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Analysis string stability of a new car-following model considering response time

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Abstract—**This study investigates the influence of the response time on the dynamics of a desired safety margin (DSM) car-following model, where the response time plays an important role in determining the qualitative dynamical of vehicles in car-following process. The stability criterion of the DSM car-following model is obtained by the linear system stability theory. Numerical simulations are in good agreement with the analytical results, which reveals that the response time would significantly influence the stability of traffic flow on a straight road. The numerical results also indicate that intelligent driving can help to reduce traffic congestion because the unstable flow can be stabilized by adopting the response time.**

Index Terms—**Safety margin, response time, stability analysis, sensitivity factor**

I. Introduction

TRAFFIC congestion is still a serious problem that has long exercised the modern society. In order to make effective policies to solve the traffic jam based on the underlying mechanism behind the phenomena of traffic flow, numerous car-following models have been proposed to describe the detailed driving behaviors of individual vehicle. As the most basic research of traffic flow analysis, car-following model has attracted extensive research interests in the past few decades. Recently, adaptive cruise control (ACC) and automotive platoon control which contribute to intelligent driving system have been widely researched, where string stability analysis of platooning vehicles plays a key role in car-following process [1]–[3]. Thus the car-following theory is extremely essential for the development of current intelligent driving system, which aims eliminating stop-and-go waves and achieving smooth traffic flows.

In 1953, Pipes [4] proposed a pioneering car following model innovatively to characterize the car-following process. Subsequently, Chandler et al. [5] proposed the first linear car-following model, which use the relative velocity as stimulus only. Later, the Gazis-Herman-Rothery (GHR) model proposed by Gazis et al. [6] make up for the disadvantages of Chandler's model where the following vehicle's motion is determined by relative velocity as well as velocity, and headway of two successive vehicles. In 1995, Bando et al developed the optimized velocity (OV) approach on the basis of drivers' desire to maintain optimal velocity in car-following process [7]. Although those reviewed models as GHR, OV, full velocity difference (FVD) and their related car-following models [8]–[16] effectively simulate car-following behaviors and determine how car following occurs in actual scenarios, the reason why vehicles follow one another in a certain manner might be unclear. Actually, driving behavior is largely affected by the risk perception of drivers in real traffic. Lu et al. [17] proposed a desired safety margin (DSM) model using the stimulus-response concept based on the risk homeostasis theory, which can not only simulate different driving behaviors as response time, sensitivity factor and safety margin, but also gives a new way to explain car-following process. This point of view has been deduced in some literatures that traffic flow is heavily influenced by response time in car-following process [18]–[20].

Therefore, this paper attempts to investigate the effect of response time on the stability of traffic flow based on the DSM car-following model. The linear stability analysis approach will be applied to obtain the stable region of the DSM model under a given DSM. Moreover, the response time will be discussed to investigate its impact on the stability of traffic flow. Finally, a comprehensive numerical simulation was carried out to validate the theoretical analytical results.

This paper is organized as follows: Section II introduces the DSM model framework. And the string stability condition of the DSM model is presented in Section III. Section IV analyzes the effect of the response time on the stability of DSM model and verifies the theoretical results by numerical simulations. Finally, conclusions are drawn in Section V.

I. The DSM Model

A new dynamical equation of car-following model is described by Lu et al. [17], namely the DSM model, which can be simply expressed as follows:

$$a_n(t+\tau)=\alpha\left(1-\frac{v_n(t)\cdot\tau_2+\left(\left[v_n(t)\right]^2/2d_n(t)\right)}{x_{n-1}(t)-x_n(t)-l_{n-1}}+\frac{\left[v_{n-1}(t)\right]^2/2d_{n-1}(t)}{x_{n-1}(t)-x_n(t)-l_{n-1}}-\chi_{DSM}\right) \tag{1}$$

where $a_n(t)$ is the acceleration of a following car at time t; α is the sensitivity constant for acceleration; the DSM χ_{DSM}

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is considered as a fixed value related to driver behavior and traffic environment, namely, desired safety margin; τ represents the response time, including the driver's response time (τ_1) and brake system's response time (τ_2); $v_n(t)$ and $v_{n-1}(t)$ denote the velocity of the n th and $(n-1)$ th vehicle at time t, respectively; $x_{n-1}(t)$ and $x_n(t)$ represents the position of the $(n-1)$ th and the n th vehicles, respectively; $d_n(t)$ and $d_{n-1}(t)$ are the deceleration of the n th and the $(n-1)$ th vehicle at time t, respectively; l_{n-1} denotes the length of the preceding vehicle.

II. Stability Analysis of the DSM Model

In order to deduce the stability condition of the DSM model for car following, it can be rewritten as follows:

$$a_n(t+\tau) = f(v_{n-1}, y_n, v_n). \qquad (2)$$

Here $y_n(t) = \tilde{y}_n(t) + y^*$ represents the spacing between vehicles $(n-1)$ and n at time t, which has the following relationship as follows:

$$\tilde{y}_n(t) + y^* = x_{n-1}(t) - x_n(t) - l_{n-1}, \qquad (3)$$

where y^* denotes the desired equilibrium separation.

Note that in steady-state uniform flow solution, all vehicles would drive at same speed $V(y^*)$ with the same time-independent separation y^*.

One approach is to consider small perturbations to the uniform flow equilibria by setting

$$y_n(t) = \tilde{y}_n(t) + y^*, v_{n-1}(t) = \tilde{v}_{n-1}(t) + v^*, v_n(t) = \tilde{v}_n(t) + v^* \qquad (4)$$

where $\tilde{y}_n(t)$ is a small separation perturbation; $\tilde{v}_{n-1}(t)$ and $\tilde{v}_n(t)$ are a small velocity perturbation for $(n-1)$ and n at time t, respectively.

Next, we apply the expansion for small τ, Eq. (2) is approximated by following first-order acceleration equation.

$$\tau \dot{a}_n(t) + a_n(t) = f(v_{n-1}, y_n, v_n). \qquad (5)$$

where the dot denotes differentiation with respect to time. Due to $f(v_{n-1}, y_n, v_n)$ is sufficiently smooth, the linearization of Eq. (2) around solution (4) leads to

$$\tau \dddot{\tilde{v}}_n(t) + \ddot{\tilde{v}}_n(t) = f_{v_{n-1}} \dot{\tilde{v}}_{n-1}(t) + f_{y_n} \dot{\tilde{y}}_n(t) + f_{v_n} \dot{\tilde{v}}_n(t), \qquad (6)$$

where the partial derivatives $f_{v_{n-1}}$, f_{y_n} and f_{v_n} are evaluated at the equilibrium arguments (v^*_{n-1}, y^*, v^*_n).

Eq. (6) can be rewritten as

$$\tau \frac{d^3\tilde{v}_n(t)}{dt^3} + \frac{d^2\tilde{v}_n(t)}{dt^2} - f_{v_n}\frac{d\tilde{v}_n(t)}{dt} + f_{y_n}\tilde{v}_n(t) = f_{v_{n-1}}\frac{d\tilde{v}_{n-1}(t)}{dt} + f_{y_n}\tilde{v}_{n-1}(t),$$

(7)The Laplace transform is taken on both sides of Eq. (7), we have

$$T(\mathrm{s}) = \frac{U_n(\mathrm{s})}{U_{n-1}(\mathrm{s})}, \qquad (8)$$

where $T(\mathrm{s})$ denotes transfer function; $L(\cdot)$ represents Laplace transformation; $U_{n-1}(\mathrm{s}) = L(\tilde{v}_{n-1}(t))$ and $U_n(\mathrm{s}) = L(\tilde{v}_n(t))$ are the output and input of car following system (7), respectively. Then the transfer function $T(\mathrm{s})$ has the following form:

$$T(\mathrm{s}) = \frac{f_{v_{n-1}} \cdot s + f_{y_n}}{\tau \cdot s^3 + s^2 - f_{v_n} \cdot s + f_{y_n}}. \qquad (9)$$

To ensure the stability of the DSM model, the corresponding transfer function (9) should satisfy

$$\|T(\mathrm{s})\|_\infty = \sup_{z\in[0,+\infty)} |T(iz)| \le 1 \qquad (10)$$

So we can obtain

$$|T(iz)| = \left| \frac{f_{v_{n-1}} \cdot iz + f_{y_n}}{-i\tau \cdot z^3 - z^2 - if_{v_n} \cdot z + f_{y_n}} \right| \le 1, z \in [0,+\infty). \qquad (11)$$

The linear stability condition of the DSM model can be derived as follows:

$$\tau < \frac{d^2\tau_2}{(1-\chi_{DSM})(2\alpha v^* + 4\tau_2 d^2)}\left(\frac{d\tau_2 + v^*}{d} + \sqrt{\tau_2^2 + \frac{2\tau_2 v^*}{d} - \frac{2v^*\tau_2}{\alpha}}\right). \qquad (12)$$

where $d_n(t) = d_{n-1}(t) = d = 7.5\ \mathrm{m/s^2},\ \tau_2 = 0.15\ \mathrm{s}$.

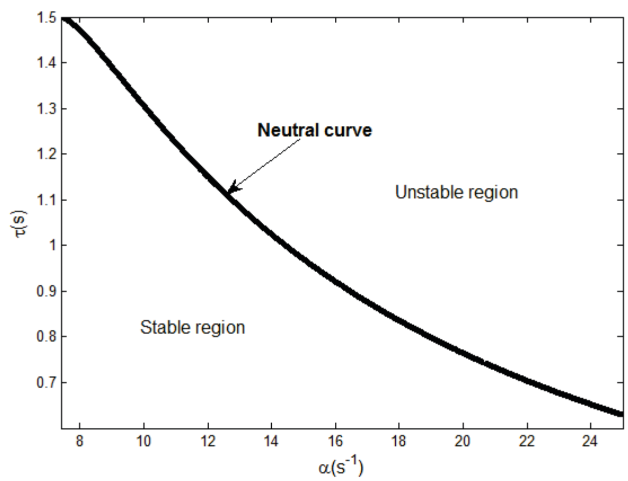


Fig. 1. Phase diagrams in (α, τ) with χ_{DSM}=0.95

From the stability condition of the DSM model as Eq. (12), one conclusion is obtained that the response time τ affects the stability of traffic flow significantly. Assuming that the leading vehicle runs at a fixed speed $v^* = 10(m/s)$, Fig.1 shows the neutral stability lines when the driver's safety margin χ_{DSM}=0.95 in the sensitivity-response time space (α, τ). In Fig.1, the sensitivity-response time space is divided into the stable region and the unstable region by the neutral curve. When (α, τ) falls to below the neutral stability curve, the DSM model is stable. Otherwise, it is unstable.

III. Numerical Simulations and Discussion

In this section, in order to check above theoretical results, numerical simulations are carried out to study the influence of response time on the stability of the DSM model.

Without loss of generality, we take $d=0.75g$, τ_2=0.15 s and the length of all vehicles is 5m. All the vehicles move on a single lane with headway L=35 m and vehicle number N=100.

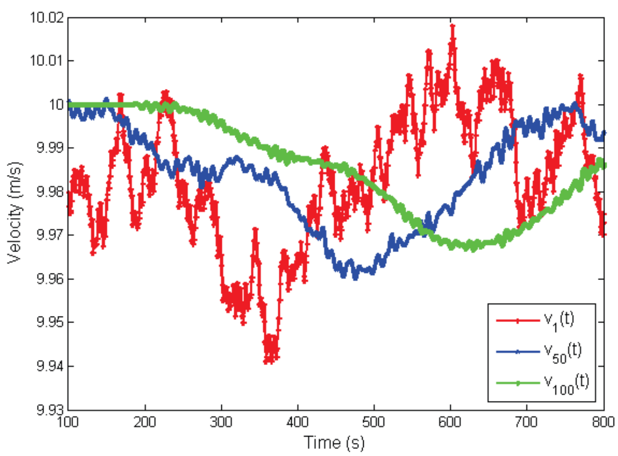
The time evolution of the velocity and headway will be investigated in the simulation initialized with the following conditions where small perturbation would be introduced.

$$\begin{cases} x_1(0)=L\cdot N, v_1(0)=10, \dot{v}_1(0)=0, \\ \dot{v}_1(t)=\varepsilon_1(\tau), t\geq\tau, \\ x_n(0)=L\cdot(N-n+1), v_{n-1}(0)=10, \dot{v}_{n-1}(0)=0, n=2,\cdots,N-1. \end{cases} \tag{13}$$

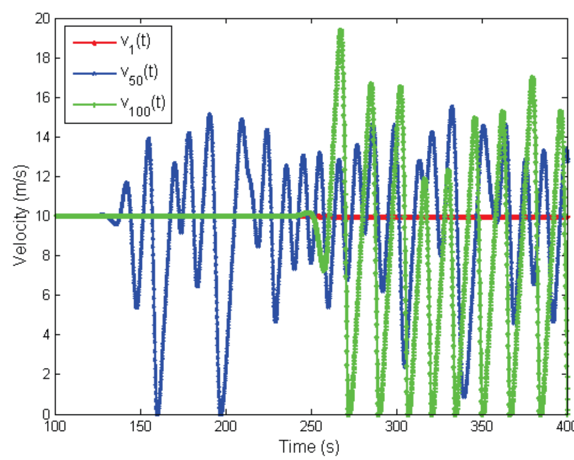
where $x_n(0)$ represents the initial position of the n th vehicle at time 0; $v_n(0)$ represents the initial velocity of the n th vehicle at time 0; $\dot{v}_n(0)$ represents the initial acceleration of the n th vehicle at time 0; $\varepsilon_1(\tau)$ is a small acceleration fluctuation of the first vehicle after time delay τ, which follows the uniform random distribution $U(-1,1)$ with amplitude 5×10^{-2}.

Firstly, to verify the stability condition (12), some numerical results based on the DSM model are given in Fig. 2. In the linear stability boundary of DSM model as Fig. 1, the Fig. 2 shows the time evolution of velocity and headway of the 1[st], 50[th] and 100[th] vehicles when $(\alpha,\tau)=(12,1)$ and $(\alpha,\tau)=(10,1.5)$, respectively.

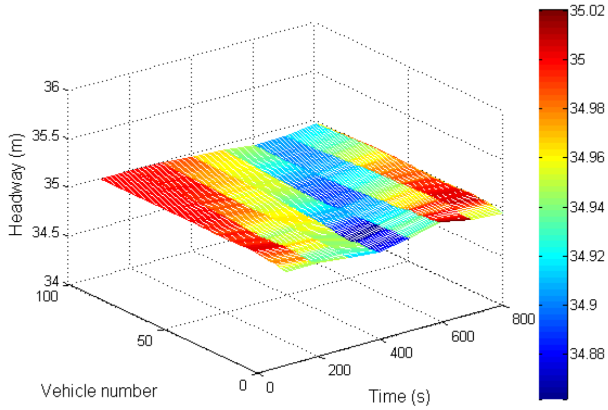
Among them, Pattern (a) and (b) in Fig. 2 corresponds to the velocity profiles of the 1[st], 50[th] and 100[th] vehicle when $(\alpha,\tau)=(12,1)$ and $(\alpha,\tau)=(10,1.5)$, and Pattern (c) and (d) in Fig. 2 display the space-time evolution of headways corresponding to the case of $(\alpha,\tau)=(12,1)$ and $(\alpha,\tau)=(10,1.5)$ with $\chi_{DSM}=0.95$, respectively. Since $(\alpha,\tau)=(12,1)$ belongs to the stability region of the DSM model, then the stability condition satisfies such that the following vehicles as the 50[th] or 100[th] vehicle almost run constantly with the initial velocity and no traffic jam occurs as shown in Fig. 2(a). Meanwhile, we can see from Fig. 2(c) that the amplitudes of the following vehicle's headway have little fluctuation. Compared with Fig. 2(a) and (c), the stability condition is unsatisfied when $(\alpha,\tau)=(10,1.5)$, and the following vehicles' velocity and headway fluctuate wildly as shown in Fig. 2(b) and (d). It indicates that a backward propagating of stop-and-go traffic jam would occur when a small disturbance is introduced into the homogeneous traffic flow. It can be concluded that the stable condition (12) holds and no traffic jam would exist if (α,τ) falls to the stable region. When (α,τ) falls to the unstable region, the stable condition (12) is unsatisfied and the density waves would appear in this situation. Therefore, it is obviously that the numerical results in Fig. 2(a)-(d) are consistent with the stable condition (12) as shown in phase diagram of stable condition.



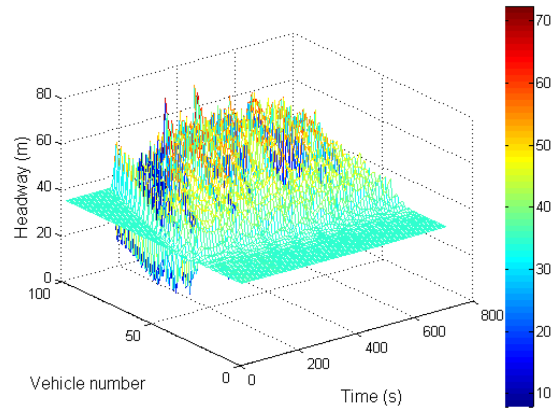
(a) $(\alpha,\tau)=(12,1)$



(b) $(\alpha,\tau)=(10,1.5)$



(c) $(\alpha,\tau)=(12,1)$

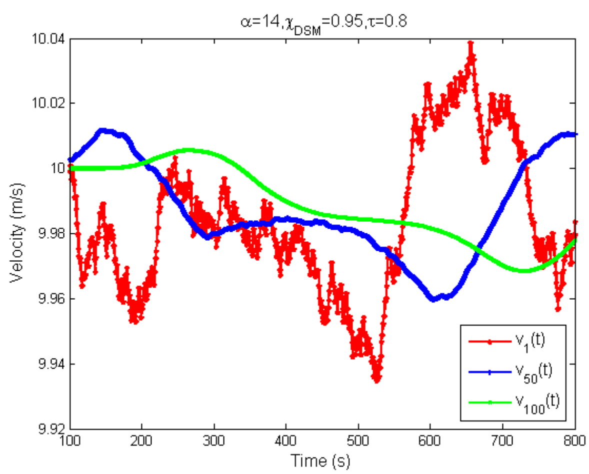


(d) $(\alpha,\tau)=(10,1.5)$

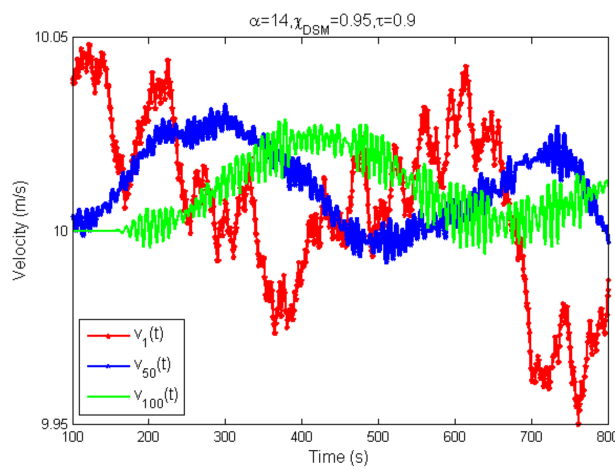
Fig. 2. The evolution of velocity and headway of three vehicles

Furthermore, we will verify the influence of the response time on the stability of traffic flow by numerical simulations. We note that the response time τ is understood as the summarized effect coming from driver's response time and mechanical response. A reasonable range of response time was permitted from actual observations. Therefore, we take response time τ as 0.8s, 0.9s, 1.2s and 1.5s to analysis the stability of DSM model under a given sensitivity factor and safety margin as $(\alpha,\chi_{DSM})=(14,0.95)$, respectively.

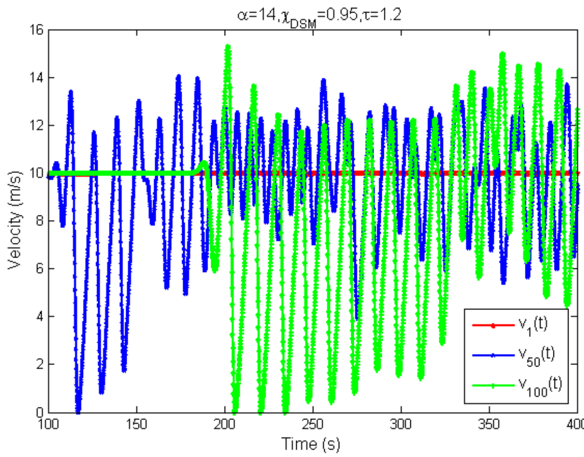
When τ is set to be a relatively larger value, that is 1.2s or 1.5s, traffic congestion clusters or rapid change of velocity would appear as shown in Fig. 3(c) and (d). However, in the cases of $\tau=0.8$ s or 0.9 s as shown in Fig. 3(a) and (b), where such patterns of congestions disappears as time goes, the stop-and-go waves may never emerge and the traffic flow becomes uniform due to the small amplitudes fluctuation of velocity in this situation. From pattern (a) and (b) in Fig. 3, we can conclude that the amplitude of the following vehicle's velocity becomes larger with the increase of the response time. That is to say, a smaller response time is more beneficial than a larger one in stabilizing traffic flow.



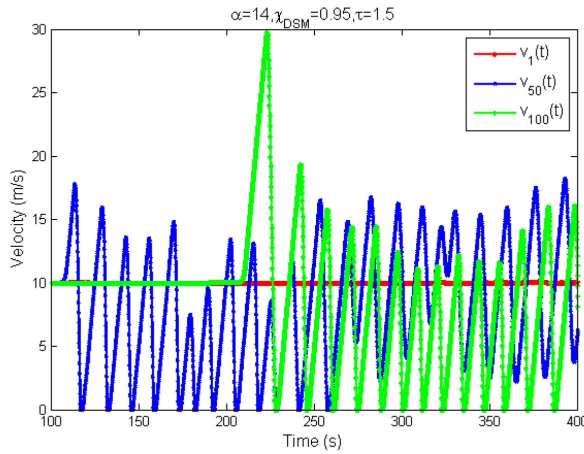
(a)



(b)



(c)



(d)

Fig. 3. The velocity profiles of the following vehicles under different response time

IV. Conclusions and Future Research

In this paper, the stability criterion of the DSM model is obtained from an analytical standpoint based on the linear stability theory. And numerical simulations are carried out to verify the analytical results. It is no doubt that the response time has significant influence on the stability of traffic flow, which may behave in different manners with different response time. A backward propagating of stop-and-go traffic jam appears with the increase of the response time and eventually leads to a inhomogeneous traffic flow. Therefore, we can arrive at the conclusion that the stability of DSM model can be maintained by a response time that is small enough. Thus the unstable flow can be stabilized by adopting the response time change in the DSM car-following model. The future work may focus on the different time-delay of each vehicle since drivers are inherently heterogeneous.

References

[1].W. Gao, Z.P. Jiang, and K. Ozbay, “Data-driven adaptive optimal control of connected vehicles,” IEEE Transactions on Intelligent Transportation Systems, vol. 99, pp. 1-12, 2016.

[2].J. Ploeg, D.P., Shukla, N.V.D., Wouw, and H., Nijmeijer, “Controller synthesis for string stability of vehicle platoons,” IEEE Transactions on Intelligent Transportation Systems, vol. 15, no. 2, pp. 854-865, 2014.

[3].W., Gao and Z.P., Jiang, “Nonlinear and adaptive suboptimal control of connected vehicles: a global adaptive dynamic programming approach,” Journal of Intelligent & Robotic Systems, pp. 1-15, 2016.

[4].L.A. Pipes, “An Operational Analysis of Traffic Dynamics,” Journal of Applied Physics, vol. 24, no. 3, pp. 274-281, 1953.

[5].R.E. Chandler and E.W. Montroll, “Traffic Dynamics: Studies in Car Following,” Operations Research, vol. 6, no. 2, pp. 165-184, 1958.

[6].D.C. Gazis, R. Herman and R.W. Rothery, “Nonlinear follow-the-leader models of traffic flow”. Operations Research, vol. 9, no. 4, pp. 545-567, 1961.

[7].M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, "Dynamical model of traffic congestion and numerical simulation," Physical Review E Statistical Physics Plasmas Fluids & Related Interdisciplinary Topics, vol. 51, p. 1035, 1995.

[8].R. Jiang, Q. Wu and Z. Zhu, “Full velocity difference model for a car-following theory,” Physical Review E Statistical Nonlinear & Soft Matter Physics, vol. 64, no. 2, 2001.

[9].T. Tang, W. Shi, H. Shang, and Y. Wang, "A new car-following model with consideration of inter-vehicle communication," Nonlinear Dynamics, vol. 76, pp. 2017-2023, 2014.

[10].L. I. Chuanyao, T. Q. Tang, H. J. Huang, and H. Y. Shang, "A New Car-Following Model with Consideration of Driving Resistance," Chinese Physics Letters, vol. 28, pp. 38902-38905(4), 2011.

[11].H.B. Zhu and S.Q. Dai, “Analysis of car-following model considering driver’s physical delay in sensing headway,” Physica A Statistical Mechanics & Its Applications, vol. 387, no. 13, pp. 3290-3298, 2008.

[12].A. Khodayari, A. Ghaffari, R. Kazemi, and R. Braunstingl, "A Modified Car-Following Model Based on a Neural Network Model of the Human Driver Effects," IEEE Transactions on Systems Man and Cybernetics - Part A Systems and Humans, vol. 42, pp. 1440-1449, 2012.

[13].L. J. Zheng, C. Tian, D. H. Sun, and W. N. Liu, "A new car-following model with consideration of anticipation driving behavior," Nonlinear Dynamics, vol. 70, pp. 1205-1211, 2012.

[14].T. Q. Tang, J. He, S. C. Yang, and H. Y. Shang, "A car-following model accounting for the driver’s attribution," Physica A Statistical Mechanics & Its Applications, vol. 413, pp. 583-591, 2014.

[15].Y. Li, L. Zhang, S. Peeta, H. Pan, T. Zheng, Y. Li, et al., "Non-lane-discipline-based car-following model considering the effects of two-sided lateral gaps," Nonlinear Dynamics, vol. 80, pp. 227-238, 2015.

[16].S. Yu and Z. Shi, “An improved car-following model considering headway changes with memory,” Physica A Statistical Mechanics & Its Applications, vol. 421, pp. 1–14, 2015.

[17].G. Lu, B. Cheng, Y. Wang, and Q. Lin, "A Car-Following Model Based on Quantified Homeostatic Risk Perception," Mathematical Problems in Engineering, vol. 2013, pp. 165-185, 2013.

[18].G. Orosz and G. Stépán, “Subcritical Hopf bifurcations in a car-following model with reaction-time delay,” Proceedings of the Royal Society A Mathematical Physical & Engineering Sciences, vol. 462, no. 2073, pp. 2643-2670, 2006.

[19]. S. Li, L. Yang, Z. Gao, and K. Li, “Stabilization strategies of a general nonlinear car-following model with varying reaction-time delay of the drivers,” Isa Transactions, vol. 53, no. 6, pp. 1739, 2014.

[20]. C. Xing, L. Yang, and Y. Zhang, “Study of Driver’s Reaction Time (DRT) during Car Following,” Applied Mechanics & Materials, vol. 713-715, pp. 2089-2092, 2015.