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L_1 NORM MINIMIZATION IN GPS NETWORKS

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ABSTRACT

The least squares method is a statistical tool for the estimation of unknown parameters. All the results which are derived from the method of least squares are deteriorated when outliers are present in the observation data. Therefore, outliers have to be detected and eliminated by using statistical tests or robust methods. For this purpose, L_1 norm minimization, which is a robust method, can be used in geodetic networks. In this paper, the formulation of L_1 norm minimization for correlated observations is presented. The method is applied to a simulated GPS network. The performances of the least squares method and L_1 norm minimization are compared in the cases of observations with or without outliers. Our example shows that L_1 norm minimization is a more successful method than the least squares method for outlier detection and the obtained coordinates are more reasonable and reliable than those from the least squares when some observations are burdened with blunders.

KEYWORDS: Least squares. L_1 norm minimization. Outliers. GPS network.

INTRODUCTION

The least squares method, which tries to arrive at a best solution by minimizing the sum of squares of weighted discrepancies between measurements, is one of the most widely used methods to acquire unique estimates for point coordinates in geodetic networks from a set of redundant measurements. The method of least squares does not require a priori knowledge of the distribution associated with the observations. If the weight matrix P is chosen to be the inverse of the variance-covariance matrix of the observations, the least squares estimate is an unbiased and minimum variance estimate. Moreover, if the observation errors have a normal distribution, the least squares estimate is the maximum likelihood estimate. The weighted least squares method tries to solve for an optimal estimate of unknown parameters by minimizing the sum of squares of the weighted residuals

$$v^T P v \rightarrow \min \tag{1}$$

where P = weight matrix of observations.

As is well known, the method of least squares is a powerful statistical technique for the estimation of unknown parameters. Nevertheless, it demands blunder-free measurement data to produce reliable results. If the observation errors are random (preferably normally distributed), it gives an optimal estimate of unknown parameters. Unfortunately, when outliers are present in an observation set, it produces poor or invalid results and makes difficult outlier detection since it spreads the corrupt effect of the outliers upon the good observations. Furthermore, the properties of least squares are no longer valid. Hence, the outliers have to be detected and eliminated by using statistical tests or robust statistical methods.

Robust techniques have been used as a statistical tool in order to discover and control the outliers [**1**], [**2**], [**5**], [**7**], [**8**] and [**9**]. Robust techniques are insensitive to outliers, i.e., they decrease the corrupt effect of outliers upon the estimated parameters. Furthermore, robust techniques are more sensitive than least squares for outlier

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detection. One of the robust techniques is L_1 norm minimization. This technique tries the minimization of the sum of the weighted residuals. Some authors applied this method to geodetic networks [**4**], [**6**], [**10**], [**11**] and [**12**]. L_1 norm minimization states that the sum of the weighted residuals should be minimal

$$\mathrm{p}^{\mathrm{T}}|\mathrm{v}| \rightarrow \min \tag{2}$$

where p=nx1 vector, which contains the diagonal elements of the matrix P. It should be noted that the L_1 norm minimization is an unbiased estimate like least squares. But, other advantages of the least squares such as minimum variance and maximum likelihood are no longer valid. However, the advantage of L_1 norm minimization compared to least squares is its robustness. That is, it is less sensitive to outliers **[12]**. Furthermore, unlike L_1 norm minimization, a lot of robust techniques such as M-estimates and Danish method are based on iteratively reweighted least squares algorithm. Unfortunately, the method of least squares smooths out blunders across the entire data set. Therefore a good observation may be pointed out as outlier or some of the outliers may be masked since the contaminated data derived from the least squares adjustment is used.

Most of the available robust techniques appearing in the literature are based on the presumably independent observations. Therefore, they are inapplicable for correlated observations. On the other hand, correlated observations are very often encountered in geodetic practice such as GPS networks. Thus, the correlations among the observations must be carefully taken into account in robust parameter estimation. The most important algorithms of robust estimation for correlated observations are IGGIII (Institute of Geodesy and Geophysics) scheme and bifactor reduction model of weight elements [**14**] and [**15**].

IGGIII is a robust estimator based on M-estimation and the principle of equivalent weight matrix. However, the symmetry of the equivalent weight matrix of IGGIII scheme is ignored. Although the parameter estimates are not affected so much due to this asymmetry, for only a few outliers are supposed to exist in observations, it causes the corresponding normal matrix and posterior variance-covariance matrix are slightly asymmetric. Thus the property of a symmetric matrix to reduce calculation or computer storage when calculating or storing the normal matrix is no longer valid [**15**]. Another robust method for correlated observations is bifactor reduction model of weight elements (see [**15**]). In this method, M-estimation has been extended into correlated observations. Bifactor reduction model of weight elements can be realized by using iteratively reweighted least squares.

This paper addresses the implementation of L_1 norm minimization in GPS networks that have correlated observations. The formulation of L_1 norm minimization in a rank deficient Gauss-Markov model was presented in [**12**]. L_1 norm minimization, however, was applied to uncorrelated observations that have a weight matrix of diagonal type in [**12**]. In order to be able to apply this method to GPS networks, correlated observations may be converted into uncorrelated ones by using Cholesky factorization. In the following section, the formulation of L_1 norm minimization, which is based on Cholesky factorization, will be presented.

FORMULATION OF L_1 NORM MINIMIZATION IN GPS NETWORKS

In the classical Gauss-Markov model, the functional and stochastic models of a geodetic network can be expressed in the following form

$$\begin{aligned} \mathrm{l} + \mathrm{v} &= \mathrm{Ax} \\ \mathrm{P} &= \sigma_0^2 \mathrm{C}_1^{-1} \\ \mathrm{D}^{\mathrm{T}}\mathrm{x} &= 0 \end{aligned} \tag{3}$$

where $\mathrm{v}_{n\mathrm{X}1}$ is the vector of residuals; $\mathrm{l}_{n\mathrm{X}1}$=vector of observations; $\mathrm{A}_{n\mathrm{X}u}$ = rank deficient design matrix; $\mathrm{P}_{n\mathrm{X}n}$ = weight matrix of observations; $\mathrm{D}_{u\mathrm{X}d}$ = datum matrix of the network; $0_{d\mathrm{X}1}$ = zero vector; $\mathrm{C}_{1(n\mathrm{X}n)}$ = covariance matrix of observations; and σ_0^2 = a priori variance factor.

The least squares adjustment or L_1 norm minimization determine the unknown parameters x based on the Gauss-Markov model according to their own objective functions given in (1) and (2), respectively.

L_1 norm minimization is a parameter estimation method, which minimizes the weighted sum of the absolute residuals. In [**12**], a full formulation of L_1 norm minimization for the general Gauss-Markov model was presented. Setting up the L_1 norm parameter estimation problem by a linear programming solution requires to develop a mathematical model where all parameters and residuals are nonnegative. The objective function (2) and the constraints (3) can be transformed into a L_1 norm parameter estimation problem by introducing slack variables that provide nonnegativity. Thus, the objective function can be written without absolute value signs. To convert the objective function and constraints into a form where there are nonnegative parameters and nonnegative residuals, two slack vectors, α and β, for the parameters, and two slack vectors, u and w, for residuals can be introduced. The parameters and the residuals may be positive or negative. Therefore, these unknowns and residuals vectors can be rewritten by using slack variables

$$\begin{aligned} \mathrm{v} &= \mathrm{u} - \mathrm{w} \quad \mathrm{u}, \mathrm{w} \geq 0 \\ \mathrm{x} &= \alpha - \beta \quad \alpha, \beta \geq 0 \end{aligned} \tag{4}$$

The original objective function (2) and constraints (3) can be rewritten in terms of slack variables

$$f = \mathrm{p}^{\mathrm{T}}|\mathrm{v}| = \mathrm{p}^{\mathrm{T}}|\mathrm{u} - \mathrm{w}| = \mathrm{p}^{\mathrm{T}}(\mathrm{u} + \mathrm{w}) \rightarrow \min \tag{5}$$

subject to

$$\begin{aligned} \mathrm{l} + \mathrm{u} - \mathrm{w} &= \mathrm{A}(\alpha - \beta) \\ \mathrm{D}^{\mathrm{T}}(\alpha - \beta) &= 0 \end{aligned} \tag{6}$$

or equivalently

$$f = \underbrace{\begin{bmatrix} 0^T & 0^T & p^T & p^T \end{bmatrix}}_{\underline{c}^T} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ w \\ u \end{bmatrix}}_{\underline{x}} \rightarrow \min \tag{7}$$

subject to

$$\underbrace{\begin{bmatrix} A & -A & I & -I \\ D^T & -D^T & Z & Z \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ w \\ u \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\underline{b}} \tag{8}$$

where Z is a zero matrix and I is a identity matrix.

L_1 norm minimization problem can be written as

$$f = \underline{c}^T \underline{x} \rightarrow \min \tag{9}$$

subject to

$$\underline{A}\underline{x} = \underline{b}; \quad \underline{x} \geq 0 \tag{10}$$

This is an optimization problem that can be solved by linear programming. Solving $\underline{x}$ will yield the vectors α, β, u and w, consequently the parameter vector x and the residual vector v can be obtained. The simplex method is generally used to solve a linear programming problem [**12**]. Interested readers refer to [**3**].

L_1 norm minimization given above works as long as the individual observations are uncorrelated, i.e., they have a weight matrix of diagonal type. If they are dependent the procedure is inapplicable. In order to be able to apply L_1 norm minimization to geodetic networks that have correlated observations, correlated observations have to be transformed into uncorrelated ones through Cholesky factorization. This happens by diagonalizing the P matrix:

$$A' = WA \tag{11}$$

$$\left(A'\right)^T A' = A^T W^T W A = A^T P A \tag{12}$$

As the covariance matrix C_l^{-1} is symmetric and positive definite, we can use the Cholesky for factorization:

$$C_l^{-1} = W^{-1} W^T \tag{13}$$

or

$$P = W^T W \tag{14}$$

The transformation is described as follows:

$$A' = WA, \qquad l' = Wl, \qquad v' = Wv \tag{15}$$

Unknown parameters x remains unchanged under this special transformation and the a priori covariance matrix of the transformed observations $C_1^{'}$ and observation weights $P^{'}$ are

$$C_1^{'} = I \tag{16}$$

$$P^{'} = I \tag{17}$$

where I is the identity matrix. This decomposition of the covariance matrix C_1^{-1} and the transformation of A and l is called decorrelation of the observations. The transformed observations obtained from this transformation are called decorrelated observations and they have the weight 1 [**13**].

Since the three baseline components in GPS networks are correlated, a covariance matrix of 3x3 is derived for each baseline as a product of the least squares adjustment of the carrier-phase measurements. This matrix is used to weight the baseline components in the network adjustment in accordance with (3). Therefore, the weight matrix for any GPS network is a block diagonal type. When more than two receivers are used, a full weight matrix that also shows the correlations among simultaneously measured baselines is used [**16**]. Therefore, L_1 norm minimization have to be adapted for GPS networks due to correlations among observations. By replacing A matrix and l vector with $A^{'}$ and $l^{'}$, $\underline{A}$ matrix and $\underline{b}$ vector in (10) can be rewritten as follows

$$\begin{bmatrix} A^{'} & -A^{'} & I & -I \\ D^{T} & -D^{T} & Z & Z \end{bmatrix} = \underline{A}^{'}; \quad \begin{bmatrix} l^{'} \\ 0 \end{bmatrix} = \underline{b}^{'} \tag{18}$$

Since the weight of the decorrelated observations is 1, the $\underline{c}$ vector in the objective function (9) is modified as

$$\begin{bmatrix} 0 \\ 0 \\ p^{'} \\ p^{'} \end{bmatrix} = \underline{c}^{'} \tag{19}$$

where $p^{'}$ is a vector of ones.

Consequently, L_1 norm minimization for the decorrelated observations can be posed as

$$f = \underline{c}^{'T} \underline{x} \rightarrow \min \tag{20}$$

subject to

$$\underline{A}^{'} \underline{x} = \underline{b}^{'}; \quad \underline{x} \geq 0 \tag{21}$$

NUMERICAL RESULTS

A GPS baseline network, which is composed of 6 IGS (International GNSS Service) stations and 13 baseline observations, is used to test the performance of L_1 norm minimization and the least squares method (see Fig. 1). The datum of the network is provided by minimal constraints; station morp has been considered as a fixed station. Degree of freedom of network is 24. The data files of these 6 points have been provided from SOPAC (Scripps Orbit and Permanent Array Center) service. In this example, the simulation procedure of [**15**] (see pp. 355) has been used.

The GAMIT software is used to estimate the station coordinates and the covariance matrix of baselines Σ. An observational vector l calculated from the estimated coordinates is used as true observational vector. By using the random number generator of MATLAB, the random error vector ε is generated from the standard normal distribution by Monte Carlo method. In order to make the simulated errors correlated in accordance with the original covariance matrix, Σ is decomposed into TT^T by using Cholesky factorization. ε is transformed into $\varepsilon^{'} = T\varepsilon$. Then the true error vector $\varepsilon^{'}$ is added to the baseline vector l to obtain the new observation vector $l^{'} = l + \varepsilon^{'}$ with its covariance matrix Σ. Three simulated outliers, which are shown in Table 1, are added to $l^{'}$

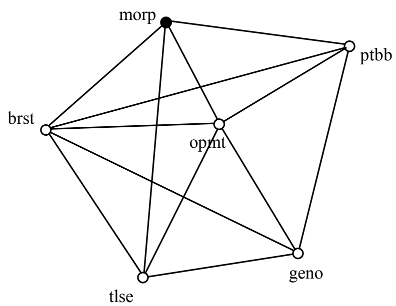

Fig. 1. GPS Network

Table 1. *Outlying Observations*

Baseline Components	Blunders
$\Delta X_{geno\text{-}ptbb}$	+2.023 m
$\Delta Y_{brst\text{-}ptbb}$	–4.998 m
$\Delta Z_{opmt\text{-}ptbb}$	+7.016 m

Two schemes of adjustment for the GPS network are performed.

Scheme 1: L_1 norm adjustment and the least squares adjustment based on the simulated observation vector $l^{'}$ without any outliers.

Scheme 2: L_1 norm adjustment and the least squares adjustment based on the observation vector $l^{'}$ with three outliers.

Recall that the implementation of L_1 norm minimization requires a linear programming solution. The subroutine “linprog.m” of MATLAB software has been used for this purpose. L_1 norm results have been obtained according to (20) and (21). The subroutine “chol.m” of MATLAB software can be used for Cholesky factorization. In this example, $\underline{A}^{'}$ is a 42x114 matrix, $\underline{b}^{'}$ is a 42x1 vector, $\underline{c}^{'}$ is a 114x1 vector and $\underline{x}$ is a 114x1 vector.

Table 2 and Table 3 show the results of adjustment by L_1 norm and the least squares of coordinates for the scheme 1 and scheme 2, respectively.

Table 2. L_1 *norm and the least squares adjusted coordinates for the scheme 1*

Point Number	Adjusted Coordinates (L_1 norm)			Adjusted Coordinates (The least squares)		
	X	Y	Z	X	Y	Z
morp	3645667.665	-107277.139	5215053.439	3645667.665	-107277.139	5215053.439
ptbb	3844059.792	709661.428	5023129.467	3844059.792	709661.425	5023129.468
geno	4507892.454	707621.328	4441603.436	4507892.450	707621.330	4441603.442
tlse	4627851.672	119640.129	4372993.463	4627851.668	119640.132	4372993.466
brst	4231162.684	-332746.821	4745130.846	4231162.682	-332746.823	4745130.846
opmt	4202777.205	171368.114	4778660.113	4202777.206	171368.111	4778660.118

Table 3. L_1 *norm and the least squares adjusted coordinates for the scheme 2*

Point Number	Adjusted Coordinates (L_1 norm)			Adjusted Coordinates (The least squares)		
	X	Y	Z	X	Y	Z
morp	3645667.665	-107277.139	5215053.439	3645667.665	-107277.139	5215053.439
ptbb	3844059.794	709661.422	5023129.465	3844059.808	709661.079	5023130.096
geno	4507892.453	707621.328	4441603.434	4507891.980	707621.919	4441603.997
tlse	4627851.672	119640.128	4372993.463	4627851.406	119640.631	4372992.917
brst	4231162.684	-332746.821	4745130.845	4231162.537	-332746.006	4745130.957
opmt	4202777.206	171368.114	4778660.113	4202777.373	171368.000	4778659.403

The adjusted residuals of the observations in the scheme 2 by L_1 norm and the least squares are shown in Table 4.

Table 4. L_1 *norm and the least squares adjusted residuals*

Residuals (m)

Observation No	L_1	Observation No	L_1	Observation No	The least squares	Observation No	The least squares
1	-0.004	20	-0.004	1	0.143	20	-0.316
2	0.008	21	0.000	2	**-0.806**	21	**-0.657**
3	-0.009	22	-0.006	3	-0.121	22	**0.636**
4	-0.001	23	-0.003	4	0.014	23	**-0.708**
5	-0.000	24	-0.010	5	-0.343	24	**-1.282**
6	-0.002	25	-0.005	6	**0.629**	25	0.163
7	***-2.016***	26	0.000	7	*-1.527*	26	-0.114
8	0.004	27	0.000	8	**-0.929**	27	**-0.709**
9	0.000	28	0.027	9	0.068	28	-0.240
10	-0.002	29	0.013	10	0.206	29	**0.516**
11	0.000	30	0.006	11	-0.088	30	**-0.540**
12	-0.004	31	0.005	12	**-1.113**	31	-0.148
13	0.002	32	-0.006	13	0.316	32	-0.234
14	0.000	33	***-7.014***	14	**-0.929**	33	*-5.674*
15	0.000	34	-0.007	15	**-0.821**	34	-0.441
16	0.006	35	-0.024	16	0.167	35	**0.593**
17	***4.996***	36	-0.000	17	*3.839*	36	0.163
18	0.001	37	-0.010	18	**0.520**	37	-0.338
19	0.003	38	0.002	19	-0.117	38	-0.222
		39	0.000			39	0.451

By implementing L_1 norm and the least squares adjustment procedures, we find the following.

1. As can be seen from Table 2, when the observations are burdened with only random errors, L_1 norm minimization yields a result close to the least squares method. As is well known, when the observation errors are random (normally distributed), the least squares method yields the most optimal solution.
2. If there are outlying observations, L_1 norm minimization provides the best possible solution that is close to results produced by least squares with blunder-free observations. On the other hand, the method of least squares produces a very highly contaminated solution, see Table 3.

3. The outliers have not been exactly reflected in the corresponding residuals in the least squares adjustment. These have been shown with italic values in the columns 6 and 8 of Table 4. There are many outlying residuals which correspond to good observations. These have been shown as bold values in the columns 6 and 8 of Table 4. On the other hand, the residuals from L_1 norm minimization have successfully shown the outlying observations. These have been shown as bold and italic values in columns 2 and 4 of Table 4.
4. Since the coordinates obtained from GAMIT are assumed as true coordinates, the coordinates calculated from both adjustment techniques can easily be analyzed by comparing the differences from the true coordinates. These differences have been shown in Table 5. As seen, the coordinates obtained from L_1 norm are more reasonable and reliable than those from least squares method even if outliers are present.

Table 5. *Differences between the true coordinates and the adjusted coordinates for the scheme 2 (in cm)*

Point Number	Coordinate differences (L_1 norm)			Coordinate differences (The least squares)		
	dX	dY	dZ	dX	dY	dZ
morp	0	0	0	0	0	0
ptbb	1.428	0.291	0.995	2.828	-34.009	64.095
geno	0.600	-0.100	0.800	-46.700	59.000	57.100
tlse	1.090	-0.362	1.019	-25.510	49.938	-53.581
brst	0.700	-0.400	0.800	-14.000	-81.900	12.000
opmt	0.439	0.943	0.763	17.139	-10.457	-70.237

DISCUSSION

Which method is more sensitive for outliers of small magnitude? To answer this question we simulated two outliers for GPS network used in our simulation example. One of these, $\Delta Z_{\text{geno-ptbb}}$, is an outlier with small magnitude (see Table 6). Outliers are added to $1'$.

Table 6. *Outlying Observations*

Baseline Components	Blunders
$\Delta Z_{\text{geno-ptbb}}$	+0.206 m
$\Delta Y_{\text{opmt-ptbb}}$	–1.080 m

Both L_1 norm minimization and the least squares method were applied to new observation set. The adjusted residuals of the observations by L_1 norm and the least squares are shown in Table 7.

As seen from Table 7, the least squares is ineffective for outlier detection. Especially, the outlying observation $\Delta Z_{\text{geno-ptbb}}$ (observation no 9) is masked. The residuals of outliers by the least squares method are indicated with italic values (see columns 6 and 8 in Table 7). There are many large residuals which correspond to good observations such as observation 29 or 39. On the other hand, L_1 norm have successfully reflect outliers in their corresponding residuals (see columns 2 and 4 with bold values in Table 7).

CONCLUSION

L_1 norm minimization is a robust method used in geodetic networks to identify outliers. This method is less sensitive to outliers than the least squares method. It produces unbiased estimates even if the outliers are present in observation data. On the other hand, it yields results close to the least squares estimates when observations are affected only by random errors. Furthermore, L_1 norm minimization does not spread the corrupt influence of the outliers upon good observations. Thus, L_1 norm minimization can be considered as a more successful method on outlier detection compared to the least squares method.

Table 7. *L_1 norm and the least squares adjusted residuals*

Residuals (m)							
Observation No	L_1	Observation No	L_1	Observation No	The least squares	Observation No	The least squares
1	-0.004	20	-0.008	1	-0.013	20	-0.216
2	0.008	21	0.000	2	0.031	21	-0.174
3	-0.009	22	-0.015	3	-0.071	22	0.012
4	0.002	23	0.001	4	-0.002	23	0.172
5	0.003	24	0.000	5	0.006	24	0.179
6	0.022	25	-0.007	6	0.026	25	-0.004
7	-0.015	26	0.000	7	0.004	26	0.041
8	0.011	27	0.000	8	0.144	27	0.001
9	**-0.170**	28	0.038	9	*0.012*	28	0.011
10	0.002	29	0.009	10	-0.001	29	-0.222
11	-0.000	30	0.006	11	-0.102	30	-0.106
12	0.006	31	0.009	12	0.073	31	0.002
13	0.000	32	-0.003	13	-0.005	32	-0.041
14	0.000	33	0.021	14	0.065	33	0.024
15	0.000	34	0.006	15	-0.062	34	-0.024
16	0.008	35	**1.060**	16	-0.004	35	*0.787*
17	-0.004	36	-0.000	17	0.023	36	-0.112
18	0.024	37	-0.003	18	-0.034	37	-0.036
19	0.014	38	-0.002	19	-0.022	38	-0.108
		39	-0.010			39	-0.252

In this paper, the formulation of L_1 norm minimization based on Cholesky factorization was presented. L_1 norm adjustment problem by a linear programming solution was applied to a GPS network that have correlated observations. The results showed that L_1 norm minimization is more sensitive than least squares for outlier detection and it is more robust than least squares when there are blunders in observations.

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