathbf{T}} \mathbf{C}_\varepsilon^{-1} \mathbf{A}_k \right)^{-1} \mathbf{A}_k{}^{T} \mathbf{C}_\varepsilon^{-1} \Delta \mathbf{y}_{\mathbf{k}}$$

2. Adjustment software for relative gravity surveys

GRANO: Compute the gravity anomalies from the adjusted gravity values (GRADJ). It requires the knowledge of the orthometric station heights (which up to now were not used).

Gravity anomaly: $$\Delta g = g(P) - \gamma(Q) = g(H+N) - \gamma(H)$$