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Effects of Communication Delay on String Stability in Vehicle Platoons

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Abstract— **The throughput of vehicles on highways can be greatly increased by forming vehicle platoons. The control law that maintains stable operation of a platoon is dependent on the lead and preceding vehicle's position, velocity and acceleration profiles. These profiles guarantee string stability of a platoon and are transmitted via wireless communication networks. Communication networks generally introduce delays and drop packets. However, these communication faults are not typically taken into account in controller designs. In this paper, we examine the robustness of current longitudinal controller designs to communication delays. The results show that string stability is seriously compromised by communication delays introduced by the network when the controllers are triggered by the receipt of either the lead vehicle information or the preceding vehicle information. We find that when all the vehicles are synchronized to update their controllers at the same time, string stability can be maintained if the delay in preceding vehicle information is small. An upper bound on the preceding vehicle information delay is derived through a simple partial fraction expansion approach. We also point out a potential problem due to the clock jitters associated with the synchronization among vehicles.**

Keywords— **communication delay, Automated Highway System(AHS), string stability, networked control**

I. Introduction

SEVERE traffic congestion is making improved highway efficiency via Automated Highway Systems (AHS) increasingly attractive. The principle motivation for AHS is increased capacity of the highways. The capacity (vehicle/lane/hour) can be increased by forming vehicle platoons and operating vehicles under automatic control at a spacing closer than is safe for human drivers [7]. Communication networks support critical information exchange between controllers in such systems. There are many control issues in an AHS design [2], [9]. In this paper, we will only look at the effects of communication delays on the string stability of the longitudinal controller. The main goal of the longitudinal controller is to safely follow its predecessor at some small given distance. The controller must not only guarantee the stability of each individual controller but also the stability of all the vehicles traveling together in the platoon, i.e., string stability. String stability requires that the maximum spacing error between vehicles in the platoon when moving away from the lead vehicle either stays constant or decreases. It was determined in [3] that for string stability the control laws must depend on both the relative position, velocity and acceleration of both the lead and preceding vehicles. These data need to be relayed via wireless communication networks. The wireless communication channel is an unpredictable and highly constrained communication medium. The scarce radio spectrum directly limits the data rates on the wireless channel. Moroever, as a signal propagates through a wireless channel, it experiences random power fluctuations over time due to changing reflections and attenuations. These power fluctuations cause time-varying data rates and intermittent connectivity. Thus, wireless communication networks inevitably introduce random delays and packet losses. Control system analysis with random delays is still an open topic [5]. In this paper, we will analyze the effects of fixed delays on string stability.

The organization of this paper is as follows. In Section 2 we describe the vehicle dynamics and use them to determine a simplified linear control model. We then review the longitudinal controller design and string stability criterion in this linear system context. In Section 3 we derive the general spacing error transfer function in the presence of communication delay. In particular, the effects of communication delay in lead and preceding vehicle information are analyzed separately when controllers are triggered by preceding vehicle information or lead vehicle information. Results show that the system goes string unstable in both cases. In Section 4 we show that the overall system can maintain string stability when all vehicles within a platoon are synchronized to update their controllers at the same time. We also derive an upper bound on the preceding vehicle information delay using a simple partial fraction expansion approach. The degradation due to clock jitter which makes perfect synchronization between vehicles impossible, is also considered in this section. In Section 5 we conclude our work and point out future research directions.

II. Controller Design and String Stability

A. Longitudinal Vehicle Model

The vehicle dynamics are highly nonlinear. However, under certain assumptions and with appropriate feedback, vehicle dynamics can be linearized. In particular, under the following assumptions:

- The intake manifold dynamics are very fast compared to the vehicle dynamics
- The torque converter is locked

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- The wheel slip is negligible
- The drive shaft is rigid

a simple vehicle dynamics model for a vehicle's longitudinal motion can be derived as [1]:

$$\dot{v}_i = k_1 T_{net}(\alpha_i, v_i) - k_2 T_L(v_i), \quad (1)$$

where T_{net} is the net engine torque and T_L, the load torque, comprises all external forces. The subscript "i" refers to the i^{th} vehicle in the platoon: v_i is the velocity of the i^{th} vehicle and α_i is its throttle angle. k_1 and k_2 reflect all terms to the vehicle's center of mass. These terms include gear ratios and moments of inertia.

The first assumption allows the net engine torque, T_{net}, to be expressed directly as a function of throttle angle α_i and engine speed ω_e. The second and third assumptions state that there is no pure rolling of the tires, therefore, the engine speed can be directly related to the vehicle's velocity by the gear ratio r_i^* as

$$v_i = r_i^* \omega_e. \quad (2)$$

If these assumptions hold an engine torque T_{net} can be produced to exactly offset the load torques and so any desired $\dot{v}_i$ can be produced. In other words, setting

$$u_i = k_1 T_{net}(\alpha_i, v_i) - k_2 T_L(v_i) \quad (3)$$

yields

$$\dot{v}_i = u_i. \quad (4)$$

This linearizes the i^{th} vehicle dynamics [8] since the i^{th} vehicle dynamics can be represented by two linear differential equations: Equation (4) and

$$\dot{x}_i = v_i, \quad (5)$$

where x_i is the position of the i^{th} vehicle.

B. Control Algorithm

The longitudinal controller under consideration is a smoothed form of sliding mode control. It is a nonlinear control approach and is known to be robust to modeling errors [2]. We define the spacing error as:

$$\varepsilon_i(t) = x_i(t) - x_{i-1}(t) + L_i, \quad (6)$$

where x_i denotes the abscissa of the rear bumper of the i^{th} vehicle and L_i is the alloted slot to i^{th} vehicle, i.e., the desired spacing between vehicle i and $i-1$ from rear bumper to rear bumper. ε_i measures the deviation in the assigned distance between vehicle i and vehicle $i-1$.

Assuming feedback information includes the relative position, velocity and acceleration of both the lead and preceding vehicles, we define

$$S_i = \dot{\varepsilon}_i + q_1\varepsilon_i + q_3(v_i - v_l) + q_4\left(x_i - x_l + \sum_{j=2}^{i} L_j\right), \quad (7)$$

where q_1, q_3 and q_4 are design parameters. S_i is a function of ε_i and we would like S_i to approach zero so that ε_i approaches zero. We set

$$\dot{S}_i = -\lambda S_i \quad (8)$$

for some $\lambda > 0$. Thus, we have a synthetic control law as

$$u_{i_d} = \frac{1}{1+q_3}[\ddot{x}_{i-1} + q_3\ddot{x}_l - (q_1+\lambda)\dot{\varepsilon}_i - q_1\lambda\varepsilon_i$$
$$-(q_4+\lambda q_3)(v_i - v_l) - \lambda q_4(x_i - x_l + \sum_{j=2}^{i} L_j)]. \quad (9)$$

Here all the variables with subscript "l" refer to the lead vehicle (the first vehicle in the platoon).

We model the actuator lag and signal processing delay as a first-order filter:

$$\tau\dot{u}_i + u_i = u_{i_d}. \quad (10)$$

τ is the time constant and we use $\tau = 0.05$. Differentiating both sides of (6), we get

$$\dot{\varepsilon}_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t) = v_i(t) - v_{i-1}(t), \quad (11)$$

$$\ddot{\varepsilon}_i(t) = \ddot{x}_i(t) - \ddot{x}_{i-1}(t) = a_i(t) - a_{i-1}(t). \quad (12)$$

Now substituting (6), (11), (12) and (4) into (10) yields

$$\tau\frac{d^3\varepsilon_i}{dt} + \ddot{\varepsilon}_i = u_{i_d} - u_{i-1_d}, \quad (13)$$

where u_{i_d} is defined by (9).

C. String Stability

String stability refers to the stability of interconnected systems. For the vehicle platoon problem, string stability requires that

$$||\varepsilon_2||_\infty > ||\varepsilon_3||_\infty > \cdots > ||\varepsilon_n||_\infty, \quad (14)$$

i.e., string stability requires that the spacing errors between vehicles attenuate moving down the platoon away from the leader [6]. Since we have linearized the system, we can use transfer function analysis to investigate string stability. In particular, we are interested in

$$G(s) = \frac{E_i(s)}{E_{i-1}(s)}, \quad (15)$$

where $E_i(s)$ is the Laplace transform of $\varepsilon_i(t)$. A well known result from linear system theory is

$$||\varepsilon_i||_\infty \le ||g(t)||_1 \cdot ||\varepsilon_{i-1}||_\infty \quad (16)$$

where $||g(t)||_1 = \int_0^\infty |g(t)|dt$. A sufficient and necessary condition that guarantees disturbances will not amplify as they propagate upstream in the platoon is

$$||g||_1 \le 1. \quad (17)$$

Another fact in linear system theory states that

$$||G||_\infty \leq ||g||_1 \quad (18)$$

where $||G||_\infty = max_w|G(jw)|$. Therefore, if $||G||_\infty > 1$ then the system is string unstable. Equality holds in (18) if $g(t)$ does not change in sign with time. If $g(t)$ is non-negative or non-positive, the equivalent frequency criterion for string stability is $||G||_\infty \leq 1$.

To examine the string stability for a system with communication delays, we first examine $||G||_\infty$. If it is greater than 1, then the system is string unstable. Otherwise, we check whether the impulse response $g(t)$ changes sign over time. If $g(t)$ is always positive(or negative), then the system is string stable. Otherwise, we need to verify (17) to determine string stability.

III. Effect of Communication Delays

A. String Stability with No Delay

Since communication networks cannot give any hard bound on delay, many networked controller designs try to minimize the amount of information exchange. However, it was shown in [3][4] that with only preceding vehicle's information (including position, velocity and acceleration), the system is string unstable independent of design parameters. Adding the lead vehicle's velocity and acceleration profiles, we can show that the system is string stable in the weak sense ($||g||_1 \geq |G(0)| = 1$ for any parameter set). After we include the lead vehicle position information, the system is made fully string stable with appropriate choice of controller parameters.

With both lead and preceding vehicle information on their position, velocity and acceleration, a synthetic control law (9) is derived through the sliding surface controller design. Then we derive the transfer function between adjacent vehicle spacing errors as

$$G(s) = \frac{\frac{1}{1+q3}(s^2 + (\lambda + q_1)s + \lambda q_1)}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}}. \quad (19)$$

Platoon stability is guaranteed if q_1, q_3, and q_4 are chosen appropriately. These parameter choices were examined in [3], where it was shown that for $\lambda = 1.0, q_1 = 0.8, q_3 = 0.5$, and $q_4 = 0.4$, $||G||_\infty < 1$. Since the impulse response $g(\tau)$ does change in sign, we numerically evaluate $||g||_1 = 0.763 < 1$. Therefore, under this choice of parameters, the system is string stable. Using a similar analysis it can be shown that with $\lambda = 0.5, q_1 = 0.72, q_3 = 0.43$, and $q_4 = 0.25$, the system is also string stable. We will use these two sets of parameters for our numerical results in the following discussion.

B. System with Communication Delay

In this section, the spacing error transfer function between vehicle i and vehicle $i-1$ is established given delays in both preceding and lead vehicle information. The time delays are defined as follows:

- $\tau_{dp}^{(i)}$ is the timing delay of the preceding vehicle information seen by vehicle i,
- $\tau_{dl}^{(i)}$ is the timing delay of the lead vehicle information seen by vehicle i.

We first derive the transfer function for the general case. By substituting (9) into (13) and taking the Laplace Transform, we get

$$H_{11}E_i(s) = \frac{1}{1+q_3}[G_1E_{i-1}(s) + G_2A_l(s) + G_3A_{i-1}(s) - G_4A_{i-2}(s)] \quad (20)$$

where

$$H_{11} = \tau s^3 + s^2 + \left(\lambda + \frac{q_1+q_4}{1+q_3}\right)s + \frac{\lambda(q_1+q_4)}{1+q_3} \quad (21)$$

$$G_1 = \lambda q_1 \quad (22)$$

$$G_2 = \frac{1}{s^2}(e^{-\tau_{dl}^{(i)}s} - e^{-\tau_{dl}^{(i-1)}s})(q_3s^2 + (q_4 + \lambda q_3)s + \lambda q_4) \quad (23)$$

$$G_3 = \frac{e^{-\tau_{dp}^{(i)}s}}{s}(s + (\lambda + q_1)) \quad (24)$$

$$G_4 = \frac{e^{-\tau_{dp}^{(i-1)}s}}{s}(s + (\lambda + q_1)). \quad (25)$$

The general solution can be manipulated to give us a transfer function of $\frac{E_i}{E_{i-1}}$. Unfortunately, the manipulation needs to be recursive due to the dependence on lead vehicle information. In order to simplify the transfer function expression, in the following discussion we break this result into two distinct cases: lead vehicle information delay and preceding vehicle information delay. This also allows us to distinguish the effects of communication delays in the lead and preceding vehicle information.

Communication delay incurred in vehicle platoons is highly dependent on the network architecture adopted and the underlying wireless channel. It also depends on how the control law is executed. If the controller is designed to update its output at the receipt of lead vehicle information, the preceding vehicle information will have longer delay, and vice versa. This is because the data sources are distributed and the capacity of the wireless channel is interference-limited. Thus, we cannot have all the vehicles transmit their data packets simultaneously and reliably. Another possibility is to install a universal clock in each vehicle in a platoon so that vehicles can synchronize to update their controllers at the same time.

In the following discussion we assume that the network architecture uses time division with token passing for multiple access. A "token" is a specific bit pattern and is circulated among all the users. A user can only transmit if he has the token. This scheme guarantees each car in the platoon an opportunity to transmit its control information

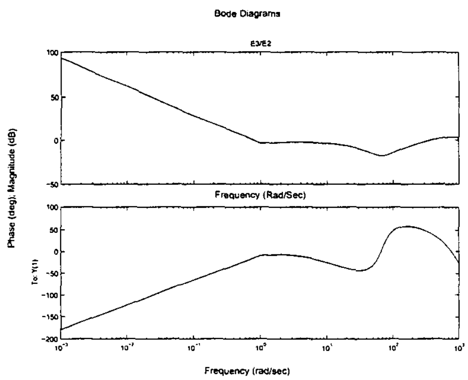

Fig. 1. Bode Plot of E_3/E_2 for Lead vehicle delay

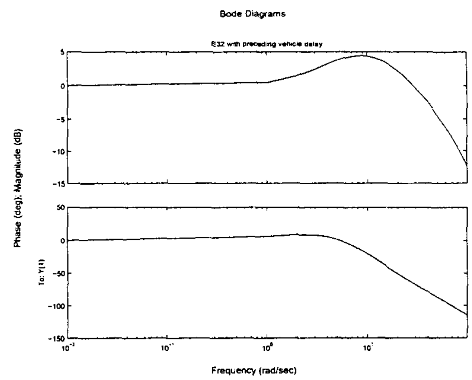

Fig. 2. Bode Plot of E_3/E_2 for preceding vehicle delay

every token cycle, which is set to be 20 milliseconds. The amount of time a user gets to transmit during a token cycle is called a time slot. The size of time slots is dependent on the platoon size, and the time slots are evenly sliced among all the vehicles in a platoon.

The first time slot is allocated to the lead vehicle. The lead vehicle broadcasts its position, velocity and acceleration plus some additional data to all the other vehicles in the platoon within its time slot. The next slot allows the second vehicle to send control messages to its follower and command messages to the lead vehicle. Then the third vehicle transmits and so on. In our analysis we assume a five-car platoon, so the size of one time slot is 4 milliseconds.

C. Lead Vehicle Delay

In this section, we consider the scenario when the control law is triggered by the receipt of preceding vehicle information. We assume that the transmission delay of the preceding vehicle information is negligible and therefore $\tau_{dp}^{(i)}$ is zero[1]. It follows that

$$E_i = HE_{i-1} + H_{3i}A_l \tag{26}$$

where

$$H = \frac{\frac{1}{1+q_3}(s^2+(\lambda+q_1)s+\lambda q_1)}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}} \tag{27}$$

and

$$H_{3i} = \frac{(e^{-\tau_{dl}^{(i)}s} - e^{-\tau_{dl}^{(i-1)}s})(q_3s^2+(q_4+\lambda q_3)s+\lambda q_4)}{(\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3})s^2}. \tag{28}$$

Since the lead vehicle is the preceding vehicle of vehicle 2, vehicle 2 does not see any delay in preceding vehicle information. Setting $i = 2$ and combining (9), (10), (11), and (12), we get

$$H_1(s) = \frac{E_2(s)}{A_l(s)} = \frac{-\tau s}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}} \tag{29}$$

[1]This is justified by the assumption that preceding vehicle information is obtained continuously by radar.

where $A_l(s)$ is the acceleration of the lead vehicle.

Setting $i = 3$ and combining (26)(27)(28)(29), we get the spacing error transfer function between vehicle 3 and vehicle 2

$$\frac{E_3(s)}{E_2(s)} = H + \frac{H_{32}}{H_1}. \tag{30}$$

For $i > 3$, we can manipulate in a similar way via recursion by noticing that

$$\frac{E_{i-1}(s)}{A_l(s)} = \frac{E_{i-1}(s)}{E_{i-2}(s)} \cdot \frac{E_{i-2}(s)}{A_l(s)}. \tag{31}$$

Since the control law is triggered once the preceding vehicle information is received, vehicle 2 sees no delay in lead vehicle information whereas vehicles 3, 4, and 5 see a 4, 8, and 12 millisecond delay, respectively, in lead vehicle information. Figure 1 is the Bode plot for $\frac{E_3(s)}{E_2(s)}$ with $\lambda = 1.0, q_1 = 0.8, q_3 = 0.5, q_4 = 0.4$, which clearly has a frequency maxima greater than 1. Bode plots for E_4/E_3 and E_5/E_4 were also generated (but not shown here) and both have a frequency maxima greater than 1. Therefore, vehicles 3, 4, 5 are not string stable with this delay.

Equation (30) actually approaches infinity as frequency approaches zero because of the double integrator term $\frac{1}{s^2}$ in (28). This seems to imply that string stability cannot hold even for infinitesimal lead vehicle information delay. This is an extremely counter-intuitive result yet to be explained.

D. Preceding Vehicle Delay

In this section, the control loop is triggered when the lead vehicle information is received. This means $\tau_{dl}^{(i)}$ equals zero and, therefore, G_2 disappears in the general transfer function (20). Then, the preceding vehicle information delay seen by vehicle 3, 4, and 5 is 16, 12, and 8 milliseconds, respectively. Manipulating the transfer function for this case is very messy, and we will omit the details here. Interested readers are referred to [4] for these details. Figure 2 shows the Bode plot for $\frac{E3}{E2}$ which has a maximum greater than 1 when $\lambda = 1.0, q_1 = 0.8, q_3 = 0.5, q_4 = 0.4$. Thus, the system is string unstable.

The two scenarios of lead and preceding vehicle information delay were simulated via Matlab and CombSim (a vehicle simulation package developed at Berkeley). Simulation results confirm the string instability caused by the lead and preceding vehicle information delay [4].

IV. A Simple Fix For Instability

The vehicle platoon is a closed-loop control system that requires highly synchronous feedback information. The asynchronous controller action with respect to lead and preceding vehicle information causes string instability. The previous section examines the string stability with a token ring communication network and longitudinal controller design for vehicle platoons. The results show that the system is string unstable when the control loop is triggered by the receipt of either the lead vehicle information or the preceding vehicle information. This seems counter-intuitive since for each individual vehicle, the optimal way for the controller to utilize sensor data is to act immediately at the receipt of any data. However, this is not true for platoons, where actions of different vehicles are coupled. If each vehicle along the platoon updates right after receipt of lead vehicle information, the error will propagate along the platoon since the vehicles father from the leader have not yet received (or acted on) this lead vehicle information. The inherent latency in the propagation of information along the string structure causes string instability. Based on this observation, we propose a simple fix for the string instability, which requires that all vehicle controllers update simultaneously.

A. Same Delay in Lead and Preceding Vehicle Information

We consider the scenario that all vehicles see the same delay in both lead vehicle information and in preceding vehicle information. This means $\tau_{dl}^{(i)} = \tau_{dl}$ and $\tau_{dp}^{(i)} = \tau_{dp}$ for all i. We can derive the transfer function under this assumption as

$$G(s) = \frac{\frac{1}{1+q_3}(\exp(-\tau_{dp}s)(s^2 + (\lambda + q_1)s) + \lambda q_1)}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}}. \qquad (32)$$

(32) indicates that the information delay in the lead vehicle information is completely cancelled out. Thus, the design criterion for the lead vehicle information delay should be derived from the controller stability criterion of each individual vehicle. We now consider how big the preceding vehicle delay can be while still maintaining string stability.

First we look at the frequency domain criterion for string stability. Figure 3 plots $||G||_\infty$ with respect to the delay τ_{dp} for the two sets of parameters. For $\lambda = 1.0, q_1 = .8, q_3 = .5, q_4 = .4$, we see from the figure that if $\tau_{dp} > 1.2$, $||G||_\infty > 1$. Therefore, the system is string unstable if the preceding vehicle delay exceeds 1.2 seconds. The other set of parameter seems to be more robust to preceding information delay since $||G||_\infty > 1$ for $\tau_{dp} > 1.33$. However, since the impulse response changes sign in time, $||G||_\infty$ is only a lower bound of $||g||_1$.

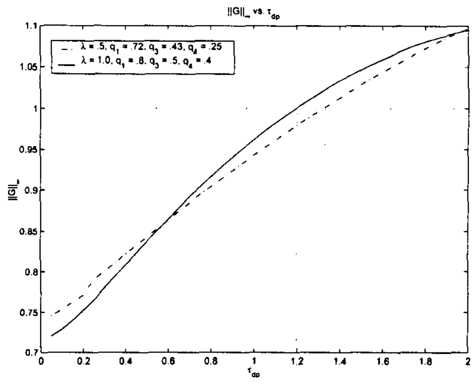

Fig. 3. $||G||_\infty$ vs. preceding vehicle delay

One way to evaluate $||g||_1$ is to numerically integrate the absolute value of the impulse response. We now consider a simple upper bound that can be used to judge the string stability criterion.

Write $G(s) = G_1(s)\exp(-\tau_{dp}s) + G_2(s)$, where

$$G_1(s) = \frac{\frac{1}{1+q3}(s^2 + (\lambda + q_1)s)}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}} = \sum_{i=1}^{3} \frac{r_i'}{s - p_i} \qquad (33)$$

$$G_2(s) = \frac{\frac{1}{1+q3}\lambda q_1}{\tau s^3 + s^2 + \frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s + \frac{\lambda(q_1+q_4)}{1+q_3}} = \sum_{i=1}^{3} \frac{r_i''}{s - p_i} \qquad (34)$$

and r_i' and r_i'' are the partial fraction expansion coefficients of $G_1(s)$ and $G_2(s)$, respectively. Therefore, the impulse response is

$$g(t) = \sum_{i=1}^{3} r_i' \exp(p_i(t - \tau_{dp}))u(t - \tau_{dp}) + \sum_{i=1}^{3} r_i'' \exp(p_i t)u(t). \qquad (35)$$

Since

$$\int_0^\infty |g(t)|dt = \int_0^{\tau_{dp}} |g(t)|dt + \int_{\tau_{dp}}^\infty |g(t)|dt, \qquad (36)$$

we can show

$$||g||_1 \leq \sum_{i=1}^{3} \left|\frac{r_i''}{p_i}\right| (1 - \exp(p_i\tau_{dp})) + \sum_{i=1}^{3} \left|\frac{r_i' + r_i'' \exp(p_i\tau_{dp})}{p_i}\right|. \qquad (37)$$

We plot this upper bound with respect to delay τ in Fig. 4 and the results show that for $\lambda = 1.0, q_1 = 0.8, q_3 = 0.5, q_4 = 0.4$, the system is stable if $\tau_{dp} < 0.075$. For parameters $\lambda = 0.5, q_1 = 0.72, q_3 = 0.43, q_4 = 0.25$, the system is string stable if $\tau_{dp} < 0.088$. Now we have provided

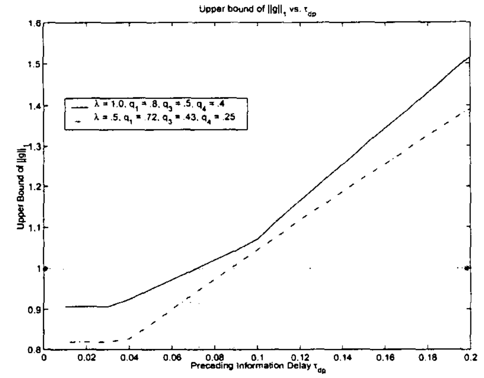

Fig. 4. Upper Bound of $||g||_1$ vs. Preceding vehicle delay

both a lower bound and an upper bound on $||g||_1$. There is a big gap between the maximal tolerable delay indicated by these two bounds: around 1.2 seconds for the lower bound but 80 milliseconds for the upper bound. The true maximum tolerable preceding vehicle delay is somewhere in between. However, the 80 millisecond bound is good enough since most current wireless LANs with a token-bus architecture have a 20 millisecond token cycle.

With the token-bus architecture, the lead vehicle information is broadcast in the first time slot and each vehicle takes turns transmitting its own information to the next vehicle in line. The scheme proposed in this section can be realized if all the vehicles do not act on the feedback information until every vehicle in the platoon gets its necessary feedback. By holding information, we improve the robustness to communication delays while sacrificing the performance of individual vehicles.

B. Clock Jitter

In practice, it is impossible to have all the vehicle controllers update at exactly the same time. Therefore, the effect of the lead vehicle information delay cannot be cancelled out completely. This is what we call the clock jitter problem. One can imagine that the implementation to force all vehicles to see the same delay in both preceding and lead vehicle information is to have all vehicles periodically update their controllers (this assumes that the communication network is reliable and delay is bounded above). Each vehicle updates its controller according to its own clock, which is not perfectly synchronized with other vehicles. This actually points out potential problems even without communication network delays. In all previous analysis, when we consider string stability under no communication delay, we make all the delay variables go to zero and their effects are cancelled in the spacing error dynamics. However, any measurement also incurs delay, not necessarily a communication delay. This kind of delay is usually very small, but it always exists.

Preliminary results using a first-order Taylor expansion for small delay jitters show that the transfer function $\frac{E_3(s)}{E_2(s)}$ has a pole on the right half plane even for very small transmission delay from the lead vehicle to the second vehicle in the platoon. Thus the clock jitter could cause string instability. This problem is currently under investigation. However, we want to emphasize that this problem is not uniquely introduced by communication delays. As a matter of fact, this kind of small delay exists even without a communication network.

V. Conclusions

We examine the effects of communication delays on the current design of longitudinal controllers in vehicle platoons. We see that string stability is seriously compromised by the communication delay introduced by the network. When all the vehicles see the same delay in both preceding and lead vehicle information, the system can maintain string stable for small preceding vehicle information delay. However, timing jitters may be a potential problem for string stability. We are currently studying the effects of clock jitters in more detail.

Our simple fix to preserve string stability is not optimal. We see that communication delays impose detrimental effects on the controller performance and may introduce instability. This comes from the fact that controller designs do not take into account the delay introduced by the communication links. In this paper we only consider a fixed communication delay, which is a very benign delay characterization for wireless networks. In practice, wireless networks inevitably introduce random delays and packet losses. In future research, we plan to examine the controller performance with random network delays and packet losses, where we hope to get some insight into controller designs that adapt to the communication delays.

Acknowledgments

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