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A model for adjustment of differential gravity measurements with simultaneous gravimeter calibration

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Abstract. A mathematical model is proposed for adjustment of differential or relative gravity measurements, involving simultaneously instrumental readings, coefficients of the calibration function, and gravity values of selected base stations. Tests were performed with LaCoste and Romberg model G gravimeter measurements for a set of base stations located along a north–south line with 1750 mGal gravity range. This line was linked to nine control stations, where absolute gravity values had been determined by the free-fall method, with an accuracy better than 10 μGal. The model shows good consistence and stability. Results show the possibility of improving the calibration functions of gravimeters, as well as a better estimation of the gravity values, due to the flexibility admitted to the values of the calibration coefficients.

Key words: Calibration – Gravimeter

1 Introduction

In order to provide good estimates for gravity differences, generally two or more measurements of each gravity interval are performed in order to reduce the influence of random errors. In addition, the inherent instability of the mechanics of differential gravimeters, which is manifested by the drift and eventual jumps in their reading systems, recommends the use of two or more of these instruments simultaneously in gravity surveys. This procedure presumes that the calibration of all gravimeters is related to the same reference.

The goal of this work is to develop a procedure for the adjustment of gravity observations, with simultaneous calibration of the gravimeters referring to the same gravity datum. This datum must be represented by at least two gravity control stations with accurately known gravity values.

The mathematical model for a gravity interval Δg is given by

$$\Delta g = k\Delta \ell' \tag{1}$$

where k is the scale factor for the gravimeter, related to the chosen gravity datum, and $\Delta \ell'$ is the reading interval in gravity units, normally mGal = 10^{-5} m/s^2 corrected for drift and Earth tides effects. In order to convert the reading interval from reading units to mGal the calibration function for the gravimeter must be applied, introducing errors related to the estimation of the calibration parameters.

In this work a linear mathematical model is presented which can be applied to any differential gravimeter, for the gravity interval that involves calibration functions of the gravimeters, gravity values, scale factors, and instrumental readings.

2 Effects on gravity measurements

The gravimeter readings at a given point on the Earth's surface are not constant. Gravity changes with time-dependent processes such as Earth tides, instrumental drifts, changes in air pressure, and groundwater mass. When relative measurements of gravity are performed on field survey, Earth tides and instrumental drift are the most notable effects. Other effects important in absolute measurements and continuous monitoring of gravity changes at a fixed point are negligible in relative gravity.

Longman (1959) proposed expressions for computing the tidal acceleration due to the Moon and Sun. In that work, Longman did not take into account the loading and attraction effects due to the oceanic tides on the intensity of gravity.

Ducarme and Melchior (1983) showed that peak-to-peak amplitude of such effects can reach 10 μGal for the ocean semi-diurnal main component M_2 in South America. The effect of ocean tide could be negligible if

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the time interval between relative measurements was reduced, and consequently Longman's expressions could be accurately applied. These expressions were used with a mean gravimetric factor equal to 1.16 (De Freitas 1993).

Instrumental drift is expressed by a slow and progressive variation in the readings with time, due to permanent accommodations in the elastic system. Thus, the drift is inherent to the instrument, and it is small and linear in a given time interval (LaCoste and Romberg 1972). However, the drift rate depends on the conditions to which the instrument is submitted in transport during the survey (Angus and Brulé 1967). Thus, it is not possible to assign the same drift rate to a gravimeter for the whole gravity survey. The usual procedure is to assume a drift rate for each gravity interval measured with that instrument. Thus the introduction of the drift rates as parameters is inadequate in the mathematical model to avoid an overparametrization process. Therefore, it would be better to remove the drift for each gravity interval prior to the adjustment. This is the procedure adopted in this paper.

Rabbel and Zschau (1985) estimated gravity variations of about $\pm 3\ \mu$Gal associated with seasonal changes in air pressure. For an extreme air pressure variation of ± 60 mbar, at the center of an (anti-) cyclone, Rabbel and Zschau estimated a gravity variation of about $\pm 20\ \mu$Gal.

Meurers (2000), using a super-conducting gravimeter, pointed out an amplitude of up to 2 μGal for the gravity variation due to the effect of atmospheric processes characterized by high vertical convection activity and heavy rainfall. The correction of these effects is important when super-conducting gravimeters are used to measure small tectonic displacements that can result in gravity variations of a few microgals. It is also important to correct these effects when free-fall apparatus is used to measure the absolute value of the gravity with an accuracy of a few microgals. However, when relative measurements are performed in a short time interval under favourable climatic conditions, and with a zero-length spring gravimeter, these effects are numerically small and negligible.

The gravity changes due to variations in groundwater masses were not taken into account in this work; It is difficult to find proper data sets indicating the true influences of these local effects on relative measurements of gravity.

3 The mathematical model

In order to convert the instrumental reading unit to the gravity unit, mGal, a calibration function is applied. Some differential gravimeters have linear calibration functions, determined by a single factor. In geodetic gravimeters, like the LaCoste & Romberg (LCR), calibration functions are not linear. They are represented by straight-line segments of a continuous polygonal line, defined in discrete intervals of equal reading amplitude.

The normal procedure for the LCR is to observe the variation in readings for a known variation in force. A standard mass is added to or removed from a pan hanging from the beam of the gravity meter (LaCoste 1991; Valliant 1991). Therefore, the calibration function is published in the form of an interval factor, as shown in Table 1.

It is important to emphasize that a low-order polynomial approximation to the calibration table will affect the gravimeter's precision specification. In contrast, the use of a high-order polynomial will introduce numerical instability. Hence, the most practical procedure is to use the coefficients provided by the manufacturer.

Let δ be the discretization interval for the calibration function, ℓ_n the instrumental readings in multiples of δ, ℓ'_n the corresponding value in milligals and α_m the calibration coefficients for the intervals. The following relation holds:

$$\ell'_n = \delta\left(\sum_{m=0}^{n-1} \alpha_m\right) \tag{2}$$

where n and m are the order of the row in the calibration table, in which $n = 1, 2, 3, \ldots,\ m = 0, 1, 2, \ldots,$ and the starting value for ℓ' is zero (see Table 1). The polygonal line represented by Eq. (2) linearizes any calibration function by parts, with a δ discretization interval.

An original reading ℓ_i, which is not multiple of δ, can be expressed as a function of the multiple of δ immediately inferior, ℓ_I, by the equation

$$\ell_i = \ell_I + \varepsilon_i = I\delta + \varepsilon_i \tag{3}$$

where I is an integer equal to the number of discretization intervals embraced by ℓ_I and $\varepsilon_i < \delta$.

Applying the linearized calibration function, Eq. (2), to Eq. (3), the value of the reading converted into mGal is

$$\ell'_i = \ell'_I + \alpha_I\varepsilon_i = \delta\left(\sum_{m=o}^{I-1} \alpha_m\right) + (\ell_i - I\delta)\alpha_I \tag{4}$$

According to Eq. (1), the calibrated instrumental readings ℓ_i and ℓ_j, observed on the stations (i) and (j) and corrected for drift and Earth tides, are related to the gravity values g_i and g_j by scale factor k_r of the gravimeter (r)

$$g_i - g_j = k_r(\ell'_i - \ell'_j) \tag{5}$$

Table 1. Part of the manufacturer's calibration table of the LCR 257 gravimeter, with $\delta = 100$ (LaCoste and Romberg 1972)

Counter reading, ℓ	Value in mGal, ℓ'	α
0	000.00	1.06825
δ	106.83	1.06823
2δ	213.65	1.06818
3δ	320.47	1.06813
4δ	427.28	1.06807
$\vdots$	$\vdots$	$\vdots$

Taking into account Eq. (4), the model for the gravity interval involving instrumental readings (ℓ), gravity values (g), calibration coefficients (α), and the datum scale factor (k) is deduced:

$$g_i - g_j + k_r \left(\ell_{j\,r}\alpha_J - \ell_{i\,r}\alpha_I - \delta J_{\,r}\alpha_J + \delta I_{\,r}\alpha_I + \delta \sum_{m=0}^{J-1} {}_r\alpha_m - \delta \sum_{m=0}^{I-1} {}_r\alpha_m \right) = 0 \tag{6}$$

where J is analogous to I in Eq. (3) for the reading ℓ'_j. Equation (6) is a mathematical model adequate for the determination of gravity differences for a gravimeter with a calibration function discretized in reading intervals with amplitude δ.

4 The constraint model

The mathematical model expressed by Eq. (6) is represented by (u) equations related to the same number of observed gravity intervals. The observables in the model are instrumental readings (ℓ) and calibration coefficients (α). The unknown parameters are the gravity values (g) on the stations and the datum scale factors k_r for each gravimeter (r). The solution of this model requires the knowledge of gravity values in at least two stations. These values can be introduced as relative constraints, weighted according to the inverse of their estimated variances, with an additional constraint model

$$g - \bar{g} = 0 \tag{7}$$

where $\bar{g}$ represents the absolute gravity observations on the control stations, and g is the same as in Eq. (6). Weighting is introduced to counteract the different precisions of the observations. Therefore, in this paper, all observations ($\ell, \alpha, \bar{g}$) were weighted according to the inverse of their estimated variance.

5 Application

A north–south test line, including 118 high-precision gravity stations in South America (Fig. 1), was selected to verify the performance of the proposed model. This line covers a gravity range of 1750 mGal and is linked to nine absolute control stations established by the JILAG-3 free-fall gravimeter, with an estimated accuracy of better than 10 μGal (Gemael et al. 1990; Torge et al. 1994). Gravity intervals between the stations were measured at least eight times, using two LCR gravimeters simultaneously in two round trips (A–B–A–B–A), where A and B are gravity stations separated by an average distance of 100 km. A total of 1554 gravity differences were performed using six LCR model G gravimeters. Table 2 shows the number of gravity differences performed with each instrument versus the magnitude of the observed interval.

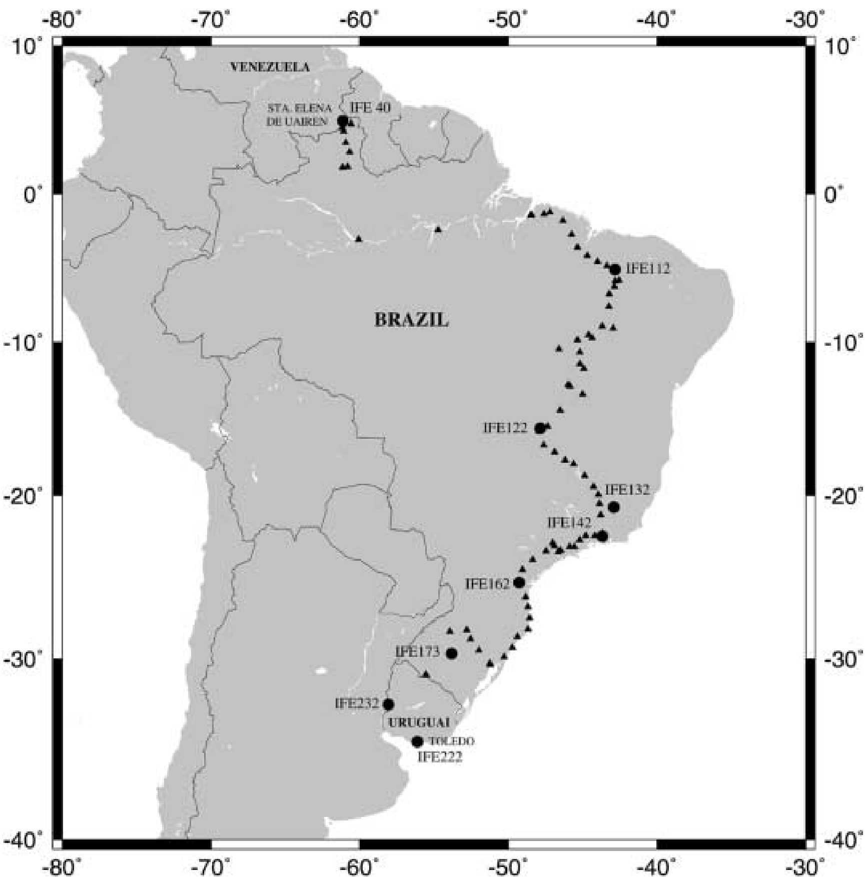

Fig. 1. The gravity line and absolute control stations from Sta. Elena de Uairen (Venezuela) to Toledo (Uruguay), numbering according to IFE Hannover. Generated with Generic Mapping Tools free software (Wessel and Smith 1991)

Table 2. Number of reading intervals obtained by individual LCR gravimeters

Interval	LCR						Total
	013	061	257	602	622	674	
$0 \leq \Delta\ell < 50$	17	176	277	160	102	38	770
$50 \leq \Delta\ell < 100$	2	24	110	132	85	20	373
$100 \leq \Delta\ell < 150$	4	17	29	18	4		72
$150 \leq \Delta\ell < 200$	4		106	57	98	63	328
$200 \leq \Delta\ell$			4	6	1		11
Total	27	217	526	373	290	121	1554

Gravity values for the absolute control stations and their standard deviations are presented in Table 3 (Torge et al. 1994).

6 Adjustment and results

In order to estimate the weights for the observables, α, ℓ and $\bar{g}_i$, their standard deviations must be available. There is no explicit indication of the precision of calibration coefficients α in the instrument calibration tables. However, truncation on the values in milligals suggests a standard deviation of $\leq$0.00005 mGal/reading unit. A trial-and-error procedure was used to determine the standard deviations for the readings ℓ. An adequate starting value is $\sigma_\ell = 0.025$. The standard deviations of the absolute gravity values $\sigma_{\bar{g}}$ are listed in Table 3. Thus, weights for the observables were established as (Dias 1997)

$$w_{\alpha_i} = \frac{1}{\sigma^2_{\alpha_i}}, \quad w_{\ell_i} = \frac{1}{\sigma^2_{\ell_i}}, \quad w_{\bar{g}_i} = \frac{1}{\sigma^2_{\bar{g}_i}}$$

A least squares (LS) adjustment gives the adjusted readings, calibration coefficients, and control gravity values with their corresponding standard deviations. Residual differences between adjusted and observed values are used for estimating the variance of the unit weight (Germael 1994)

$$\widehat{\sigma}^2_o = \frac{\mathbf{V}^T\mathbf{P}\mathbf{V}}{v} \tag{8}$$

where $\mathbf{V}$ is the array of residuals, v is the degree of freedom for the adjustment, and $\mathbf{P}$ is the weighting matrix

Table 3. Gravity values and standard deviations at the control stations

Station	Gravity (mGal)	σ
IFE040	977 822.084	0.01
IFE112	978 016.343	0.01
IFE122	978 048.798	0.01
IFE132	978 460.230	0.01
IFE142	978 637.581	0.01
IFE162	978 760.387	0.01
IFE173	979 261.636	0.01
IFE222	979 715.855	0.01
IFE232	979 523.526	0.01

$$\mathbf{P} = \sigma^2_o \begin{pmatrix} w_{\bar{g}_1} & 0 & \dots & \dots & 0 & 0 \\ 0 & \ddots & & & & 0 \\ \vdots & & & w_{\alpha_1} & & \vdots \\ 0 & & & & \ddots & 0 \\ 0 & 0 & \dots & \dots & 0 & w_{\ell_n} \end{pmatrix} \tag{9}$$

As the observations were weighted according to the inverse of their estimated variance, σ^2_o was assumed to be equal to 1.

Some preliminary adjustments were performed in order to determine a better combination for the weights of the calibration coefficients and readings. Figure 2 shows the correlation between σ_α, σ_ℓ, and $\widehat{\sigma}^2_o$, as found in the adjustments with different values for σ_α and σ_ℓ.

The best fit to $\widehat{\sigma}^2_o = 1$ was obtained for $\sigma_\alpha = 0.00004$ mGal/reading unit and $\sigma_\ell = 0.019$ mGal, with $\widehat{\sigma}^2_o = 0.9958$. The largest residual of the absolute values is 9 μGal, at the station IFE222, which is less than the estimated standard deviation for the gravity values on the control stations (10 μGal).

It should be emphasized that no systematic trend can be observed in the distribution of the residuals (Fig. 3), which confirms that this mathematical model performs well with respect to the gravity values of the control stations. The distribution of the residuals obtained for the readings, over that gravity range (1750 mGal), is

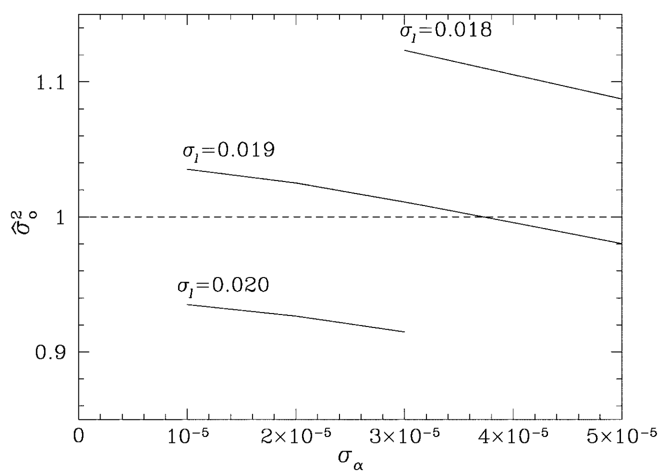

Fig. 2. Solutions for the standard deviations of the calibration coefficients and the readings, σ_α and σ_ℓ, with respect to the relation between a posteriori and a priori variance of the weight unit, $(\widehat{\sigma}^2_o/\sigma^2_o) = 1$

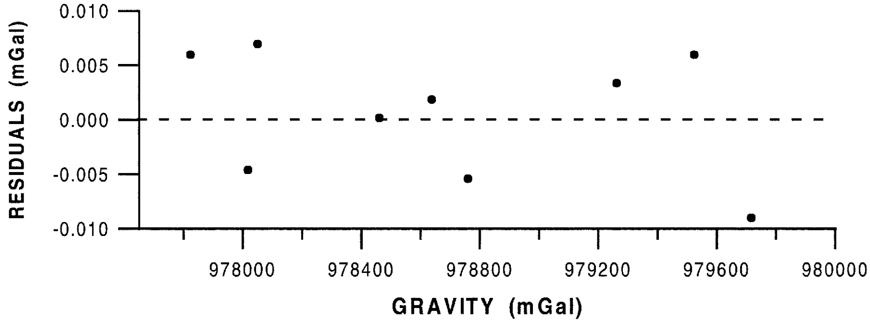

Fig. 3. Gravity residuals at the control stations

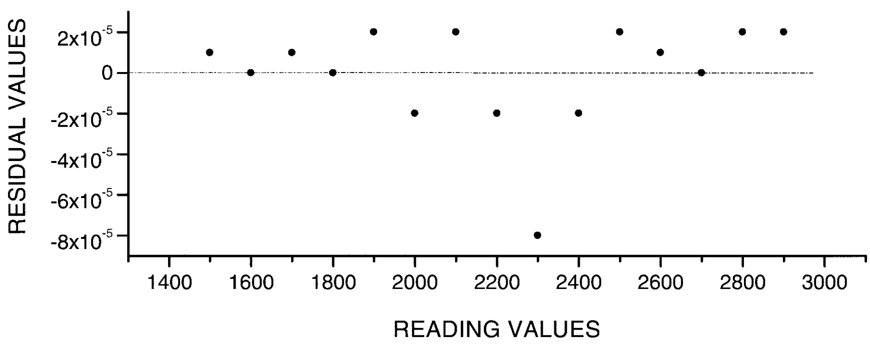

Fig. 4. Distribution of reading residuals

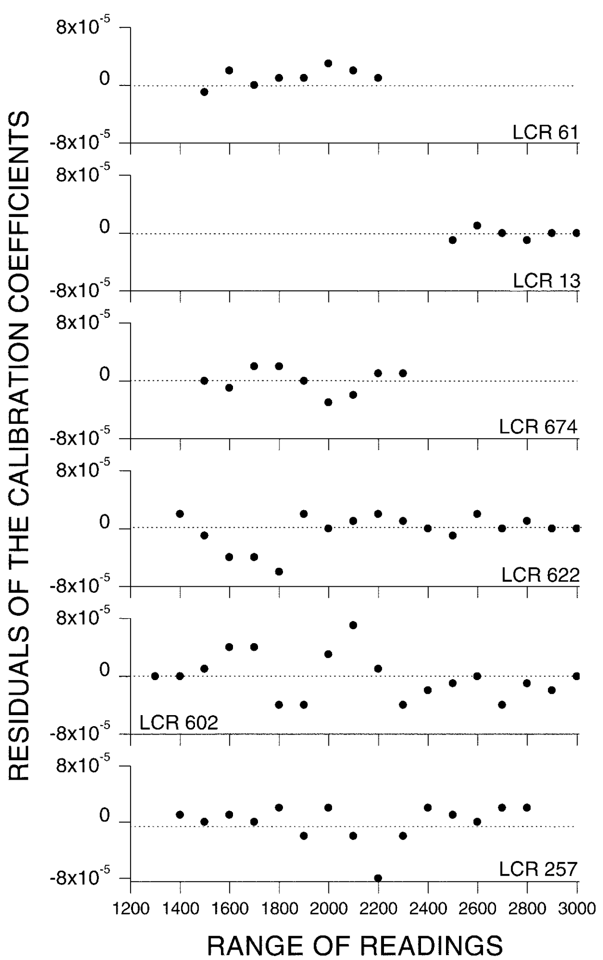

Fig. 5. Calibration coefficient residuals for the LCR gravimeters

Table 4. Datum scale factors for the gravimeters

LCR	k	σ_k	k (Escobar et al. 1996)	σ_k
013	0.998834	0.000077	–	–
061	1.000007	0.000004	–	–
257	1.000328	0.000034	1.000317	0.000021
602	1.000470	0.000036	1.000471	0.000026
622	1.000401	0.000037	1.000448	0.000027
674	1.000552	0.000042	1.000476	0.000030

given in Fig. 4. The largest residual value was 0.0305 mGal, which is less than $2\sigma_\ell$ ($\sigma_\ell = 0.019$ mGal). Again, no systematic trend can be observed in the residuals' distribution.

The distributions of the calibration coefficient residuals for all gravimeters used here are presented in Fig. 5. The largest residual is 0.000082 mGal/reading unit, which is approximately $2\sigma_\alpha$ ($\sigma_\alpha = 0.00004$). An analysis of the residuals shows that there is no trend for none of the gravimeters.

It should be noted that the significance of the adjusted calibration coefficients depends on the number of observations that were performed on the respective reading interval with a given gravimeter. The LS estimates for the datum scale factors, k, are given in Table 4, which includes the values of k obtained previously for the four gravimeters used here (Escobar et al. 1996). Those values were obtained using a mathematical model without α, and the results for k in both adjustments are consistent.

Another value for the datum scale factor of LCR 257 was estimated previously, in which $k_{257} = 1.000317 \pm 0.000021$ (Escobar 1995). This value also was obtained using a mathematical model without α, applied to more than 6500 observations at 564 stations, of the adjustment of the Brazilian Gravity Standardization Net (BGSN). The value obtained in this paper, $k_{257} = 1.000328 \pm 0.000034$, was obtained using less than 25% of the BGSN observations. Furthermore, a comparison of the gravity values obtained at BGSN stations and in the present investigation, results in difference values of less than 0.025 mGal, which confirms the agreement with that work.

7 Conclusions

The model proposed here can be used in the calibration of differential gravimeters with geodetic range, where the calibration function can be discretized by the adoption of a reading interval. If applied to the adjustment of gravimetric nets or lines, the model can provide estimates for the interval calibration coefficients, simultaneously with the other unknown parameters.

Once a suitable weighting is adopted, the introduction of calibration coefficients as observations instead of parameters enhances the importance of the values determined by the manufacturer, and avoids large discrepancies between known and adjusted values. Nevertheless, the accuracy of the adjusted values depends on the quantity of observations and their quality, and the fit of the calibration table can lead to a less-smoothed calibration function.

The procedure used can improve the quality of the results, due to the flexible handling of the calibration coefficients introduced in the model as observables. In practice, when two or more gravimeters are used on the same station, this method can improve the precision of a gravimetric network, by a simple analysis of the standard deviations and residuals.

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