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A FORTRAN IV PROGRAM FOR A LEAST-SQUARES GRAVITY BASE-STATION NETWORK ADJUSTMENT

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Abstract—An analysis of a least-squares adjustment of a gravity base-station network is outlined followed by an error estimate of the value of each base. Subsequently, a FORTRAN IV program is presented, in which adjusted values of the gravity bases are calculated with their standard deviations and standard errors followed by a histogram display of residuals. The application of this program to adjust the values of gravity networks in Scotland and Greece is discussed and the results of the adjustment of a local network in central Greece finally are outlined.

Key Words; Data processing, Least squares, Geophysics, Network adjustment, Gravity prospecting.

INTRODUCTION

It is well known that a gravity meter only measures differences in gravity and if a gravity survey is extended outwards in a uncontrolled manner, the errors will accumulate with distance. The network adjustment consists of correcting each measured difference between adjacent bases until the cumulative gravity difference between any two bases on the netword is the same for all routes connecting them.

A few workers in the past have attempted to develop an algorithm for the adjustment of a gravity base-station network. Pentz (1952) developed one which gives the most probable value for an arbitrary number of base stations in gravity network, which is expanded when a value of a new base is assigned from two existing bases. As long as the number of conditional equations required to produce the necessary accuracy is maintained, the network of finite but unknown bases is determined fully, when the conditions for the solution of normal equations are satisfied.

Smith (1950), based on Gibson's (1937) paper, developed a satisfactory graphical method for network adjustment. However, it has the disadvantage of being a slow graphical method.

Searle (1969) developed an analysis for altimetric traverses, but which could be used also for a gravity network, based on minimizing the quantity $\Sigma w_i Q_i^2$, where $w_i \equiv$ weighting factor $= 1/N^{1/2}$ and $N =$ number of height differences in each traverse;

$$Q_i = q_i' - q_i,$$

where q_i' and q_i are, respectively, the observed and adjusted height differences in a closing direction along the ith traverse. In none of these papers was the instrumental drift taken into consideration.

An analysis of a least-squares adjustment of a gravity base-station network is outlined in which the instrumental drift has been considered. Subsequently, a FORTRAN IV program of this algorithm is presented, which can be of great help in geophysical (gravity) prospecting.

DEFINITIONS

Base-station is any point at which repeated observations have taken place.

Traverse is a sequence of observations to which a single drift curve is to be fitted.

Gravity observation (g) is obtained by converting the meter dial turns to gravity units and correcting for Earth tides. An (arbitrary) datum value and, possibly, a linear drift correction, which are the same for all stations on the same traverse, may have been applied.

OBSERVATIONAL EQUATION

For each observation, p, at base m on traverse k, the observed value of gravity g_{pkm} has the form:

$$g_{pkm} = G_m - a_k + b_k t_{pkm} + \xi_{pkm} \qquad (1)$$

where t_{pkm} and ξ_{pkm} are the time and error for that particular observation.

VARIANCE

From equation (1) summing the square of ξ_{pkm} for all the observations at all bases on all traverses, the variance of the network is:

$$\sum(\xi^2) \equiv \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{p=1}^{P_{km}} (\xi_{pkm})^2$$

$$= \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{p=1}^{P_{km}} (g_{pkm} - G_m + a_k - b_k t_{pkm})^2 \qquad (2)$$

NORMAL EQUATIONS

The quantity $\Sigma\xi^2$ has to be minimized. Therefore, three sets of equations are produced by setting the partial derivatives of the variance equal to zero. From $\partial\Sigma\xi^2/\partial a_k$, we have K equations where $k = 1, 2, \ldots, K$:

$$\left[\sum_{m=1}^{M}\sum_{p=1}^{P_{km}} g_{pkm}\right] = \sum_{m=1}^{M}[p_{km}]G_m + [-R_k]a_k + \left[\sum_{m=1}^{M}\sum_{p=1}^{P_{km}} t_{pkm}\right]b_k. \quad (3)$$

In the previous equations, we shall replace the sums

$$\sum_{p=1}^{P_{km}} \text{ and } \sum_{k=1}^{K} \text{ and } \sum_{m=1}^{M} \text{ by } \sum_p \text{ and } \sum_k \text{ and } \sum_m,$$

respectively, and thereafter this notation will be held in the following.

From $\partial\Sigma\xi^2/\partial b_k$, we have K equations for $k = 1,2,\ldots,K$:

$$\left[\sum_m\sum_p g_{pkm}t_{pkm}\right] = \sum_m\left[\sum_p t_{pkm}\right]G_m - \left[\sum_m\sum_p t_{pkm}\right]a_k + \left[\sum_m\sum_p (t_{pkm})^2\right]b_k. \quad (4)$$

From $\partial\Sigma\xi^2/\partial G_m$, we have M equations, when $m = 1,2,\ldots,M$:

$$\left[\sum_k\sum_p g_{pkm}\right] = [Q_m]G_m + \sum_k[-R_k]a_k + \sum_k\left[\sum_p t_{pkm}\right]b_k. (5)$$

Considering equations (3)–(5), we have a total number of $M + 2K$ equations, which equals the number of unknowns G_m, a_k, b_k of which only $(M + 2K) - 1$ of them are independent. For, the sum over all bases ($\Sigma_{m=1}^{M}$) on set equation (5) is equal to the sum for all traverses ($\Sigma_{k=1}^{K}$) on set equation (4). It appears then that the whole set of equations is under-determined, for there are $(2K + M) - 1$ linearly independent equations with $(2K - M)$ unknowns. An additional equation is required therefore.

This is provided by defining the datum for the survey, which otherwise is entirely relative and could not determine absolute values G_m.

If the base station $m = 1$ has the (absolute) gravity value ζ_0, the additional required equation is simply $G_1 = \zeta_0$ and the equation with $m = 1$ from equation (3) can be omitted.

Summarizing the previous discussion, we have that the $(M + 2K)$ set of normal equations has the form:

$$[1]G_1 = [\zeta_0]$$

$$[0]G_1 + [Q_2]G_2 + \cdots + [-P_{21}]a_1 + [-P_{22}]a_2 + \cdots + \left[\sum_p t_{p12}\right]b_1 + \left[\sum_p t_{p22}\right]b_2 + \cdots = \left[\sum_k\sum_p g_{pk2}\right]$$

$$[0]G_1 + [0]G_2 + [Q_3]G_3 + \cdots + [-P_{31}]a_1 + [-P_{32}]a_2 + [-P_{33}]a_3 + \cdots + \left[\sum_p t_{p13}\right]b_1 + \left[\sum_p t_{p23}\right]b_2 + \cdots = \left[\sum_k\sum_p g_{pk3}\right]$$

. . . .

$$[P_{11}]G_1 + [P_{12}]G_2 + \cdots + [-R_1]a_1 + [0]a_2 + [0]a_3 + \cdots + \left[\sum_m\sum_p t_{plm}\right]b_1 + [0]b_2 + [0]b_3 + \cdots = \left[\sum_m\sum_p g_{plm}\right]$$

$$[P_{21}]G_1 + [P_{22}]G_2 + \cdots + [0]a_1 + [-R_2]a_2 + [0]a_3 + \cdots + [0]b_1 + \left[\sum_m\sum_p t_{p2m}\right]b_2 + [0]b_3 + \cdots = \left[\sum_m\sum_p g_{p2m}\right]$$

. . . .

$$\left[\sum_p t_{p11}\right]G_1 + \left[\sum_p t_{p22}\right]G_2 + \cdots + \left[-\sum_m\sum_p t_{plm}\right]a_1 + [0]a_2 + \cdots + \left[\sum_m\sum_p (t_{p1m})^2\right]b_1 + [0]b_2 + [0]b_3 + \cdots = \left[\sum_m\sum_p g_{p1m}t_{p1m}\right]$$

$$\left[\sum t_{p21}\right]G_1 + \left[\sum_p t_{p22}\right]G_2 + \cdots + [0]a_1 + \left[-\sum_m\sum_p t_{2pm}\right]a_2 + \cdots + [0]b_1 + \left[\sum_m\sum_p (t_{p2m})^2\right]b_2 + \cdots = \left[\sum_m\sum_p g_{p2m}t_{p2m}\right]$$

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ERROR ANALYSIS

Considering equation (1) where G_m is a function of the uncorrelated measured variables g_{pkm} and t_{pkm}, we have (Bevington, 1969):

$$(\xi_{pkm})^2 = \left(\frac{\partial G_m}{\partial g_{pkm}}\right)^2(\xi_{g_{pkm}})^2 + \left(\frac{\partial G_m}{\partial t_{pkm}}\right)^2(\xi_{t_{pkm}})^2. \quad (6)$$

Assuming that all the observations of gravity (g_{pkm}) and time (t_{pkm}) at all bases on all traverses, belong to the same population then the expected value of any $\xi_{g_{pkm}}$ and any $\xi_{t_{pkm}}$ will be as follows:

$$\langle\xi^2_{g_{pkm}}\rangle = S^2_{g_m} \text{ and } \langle\xi^2_{t_{pkm}}\rangle = S^2_{t_m} \quad (7)$$

for any observation on any traverse; ξ_{g_m} and ξ_{t_m} are the standard deviations of the gravity observations and time at a base m, respectively.

Assuming P_{km} observations at the mth base station for the kth traverse, a total number of Q_m observations at the same base for all the traverses, and summing for all traverses and observations, from equation (6) we have:

$$\sum_k\sum_p(\xi_{pkm})^2 = \sum_k\sum_p\left(\frac{\partial G_m}{\partial g_{pkm}}\right)^2(\xi_{g_{pkm}})^2$$

$$+\sum_k \sum_p \left(\frac{\partial G_m}{\partial t_{pkm}}\right)^2 (\xi_{t_{pkm}})^2 \tag{8}$$

and from equation (7)

$$\sum_k \sum_p (\xi_{pkm})^2 = Q_m S_{g_m}^2 + \left(-\sum_k P_{km} b_k\right)^2 S_{t_m}^2. \tag{9}$$

Because the contribution of the second term of equation (9) is negligible, it can be omitted.

Hence, the error for each base is expressed in terms of the total number of observations at that base station. The greater the number of visits to a base, the better the estimation of its error. The root mean square error of the adjustment S_{RMS} can be derived as the square root of the total variance divided by the total number of observations on all the bases, NOBS, minus one:

$$S_{RMS} = \left(\frac{\Sigma \xi^2}{NOBS - 1}\right)^{1/2}. \tag{10}$$

Because the number of observations at almost any base is relatively small, it is more desirable to estimate confidence limits on the adjusted value, G_m, rather than simply quote an apparent standard deviation or standard error.

For example, over what range is there a 95% probability that the true mean μ_{G_m} will be within this confidence interval on either side of our adjusted mean G_m.

Thus, the confidence interval within which μ_{G_m} falls with $100(1 - a)\%$ confidence is (Bendat and Piersol, 1971):

$$G_m - \frac{S_{g_m} t_{n,a/2}}{\sqrt{Q_m}} \leq \mu_{G_m} < G_m + \frac{S_{g_m} t_{n,a/2}}{\sqrt{Q_m}} \tag{11}$$

and the true variance $\sigma_{G_m}^2$ of μ_{G_m} based upon the standard deviation S_{g_m} of G_m is:

$$\frac{ns_{g_m}^2}{\chi_{n,a/2}^2} \leqslant \sigma_{G_m}^2 < \frac{ns_{g_m}^2}{\chi_{n,1-a/2}^2} \tag{12}$$

where $n = Q_m - 1$ and a can be obtained from tables showing the percentage points of "Student's t" distribution (see Bendat and Piersol (1971), p. 389) and χ-square distribution.

Taking the square root on both sides of equation (12), we can obtain an estimate of a lower and upper limit of the standard deviation of μ_{G_m}. This was performed with a 95% confidence interval for the μ_{G_m} and σ_{G_m}, taking the values of the "Student t" and χ-square distribution from tables given by Bendat and Piersol (1971).

NETWORK COMPUTER PROGRAM

NETWORK is a FORTRAN IV computer program which performs a least-square adjustment of a gravity base station network.

With the first READ statement, the total number of traverses (K), bases (M), observations ($NOBS$), and the datum of the network ($G1$), are read from channel 7. A serial reference number ($NBASE$) must be assigned to each of the base stations, beginning at 1 for the site of network datum $G1$.

With the second and third READ statements the rest of the required data are read from channel 8; that is, the reference number of a base ($NBASE$), the observation of its gravity value before adjustment ($GRAV$), and the time of observation ($TIME$) given in days and decimals of a day. Traverses values optionally may have been corrected prior to the adjustment by any linear drift function; traverses need not be closed loops, but if not, the same set of trial base station values must have been used in the initial reduction. Subsequently, the arrays A and B of the $(M + 2K)$ normal equations are constructed: $AX = B$, corresponding to equations (3)–(5). The solution of the latter set of simultaneous equations is executed by subroutine SOLVE, which is a modified version of the SIMQ subroutine (IBM Scientific Subroutine Package, 1969), using the method of Gauss elimination. This routine returns the unknowns X in the array B, with the first M elements as the adjusted base values and the remaindering K elements as estimates of, or corrections to, the drift rate during each traverse.

The standard deviation ($DGRAV$) and standard error ($STEER$) of each base as well as the RMS residual of the network adjustment ($SIGMA$) are calculated according to the error analysis presented previously. Also, the lower and upper limits of the adjusted values of the gravity bases and their standard deviations at a 95% confidence interval, applying Student's "t" test, is calculated according to equations (10) and (11). All these results are output to channel 6.

After adjustment, the residuals ($ERROR$) from all the observational equations are prepared for display as a histogram on a line printer and output to channel 6. For this purpose, the subroutine HIST of IBM Scientific Subroutine Package (1969) originally was employed, and which subsequently, was modified in part. The histogram display of the residuals is prepared in such a manner so that, after the determination of minimum and maxiumu values of the fluctuation of residuals, the class interval is always one $SIGMA$ (the RMS error of the adjustment of the network).

The percentage points of "t" and χ-square distributions at 95% confidence, which are used for the evaluation of the lower and supper limits of the adjusted values of gravity as well as their standard deviations, have been taken from the tables by Bendat and Piersol (1971). Data from these tables precede the READ statements.

APPLICATIONS—RESULTS

NETWORK originally was applied to adjust the gravity values of the bases of a gravity survey in SE Scotland (Lagios, 1979). It was discovered that for a

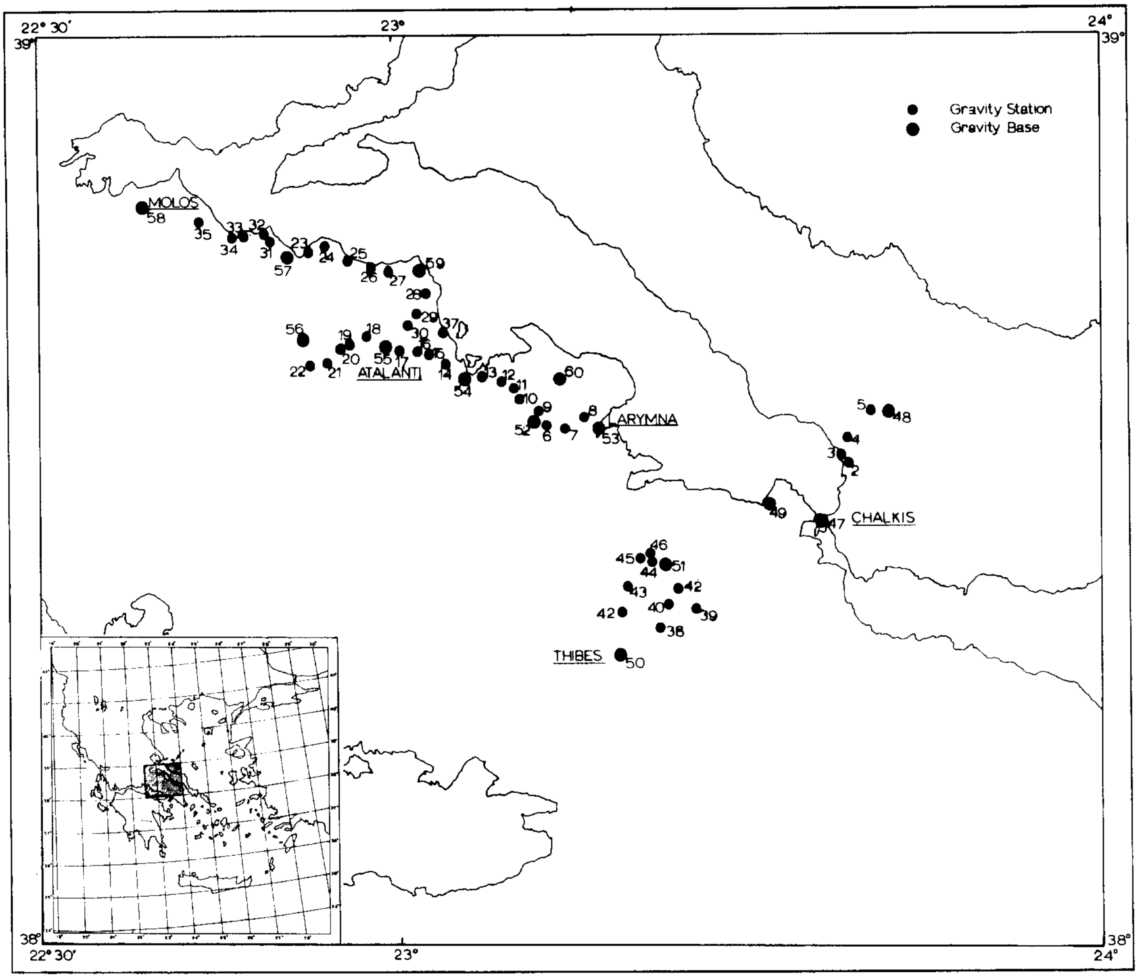

Figure 1. Gravity station distribution of a network in Central Greece.

total number of 57 (M) bases and 82 (K) traverses with a total number of 364 ($NOBS$) observations, it took less than 6 mins running time on a 4-75 ICL computer and 40 secs on the 2980 ICL machine. The RMS error of the adjustment of this network was determined to be only 9.6 μgals, an indication of a survey with high accuracy observations, compared to the National Gravity Reference Net (NGRN) of 1973 (Masson-Smith, Howell and Abernathy-Clar, 1974). The results of the adjustment of the Scottish network have been presented elsewhere (Lagios, 1979; Lagios and Hipkin, 1980). Moreover, a comparison with local NGRN'73 values already has been outlined and discussed by Lagios and Hipkin (1981).

Recently, NETWORK was used to adjust the gravity observations of a network of 60 stations in Atalanti region, Central Greece (Fig. 1). Even though these results will be presented elsewhere in detail (Lagios, *et al.*, in preparation), nevertheless, the adjusted values of stations are outlined in Tables 1–3. Analytically, Table 1 shows the adjusted values of bases and stations of the Greek network with their estimated standard deviations and standard errors expressed in gravity units (*gu*); $1 gu = 10^{-1}$ mgal. Table 2 demonstrates the statistically calculated lower and upper limits of the adjusted gravity values. Similarly, Table 3 outlines upper and lower limits of the calculated standard deviations at 95% confidence. The RMS error of the Greek network adjustment was 10 μgals here. Finally, the distribution of the residuals ($ERROR$) from every observational equation is displayed in a histogram with class interval of one $SIGMA$, which was determined to be 10 μgals (Fig. 2).

```
                          HISTOGRAM

FREQUENCY      2     0     2     9    59   151    70    10     0     0     2
-----------------------------------------------------------------------------
EACH *EQUALS 4 POINTS

       148                                   *
       144                                   *
       140                                   *
       136                                   *
       132                                   *
       128                                   *
       124                                   *
       120                                   *
       116                                   *
       112                                   *
       108                                   *
       104                                   *
       100                                   *
        96                                   *
        92                                   *
        88                                   *
        84                                   *
        80                                   *
        76                                   *
        72                                   *
        68                                   *     *
        64                                   *     *
        60                             *     *     *
        56                             *     *     *
        52                             *     *     *
        48                             *     *     *
        44                             *     *     *
        40                             *     *     *
        36                             *     *     *
        32                             *     *     *
        28                             *     *     *
        24                             *     *     *
        20                             *     *     *
        16                             *     *     *
        12                             *     *     *
         8                       *     *     *     *     *
         4                       *     *     *     *     *
-----------------------------------------------------------------------------
INTERVAL       1     2     3     4     5     6     7     8     9    10    11
 CLASS
```

Figure 2. Histogram showing distribution of residuals. Class interval equals one *SIGMA* (10 gu).

Table 1. Adjusted values of Atalanti gravity network

STATION	ADJUSTED VALUE	STAND. DEVIATION	STAND. ERROR
1 (6)*	9800492.83	0.02	0.01
2 (4)	9800927.79	0.03	0.02
3 (4)	9800998.15	0.02	0.01
4 (4)	9800994.96	0.09	0.04
5 (4)	9801008.34	0.02	0.01
6 (4)	9800630.27	0.02	0.01
7 (3)	9800947.92	0.10	0.06
8 (4)	9800974.71	0.06	0.03
9 (4)	9800595.19	0.15	0.08
10 (4)	9800575.87	0.02	0.01
11 (4)	9800893.98	0.17	0.09
12 (4)	9801023.91	0.08	0.04
13 (4)	9801012.18	0.03	0.01
14 (4)	9801020.26	0.06	0.03
15 (4)	9800996.33	0.07	0.04
16 (4)	9800930.50	0.06	0.03
17 (4)	9800915.83	0.04	0.02
18 (4)	9800687.30	0.01	0.00
19 (4)	9800509.79	0.01	0.00
20 (4)	9800421.17	0.08	0.04
21 (4)	9800366.89	0.07	0.03
22 (4)	9800174.95	0.08	0.04
23 (4)	9800751.85	0.06	0.03
24 (4)	9800749.60	0.15	0.07
25 (4)	9800852.34	0.53	0.27
26 (4)	9800877.25	0.02	0.01
27 (4)	9800914.11	0.05	0.03
28 (4)	9800948.78	0.04	0.02
29 (4)	9800969.56	0.06	0.03
30 (4)	9800945.23	0.05	0.02
31 (4)	9800724.20	0.10	0.05
32 (4)	9800676.38	0.03	0.02
33 (4)	9800694.30	0.02	0.01
34 (4)	9800699.22	0.08	0.04
35 (4)	9800498.38	0.06	0.03
36 (4)	9800374.65	0.05	0.02
37 (4)	9801104.27	0.04	0.02
38 (4)	9800467.79	0.09	0.04
39 (4)	9800464.25	0.01	0.00
40 (4)	9800562.61	0.10	0.05
41 (4)	9800399.89	0.06	0.03
42 (4)	9800490.77	0.01	0.00
43 (4)	9800609.31	0.04	0.02
44 (4)	9800658.68	0.06	0.03
45 (4)	9800686.51	0.03	0.02
46 (4)	9800642.23	0.08	0.04
47 (5)	9800973.89	0.09	0.04
48 (6)	9800928.35	0.06	0.02
49 (6)	9800908.67	0.05	0.02
50 (12)	9800350.29	0.07	0.02
51 (4)	9800655.59	0.02	0.01
52 (10)	9800495.96	0.10	0.03
53 (3)	9801124.97	0.14	0.08
54 (12)	9801057.74	0.14	0.04
55 (20)	9800848.88	0.08	0.02
56 (8)	9799871.45	0.04	0.02
57 (10)	9800715.73	0.12	0.01
58 (8)	9800523.08	0.02	0.01
59 (8)	9800907.11	0.06	0.02
60 (6)	9800624.25	0.18	0.08

* Values in parenthesis are number of visits at this station
Values of gravity are expressed in gravity units (gu).

Table 2. Lower and upper limits of adjusted values at 95% confidence

BASE	LOWER LIMIT	ABS. GRAVITY	UPPER LIMIT
1 (6)*	9800492.81	9800492.83	9800492.85
2 (4)	9800927.75	9800927.79	9800927.84
3 (4)	9800998.13	9800998.15	9800998.17
4 (4)	9800994.85	9800994.96	9800995.08
5 (4)	9801008.31	9801008.34	9801008.37
6 (4)	9800630.25	9800630.27	9800630.29
7 (3)	9800947.75	9800947.92	9800948.09
8 (4)	9800974.63	9800974.71	9800974.78
9 (4)	9800594.99	9800595.19	9800595.39
10 (4)	9800575.85	9800575.87	9800575.90
11 (4)	9800893.76	9800893.98	9800894.20
12 (4)	9801023.81	9801023.91	9801024.01
13 (4)	9801012.15	9801012.18	9801012.21
14 (4)	9801020.18	9801020.26	9801020.33
15 (4)	9800996.24	9800996.33	9800996.42
16 (4)	9800930.43	9800930.50	9800930.58
17 (4)	9800915.78	9800915.83	9800915.88
18 (4)	9800687.29	9800687.30	9800687.31
19 (4)	9800509.78	9800509.79	9800509.80
20 (4)	9800421.06	9800421.17	9800421.27
21 (4)	9800366.81	9800366.89	9800366.98
22 (4)	9800174.85	9800174.95	9800175.05
23 (4)	9800751.78	9800751.85	9800751.93
24 (4)	9800749.40	9800749.60	9800749.79
25 (4)	9800851.64	9800852.34	9800853.03
26 (4)	9800877.22	9800877.25	9800877.27
27 (4)	9800914.04	9800914.11	9800914.18
28 (4)	9800948.73	9800948.78	9800948.83
29 (4)	9800969.48	9800969.56	9800969.65
30 (4)	9800945.17	9800945.23	9800945.29
31 (4)	9800724.07	9800724.20	9800724.33
32 (4)	9800676.34	9800676.38	9800676.42
33 (4)	9800694.27	9800694.30	9800694.33
34 (4)	9800699.12	9800699.22	9800699.33
35 (4)	9800498.30	9800498.38	9800498.45
36 (4)	9800374.59	9800374.65	9800374.71
37 (4)	9801104.22	9801104.27	9801104.31
38 (4)	9800467.68	9800467.79	9800467.91
39 (4)	9800464.24	9800464.25	9800464.26
40 (4)	9800582.49	9800582.61	9800582.73
41 (4)	9800399.82	9800399.89	9800399.96
42 (4)	9800490.76	9800490.77	9800490.77
43 (4)	9800609.26	9800609.31	9800609.35
44 (4)	9800658.60	9800658.68	9800658.75
45 (4)	9800686.47	9800686.51	9800686.55
46 (4)	9800642.12	9800642.23	9800642.33
47 (5)	9800973.79	9800973.89	9800974.00
48 (6)	9800928.29	9800928.35	9800928.41
49 (6)	9800908.62	9800908.67	9800908.72
50 (12)	9800350.23	9800350.29	9800350.36
51 (4)	9800655.56	9800655.59	9800655.62
52 (10)	9800495.89	9800495.98	9800496.07
53 (3)	9801124.72	9801124.97	9801125.21
54 (12)	9801057.61	9801057.74	9801057.87
55 (20)	9800848.82	9800848.88	9800848.95
56 (8)	9799871.41	9799871.45	9799871.50
57 (10)	9800715.62	9800715.73	9800715.84
58 (8)	9800523.05	9800523.08	9800523.10
59 (8)	9800907.05	9800907.11	9800907.16
60 (6)	9800624.05	9800624.25	9800624.44

* Values in parenthesis are number of visits at this station
Values of gravity are in gravity units (gu).

Table 3. Lower and upper limits of standard deviation at 95% confidence

STATION	LOWER LIMIT	ST. DEVIATION	UPPER LIMIT
1 (6)*	0.01	0.02	0.06
2 (4)	0.02	0.03	0.13
3 (4)	0.01	0.02	0.06
4 (4)	0.05	0.09	0.33
5 (4)	0.01	0.02	0.08
6 (4)	0.01	0.02	0.06
7 (3)	0.05	0.10	0.60
8 (4)	0.03	0.06	0.22
9 (4)	0.09	0.15	0.56
10 (4)	0.01	0.02	0.07
11 (4)	0.10	0.17	0.64
12 (4)	0.04	0.08	0.28
13 (4)	0.01	0.03	0.10
14 (4)	0.03	0.06	0.21
15 (4)	0.04	0.07	0.27
16 (4)	0.03	0.06	0.22
17 (4)	0.02	0.04	0.15
18 (4)	0.00	0.01	0.03
19 (4)	0.00	0.01	0.03
20 (4)	0.05	0.08	0.31
21 (4)	0.04	0.07	0.25
22 (4)	0.04	0.08	0.28
23 (4)	0.03	0.06	0.21
24 (4)	0.08	0.15	0.55
25 (4)	0.30	0.53	1.99
26 (4)	0.01	0.02	0.07
27 (4)	0.03	0.05	0.20
28 (4)	0.02	0.04	0.14
29 (4)	0.04	0.06	0.24
30 (4)	0.03	0.05	0.17
31 (4)	0.06	0.10	0.37
32 (4)	0.02	0.03	0.12
33 (4)	0.01	0.02	0.08
34 (4)	0.05	0.08	0.30
35 (4)	0.03	0.06	0.23
36 (4)	0.03	0.05	0.17
37 (4)	0.02	0.04	0.13
38 (4)	0.05	0.09	0.32
39 (4)	0.00	0.01	0.03
40 (4)	0.05	0.10	0.36
41 (4)	0.03	0.06	0.21
42 (4)	0.00	0.01	0.02
43 (4)	0.02	0.04	0.14
44 (4)	0.03	0.06	0.22
45 (4)	0.02	0.03	0.12
46 (4)	0.04	0.08	0.30
47 (5)	0.06	0.09	0.27
48 (6)	0.03	0.06	0.14
49 (6)	0.03	0.05	0.12
50 (12)	0.05	0.07	0.12
51 (4)	0.01	0.02	0.09
52 (10)	0.07	0.10	0.17
53 (3)	0.07	0.14	0.89
54 (12)	0.10	0.14	0.24
55 (20)	0.06	0.08	0.11
56 (8)	0.03	0.04	0.09
57 (10)	0.09	0.12	0.23
58 (8)	0.02	0.02	0.05
59 (8)	0.04	0.06	0.12
60 (6)	0.12	0.18	0.45

* Values in parenthesis are number of visits at this station
Values of gravity are always in gravity units (gu).

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NOTATION

The following notation has been used:

M	total number of base stations
m	index specifying the base station ($m = 1, 2, \ldots, M$)
K	total number of traverses
k	index specifying the traverse ($k = 1, 2, \ldots, K$)
P	index specifying an observation on kth traverse at station m
P_{km}	number of observations at mth base station during the kth traverse
$Q_m = \Sigma_{k=1}^{K} P_{km}$	total number of observations at mth base station
$R_k = \Sigma_{m=1}^{M} P_{km}$	total number of observations on kth traverse
G_m	adjusted value of gravity at mth base station
a_k	datum constant of kth traverse
b_k	drift rate for kth traverse
[ ]	enclose observed quantities
$\langle\ \rangle$	indicate mean value

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APPENDIX

```
C
C
C                              NETWORK
C                        FORTRAN - IV  PROGRAM
C
C
C          PERFORMS LEAST-SQUARE ADJUSTMENT OF A GRAVITY NETWORK
C
C
C
C
C
C
C
      REAL*8 ERROR(324),ERRSQ,SERSQ,TEME2,GT,A(83,83),B(83),SIGMA
     1,ERSQM(61),DGRAV(61),STERR(61),CNIV(61),GLL(61),VUL(61)
     2,VLL(61),GPR(324),GAUSPR,NBST(61)
      DIMENSION NBASE (324),GRAV(324),TIME(324),EREQ(20),
     1T(50),X025(50),X975(50)
      COMMON A,ERROR,B,ERSQM,DGRAV,STERR,GRAV,TIME,NBASE,CNIN,GUL,
     1GLL,VUL,VLL,GPR
C                  PERCENTAGE POINTS OF T-DISTRIBUTION
C                              FOR 95% CONFIDENCE
      DATA T/12.706,4303,3.182,2.776,2.571,2.447,2.365,2.306,2.262,2.2
     128,2.201,2.179,2.160,2.145,2.131,2.120,2.110,2.101,2.093,2.086,2.0
     280,2.074,2.069,2.064,2.060,2.056,2.052,2.048,2.045,2.042,2.040,2.0
     338,2.036,2.034,2.034,2.032,2.030,2.028,2.026,2.021,2.020,2.019,2.0
     418,2.017,2.016,2.015,2.014,2.013,2.012,2.011/
C             PERCENTAGE POINTS FOR CHI-SQUARE DISTRIBUTION
C                            FOR  95%  CONFIDENCE
      DATA X975/.===)(",.0506356,0.216,0.484,0.831,1.237,1.689,2.179,2.7
     100,3247,3.816,4.403,5.008,5.628,6.262,6.907,7.564,8.230,8.906,9.
     2590,10.282,10.982,11.688,12.401,13.197,13.843,14.573,15.308,16.04
     37,16.790,17.5,18.35,19.1,19.9,20.7,21.5,22.3,23.1,23.8,24.4331,25.
     42,26.0,26.8,27.5,28.3,29.1,29.9,30.7,31.5,32.32,3574/
      DATA X025/5.0238,7.377,9.348,11.143,12.832,14.449,16.012,17.534,1
     19.022,20.483,21.920,23.336,24.735,26.119,27.448,28.845,30.191,31.
     2526,32.852,34.169,35.478,36.780,38.075,39.364,40.646,41.923,43.19
     34,44.460,45.722,46.979,48.1,49.3,50.5,51.7,52.9,54.1,55.3,56.5,57
     4.9,59.341,60.5,61.7,62.9,64.3,65.6,66.9,68.2,69.4,70.5,71.42/
```

```
C
C           K=NO OF TRAVERCES, M=NO OF BASES, NOBS=TOTAL NO OF
C               OBSERVATIONS, G1=ABSOLUTE VALUE OF GRAVITY
       READ(7,100) K,M,NOBS,G1
100    FORMAT(2I3,I4,2F13.3)
      NN=M+2*K
      NS=NN*NN
C           NFREE IS THE DEGREE OF FREEDOM VARIABLE
C
C
      NFREE=NOBS-NN
C
C           INITIALIZATION OF THE ARRAYS A(I,J), B(J)
C
      DO 2 I=1,NN
      DO 2 J=1,NN
      B(J)=0.
     2A(I,J)=0.
      N=0
      DO 1 I=1,K
      L=M+I
      LN=M+K+I
C
C             IK IS THE NUMBER OF OBSERVATIONS AT EACH TRAVERSE
C
      READ(8,200) IK
200   FORMAT(I3)
       A(L,L)=-IK
       DO 1 NP=1,IK
        N=N+1
C
C          NBASE, GRAV, TIME, REPRESENT ARRAYS OF GRAVITY READINGS,TAKEN
C                      A CERTAIN TIME ON EACH BASE,RESPECTIVELY.
C
C
       READ (8,300) NBASE(N),GRAV(N),TIME(N)
300     FORMAT(I7,F13.2,F15.5)
        TIMEN=TIME(N)
        J=NBASE(N)
         GRAVN=GRAV(N)
        TIME2=TIMEN*TIMEN
       GT=GRAVN*TIMEN
       A(L,J)=A(L,J)+1
       A(J,L)=A(J,L)-1
       A(J,J)=A(J,J)+1
       A(J,LN)=A(J,LN)+TIMEN
       A(LN,J)=A(LN,J)+TIMEN
       A(L,LN)=A(L,LN)+TIMEN
       A(LN,LN)=A(LN,LN)+TIME2
       A(LN,L)=A(LN,L)+TIMEN
       B(J)=B(J)+GRAVN
       B(L)=B(L)+GRAVN
       B(LN)=B(LN)+GT
     1 CONTINUE
       DO £ J=1,NN
3       A(1,J)=0.
       A(1,1)=1.
C
C    DETERMINATION OF THE ABSOLUTE DATUM OF THE SURVEY BY ASSIGNING B(1)=G1
C
C
        B(1)=G1
C
C
C         SOLUTION OF SIGMULTANEOUS EQUATIONS
C
C
           CALL SOLVE(A,B,NN,IER)
C         IF IER=0  ACCEPTABLE SOLUTION OF NORMAL EQUATIONS
           WRITE(6,888) IER
888       FORMAT('IER=',I2)
C
C    OUTPUT OF THE ADJUSTED VALUES OF THE BASES WITH THEIR REFERENCE NUMBER
C
C         WRITE(6,999) (I,B(I),I=1,NN)
      ERSUM=0.
      SERSQ=0.
      MK=M+K
C
C
```

```
C      INITIALIZATION OF THE ARRAYS FOR THE COMPUTATION OF THE ERRORS
C
C
C       DO 51 N=1,M
        ERSQM(N)=0.
         STERR(N)=0.
C
C      NBST IS THE ARRAY OF THE TOTAL NUMBER OF VISITS FOR EACH BASE STATION
C
        NBST(N)=0.
     51 DGRAV(N)=0.
         DO 55 N=1,NOBS
     55  ERROR(N)=0.
        N=0
C
C
C        RE-READING OF THE ORIGINAL DATA FILE FROM CHANNEL 8
C
C
         REWIND 8
         DO 11 KK=1,K
         L1=M+KK
         L2=MK+KK
C
C        IK = NUMBER OF OBSERVATIONS AT TRAVERSE K
C
          READ(8,200) IK
          DO 11 NP=1,IK
          N=N+1
C
C        READING THE REFERENCE NUMBER (J), GRAVITY READING (GRAVN), AND TIME
C
          READ(8,300) J,GRAVN,TIMEN
         NBST(J)=NBST(J)+1
         ERROR(N)=ERROR(N)+GRAVN-B(J)+B(L1)-TIMEN*B(L2)
         ERSUM=ERSUM+ERROR(N)
         ERRSQ=ERROR(N)*ERROR(N)
         ERSQM(J)=ERSQM(J)+ERRSQ
         SERSQ=SERSQ+ERRSQ
      11 CONTINUE
         DO 52 N=1,M
C
C     IF THE NUMBER OF VISITS AT A CERTAIN BASE IS ONLY ONE, THEN NO STANDARD
C        DEVIATION (DGRAV) OR STANDARD ERROR (STERR) IS CALCULATED.
C
         IF(NBST(N).EQ.1)GO TO 133
         GO TO 134
     133 DGRAV(N)=0.
         STERR(N)=0.
           GO TO 52
C
C        CALCULATION OF STANDARD DEVIATION & STANDARD ERROR
C                  OF EACH BASE
C
C
 134     DGRAV(N)=DGRAV(N)+DSQRT(ERSQM(N)/(NBST(N)-1))
         STERR(N)=STERR(N)+DGRAV(N)/DSQRT(NBST(N))
  52         CONTINUE
C
C
C
C            EVALUATION OF UPPER & LOWER LIMIT OF GRAVITY & ITS STANDARD
C                  DEVIATION ACCORDING TO STUDENT 'T' TEST
C
             DO 94 I=1,M
             L=NBST(I)-1
             iF(L) 95,95,96
C
C        GUL AND GLL IS THE UPPER AND LOWER LIMIT OF GRAVITY, RESPECTIVELY,
C                        FOR EACH ADJUSTED VALUE OF A BASE.
C
  96         CNIN(I)=DGRAV(I)*T(L)/SQRT(FLOAT(NBST(1)))
             GUL(I)=B(I)+CNIN(I)
             GLL(I)=B(I)-CNIN(I)
C
C        VLL AND VUL IS THE STATISTICALLY CALCULATED LOWER AND UPPER LIMIT
C                                OF THE STANDARD DEVIATION
C
             VLL(I)=DGRAV(I)*SQRT(FLOAT(L)/X025(L))
```

```
          VUL(I)=DGRAV(I)*SQRT(FLOAT(L)/X975(L))
        GO TO 94
   95   GUL(I)=0.
        GLL(I)=0.
        VLL(I)=0.
   94     CONTINUE
C
C       OUTPUT OF THE ADJUSTED VALUES OF THE BASES WITH THEIR STANDARD DEVI
C                          AND STANDARD ERROR.
C
        WRITE (6,5000)
 5000   FORMAT (1H1)
        WRITE(6,997)
        WRITE(6,999)(I,NBST(I),B(I),DGRAV(I),STERR(I),I=1,M)
C
        WRITE (6,5000)
        WRITE (6,718)
  718   FORMAT (9X,'BASE',3X,'LOWER LIMIT',2X,'ABS.GRAVITY',2X,
       1'UPPER LIMIT')
C
C       OUTPUT OF THE STATISTICALLY CALCULATED LOWER AND UPPER LIMIT FOR EACH
C              ADJUSTED VALUE OF EACH BASE STATION.
C
        WRITE(6,719)  (I,NBST(I);GLL(I),B(I),GUL(I),I=1,M)
  719   FORMAT (10X,I3,'(',I2,')',1X,F10.2,4X,F10,2,4X,F10.2/)
C
C
C       OUTPUT OF THE STATISTICALLY CALCULATED LOWER AND UPPER LIMIT OF THE
C              CALCULATED STANDARD DEVIATION OF EACH BASE STATION.
C
C       WRITE (6,5000)
        WRITE (6,494)
  494   FORMAT (9X,'BASE',3X,'LOWER LIMIT',2X,'ST DEVIATION',2X
        1'UPPER LIMIT')
        WRITE (6,729) (I,NBST(I),VLL(I),DGRAV(I),VUL(I),I=1,M)
  729   FORMAT (10X,I3,'(',I2,')',1X,F10.2,4X,F10.2,4X,F10.2/)
  997   FORMAT(10X,'BASE STATION',2X,'ADJUSTED VALUE',2X,'STAND.DEVIATION
       1X',2X,'STAND.ERROR')
C
C       CALCULATION OF MEAN & RMS ERROR OF NETWORK
C
        EMEAN=ERSUM/FLOAT(NOBS)
        SIGMA=DSQRT(SERSO/FLOAT(NFREE))
        WRITE(6,703) SIGMA
 703    FORMAT('0',20X,'SIGMA = ',F10.5)
    999 FORMAT(10X,I3,'(',I2,')',3X,F14.2,11X,F5.2,12X,F5.2/)
C
C                    HISTOGRAM
C
C
C         FIND MINIMUM,MAXIMUM VALUE OF ERROR
C
        ERMAX=-100000000.0
        ERMIN=-ERMAX
        DO 24 I=1,NOBS
        XI=ERROR(I)
        IF(XI.GT.ERMAX) ERMAX=XI
        IF(XI.LT.ERMIN) EPMIN=XI
     24 CONTINUE
C
C            DETERMINATION OF NO OF INTERVALS
C
C            INETRVAL (INCREMENT) OF THE HISTOGRAM EQUALS TO SIGMA
C
C
        I=1
        INC=0
 5981   INC=INC+2+I
        HL=INC*SIGMA/2.
         IF (HL.LT.ABS(ERMIN).AND.HL.LT.ERMAX) GO TO 5981
        INC=INC+1
         X1=-HL-HL/2.
         X2=HL+HL/2.
C         WRITE UPPER & LOWER LIMIT OF ERRORS INTERVAL
C
C
          WRITE(6,15206) X1,X2,SIGMA
15206       FORMAT('  X1 = ',F15.9,' X2 = ',F15.9,' HISTOGRAM INTERVAL = ',
         1F5.3)
```

```
C              EVALUATION OG THE FREQUENCY NUMBER
          DO 71 J=1,INC
   71     FREQ(J)=0.
         X12=X2-X1
         DO 26 I=1,NOBS
         J=IDINT((ERROR(I)-X1/x12)*INC)+1
         FREQ(J)=FREQ(J)+1.
   26    CONTINUE
C
C
C                           OUTPUT OF THE HISTOGRAM
C
C
         WRITE(6,331)(J,FREQ(J),J=1,INC)
  331    FORMAT(I2,5X,F5.0)
         CALL HIST(1,FREQ,INC)
         STOP
         END
C
C
C
C
C              SUBROUTINE SOLVE IS A MODIFIED VERSION OF SIMQ ROUTINE
C             OF IBM SCIENTIFIC SUBROUTINE PACKAGE FOR THE SOLUTION
C                           OF THE NORMAL EQUATIONS
C
C
C
C
          SUBROUTINE SOLVE(A,B,N,KS)
          REAL*8 A(6889),B(83),BIGA,SAVE
          TOL=0.0
          KS=0
          JJ=-N
         DO 65 J=1,N
          JY=J+1
          JJ=JJ+N+1
          BIGA=0.
          IT=JJ-J
          DO 30 I=J,N
          IJ=IT+I
          IF(DABS(BIGA)-DABS(A(IJ))) 20,30,30
   20      BIGA=A(IJ)
          IMAX=I
   30     CONTINUE
          IF(DABS(BIGA)-TOL) 35,35,40
   35     KS=1
          RETURN
   40     I1=J+N*(J-2)
          IT=IMAX-J
            DO 50 K=J,N
            I1=I1+N
            I2=I1+IT
            SAVE=A(I1)
            A(I1)=A(I2)
            A(I2)=SAVE
   50       A(I1)=A(I1)/BIGA
          SAVE=B(IMAX)
          B(IMAX)=B(J)
          B(J)=SAVE/BIGA
          IF(J-N) 55,70,55
   55     IQS=N*(J-1)
          DO 65 IX=JY,N
          IX=IQS+IX
          IT=J-IX
          DO 60 JX=JY,N
        IXJX=N*(JX-1)+IX
        JJX=IXJX+IT
   60   A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65   B(IX)=B(IX)-(B(J)*A(IXJ))
   70   NY=N-1
        IT=N*N
             DO 80 J=1,NY
               IA=IT-J
        IB=N-J
        IC=N
        DO 80 K=1,J
        B(IB)=B(IB)-A(IA)*B(IC)
        IA=IA-N
```

```
   80   IC=IC-1
        RETURN
        END
C
C
C
C
C
C       SUBROUTINE HIST IS A MODIFIED VERSION OF HIST ROUTINE OF
C          IBM SCIENTIFIC SUBROUTINE PACKAGE FOR THE EVALUATION
C             AND DISPLAY OF THE HISTOGRAM OF THE RESIDUALS
C
C
C
        SUBROUTINE HIST(NU,FREQ,IN)
        DIMENSION JOUT(20),FREQ(20)
          DATA K/'*'/,NOTH/''/
    1   FORMAT('EACH ',A1,'EQALS ',I2,'POINTS',/)
    2   FORMAT(I6,4X,20(4X,19(I2,3X),I2)
    3   FORMAT('INTERVAL',4X,19(I2,3X),I2)
    4   FORMAT(1H1,47X,' HISTOGRAM ',I3)
    5   FORMAT('FREQUENCY',20I5)
    6   FORMAT(' CLASS')
    7   FORMAT(113('-'))
        WRITE(6,4) NU
        DO 12 I=1,IN
   12   JOUT(I)=FREQ(I)
               WRITE(6,5)(JOUT(I),I=1,IN)
         WRITE(6,7)
        FMAX=0.0
        DO 20 I=1,IN
        IF(FREQ(I)-FMAX) 20,20,15
   15   FMAX=FREQ(I)
   20   CONTINUE
        JSCAL=1
        IF(FMAX-50.) 40,40,30
   30   JSCAL=(FMAX+49.0)/50.0
        WRITE(6,1) K,JSCAL
   40    DO 50 I=1,IN
   50    JOUT(I)=NOTH
         MAX=FMAX/FLOAT(JSCAL)
         DO 80 I=1,MAX
         X=MAX-(I-1)
            DO 70 J=1,IN
            IF(FREQ(J)/FLOAT(JSCAL)-X) 70,60,60
    60      JOUT(J)=K
    70      CONTINUE
            IX=X*FLOAT(JSCAL)
    80      WRITE(6,2) IX,(JOUT(J),J=1,IN)

            DO 90 I=1,IN
    90      JOUT(I)=I
            WRITE(6,7)
            WRITE(6,3)(JOUT(J),J=1,IN)
            WRITE(6,6)
            RETURN
            END
```