Marine Gravity Network Adjustment in the North Atlantic

Addisu Hunegnaw and Roger Hipkin, School of GeoSciences, University of Edinburgh, Edinburgh, EH9 3JW, UK

Abstract. We report the development of software not only to identify and correct those aspects of marine gravity data that are unreliable, but to do so in a way that can be applied to very large, ocean-wide data sets. International compilations of marine gravity, such as those by the International Gravity Bureau (BGI) and the National Geophysical Database (GEODAS) contain some tens of millions of point data. Nevertheless, Lemoine *et al.* (1998) chose not to include any marine gravity when forming the 30' mean gravity anomalies used in the construction of the global gravity model EGM96. They used synthetic anomalies derived from altimetry instead so that no further information about dynamic ocean topography can be deduced. This paper demonstrates that this judgement was not well-founded. First, we fit a high-degree series of Chebyshev polynomials, whose misfit standard deviation is known to each ship-track that is straight-line. Then, network adjustment determines how the gravity datum is offset for each survey. A free least squares adjustment minimises the gravity anomaly mismatch at line-crossing points, using the known standard deviation to weight the estimate for each line. For some 42000 cross-over points, network adjustment reduces the weighted rms cross-over errors from 1.32mGal to 0.39 mGal. This weighted statistic is an indicator of how *long wavelength* geoid distortion accumulates. The unweighted standard deviation, which the adjustment reduces from 4.03 mGal to 1.58 mGal, translates to *local* geoid errors. Similarly the difference between altimetric anomalies and shipborne and airborne data improved, with the adjustment reducing the standard deviation of the differences from 7.17 to 4.53 mGal. The overall mean difference between surface observations and the GRACE model ggm01s evaluated along ship-tracks is 0.009 mGal.

Keywords: Marine gravity, network adjustment

1 Introduction

International compilations of marine gravity, such as those by the International Gravity Bureau (BGI) and the National Geophysical Database (GEODAS) include some tens of millions of point data. Nevertheless, the compilers of the 1996 Earth Gravity Model (Lemoine et al. 1998) decided that marine gravity data were too unreliable to be included and discarded the lot. The purpose of this paper is to mitigate this decision. We report the development of software not only to identify and correct those aspects of marine gravity data that are unreliable but to do so in a way that can be applied to very large, ocean-wide data sets without excessive manual intervention. These corrections are essential for computing a high precision geoid or dynamic sea surface topography.

Although synthetic free air gravity anomaly fields derived from sea surface satellite altimetry have become a widely used tool for investigating marine geophysics and tectonics, their derivation ignores some scales of sea surface topography, making them inconsistent with applications to oceanography and vertical reference systems. While the future GOCE gravity satellite mission will extend the resolution of real gravity fields derived from space beyond that already achieved by CHAMP and GRACE, marine and airborne surveys remain the only way of providing shorter wavelength information.

Marine gravity measurements are, in principle, very precise – military trials of a type of gravity sensor now commercially available achieved RMS cross-over errors of only 0.1 - 0.2 mGal in rough seas (M. Perrin, *pers. comm.*, 1995). However, large biases and tilts occur along the ship-tracks due to a combination of gravimeter drift with infrequent crossovers, with others due to systematic instrument errors (Wessel & Watts, 1988).

Our strategy involves pre-processing the raw gravity data followed by network adjustment. Pre-processing aims to reduce the dynamical errors associated with courses changes, smooth out highfrequency noise, and remove spikes and gross blunders. Network adjustment aims to remove the systematic effects of datum offsets, different gravity reference systems and drift in the gravity meter zero.

The northern Atlantic was used to test the new methodology because its coverage of surface ship gravity data is amongst the densest in the world. It has benefited from a coincidence of oil exploration off the North West European shelf and the seminal role played by surveys of the Rejkjanes Ridge in the development of plate tectonics. Later academic interest in the Iceland hotspot, together with the Shetland-Faroes-Iceland-Greenland Ridge marking its bathymentric trail, extended detailed coverage beyond regions with petroleum potential. Finally some civil benefit has derived from the intensive mapping of the northern Atlantic as a Cold-War submarine hide-away.

For many parts of the deep ocean, coverage by marine or airborne surveys is not dense enough. This paper only deals with optimising the recovery of along-track gravity.

2 Archives, 'surveys' and the choice of adjustment model.

Our adjustment involves a model with at least one parameter for every 'survey', corresponding to an optimal shift in its gravity datum. For long, well-crossed surveys, the model may add a second parameter, representing instrumental drift. However, we are not usually able to retrieve all the information needed to identify the real operational unit of data collection, so what we are forced to call a 'survey' has to be pragmatic and may include a range of different types of data set.

A marine survey is operationally hierarchical: a '*cruise*' involves a particular ship, gravity meter and survey institution. It starts at a home port, may call in at a sequence of other ports and then return to its home port at the end of a season. The data component collected between successive port visits – places where the gravity datum and instrumental drift rate may have been adjusted – is known as one '*leg*' of the cruise. Each leg may consist of many '*lines*'.

Ideally, the unit of data for which the adjustment model uses the same parameters corresponds to a *leg*. Throughout a leg, the gravity datum and reference system, together with the behaviour of the gravity meter, should be consistent. Ideally, what we call a '*survey*' should be the same as a *leg*. However, even good archiving structures may not record port visits, so our analysis may not be able to identify legs and, in practice, the whole cruise may be the only data component that can be distinguished objectively.

A more serious difficulty is posed by data archives that list nothing else but gravity and position in geographically sorted bins. In most cases these data have to be discarded but, where alternative coverage is lacking, very considerable computational effort is needed to identify linear segments. Some manual intervention and judgements that are more subjective are usually needed. Even if a probable 'ship track' can be identified and isolated, the direction of travel remains unknown. More seriously, not knowing how to construct a survey as a time-ordered sequence of lines means that every empirically-identified line-segment has to be treated as a one-line survey. This can greatly increase the number of free parameters in the adjustment model, as well as decreasing its stability.

3 Line segments and curve-fitting

The basic component of our pre-processing algorithm is the '*line-segment*'. A line segment is a component of a survey where the ship's course is adequately straight. Point-to-point vectors are compared with chosen criteria for breaking surveys into line-segments: a break can be triggered by a large change in course azimuth or an excessive gap between points.

By using only line-segment data, we improve the reliability of the data used for the network adjustment stage. During a course change, and for a period after it, the stabilised platform housing the sensor may introduce transient gravity errors. They should not modify either the instrument's long-term datum or its drift rate, so it remains appropriate to use the same drift model for the many line segments that make up a survey. However, data collected during and soon after course changes may be unreliable: we discard them.

Apart from an unknown datum shift and possibly an unknown linear slope, we suppose that the gravity anomaly profile along one line segment involves 'noise' superimposed on a smooth function of along-track distance. The *shape* of this function should correspond to 'true' gravity. In practice, this noise results from a variety of sources – the more significant ones include failure of the stabilisation system to deal with bad sea-states and, for early surveys, digitisation binning and failure to re-process dynamical corrections once *post-facto* navigation corrections became known. We suppose that these components of noise can be modelled as spatially random. In addition to random noise, recording blunders and instrumental malfunction may generate spikes or unrealistic excursions.

For each line-segment, the pre-processing routine involves three operations. First, it must parameterise the smooth curve representing ‘true’ gravity; second, it must quantify the random noise, and, third, it must identify and remove ‘blunders’. For the deep-water parts of the North Atlantic, we have some 12000 line-segments, so these procedures have to be largely automatic.

The free air anomaly was fitted by an appropriately high degree series of Chebyshev polynomials. This generated a continuous, analytical function to represent the ‘true’ shape of each gravity profile together with the standard deviation of the misfit. Here the ‘trick’ is to devise rules that do not overfit noisy data but still give a faithful representation of ‘true’ gravity over the shelf-edge, canyons, sea mounts, the mid-ocean-ridge and other real features with high amplitudes and relatively short wavelengths. For any segment where the fitting criteria are not met, the software displays a profile showing the observed points, the polynomial curve and a curve interpolated from KMS02 altimetric gravity anomalies (Anderson et al. 2003). At this stage, data spikes and blunders are identified and eliminated by manual editing.

4 Line crossing points and connectivity

A typical geodetic network is *designed*. Network design aims to achieve a particular level of accuracy, reliability and cost by choosing what density of framework needs to be observed, where bracing links are needed and what absolute constraints should be imposed and where.

In contrast, an ocean-wide marine gravity ‘network’ is not ‘designed’, but comes about as a chance assembly of unrelated ship tracks. The network is rarely strongly braced, so single ‘passage-lines’ become structurally important but may involve unfavourably acute angles for track intersections.

The first task is to find all the points where two line-segments cross and, at each of these points, to find the free air anomaly estimated by the two lines-segments.

Once the crossing point has been determined, we use the Chebyshev polynomials that describe the free air anomaly as a continuous function of along-track distance to estimate gravity at the intersection point. Some cross-over analysis routines use piecewise-linear interpolation to calculate this from the points along the ship track where it is measured. However, this process can alias random errors and has been shown to generate systematic geoid effects up to 50 cm over 500 km (Hipkin et al. 2004). Our routine avoids aliasing.

5 Network adjustment model

If the pre-processing stages have been successful, the gravity values obtained at cross-over points need only a *survey-dependent* modification to correct for different gravity reference systems, bad connections to harbour base stations and instrumental drift. Determining these is the purpose of network adjustment, where estimates for each survey are modified by a one or two parameter model:

$$\mathbf{g}_{km} - \mathbf{G}_m - \mathbf{a}_k - \mathbf{b}_k \mathbf{s}_{km} = \mathbf{v}'_{km} \qquad (1)$$

Where $\mathbf{g}_{km}$ is the free air anomaly provided by Chebyshev polynomial fitting at intersection point *m* on survey *k*, $\mathbf{v}'_{km}$ is the residual for the observation on survey *k* at cross-over point *m*, $\mathbf{G}_m$ is the ‘true’ free anomaly at cross-over point *m*, $\mathbf{a}_k$ is the datum shift for survey *k*, $\mathbf{b}_k$ is the drift rate (mGal km^{-1}) for survey *k*, and $\mathbf{s}_{km}$ is the cumulative along-track distance from the beginning of survey *k* to cross-over point *m*.

It is self-evident that the set of equations (1) cannot be solved for the unknown parameters $\{\mathbf{G}_m, \mathbf{a}_k, \mathbf{b}_k\}$: if one arbitrary constant is added to all the site gravity values $\mathbf{G}_m$ and the same one subtracted from all the survey gravity datum shifts $\mathbf{a}_k$, the ‘observations’ $\mathbf{g}_{km}$ remain unchanged. Prescribing the free air anomaly for one of the cross-over sites, or fixing the datum shift for one of the surveys are alternative ways of converting this indeterminacy to a *free adjustment solution*. Fixing two or more site values could be used to generate a *constrained adjustment solution*. We have chosen a free adjustment and define the datum on one long tie line to be zero. In what follows, it is assumed that the equation set (1) has been modified to incorporate the imposition of $\mathbf{a}_1 \equiv 0$. We note ways of testing this proposition later.

For any cross-over site *m*, there will be an equation like (1) for each of the two surveys whose line-segments intersect there. Commonly, cross-over adjustment algorithms subtract the two and so eliminate the cross-over point value $\mathbf{G}_{km}$. However, the residuals on different surveys do not necessary belong to the same population of errors, so their difference will form a disparate population for which ‘minimising the sum of the squares of the residuals’ may not have much meaning. Our approach uses the earlier curve fitting stage to provide an *a priori* estimate of the standard deviation s_{km} of each error population. We thereby convert all residuals to common population of normalised residuals v_{km} :

$$\frac{g_{km}}{s_{km}} - \frac{G_m}{s_{km}} - \frac{a_k}{s_{km}} - \frac{b_k s_{km}}{s_{km}} = \frac{v'_{km}}{s_{km}} \quad {}^{o} \quad v_{km} \qquad (2)$$

Note that our approach allows the characteristics of the error population to vary during one survey: the standard deviation is estimated separately for each of its line-segments. If blunder and spike detection have been successful, residuals from the *n* observational equations will have zero mean. If the adjustment model is appropriate, residuals will be uncorrelated. The root mean square residual will estimate the standard error of the adjustment, s_{o}^{2} and, if the normalisation has been successful in scale as well as functional form, we expect $s_{o}^{2} \approx 1$.

The matrix representation of the *N* observation equations, each of which depends on *p* unknown parameters, is

$$\mathbf{A\,X} = \mathbf{B} \qquad (3)$$

In principle, their solution is found from

$$\mathbf{X} = \mathbf{A}^{-1}\,\mathbf{B} \qquad (4)$$

In practice, the size of the normal equation matrix $\mathbf{A}$ is too large for its inverse to be computed. For our work with North Atlantic data, there are about 49000 unknown parameters, so simply writing one double precision matrix with the size of $\mathbf{A}$ would occupy about 20 Gbyte, beyond the capacity of the existing processor RAM. Computation of the full inverse matrix $\mathbf{A}^{-1}$ would be entirely prohibitive and even a direct solution of the normal equations by, for example, Cholesky LU decomposition (Golub & van Loan 1989) remains impractical.

6 Solving the normal equations

Our algorithm avoids computing the design matrix and computes individual non-zero elements of the normal equation $\mathbf{A}$ directly from the output of the line-crossing program, storing the result using the Stanford row-indexed sparse-matrix algorithm (Press, et al, 1986). We assign a drift parameter only if the number of other surveys that are intersected exceeds a chosen value and if the along-track distance between the first and last cross-over point exceeds another chosen value. Thus the number of unknown parameters finally appearing in the normal equations is determined dynamically as the processing progresses. Writing the normal equation matrix directly with the correct indexing for the sparse matrix storage scheme requires some subtleties.

The normal equations are solved using the bi-conjugate gradient with pre-conditioning (PBCG) algorithm (Press *et al*. 1986).

7 Internal error estimation

Residuals should have zero mean and be uncorrelated if our pre-processing has been successful and our adjustment model corresponds to reality. Even though the model may itself be functionally correct, the procedure for estimating its parameters may still be imperfect. If, for example, the datum shift and slope of the drift curve of one survey are estimated wrongly, residuals at successive cross-over points along that survey will no longer be uncorrelated. In addition, this failure will bias the estimate of the datum and drift parameters inferred for surveys that it crosses. One approach aims to improve the parameter estimation process by changing the way the data are weighted. Here we discuss why we did not attempt to implement a formal variance-covariance matrix approach, even though the algorithms have become relatively standard.

With our procedure, known, *a priori* weights are introduced by pre-multiplying the observational matrix $\mathbf{D}'$ by an *N* by *N* matrix $\mathbf{P}$ that is *diagonal* and has elements $\mathbf{P}_{ii} = 1/s_{km}$. The normal equation matrix becomes

$$\mathbf{A} = \mathbf{D}'^{T}\,\mathbf{p}\,\mathbf{p}^{T}\,\mathbf{D}' \qquad (5)$$

An extension of this procedure that is claimed to improve parameter estimation is to include *off-diagonal* elements – *in extremis* to make the weight matrix full. In principle, the off-diagonal weight elements are *unknowns* to be determined from the data but, in practice, are usually imposed via a variance-covariance model. However it is determined, reciprocity requires that the weight matrix remains symmetrical.

Even if some physically sensible way of modelling the error covariances did suggest itself for our problem – and so far it has not – most formal procedures then need to evaluate the inverse of the normal matrix $\mathbf{A}$: for us this remains computationally prohibitive.

We are thus unable to determine formal *internal* errors on the output of our network adjustment. Fortunately, there are a number of external comparisons against which it can be tested. In addition, there are some internal tests of the validity of the model and the hypotheses used to implement it.

8 Crossover residuals

Measures of self-consistency depend on the mismatch at points where different ship tracks cross. Without some kind of processing to interpolate between points where the ship recorded gravity, the cross-over error cannot be estimated; thus, this statistic is not available for raw data. With our algorithm, we only find out where these points are after all processes of breaking up ship tracks into line-segments, discarding sections related to course changes, and then fitting continuous functions to estimate the measurement position and 'true' gravity.

The most primitive cross-over statistic comes directly from the two estimates of free air anomaly at site $\mathbf{m}$ $\mathbf{g_{m1}}$ and $\mathbf{g_{m2}}$, each of which comes with an estimate of its standard deviation, $\sigma_{\mathbf{m1}}$ and $\sigma_{\mathbf{m2}}$. With no weighting, the only estimate for site gravity is

$$\mathbf{G_m} = \tfrac{1}{2}\left(\mathbf{g_{m1}} + \mathbf{g_{m2}}\right) \tag{6}$$

The root mean square residual for $\mathbf{M}$ cross-over points is

$$\sigma_{\mathbf{r}} = \sqrt{\frac{\sum_{\mathbf{m}=1}^{\mathbf{M}} \tfrac{1}{2}\left(\mathbf{g_{m1}} - \mathbf{g_{m2}}\right)^2}{\mathbf{M}}} \tag{7}$$

With weighting, site gravity is estimated as

$$\mathbf{G_m} = \frac{\sigma_{\mathbf{m2}}{}^2\,\mathbf{g_1} + \sigma_{\mathbf{m1}}{}^2\,\mathbf{g_2}}{\sigma_{\mathbf{m1}}{}^2 + \sigma_{\mathbf{m2}}{}^2} \tag{8}$$

The root mean square weighted residual is

$$\sigma_{\mathbf{rw}} = \sqrt{\frac{\sum_{\mathbf{m}=1}^{\mathbf{M}} \frac{\left(\mathbf{g_1} - \mathbf{g_2}\right)^2}{\sigma_{\mathbf{m1}}{}^2 + \sigma_{\mathbf{m2}}{}^2}}{\sum_{\mathbf{m}=1}^{\mathbf{M}} \frac{1}{\sigma_{\mathbf{m1}}{}^2} + \frac{1}{\sigma_{\mathbf{m2}}{}^2}}} \tag{9}$$

Table 1. Statistics of the raw data and crossover point values before and after the network adjustment (units in mGal).

	before processing	Cross-over points Before adjst	after adjst
No.of pts.	1293236	83120	83120
Mean	-7.66	0.86	-0.009
St. Dev		4.03	1.58
Wt.St.Dev	61.98	1.32	0.39
Min	-84.77	-69.87	-37.73
Max	205.82	68.64	38.24

Table 1 shows the large improvement due to cleaning the input data compared with its raw state and then a further big improvement due to the network adjustment. Figure 3 shows the residual distributions before and after adjustment. Achieving data with zero mean is a necessary feature of least-squares analysis, so here the value of -0.009 mGal is not indicative of quality. However, the improvement in the unweighted standard deviation from 4 mGal to 1.6 mGal measures the scatter remaining in the data synthesised from the Chebyshev polynomials. Since applications use *unweighted* gravity anomalies, this is a measure of *local* noise introduced into for example geoid computation. On a *regional* scale, network adjustment seeks to control the way errors accumulate, which translates into long-wavelngth geoid errors. Error accumulation depends on the *weighted* standard deviation, here 0.39 mGal.

Table 2. Statistics of the difference between surface gravity and external data, both evaluated at 2 km intervals along observation tracks (mGal).

	Altimetric freeair anomalies		Grace freeair amomalies Before adjst.	after adjst
No. pts	81761	80172	81761	80172
Mean	2.27	0.009	2.31	0.007
St. DEV	7.17	4.53	17.68	16.82
Min	-84.29	-64.68	-102.94	107.76
Max	70.44	59.47	205.82	203.76

9 External comparisons

Here we present two comparisons with external data not included in the adjustment. The first is the KMS02 synthetic free air gravity anomaly model derived from satellite altimetry. We have interpolated this on to along-track measurement points at 2 km intervals. A synthetic altimetric anomaly will have regional biases due to the effect of mean dynamic sea surface topography. Using dynamic ocean topography models derived from hydrographic data shows that the equivalent gravity signal has a range of about 2 mGal and that the biases should average to zero on a basin-wide scale. Thus comparison of well adjusted real gravity data with a good altimetric gravity model should have a mean close to zero and a standard deviation no more than 2 mGal worse than the errors in the real data.

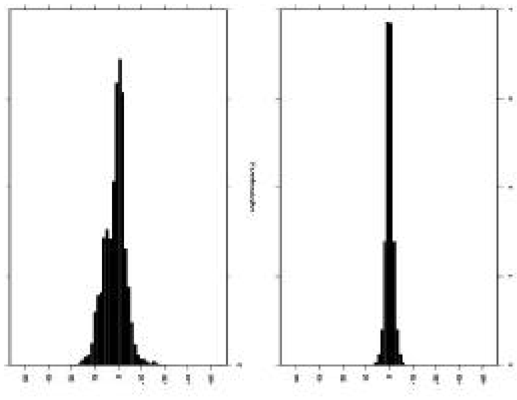

Fig. 3. Histogram of cross-over errors before (left) and after (right) adjustment.

The left-hand side of Table 2 shows that the mean has indeed been reduced nearly to zero and the standard deviation has been reduced. The extrema are larger than anticipated but mainly due to localised very high amplitude anomalies like sea mounts where neither data set will necessarily represent the correct position or amplitude of the peak.

A second comparison uses the GRACE global gravity model. This does not resolve features with wavelength less than ~350 km and still has significant errors at ~ 500 km. However, it does contain the best currently available gravity information still at longer wavelengths. In interpolating GRACE on to the measured along-track points, there will be a gross mismatch in scale – 4 km compared with 500 km. Thus most of the residuals will represent omission errors in GRACE. Because the along-track data is not available on a complete grid, it cannot be smoothed reliably to make it represent only equivalent long wavelengths. However, the surface data are available in a region about 3000 km square, large enough for the mean contribution of shorter wavelength features to vanish. We therefore expect that comparison with GRACE will identify any datum error in the adjustment values, but that little information will be contained in the standard deviations, or extrema. The right hand side of Table 2 confirms these expectations. Note that we have carried out a free adjustment, in which the datum correction of only one ship-track fixes the level of the whole of the northern Atlantic. This survey was chosen carefully: although it is an unidentified military passage line, it runs from Svarlbard, via the Faroes to Iceland, thereby controlling most of the central part of the Nordic Seas. This datum differs from the global datum of GRACE by only 0.0068 mGal.

10 Discussion

With our systematic approach to cleaning and adjusting very large compilations of marine gravity data, and by adding the control provided by a few modern airborne gravity lines, we have generated a very accurate ocean-wide gravity data set whose quality has been demonstrated. This verification has had to be by external comparison, because the size of the matrix adjustment stage made formal internal error estimation and propagation impossible.

The work described in this paper solves only one of the two problems associated with marine geoid computation. Even in the unusually favourable circumstances of the northern Atlantic, there are still large data gaps. Because geoid computation involves a surface integral of gravity anomalies, some way must be found to patch these gaps before a good geoid can be computed. However, the additional stage of carrying out the patching process more rigorously improves the result much more. With the new method of 'patching', gravity interpolation into data gaps and the determination of mean dynamic ocean topography are estimated together in a self-consistent way. This brings the standard deviation in the ocean-wide geoid down to only 3 cm, equivalent to the best results found for continental regions. We conclude that, when properly cleaned and adjusted, marine gravity data are at least as good as those on land. The EGM96 decision to discard them was unwarranted.

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