

Evaluation of Recent GOCE Geopotential Models in South America

Ana Cristina Oliveira Cancoro de Matos, Denizar Blitzkow

Laboratory of Surveying and Geodesy, Department of Transportation, University of São Paulo, EPUSP-PTR, Postal Code 61548, Zipcode: 05424-970, São Paulo, São Paulo, Brazil,
Email: dblitzko@usp.br, acocmatos@gmail.com

Gabriel do Nascimento Guimarães

Institute of Geography, University Federal of Uberlândia, IGUFU, Zipcode: 38500-000, Monte Carmelo, Minas Gerais, Brazil, Email: gabriel@ig.ufu.br

Maria Cristina B. Lobianco

Brazilian Institute of Geography and Statistics (IBGE), Av. Brasil, 15671/Bl.III-B/3º andar, Parada de Lucas, Rio de Janeiro-RJ, Brazil, Email: lobianco@ibge.gov.br

Ilce de Oliveira Campos

Federal University of Rondônia, Academic Department of Civil Engineering. Rodovia BR 364 Km 9,5 sentido Acre, Zona Rural- Zipcode: 76801-974 - Porto Velho, RO – Brazil,
Email: ilce.campos@gmail.com

Abstract

Several global geopotential models based on Gravity field and steady-state Ocean Circulation Explorer (GOCE) satellite have been published in the last few years. Some of these models use combinations of different satellite missions, while others use only GOCE data. This paper presents the evaluation and analysis of six Global Gravity Models (GGMs) using GOCE data, in South America.

1 Introduction

Nowadays, with dedicated satellite gravity missions as Challenging Minisatellite Payload (CHAMP), Gravity Recovery and Climate Experiment (GRACE) and Gravity Field and Steady-State Ocean Circulation (GOCE) are allowing the best knowledge of the long and medium wavelength components of the Earth gravitational field.

This paper focuses on GOCE GGMs. Table 1 shows the characteristics of the models considered: name, year of GGMs publication, maximum spherical harmonic degree and input data information. GO_CONS_GCF_2_DIR_R5 (DIRR5) is a satellite-only model based on a full combination of GOCE-SGG with GRACE and LAGEOS. It was produced by GFZ German Research Centre (GFZ) for Geosciences Potsdam and *Groupe de Recherche de Géodésie Spatiale* (GRGS)/CNES, Toulouse (Bruinsma et al, 2013). GO_CONS_GCF_2_TIM_R5 (TIMR5) is the 5th release of the GOCE gravity field model computed by time-wise approach. It was produced by Graz University of Technology, Institute for Theoretical and Satellite Geodesy University of Bonn, Institute of Geodesy and Geoinformation TU München, Institute of Astronomical and Physical Geodesy (IAPG) (Pail et al, 2011). GFZ and GRGS/CNES produced EIGEN-6C4, which is a global combined gravity field model (Shako et al, 2014; Förste et al, 2014). The others satellite-only models studied are GOGRA04S and JYY_GOCE04S, produced by IAPG, TU München (Yi et al, 2013). Finally, GOCO03S model has been produced by the Gravity Observation Combination (GOCO) in 2012. It is an initiative of TU München, Institute of Astronomical and Physical Geodesy; Univ. Bonn, Institute of Geodesy and Geoinformation; TU Graz, Institute of Theoretical

and Satellite Geodesy; Austrian Academy of Sciences, Space Research Institute; Univ. Bern, Astronomical Institute. It is a satellite-only model and uses GOCE and GRACE satellites, as well as geodetic laser satellite data (Mayer-Gürr et al., 2012).

Geoidal heights derived from GPS/BM and terrestrial gravity data are used to evaluate these models for South America. The EGM2008 was used too for comparison of GOCE GGMs. It is a combined solution based on ITG-GRACE03S (Mayer-Guerr, 2007), surface gravity and altimeter data over the ocean. It was calculated and made available to the scientific community by the development team from the Earth Gravitational Model (EGM), NASA (National Aeronautics and Space Administration) / NGA (Pavlis et al, 2008; Pavlis et al, 2012). The model is completed to degree and order 2159 and contains additional spherical harmonic coefficients up to degree 2190 and order 2159. The GGMs are tide free system and in this study no zero degree term on the height anomaly was added.

This paper shows also the comparison of the geoidal heights derived from GPS/BM with the geoidal model GEOID2014 (Blitzkow et al, 2014).

Table 1: GGMs used

Model	Year	Degree	Data
EIGEN-6C4	2014	2190	S(Goce,Grace,Lageos),G,A
TIMR5	2014	280	S(Goce)
DIRR5	2014	300	S(Goce,Grace,Lageos)
JYY_GOCE04S	2014	230	S(Goce)
GOGRA04S	2014	230	S(Goce,Grace)
GOCO03S	2012	250	S(Goce,Grace,...)
EGM2008	2008	2190	S(Grace),G,A

Source: International Centre for Global Earth Models (ICGEM) - Satellite (S); airborne and terrestrial gravity (G); Altimetry (A) survey.

2 Geoid model

The Laboratory of Surveying and Geodesy (LTG-EPUSP) computed recently GEOID2014 (Blitzkow et al, 2014). The area limited by 15° N and 57° S in latitude and 30° W and 95° W in longitude is involved by the model. The terrestrial gravity data for the continent have been updated with the most recent surveys. A total of 892,604 gravity points in South America (Figure 1) were collected. The complete Bouguer and Helmert gravity anomalies have been derived through the Canadian package SHGEO (Ellmann and Vaníček, 2007). The oceanic area was completed with the mean free-air gravity anomalies derived from a satellite altimetry model by the Danish National Space Center, called DTU10 (Andersen, 2010). The short wavelength component was estimated via FFT with the modified Stokes kernel proposed by Featherstone (2003). The model was based on EIGEN-6C3stat up to degree and order 200 as a reference field (Shako et al, 2014). A zero degree term of -0.41 m was added, see Figure 2. This converts geoid undulations that are intrinsically referred to an ideal mean-Earth ellipsoid into undulations that are referred to WGS 84.

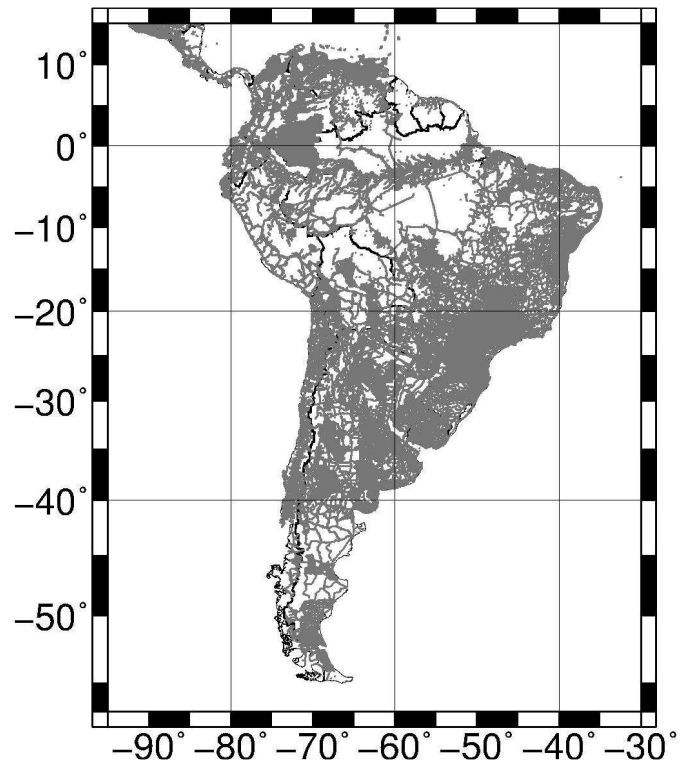


Figure 1: Gravity data in South America.

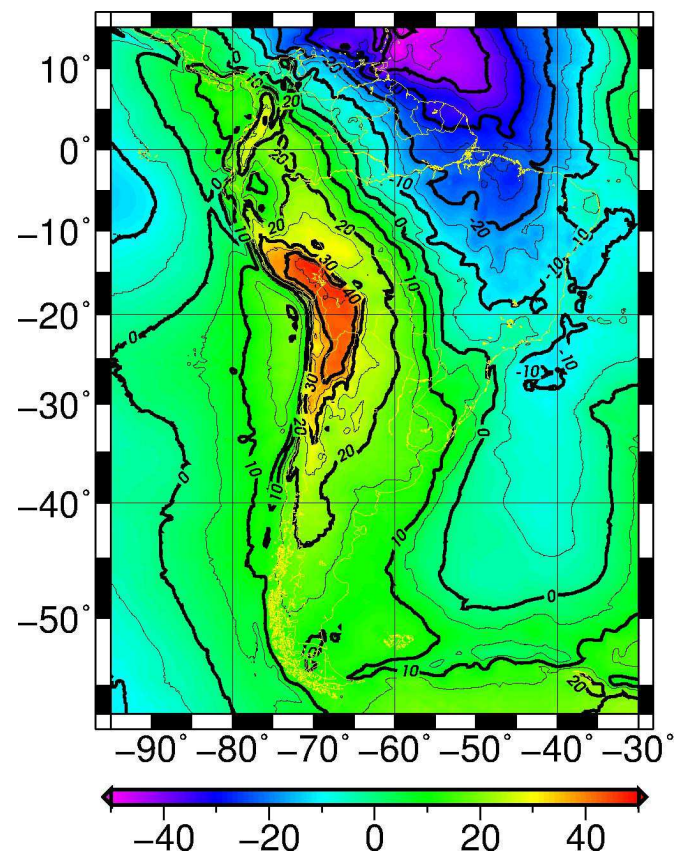


Figure 2: Model GEOID2014 (meter)

3 GPS data on benchmark

GPS observations carried out on benchmarks of the spirit levelling network in South America, which have been delivered under the SIRGAS (Geocentric Reference System for Americas) project (Hoyer et al, 1998; SIRGAS, 1997), were used for testing the selected GGMs and the geoid model. At the moment there are GPS/BM data available from the following countries: Argentina, Brazil, Chile, Ecuador, Uruguay and Venezuela, in a total of 1,861 points (Figure 7). Table 2 shows this information for each country: number of points; vertical and horizontal datum; and web site data.

The geoidal heights associated with GPS/BM have their inaccuracies due to the error of the spirit levelling as well as of the GPS. Figure 3 shows the standard deviation of the ellipsoidal and the orthometric heights in 673 points in Brazil, as a function of the latitude. This result was derived from the last vertical network adjustment undertaken by IBGE. The results of these points are on the decimetre level accuracy for most of the country as shown by Figure 3. The standard deviation of ellipsoidal and orthometric heights of the GPS/BM is only provided by IBGE. The institutions of other countries do not provide this information.

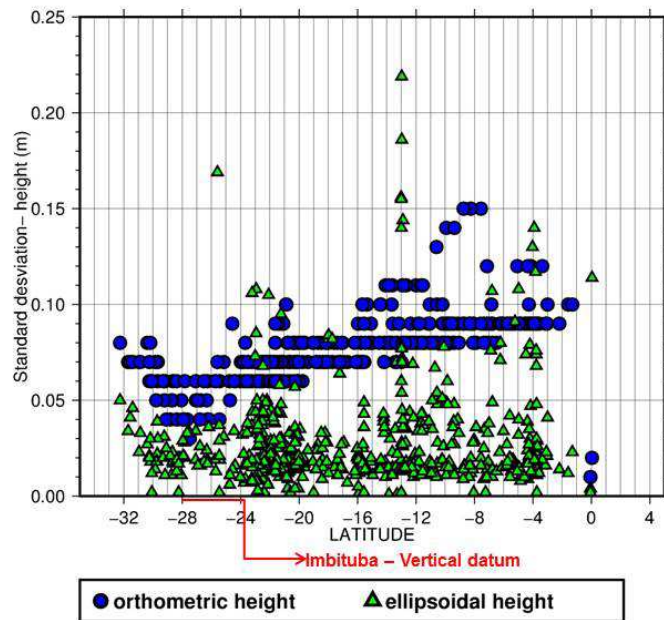


Figure 3: Standard deviation of ellipsoidal and orthometric heights as a function of latitude.

The mentioned comparison is very much useful to look after the consistency between the two heights. The original ellipsoidal heights derived from the GPS measurements refer in principle to a tide-free (*tf*) system in terms of the treatment of the permanent tide effect (Poutanen et al, 1996). However, as no tidal correction was applied to the height observations of the levelling network, the available normal orthometric heights refer, in principle, to a mean-tide system (*mt*) (Ferreira et al, 2013).

For the present analysis, these values were transformed into the tide-free system by using the formula (Tenzer et al, 2010):

$$H_{tf} = H_{mr} + \left\{ (1 + k - h) \left[-0.198 \left(\frac{3}{2} \sin^2 \varphi - \frac{1}{2} \right) \right] \right\} \quad (1)$$

where k and h are the tidal Love numbers and their values are 0.3 and 0.62, respectively, and φ is the geocentric latitude. This was necessary because the GPS and the applied GGMs are related to a tide-free system.

Table 3 shows the results in terms of mean value, RMS difference, standard deviation (σ) difference, extreme values of the differences among height anomalies of several GGMs (maximum degree) and GEOID2014 geoidal heights with GPS/BM geoidal heights.

Figures 4 and 5 show the histograms of the discrepancies between GGMs (EIGEN6C4 and DIRR5) and GPS/BMs, respectively. EIGEN6C4 is more consistent with GPS/BM than the other GGMs, due to the inclusion of terrestrial gravity data in its computation. Figures 7 and 8 show the GPS/BM distribution with a color palette for differences between GPS/BM geoidal heights and EIGEN6C4 and DIRR5 height anomalies, respectively. Figures 6 and 9 show the histogram and the map of the discrepancies between GPS/BM and GEOID2014 model, respectively. Almost 50% of the discrepancies in absolute terms are around 0.2 meters, which is within the GPS/BM points inaccuracies (Figure 3).

Table 4 shows RMS differences among GPS/BM geoidal heights with GGMs height anomalies (max degree) and GEOID2014 geoidal heights for each country. It is possible to observe that the zero degree term added in the geoid model shows a worse result for Argentina and Ecuador, not for other countries. For example, in Argentina (Figure 9 and Table 4), the RMS difference between GPS/BM and GEOID2014 is 0.60 m (Table 4). But, RMS difference with respect to GEOID2014, without zero degree term, is 0.30 m and, just in the Buenos Aires province, is 0.21 m. The vertical datum is not the same for different countries (Table 2). For example, the vertical datum discrepancy between Brazil and Argentina is higher than 20 cm, and Brazil and Ecuador is higher than 80 cm (Sánchez and Brunini, 2009; Sánchez, 2005). The height difference of each country was not corrected for the discrepancies. Although zero degree term has no relation with the difference between the vertical datum of each country, it emphasizes eventually these differences.

The GPS/BM information is still sparse, without a homogeneous distribution, so that this result is geographically limited, but Figure 10 shows the graphical differences for Argentina (296 points), Brazil (1,112 points), Chile (173 points), Ecuador (60 points), Uruguay (11 points) and Venezuela (187 points).

The GPS/BM for Argentina are concentrated in Buenos Aires province where the GGMs show good agreement up to degree 200. Out of that, TIMR5 GGM presents the best agreement up to 260. The GGMs EGM2008 and EINGEN6C4, to which terrestrial gravity data are included, show best agreement above degree 260.

For Brazil (Figure 10), most of these points are concentrated in the Southeast and South and all GGMs show good agreement up to 250 degree. Out of that, the complete GGMs are the best. Many differences are in the border between Roraima, Amazonas and Rondônia states (Figure 7 to 9, black North-South profile) probably there is a leveling bias on the benchmarks. Some points in Piauí state show high differences too. These points need to be reviewed.

In the same analysis for Chile, Ecuador and Uruguay, the GOCE GGMs only-satellites (TIMR5, DIRR5, JYY_GOCE04S, GOGRA04S) shows good agreement with GPS/BM. The EGM2008 presents better result than EIGEN6C4 for Chile and Uruguay. Figure 10 shows that all GGMs are good agreement with GPS/BM up to 230 in Venezuela. Out of that, the complete GGMs are the best too.

Table 2: GPS/BM information

	Number points GPS/BM	Institution	Datum horizontal	Datum vertical tide gauge	Web Site Data
Argentina	296	Instituto Geográfico Nacional/IGN and Universidade Nacional de Rosário/UNR	POSGAR07, ITRF 2005 época 2006.632 ⁽¹⁾	Mar del Plata, Province Buenos Aires, 1949 (Lauría et al, 2002)	No
Brazil	1112	IBGE, LTG, Instituto de Astronomia, Geofísica e Ciências Atmosféricas/IAG-USP, Universidade Estadual Paulista "Júlio de Mesquita Filho"/UNESP and Universidade Federal do Paraná/UFPR	SIRGAS2000, ITRF 2000, epoch 2000.4 ⁽²⁾	Imbituba, Santa Catarina state, 1958 ⁽³⁾	Yes ⁽⁴⁾ (IBGE)
Chile	173	Instituto Geográfico Militar/IGM	SIRGAS2000, ITRF 2000, epoch 2002.0 (Sánchez and Brunini, 2009)	Arica, Antofagasta, Valparaíso (mainly), San Antonio, Talcahuano, Puerto Montt and Punta Arenas (Sánchez, 2005)	No
Ecuador	60	Instituto Geográfico Militar/IGM	SIRGAS 95 ITRF1995.4 (Sánchez and Brunini, 2009)	La Libertad, Province del Guayas, 1959 (Sánchez, 2005)	No
Uruguay	11	Servicio Geográfico Militar/SGN	SIRGAS 95, epoch 1995.4 (Sánchez and Brunini, 2009)	Montivideo, Capital, 1948 (Piña et al, 2002)	No
Venezuela	187	Instituto Geográfico de Venezuela Simón Bolívar/IGVSB and Escuela de Ingeniería Geodésica (Facultad de Ingeniería, Universidad del Zulia)	SIRGAS 95, epoch 1995.4 (Sánchez and Brunini, 2009)	La Guaira, 1953 (Hernández et al, 2002)	No

(1) <http://www.ign.gob.ar/NuestrasActividades/Geodesia/Posgar07>;

(2) http://www.ibge.gov.br/home/geociencias/noticia_sirgas.shtm ;

(3) http://www.ibge.gov.br/home/geociencias/geodesia/rmpg/default_rmpg_int.shtm?c=10;

(4) <http://www.visualizador.inde.gov.br>

Table 3: Statistics of the differences between GPS/BM geoidal heights and height anomalies of the GGMs (max degree) for South America in meters.

	EGM2008	GOCO03S	JYY_GOCE04S	GROGA04S	TIMR5	DIRR5	EIGEN6C4	GEOID2014
Mean	-0.31	-0,28	-0,29	-0,29	-0,32	-0,32	-0,32	0,17
σ diff	0.46	0,61	0,59	0,58	0,54	0,54	0,44	0,52
RMS diff	0.55	0,67	0,65	0,65	0,63	0,63	0,55	0,55
Max.	2.10	2,57	2,46	2,47	2,48	2,58	2,09	2,24
Min.	-3.42	-2,80	-2,88	-2,88	-2,91	-2,94	-3,74	-2,55

Table 4: RMS difference between GPS/BM geoidal heights and height anomalies of the GGMs (max degree) for each country in meters.

	EGM2008	GOCO03S	JYY_GOCE04S	GROGA04S	TIMR5	DIRR5	EIGEN6C4	GEOID2014
Argentina	0.30	0.34	0.34	0.34	0.32	0.33	0.29	0.60
Brazil	0.57	0.64	0.64	0.63	0.64	0.64	0.57	0.44
Chile	0.65	0.94	0.64	0.79	0.70	0.68	0.76	0.76
Ecuador	0.80	1.158	1.12	1.125	1.06	1.07	0.72	1.18
Uruguay	0.63	0.65	0.58	0.59	0.63	0.63	0.65	0.67
Venezuela	0.49	0.82	0.85	0.85	0.77	0.76	0.49	0.47

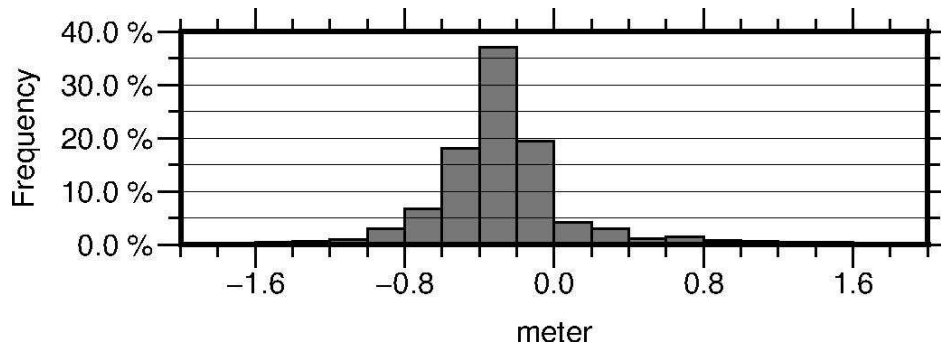


Figure 4: Histogram of the discrepancies between GPS/BM geoidal heights and EIGEN6C4 (max. degree) height anomalies.

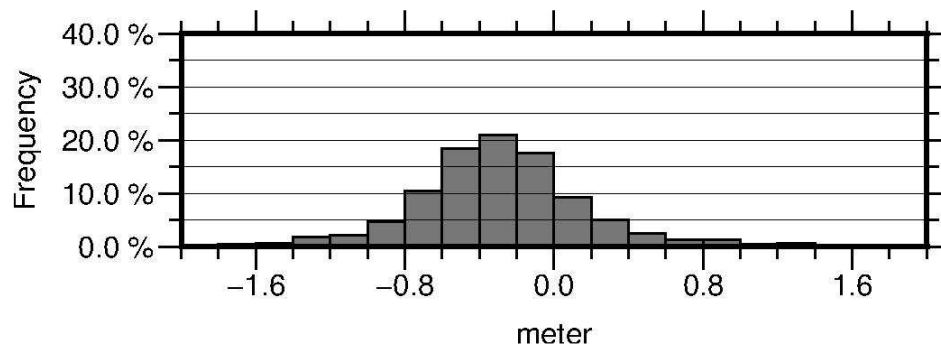


Figure 5: Histogram of the discrepancies between GPS/BM geoidal heights and DIRR5 (max. degree) height anomalies.

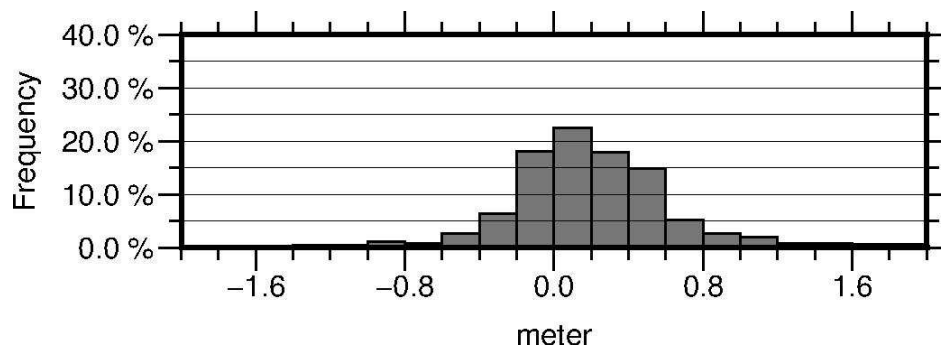


Figure 6: Histogram of the discrepancies between GPS/BM and GEOID2014 geoidal heights.

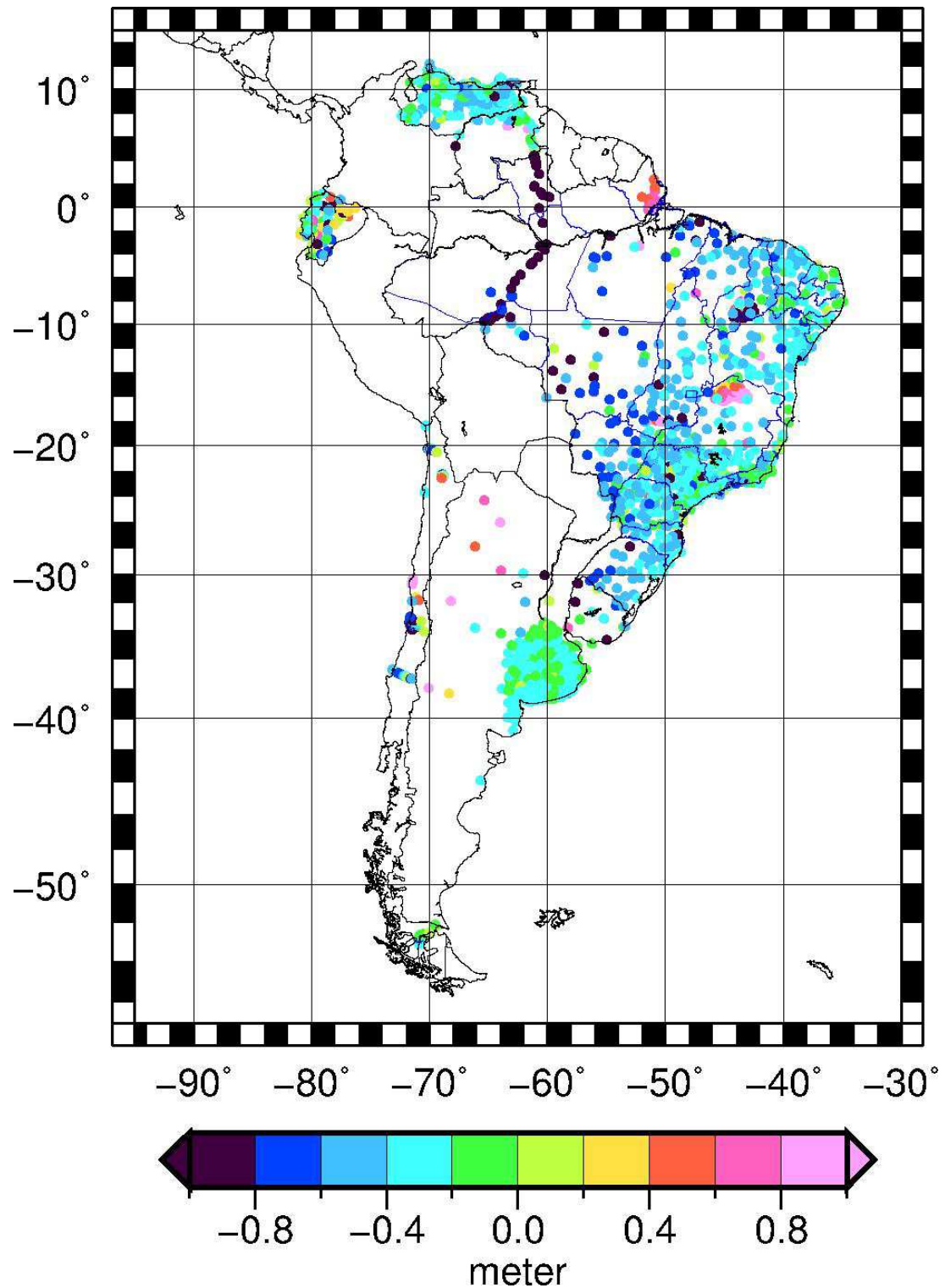


Figure 7: Distribution of the GPS/BMs and illustration of the differences between GPS/BM geoidal heights and EIGEN6C4 (max. degree) height anomalies.

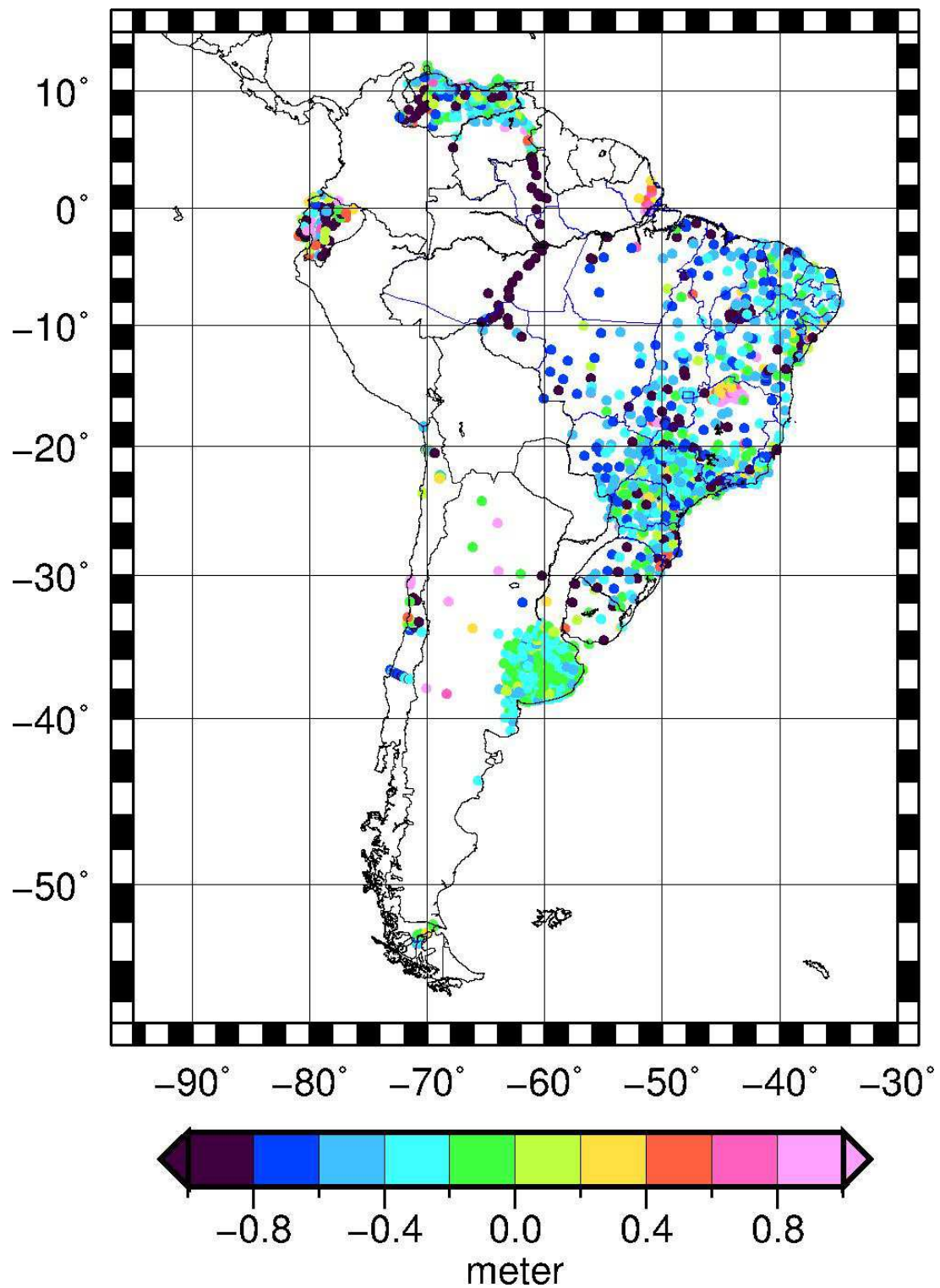


Figure 8: Distribution of the GPS/BMs and illustration of the differences between GPS/BM geoidal heights and DIRR5 (max. degree) height anomalies.

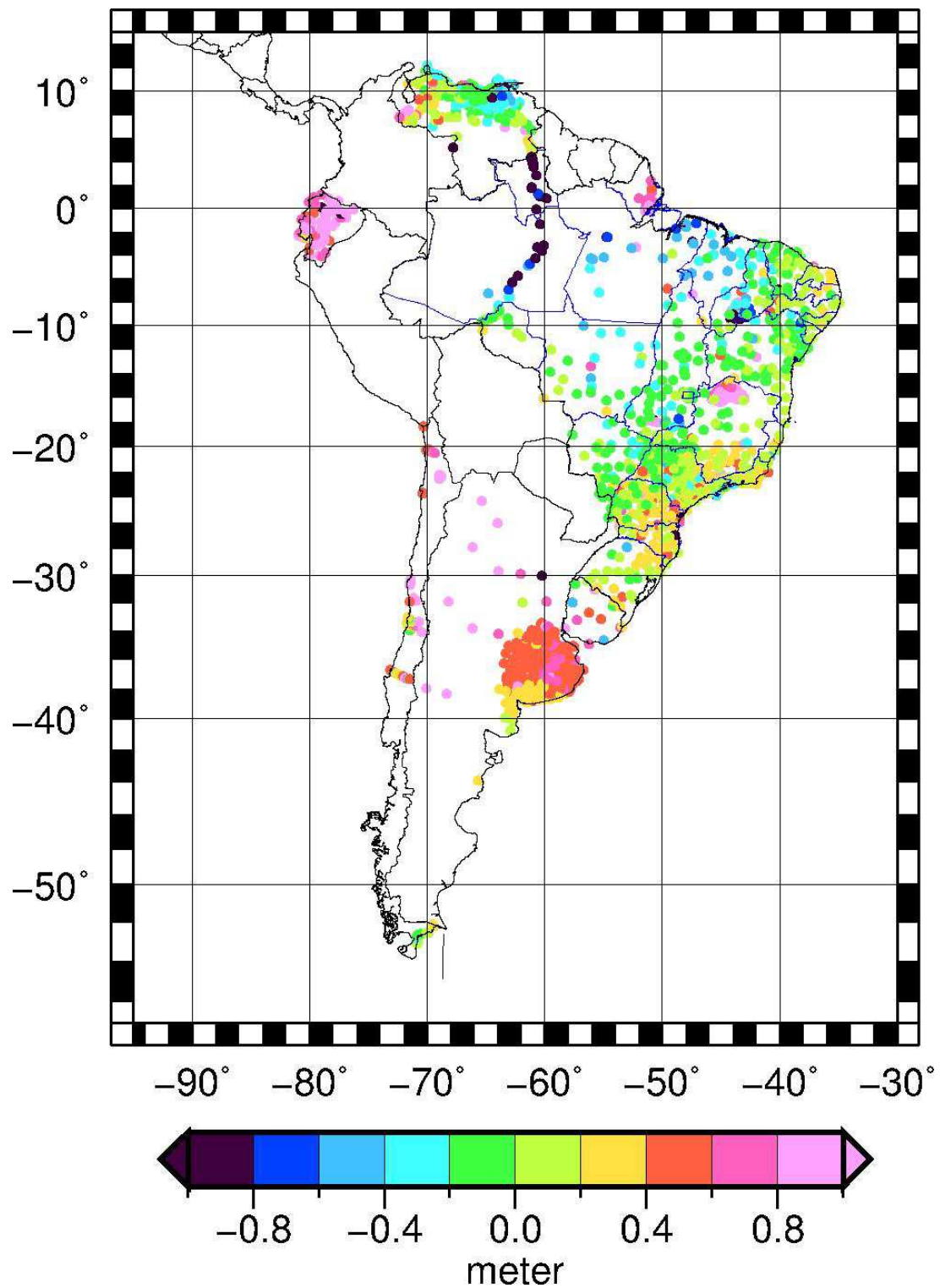


Figure 9: Distribution of the GPS/BMs and illustration of the differences between GPS/BM and GEOID2014 geoidal heights.

4 Terrestrial gravity data

South American Gravity Project (Green and Fairhead, 1991) was the first great effort in collecting and validating gravity data over the continent. This initiative was important to indicate the terrestrial and marine gravity distribution and to identify the major gaps. In 1991, the Anglo-Brazilian Gravity Project (ABGP) started some new efforts to infill the gaps in Brazil. This project was a cooperation program between LTG/EPUSP, IBGE and Geophysical Exploration Technology (GETECH), supported by U.S. National Geospatial-Intelligence Agency (NGA). After seven years of activities this project was responsible for an outstanding improvement on the gravity point distribution, mainly in the Amazon region, including rivers and airstrips along small villages. The activities of ABGP were extended to other countries in the continent in 2000 as South America Gravity Studies (SAGS). Presently a total of 892,604 terrestrial gravity points are available in South America.

The gravity anomalies derived from terrestrial gravity data are compared with gravity disturbances derived from GGMs. Table 5 shows the results in terms of mean value, standard deviation (σ) difference, RMS difference and extreme values of the differences between gravity anomalies derived from terrestrial gravity data and gravity disturbances derived from GGMs. Figures 11 and 12 show the histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGEN6C4 and DIRR5, respectively. One can see that EIGEN6C4 is better adjusted to the terrestrial gravity data than the other GGMs.

Figures 13 and 14 show the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGEN6C4 and DIRR5 in map of color palette, respectively. Nevertheless, the main discrepancies are correlated with high and rough topography, especially over Andes Mountains. In Brazil, the Minas Gerais, Rio de Janeiro and Bahia states (mountainous regions) are worse. The rain forest area that is located next to the coast of Paraná and Santa Catarina also shows high differences.

Figure 15 shows the RMS difference between terrestrial gravity anomalies and gravity disturbances derived by GGMs (180 to 360 degree) for South America. All GGMs present similar result up to 190 degree. After that up to 240 degree, the GOCE GGMs only-satellites (TIMR5, DIRR5, JYY_GOCE04S, GOGRA04S) are those with smallest RMS differences. The EIGEN6C4 shows smaller RMS differences than EGM2008 up to 320 degree.

Figure 16 shows in detail the discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs for Argentina (123,944 gravity points), Bolivia (99,733 gravity points), Brazil (444,608 gravity points), Chile (35,392 gravity points), Colombia (69,101 gravity points), Ecuador (13,222 gravity points), Paraguay (7,464 gravity points), Peru (39,999 gravity points), Uruguay (2,297 gravity points), and Venezuela (56,712 gravity points).

Among the countries, Chile, Colombia, Ecuador, Peru and Venezuela have the highest RMS differences. Brazil, Chile, Colombia and Venezuela all GGMs up to 200 are similar adjusted. Out of that, the complete GGMs present smallest RMS differences; the behavior is similar with Figure 15. EGM2008's RMS difference is the smallest for Argentina. GGMs only-satellite show smallest RMS differences to Bolivia, Ecuador and Paraguay. This analysis for Peru shows smallest with the EIGEN4C6 up to 230, after that GGMs only-satellite show smallest results.

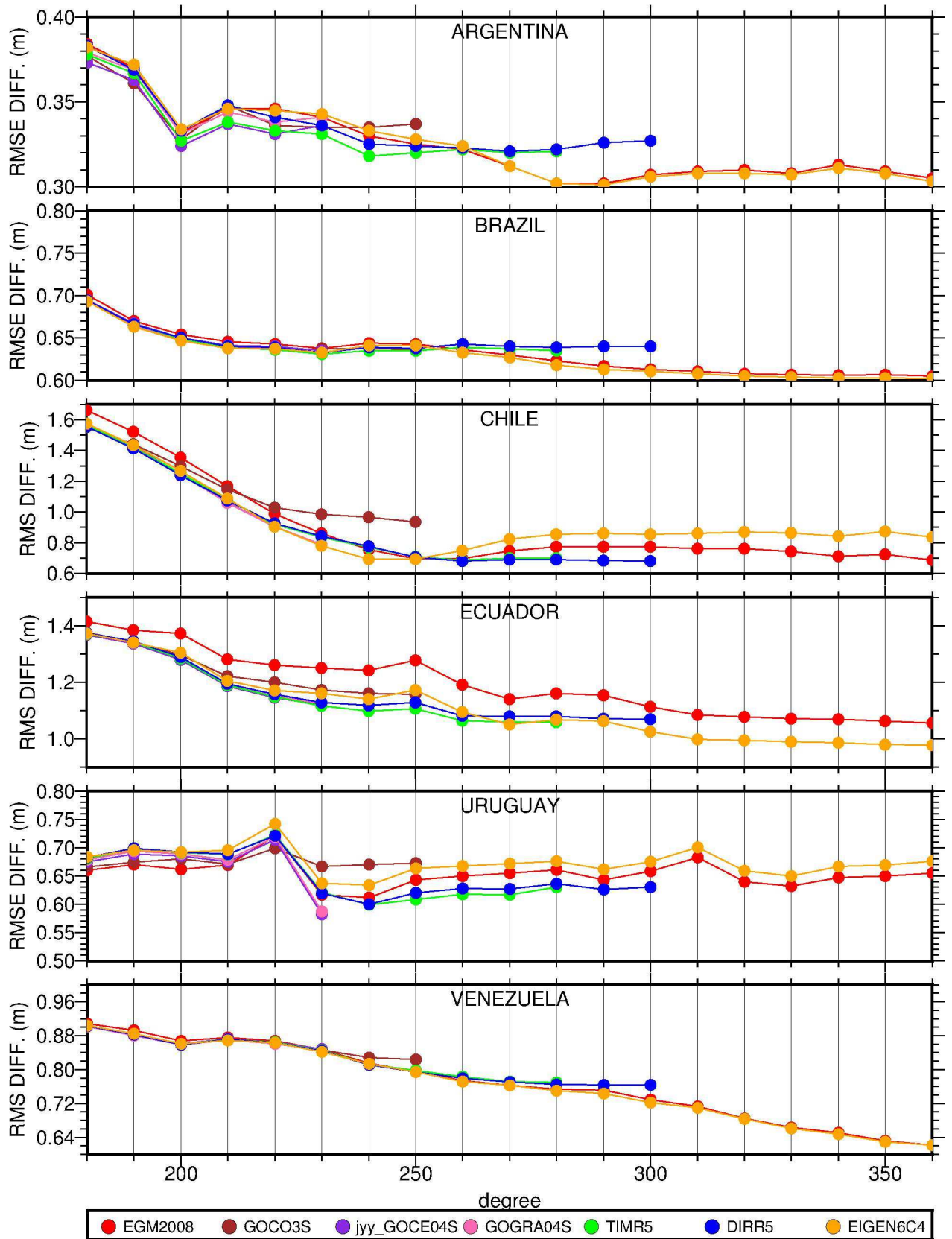


Figure 10: RMS difference between the geoid height (GPS/leveling) minus height anomalies of the GGMs (180 to 360 degree) for Argentina (296 points), Brazil (1,112 points), Chile (173 points), Ecuador (60 points), Uruguay (11 points) and Venezuela (187 points) in meter.

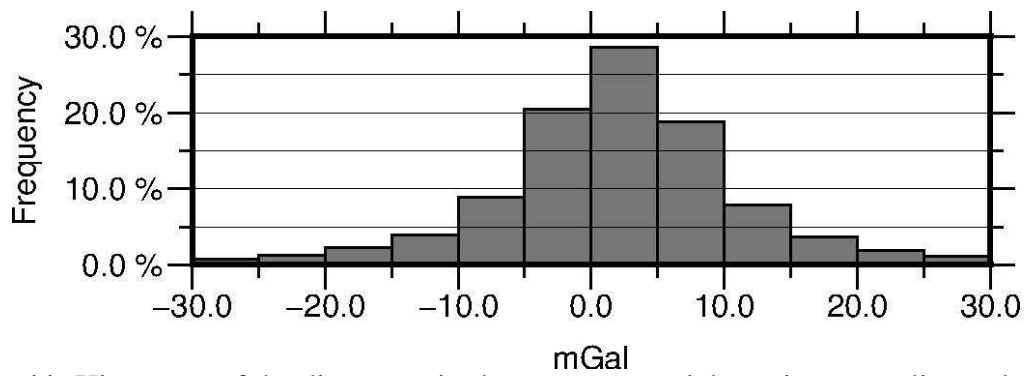


Figure 11: Histogram of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGE6C4 (max. degree).

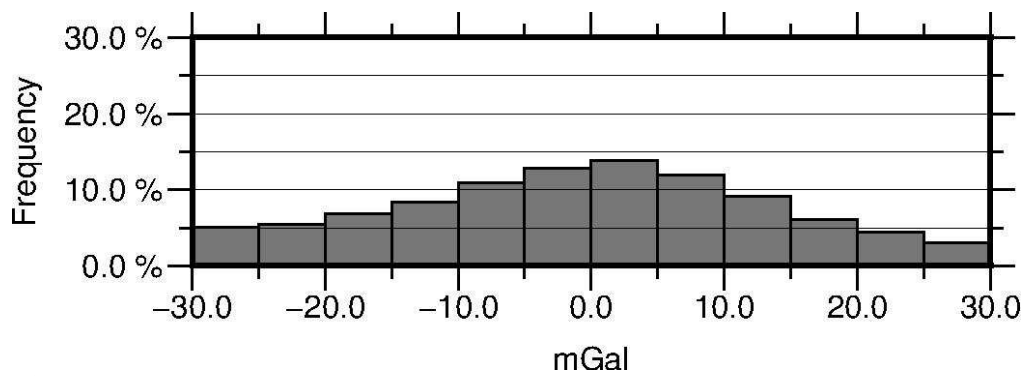


Figure 12: Histogram of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from DIRR5 (max. degree).

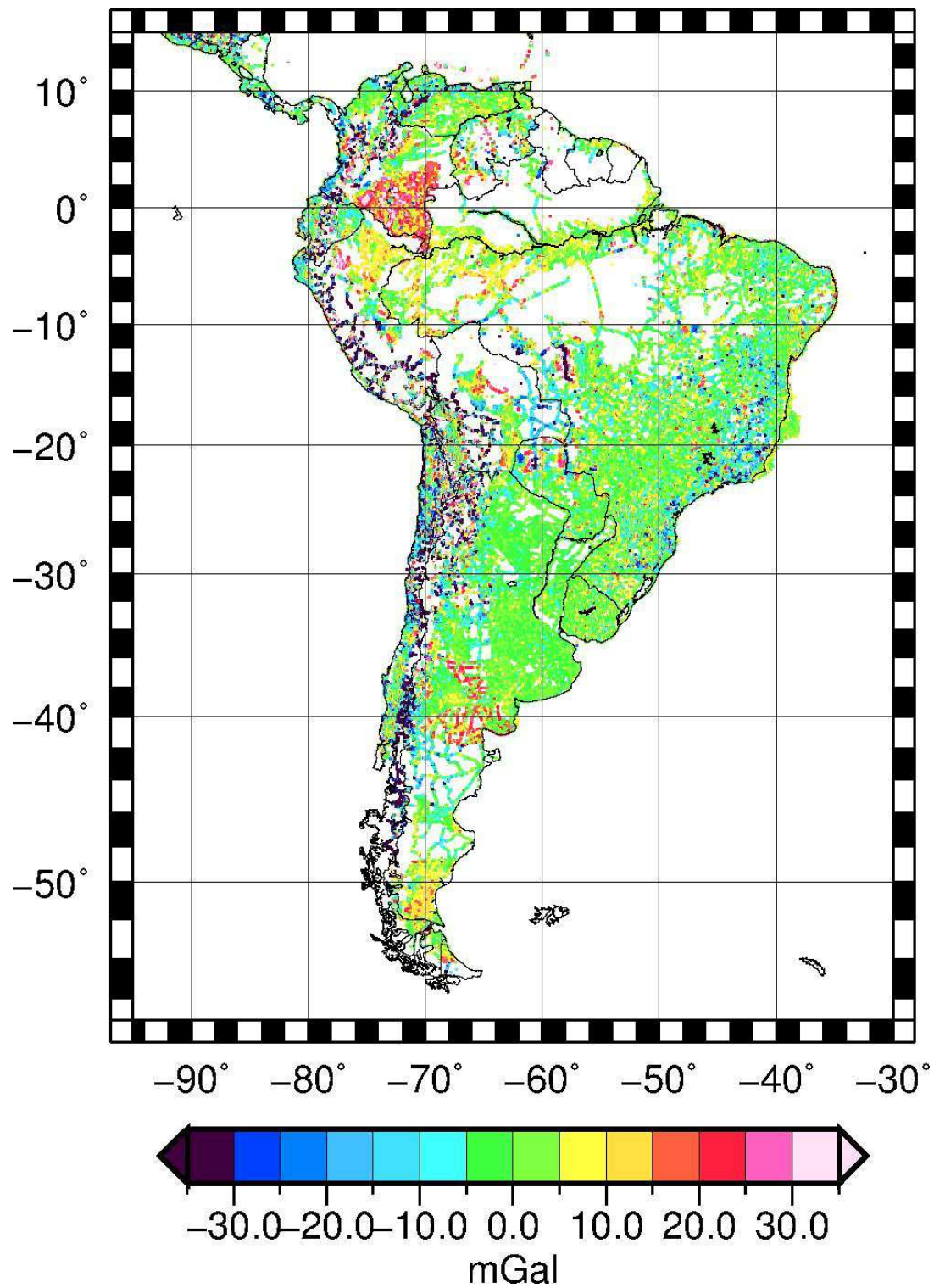


Figure 13: Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGEN6C4 (max. degree).

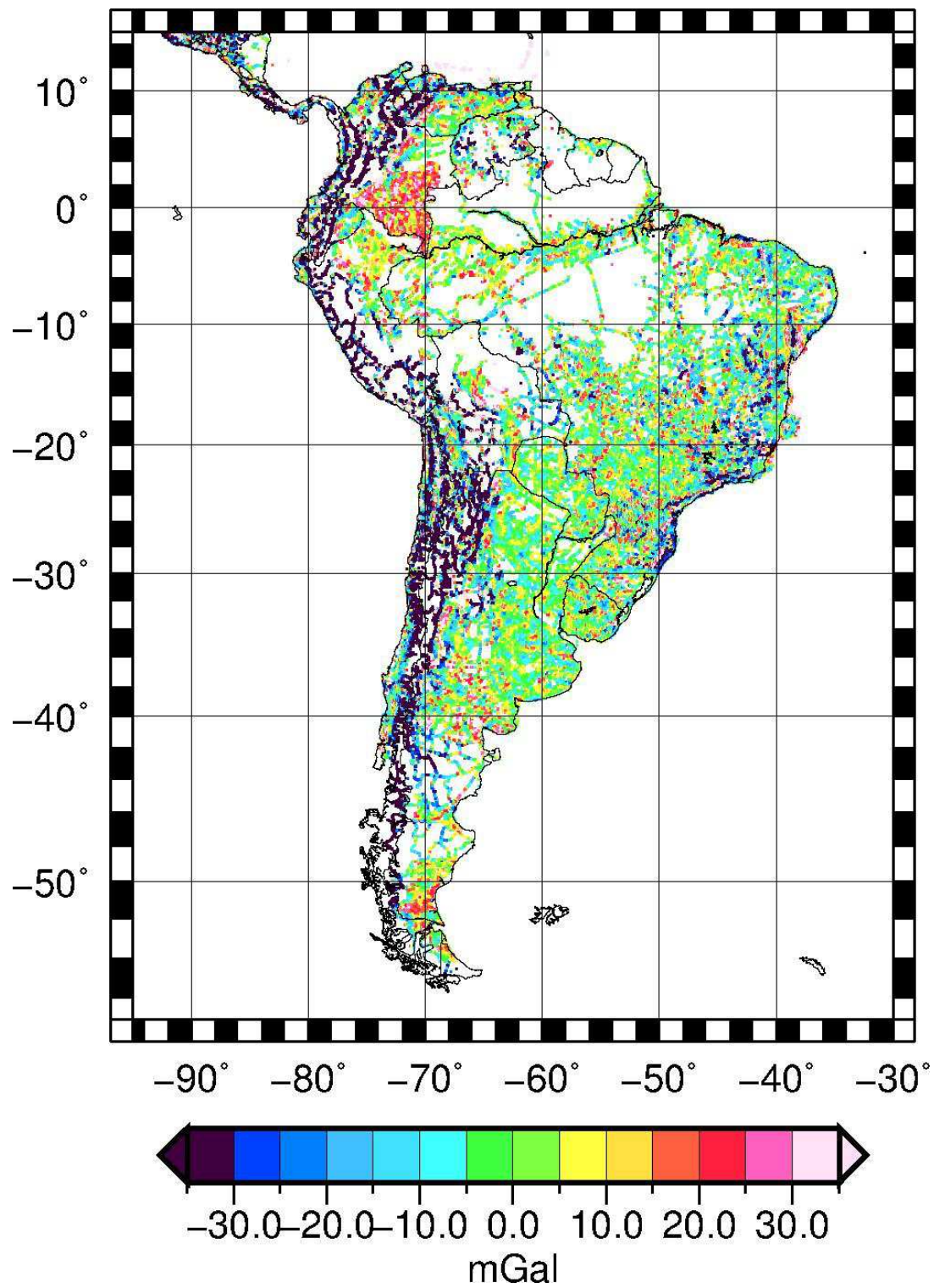


Figure 14: Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from DIRR5 (max. degree).

Table 5: Statistics for the discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs (max degree) in mGal.

	EGM2008	GOCO03S	JYY_GOCE04S	GROGA04S	TIMR5	DIRR5	EIGEN6C4
Mean	0.97	-5.82	-5.72	-5.73	-5.14	-5.19	1.81
σ diff	14.38	25.83	25.53	25.53	24.71	24.51	14.48
RMS diff	14.41	26.48	26.17	26.17	25.24	25.06	14.59
Max.	301.59	282.20	284.27	284.39	285.42	286.53	304.81
Min.	-369.09	-369.18	-360.03	-360.21	-358.51	-351.16	-518.32

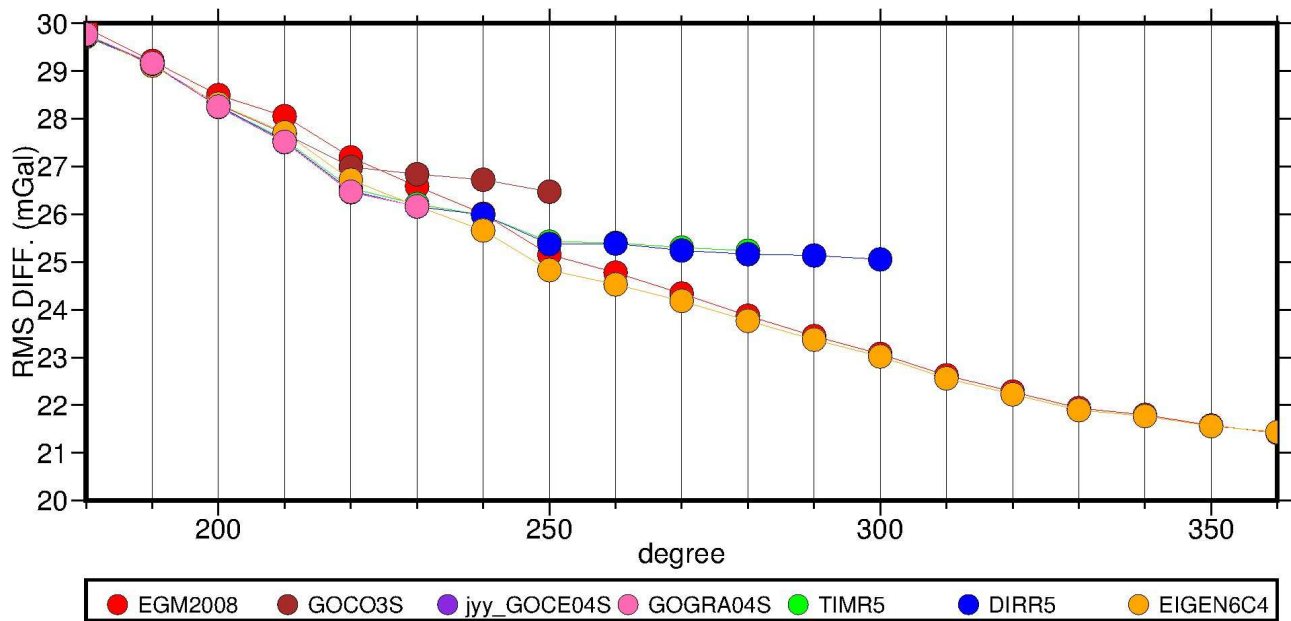
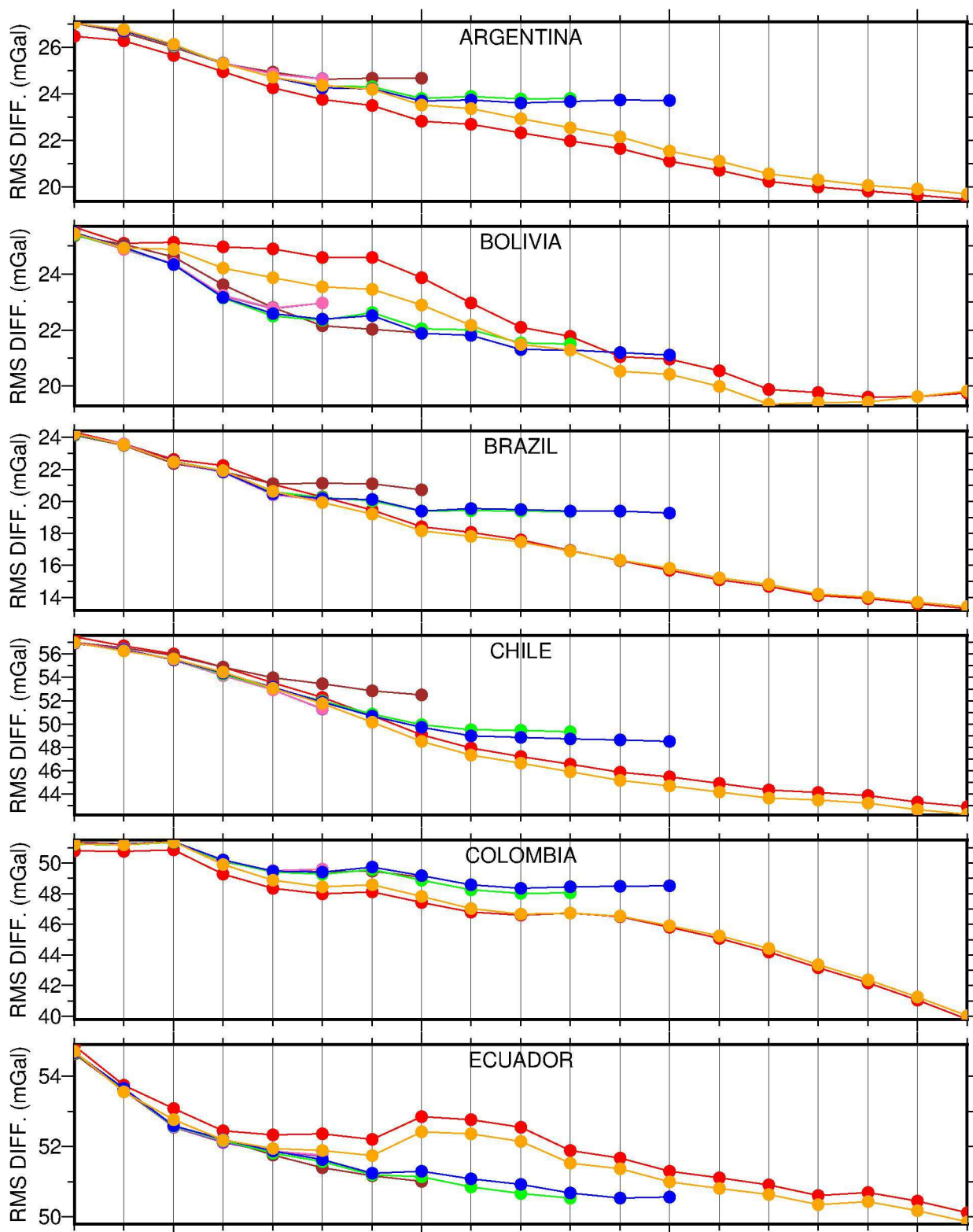


Figure 15: The discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs (180 to 360 degree) for South America in mGal.



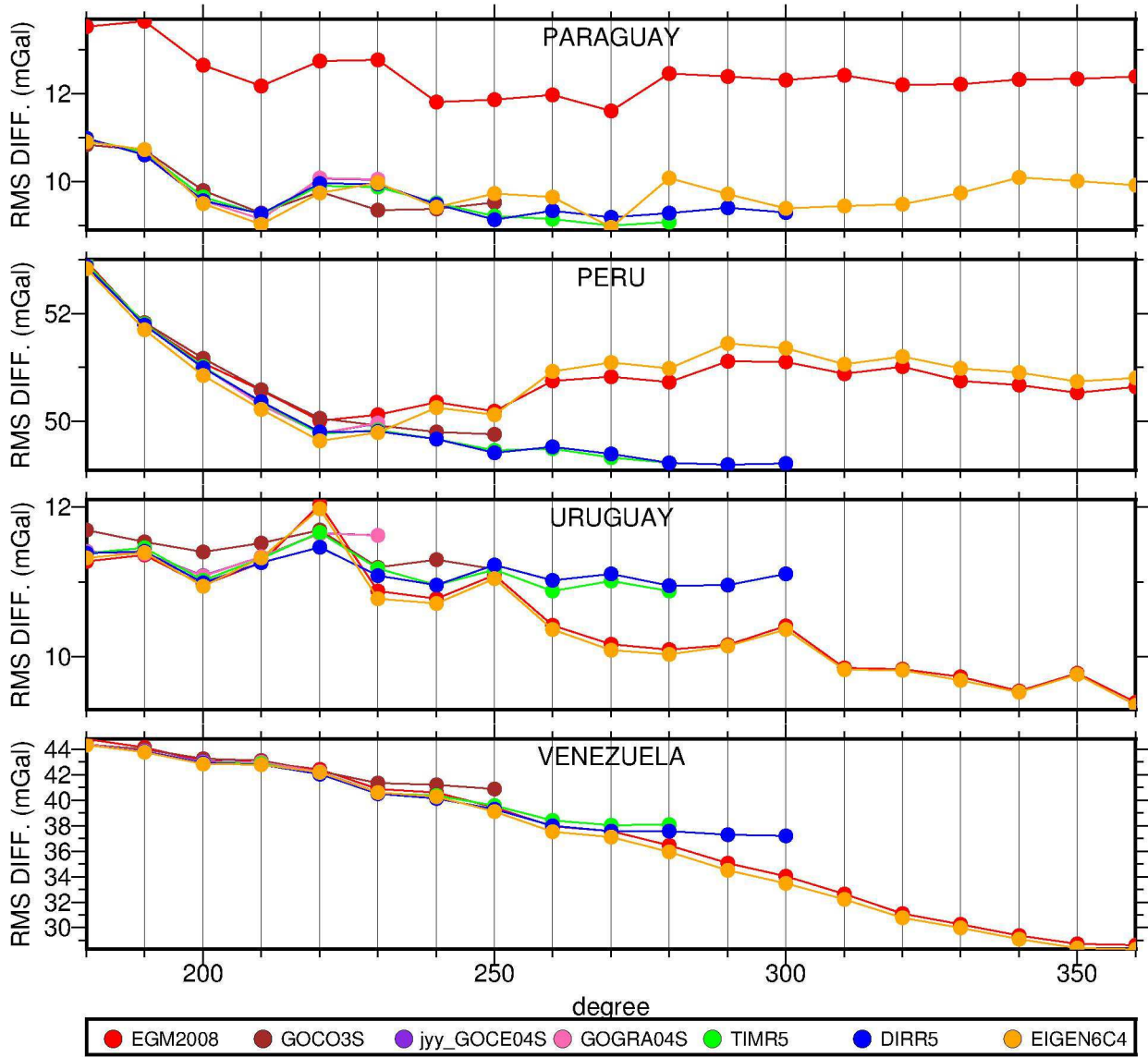


Figure 16: The discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs (180 to 360 degree) for Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay nad Venezuela in mGal.

5 Conclusions

The geoid undulation (height anomaly) derived from GGMs have two types of errors:

- 1- Commission error: due to the implied error of the coefficients of the GGMs;
- 2- Omission error: it is defined as neglecting harmonic coefficients higher than the highest degree in in the reference model and also due the discreteness of, or a lack of resolution, the gravimetric data (Jekeli et al, 2009; Yang, 2013).

Today's instruments are quite accurate and different sources of data are generally integrated to yield the geoid undulation, thus reducing the impact of the commission error. On the other hand, the omission error tells the resolution of gravity data (spacing between points) in a region and the

inherent accuracy. The gravity data of South America have a non-homogeneous distribution and spacing of 10 km or greater between points, out of São Paulo State (5 or 8 km) and some regions with airborne gravity surveys (Colombia). The ideal would be to have more gravity data in rough areas than smooth areas, but it is exactly the opposite that occurs due to the difficult access and high cost for such campaigns. In the Andes, aerial gravimetry is the only possibility which implies in a high cost for research institutions. The Amazon region, flat area, the difficulty is due to very dense tropical forest and the restriction to access indigenous and forest reserves. Due to all these difficulties, South America has large areas without information or few data, so it is expected a quite high omission error difficult to estimate.

This paper shows the validation of the GGMs over South America, but the errors mentioned above were disregarded. The selected GGMs are evaluated for various degrees and orders.

The statistical analysis of the differences between GPS/BM geoid heights and height anomalies of the EGM2008 and EIGEN6C4 in the Table 3 show close results. The same is observed between TIMR5 and DIRR5 and between JYY_GOCE04S and GOGA04S, respectively. The GEOID2014 has the same RMS difference that EGM2008 and EIGEN6C4 but a lower mean value.

EGM2008 shows the best fit for Chile and Uruguay (Table 4). For Argentina and Ecuador, the same occurs for EIGEN6C4. Figure 10 shows in more detail the evolution of these differences for various order and degree of GGMs studied in this paper. The GEOID2014 shows the best results for Brazil and Venezuela.

Section 4 addressed the attention to the study of discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs. EGM2008 has better results than EIGEN6C4 (Table 5 and Figure 15). TIMR5 and DIRR5 have similar results and as well as GOCO03S, JYY_GOCE04S and GROGA04S. Figure 16 shows the same discrepancies but in greater detail to the degrees and orders ranging from 180 to 360 degree for Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay and Venezuela. It is also observed that Andes show the highest differences (Figures 13 and 14). Colombia shows over 20 mGal differences in the region with data obtained from aerial gravimetry.

The general conclusion is that the recent geopotential models with GOCE/GRACE information represent an important improvement on the knowledge of the gravitational potential for South America.

References

- Andersen, O.B. (2010). *The DTU10 Gravity field and Mean sea surface*. Second International Symposium of the Gravity Field Service – IGFS2 20–22 September 2010 Fairbanks, Alaska.
- Blitzkow, D.; Matos, A. C. O. C.; Costa, D. S.; Guimarães, G. N.; Pacino, M. C.; Lauria, E. A.; Castro JR., C. A. C. E.; Mesquita, A. R. (2014). *Gravity surveys and quasi-geoid model for South America*. In: The 3rd International Gravity Field Service (IGFS) General Assembly, June 30-July 6, 2014, Shanghai, China.
- Bruinsma, S.; Foerste, C.; Abrikosov, O.; Marty, J.-C.; Rio, M.-H.; Mulet, S.; Bonvalot, S. (2013). *The new ESA satellite-only gravity field model via the direct approach*, Geophysical Research Letters, 40, 14:3607-3612. doi.org/10.1002/grl.50716.

- Ellmann, A.; Vaníček, P. (2007). *UNB applications of Stokes-Helmert's approach to geoid computation*. Journal of Geodynamics, 43, p. 200-213.
- Featherstone, W.E. (2003) Software for computing five existing types of deterministically modified integration kernel for gravimetric geoid determination, Computers and Geosciences 29(2): 183-193, doi: 10.1016/S0098-3004(02)00074-2.
- Ferreira, V. G.; Zhang, Y.; de Freitas, S. R. C. (2013). *Validation of GOCE gravity field models using GPS-leveling data and EGM08: a case study in Brazil*. Journal of Geodetic Science. 3(3):209–218, ISSN (Online) 2081-9943, ISSN (Print) 2081-9919, DOI: 10.2478/jogs-2013-0027.
- Förste, Ch.; Bruinsma, S.L.; Abrikosov, O.; Lemoine, J.- M.; Schaller, T.; Götze, H.- J.; Ebbing, J.; Marty, J.C.; Flechtner, F.; Balmino, G.; Biancale, R. (2014). *EIGEN-6C4 The latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse*. 5th GOCE User Workshop, November, 25–28.11, 2014, Paris.
- Green, C. M.; Fairhead, J. D. (1991) *The South American Gravity Project*. In: Torge, W. (Ed.). Recent geodetic and gravimetric research. [S.l.]: Springer-Verlag, p. 82–94.
- Hernández, J. N.; Blitzkow, D.; Luz, R. T.; Sánchez, L.; Sandoval, P.; Drewes, H. *Connection of the Vertical Control Networks of Venezuela, Brazil and Colômbia*. In: Drewes, H.; Dodson, A. H.; Fortes, L. P. S.; Sánchez, L.; Sandoval, P. (Ed.). Vertical Reference Systems. IAG Symposium 124, Cartagena, Colômbia. Berlin: Springer-Verlag, 2002. p. 324-327.
- Hoyer, M.; Arciniegas, S. ; Pereira, K.; Fagard, H.; Maturana, R.; Torchetti, R.; Drewes, H.; Kumar, M.; Seeber, G. (1998). *The definition and realization of the reference system in the SIRGAS project*, Springer; IAG Symposia; No. 118, 167–173.
- Jekeli, C.; Yang, H. J.; Kwon, J. H. (2009). Using gravity and topography-implied anomalies to assess data requirements for precise geoid computation, Journal of Geodesy, 83, 12, 1193
- Lauría, E. A.; Galbán, F. M.; Brunini, C.; Font, G.; Rodríguez, R.; Pacino, C. (2002). *The Vertical Reference System in the Argentine Republic*. In: Drewes, H.; Dodson, A. H.; Fortes, L. P. S.; Sánchez, L.; Sandoval, P. (Ed.). Vertical Reference Systems. IAG Symposium 124, Cartagena, Colômbia. Berlin: Springer-Verlag, 2002. p. 11-15.
- Mayer-Gürr, T. and the GOCO consortium (2012). *The new combined satellite only model GOCO03s*. Presented at International Symposium on Gravity, Geoid and Height Systems GGHS 2012, October 9-12, 2012, Venice.
- Mayer-Gürr, T. (2007). *ITG-Grace03s: The latest GRACE gravity field solution computed in Bonn*. Joint International GSTM and DFG SPP Symposium, 15-17 Oct 2007, Potsdam.
- Pail, R.; Bruinsma, S.; Migliaccio, F.; Foerste, C.; Goiginger, H.; Schuh, W.-D.; Hoeck E.; Reguzzoni, M.; Brockmann, J.M.; Abrikosov, O.; Veicherts, M.; Fecher, T.; Mayrhofer, R.; Krasbutter, I.; Sanso, F.; Tscherning, C.C. (2011). *First GOCE gravity field models derived by three different approaches*. Journal of Geodesy, 85, 11: 819-843.
- Pavlis, N.K.; Holmes, S.A.; Kenyon, S.C.; Factor, J.K. (2008) *An Earth Gravitational Model to Degree 2160: EGM2008*- European Geosciences Union General Assembly, Vienna, Austria, April 13-18, 2008. <http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008>.
- Pavlis, N.K.; Holmes, S.A.; Kenyon, S.C.; Factor, J.K. (2012). *The development and evaluation of the Earth Gravitational Model 2008 (EGM2008)*. Journal of Geophysical Research: Solid Earth, 117: 2156-2202, B04406. doi:10.1029/2011JB008916.
- Piña, W. H. S.; Di Landro, H. R.; Turban, L. (2002). *The Vertical datum and Local Geoid Models in Uruguay*. In: Drewes, H.; Dodson, A. H.; Fortes, L. P. S.; Sánchez, L.; Sandoval, P. (Ed.). Vertical Reference Systems . IAG Symposium 124, Cartagena, Colômbia. Berlin: Springer-Verlag, 2002. p. 169-175.

- Poutanen, M.; Vermeer, M.; Mäkinen, J. (1996). *The permanent tide in GPS positioning*. J. Geod., 70, 8, 499–504.
- Sánchez, L.; Brunini, C. (2009) *Achievements and challenges of SIRGAS*. In: Drewes H. (Ed.): Geodetic Reference Frames, IAG Symposia 134: 161-166, Springer, 10.1007/978-3-642-00860-3_25.
- Sánchez, L. (2005). *GTIII SIRGAS: Datum Vertical – Reporte 2005*. Caracas, Venezuela, November 17 and 18, 2005. 37p.
- Shako, R.; Förste, C.; Abrykosov, O.; Bruinsma, S.; Marty, J.-C.; Lemoine, J.-M.; Flechtner, F.; Neumayer, K.-H.; Dahle, C. (2014). *EIGEN-6C: A High-Resolution Global Gravity Combination Model Including GOCE Data* - In: Flechtner, F., Sneeuw, N., Schuh, W.-D. (Eds.), *Observation of the System Earth from Space - CHAMP, GRACE, GOCE and future missions*, (GEOTECHNOLOGIEN Science Report; No. 20; Advanced Technologies in Earth Sciences), Berlin [u.a.]: Springer, 155-161.DOI 10.1007/978-3-642-32135-1_20, Print ISBN 978-3-642-32134-4 Online ISBN 978-3-642-32135-1
- SIRGAS Project Committee: *SIRGAS Final Report* (1997); Working Groups I and II, IBGE, Rio de Janeiro, 96 pp.
- Tenzer, R.; Vatr, V.; Abdalla, A.; Dayoub, N. (2010). Assessment of the LVD offsets for the normal-orthometric heights and different permanent tide systems - a case study of New Zealand. Appl. Geom., 3(1): 1–8.
- Yang, H. J. (2013). Geoid Determination Based on a Combination of Terrestrial and Airborne Gravity Data in South Korea, report No. 507, Geodetic Science, v+98 pp, December 2013.
- Yi, W.; Rummel, R.; Gruber, Th.(2013). *Gravity field contribution analysis of GOCE gravitational gradient components*; Studia Geophysica et Geodaetica, 57(2):174-202, Springer Netherlands, ISSN 0039-3169, ISSN (Online) 1573-1626, DOI: 10.1007/s11200-011-1178-8.

Acknowledgements

The authors acknowledge the contribution of the ICGEM, IBGE, GETECH, NGA and the Civil and Military organizations in the South America (Argentina, Brazil, Chile, Colombia, Ecuador, Paraguay, Uruguay and Venezuela) for the efforts to provide data.