3 RADIOACTIVE DECAY CHAINS

As we move further and further from stability, the radioactive nuclei we observe also have radioactive daughters, granddaughters etc. Proceeding down the generations requires a description of the *radioactive decay chains*. As we discussed in class, the decay laws and decay statistics do not depend on the type of radioactive decay they are describing. Therefore, one can solve the coupled differential equations for a decay chain with k generations (also see the Bateman equations for nuclear decay). The general solution to this (for 3 species) was found to be of the form (see lecture notes, and Heyde Pg. 73):

$$
\begin{aligned}
N_1(t) &= a_{11}e^{-\lambda_1 t}\\
N_2(t) &= a_{21}e^{-\lambda_1 t} + a_{22}e^{-\lambda_2 t}\\
N_3(t) &= a_{31}e^{-\lambda_1 t} + a_{32}e^{-\lambda_2 t} + a_{33}e^{-\lambda_3 t}
\end{aligned}
$$

Considering the following β decay chain with **4 members** (where ^{56}Fe is stable), answer the questions below:

$$^{56}\mathrm{V} \xrightarrow{T_{1/2}=0.23\ \mathrm{s}} {}^{56}\mathrm{Cr} \xrightarrow{T_{1/2}=5.9\ \mathrm{min}} {}^{56}\mathrm{Mn} \xrightarrow{T_{1/2}=2.58\ \mathrm{h}} {}^{56}\mathrm{Fe}$$

1. Write a program to calculate the coefficients a_{ki} (start with $a_{11} = N_{10} = 1000$) (include your source code). What are the numerical values for all of the a_{ki}?

2. Plot the evolution of the *number of radioactive nuclei* $N(t)$ as a function of time from $t = 0 \to 2\times10^5$ s for all species on the same graph. Use a different line colour or style for each species. Since the decays have very different $T_{1/2}$ values, also make sure that you use a log scale for your x axis. To make sure your result is correct, check the following:

   i) $N_1(t=0) = N_{10} = 1000$

   ii) $N_1(t), N_2(t)$, and $N_3(t)$ should be roughly zero at $t = 2\times10^5$ s, and $N_4(t)$ should be roughly 1000.

3. Now plot the *activity* $A(t)$ as a function of time using the same plotting parameters for each species as you did above, except you should also now use a logarithmic axis for both X and y. To make sure your result is correct, check the following:

   i) The maximum activity for a given species i is reached when $A_i(t) = A_{i-1}(t)$

   ii) $A_4(t) = 0$

4. From your plot above, estimate the maximum activity reached by species 1, 2, and 3 in units of Bq. At what time do these maxima occur?