Cooperative Spectrum Sensing for Cognitive Radios under Bandwidth Constraints

Chunhua Sun, Wei Zhang, and Khaled Ben Letaief, Fellow, IEEE

Department of Electronic and Computer Engineering

The Hong Kong University of Science and Technology

Kowloon, Hong Kong

Email: {sunnyhua, eewzhang, eekhaled}@ust.hk

Abstract—In cognitive radio systems, cooperative spectrum sensing is conducted among the cognitive users so as to detect the primary user accurately. However, when the number of cognitive users tends to be very large, the bandwidth for reporting their sensing results to the common receiver will be very huge. In this paper, we employ a censoring method with quantization to decrease the average number of sensing bits to the common receiver. By censoring the collected local observations, only the users with enough information will send their local one bit decisions (0 or 1) to the common receiver. The performance of spectrum sensing is investigated for both perfect and imperfect reporting channels. Numerical results will show that the average number of sensing bits decreases greatly at the expense of a little sensing performance loss.

I. Introduction

Cognitive radio has been under active consideration in recent years to deal with the conflict between the steady spectrum demand of unlicensed users (cognitive users) and the inefficient spectrum utilization of the licensed users (primary users) [1]–[3]. Spectrum sensing must be performed before the cognitive users access the licensed spectrum in order to limit the interference to the primary user [4]. However, due to the fading of the channels and the shadowing effects, the sensing performance for one cognitive user will be degraded. To enhance the sensing performance, cooperative spectrum sensing has been proposed [4], [5], which is usually conducted in two successive stages: sensing and reporting. In the sensing stage, every cognitive user performs spectrum sensing independently using some detection methods and gets an observation. In the reporting stage, all the local sensing observations are reported to a common receiver and then a final decision will be made to indicate the absence (H_0) or the presence (H_1) of the primary

It has been shown that cooperative spectrum sensing needs a control channel for each cognitive radio to report its sensing result and the control channel is usually bandwidth limited [4]. If every cognitive radio transmits the real value of its sensing observation, infinite bits are required and this will result in a large communication bandwidth. Quantization of local observations has attracted much research interest even though it introduces additional noise and a signal-to-noise ratio (SNR) loss at the receiver [6]. A lot of work has been done on the quantization for the signal detection but most of them focused on the optimal design of the quantizer [7], [8].

It was shown that two or three bits quantization was most appropriate without noticeable loss in the performance [9]. It has been claimed that identical binary quantization, i.e., one bit quantization, performs asymptotically optimal as the number of users goes to infinity [10]. However, when the number of cognitive radios is very large, the total number of sensing bits transmitted to the common receiver is still very huge. Recently, censoring sensors have attracted a lot of attentions in decentralized detection under communication constraints [11], [12]. In their systems, only the likelihood ratios (LR) with enough information are allowed to send to the common receiver via perfect reporting channels. However, in [11], the quantization of the LR was not considered and only the case of perfect reporting channel was studied. In [12], although the quantization and imperfect reporting channels were taken into consideration, only the special case that the probability of the presence of the primary user was sufficiently small was investigated.

In this paper, we consider cooperative spectrum sensing with 1 bit quantization. Every cognitive user will firstly obtain an observation independently and then determine the *reliability* of its information. After *censoring* their observations, only the users with reliable information are allowed to report their local binary decisions (0 or 1) to the common receiver while the others will not make any decision during the reporting stage. The performance of spectrum sensing is studied in both perfect and imperfect reporting channels. Analytical results will show that the average number of sensing bits decreases greatly with a little loss of sensing performance.

The rest of the paper is organized as follows. In Section III, the system model is briefly introduced. In Section III, the sensing performance is analyzed for both perfect and imperfect reporting channels. The simulation results are shown in Section IV. Finally, we draw our conclusions in Section V.

II. SYSTEM MODEL

In cognitive radio systems, cooperative spectrum sensing has been widely used to detect the primary user with a high agility and accuracy. Every cognitive user conducts its individual spectrum sensing using some detection method and then sends a binary local decision to the common receiver. Usually, the local decision is made by comparing the observation with a pre-fixed threshold. For example, the energy

1

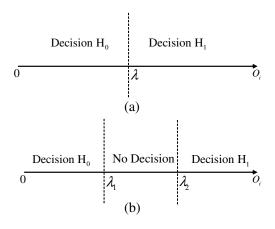


Fig. 1. (a) Conventional detection method with one threshold for the *i*th user. (b) Censoring detection method with bi-thresholds for the *i*th user.

detection for the *i*th cognitive user is illustrated in Fig. 1 (a). When the collected energy O_i exceeds the threshold λ , decision H_1 will be made which assumes that the primary user is present. Otherwise, decision H_0 will be made. Compared to the conventional method, the system model of our interest is shown in Fig. 1 (b). Two thresholds λ_1 and λ_2 are used to measure the reliability of the collected energy. "Decision H_0 " and "Decision H_1 " represent the absence and the presence of primary user, respectively. "No Decision" means that the observation is not reliable enough and the *i*th cognitive user will send nothing to the common receiver.

Based on the above censoring method, cooperative spectrum sensing can be performed as follows

1) Every cognitive user i, for $i=1,\cdots,N$, conducts spectrum sensing individually and collects the energy O_i . If the energy O_i is located in the "No Decision" region, i.e., $\lambda_1 < O_i < \lambda_2$, then the ith cognitive user sends nothing. Otherwise, it will report to the common receiver a local decision D_i , which is given by

$$D_i = \begin{cases} 0, & 0 \le O_i \le \lambda_1, \\ 1, & O_i \ge \lambda_2. \end{cases} \tag{1}$$

Note that given an instantaneous SNR γ , O_i follows the distribution [13]

$$f(O|\gamma) \sim \begin{cases} \chi_{2u}^2, & H_0, \\ \chi_{2u}^2(2\gamma), & H_1, \end{cases}$$
 (2)

where γ is exponentially distributed with the mean value $\bar{\gamma}$, u is the time bandwidth product of the energy detector, χ^2_{2u} represents a central chi-square distribution with 2u degrees of freedom and $\chi^2_{2u}(2\gamma)$ represents a non-central chi-square distribution with 2u degrees of freedom and a non-centrality parameter 2γ .

2) Assume that the common receiver receives K out of N local decisions reported from the cognitive users, it then makes a final decision H according to the fusion

function ψ , as follows

$$H = \psi(\hat{D}_1, \hat{D}_2, \cdots, \hat{D}_K), \tag{3}$$

where $\hat{D}_1, \hat{D}_2, \dots, \hat{D}_K$ denote the decoded signals of D_1, D_2, \dots, D_K after passing through the reporting channels, respectively.

To further limit the interference to the primary user, the spectrum is assumed to be available only when all the K reporting decisions are 0. Thus, $\overline{\text{OR-rule}}$ is used in the common receiver.

Let \bar{K} denote the normalized average number of sensing bits, i.e.,

$$\bar{K} = \frac{K_{avg}}{N}, \tag{4}$$

where K_{avg} is the average number of sensing bits. Let T_K and $\bar{T}_{(N-K)}$ represent the event that there are K cognitive users reporting and (N-K) users not reporting to the common receiver, respectively. Then $P\{T_K\} = (1-P\{\lambda_1 < O < \lambda_2\})^K$ and $P\{\bar{T}_{(N-K)}\} = P\{\lambda_1 < O < \lambda_2\}^{(N-K)}$, where $P\{\cdot\}$ stands for the probability. Further let $P_0 = P\{H_0\}$ and $P_1 = P\{H_1\}$. Then, K_{avg} can be calculated as

$$K_{avg} = P_0 \sum_{K=1}^{N} K \binom{N}{K} P\{T_K | H_0\} P\{\bar{T}_{(N-K)} | H_0\} + P_1 \sum_{K=1}^{N} K \binom{N}{K} P\{T_K | H_1\} P\{\bar{T}_{(N-K)} | H_1\}.$$
 (5)

Consequently, the normalized average number of sensing bits is

$$\bar{K} = 1 - P_0 \Delta_0 - P_1 \Delta_1,\tag{6}$$

where $\Delta_0 = P\{\lambda_1 < O < \lambda_2 | H_0\}$ and $\Delta_1 = P\{\lambda_1 < O < \lambda_2 | H_1\}$ represent the probability of "No Decision" for one cognitive user under hypothesis H_0 and H_1 , respectively.

From (6), it can be seen that, due to the factors Δ_0 and Δ_1 the normalized average number of sensing bits \bar{K} is always smaller than 1. Therefore, in our proposed method, the average number of sensing bits of the cooperative spectrum sensing is decreased as opposed to that of the conventional method.

III. SPECTRUM SENSING PERFORMANCE ANALYSIS

In this section, we will analyze the spectrum sensing performance of the proposed method.

Let $F(\lambda)$ and $G(\lambda)$ be the cumulative probability function (CDF) of the collected energy O under hypothesis H_0 and H_1 , respectively. Then, we have [13]

$$F(\lambda) = \int_{0}^{\lambda} f(O|H_{0})dO = 1 - \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)},$$

$$G(\lambda) = \int_{0}^{\lambda} f(O|H_{1})dO$$

$$= 1 - e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^{n} + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{u-1}$$

$$\times \left[e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})}\right)^{n}\right].$$
 (8)

Immediately, Δ_0 and Δ_1 can be written as $\Delta_0 = F(\lambda_2) - F(\lambda_1)$ and $\Delta_1 = G(\lambda_2) - G(\lambda_1)$.

If no any local decision is reported to the common receiver, i.e., K=0, we call it *fail sensing*. In this case, the common receiver will request all the cognitive users to perform spectrum sensing again. Let β_0 and β_1 denote the probability of fail sensing under hypothesis H_0 and H_1 , respectively. Then,

$$\beta_0 \triangleq P\{K = 0 | H_0\} = (F(\lambda_2) - F(\lambda_1))^N,$$
 (9)

$$\beta_1 \triangleq P\{K = 0|H_1\} = (G(\lambda_2) - G(\lambda_1))^N.$$
 (10)

Obviously, $\beta_0 = \Delta_0^N$ and $\beta_1 = \Delta_1^N$. When there is no fail sensing, i.e., $K \geq 1$, the false alarm probability Q_f , the detection probability Q_d , and the missing probability Q_m will be given as follows

$$Q_{f} = P\{H = 1, K \ge 1 | H_{0}\}$$

$$= P\{K \ge 1 | H_{0}\} P\{H = 1 | H_{0}, K \ge 1\}$$

$$= (1 - \beta_{0})(1 - P_{A}), \qquad (11)$$

$$Q_{d} = P\{H = 1, K \ge 1 | H_{1}\}$$

$$= P\{K \ge 1 | H_{1}\} P\{H = 1 | H_{1}, K \ge 1\}$$

$$= (1 - \beta_{1})(1 - P_{B}), \qquad (12)$$

$$Q_{m} = P\{H = 0, K \ge 1 | H_{1}\} = 1 - \beta_{1} - Q_{d}, \qquad (13)$$

where $P_A = P\{H = 0 | H_0, K \ge 1\}$ and $P_B = P\{H = 0 | H_1, K \ge 1\}$. In the following, we will only focus on the calculation of P_A and P_B for both perfect and imperfect reporting channels, respectively.

A. Perfect Reporting Channel

If the channels between the cognitive users and the common receiver are perfect, the local decisions will be reported without any error. In this case, $P_A = P\{H=0|H_0,K\geq 1\}$ characterizes the probability of the event that under hypothesis H_0 , all the K users claim H_0 and other N-K users make no local decisions. Then,

$$P_{A} = \sum_{K=1}^{N} {N \choose K} F(\lambda_{1})^{K} (F(\lambda_{2}) - F(\lambda_{1}))^{N-K}$$

$$= \sum_{K=0}^{N} {N \choose K} F(\lambda_{1})^{K} (F(\lambda_{2}) - F(\lambda_{1}))^{N-K}$$

$$- (F(\lambda_{2}) - F(\lambda_{1}))^{N}$$

$$= F(\lambda_{2})^{N} - \beta_{0}.$$
(14)

Likewise, we will have

$$P_B = G(\lambda_2)^N - \beta_1. \tag{15}$$

By substituting (14) and (15) into (11) and (12), we will obtain the false alarm probability Q_f , the detection probability Q_d and the missing probability Q_m for cooperative spectrum sensing, respectively.

It can be observed that when $\beta_0 = 0$, the performance in our proposed method will become the same as that of the conventional method. Note that $0 \le F(\lambda) \le 1$ and $F(\lambda)$ is a monotone increasing function since it is the CDF of λ .

Furthermore, since $F(\lambda_2) - F(\lambda_1) = \Delta_0$, it can be known that $\Delta_0 \leq F(\lambda_2) \leq 1$. Therefore, for a given β_0 , from (11) and (14), we can notice that Q_f is upper bounded by \bar{Q}_f and lower bounded by Q_f where

$$\bar{Q}_f = \lim_{F(\lambda_2) \to \Delta_0} Q_f = 1 - \beta_0,
\underline{Q}_f = \lim_{F(\lambda_2) \to 1} Q_f = \beta_0 (1 - \beta_0).$$
(16)

However, for a fixed Δ_0 , when the number of cognitive users is very large, the bound of Q_f can be neglected because $\beta_0 = \Delta_0^N$ and $\beta_1 = \Delta_1^N$ are extremely small for a very large N.

B. Imperfect Reporting Channel

It is not realistic that the reporting channel between the cognitive user and the common receiver is assumed to be perfect since it is usually subject to fading [14]. Due to the reporting error introduced by the imperfect channel, the reported local decisions should be firstly decoded in the common receiver before the final decision is made. Let $P_{e,i}$ denote the reporting error between the *i*th cognitive user and the common receiver, for $i=1,\cdots,K$. For simplicity, we assume that all the reporting channels are independent and identical, i.e., $P_{e,i}=P_e$. At the common receiver, the local decision will be recovered as 0 in two cases: a), the cognitive user transmits H_0 and the it is decoded as H_0 ; b), the cognitive user transmits H_1 while the it is decoded as H_0 because of the reporting error. Therefore, the probability P_A can be obtained as follows:

$$P_{A} = \sum_{K=1}^{N} {N \choose K} (F(\lambda_{2}) - F(\lambda_{1}))^{N-K}$$

$$\times ((1 - P_{e})F(\lambda_{1}) + P_{e}(1 - F(\lambda_{2})))^{K}$$

$$= (F(\lambda_{2}) + P_{e}(1 - F(\lambda_{2}) - F(\lambda_{1})))^{N} - \beta_{0}.(17)$$

Likewise, P_B can be given by

$$P_B = (G(\lambda_2) + P_e(1 - G(\lambda_2) - G(\lambda_1)))^N - \beta_1.(18)$$

By substituting (17) and (18) into (11) and (12), we will obtain the false alarm probability Q_f , the detection probability Q_d and the missing probability Q_m for imperfect reporting channel, respectively.

In the special case when the reporting error $P_e = 0$, (17) and (18) will be equivalent to (14) and (15), respectively.

Due to the existence of reporting errors, the sensing performance is decreased compared with that in the perfect channel. We can also observe that Q_f has some fixed bound values, namely, upper bound \bar{Q}_f and lower bound \underline{Q}_f , which can be expressed as

$$\bar{Q}_f = (1 - \beta_0)(1 + \beta_0 - (\sqrt[N]{\beta_0} + P_e - P_e \sqrt[N]{\beta_0})^N),
\underline{Q}_f = (1 - \beta_0)(1 + \beta_0 - (1 - P_e + P_e \sqrt[N]{\beta_0})^N).$$
(19)

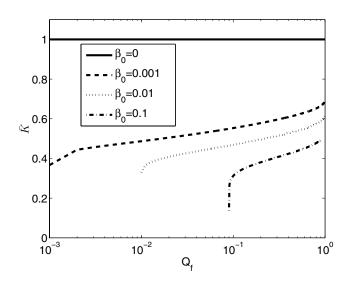


Fig. 2. The performance of the normalized average number of sensing bits, \bar{K} vs. Q_f , N=10, $\beta_0=0,0,001,0.01,0.1$ and SNR=10 dB.

IV. SIMULATION RESULTS

Simulation results are presented in this section to demonstrate the performance of cooperative spectrum sensing under bandwidth constraints. The results of the conventional method, i.e., $\beta_0=0$, are also shown for a comparison. We assume that there are ten users in the system and the average SNR between the primary user and any cognitive user is $10~\mathrm{dB}$. We use $P_0=0.5$.

Fig. 2 shows the decrease of the normalized transmission bits in the perfect reporting channel for different values of fail sensing, $\beta_0 = 0, 0.001, 0.01, 0.1$. It can be observed that, compared with the conventional method, i.e., $\beta_0 = 0$, the false alarm probability Q_f is upper bounded and lower bounded which has been discussed in Section III. For example, when $\beta_0 = 0.01$, we can see that the upper and lower bound of Q_f are 0.99 and 0.0099, respectively, which justifies the analysis of Section III. The false alarm probability Q_f is bounded because of the fail sensing and based on (9), one can find that the loss of Q_f is caused by the large "No Decision" region which is $\triangle_0 = \sqrt[N]{\beta_0} = 0.6310$, for $\beta_0 = 0.01$. However, the number of sensing bits is dramatically decreased, by at least 40%, compared with conventional method, where $\beta_0 = 0$. Furthermore, the reduction becomes larger when β_0 is larger. We can also notice that, when $Q_f \rightarrow \bar{Q}_f$ and $Q_f \rightarrow Q_f$, \bar{K} goes to some fixed values for a particular β_0 . This can be explained as follows:

$$\lim_{Q_f \to \bar{Q}_f} \bar{K} = \lim_{F(\lambda_2) \to \Delta_0} \bar{K}$$

$$= 1 - P_0 \sqrt[N]{\beta_0} - P_1 G(F^{-1}(\sqrt[N]{\beta_0})), \qquad (20)$$

$$\lim_{Q_f \to \bar{Q}_f} \bar{K} = \lim_{F(\lambda_2) \to 1} \bar{K}$$

$$= 1 - P_0 \sqrt[N]{\beta_0} - P_1 (1 - G(F^{-1}(1 - \sqrt[N]{\beta_0})))$$
(21)

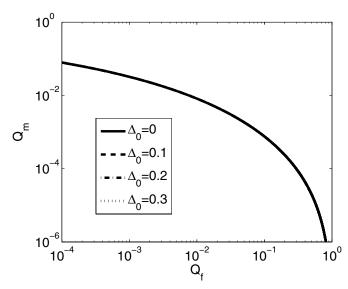


Fig. 3. Complementary receiver operating characteristic performance (Q_m vs. Q_f) of cooperative spectrum sensing, $N=10,\,\Delta_0=0,0,1,0.2,0.3$ and SNR=10 dB.

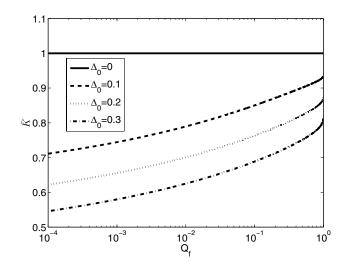


Fig. 4. The performance of the normalized average number of sensing bits, \bar{K} vs. Q_f , N=10, $\Delta_0=0,0,1,0.2,0.3$ and SNR=10 dB.

where $F^{-1}(\lambda)$ is the inverse function of $F(\lambda)$.

Fig. 3 illustrates the complementary receiver operating characteristic performance $(Q_m \text{ vs. } Q_f)$ of cooperative spectrum sensing, for different values of Δ_0 , $\Delta_0=0,0.1,0.2,0.3$. It can be seen that the four curves are almost the same which means there is very little performance loss of spectrum sensing between our method and conventional method. This is because, in this case, the probability of fail sensing, i.e., $\beta_0=\Delta_0^{10}$ and $\beta_1=\Delta_1^{10}$ are extremely small.

Fig. 4 depicts the normalized average number of sensing bits in terms of Q_f , i.e., \bar{K} vs. Q_f , for $\triangle_0=0,0.1,0.2,0.3$. We can notice that, there is a significant decrease of the normalized average transmission bit in comparison with $\triangle_0=0$. For

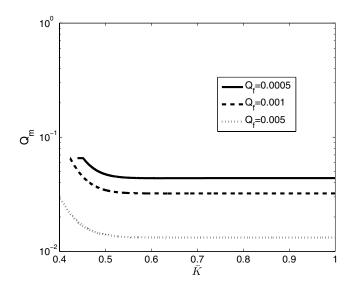


Fig. 5. Q_m vs. \bar{K} in perfect reporting channels, for Q_f 0.0005, 0.001, 0.005, N=10, SNR=10 dB.

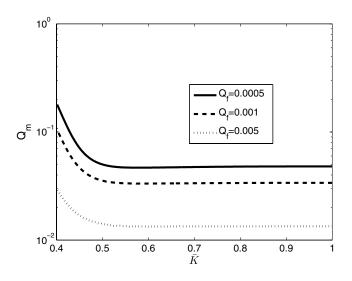


Fig. 6. Q_m vs. \bar{K} in imperfect reporting channels, for $Q_f=0.0005,0.001,0.005,$ N=10, SNR=10 dB and $P_e=10^{-5}$.

example, when $Q_f=0.01$, almost 20% and 40% reduction can be obtained for $\Delta_0=0.1$ and $\Delta_0=0.3$, respectively. Besides, for the same Q_f , the decrease will become larger with the increase of Δ_0 .

Fig. 5 shows the tradeoff between the spectrum sensing performance and the average number of sensing bits, i.e., Q_m vs. \bar{K} , for given false alarm probability, $Q_f=0.0005, 0.001, 0.005$, respectively. It can be observed that, for a fixed false alarm probability, the missing probability Q_m changes a little when \bar{K} varies from 0.5 to 1, which means that we can achieve a large reduction of number of sensing bits at a very little expense of performance loss.

Fig. 6 is simulated to show Q_m vs. \bar{K} in imperfect reporting channel, for the given false alarm probability, $Q_f =$

0.0005, 0.001, 0.005. The reporting error is assumed to be $P_e = 10^{-5}$. We can also observe that for a fixed false alarm probability Q_f , compared with the conventional method, i.e., when $\bar{K}=1$, there is a very little loss of the missing probability Q_m while the normalized average number of sensing bits decrease greatly.

V. CONCLUSION

Cooperative spectrum sensing for cognitive radio systems has been studied under bandwidth constraints in this paper. To decrease the average number of sensing bits to the common receiver, a censoring method was developed. Analytical performance results of the proposed cooperative spectrum sensing method under bandwidth constraints were studied for both perfect reporting channels and imperfect reporting channels. In addition, the normalized average number of sensing bits has been derived. Simulation results showed a great decrease of the average number of sensing bits to the common receiver at the expense of a little performance loss.

ACKNOWLEDGEMENT

This work was partially supported by a research grant awarded by the Hong Kong Research Grant Council.

REFERENCES

- Federal Communications Commission, "Spectrum Policy Task Force," Rep. ET Docket no. 02-135, Nov. 2002.
- [2] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, pp. 13–18, Aug. 1999.
- [3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," IEEE J. Select. Areas Commun., vol. 23, pp. 201-220, Feb. 2005.
- [4] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. of Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Nov. 7-10, 2004, pp. 772 - 776.
- [5] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. 1st IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, USA, Nov. 8–11, 2005, pp. 131–136.
- [6] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proc. of Allerton Conf. Signals, Systems, and Computers*, Monticello, Oct. 2004.
- [7] C. W. Helstronm, "Improved multilevel quantization for detection of narrowband signals," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 24, no. 2, pp. 142-147, Mar. 1988.
- [8] W. A. Hashlamoun and P. K. Varshney, "Near-optimum quantization for signal detection," *IEEE Trans. Commun.*, vol. 44, pp. 294-297, Mar. 1996.
- [9] R. S. Blum, "Distributed detection for diversity reception of fading signals in noise," *IEEE Trans. Inf. Theory*, vol. 45, pp. 158-164, Jan. 1999.
- [10] J. F. Chamberland and V. V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Trans. Signal Processing*, vol. 51, pp. 407-416, Feb. 2003.
- [11] C. Rago, P. Willett, and Y. Bar-Shalom, "Censoring sensors: a low communication-rate scheme for distributed detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, pp. 554-568, Apr. 1996.
- [12] R. X. Jiang and B. Chen, "Fusion of censored decisions in wireless sensor networks," *IEEE Trans. on Wireless Commun.*, vol. 4, pp. 2668-2673, Nov. 2005.
- [13] F. F. Digham, M. -S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," in *Proc. IEEE ICC*, Anchorage, AK, USA, May 11-15, 2003, pp. 3575–3579.
- [14] J. F. Chamberland and V. V. Veeravalli, "The impact of fading on decentralized detection in power constrained wireless sensor networks," in *Proc. IEEE ICASSP*, Montreal, Cananda, May 17-21, 2004, pp. 837-840.