TTK4115 - Linear System Theory

Helicopter Lab Report

Group 70

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1 Part I - Mathematical modeling

The purpose of this part of the assignment is to derive a mathematical model of the system (the helicopter), that can be used for control purposes.

1.1 Problem 1

The model of the helicopter is depicted in figure 1. The cubes represent the point masses. The cylinders represent the helicopter joints.

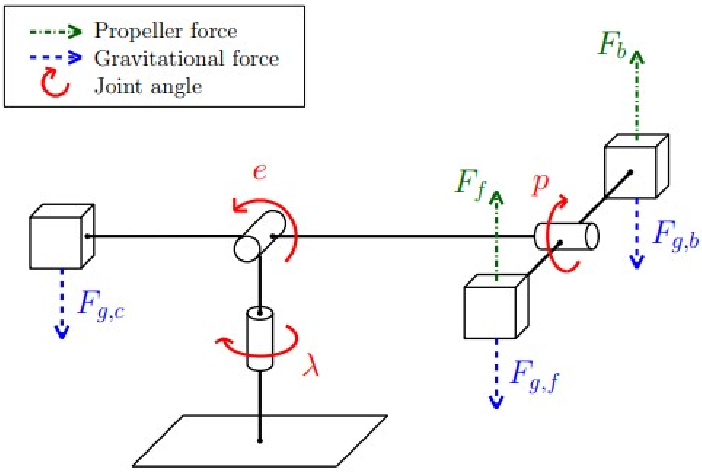

Figure 1: Helicopter model with masses and distances

Equations of motion is mathematically derived by using Newton's second law for rotation. Where α is angular acceleration, τ is torque and I is rotational inertia. Gravitational forces always point in a vertical direction, while the direction of the propeller forces is dependent on the joint angles.

$$\alpha = \frac{\sum \tau}{I} \Leftrightarrow I\alpha = \sum \tau \qquad \tau = Fr\sin\theta$$

The helicopter head can tilt with respect to the arm. This will be referred to as the pitch of the helicopter. The difference between the forces generated by the two propellers will result in rotation about the pitch axis.

$$\begin{aligned} J_p\ddot{p} &= F_f l_p \sin 90 - F_b l_p \sin 90 + F_{g,f}\, l_p - F_{g,b}\, l_p \\ &= F_f l_p - F_b l_p \\ &= K_f V_f l_p - K_f V_b l_p \\ &= K_f l_p (V_f - V_b) \end{aligned}$$

$$J_p\ddot{p} = L_1 V_d \qquad L_1 = K_f l_p \tag{1}$$

The helicopter head can move up and down with respect to the base. This will be referred to as the elevation of the helicopter. The sum of the forces developed by the two propellers determines the lift of the helicopter.

$$\begin{aligned}
J_e\ddot{e} &= F_{g,c}\, l_c \cos e - (F_{g,f} + F_{g,b})l_h \cos e + (F_f + F_b)l_h cosp \\
&= (m_c l_c - 2m_p l_h)g \cos e + (K_f V_f + K_f V_b)l_h \cos p \\
&= (m_c l_c - 2m_p l_h)g \cos e + (V_f + V_b)K_f l_h \cos p
\end{aligned}$$

$$J_e\ddot{e} = L_2 \cos e + L_3 V_s \cos p \qquad L_2 = (m_c l_c - 2m_p l_h)g \qquad L_3 = K_f l_h \tag{2}$$

The helicopter arm can rotate about the vertical axis. This will be referred to as the travel of the helicopter.Travel is dependent on the pitch and the elevation, since those angles effects the length of the arm.

$$\begin{aligned}
J_\lambda\ddot{\lambda} &= -(F_f + F_b)l_h \cos e \sin p \\
&= -(K_f V_f + K_f V_b)l_h \cos e \sin p \\
&= -(V_f + V_b)K_f l_h \cos e \sin \\
&= -K_f l_h V_s \cos e \sin p
\end{aligned}$$

$$J_\lambda\ddot{\lambda} = L_4 V_s \cos e \sin p \qquad L_4 = -K_f l_h \tag{3}$$

1.2 Problem 2

In order to design a linear controller for the system, we have to linearize the nonlinear pitch, elevation and travel equations derived in the previous section.

We want to linearize the equations of motions around the point $(p, e, \lambda)^T = (p^*, e^*, \lambda^*)^T$, with $p^* = e^* = \lambda^* = 0$. In order to do so we need to determine the voltages V_S^* and V_d^* such that $(p^*, e^*, \lambda^*)^T$ is an equilibrium point of the system.

$$\begin{aligned}
J_p\ddot{p} &= L_1 V_d \\
0 &= L_1 V_d^*
\end{aligned}$$

$$V_d^* = 0 \tag{4}$$

$$\begin{gathered}
J_e\ddot{e} = L_2 \cos e + L_3 V_s \cos p \\
0 = L_2 \cos 0 + L_3 V_s \cos 0 = L_2 + L_3 V_s
\end{gathered}$$

$$V_s^* = -\frac{L_2}{L_3} \tag{5}$$

The following coordinate transformation is introduced to simplify the further analysis, as the equilibrium point of this new coordinate system is located in the origin.

$$\begin{bmatrix}\tilde{p}\\ \tilde{e}\\ \tilde{\lambda}\end{bmatrix} = \begin{bmatrix}p\\ e\\ \lambda\end{bmatrix} - \begin{bmatrix}p^*\\ e^*\\ \lambda^*\end{bmatrix} \to \begin{bmatrix}p\\ e\\ \lambda\end{bmatrix} = \begin{bmatrix}\tilde{p}\\ \tilde{e}\\ \tilde{\lambda}\end{bmatrix} + \begin{bmatrix}p^*\\ e^*\\ \lambda^*\end{bmatrix} = \begin{bmatrix}\tilde{p}\\ \tilde{e}\\ \tilde{\lambda}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\tilde{V_s}\\ \tilde{V_d}\end{bmatrix} = \begin{bmatrix}V_s\\ V_d\end{bmatrix} - \begin{bmatrix}V_s^*\\ V_d^*\end{bmatrix} \to \begin{bmatrix}V_s\\ V_d\end{bmatrix} = \begin{bmatrix}\tilde{V_s}\\ \tilde{V_d}\end{bmatrix} + \begin{bmatrix}V_s^*\\ V_d^*\end{bmatrix} = \begin{bmatrix}\tilde{V_s}\\ \tilde{V_d}\end{bmatrix} + \begin{bmatrix}-L_2/L_3\\ 0\end{bmatrix}$$

$$J_p\ddot{\tilde{p}} = L_1\tilde{V_d}$$
$$J_e\ddot{\tilde{e}} = L_2\cos\tilde{e} + L_3(\tilde{V_s} - L_2/L_3)\cos\tilde{p}$$
$$J_\lambda\ddot{\tilde{\lambda}} = L_4(\tilde{V_s} - L_2/L_3)\cos\tilde{e}\sin\tilde{p}$$

We introduce the state vector $x = (x_1, x_2, x_3, x_4, x_5, x_6) = (\tilde{p}, \dot{\tilde{p}}, \tilde{e}, \dot{\tilde{e}}, \tilde{\lambda}, \dot{\tilde{\lambda}})$ and the input vector $u = (u_1, u_2) = (\tilde{V_s}, \tilde{V_d})$ and rewrite the equations of motion.

$$\begin{aligned}
\dot{x_1} &= x_2\\
\dot{x_2} &= (L_1/J_p)V_d\\
\dot{x_3} &= x_4\\
\dot{x_4} &= (L_2/J_e)\cos x_3 + (L_3/J_e)V_s\cos x_1\\
\dot{x_5} &= x_6\\
\dot{x_6} &= (L_4/J_e)V_s\cos x_3\sin x_1
\end{aligned}$$

The system is linearized by finding the jacobians of h with respect to the state and the input and then insert the equilibrium values.

$$A = \frac{\partial h}{\partial x}\Big| = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0\\ \frac{L_4L_2}{J_\lambda L_3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{B} = \frac{\partial h}{\partial u\Big| = \begin{bmatrix} 0 & 0\\ 0 & L_1/J_p\\ 0 & 0\\ L_3/J_e & 0\\ 0 & 0\\ 0 & 0 \end{bmatrix}}$$

The linearized system of equations put in a state-space formulation.

$$\begin{bmatrix} \dot{\tilde{x_1}} \\ \dot{\tilde{x_2}} \\ \dot{\tilde{x_3}} \\ \dot{\tilde{x_4}} \\ \dot{\tilde{x_5}} \\ \dot{\tilde{x_6}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{L_4 L_2}{J_\lambda L_3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x_1} \\ \tilde{x_2} \\ \tilde{x_3} \\ \tilde{x_4} \\ \tilde{x_5} \\ \tilde{x_6} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & L_1/J_p \\ 0 & 0 \\ L_3/J_e & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix}$$

We have obtained the linear equations. We can assume that the nonlinear system will behave similarly close to the equilibrium point.

$$\ddot{\tilde{p}} = (L_1/J_p)\tilde{V}_d = K_1\tilde{V}_d \qquad K_1 = L_1/J_p \tag{6}$$

$$\ddot{\tilde{e}} = (L_3/J_e)\tilde{V}_s = K_2\tilde{V}_d \qquad K_2 = L_3/J_p \tag{7}$$

$$\ddot{\tilde{\lambda}} = (L_4 L_2/J_\lambda L_3)\tilde{p} = K_3\tilde{p} \qquad K_3 = (L_4 L_2/J_\lambda L_3) \tag{8}$$

1.3 Problem 3

In this part of the assignment, we made the first attempt to control the helicopter. It was rather difficult to control the helicopter using feed forward.

The physical behavior of the helicopter deviates from the theoretical models. When we made the model, we made many simplifications, thus the model will be a simplified version of reality. Since the physical system is not linear, the linear assumption breaks down when the system is too far from this equilibrium point.

1.4 Problem 4

In figure 1, all joint angles is zero. It was necessary to subtract approximately 30 degrees from the elevation to make the output zero when the arm between the elevation axis and the helicopter head is horizontal. Added gain, $\pi/180$, to convert the encoder output values form degrees to radius.

The voltage V_s^* required to make the helicopter maintain the equilibrium was found by measurement to be 7V. The motor force constant K_f calculated.

$$K_f = -\frac{(m_c l_c - 2m_p l_h)g}{V_s^* l_h} \tag{9}$$

2 Part II - Monovariable control

In this part we design a PD-controller to control the pitch and a P-controller to control the travel rate.

2.1 Problem 1

A PD controller is implemented to control the pitch angle p. The following equation is used to determine the controller:

$$\tilde{V_d} = K_{pp}(\tilde{p_c} - \tilde{p}) - K_{pd}\dot{\tilde{p}} \tag{10}$$

To find the transfer function , it is necessary to replace $\tilde{V_d}$ with equation

$$\ddot{\tilde{p}} = K_1\tilde{V_d}$$

And by some extra mathematical expanding, we get:

$$\ddot{\tilde{p}} + K_1K_{pd}\dot{\tilde{p}} + K_1K_{pp}\tilde{p} = K_1K_{pp}\tilde{p_c}$$

With the use of Laplace transform, and assuming $\tilde{p}(0) = 0$, we can arrive at:

$$s^2\tilde{p}(s) + sK_1K_{pd}\tilde{p}(s) + K_1K_{pp}\tilde{p}(s) = K_1K_{pp}\tilde{p_c}(s)$$

Finally, the transfer function:

$$\frac{\tilde{p}(s)}{\tilde{p_c}(s)} = \frac{K_1K_{pp}}{s^2 + K_1K_{pd}s + K_1K_{pp}} \tag{11}$$

To determined expression for K_{pp} and K_{pd}, the transfer function of the linearized system is compared with a general expression for a second order linear system.

$$h(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

By the comparing, the following expressions is determined:

$$\omega_0 = \sqrt{K_1K_{pp}}$$

$$2\zeta\omega_0 = K_1K_{pd}$$

If value of ζ is set to high it will give a more damped response our opposite to low, the result will be a less damping response. By setting the damping ratio $\zeta = 1$, gives the system critical damping. We get the following equations:

$$K_{pp} = \frac{\omega_0^2}{K_1} \tag{12}$$

$$K_{pp} = 2\frac{\sqrt{K_1K_{pp}}}{K_1} \tag{13}$$

After some tuning by testing different values for ω_0, we noticed that values between 2-3 made a good response. Lower values made the pitch response slow. Values over 3 made helicopter more unstable, and the motors started turned on and off gradually. We ended up choosing $\omega_0 = 3$. This gave us $K_{pp} = 15.89$ and $K_{pd} = 10.59$.

2.2 Problem 2

The travel rate $\dot{\tilde{\lambda}}$ is to be controlled using a P controller. The following equation is used to determine the controller:

$$\tilde{p} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{14}$$

The constant $K_{rp} < 0$, assuming $\tilde{p} = \tilde{p}_c$ and by use of $\ddot{\tilde{\lambda}} = K_3\tilde{p}$ we can arrive at:

$$\ddot{\tilde{\lambda}} + K_3 K_{rp}\dot{\tilde{\lambda}} = K_3 K_{rp}\dot{\tilde{\lambda}}_c \tag{15}$$

With the use of Laplace transform, and assuming $\tilde{\lambda}(0) = 0$ and $\dot{\tilde{\lambda}}(0) = 0$, gave us the following expression:

$$\rho = K_3 K_{rp}, \frac{\dot{\tilde{\lambda}}(s)}{\dot{\tilde{\lambda}}_c(s)} = \frac{K_3 K_{rp}}{s + K_3 K_{rp}} = \frac{\rho}{s + \rho} \tag{16}$$

By implementing the P-controller, and after some tuning, the system with the controller made the helicopter response faster and more accurate. Limiting the joystick sensitivity was done by adding a gain on the output x off the joystick. After trying different values for K_{rp} and the gain block, our tuning ended up with $K_{rp} = -1$ and the $gain = 0,8$.

3 Part III - Multivariable control

This part will focus on multivariable control structure, where the group control the pitch angle p̃ and the elevation rate $\dot{\tilde{e}}$. The referenece for the pitch angle and the elevation rate are provided by x- and y- axis of the joystick.

3.1 Problem 1

$$\dot{x} = Ax + Bu \tag{17}$$

Where A and B are matrices on the state-space form. The input vectors are x and u:

$$x = \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} \qquad and \qquad u = \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} \tag{18}$$

The state space model:

$$\dot{x} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} \tag{19}$$

3.2 Problem 2

The group aimed to track the reference $r = |\tilde{p_e}, \dot{\tilde{e_e}}|^T$ for the pitch angle p̃ and the elevation rate, $\dot{\tilde{e}}$, which was given by the joystick output. The x-axis tracked the pitch angle, and the y-axis tracked the elevation rate.

$$\mathbf{C} = \begin{bmatrix} B & AB & AB^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & K_1 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

When analyze the controllability matrix, it has full rank, which mean it is theoretical possible to control.
The assignment give us the controller form **u** = **Pr - Kx**. Where the matrix K, corresponds to the linear quadratix regulator (LQR) for which the controller input **u** = **-Kx** optimizes the cost function J. See equation nr 21.

$$J = \int_0^\infty (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + u^T(t)\mathbf{R}\mathbf{u}(t))dt \tag{21}$$

For simplicity, the weighting matrices Q and R are set diagonal. The Q and R matrices are choosen so the response of the helicopter is fast and

accurate. In order to find the K matrix the group used the MATLAB command $K = LQR(A, B, Q, R)$
The P matrix was chosen such that $\lim_{t\to\infty} \tilde{p}(t) = \tilde{p}_e$ and $\lim_{t\to\infty} \dot{\tilde{e}}(t) = \dot{\tilde{e}}_e$ for fixed values of $\tilde{p}_e$ and $\dot{\tilde{e}}_e$.

$$\dot{x} = Ax + Bu \tag{22}$$

$$u = Pr - Kx \tag{23}$$

$$\dot{x} = Ax + B(Pr - Kx) \tag{24}$$

$$P = |C(BK - A)^{-1}B|^{-1} \tag{25}$$

When setting up the R and Q matrix, the group first chose some values, then the group experimented with the matrices and ended up with:

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 10 & 0 \\ 100 & 0 & 100 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{26}$$

After plotting the LQR in matlab the group got the K matrix:

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 10 \\ 10 & 6.73 & 0 \end{bmatrix} \tag{27}$$

To find the P matrix, the group used matrix nr 25.

$$P = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix} \tag{28}$$

3.3 Problem 3

The group modified the controller from the previous problem to include an integral effect for the elevation rate and the pitch angle. In order to fill the two extra states needed for the integral the group now also have the states γ and ζ.

$$\dot{\gamma} = \tilde{p} - \tilde{p}_e \tag{29}$$

$$\dot{\zeta} = \dot{\tilde{e}} - \dot{\tilde{e}}_e \tag{30}$$

The state vector and the input vector is changed into:

$$x = \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \\ \gamma \\ \zeta \end{bmatrix} \qquad and \qquad u = \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} \tag{31}$$

The A, B and C matrices also had to be adjusted to work with the integrator.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{32}$$

Since the dimention of the Q and R matrices also need matching because of the two extra input vectors. For the tuning, the group sat a starting point quite similar to the tuning from 3.2, and try to tune it fast and responsive. Because of the integrator, it was not as fast as with the regulator from 3.2. Here are the R and Q matrices the group ended up with:

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad and \qquad Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \tag{33}$$

For the K matrix, the group used the MATLAB command LQR(A,B,Q,R). And for the P matrix, the group did not get any good answer. After analysing the similarities between the K and P matrices in 3.2, the group figured out that the P matrix uses some of the K matrix, and therefor the group plotted some parts of the K matrix into the P matrix as shown in equation 34.

$$K = LQR(A, B, Q, R) = \begin{bmatrix} 0 & 0 & 13.02 & 0 & 3.16 \\ 12.08 & 7.26 & 0 & 3.16 & 0 \end{bmatrix}$$

$$\text{P}= \begin{bmatrix} 0 & K(1.3) \\ K(2.1) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 13.02 \\ 12.08 & 0 \end{bmatrix} \tag{34}$$

4 Part IV - State estimation

In this part we implemented state estimators and reused the LQR controller from part III. The goal is use an observer in order to estimate these non measured states.

4.1 Problem 1

By the use of the equations $\ddot{\tilde{p}} = K_1\tilde{V}_d$, $\ddot{\tilde{e}} = K_2\tilde{V}_s$ and $\ddot{\tilde{\lambda}} = K_3\tilde{p}$ and a set of first degree equations, we can derive a state-space formulation by using the included vectors.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{35}$$

This gives:

$$\dot{x} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ K_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ 0 & 0 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} \tag{36}$$

4.2 Problem 2

System observability were calculated using "obsy(A,C)" function in MATLAB. The matrix has full rank(rank=6), thereby the system is observable. The following linear observer for the system were used:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{37}$$

Since the system is observable,it is possible to place the poles of the estimator arbitrarily by choosing an appropriate gain matrix, L. Because of the pole placement influence the system behavior, it is important to keep that in mind when tuning the helicopter to become stable and responsive.

Whem chosing poles, we made the amplitude aprox. ten times bigger then the system value, and spred them out in a "fan-form" by basing our poles on complex-conjured butterworth poles. The group tested first poles at: $-80 \pm 157.5°, -80 \pm 135° and - 80 \pm 102°$. After some testing, the group figured out there was to much overshoot and changed the $-80 \pm 102°$ poles into $-80 \pm 122.7°$. After adjusting the most "unstable" pole a bit more stable, less imaginary value and larger real value, the estimator got quite fast and

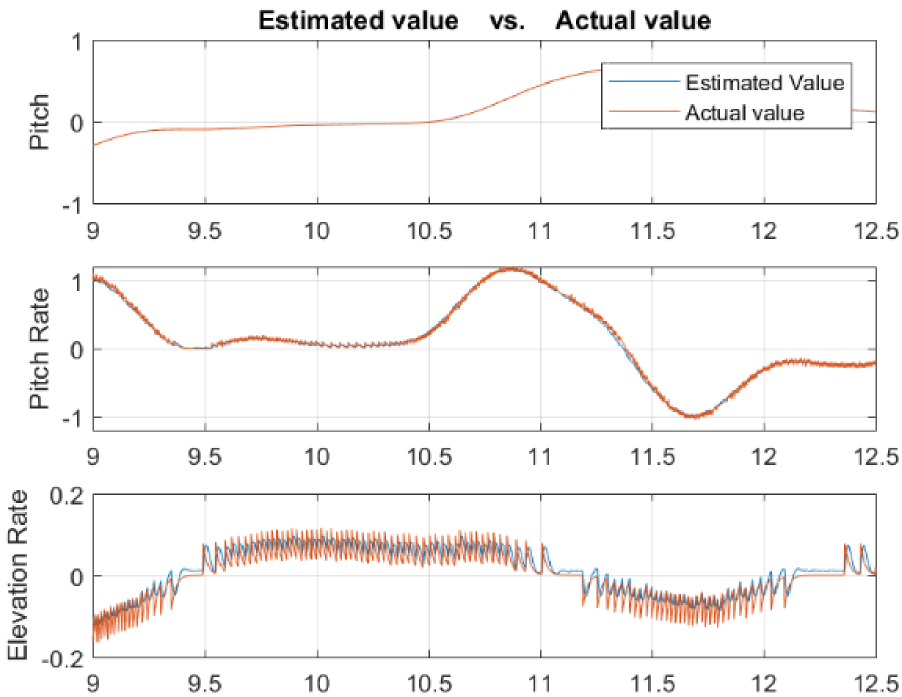

Figure 2: Problem 2 - LQR with estimator

responsive, as showed in figure 2 and 3 . Our final poles can be seen in equation 38. After finding the poles, the group used the MATLAB-function "$L = place(A^T, C^T, poles)^T$" to make it possible to calculate the gain matrix L.

$$\lambda = \begin{bmatrix} -43+67i & -43-67i & -57+57i & -57-57i & -74+31i & -74-31i \end{bmatrix} \tag{38}$$

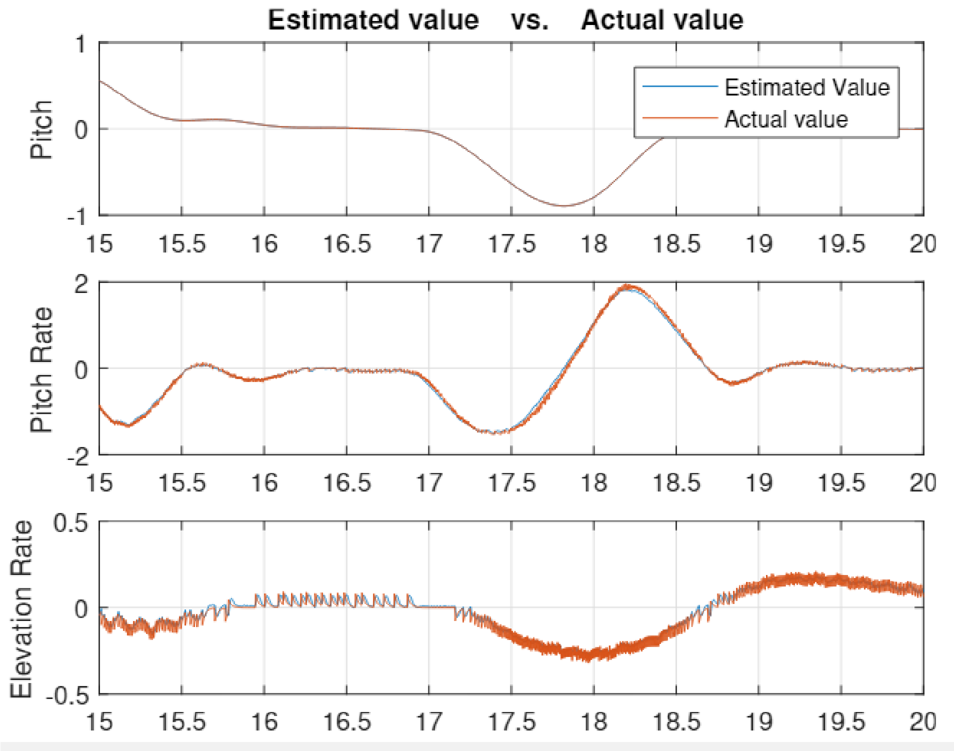

Figure 3: LQR with integrator and estimator

4.3 Problem 3

When only measuring $\tilde{e}$ and $\tilde{\lambda}$, the MATLAB function "obsy(A,C)" gives the rank=6. This is full rank, and the system is observable. However when only $\tilde{p}$ and $\tilde{e}$ being measured, the MATLAB function "obsy(A,C)" gives the rank=4. This is not full rank, and the system is not observable. A new observer was created after the following equation:

$$y = \begin{bmatrix} \tilde{e} \\ \tilde{\lambda} \end{bmatrix} \tag{39}$$

This problem were realized with the P controller from part 3 problem 2. Although the system is observable with the parameters $\tilde{e}$ and $\tilde{\lambda}$ it didn't made the helicopter easy to control. Because of the unmeasured state pitch, it was difficult to make the helicopter respond fast and were behaving pour to sudden changes in travel. In the case of sudden changes, the helicopter becomes uncontrollable and sometimes ending up crashes.

At first the poles from problem 4.2 tried, but those poles were too quick, making the estimator unstable. The group understood that the easiest were to use "over-dampened poles", meaning a slow estimator, but quite stable.

The group figured out from the MATLAB scope, that the estimator was ok, but since it was yet quite difficult to control, we changed the cost matrix, Q, to get better control. After a reconfiguration of the Q matrix, the group were happy with the result. The final Q and λ equation is found in equation nr 40 and 41 .

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 100 \end{bmatrix} \tag{40}$$

$$\lambda = \begin{bmatrix} -5 & -5 & -10 & -15 & -15 & -20 \end{bmatrix} \tag{41}$$

The physical reason for why it is possible to use the first two states, but not the second pair is because of the equation

$$\ddot{\tilde{\lambda}} = K_3 \tilde{p} \tag{42}$$

Information get lost during differentiating and can not be obtained. This will result in a unknown constant.

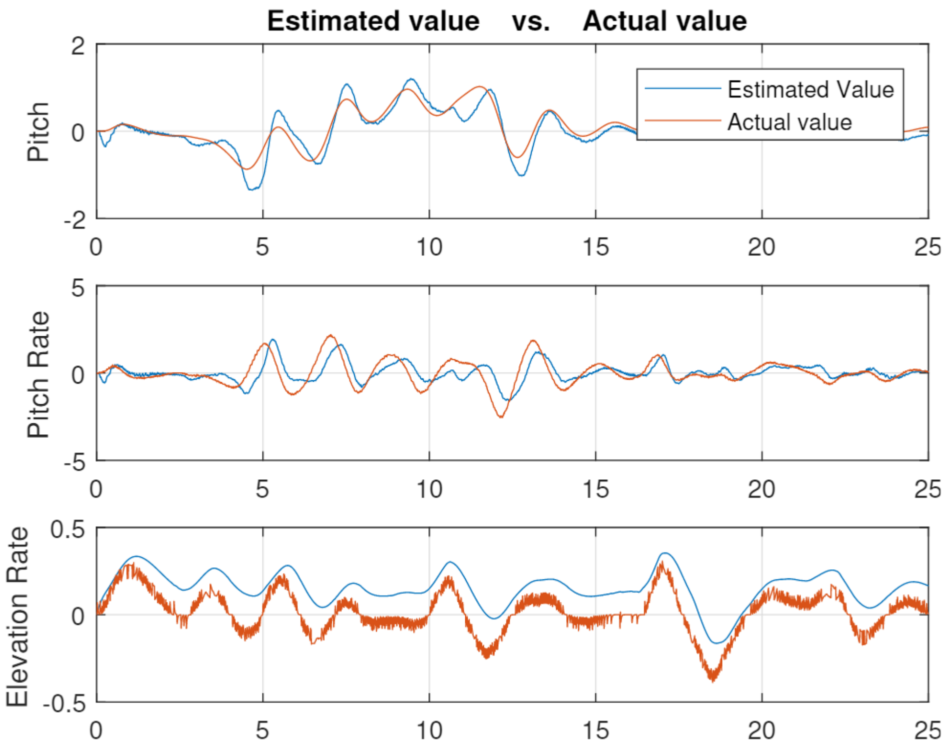

Figure 4: Plot problem 3

A Simulink Models

A.1 Pitch controller

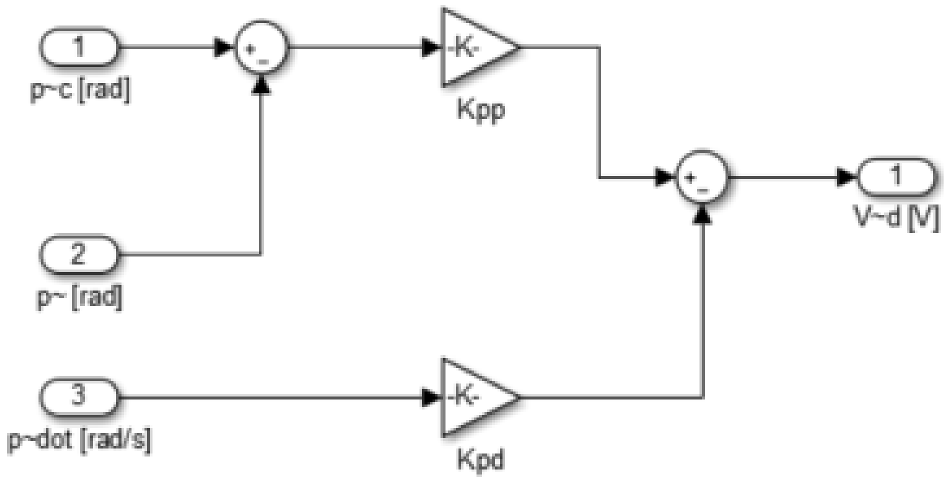

Figure 5: shows the pitch controller used in part 2 problem 1.

A.2 LQR controller

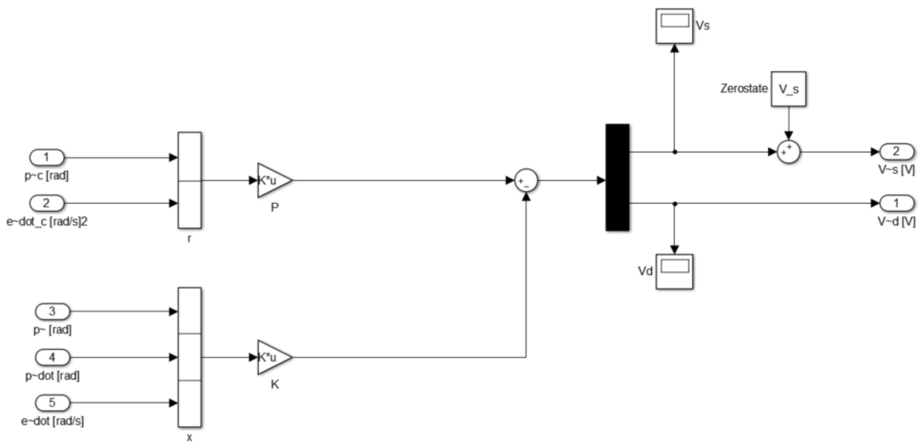

Figure 6: shows the LQR controller used in part 3 problem 2.

A.3 LQR controller with integrator

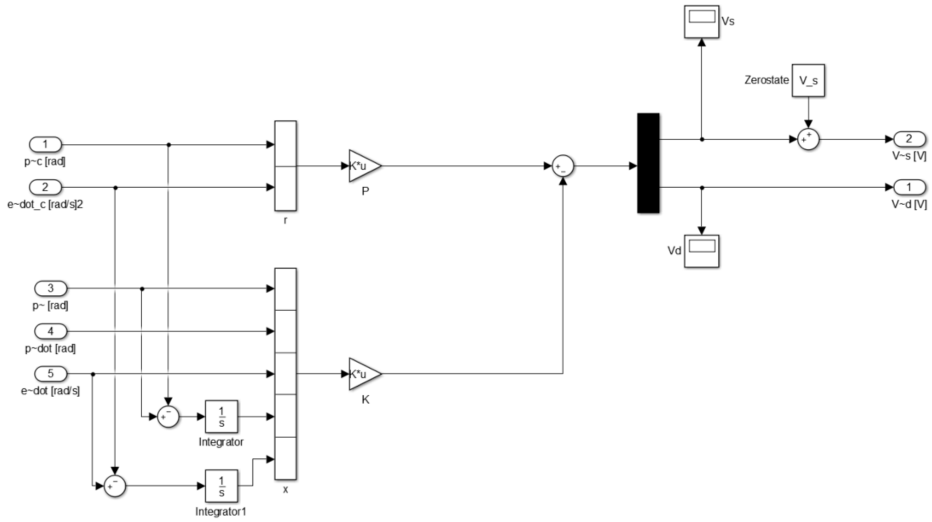

Figure 7: shows the LQR controller used in part 3 problem 3.

A.4 Estimator p4p2

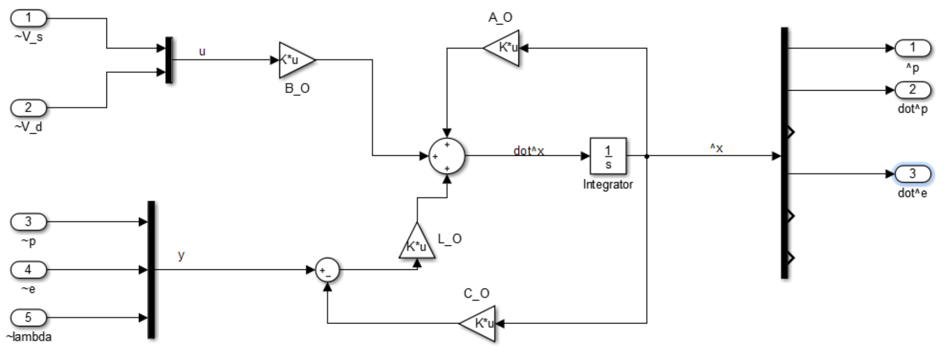

Figure 8: shows the estimator used in part 4 problem 2.

A.5 Estimator p4p3

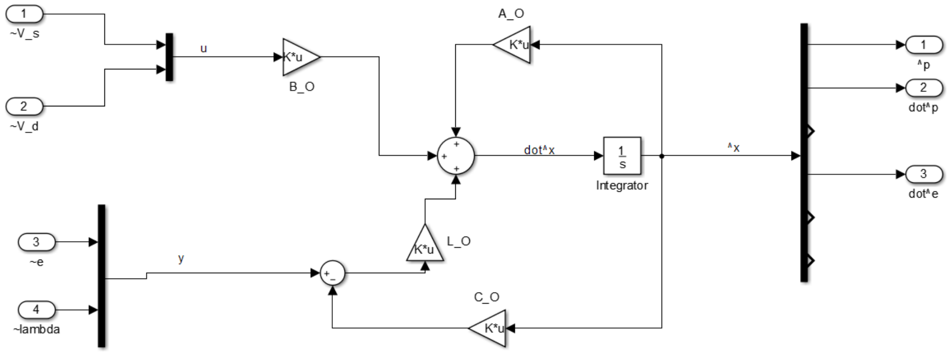

Figure 9: shows the estimator used in part 4 problem 3.