

Anti-Jamming Transmission Stackelberg Game With Observation Errors

Liang Xiao, Tianhua Chen, Jinliang Liu, and Huaiyu Dai

Abstract—As smart jammers that can analyze the ongoing radio transmission with flexible and powerful control on jamming signals throw serious threats on cognitive radio networks, game theory provides a powerful approach to study the interactions between smart jammers and secondary users (SUs). In this work, the power control strategy of an SU against a smart jammer under power constraints is formulated as a Stackelberg game. The jammer as the follower of the game chooses the jamming power according to the observed ongoing transmission, while the SU as the leader determines its transmit power based on the estimated jamming power. The impact of the observation accuracy of the jammer regarding the transmit power of the SU is investigated. The Stackelberg equilibrium of the anti-jamming game is derived and compared with the Nash equilibrium of the game. Simulation results show that the transmission of an SU benefits from the observation error of the jammer with a higher signal-to-interference-plus-noise ratio and utility.

Index Terms—Jamming, Stackelberg game, transmit power, observation error, Nash equilibrium.

I. INTRODUCTION

GAME THEORY as a powerful tool in strategic decision making has shown strength to address jamming attacks in wireless communications [1]. For example, the Nash equilibrium (NE) of a zero-sum jamming game in cognitive radio networks (CRNs) with perfect channel information was investigated in [2]. Anti-jamming games with inaccurate knowledge on the number of attackers and environmental parameters were analyzed in [3]. In [4], a stochastic jamming game was formulated for multi-carrier CRNs. A non-cooperative random access game was investigated to address jamming attacks with unknown system parameters in [5]. A zero-sum game with unknown jamming strategy for sub-carriers and fading channel gains was investigated in [6], while a nonzero-sum matrix game with inaccurate histories of the opponents was analyzed in [7]. The saddle-point strategy of a dynamic zero-sum game with asymmetric information between a transmitter and a jammer was provided in [8]. In [9], a Bayesian jamming game was investigated with incomplete information regarding the traffic dynamics, channel states, and the identities, costs and rewards of the opponents.

With the development of cognitive radio techniques, the emerging smart jammers are capable of learning the transmission patterns of users and adjusting their jamming strategies to

exacerbate the damage [10], thus throwing serious threats to wireless networks. Being able to describe the hierarchical behaviors among players, Stackelberg game provides a powerful method to address smart jammers. For instance, the jamming power control was formulated as a Stackelberg game in [10], in which the jamming power is chosen according to the ongoing transmit power. In addition, the Stackelberg equilibrium (SE) of a game between a primary user (PU) and a secondary user (SU) was presented in [11]. However, the existing works on anti-jamming Stackelberg games assume that the jammer can accurately measure the transmit power of the user, which does not always hold due to the uncertainty regarding the time variant channel states.

Therefore, we evaluate the impact of the observation error of a smart jammer on the performance of a Stackelberg anti-jamming game. More specifically, the power control interaction between an SU and a smart jammer in CRNs is formulated as a Stackelberg game, in which the SU as the leader first determines its transmit power while the jammer as the follower chooses its jamming power based on its observed action of the SU. Similarly to [10], the jammer has a goal of minimizing the signal-to-interference-plus-noise ratio (SINR) of the ongoing transmission. The jammer is assumed to have measurement error regarding the SU's action. The transmissions of both players are restricted by the maximum transmit powers. Moreover, the jammer also aims to maximize the power consumption of the SU with the least jamming cost. The SE of the anti-jamming game under various conditions is presented and compared with the NE in a counterpart jamming game with a non-reactive jammer. The impact of the observation error of the jammer in the game is evaluated via simulations.

The rest of the paper is organized as follows. We introduce the anti-jamming game in Section II and provide the SE and NE of the game in Section III. We present simulation results in Section IV, and draw conclusions in Section V.

II. ANTI-JAMMING TRANSMISSION STACKELBERG GAME

The power control strategies of an SU and a smart jammer in CRNs are analyzed through the Stackelberg game framework. In the anti-jamming game, the SU as the leader first transmits with power $P_s \in [0, P_s^M]$, and then the smart jammer as the follower chooses its jamming power $P_j \in [0, P_j^M]$, where P_x^M is the maximum power of Player x , with $x = s$ (i.e., SU) or j (i.e., jammer). Let h_s (or h_j) denote the channel power gain between the transmitter (or jammer) and the legitimate receiver, C_x be the transmission cost per unit power of Player x , and σ be the noise power. The presence indicator of PUs is denoted as α , which equals one in the absence of PU and zero otherwise. Similarly to [10], both players are assumed to know these system parameters, possibly by learning from the interaction history.

The SU (or jammer) takes action P_s (or P_j) to maximize its individual utility, denoted by U_s (or U_j). Unlike [10] that

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discussed generic wireless networks, we consider CRNs in which both SUs and jammers avoid interfering with the PUs, and assume that the jammers aim to deplete the SUs' battery levels. The utility functions based on the SINR at the legitimate receiver and the transmission costs are modeled as

$$U_s(P_s, P_j) = \frac{h_s P_s \alpha}{\sigma + h_j P_j} - C_s P_s \quad (1)$$

$$U_j(P_s, P_j) = -\frac{h_s P_s \alpha}{\sigma + h_j P_j} + C_s P_s - C_j P_j, \quad (2)$$

where $C_s P_s$ in U_j is introduced for the jammer with a goal of depleting the SU's battery level. α introduced here indicates the access priority of the PU. If the PU is present on the channel, i.e., $\alpha = 0$, U_s and U_j decrease with P_s and P_j respectively and then both the SU and the jammer stop transmitting. Otherwise, the SU and the jammer compete for higher individual utilities.

In summary, the anti-jamming game is denoted by $\mathbf{G} = \langle \{s, j\}, \{P_s, P_j\}, \{U_s, U_j\} \rangle$, in which the leader is the SU and the follower is the jammer; the actions of the players follow the power constraints; and the utilities of the players U_s and U_j , are defined in (1) and (2). The relative observation error of the jammer, denoted by ϵ , is defined as $\epsilon = |\hat{P}_s - P_s|/P_s$, where \hat{P}_s represents the observed transmit power of the SU.

The jammer is assumed to ignore its unknown observation error when choosing its jamming power to maximize its expected utility, which is obtained from (2) by replacing P_s with \hat{P}_s , i.e.,

$$\hat{U}_j(\hat{P}_s, P_j) = -\frac{h_s \hat{P}_s \alpha}{\sigma + h_j P_j} + C_s \hat{P}_s - C_j P_j. \quad (3)$$

Meanwhile, as the leader, the SU assumes that the jammer can exactly observe its transmit power, thus decides its transmit power by first estimating the most powerful jamming strategy \hat{P}_j with detail given below, and substituting it in (1) to obtain an estimated utility \hat{U}_s , which serves as the basis for its decision. Thus we have

$$\hat{U}_s(P_s, \hat{P}_j) = \frac{h_s P_s \alpha}{\sigma + h_j \hat{P}_j} - C_s P_s. \quad (4)$$

III. STACKELBERG EQUILIBRIUM IN THE GAME

At the SE in the anti-jamming transmission game \mathbf{G} , the SU chooses its transmit power to maximize its expected utility according to the estimated jamming strategy \hat{P}_j , while the jamming power is selected basing on the observed transmit power of the SU, \hat{P}_s . More specifically, the Stackelberg equilibrium of the game \mathbf{G} under the power constraints, denoted by (P_s^{SE}, P_j^{SE}) , are given by

$$P_s^{SE} = \arg \max_{0 \leq P_s \leq P_s^M} \hat{U}_s(P_s, \hat{P}_j) \quad (5)$$

$$P_j^{SE} = \arg \max_{0 \leq P_j \leq P_j^M} \hat{U}_j(\hat{P}_s, P_j). \quad (6)$$

Lemma 1: The optimal jamming strategy of the Stackelberg anti-jamming game \mathbf{G} is given by

$$P_j^{SE} = \begin{cases} 0, & \alpha = 0, \text{ or } \hat{P}_s \leq \frac{\sigma^2 C_j}{h_s h_j} \\ P_j^M, & \alpha = 1, \hat{P}_s \geq \frac{C_j(P_j^M h_j + \sigma)^2}{h_s h_j} \\ \frac{1}{h_j} \left(\sqrt{\frac{h_s h_j \hat{P}_s}{C_j}} - \sigma \right), & \text{otherwise.} \end{cases} \quad (7)$$

Proof: If the PU is transmitting on the channel, i.e., $\alpha = 0$, $\hat{U}_j(\hat{P}_s, P_j) = C_s \hat{P}_s - C_j P_j$ decreases with P_j and $P_j^{SE} = 0$. Otherwise, if the PU is absent, i.e., $\alpha = 1$, by (3), we have $\frac{\partial \hat{U}_j}{\partial P_j} = \frac{h_s h_j \hat{P}_s}{(P_j h_j + \sigma)^2} - C_j$, and $\frac{\partial^2 \hat{U}_j}{\partial P_j^2} = \frac{-2h_s h_j^2 \hat{P}_s}{(P_j h_j + \sigma)^3} \leq 0$. Thus $\hat{U}_j(\hat{P}_s, P_j)$ is concave with respect to P_j .

By $\partial \hat{U}_j(\hat{P}_s, \tilde{P}_j)/\partial P_j = 0$, we have $\tilde{P}_j = \frac{1}{h_j} (\sqrt{\frac{h_s h_j \hat{P}_s}{C_j}} - \sigma)$, which maximizes \hat{U}_j if $\tilde{P}_j \in (0, P_j^M)$. Thus, $P_j^{SE} = \tilde{P}_j$ if, $\frac{\sigma^2 C_j}{h_s h_j} < \hat{P}_s < \frac{C_j(P_j^M h_j + \sigma)^2}{h_s h_j}$. If $\tilde{P}_j \leq 0$, U_j decreases with P_j for $0 \leq P_j \leq P_j^M$, yielding $P_j^{SE} = 0$. If $\tilde{P}_j \geq P_j^M$, U_j increases with P_j and thus $P_j^{SE} = P_j^M$. ■

The jammer intends to interrupt the communication of the SU, and if the ongoing transmission is observed to have low power and thus small transmission rate compared with the jamming cost, i.e., $\hat{P}_s \leq \sigma^2 C_j / (h_s h_j)$, the optimal jamming strategy is to ignore the current transmission. As another extreme case, if the jamming cost is negligible compared with the powerful ongoing transmission, i.e., $\hat{P}_s \geq C_j(P_j^M h_j + \sigma)^2 / (h_s h_j)$, the optimal strategy is to apply the highest jamming power to block the ongoing transmission. Otherwise, if $\frac{\sigma^2 C_j}{h_s h_j} < \hat{P}_s < \frac{C_j(P_j^M h_j + \sigma)^2}{h_s h_j}$, the optimal strategy is to adjust the jamming power according to the observed transmit power.

Lemma 2: The optimal SU's strategy of the Stackelberg anti-jamming game \mathbf{G} is given by

$$P_s^{SE} = \begin{cases} 0, & \Pi_1 \\ \frac{h_s C_j}{4h_j C_s^2}, & \Pi_2 \\ \frac{\sigma^2 C_j}{h_s h_j}, & \Pi_3 \\ P_s^M, & \Pi_4, \end{cases} \quad (8)$$

where Condition Π_1 : $\alpha = 0$, or $C_s > \frac{h_s}{\sigma}$; Condition Π_2 : $\alpha = 1$, $\frac{h_s}{P_j^M h_j + \sigma} \leq C_s < \frac{h_s}{2\sigma}$, $C_j < \frac{4h_j C_s^2 P_s^M}{h_s}$, or $\alpha = 1$, $C_s < \min(\frac{h_s}{2\sigma}, \frac{h_s}{P_j^M h_j + \sigma})$, $C_j < \frac{4h_j C_s^2 P_s^M}{h_s}$, or $\alpha = 1$, $C_s < \min(\frac{h_s}{2\sigma}, \frac{h_s}{P_j^M h_j + \sigma})$, $\frac{4P_s^M h_j C_s}{h_s} (\frac{h_s}{P_j^M h_j + \sigma} - C_s) < C_j < \frac{4P_s^M C_s^2 h_j}{h_s}$; Condition Π_3 : $\alpha = 1$, $\max(\frac{h_s}{2\sigma}, \frac{h_s}{P_j^M h_j + \sigma}) \leq C_s \leq \frac{h_s}{\sigma}$, $C_j < \frac{P_s^M h_s h_j}{\sigma^2}$, or $\alpha = 1$, $\frac{h_s}{2\sigma} < C_s < \frac{h_s}{P_j^M h_j + \sigma}$, $\frac{P_s^M h_s h_j}{h_s \sigma - \sigma^2 C_s} (\frac{h_s}{P_j^M h_j + \sigma} - C_s) < C_j < \frac{P_s^M h_s h_j}{\sigma^2}$; and Condition Π_4 corresponds to the other case.

Proof: If $\alpha = 0$, $\hat{U}_s(P_s) = -C_s P_s$ decreases with P_s , and $P_s^{SE} = 0$. If $\alpha = 1$, by substituting \hat{P}_j in (4) with the optimal jamming strategy with accurate observation of P_s , which is given by (7) with $\hat{P}_s = P_s$, we have

$$\hat{U}_s(P_s) = \begin{cases} (\frac{h_s}{\sigma} - C_s) P_s, & P_s \leq \frac{\sigma^2 C_j}{h_s h_j} \\ \left(\frac{h_s}{P_j^M h_j + \sigma} - C_s \right) P_s, & P_s \geq \frac{C_j(P_j^M h_j + \sigma)^2}{h_s h_j} \\ \sqrt{\frac{h_s P_s C_j}{h_j}} - C_s P_s, & \text{otherwise.} \end{cases} \quad (9)$$

Let $F(P_s) = \sqrt{h_s P_s C_j / h_j} - C_s P_s$, and we have $\frac{dF}{dP_s} = 0.5 \sqrt{\frac{h_s C_j}{h_j P_s}} - C_s$ and $\frac{d^2 F}{dP_s^2} = -\frac{1}{4P_s} \sqrt{\frac{h_s C_j}{h_j P_s}} \leq 0$. Thus $F(P_s)$ is concave and is maximized by $\hat{P}_s = h_s C_j / (4h_j C_s)$ with $\max F = C_j h_s / (4h_j C_s)$.

We consider the following cases:

- 1) $C_s > \frac{h_s}{\sigma}$: As $\frac{h_s}{P_j^M h_j + \sigma} - C_s < \frac{h_s}{\sigma} - C_s < 0$, $\hat{U}_s(P_s)$ given by (9) decreases with P_s , if

$$P_s \leq \frac{\sigma^2 C_j}{h_s h_j} \quad \text{or} \quad P_s \geq \frac{C_j (P_j^M h_j + \sigma)^2}{h_s h_j}.$$

As $\tilde{P}_s < \sigma^2 C_j / (h_s h_j)$, $\hat{U}_s(P_s)$ decreases with P_s , if $\frac{\sigma^2 C_j}{h_s h_j} < P_s < \frac{C_j (P_j^M h_j + \sigma)^2}{h_s h_j}$. Thus by (5), $P_s^{SE} = 0$ (in Condition II₁).

- 2) $\frac{h_s}{P_j^M h_j + \sigma} \leq C_s \leq \frac{h_s}{\sigma}$: By (9), $\hat{U}_s(P_s)$ is non-decreasing if $P_s \leq \sigma^2 C_j / (h_s h_j)$, and is non-increasing if $P_s \geq C_j (P_j^M h_j + \sigma)^2 / (h_s h_j)$. Meanwhile, $\tilde{P}_s < C_j (P_j^M h_j + \sigma)^2 / (h_s h_j)$. If $C_s < \frac{h_s}{2\sigma}$ and $C_j < \frac{4h_j C_s^2 P_s^M}{h_s}$, we have $\frac{\sigma^2 C_j}{h_s h_j} < \tilde{P}_s < P_s^M$, and thus $P_s^{SE} = \tilde{P}_s$ (in II₂). If $C_s \geq \frac{h_s}{2\sigma}$ and $C_j < \frac{h_s h_j P_s^M}{\sigma^2}$, we have $\tilde{P}_s \leq \frac{\sigma^2 C_j}{h_s h_j} < P_s^M$, and thus $P_s^{SE} = \sigma^2 C_j / (h_s h_j)$ (in II₃). Otherwise, $P_s^{SE} = P_s^M$ (in II₄).

- 3) $C_s < \frac{h_s}{P_j^M h_j + \sigma}$: As $0 < \frac{h_s}{P_j^M h_j + \sigma} - C_s < \frac{h_s}{\sigma} - C_s$, $\hat{U}_s(P_s)$ increases with P_s , if $P_s \leq \frac{\sigma^2 C_j}{h_s h_j}$ or $P_s \geq \frac{C_j (P_j^M h_j + \sigma)^2}{h_s h_j}$.

3.1) $C_s < \frac{h_s}{2\sigma}$: As $\tilde{P}_s > \frac{\sigma^2 C_j}{h_s h_j}$, we have $P_s^{SE} = \tilde{P}_s$, if $\tilde{P}_s < P_s^M < \frac{C_j h_s}{4h_j C_s (\frac{h_s}{P_j^M h_j + \sigma} - C_s)}$, i.e., $\frac{4P_s^M h_j C_s}{h_s} (\frac{h_s}{P_j^M h_j + \sigma} - C_s) < C_j < \frac{4P_s^M C_s^2 h_j}{h_s}$ (in II₂). Otherwise, $P_s^{SE} = P_s^M$ (in II₄).

3.2) $C_s \geq \frac{h_s}{2\sigma}$: As $\tilde{P}_s \leq \frac{\sigma^2 C_j}{h_s h_j}$, we have $P_s^{SE} = \frac{\sigma^2 C_j}{h_s h_j}$, if $\frac{\sigma^2 C_j}{h_s h_j} < P_s^M < \frac{(h_s - C_s \sigma) C_j \sigma}{(\frac{h_s}{P_j^M h_j + \sigma} - C_s) h_s h_j}$, i.e.,

$$\frac{P_s^M (\frac{h_s}{P_j^M h_j + \sigma} - C_s) h_s h_j}{h_s \sigma - \sigma^2 C_s} < C_j < \frac{P_s^M h_s h_j}{\sigma^2} \quad (\text{in II}_3). \quad \text{Otherwise, } P_s^{SE} = P_s^M \quad (\text{in II}_4). \quad \blacksquare$$

The SE of the anti-jamming game \mathbf{G} , denoted by (P_s^{SE}, P_j^{SE}) , is given by Lemma 1 and Lemma 2. If the transmission cost is large enough (i.e., $C_s \geq h_s/\sigma$), the optimal SU's strategy is to stop transmitting. As another extreme case with very small transmission cost compared with the jamming cost and channel conditions (i.e., Condition II₄), the optimal SU's strategy is to maximize its power to increase the SINR. If the transmission cost is relatively small compared with the channel conditions (i.e., Condition II₃), the transmit power of the SU mostly depends on the channel conditions instead of C_s . Both the SU and jammer stop transmitting in the presence of PUs to avoid interfering with PUs.

For comparison, the Nash equilibrium of the anti-jamming game, denoted by (P_s^{NE}, P_j^{NE}) , corresponds to a non-reactive jammer without knowledge about the ongoing transmission. The NE can be viewed as the best response strategy of a static game, in which both the SU and jammer choose transmit powers simultaneously, i.e., $\forall 0 \leq P_s \leq P_s^M$ and $0 \leq P_j \leq P_j^M$,

$$U_s(P_s^{NE}, P_j^{NE}) \geq U_s(P_s, P_j^{NE}) \quad (10)$$

$$U_j(P_s^{NE}, P_j^{NE}) \geq U_j(P_s^{NE}, P_j). \quad (11)$$

Lemma 3: The NE of the anti-jamming game \mathbf{G} is given by

$$(P_s^{NE}, P_j^{NE}) = \begin{cases} (0, 0), & I_1 \\ \left(\frac{C_j h_s}{C_s^2 h_j}, \frac{1}{h_j} \left(\frac{h_s}{C_s} - \sigma \right) \right), & I_2 \\ (P_s^M, P_j^M), & I_3 \\ (P_s^M, 0), & I_4 \\ \left(P_s^M, \frac{1}{h_j} \left(\sqrt{\frac{h_s h_j P_s^M}{C_j}} - \sigma \right) \right), & I_5, \end{cases} \quad (12)$$

where the conditions are given by

$$I_1 : \alpha = 0, \text{ or } C_s > \frac{h_s}{\sigma};$$

$$I_2 : \alpha = 1, \frac{h_s}{P_j^M h_j + \sigma} \leq C_s \leq \frac{h_s}{\sigma}, C_j \leq \frac{C_s^2 P_s^M h_j}{h_s};$$

$$I_3 : \alpha = 1, C_s < \frac{h_s}{P_j^M h_j + \sigma}, C_j < \frac{h_s P_s^M h_j}{(P_j^M h_j + \sigma)^2};$$

$$I_4 : \alpha = 1, C_s < \frac{h_s}{P_j^M h_j + \sigma}, C_j > \frac{h_s P_s^M h_j}{\sigma^2},$$

$$\text{or } \frac{h_s}{P_j^M h_j + \sigma} \leq C_s \leq \min \left(\frac{h_s}{\sigma}, \sqrt{\frac{C_j h_s}{P_s^M h_j}} \right);$$

$$I_5 : \alpha = 1, C_s < \frac{h_s}{P_j^M h_j + \sigma}, \frac{h_s P_s^M h_j}{(P_j^M h_j + \sigma)^2} \leq C_j \leq \frac{h_s P_s^M h_j}{\sigma^2}.$$

Proof: When $\alpha = 0$, it's obvious that $(P_s^{NE}, P_j^{NE}) = (0, 0)$ according to the utility functions given in (1), (2). When $\alpha = 1$, by (1), (2), we have

$$\frac{\partial U_s(P_s, P_j)}{\partial P_s} = \frac{h_s}{\sigma + h_j P_j} - C_s \quad (13)$$

$$\frac{\partial U_j(P_s, P_j)}{\partial P_j} = \frac{h_s P_s h_j}{(\sigma + h_j P_j)^2} - C_j \quad (14)$$

$$\frac{\partial^2 U_j(P_s, P_j)}{\partial P_j^2} = \frac{-2h_s P_s h_j^2}{(\sigma + h_j P_j)^3} \leq 0. \quad (15)$$

Thus U_j is concave with respect to P_j . We consider the following cases:

- 1) $C_s > \frac{h_s}{\sigma}$: As $\partial U_s / \partial P_s < 0$ and $\partial U_j(P_s^{NE}, P_j) / \partial P_j = -C_j < 0$, $(0, 0)$ is the NE (in Condition I₁).

- 2) $\frac{h_s}{P_j^M h_j + \sigma} \leq C_s \leq \frac{h_s}{\sigma}$: Let $\partial U_s(P_s, \tilde{P}_j) / \partial P_s = 0$, and we have $\tilde{P}_j = \frac{h_s - \sigma C_s}{h_j C_s} \in [0, P_j^M]$. Let $\partial U_j(\tilde{P}_s, \tilde{P}_j) / \partial P_j = 0$ and we have $\tilde{P}_s = C_j h_s / (C_s^2 h_j)$. 2.1) If $\tilde{P}_s \leq P_s^M$, i.e., $C_j \leq C_s^2 P_s^M h_j / h_s$ (in I₂), according to (14) and \tilde{P}_j , we have $P_s^{NE} = \frac{C_j h_s}{C_s^2 h_j}$ and $P_j^{NE} = \frac{h_s - \sigma C_s}{h_j C_s}$. 2.2) If $\tilde{P}_s > P_s^M$, i.e., $C_j > C_s^2 P_s^M h_j / h_s$, yielding $C_s < \sqrt{C_j h_s / (P_s^M h_j)}$. According to $\frac{h_s}{P_j^M h_j + \sigma} \leq C_s \leq \frac{h_s}{\sigma}$,

we obtain a constraint in I₄. Thus we have $\frac{\partial U_j(P_s^M, P_j)}{\partial P_j} \leq 0$ and $\frac{\partial U_s(P_s, 0)}{\partial P_s} \geq 0$, indicating that $(P_s^M, 0)$ is the NE of the game (in I₄).

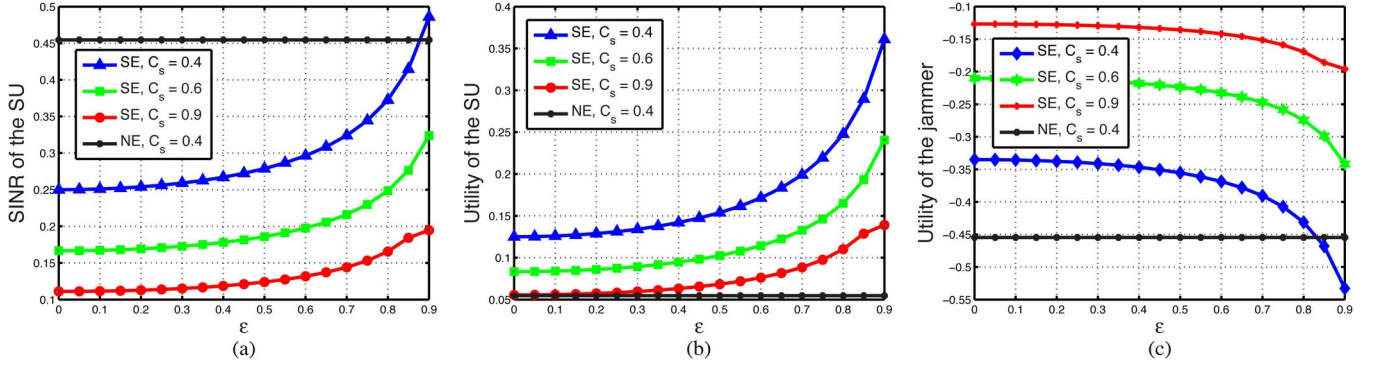


Fig. 1. Performance of the game G , vs. relative observation error of the jammer, ϵ , with $\alpha = 1$, $h_s = h_j = 0.5$, $C_j = 0.2$, $P_s^M = P_j^M = 5$ and $\sigma = 0.1$. (a) SINR of the SU; (b) Utility of the SU; (c) Utility of the jammer.

- 3) $C_s < \frac{h_s}{P_j^M h_j + \sigma}$: By (13), we have $\partial U_s / \partial P_s > 0$, yielding $P_s^{NE} = P_s^M$. 3.1) If $\frac{h_s P_s^M h_j}{(P_j^M h_j + \sigma)^2} > C_j$, i.e., $\frac{\partial U_j(P_s^{NE}, P_j)}{\partial P_j} > 0$, we have $P_j^{NE} = P_j^M$ (in I_3). 3.2) If $\frac{h_s P_s^M h_j}{\sigma^2} < C_j$, we have $\frac{\partial U_j(P_s^{NE}, P_j)}{\partial P_j} < 0$, yielding $P_j^{NE} = 0$ (in I_4). 3.3) Otherwise, $P_j^{NE} = \frac{1}{h_j} \left(\sqrt{\frac{h_s h_j P_s^M}{C_j}} - \sigma \right)$ (in I_5). ■

IV. SIMULATION RESULTS

Simulations have been performed to evaluate the anti-jamming game with $\alpha = 1$, $h_s = h_j = 0.5$, $C_j = 0.2$, $P_s^M = P_j^M = 5$ and $\sigma = 0.1$. As shown in Fig. 1(a), the SINR of the SU at the SE increases with the observation error of the jammer ϵ , because the jammer deviates from the optimal jamming power due to the estimation error of the SU's transmit power. In most cases, the SINR of the SU at the NE is larger than that at the SE, because a smart jammer can block the ongoing transmission more efficiently even with some observation error. As indicated in Fig. 1(b) and (c), the SU's utility at the SE increases with the observation error of the jammer, while the jammer's utility decreases with it. The SU at the NE has a lower utility than that at the SE, because at the latter the SU knows the existence of a jammer and utilizes its transmit power more efficiently. Similarly, in most cases a jammer obtains a higher utility at the SE than that at the NE.

V. CONCLUSION

The power control of an SU against a smart jammer with inaccurate observation of the ongoing transmission has been formulated as a Stackelberg game. The Stackelberg equilibrium and Nash equilibrium of the game under power constraints have

been compared. Simulation results show that the SU's utility increases with the relative observation error of the jammer, and the SU obtains a higher SINR if the jammer inaccurately estimates the transmit power of the SU. A jammer at the SE of the game results in a lower SINR of the SU than the jammer at the NE, even in the presence of some observation error.

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