# Energy Detection Based Cooperative Spectrum Sensing in Cognitive Radio Networks

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Abstract—Detection performance of an energy detector used for cooperative spectrum sensing in a cognitive radio network is investigated over channels with both multipath fading and shadowing. The analysis focuses on two fusion strategies: data fusion and decision fusion. Under data fusion, upper bounds for average detection probabilities are derived for four scenarios: 1) single cognitive relay; 2) multiple cognitive relays; 3) multiple cognitive relays with direct link; and 4) multi-hop cognitive relays. Under decision fusion, the exact detection and false alarm probabilities are derived under the generalized "k-outof-n" fusion rule at the fusion center with consideration of errors in the reporting channel due to fading. The results are extended to a multi-hop network as well. Our analysis is validated by numerical and simulation results. Although this research focuses on Rayleigh multipath fading and lognormal shadowing, the analytical framework can be extended to channels with Nakagami-m multipath fading and lognormal shadowing as well.

Index Terms—Cognitive radio, cooperative spectrum sensing, data fusion, decision fusion, energy detection.

## I. INTRODUCTION

RADIO spectrum, an expensive and limited resource, is surprisingly underutilized by licensed users (*primary users*). Such spectral under-utilization has motivated cognitive radio technology which has built-in radio environment awareness and spectrum intelligence [1]. Cognitive radio enables opportunistic access to unused licensed bands. For instance, unlicensed users (*secondary users* or *cognitive users*) first sense the activities of primary users and access the spectrum holes (*white spaces*) if no primary activities are detected. While sensing accuracy is important for avoiding interference to the primary users, reliable spectrum sensing is not always guaranteed, due to the multipath fading, shadowing and hidden terminal problem. *Cooperative spectrum sensing* has thus been introduced for quick and reliable detection [2]–[5].

Among spectrum sensing techniques such as the matched filter detection (coherent detection through maximization of the signal-to-noise ratio (SNR)) [6] and the cyclostationary feature detection (exploitation of the inherent periodicity of primary signals) [7], energy detection is the most popular method addressed in the literature [8]. Measuring only the

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received signal power, the energy detector is a non-coherent detection device with low implementation complexity. The performance of an energy detector has been studied in some research efforts [9]–[14]. It performs poorly in low SNRs, and the estimation error due to noise may degrade detection performance significantly [15].

When energy detection is utilized for cooperative spectrum sensing, secondary users report to a fusion center their sensing results, in either of the following methods.

Data fusion: Each cognitive user simply amplifies the received signal from the primary user and forwards to the fusion center [2], [4], [16]. Although secondary users do not need complex detection process, the reporting channel bandwidth should be at least the same as the bandwidth of the sensed channel. At the fusion center, different fusion techniques can be applied, such as maximal ratio combining (MRC) and square-law combining (SLC) [10]. When MRC is used, channel state information (CSI) from the primary user to secondary users and from each secondary user to the fusion center is needed. When SLC is used with fixed amplification factor at each secondary user, only CSI from secondary users to the fusion center is needed. However, if variable amplification factor is applied. CSI from the primary user to secondary users and from secondary users to the fusion center is needed [17]. The framework for two-user and multiple-user cooperative spectrum sensing with data fusion was introduced in [2], [3]. However, an analytical study for the detection capability in the cooperative spectrum sensing has not been addressed.

Decision fusion: Each secondary user makes a decision (probably a binary decision) on the primary user activity, and the individual decisions are reported to the fusion center over a reporting channel (which can be with a narrow bandwidth). Capability of complex signal processing is needed at each secondary user. The fusion rule at the fusion center can be OR, AND, or Majority rule, which can be generalized as the "k-out-of-n rule". Two main assumptions are made in the literature for simplicity: 1) the reporting channel is error-free [18], [19]; and 2) the SNR statistics of the received primary signals are known at secondary users [20]. However, these assumptions are not practical in a real cognitive network. In [21], detection performance has been investigated by considering reporting errors with OR fusion rule under Rayleigh fading channels.

To fill the research gaps in cooperative spectrum sensing with data fusion and decision fusion, in this paper, we provide a rigorous analytical framework for cooperative spectrum sensing with data fusion, and investigate the detection performance with decision fusion in scenarios with reporting

errors and without SNR statistics of received primary signals. The rest of this paper is organized as follows. Section II briefly discusses preliminaries of energy detection and channel models. Sections III and IV are devoted to the analysis of cooperative spectrum sensing with data fusion and decision fusion, respectively. Section V presents our numerical and simulation results, followed by concluding remarks in Section VI.

# II. PRELIMINARIES OF ENERGY DETECTION AND CHANNEL MODELS

When a primary signal, x(t), is transmitted through a wireless channel with channel gain h, the received signal at the receiver, y(t), which follows a binary hypothesis:  $\mathcal{H}_0$  (signal absent) and  $\mathcal{H}_1$  (signal present), can be given as

$$y(t) = \begin{cases} w(t) & : \mathcal{H}_0, \\ hx(t) + w(t) & : \mathcal{H}_1, \end{cases}$$
 (1)

where w(t) is the additive white Gaussian noise (AWGN), which is assumed to be a circularly symmetric complex Gaussian random variable with mean zero and one-sided power spectral density  $N_0$  (i.e.,  $w(t) \sim \mathcal{CN}(0, N_0)$ ).

# A. Energy Detector over AWGN Channels

In energy detection, the received signal is first pre-filtered by an ideal bandpass filter which has bandwidth W, and the output of this filter is then squared and integrated over a time interval T to produce the test statistic. The test statistic  $\Lambda$  is compared with a predefined threshold value  $\lambda$  [8]. The probabilities of false alarm  $(P_f)$  and detection  $(P_d)$  can be evaluated as  $Pr(\Lambda > \lambda | \mathcal{H}_0)$  and  $Pr(\Lambda > \lambda | \mathcal{H}_1)$ , respectively, to yield [9]

$$P_f = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \tag{2}$$

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}),\tag{3}$$

where u=WT,  $\gamma$  is SNR given as  $\gamma=E_s|h|^2/N_0$ ,  $E_s$  is the power budget at the primary user,  $Q_u(\cdot,\cdot)$  is the uth order generalized Marcum-Q function,  $\Gamma(\cdot)$  is the gamma function, and  $\Gamma(\cdot,\cdot)$  is the upper incomplete gamma function. Probability of false alarm  $P_f$  can easily be calculated using (2), because it does not depend on the statistics of the wireless channel. In the sequel, detection probability is focused. The generalized Marcum-Q function can be written as a circular contour integral within the contour radius  $r \in [0,1)$ . Therefore, expression (3) can be re-written as [22]

$$P_d = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \oint_{\Omega} \frac{e^{(\frac{1}{z}-1)\gamma + \frac{\lambda}{2}z}}{z^u(1-z)} dz,\tag{4}$$

where  $\Omega$  is a circular contour of radius  $r \in [0,1)$ . The moment generating function (MGF) of received SNR  $\gamma$  is  $\mathcal{M}_{\gamma}(s) = \mathbb{E}(e^{-s\gamma})$ , where  $\mathbb{E}(\cdot)$  means expectation. Thus, the average detection probability,  $\overline{P_d}$ , is given by

$$\overline{P_d} = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \oint_{\Omega} g(z)dz,\tag{5}$$

where

$$g(z) = \mathcal{M}_{\gamma} \left( 1 - \frac{1}{z} \right) \frac{e^{\frac{\lambda}{2}z}}{z^{u}(1-z)}.$$

Since the Residue Theorem [23] in complex analysis is a powerful tool to evaluate line integrals and/or real integrals of functions over closed curves, it is applied for the integral in (5), with details given in Appendix A.

### B. Average Detection Probability with Fading and Shadowing

1) With Small Scale Fading: The Rayleigh channel model is one of the common and simple models for multipath fading. For Rayleigh fading, the MGF is  $\mathcal{M}_{\gamma}(s)=1/(1+\overline{\gamma}s)$ , where  $\overline{\gamma}$  is the average SNR. The average detection probability,  $\overline{P_d}$ , under Rayleigh fading can be written in the form of expression (5) with

$$g(z) = \frac{e^{\frac{\lambda}{2}z}}{(1+\overline{\gamma})z^{(u-1)}(1-z)(z-\frac{\overline{\gamma}}{1+\overline{\gamma}})}.$$

In radius  $r \in [0,1)$ , there are (u-1) poles at the origin (z=0) and one pole at  $z = \overline{\gamma}/(1+\overline{\gamma})$ . Thus  $\overline{P_d}$  under Rayleigh fading can be derived as

$$\overline{P_d} = \begin{cases}
e^{-\frac{\lambda}{2}} \left( \operatorname{Res}(g; 0) + \operatorname{Res}\left(g; \frac{\overline{\gamma}}{1 + \overline{\gamma}}\right) \right) : & u > 1 \\
e^{-\frac{\lambda}{2(1 + \overline{\gamma})}} : & u = 1,
\end{cases}$$
(6)

where Res (g; 0) and Res  $(g; \overline{\gamma}/(1+\overline{\gamma}))$  denote the residues of the function g(z) at the origin and at  $z = \overline{\gamma}/(1+\overline{\gamma})$ , respectively, which are evaluated in Appendix A. An alternative expression for  $\overline{P_d}$  under Rayleigh fading has been obtained in [10], which is numerically equivalent with (6).

2) With Composite Multipath Fading and Shadowing: Shadowing effect can be modeled as a lognormal distribution (for signal amplitude). The SNR of the composite Rayleigh-lognormal or Nakagami-lognormal channel model follows a gamma-lognormal distribution, which does not have a closed-form expression [24]. Therefore, we have accurately approximated the composite Rayleigh-lognormal channel model by a mixture of gamma distributions as [25]

$$f_{\gamma_N}(x) = \sum_{i=1}^N \alpha_i e^{-\zeta_i x}, \quad x \ge 0, \ \alpha_i > 0, \ \zeta_i > 0,$$
 (7)

where  $\alpha_i = \frac{w_i e^{-(\sqrt{2}\sigma t_i + \mu)}}{\sqrt{\pi} \sum_{i=1}^N w_i}$ ,  $\zeta_i = \frac{e^{-(\sqrt{2}\sigma t_i + \mu)}}{\rho}$ , N is the number of terms in the mixture,  $t_i$  and  $w_i$  are abscissas and weight factors for the Gaussian- Laguerre integration,  $\mu$  and  $\sigma$  are the mean and the standard deviation of the lognormal distribution, respectively, and  $\rho$  is the unfaded SNR defined as  $\rho \triangleq E_s/N_0$ . The MGF of  $\gamma_N$  is  $\mathcal{M}_{\gamma_N}(s) = \sum_{i=1}^N \alpha_i/(s+\zeta_i)$ . Therefore, the average detection probability,  $P_d$ , under composite multipath and shadowing can be evaluated in closed-form as

$$\overline{P_d} = \begin{cases} e^{-\frac{\lambda}{2}} \sum_{i=1}^{N} \frac{\alpha_i}{1+\zeta_i} \left( \operatorname{Res}\left(g_i; 0\right) + \operatorname{Res}\left(g_i; \frac{1}{1+\zeta_i}\right) \right) : \\ u > 1 \\ \sum_{i=1}^{N} \frac{\alpha_i}{\zeta_i} e^{-\frac{\lambda}{2} \frac{\zeta_i}{1+\zeta_i}} : u = 1, \end{cases}$$
(8)

where

$$g_i(z) = \frac{e^{\frac{\lambda}{2}z}}{z^{(u-1)}(1-z)(z-\frac{1}{1+\zeta_i})},$$

and residues Res  $(g_i; 0)$  and Res  $(g_i; 1/(1 + \zeta_i))$  are given in Appendix A. Because of possibly severe multipath fading and

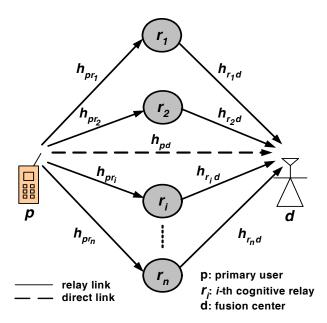


Fig. 1. Illustration of a multiple-cognitive relay network.

shadowing effects, the decision made by a single secondary user is not always reliable. In the following, cooperative spectrum sensing with data fusion and decision fusion is investigated, respectively.

# III. COOPERATIVE SPECTRUM SENSING WITH DATA FUSION

Without complex signal processing at secondary users, each secondary user acts as a relay node to the fusion center, referred to as a *cognitive relay*. This means a secondary user simply amplifies the received signal and then, forwards the amplified signal to the fusion center.

### A. Cooperative Scheme

We consider a cognitive radio network (Fig. 1), with a number n of cognitive relays (named  $r_1, r_2, ..., r_n$ ). In the first phase, all cognitive relays listen to the primary user signal. Instead of making individual hard decision about the presence or absence of the primary user, each cognitive relay amplifies and forwards the noisy version of its received signal to the fusion center in the second phase. We assume that the communication channels between cognitive relays and fusion center are orthogonal to each other (e.g., based on time division multiple access (TDMA)). In orthogonal channels, the fusion center receives independent signals from cognitive relays. The fusion center is equipped with an energy detector which compares the received signal energy with a pre-defined threshold  $\lambda$ .

# B. Single-Cognitive Relay Network

First we consider a single-cognitive relay network. In this case, we have three nodes, i.e., the primary user, the cognitive relay, and the fusion center. The cognitive relay continuously monitors the signal received from the primary user. The received signal by the cognitive relay at time t, denoted  $y_{pr}(t)$ , is given by  $y_{pr}(t) = \theta h_{pr} x(t) + w_r(t)$  where  $\theta$  denotes

the primary activity indicator, which is equal to 1 at the presence of primary activity, or equal to 0 otherwise, x(t) is the transmitted signal from the primary user with power  $E_s$ ,  $h_{pr}$  is the flat fading channel gain between the primary user and relay, and  $w_r(t)$  is the AWGN at the cognitive relay with variance  $N_{0,r}$ . Let G be the amplification factor at the cognitive relay. Thus, the received signal at the decision maker (i.e., the fusion center), denoted  $y_{rd}(t)$ , is given by

$$y_{rd}(t) = Gh_{rd}y_{pr}(t) + w_{d}(t)$$
  
=  $\theta Gh_{pr}h_{rd}x(t) + Gh_{rd}w_{r}(t) + w_{d}(t)$  (9)  
=  $\theta hx(t) + w(t)$ ,

where  $h_{rd}$  is the channel gain between the relay and the fusion center, and  $w_d(t)$  is the AWGN at the fusion center. Further,  $h=Gh_{pr}h_{rd}$  and  $w(t)=Gh_{rd}w_r(t)+w_d(t)$  can be interpreted as the equivalent channel gain between the primary user and the fusion center and the effective noise at the fusion center, respectively.

- 1) Amplification Factor: In amplify-and-forward (AF) relay communications, it can be assumed that the relay node has its own power budget  $E_r$ , and the amplification factor is designed accordingly. First the received signal power is normalized, and then it is amplified by  $E_r$ . The CSI requirement depends on the AF relaying strategy, in which two types of relays have been introduced in the literature [17], [26]:
  - Non-coherent power coefficient: the relay has knowledge of the average fading power of the channel between the primary user and itself, i.e.,  $\mathbb{E}[|h_{pr}|^2]$ , and uses it to constrain its average transmit power. Therefore, G is given as

$$G = \sqrt{\frac{E_r}{N_{0,r} + E_s \mathbb{E}[|h_{pr}|^2]}}.$$

• Coherent power coefficient: the relay has knowledge of the instantaneous CSI of the channel between the primary user and itself, i.e.,  $h_{pr}$ , and uses it to constrain its average transmit power. Therefore, G is given as

$$G = \sqrt{\frac{E_r}{N_{0,r} + E_s |h_{pr}|^2}}.$$

An advantage of the non-coherent power coefficient over the coherent one is in its less overhead, because it does not need the instantaneous CSI, which requires training and channel estimation at the relay. In this research, we use the non-coherent power coefficient which is also called as fixed-gain relay. The amplification factor G can also be written as  $G^2 = E_r/(CN_{0,r})$  where  $C = 1 + E_s \mathbb{E}[|h_{pr}|^2]/N_{0,r}$  which is a constant.

2) Receiver Structure: The same receiver structure as in reference [9] is used. The total effective noise in (9) can be modeled as  $w|_{h_{rd}} \sim \mathcal{CN}\left(0, (G^2|h_{rd}|^2+1)N_0\right)$ . The received signal is first filtered by an ideal band-pass filter. The filter limits the average noise power and normalizes the noise variance. The output of the filter is then squared and integrated over time T to form decision statistic  $\Lambda$ . Therefore, the false alarm probability and detection probability can be written as

<sup>&</sup>lt;sup>1</sup>Note that cooperative spectrum sensing is not considered in reference [9].

(2) and (3), respectively, with  $\gamma = (\gamma_{pr}\gamma_{rd})/(C+\gamma_{rd})$  where  $\gamma_{pr}$  and  $\gamma_{rd}$  are SNRs of the links from the primary user to the cognitive relay and from the cognitive relay to the fusion center, respectively.

### C. Multiple Cognitive Relays

A multiple-cognitive relay network is shown in Fig. 1. We have n cognitive relays between the primary user and the fusion center, and  $h_{pr_i}$ ,  $h_{pd}$  and  $h_{r_id}$  denote the channel gains from the primary user to the ith cognitive relay  $r_i$ , from the primary user to the fusion center, and from the ith cognitive relay  $r_i$  to the fusion center, respectively. All cognitive relays receive primary user's signal through independent fading channels simultaneously. Each cognitive relay (say relay  $r_i$ ) amplifies the received primary signal by an amplification factor  $G_{r_i}$  given as  $G_{r_i}^2 = E_{r_i}/(C_iN_{0,r_i})$  (where  $E_{r_i}$  is the power budget at relay  $r_i$ ,  $C_i = 1 + E_s\mathbb{E}[|h_{pr_i}|^2]/N_{0,r_i}$ , and  $N_{0,r_i}$  is the AWGN power at relay  $r_i$ ) and forwards to the fusion center over mutually orthogonal channels.

In [27], we consider MRC at the fusion center with known CSI. Therefore, the CSI of channels in the first hop should be forwarded to the fusion center. On the other hand, CSI may not be available for energy detection (which is noncoherent). In contrast to MRC, receiver with SLC (which is a non-coherent combiner) does not need instantaneous CSI of the channels in the first hop, and consequently results in a low complexity system. CSI of channels in the second hop is available at the filters for the noise normalization. The number n of outputs from all the branches in the SLC, denoted  $\{y_i\}_{i=1}^n$ , are combined to form the decision statistic  $\Lambda_{SLC} = \sum_{i=1}^n y_i$ . Under AWGN channels,  $\Lambda_{SLC}$  follows a central chi-square distribution with nu degrees of freedom (DoF) and unit variance under  $\mathcal{H}_0$ , and a non-central chisquare distribution with nu DoF under  $\mathcal{H}_1$ . Further, effective SNR after the combiner is  $\gamma_{SLC} = \sum_{i=1}^n \gamma_i$  where  $\gamma_i$  is the equivalent SNR of the ith relay path. The non-centrality parameter under  $\mathcal{H}_1$  is  $2\gamma_{SLC}$ . The false alarm and detection probabilities can be calculated using (2) and (3) by replacing u by nu and  $\gamma$  by  $\gamma_{SLC}$ .

#### D. Detection Analysis for a Multiple-Cognitive Relay Network

A closed-form solution for the exact average detection probability  $\overline{P_d}$  in (5) seems analytically difficult with the MGF of  $\gamma$  which is given in [26, eq. (12)]. However, efficient numerical algorithms are available to evaluate a circular contour integral. We can use MATHEMATICA or MATLAB software packages that provide adaptive algorithms to recursively partition the integration region. With a high precision level, the numerical method can provide an efficient and accurate solution for (5). In the following, we derive an upper bound of average detection probability.

1) Upper Bound of  $\overline{P_d}$ : The total SNR  $\gamma$  of a multiple-cognitive relay network can be upper bounded by  $\gamma_{\rm up}$  as  $\gamma \leq \gamma_{\rm up} = \sum_{i=1}^n \gamma_i^{\rm min}$ , where  $\gamma_i^{\rm min} = \min(\gamma_{pr_i}, \gamma_{r_id})$ , and  $\gamma_{pr_i}$  and  $\gamma_{r_id}$  are the SNRs of the links from the primary user to cognitive relay  $r_i$  and from cognitive relay  $r_i$  to the fusion center, respectively. Therefore, for independent channels,

MGF of  $\gamma_{up}$  can be written as  $\mathcal{M}_{\gamma_{up}}(s) = \prod_{i=1}^n \mathcal{M}_{\gamma_i^{\min}}(s)$  (note that  $\mathcal{M}_{\gamma_i^{\min}}(s)$  is available in [27, eq. (20)]), to yield

$$\mathcal{M}_{\gamma_{\text{up}}}(s) = \prod_{i=1}^{n} \frac{\overline{\gamma}_{pr_i} + \overline{\gamma}_{r_i d}}{\overline{\gamma}_{pr_i} \overline{\gamma}_{r_i d}} \frac{1}{\left(s + \frac{\overline{\gamma}_{pr_i} + \overline{\gamma}_{r_i d}}{\overline{\gamma}_{pr_i} \overline{\gamma}_{r_i d}}\right)}, \tag{10}$$

where  $\overline{\gamma}_{pr_i}$  and  $\overline{\gamma}_{r_id}$  are the average SNRs for links from the primary user to cognitive relay  $r_i$  and from cognitive relay  $r_i$  to the fusion center, respectively. Substituting (10) into (5), an upper bound of  $\overline{P_d}$ , denoted  $\overline{P_d^{\text{up}}}$ , can be re-written as (5) with

$$g(z) = \frac{e^{\frac{\lambda}{2}z}}{z^{u-n}(1-z)} \prod_{i=1}^{n} \frac{1-\Delta_i}{z-\Delta_i},$$

and

$$\Delta_i = \frac{\gamma_{pr_i} \gamma_{r_i d}}{\gamma_{pr_i} + \gamma_{r_i d} + \gamma_{pr_i} \gamma_{r_i d}}.$$

Two scenarios need to be considered: 1) When u > n, there are u-n poles at the origin and n poles for  $\Delta_i$ 's (i=1,..,n) in radius  $r \in [0,1)$ , and 2) when  $u \leq n$ , there are n poles at  $\Delta_i$ 's (i=1,..,n) in radius  $r \in [0,1)$ . Therefore,  $\overline{P_d^{\rm up}}$  can be derived as

$$\overline{P_d^{\text{up}}} = \begin{cases} e^{-\frac{\lambda}{2}} \left( \text{Res} \left( g; 0 \right) + \sum_{i=1}^n \text{Res} \left( g; \Delta_i \right) \right) & : u > n \\ e^{-\frac{\lambda}{2}} \sum_{i=1}^n \text{Res} \left( g; \Delta_i \right) & : u \leq n, \end{cases}$$

$$(1)$$

where Res (g; 0) and Res  $(g; \Delta_i)$  denote the residue of the function g(z) at the origin and  $\Delta_i$ , respectively. We refer readers to Appendix A for the details of the derivation of residue calculations Res $(g; \cdot)$ .

### E. Incorporation with the Direct Link

In preceding subsections, the fusion center receives only signals coming from cognitive relays. If the primary user is close to the fusion center, the fusion center can however have a strong direct link from the primary user. The direct signal can also be combined at the SLC together with the relayed signals. Then the total SNR at the fusion center can be written as  $\gamma^{\dagger} = \gamma_d + \sum_{i=1}^n \gamma_i$ , where  $\gamma_d$  is the SNR of the direct path. Assuming independent fading channels, the MGF of  $\gamma^{\dagger}$  can be written as

$$\mathcal{M}_{\gamma^{\dagger}}(s) = \mathcal{M}_{\gamma_d}(s) \prod_{i=1}^n \mathcal{M}_{\gamma_i}(s),$$

where  $\mathcal{M}_{\gamma_d}(s)$  is given by  $1/(1+\overline{\gamma}_d s)$  with  $\overline{\gamma}_d=\mathbb{E}(\gamma_d)$ , and  $M_{\gamma_i}(s)$  is given in [26, eq. (12)]. As in preceding subsections, we can find accurate average detection probability using numerical integration. An upper bound is derived in the following. In this case, g(z) in (5) can be written as

$$g(z) = \frac{(1-\Delta)e^{\frac{\lambda}{2}z}}{z^{u-n-1}(1-z)(z-\Delta)} \prod_{i=1}^{n} \frac{1-\Delta_i}{z-\Delta_i}$$
(12)

where  $\Delta = \overline{\gamma}_d/(1+\overline{\gamma}_d)$ . When u > n+1, there are u-n-1 poles at the origin, one pole at  $\Delta$  and n poles for  $\Delta_i$ 's (i=1,...,n) in radius  $r \in [0,1)$ . When  $u \leq n+1$ , there are one pole at  $\Delta$  and n poles for  $\Delta_i$ 's (i=1,...,n) in radius  $r \in [0,1)$ .

Therefore, a tight upper bound of the detection probability, denoted  $\overline{P_d^{\dagger,\mathrm{up}}}$ , can be derived in closed-form as

$$\overline{P_d^{\dagger, \text{up}}} = \begin{cases}
e^{-\frac{\lambda}{2}} \left( \text{Res}(g; 0) + \text{Res}(g; \Delta) \right) \\
+ \sum_{i=1}^n \text{Res}(g; \Delta_i) : u > n+1 \\
e^{-\frac{\lambda}{2}} \left( \text{Res}(g; \Delta) + \sum_{i=1}^n \text{Res}(g; \Delta_i) \right) : u \leq n+1.
\end{cases} (13)$$

All residues, Res (g; 0), Res  $(g; \Delta)$  and Res  $(g; \Delta_i)$ , are calculated in Appendix A.

# F. Multi-hop Cognitive Relaying

Multi-hop communication is introduced as a smart way of providing a broader coverage in wireless networks. We exploit the same idea in a cognitive radio network because the coverage area can be broadened with less power consumption. Fixed-gain non-regenerative cognitive relays are considered in Rayleigh fading channels. Channels over different hops are non-identical.<sup>2</sup>

Consider a cognitive radio network with M hops between the primary user and the fusion center. There are M-1 relays  $(r_1,\ r_2,\ ...,\ r_{M-1})$  between the primary user and the fusion center. The end-to-end SNR can be expressed as

$$\gamma = \left(\sum_{i=1}^{M} \prod_{j=1}^{i} \frac{C_{j-1}}{\gamma_j}\right)^{-1}$$

where  $\gamma_i$  is the instantaneous SNR of the ith hop and  $C_j$  is the fixed-gain constant in relay  $r_j$  and  $C_0=1$ . Further,  $\gamma$  can be upper bounded as

$$\gamma_{
m up} = \mathcal{Z}_M \prod_{i=1}^M \gamma_i^{rac{M+1-i}{M}}$$

where  $\mathcal{Z}_M=\left(\prod_{i=1}^M C_i^{-(M-i)/M}\right)/M$  [28]. In [29], the MGF of  $\gamma_{\rm up}$  is expressed using the Padé approximation method as

$$\mathcal{M}_{\gamma_{ ext{up}}}(s)\cong\sum_{i=1}^{Q}rac{\mu_{i}s^{i}}{i!}+\mathcal{O}(s^{Q+1})$$

where Q is a finite number of terms of the truncated series,

$$\mu_i = \mathcal{Z}_M^i \prod_{j=1}^M \bar{\gamma_j}^{\frac{i(M-j+1)}{M}} \Gamma\left(\frac{i(M-j+1)}{M}\right)$$

is the ith moment of  $\gamma_{\rm up}$  (here  $\bar{\gamma_j}$  is the average SNR over the jth hop), and  $\mathcal{O}(s^{Q+1})$  is the remainder of the truncated series. After applying the Padé approximation,  $\mathcal{M}_{\gamma_{\rm up}}(s)$  is given as

$$\mathcal{M}_{\gamma_{\text{up}}}(s) \cong \frac{\sum_{i=0}^{A} a_i s^i}{1 + \sum_{i=1}^{B} b_i s^i} = \sum_{i=1}^{B} \frac{q_i}{s + p_i}$$
 (14)

where A and B are specified orders of the numerator and the denominator of the Padé approximation,  $a_i$  and  $b_i$  are approximated coefficients, and  $p_i$  and  $q_i$  can be obtained based

on the second equality in (14), as detailed in [29, Sec. II-C], [30, Sec. IV].

Since  $\mathcal{M}_{\gamma_{up}}(s)$  in (14) is a sum of rational functions, an upper bound of the average detection probability can be written using (5) to yield

$$\overline{P_d^{\text{up}}} = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \sum_{i=1}^{B} \frac{q_i}{1+p_i} \oint_{\Omega} g_i(z) dz, \tag{15}$$

where

$$g_i(z) = \frac{e^{\frac{\lambda}{2}z}}{\left(z - \frac{1}{1+p_i}\right)z^{u-1}(1-z)}.$$

When u > 1, there is a pole at  $z = 1/(1 + p_i)$  and (u - 1) poles at the origin, and when u = 1, there is only a pole at  $z = 1/(1 + p_i)$  of  $g_i(z)$ . Thus,  $\overline{P_d^{\rm up}}$  can be written as

$$\overline{P_d^{\text{up}}} = \begin{cases}
e^{-\frac{\lambda}{2} \sum_{i=1}^{B} \frac{q_i}{1+p_i} \left( \text{Res} \left( g_i; 0 \right) + \text{Res} \left( g_i; \frac{1}{1+p_i} \right) \right) : \\
u > 1, \\
e^{-\frac{\lambda}{2} \sum_{i=1}^{B} \frac{q_i}{1+p_i} \text{Res} \left( g_i; \frac{1}{1+p_i} \right) : u = 1,
\end{cases}$$
(16)

where Res  $(g_i; 0)$  and Res  $(g_i; 1/(1+p_i))$  are given in Appendix A. We have also derived  $\mathcal{M}_{\gamma_{up}}(s)$  in closed-form in Appendix B. To the best of the authors' knowledge, an exact closed-form expression for  $\mathcal{M}_{\gamma_{up}}(s)$  is not available in the literature. The result will be helpful for exact numerical calculation of the upper bound for average detection probability, and other performance evaluation in multi-hop relaying.

# IV. COOPERATIVE SPECTRUM SENSING WITH DECISION FUSION

Each cognitive relay makes its own one-bit hard decision: '0' and '1' mean the absence and presence of primary activities, respectively. The one-bit hard decision is forwarded independently to the fusion center, which makes the cooperative decision on the primary activity.

# A. k-out-of-n Rule

We assume that the decision device of the fusion center is implemented with the k-out-of-n rule (i.e., the fusion center decides the presence of primary activity if there are k or more cognitive relays that individually decide the presence of primary activity). When  $k=1,\ k=n$  and  $k=\lceil n/2\rceil$ , the k-out-of-n rule represents OR rule, AND rule and Majority rule, respectively. In the following, for simplicity of presentation, we use  $p_f$  and  $p_d$  to represent false alarm and detection probabilities, respectively, for a relay, and use  $P_f$  and  $P_d$  to represent false alarm and detection probabilities, respectively, in the fusion center.

1) Reporting Channels without Errors: If the sensing channels (the channels between the primary user and cognitive relays) are identical and independent, then every cognitive relay achieves identical false alarm probability  $p_f$  and detection probability  $p_d$ . If there are error free reporting channels (the channels between the cognitive relays and the fusion center),  $P_f$  and  $P_d$  at the fusion center can be written as

$$P_{\chi} = \sum_{i=1}^{n} \binom{n}{i} (p_{\chi})^{i} (1 - p_{\chi})^{n-i}$$
 (17)

<sup>&</sup>lt;sup>2</sup>Note that when we say two channels are identical, we mean they are identically distributed.

where the notation ' $\chi$ ' means 'f' or 'd' for false alarm or detection, respectively.

2) Reporting Channels with Errors: Because of the imperfect reporting channels, errors occur on the decision bits which are transmitted by the cognitive relays. Assume bit-by-bit transmission from cognitive relays. Thus, each identical reporting channel can be modeled as a binary symmetric channel (BSC) with cross-over probability  $p_e$  which is equal to the bit error rate (BER) of the channel.

Consider the ith cognitive relay. When the primary activity is present (i.e., under  $\mathcal{H}_1$ ), the fusion center receives bit '1' from the ith cognitive relay when (1) the one-bit decision at the ith cognitive relay is '1' and the fusion center receives bit '1' from the reporting channel of the ith relay, with probability  $p_d(1-p_e)$ ; or (2) the one-bit decision at the ith cognitive relay is '0' and the fusion center receives bit '1' from the reporting channel of the ith relay, with probability  $(1 - p_d)p_e$ . On the other hand, when the primary activity is absent (i.e., under  $\mathcal{H}_0$ ), the fusion center receives bit '1' from the ith cognitive relay when (1) the one-bit decision at the ith cognitive relay is '1' and the fusion center receives bit '1' from the reporting channel of the ith relay, with probability  $p_f(1-p_e)$ ; or (2) the one-bit decision at the ith cognitive relay is '0' and the fusion center receives bit '1' from the reporting channel of the ith relay, with probability  $(1 - p_f)p_e$ . Therefore, the overall false alarm and detection probabilities with the reporting error can be evaluated as

$$P_{\chi} = \sum_{i=k}^{n} \binom{n}{i} (p_{\chi,e})^{i} (1 - p_{\chi,e})^{n-i}$$
 (18)

where  $p_{\chi,e}=p_\chi(1-p_e)+(1-p_\chi)p_e$  is the equivalent false alarm (' $\chi$ ' is 'f') or detection (' $\chi$ ' is 'd') probabilities of the ith relay.

Note that  $p_e$  is the cross-over probability of BSC. It is typically taken as a constant value (e.g.,  $p_e=10^{-1},10^{-2},10^{-3})$  in a network with AWGN channels. In our system model, we can calculate  $p_e$  analytically as BER calculation of different modulation schemes under multipath fading and shadowing effects. For binary phase shift keying (BPSK), BER can be calculated as  $p_e=\frac{1}{\pi}\int_0^{\pi/2}\mathcal{M}_{\gamma}\left(1/\sin^2\theta\right)d\theta$  to yield

$$p_e = \frac{1}{2} \left( 1 - \sqrt{\frac{\overline{\gamma}}{1 + \overline{\gamma}}} \right)$$

and

$$p_e = \frac{1}{2} \sum_{i=1}^{N} \frac{\alpha_i}{\zeta_i} \left( 1 - \sqrt{\frac{1}{1 + \zeta_i}} \right)$$

for Rayleigh fading and composite Rayleigh-lognormal fading, respectively. Here  $\alpha_i$  and  $\zeta_i$  are defined in (7).

### B. Multi-hop Cognitive Relaying

Consider a multi-hop wireless network for both identical and non-identical channels. Each cognitive relay makes a decision on the presence or absence of the primary activity and forwards the one-bit decision to the next hop. Each hop is modeled as BSC. We assume that there are M hops (i.e., (M-1) relays) between the primary user and the fusion center. A channel with (M-1) non-identically cascaded BSCs, which

is equivalent to a single BSC with 1) effective cross-over probability  $P_e$  given as (see Appendix C for the derivation)

$$P_e = \frac{1}{2} \left( 1 - \prod_{i=1}^{M-1} (1 - 2p_{e,i}) \right)$$

where  $p_{e,i}$  is the cross-over probability of the ith BSC and 2) the approximately equivalent average SNR being the average SNR of the (M-1) BSCs. A channel with (M-1) identically cascaded BSCs, which is equivalent to a single BSC with effective cross-over probability  $P_e = \frac{1}{2}[1-(1-2p_e)^{M-1}]$  and the average SNR being the average SNR of any BSC. Based on the channel gain of the equivalent single BSC, the detection and false alarm probabilities,  $p_d$  and  $p_f$ , of the BSC can be derived, similar to the derivation in Section II. Then, the detection and false alarm probabilities under a multi-hop cognitive relay network can be given as  $P_d = p_d(1-P_e) + (1-p_d)P_e$  and  $P_f = p_f(1-P_e) + (1-p_f)P_e$ , respectively.

### V. NUMERICAL AND SIMULATION RESULTS

This section provides analytical and simulation results to verify the analytical framework, and to compare the receiver operating characteristic (ROC) curves [8] of different scenarios that are presented in the previous sections. Note that each of the following figures contains both analytical result and simulation result, which are represented by lines and discrete marks, respectively.

We first show the performance of the energy detector in non-cooperative cases (as discussed in Section II), which is an important starting point of the investigation in the cooperative cases. Fig. 2 thus illustrates ROC curves for small scale fading with Rayleigh channel and composite fading (multipath and shadowing) with Rayleigh-lognormal channel. We take N=10 in (7), which makes the mean square error (MSE) between the exact gamma-lognormal channel model and the approximated mixture gamma channel model in (7) less than  $10^{-4}$ . The numerical results match well with their simulation counterparts, confirming the accuracy of the analysis. The energy detector capabilities degrade rapidly when the average SNR of the channel decreases from 10 dB to -5 dB. Further, there is a significant performance degradation of the energy detector due to the shadowing effect in higher average SNR (e.g.,  $\overline{\gamma} = 10$  dB and  $\eta = 1$ ).

Second, we focus on cooperative cases (discussed in Sections III and IV). We are interested in the impact of the number of relays on detection capability. The upper bound of average detection probability, based on (11), and simulation results are shown in Fig. 3. Note that the bound is tight for all the cases, and it is tighter when the number of relays increases. Increasing the number of cognitive relays improves the detection capability dramatically. The presence of the direct path can significantly improve the detection performance. Thus, Fig. 4 shows the impact of the direct path on the detection capability. The direct path has an average SNR value as -5 dB, -3 dB, 0 dB, 3dB or 5 dB. We consider a network with n = 1 or n = 3relays. The average SNR for other channels (from the primary user to each cognitive relay and from each cognitive relay to the fusion center) is 5 dB. When the average SNR of the direct link is improved from -5 dB to 5 dB, ROC curves move

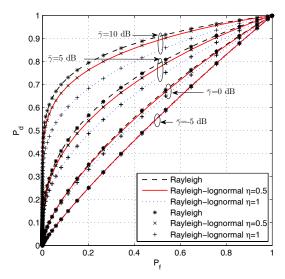


Fig. 2. ROC curves of an energy detector over Rayleigh and Rayleighlognormal fading channels.

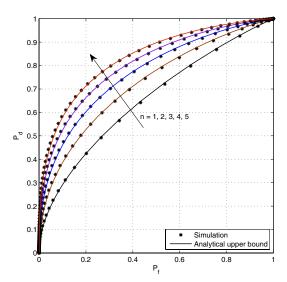


Fig. 3. ROC curves for different number of cognitive relays over Rayleigh fading channels  $\overline{\gamma}$  = 5 dB.

rapidly to the left-upper corner of the ROC plot, which means better detection capability. Therefore, it is better to utilize the direct link for spectrum identification in the mobile wireless communication networks with data fusion strategy, because there is a possibility for the fusion center and the primary user to be close to each other. The bound is much tighter when a stronger direct path exists.

Fig. 5 and Fig. 6 show the ROC curves for *k*-out-of-*n* rule in decision fusion strategy for error-free and erroneous reporting channels, respectively. Three fusion rules: OR, AND, and Majority rules, are considered. The average SNR in each link (from the primary user to each relay, and from each relay to the fusion center) is 5 dB. With error-free reporting channels, OR rule always outperforms AND and Majority rules, and Majority rule has better detection capability than AND rule. With erroneous reporting channels, the comparative performances of the three fusion rules are not as clearcut. However, OR rule outperforms AND and Majority rules in

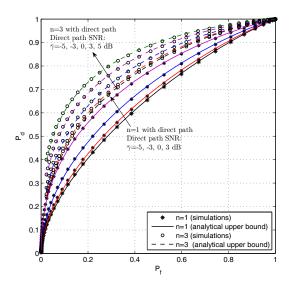


Fig. 4. ROC curves with the direct link over Rayleigh fading channels.

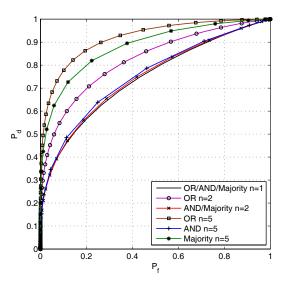


Fig. 5. ROC curves for OR, AND and Majority fusion rules with error-free reporting channels.

lower detection threshold  $(\lambda)$  values (i.e., higher  $P_d$  and  $P_f$ ). As shown in Fig. 6, when n=5, OR rule has better performance than Majority rule and AND rule when  $\lambda < 12.3$  dB and  $\lambda < 14.3$  dB, respectively. With the erroneous reporting channels, we cannot expect  $(P_f, P_d) = (1,1)$  at  $\lambda = 0$  and  $(P_f, P_d) \to (0,0)$  when  $\lambda \to \infty$  on the ROC plot. When  $\lambda = 0$ ,  $P_f = P_d = \sum_{i=k}^n \binom{n}{i} (1-p_e)^i p_e^{n-i}$ ; and when  $\lambda \to \infty$ ,  $P_f$  and  $P_d$  approaches  $\sum_{i=k}^n \binom{n}{i} p_e^i (1-p_e)^{n-i}$ . In both scenarios, the values of  $P_d$  and  $P_f$  depend only on the error probabilities of the reporting channels. We do not get reliable decision in these cases.

In Fig. 7, we consider a multi-hop cognitive relay network and its detection capability over Rayleigh fading. The average SNR in each hop is 5 dB. Note that for data fusion strategy, each ROC curve starts from (1, 1) when  $\lambda=0$  to (0, 0) when  $\lambda$  goes to infinity. On the other hand, for decision fusion strategy with erroneous reporting channels, when  $\lambda=0$ , we have  $P_d=P_f=1-P_e$ , and when  $\lambda$  goes to infinity,  $P_d$  and  $P_f$  approaches  $P_e$ . Fig. 7 shows that the detection performances of

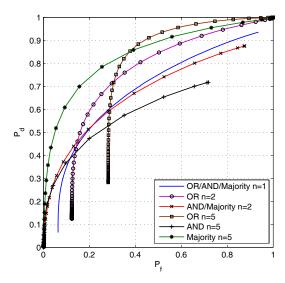


Fig. 6. ROC curves for OR, AND and the Majority fusion rules with Rayleigh faded reporting channels.

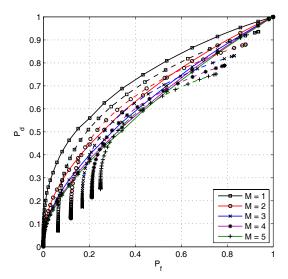


Fig. 7. ROC curves for a multi-hop cognitive relay network.

both data fusion strategy (represented by continuous lines) and decision fusion strategy (represented by dashed lines) degrade rapidly as the number of hops increases.

### VI. CONCLUSION

Detection performance of cooperative spectrum sensing is studied for data fusion and decision fusion strategies. A new set of results for the average detection probability is derived over Rayleigh and Raleigh-lognormal fading channels. For the data fusion strategy, the MGFs of received SNR of the primary user's signal at the fusion center are utilized to derive tight bounds of the average detection probability. The results show that the detection capability increases with spatial diversity due to multiple cognitive relays. Further, the direct link has a major impact on the detection probability even for low average SNRs. For the decision fusion strategy in the cooperative spectrum sensing, the generalized k-out-of-n fusion rule is considered, with particular focus on the OR, AND, and Majority rules. OR rule always outperforms AND and Majority

rules, and Majority rule has better detection capability than AND rule with error-free reporting channels. However, given a probability of reporting error, the performance is limited by the reporting error. The detection performance of both strategies degrade rapidly when the number of hops increases. With reporting errors, the ROC curve of the decision fusion strategy cannot reach (1, 1) at the right upper corner and cannot reach (0, 0) at the left lower corner. Although the Rayleigh and Rayleigh-lognormal fading channels are considered here, the same analytical framework can be extended to Nakagami-m and Nakagami-lognormal fading channels with integer fading parameter m.

In this work, cognitive relays use orthogonal channels (e.g., based on TDMA) to forward received signal to the fusion center. The channel detection time may increase with the number of cognitive relays. Note that IEEE 802.22 standard allows for the maximum channel detection time as 2 seconds. Therefore, we recommend a moderate number of cooperative relays be utilized, based on the specific maximum channel detection time requirement.

#### **APPENDIX**

# A. Calculation of $\overline{P_d}$

If g(z) has the Laurent series representation, i.e.,  $g(z) = \sum_{i=-\infty}^{\infty} a_i (z-z_0)^i$  for all z, the coefficient  $a_{-1}$  of  $(z-z_0)^{-1}$  is the residue of g(z) at  $z_0$  [23]. For g(z) given in (5), assume there are k different poles at  $z=\eta_i$  (i=1,2,...,k) and  $n_i$  poles at  $z=\eta_i$ . With the Residue Theorem,  $\overline{P_d}$  can be calculated as  $\overline{P_d}=e^{-\frac{\lambda}{2}}\sum_{i=1}^k \mathrm{Res}(g;\eta_i)$ , where

Res 
$$(g; \eta_i) = \frac{D^{n_i - 1} (g(z)(z - \eta_i)^{n_i}) \big|_{z = \eta_i}}{(n_i - 1)!},$$

and  $D^n(f(z))$  denotes the *n*th derivative of f(z) with respect to z.

1) Residues of equation (6):

$$\operatorname{Res}\left(g;0\right) = \frac{D^{u-2}\left(\frac{e^{\frac{\lambda}{2}z}}{(1-z)\left(z-\frac{\overline{\gamma}}{1+\overline{\gamma}}\right)}\right)\bigg|_{z=0}}{(1+\overline{\gamma})\left(u-2\right)!},$$

$$\operatorname{Res}\left(g;\frac{\overline{\gamma}}{1+\overline{\gamma}}\right) = \left(\frac{1+\overline{\gamma}}{\overline{\gamma}}\right)^{u-1}e^{\frac{\lambda}{2}\cdot\frac{\overline{\gamma}}{1+\overline{\gamma}}}.$$

2) Residues of equation (8):

$$\operatorname{Res}(g_i; 0) = \frac{D^{u-2} \left( \frac{e^{\frac{\lambda}{2}z}}{(1-z)\left(z - \frac{1}{1+\zeta_i}\right)} \right) \bigg|_{z=0}}{(u-2)!},$$

$$\operatorname{Res}\left(g_i; \frac{1}{1+\zeta_i}\right) = \frac{(1+\zeta_i)^u e^{\frac{\lambda}{2}\frac{1}{1+\zeta_i}}}{\zeta_i}.$$

3) Residues of equation (11):

for j = 1, ...., n.

$$\operatorname{Res}\left(g;0\right) = \frac{D^{u-n-1}\left(\frac{e^{\frac{\lambda}{2}z}}{(1-z)}\prod_{i=1}^{n}\left(\frac{1-\Delta_{i}}{z-\Delta_{i}}\right)\right)\bigg|_{z=0}}{(u-n-1)!},$$

$$\operatorname{Res}\left(g;\Delta_{j}\right) = \frac{e^{\frac{\lambda}{2}\Delta_{j}}}{\Delta_{j}^{u-n}}\prod_{i=1,i\neq j}^{n}\left(\frac{1-\Delta_{i}}{\Delta_{j}-\Delta_{i}}\right)$$

4) Residues of equation (13):

$$\operatorname{Res}\left(g;0\right) = \frac{D^{u-n-2}\left(\frac{(1-\Delta)e^{\frac{\lambda}{2}z}}{(1-z)(z-\Delta)}\prod_{i=1}^{n}\frac{1-\Delta_{i}}{z-\Delta_{i}}\right)\bigg|_{z=0}}{(u-n-2)!},$$

$$\operatorname{Res}\left(g;\Delta\right) = \frac{e^{\frac{\lambda}{2}\Delta}}{\Delta^{u-n-1}}\prod_{i=1}^{n}\frac{1-\Delta_{i}}{\Delta-\Delta_{i}},$$

$$\operatorname{Res}\left(g;\Delta_{j}\right) = \frac{(1-\Delta)e^{\frac{\lambda}{2}\Delta_{j}}}{(\Delta_{j}-\Delta)\Delta_{j}^{u-n-1}}\prod_{i=1}^{n}\prod_{i\neq j}\left(\frac{1-\Delta_{i}}{\Delta_{j}-\Delta_{i}}\right),$$

for j = 1, ...., n.

5) Residues of equation (16):

$$\operatorname{Res}(g_i; 0) = \frac{1}{(u-2)!} D^{u-2} \left( \frac{e^{\frac{\lambda}{2}z}}{(1-z)\left(z - \frac{1}{1+p_i}\right)} \right) \Big|_{z=0},$$

$$\operatorname{Res}\left(g_i; \frac{1}{1+p_i}\right) = \frac{e^{\frac{\lambda}{2}\frac{1}{1+p_i}}}{p_i(1+p_i)^{-u}}.$$

# B. Exact Closed-form Expression for $\mathcal{M}_{\gamma_{un}}(s)$ in Section III-F

With the aid of PDF of  $\gamma_{\rm up}$  given in [28, eq. (20)] and the Laplace transform given in [31, eq. (07.34.22.0003.01)],  $\mathcal{M}_{\gamma_{\rm un}}(s)$  can be evaluated in closed-form as

$$\mathcal{M}_{\gamma_{\text{up}}}(s) = \frac{\sqrt{n}\mathcal{P}}{(2\pi)^{\frac{n-1}{2}}} G_{n,v}^{v,n} \left[ \frac{\mathcal{R}n^n}{s^n} \left| \begin{array}{c} \frac{1}{n}, \frac{2}{n}, \dots \frac{n}{n} \\ \Lambda_1, \Lambda_2, \dots, \Lambda_n \end{array} \right] \right]$$
(19)

where

$$\begin{array}{rcl} v & = & \frac{n(n+1)}{2}, \\ \\ \mathcal{P} & = & \prod_{i=1}^n (2\pi)^{-\frac{n(n-1)}{4}} (n+1-i)^{-\frac{1}{2}}, \\ \\ \Lambda_i & = & \Delta(n+1-i,1), \\ \\ \mathcal{R} & = & n^n \prod_{i=1}^n C_i^{n-i} \prod_{i=1}^n (\overline{\gamma}(n+1-i))^{-(n+1-i)}, \\ \\ \Delta(\kappa,\vartheta) & \triangleq & \left(\frac{\vartheta}{\kappa},\frac{\vartheta+1}{\kappa},...,\frac{\vartheta+\kappa-1}{\kappa}\right), \end{array}$$

 $G[\cdot]$  is the Meijer's G-function, and  $\vartheta$  is real.

### C. Cascaded BSC

The transition probability matrix  $T_i$  of the ith hop can be written using the singular value decomposition as  $T_i = P^{-1}Q_iP$ , where

$$P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

$$Q_{i} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - 2p_{e,i} \end{pmatrix},$$

$$T_{i} = \begin{pmatrix} 1 - p_{e,i} & p_{e,i} \\ p_{e,i} & 1 - p_{e,i} \end{pmatrix}.$$

Then, the equivalent transition probability matrix T for n-cascaded BSCs can be evaluated as  $T = \prod_{i=1}^{n} T_i$  to yield

$$T = \frac{1}{2} \left( \begin{array}{cc} 1 + \prod_{i=1}^{n} (1 - 2p_{e,i}) & 1 - \prod_{i=1}^{n} (1 - 2p_{e,i}) \\ 1 - \prod_{i=1}^{n} (1 - 2p_{e,i}) & 1 + \prod_{i=1}^{n} (1 - 2p_{e,i}) \end{array} \right).$$

Therefore, the effective cross-over probability  $P_e$  is given as

$$P_e = \frac{1}{2} \left( 1 - \prod_{i=1}^{n} (1 - 2p_{e,i}) \right).$$

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