Proceedings of the American Control Conference
Philadelphia, Pennsylvania • June 1998

Stability of an Automated Vehicle Platoon

Yibing Wang* Zengjin Han

Department of Automation
Tsinghua University
Beijing 100084 P. R. China
*wyb@mail.au.tsinghua.edu.cn

Abstract--In recent years much attention has been focused on the development of Automated Highway Systems (AHS), where the concept of automated vehicle platoons is crucially relied on. In the view of control an automated vehicle platoon can be regarded as an interconnected nonlinear system. Among a variety of issues involved in the design and operation of this system, the stability issue is of much concern both in theory and practice. The purpose of this paper is to derive some general conditions under which every intra-platoon vehicle is individual stable and the overall platoon is string stable. By considering stability properties of nonlinear systems with slowly varying inputs, we present two individual stability theorems for the intra-platoon vehicles and a string stability theorem for the whole platoon. It is demonstrated that a stable automated vehicle platoon can be achieved if the vehicular controllers are suitably designed.

1. INTRODUCTION

In traffic engineering, especially in car-following theory, local stability and asymptotic stability are defined for a platoon of moving vehicles [5,6]. If the (speed and hence spacing) response of a vehicle to the speed perturbation of its preceding vehicle decays, then (the motion of) the vehicle is said to be **locally stable**. On the other hand, if the speed perturbation in the platoon is not magnified as it propagates down the platoon, then (the motion of) the platoon is said to be **asymptotically stable**. It has been shown that a manually-driving platoon may not be asymptotically stable even if each intra-platoon vehicle is locally stable. In traffic engineering this phenomenon is called the **slinky effect** [9,12,13], which is due to existence of human delay and lack of information exchange among vehicles. The slinky effect reflects the inherent instability of the motion of a manually-driving platoon.

Traffic congestion has been a global problem and traditional approaches to it are getting no more practicable. In recent years an increasing attention has been paid to Automated Highway Systems (AHS) [7,8], where moving vehicles are formed into platoons with small intra-platoon but larger inter-platoon spacing by applying control and communication technologies [7-11]. In a lot of literature this vehicular operation is called platooning or automatic vehicle following, and in this paper the corresponding platoon is called an automated vehicle platoon (hereafter called platoon for short). Considering the slinky effect, we can see that the stability issue of automated vehicle platoons is of great importance to the AHS development since an unstable platoon is of no use. It has been pointed out in [12] that asymptotic stability is not automatically provided when a group of good automated vehicles are simply operated in a platoon. So, the stability of such platoon must be specially investigated.

An automated vehicle platoon can be thought of as a special interconnected nonlinear system. In this paper the stability of the platoon is defined as **individual stability** and **string stability**. We first deal with the individual stability of the intra-platoon vehicles, then consider the string stability of the platoon. It is shown that the platoon is string stable if the vehicular controllers are suitably designed and the intra-platoon vehicles are all individual stable. As a result, the slinky effect is effectively suppressed or eliminated.

The proofs in this paper have been omitted due to space limitations. They can be found in [17] and some unpublished papers, which may be requested from the first author.

2. MODELING AND STABILITY DESCRIPTION OF AN AUTOMATED VEHICLE PLATOON

2.1. Platoon Model

We consider a platoon consisting of N+1 identical automated vehicles traveling in a lane of a straight highway. The platoon is composed of one leader V_0 and N followers V_n ($n=1,\cdots,N$). According to the present design [7-11], each vehicle has timely access to the following information: (a) its velocity and acceleration (b) its headway (distance to the preceding vehicle) (c) velocity, acceleration and headway of the preceding vehicle (d) velocity and acceleration of the leader. In the paper we focus our attention to how the perturbation in the speed and acceleration of the leader influences the motion of the followers. Hence the platoon discussed hereafter does not include the leader.

The automated vehicle platoon is a special interconnected nonlinear system, where each vehicle is regarded as a nonlinear subsystem. Some vehicular dynamic models can be found in [7,8]. Considering these models, design of decentralized controllers for the platoon yields the following systems [10,11]:

$$\dot{x}_1(t) = f_1(x_1(t), u(t)) \tag{1}$$

$$\dot{x}_{n+1}(t) = f(x_{n+1}(t), x_n(t), u(t)) \quad n=1,\cdots,N-1 \tag{2}$$

where (1) represents the follower V_1, (2) represents the follower V_n (n=2,⋯ N); $f_1(\cdot)$ and $f(\cdot)$ are nonlinear functions; $x_{n+1}(t)$ denotes the state vector, its components including the spacing Δ_{n+1} between V_{n+1} and V_n, the speed v_{n+1} and acceleration a_{n+1} of V_{n+1}; $u(t)$ denotes the input vector consisting of the velocity and acceleration of the leader. Obviously, $u(t)$ is an exogenous signal to each subsystem, and the platoon is an open-loop interconnected system. In the paper we only consider the case that $u(t)$ is slowly varying.

2.2. Stability Problems and Definitions

Definition 1: $u(t)$ is said to be **"frozen"** at the moment t' if $\dot{u}(t) \neq 0$ for $t_0 \le t < t'$ and $\dot{u}(t) = 0$, $t \ge t'$; and **"asymptotically frozen"** at the moment t' if $\dot{u}(t) \neq 0$ for $t_0 \le t < t'$ and $\lim_{t \to t'} \dot{u}(t) = 0$, $\dot{u}(t) = 0$, $t \ge t'$.

Definition 2 — sink [14]: Consider the nonlinear system (1), where f_1 is $\mathbf{C}^2$, $x_1(t) \in U$ is the state vector, $u(t) \in V$ is the input vector, U and V are open in R^p and R^m respectively. $\sigma[\cdot]$ denotes the spectrum of a matrix. If, for each fixed $t \ge t_0$, a point $(x_e(t), u(t)) \in U \times V$ satisfies

(a) $$f_1(x_e(t), u(t)) = 0 \tag{3}$$

(b) $$\operatorname{Re}\sigma\left[\tfrac{\partial}{\partial x_1} f_1(x_e(t), u(t))\right] < 0 \tag{4}$$

then we call $x_e(t)$ the sink of (1) corresponding to $u(t)$.

Remarks: (a) If $u(t)$ is "frozen" at the moment t', then $x_e(t')$ is a locally exponentially stable equilibrium of the corresponding frozen system.

(b) To keep platoon stability, the sink conditions of (2) should be $f(x_e(t), x_e(t), u(t)) = 0$ and $\operatorname{Re}\sigma\left[\frac{\partial}{\partial x_{n+1}} f(x_e(t), x_e(t), u(t))\right] < 0$ (see Section 4). We suppose the vehicular controllers can be suitably designed to guarantee all the sink conditions.

(c) In terms of the platoon, $x_e(t)$ is the **nominal trajectory** of the platoon corresponding to $u(t)$. If $x_n(t)$ is written as $(\Delta_n(t), v_n(t), a_n(t))$, then $x_e(t)$ can be written as $(\Delta(t), v(t), a(t))$. If $u(t)$ is "frozen" or "asymptotically frozen" at the moment t', then $x_e(t') = (\Delta(t'), v(t'), 0)$ can be regarded as a steady state of the platoon. Therefore, $x_n(t)$ is always expected to be close to $x_e(t)$ and tend to it after $u(t)$ is "frozen" or "asymptotically frozen".

Invoking the above definitions and remarks, we mainly consider the following stability problems of the platoon. Assume that the platoon is in the steady state $(\Delta(t_0), v(t_0), 0)$ before the moment t_0: all vehicles moving with a constant speed $v(t_0)$ and a constant headway $\Delta(t_0)$. ① If the leader V_0 begins to accelerate or decelerate at t_0, and from a later moment t' on, moves with a new constant speed ($u(t)$ is "frozen" or "asymptotically frozen" at t'), then, is it possible that the followers can successively approach to the new steady state $(\Delta(t'), v(t'), 0)$ after t'? ② If the leader keeps on accelerating or decelerating ($\dot{u}(t) \neq 0$ for $t > t_0$), how far does the states of the followers deviate from $(\Delta(t), v(t), a(t))$?

Definition 3: If the state deviation of a follower from the nominal trajectory is bounded all the time, then the (motion of) follower is said to be **individual stable**. If, in addition, the state of the follower tends exponentially to a steady state once the leader moves with a constant speed, then the follower is said to be **exponentially individual stable**. If the state deviations of all the followers from the nominal trajectory are uniformly bounded all the time, then the platoon is said to be **string stable**. If, in addition, the peak deviations in the states of the followers from the steady state decrease monotonically from the front to the back of the platoon once the leader moves with a constant speed, then the platoon is said to be **monotonically string stable**. If the platoon is monotonically string stable with each follower exponentially individual stable, then the platoon is said to be **exponentially monotonically string stable**. Obviously, local stability and asymptotic stability discussed in traffic engineering are roughly equivalent to exponentially (maybe only asymptotically) individual stability and monotonically string stability defined here.

2.3. Stability Results

Several stability results are established for the platoon:

(a) If the initial state deviation of a follower from the nominal trajectory $(\Delta(t), v(t), a(t))$ is small, $u(t)$ is sufficiently slowly varying, moreover, the preceding vehicle is individual stable for $t \ge t_0$, then the follower is also individual stable for $t \ge t_0$. The oscillation in its state response is effectively suppressed.

(b) If a follower is individual stable for $t \ge t_0$. $u(t)$ is "frozen" or "asymptotically frozen" at the moment t', and if the preceding vehicle is exponentially individual stable for $t \ge t'$, then the follower is also exponentially individual stable for $t \ge t'$.

(c) If the initial state deviations of all the followers are uniformly bounded, and $u(t)$ is slowly varying, then the platoon is string stable for $t \ge t_0$. The slinky effect and oscillation are suppressed effectively.

(d) If the platoon is string stable, and $u(t)$ is "frozen" or "asymptotically frozen" at t', then the platoon is monotonically string stable for $t \ge t'$. The slinky effect is eliminated as a result.

(e) If, in addition to the hypotheses in (d), some other appropriate conditions are satisfied, then the platoon is exponentially monotonically string stable. The slinky effect does not appear.

3. INDIVIDUAL STABILITY OF THE FOLLOWER V_1

In this section we discuss the individual stability of the follower V_1, namely local uniform stability of the sink of nonlinear systems (1) with slowly varying inputs. Throughout the paper, $\|\cdot\|$ represents l_2 norm on a time-varying vector as well as the corresponding induced norm on matrices.

Theorem 1: Suppose that U and V are open in R^p and R^m respectively, U is convex; f_1: $U \times V \to R^p$ is C^2 such that $M_1 = \{(x_e(t), u(t)) \in U \times V \mid x_e(t)$ is a sink of (1) corresponding to $u(t)\}$ is not empty; Q_1 is an open connected subset of M_1, relatively compact in M_1; $x_e:(t_0,\infty) \to U$, $u:(t_0,\infty) \to V$ are C^1 such that $(x_e(t), u(t)) \in Q_1$ for $t \ge t_0$. Let $x_1(t)$ be the solution of (1). Then, for any $T>0$ and $\varepsilon>0$, there exist $\delta_u^1>0$, $\delta_0^1>0$ independent of t_0, $u(\cdot)$, $x_e(\cdot)$ such that if

(a) $\|x_1(t_0) - x_e(t_0)\| \le \delta_0^1$ (5)

(b) $\frac{1}{T}\int_t^{t+T} \|\dot{u}(s)\| ds \le \delta_u^1$, $t \ge t_0$ (6)

then

$$\|x_1(t) - x_e(t)\| < \varepsilon, \quad t \ge t_0 \tag{7}$$

Remark: For the follower V_1, Theorem 1 establishes that if its state is sufficiently close to the nominal trajectory of the platoon at $t = t_0$, and the speed and acceleration of the leader are sufficiently slowly varying for $t \ge t_0$, then the state of V_1 remains close to the nominal trajectory for all $t \ge t_0$. V_1 is **individual stable.**

Corollary 1: In addition to the hypotheses of Theorem 1,

(a) if $u(t)$ is "frozen" at t', then $x_1(t') \in \mathbf{D_r}(x_e(t'), u(t'))$, where $\mathbf{D_r}(x_e(t'), u(t'))$ denotes the attractive region of $x_e(t')$ of the system (1) with input $u(t')$.

(b) if $u(t)$ satisfies

$$\lim_{t\to\infty} \dot{u}(t) = 0, \qquad \lim_{t\to\infty} u(t) = u_\infty \in V \tag{8}$$

then

$$\lim_{t\to\infty} x_1(t) = x_e(u_\infty) \tag{9}$$

where $x_e(u_\infty)$ is the sink of the system (1)(2) corresponding to u_∞. Moreover, if $u(t)$ is "asymptotically frozen" at t' ($t' > t_0$), namely $u(t) = u_\infty$, $t \ge t'$, then $x_1(t') \in \mathbf{D_r}(x_e(u_\infty), u_\infty)$.

Remark: It is ensured by Corollary 1 that if the leader moves at a constant velocity from the moment t' on ($t' > t_0$), then for $t \ge t'$ the state of V_1 tends exponentially to the steady state $x_e(t')$ or $x_e(u_\infty)$, namely $(\Delta(t'), v(t'), 0)$. The follower V_1 is **exponentially individual stable** for $t \ge t'$.

4. INDIVIDUAL STABILITY OF THE FOLLOWER V_n (n=2,⋯,N)

After discussing the individual stability of V_1, we shall prove that the followers are all individual stable if some appropriate conditions are satisfied. Obviously, every subsystem of the interconnected system (1)(2) has its own sink corresponding to the given input. However, designing a stable platoon requires these sinks are the same. Since we have assumed that the platoon consists of identical vehicles, this requirement is not difficult to fulfill (Even if the platoon consists of different types of vehicle, this requirement should be guaranteed by the special design of the control system). Hence

$$f(x_e(t), x_e(t), u(t)) = 0, \text{ for } n = 2,\cdots,N \tag{10}$$

$$\mathrm{Re}\,\sigma[D_1 f(x_e(t), x_e(t), u(t))] < 0 \tag{11}$$

Theorem 2: Suppose U and V are open in R^p and R^m respectively, U is convex; f: $U \times U \times V \to R^p$ is C^2 such that $M_2 = \{(x_e(t), x_e(t), u(t)) \in U \times U \times V \mid x_e(t)$ is a sink of (2) corresponding to $(x_e(t), u(t))\}$ is not empty; Q_2 is an open connected subset of M_2, relatively compact in M_2; $x_e:(t_0,\infty) \to U$, $u:(t_0,\infty) \to V$ are C^1 such that

$(x_e(t), u(t)) \in Q_1$ and $(x_e(t), x_e(t), u(t)) \in Q_2$ for $t \geq t_0$. Let $x_{n+1}(t)$ be the solution of (2) and $\rho(\cdot)$ denote the spectral radius of a matrix. Then, for any $T > 0$, $\varepsilon > 0$, there exist $\delta_u^2 > 0$, $\delta_0^2 > 0$ independent of t_0, $u(\cdot)$, $x_e(\cdot)$ such that if for $t \geq t_0$

(a) $\|x_{n+1}(t_0) - x_e(t_0)\| \leq \delta_0^2$ (12)

(b) $\|x_n(t) - x_e(t)\| \leq \varepsilon$ (13)

(c) $\frac{1}{T}\int_t^{t+T} \|\dot{u}(s)\| ds \leq \delta_u^2$ (14)

(d) $\rho\left[\left(\frac{\partial}{\partial x_{n+1}} f(x_{n+1}, x_n, u)\right)^{-1} \left(\frac{\partial}{\partial x_n} f(x_{n+1}, x_n, u)\right)\right] < 1$ (15)

then

$$\|x_{n+1}(t) - x_e(t)\| \leq \varepsilon, \quad t \geq t_0 \tag{16}$$

Remarks: (a) Theorem 2 indicates that if the state of V_{n+1} is sufficiently close to the nominal trajectory at $t = t_0$, the speed and acceleration of the leader are sufficiently slowly varying for $t \geq t_0$, and the follower V_n is individual stable, then V_{n+1} is also individual stable. for $t \geq t_0$.

(b) $\|\dot{u}(t)\|$ is required to be sufficiently small and $\max_{t \geq t_0} \|\dot{x}_e(t)\|$ is supposed to be known when a similar problem is discussed in [10,11], while in Theorem 2 $\|\dot{u}(t)\|$ is only required to be sufficiently small in the average sense, as indicated by (14). Clearly, the result in [10,11] is a limiting case of Theorem 2 when $T \to 0$. Moreover, Theorem 2 yields more systematic results.

Corollary 2: If, in addition to the hypotheses of Theorem 2, $u(t)$ and $x_n(t)$ satisfies

$$\lim_{t\to\infty} \dot{u}(t) = 0, \quad \lim_{t\to\infty} u(t) = u_\infty \in V \tag{17}$$

$$\lim_{t\to\infty} x_n(t) = x_e(u_\infty) \tag{18}$$

then

$$\lim_{t\to\infty} x_{n+1}(t) = x_e(u_\infty) \tag{19}$$

where $x_e(u_\infty)$ is the sink of the system (1)(2) corresponding to u_∞. Let $\mathbf{D}_r(x_e(u_\infty), x_e(u_\infty), u_\infty)$ denote the attractive region of $x_e(u_\infty)$ of the system (2) corresponding to $(x_e(u_\infty), u_\infty)$. If $u(t)$ is "asymptotically frozen" at the moment t', and $x_n(t') \in \mathbf{D}_r(x_e(u_\infty), u_\infty)$ or $x_n(t') \in \mathbf{D}_r(x_e(u_\infty), x_e(u_\infty), u_\infty)$ (n=1 or n=2,$\cdots$, N-1), then $x_{n+1}(t') \in \mathbf{D}_r(x_e(u_\infty), x_e(u_\infty), u_\infty)$. A similar result can be established if the system (1)(2) is "frozen" at the moment t'.

Remark: Corollary 2 implies that if the platoon leader moves with a constant speed from the moment t' on ($t' \geq t_0$), and V_n is exponentially individual stable for $t \geq t'$, then V_{n+1} is also exponentially individual stable for $t \geq t'$.

5. String Stability of the Platoon

By Theorem 1 and 2, we have discussed the individual stability of the followers. Although drivers can sometimes guarantee local stability of their vehicles independently, it is almost at a cost of destroying asymptotic stability and producing the slinky effect. In this section we show that local stability and asymptotic stability can be harmoniously achieved in an automated vehicle platoon. In this case, the platoon is exponentially monotonically string stable.

Definition 4 — string stability: Consider an interconnected nonlinear system

$$\dot{x}_n(t) = f_n(x_n(t), \cdots, x_{n-r_n+1}(t), u(t)) \tag{20}$$

where $f_n : \underbrace{R^p \times \cdots \times R^p}_{r_n} \times R^m \to R^p$, $f_n\left(\underbrace{x_e, \cdots, x_e}_{r_n}, u\right) = 0$, $1 \leq n \leq \infty$, $1 \leq r_n \leq n$. Let $X = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots)^{\mathrm{T}}$, then $X \in R^p \times R^p \times \cdots$, where the superscript T represents the matrix transpose, X and x_i are all column vectors. The sink of the system (20) is string stable if for any $T > 0$, $\varepsilon > 0$, there exist $\delta_0 > 0$ and $\delta_u > 0$ such that if

(a) $\|X_\delta(t_0)\|_\infty \leq \delta_0$ (21)

(b) $\frac{1}{T}\int_t^{t+T} \|\dot{u}(s)\| ds \leq \delta_u, \quad t \geq t_0$ (22)

then

$$\|X_\delta(t)\|_\infty < \varepsilon, \quad t \geq t_0 \tag{23}$$

where $\|X_\delta(t_0)\|_\infty = \sup_n \|x_n(t_0) - x_e(t_0)\|$,

$\|X_\delta(t)\|_\infty = \sup_n \sup_{t \geq t_0} \|x_n(t) - x_e(t)\|$ □

Remarks: (a) String stability implies uniform boundedness of the states of interconnected system (20) with respect to its sink for all time. Strictly speaking, the string stability defined here is l_∞ string stability [16].

(b) The automated vehicle platoon (1)(2) is a special case of the system (20), that is, $r_1 = 1$, $r_n = 2$, $f_n = f$, $(2 \leq n \leq N)$, $X = (x_1^{\mathrm{T}}, \cdots, x_N^{\mathrm{T}})^{\mathrm{T}}$, $X \in U_Q^1 \times \overbrace{U_Q^2 \times \cdots \times U_Q^2}^{N-1}$ (For U_Q^1 and U_Q^2, see the proof of Theorem 1 and 2). Since $U_Q^1 \times \overbrace{U_Q^2 \times \cdots \times U_Q^2}^{N-1}$ is relatively compact in U^N, there exist compact sets Z_1 and Z_2 satisfying

$$U_Q^1 \times \overbrace{U_Q^2 \times \cdots \times U_Q^2}^{N-1} \in \mathring{Z}_1 \times \overbrace{\mathring{Z}_2 \times \cdots \times \mathring{Z}_2}^{N-1} \in Z_1 \times \overbrace{Z_2 \times \cdots \times Z_2}^{N-1} \in U^N$$

Theorem 3: The platoon is string stable.

Corollary 3.1: Suppose the platoon is string stable. If $u(t)$ is "asymptotically frozen" at t' ($t' \geq t_0$), then $X(t')$ is in the attractive region of $(\overbrace{x_e^T(u_\infty), x_e^T(u_\infty), \cdots, x_e^T(u_\infty)}^{N})^T$. If the platoon is "frozen" at t', then $X(t')$ is in the attractive region of $(\overbrace{x_e^T(t'), x_e^T(t'), \cdots, x_e^T(t')}^{N})^T$.

Corollary 3.2: Suppose the platoon is string stable and some appropriate conditions are satisfied. Then the platoon is monotonically string stable or exponentially monotonically string stable.

Remarks: (a) Theorem 3 implies that the state deviations of the followers from the nominal trajectory can be guaranteed to be uniformly bounded under disturbance caused by the leader. This answers the problem ② put forward in section 2.2.

(b) Corollary 3.1 implies that after t', the follower tends exponentially to the steady state, consecutively. This answers the problem ① in section 2.2.

(c) Corollary 3.2 implies that after t', the peak deviation in the state of each follower from $x_e(u_\infty)$ or $x_e(t')$ decreases monotonically from the front to the back of the platoon. The slinky effects are eliminated.

(d) In theory, the stability properties of the platoon are not related to N, the length of the platoon.

(e) The stability of an automated vehicle platoon is investigated under the condition that $\frac{1}{T}\int_t^{t+T}\|\dot{u}(s)\|ds$ is sufficiently small. From this we know that the leader's sudden acceleration or deceleration maneuver of short-duration (such as slamming on the brakes) does not have influence on the stability of the platoon. However, the condition that $\|\dot{u}(t)\|$ is sufficiently small requires that no jump in acceleration arise during the leader moves [10,11].

6. CONCLUSIONS

We have studied the stability of the automated vehicle platoon from the viewpoint of interconnected nonlinear systems with slowly varying inputs. These stability results can provide theoretical support for the system design of the platoon. However, The platoon considered here is an open-loop system on the whole (The subsystem is close-looped individually). The stability properties of such system are mainly dependent on its system structure. Therefore the design of vehicular controllers (to achieve this structure) is very important. Furthermore, in order to improve the stability and other performance of the platoon, it is necessary to investigate a close-loop platoon in the future work.

REFERENCES

1. Dieudonne, J., Foundations of modern analysis. Academic Press, 1960.
2. Horn, R. A., Matrix Analysis. Cambridge University Press, 1985.
3. Desoer, C. A., "slowly varying system $\dot{x} = A(t)x$," IEEE Trans. on AC., Vol. 14, pp. 780-781, 1969.
4. Brockett, R. W., Finite dimensional linear systems. John Wiley and Sons, Inc., 1970.
5. Herman, R. C., et al., "Traffic dynamics: Analysis of stability in Car following," Opns. Res., 86-106, 1959.
6. Gazis, D. C., et al., "Car following theory of stability-state traffic flow," Opns. Res., 499-505, 1959.
7. Shladover, S. E., et al., "Automatic vehicle control developments in the PATH program," IEEE Trans. on Vehicular Technology, no. 1, 1991.
8. Hedrick, J. K., Tomizuka, M. and Varaiya, P., "Control issues in automated highway systems," IEEE Control Systems, no. 6, 1994.
9. Sheikholeslam, S. and Desoer, C. A., "Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study," IEEE Trans. on Vehicular Technology, no. 4, 1993.
10. Sheikholeslam, S. and Desoer, C. A., "Control of interconnected nonlinear dynamic systems: platoon problem," IEEE Trans. on AC., no.6, 1992.
11. Sheikholeslam, S. and Desoer, C. A., "Indirect adaptive control of a class of interconnected nonlinear dynamic systems," Intern. J. Contr., Vol. 57, no. 3, 1993.
12. Stotsky, A. and Ioannou, P., "Robust platoon-stable controller design for autonomous intelligent vehicles," IEEE Proceedings of 33rd Conference on Decision and Control, 1994.
13. Ioannou, P. and Chien, C. C., "Autonomous intelligent cruise control," IEEE Trans. on Vehicular Technology, no. 4, 1993.
14. Kelemen, M., "A stability property," IEEE Trans. on AC., no. 8, 1986.
15. Lawrence, D. A. and Rugh, W. J., "On a stability theorem for nonlinear systems with slowly varying inputs," IEEE Trans. on AC., no. 7, 1990.
16. Swaroop, D. and Hedrik, J. K., "String stability of interconnected systems," IEEE Trans. on AC., no. 3, 1996.
17. Wang, Yibing and Han, Zengjin, "An extended stability theorem for nonlinear systems subject to slowly-varying inputs," 1998 American Control Conference.