2005 International Conference on Control and Automation (ICCA2005)
June 27-29, 2005, Budapest, Hungary

Robust String Stability Analysis of Autonomous Intelligent Vehicles

Lijuan Wu[1,2], Xue-Bo Chen[2], Yuanwei Jing[1] and Wei Wang[3]

1. Northeast University, Shenyang, 110004, P.R.China, E-mail:wuleejuan@yahoo.com.cn, ywjjing@peoplemail.com.cn
2. Anshan University of Science and Technology, Anshan, 114044, P.R.China, E-mail:xuebochen@sohu.com
3. Dalian University of Technology, Dalian, 116023, P.R.China

Abstract—**This paper is devoted to robust stable control for a platoon system with mismatched parametric uncertainty. A stable sufficient condition of the platoon of vehicles is given in terms of linear matrix inequalities, based on which the corresponding controller is also developed. The results are illustrated by an example.**

I. INTRODUCTION

An Autonomous Intelligent Vehicle is assumed to be capable of measuring necessary dynamical information from the immediate front vehicle by its on-board sensors. The computer in the vehicle processes the measured date and generates proper input actions for controlling the vehicle's movement under the constrains of safety and ride comfort [1].

Autonomous Intelligent Vehicles are spaced closely and can be called a platoon commonly. The research on a platoon of Autonomous Intelligent Vehicles has drawn much concern in recent years since they are assumed to be safe and effective methods to meet the increasing traffic demand and resultant problems such as traffic accidents, traffic congestion and air pollution. The two major advantages of the platooning system are increased safety and highway capacity resulting from the automation and close coordination of vehicles.

A platoon is one or more vehicles traveling together as a group with relatively small spacing to improve capacity and to reduce relative velocity in case of accidents [2]. Inter-platoon spacing is large enough to allow emergency stopping. By using advanced control, communication, and computing technologies, dynamic lane changing, merging, and exiting problems can solved, and it appears that platooning is most feasible [3].

Manuscript received November 25, 2004. This work was supported in part by NSFC of China under grant No.60074002 and by the USRP of Liaoning Education Department under grant No.202192057.

The platoon control structure can be decomposed into two subsets of control: longitudinal control and lateral control. Longitudinal control deals with the spacing regulation without considering the steering, while lateral control takes care of the steering to maintain the vehicle in its lane. In this paper we consider only longitudinal control, the platoon is moving in a straight line.

The platoon control system must be designed to take into account the string stability of a vehicle stream, in addition to good vehicle following performance. The 'string stability ' of a vehicle platoon under automatic control has been an active topic of research in recent decades [4].

A platoon of vehicles is said to be string stable if the spacing errors do not amplify upstream from one vehicle to another in the string. Mathematically, if the transfer function from the range error of a vehicle to that of its following vehicle has a magnitude of less than or equal to 1, then it is string stability. This work was developed by Stankovic [5] and Chen [6], in their papers, they designed a decentralized controller using LQ approach and analyzed the platoon string stability by its transfer function. Swaroop [4] studied the string stability of a countably infinite interconnected of a class of nonlinear systems. Lee [7] adopted fuzzy-sliding mode algorithm to study the string stability of platoon system.

In this paper, the string stability of a platoon system is studied based on LMI approach. The sufficient condition of stability was obtained first, then based on which the corresponding controller is also developed, and at last the unstable bounds can be obtained from it. The paper is organized as follows: The following section presents the model structure of systems. The string stability analysis method is given in section 3. Based on this method, we make a simulation in section 4. We present conclusion in Section 5.

II. MODEL STRUCTURE OF SYSTEM

In this paper, it will be supposed that the i-th vehicle in a close platoon consisting of N vehicles can be represented by the following nonlinear third-order model [5]:

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= f_i(v_i, a_i) + g_i(v_i)\eta_i \\ i &= 1, 2, \ldots, N \end{aligned} \tag{1}$$

where $d_i = x_i - x_{i-1}$ is the distance between two consecutive vehicles, x_{i-1} and x_i being their position, v_i and a_i are the velocity and acceleration respectively, while η_i is the engine input. Functions $f_i(.,.)$ and $g_i(.,.)$ are given by

$$\begin{aligned} f_i(v_i, a_i) &= -\frac{2k_{di}}{m_i} v_i a_i - \frac{1}{\tau_i(v_i)}[a_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i}], \\ g_i(v_i) &= \frac{1}{m_i \tau_i(v_i)} \end{aligned} \tag{2}$$

where m_i represent the vehicle mass, τ_i is the time-constant of the engine, K_{di} the aerodynamic drag coefficient and d_{mi} the mechanical drag. Assuming that the parameters in (2) are a priori known, we shall adopt the following control law structure:

$$\eta_i = m_i u_i + k_{di} v_i^2 + d_{mi} + 2\tau_i k_{di} v_i a_i \tag{3}$$

where u_i is the input signal. After introducing (3), the third equation in (1) becomes

$$\dot{a}_i = -\tau_i^{-1} a_i + \tau_i^{-1} u_i \tag{4}$$

Assuming that every vehicle have the same τ_i, so τ_i can be replaced by τ. Then (1) will be

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= -\tau^{-1} a_i + \tau^{-1} u_i \end{aligned} \tag{5}$$

The resulting linearized vehicle model is the basis of realizing the platoon control strategies.

We change the model by the following expressions

$$\Delta d_i = d_i - d_r, \Delta v_i = v_i - v_r, \Delta a_i = a_i - a_r \tag{6}$$

where d_r is the reference value of distance between two consecutive vehicles, v_r, a_r are the reference values of velocity and acceleration, respectively. When the vehicle is stable, we demand d_r , v_r are constant values, so a_r =0. Obviously $\Delta d_i, \Delta v_i, \Delta a_i$ are the deviation values of the corresponding values.

From (5) and (6) we get the deviation state equation of vehicle

$$\begin{aligned} \Delta\dot{d}_i &= \Delta v_{i-1} - \Delta v_i \\ \Delta\dot{v}_i &= \Delta a_i \\ \Delta\dot{a}_i &= -\tau^{-1}\Delta a_i + \tau^{-1} u_i \end{aligned} \tag{7}$$

or

$$\dot{x}_i = A_v x_i + B_v u_i$$

where

$$x_i^T = [\Delta d_i \quad \Delta v_i \quad \Delta a_i];$$

$$x_1^T = [\Delta v_1 \quad \Delta a_1];$$

$$\mathbf{A}_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{bmatrix}; \mathbf{B}_v = \begin{bmatrix} 0 \\ 0 \\ \tau^{-1} \end{bmatrix}.$$

The platoon configuration is shown in fig.1.

The following state model of the entire platoon can be formulated

$$\text{S:} \quad \begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdots \\ \dot{x}_n \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_v & 0 & \cdots & & 0 \\ \mathbf{A}_d & \mathbf{A}_v & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \mathbf{A}_d & \mathbf{A}_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{B}_v & 0 & \cdots & 0 \\ 0 & \mathbf{B}_v & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathbf{B}_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{bmatrix}. \end{aligned} \tag{8}$$

Where

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

A_d is the interconnection between the vehicles.

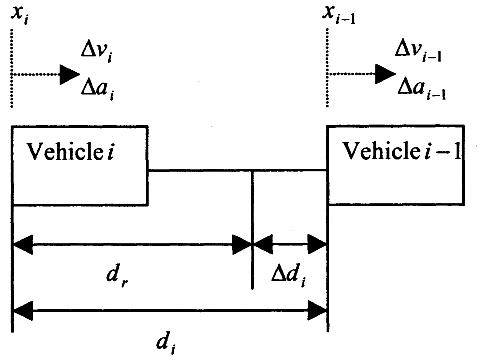


Fig.1. Platoon configuration.

III. CONTROL METHODOLOGY AND DESIGN

The platoon control system is modeled from the outset of stability analysis as an interconnection of a number of subsystems. The problem of structural perturbations arises in a natural way.

Let's consider the interconnected system [8]

$$\dot{x} = A_i x_i + B_i u_i + h_i(t,x), \quad i \in N \tag{9}$$

which is composed of N linear time-invariant subsystems, which model is the same as equation (8), we list it again in the following:

$$\dot{x}_i = A_i x_i + B_i u_i, \quad i \in N$$

where $x_i \in R^{ni}$ are the states, $u_i \in R^{mi}$ are the inputs, $h_i : R^{n+1} \to R^{ni}$ are the interconnection, and $N = \{1,2,\ldots,N\}$. The state of the overall system $x = (x_1{}^T, x_2{}^T, \cdots, x_N{}^T)^T$, $\sum_{i=1}^{N} n_i = n$, $A_i = A_v, B_i = B_v$, the subsystems are disjoint.

For the linear part of the system we require that all pairs $\{A_i, B_i\}$ be stablizable. To the nonlinear interconnection, we require that they all satisfy the quadratic constraints

$$h_i^T(t,x)h_i(t,x) \le \alpha_i^2 x^T H_i^T H_i x \tag{10}$$

where $\alpha_i > 0$ are interconnection bounds. The bounding matrices H_i are constant and are a necessary ingredient in formulating the connective stabilization problem of the overall system.

The constraints (10) can further be interpreted as

$$\|h_i(t,x)\| \le \alpha_i \|H_i x\| \tag{11}$$

where $\|\bullet\|$ is the Euclidean norm. If we define the constant matrix H_i as a block matrix

$$H_i = [H_{i1}, \ H_{i2}, \ \cdots, \ H_{iN}] \tag{12}$$

with the blocks H_{ij} compatible with the subsystems state vectors x_i, the constraints (11) can be rewritten as

$$\|h_i(t,x)\| \le \alpha_i \left\| \sum_{j=1}^{N} H_{ij} x_j \right\| \le \alpha_i \sum_{j=1}^{N} \|H_{ij}\| \|x_j\| \tag{13}$$

and arrive at the inequality

$$\|h_i(t,x)\| \le \alpha_i \sum_{j=1}^{N} \xi_{ij} \|x_j\| \tag{14}$$

which is the standard interconnection constraint with

$$\xi_{ij} = \|H_{ij}\|$$

The overall interconnected system can be rewritten in a compact form

$$\dot{x} = A_D x + B_D u + h(t,x) \tag{15}$$

where $x \in R^n$ is the state, $u \in R^m$ is the input, and

$$A_D = diag\{A_1, \ A_2, \ \cdots, \ A_N\},$$
$$B_D = diag\{B_1, \ B_2, \ \cdots, \ B_N\}$$

are constant matrices of appropriate dimensions. We assume that the subsystems are disjoint, that is,

$$x = (x_1{}^T, \ x_2{}^T, \ \cdots, \ x_N{}^T)^T$$
$$u = (u_1{}^T, \ u_2{}^T, \ \cdots, \ u_N{}^T)^T.$$

In the compact notation (15), the interconnection function

$$h : R^{n+1} \to R^n,$$
$$h = (h_1{}^T, \ h_2{}^T, \ \cdots, \ h_N{}^T)^T,$$

is constrained as

$$h^T(t,x)h(t,x) \le x^T\left(\sum_{i=1}^{N}\alpha_i^{2}H_i^{T}H_i\right)x \qquad (16)$$

Assume each subsystem is controlled by only its locally available state. The assumption implies that the i-th subsystem is controlled by the local control law:

$$u_i(x_i) = K_i x_i, \quad i \in N \qquad (17)$$

where K_i is a $m_i \times n_i$ constant matrix. The control law for the overall system has the familiar block-diagonal form

$$u(x) = K_D x \qquad (18)$$

where $K_D = diag\{K_1, K_2, \cdots, K_N\}$ is a $m \times n$ constant matrix with diagonal blocks compatible with those of A_D and B_D.

To compute the gain matrix K_D, so that the closed-loop system

$$\dot{x} = (A_D + B_D K_D)x + h(t,x). \qquad (19)$$

is robustly string stable in the large under the constraint (16) on the interconnection function $h(t,x)$, we use the change of variables

$$K_D Y_D = L_D, \qquad (20)$$

and express K_D as

$$K_D = L_D Y_D^{-1} \qquad (21)$$

Then, using the results of [9], we formulate the following optimization problem:

minimize $\sum_{i=1}^{N}\gamma_i$,

subject to $Y_D > 0$,

$$\begin{bmatrix} A_D Y_D + Y_D A_D^T + B_D L_D + L_D^T B_D^T & I & Y_D H_1^T & \cdots & Y_D H_N^T \\ I & -I & 0 & \cdots & 0 \\ H_1 Y_D & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_N Y_D & 0 & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \qquad (22)$$

where $\gamma_i = 1/\alpha_i^2$.

We have the following:

Theorem 1. The interconnected closed-loop system (15) is robustly string stable with degree vector $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N)$ by control law (18), if the problem (22) is feasible.

IV. SIMULATION

In the simulation, we set N=3 and we consider the uncertainty of the vehicle model parameters. The model of every subsystem (every vehicle) can represent as

$$\dot{x}_i = A_v x_i + A_d e(t,x)x_{i-1} + B_v u_i \qquad (23)$$

The interconnection between the i-th vehicle and $(i$-1)th vehicle is

$$h_i(t,x) = A_d e(t,x), \quad 2 \le i \le N$$
$$h_1(t,x) = 0 \qquad (24)$$

According the method described in section 3, and set the reference values of velocity and acceleration are $v_r = 20km/h, a_r = 0m/s^2$ respectively, the reference value of distance between two consecutive vehicles $d_r = 10m$. At $t = 0$, all the vehicle start with the maximum acceleration and they are positioned with zero initial headway spacing. So the beginning distance deviation is $10m$.

We can solve problem (22) to get the feedback gains of the subsystems as follow

$$K_1 = [-1.9070 \quad -0.1172]$$
$$K_2 = [2.8687 \quad -3.6291 \quad -0.2420]$$
$$K_3 = [0.0000 \quad -0.0149 \quad -0.0015]$$

Thus we get the decentralized feedback control law

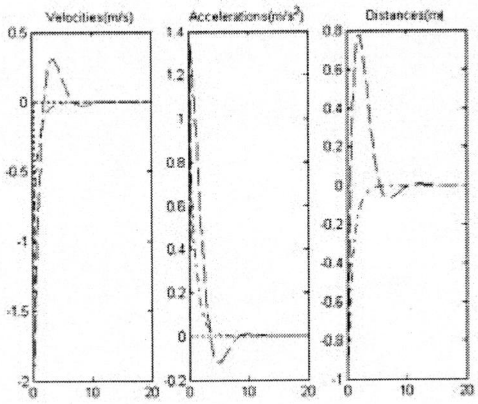
$$u_i(x_i) = K_i x_i \qquad (25)$$

At the same time, the robust stability bounds are obtained.

$$\alpha_1 = 17.8735, \alpha_2 = 9.4978, \alpha_3 = 0.3221$$

Finally, the result of the simulation is shown in Fig. 2. From the result it can be seen that the string stability is achieved.

The three lines in Fig.2 (a) and (b) represent the velocities and accelerations deviation of three vehicles, respectively. In Fig.1(c) the two lines represent the distances between the consecutive vehicles. Good velocity tracking and very small transient distance errors are achieved for each vehicle.



(a) (b) (c)

Fig.2.Three-vehicle platoon with $\tau = 0.1$

V. STRING STABILITY ANALYSIS

String stability, which ensures attenuation of errors as they propagate upstream in the platoon, is one of the most important issues related to platoon safety and performance. Until now, most studies have dealt with string stability without considering performance deviation of each vehicle in the platoon. However, it is very difficult to design each controller so that teach vehicle has the same performance. Besides, small charges in the individual vehicles can result in different velocity/acceleration/distance tracking performance of vehicles. Consequently, string stability, which is assured under the assumption of the same tracking performance, may be violated in a real platoon system.

In addition that there are different kinds of vehicles in the platoon, they may have different model structures and parameters. Designing a controller for every vehicle becomes a heavy burden to the designer. Due to the model mismatches, parametric uncertainties or disturbances, in this paper, we just deal with them as the interconnection function $h(t,x)$ between the vehicles. So far as it meets the condition of Theorem 1, the platoon is string stability.

VI. CONCLUSION

In this paper, a new methodology for control design of platoon of vehicles is proposed. It is based on LMI approach. The decentralized controllers can make the platoon system with string stability; At the same time also can get the unstable bounds. A simulation has been carried out and theresult has been given. The result shows the proposed controller achieve good velocity tracking performance and small distance error while assuring platoon safety.

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