# String Stable Model Predictive Cooperative Adaptive Cruise Control for Heterogeneous Platoons

Ellen van Nunen, Joey Reinders, Elham Semsar-Kazerooni, and Nathan van de Wouw

***Abstract*—Cooperative adaptive cruise control (CACC) is a potential solution to decrease traffic jams caused by shock waves, increase the road capacity, decrease fuel consumption and improve safety. This paper proposes an integrated solution to a combination of four challenges in these CACC systems. One of the technological challenges is how to guarantee string stability (the ability to avoid amplification of dynamic vehicle responses along the string of vehicles) under nominal operational conditions. The second challenge is how to apply this solution to heterogeneous vehicles. The third challenge is how to maintain confidentiality of the vehicle parameters. Finally, the fourth challenge is to find a method which improves robustness against wireless packet loss. This paper proposes a model predictive control approach in combination with a feed-forward control design, which is based on a shared vector of predicted accelerations over a finite time horizon. This approach is shown to be applicable to a heterogeneous sequence of vehicles, while the vehicle parameters remain confidential. In previous works such an approach has shown to increase robustness against packet losses. Conditions for string stability are presented for the nominal operational conditions. Experimental results are presented and indeed demonstrate string stable behavior.**

***Index Terms*—Heterogeneous platooning, cooperative adaptive cruise control, model predictive control, string stability.**

## I. Introduction

WITH the world-wide increase of traffic density over the years, transportation times and $CO_2$ emissions caused by traffic congestion are rising. In daily traffic, ghost traffic jams occur, which are caused by an upstream amplification of a slight braking action, leading to a traffic jam. This phenomenon is called string instability. String stable short inter-vehicle distance platooning (automated vehicle following), is considered as a solution to the aforementioned problems, allowing for an increase in road capacity and a decrease in traffic jams [1], [2]. Furthermore, especially for trucks, aerodynamic benefits are found for short inter-vehicle distances, resulting in lower fuel consumptions [3], [4].

The considered platooning system is based on Cooperative Adaptive Cruise Control (CACC) to longitudinally control a vehicle to follow the preceding vehicle at a certain desired distance. Within the design of CACC, the property of string stability is desired to suppress shock waves, [5]–[7]. The control design of a CACC-system typically consists of a feedback part, based on on-board sensors such as a radar, and a feed-forward part based on shared information describing the intended dynamic behavior of a preceding vehicle. CACC utilizes wireless inter-vehicle communication to share this additional key information. As a consequence, safety and string stability become strongly dependent on the wireless link [8]. With the increase of communicating nodes, i.e., vehicles, the probability of packet loss increases as well, [9], [10].

A promising solution to increase the robustness against packet loss in terms of control performance is presented in [11] and applied to CACC in [8]. In the latter, a Model Predictive Control (MPC) controller is implemented and a vector of *predicted* intended accelerations is shared by the Vehicle-to-Vehicle (V2V) communication. This vector is stored in a buffer in the following, i.e., receiving, vehicle, and in case of packet dropouts (up to a certain fixed number), the vector of predictions is utilized.

To further exploit this benefit of robustness for packet dropouts, and to allow the addition of state and input constraints, this paper focuses on an MPC approach for string stable CACC. A strategy to include string stability for an MPC-based CACC by means of constraints is proposed in [12]. Here, the constraints are based on the assumption that the following vehicle should mimic the behavior of its predecessor. [13] states that the (predicted) acceleration of the following vehicle should be smaller than the maximal acceleration of its predecessor in the (recent) past. Based on experiments, it is observed that for certain situations this approach results in string stable behavior, but no proof of string stability is presented. Further, the forecasted intentions are not shared, which makes the approach less robust against packet loss. In [14], also the frequency-domain string stability properties of the unconstrained controller are considered. During the design of this MPC controller, the direct relation between the

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design parameters of the controller (the weightings in the cost function) and the string stability property is lost, which makes it difficult to tune, while maintaining string stability.

Most of the above mentioned research assumes homogeneous traffic. However, it is essential to focus on heterogeneous traffic in order to support application of CACC in a realistic scenario. [15] proposes to share dynamic vehicle properties to allow for heterogeneous platooning. However, in general, truck and car manufactures require that these parameters are to be treated confidentially. [16] applies a distributed MPC approach to a simulated heterogeneous platoon, but does not consider string stability. The combination of multi-brand platooning and string stability has been investigated by [17]–[19]. However, robustness against packet loss is not included in the control approaches and no experimental results are presented in these works. [20] proposes to share prediction horizons between vehicles, and in [21] this prediction is used in an MPC design for heterogeneous trucks with the focus on reducing fuel consumption, but the string stability property is not studied. [22] studies string stability for heterogeneous platoons and applies an MPC approach, but requires leader to platoon communication at initialization, which might be impractical, especially when considering large platoons.

To summarize, the main contribution of this paper is an integrated control approach and its experimental results, where this integrated approach combines solutions aiming at: 1) string stability for nominal operational conditions, 2) applicability in heterogeneous platoons, 3) maintaining confidentiality of dynamic vehicle parameters and 4) improving robustness against (short periods of) packet loss. Under nominal conditions, it is assumed that the vehicle does not exceed actuator constraints or safety distances. In case of non-nominal conditions, in which, e.g., critical safety constraints becomes active, string stability is considered to be of lower priority. Hereto, conditions for both $\ell_2$ and $\ell_\infty$ string stability are derived and design rules for the MPC approach, guaranteeing string stability, are provided. Finally, this controller is implemented in experimental vehicles, which shows, also in practice, string stable behavior.

The paper is organised as follows. The control problem description is given in Section II. Next, the model predictive controller design is presented in Section III, followed by the derived string stability conditions in Section IV. Illustrative simulation studies are presented in Section V and experimental results are discussed in Section VI. Finally, the conclusions are given in Section VII.

## II. Control Problem Description

Let us consider a platoon of $S_p \in \mathbb{N}$ heterogeneous vehicles. Each vehicle is indexed with $i \in S_n$ with

$$S_n = \{i \in \mathbb{N} | 1 \leq i \leq S_p\}.$$

Vehicle $i$ directly follows vehicle $i-1$. This paper focuses only on the longitudinal vehicle dynamics, which is based upon the driveline dynamics and an actuator delay. The adopted vehicle model of vehicle $i$ in continuous time is defined as follows:

$$\dot{a}_i(t) = -\frac{1}{\tau_i} a_i(t) + \frac{1}{\tau_i} u_i(t - \phi_i), \tag{1}$$

with $\dot{a}_i(t)$ the time-derivative of the acceleration at time $t$, $a_i(t)$ the acceleration, $\tau_i$ a time constant representing the (heterogeneous) drive-line dynamics, $\phi_i$ the (heterogeneous) actuator delay and $u_i$ the intended acceleration, i.e., the input to the driveline. Each vehicle has minimal and maximal acceleration constraints:

$$a_{\min,i} \leq a_i(t) \leq a_{\max,i}, \forall i \in S_n \text{ and } \forall t \in \mathbb{R}, \tag{2}$$

with $a_{\min,i}$ and $a_{\max,i}$, respectively, the physical maximum deceleration and maximum acceleration of vehicle $i$. The velocity of vehicle $i$ is defined as $v_i$. Further, the vehicles are assumed to move forwards and to exhibit a maximum allowed velocity $v_{\max,i}$, which is formalized by the following constraint:

$$0 \leq v_i(t) \leq v_{\max,i}, \quad \forall i \in S_n \text{ and } \forall t \in \mathbb{R}. \tag{3}$$

The distance $d_i$ is defined between the center front bumper of a following vehicle $i$ and the center rear bumper of its preceding vehicle $i-1$. For each vehicle $i$, the main objective is to longitudinally follow the preceding vehicle $i-1$ at the desired distance $d_{r,i}$, with

$$d_{r,i}(t) = r_i + h_i v_i(t).$$

and $r_i \in \mathbb{R}^+$ the desired standstill distance and $h_i \in \mathbb{R}^+$ the desired time gap towards vehicle $i-1$. In [23], it is shown that this spacing policy improves string dynamics (i.e., string stability properties) when compared to a constant distance spacing policy. The first vehicle in the platoon ($i = 1$), follows a mixed traffic participant (a non-equipped vehicle) if it exists, otherwise, it follows a desired constant cruise speed.

The control error $e_i(t)$ can now be introduced as:

$$e_i(t) = d_i(t) - d_{r,i}(t) = d_i(t) - r_i - h_i v_i(t). \tag{4}$$

The objective is to asymptotically realize the desired spacing for the vehicles described with (1)–(3):

$$\lim_{t \longrightarrow \infty} e_i(t) = 0 \;\; \forall i \in S_n,$$

while safety is realized by demanding a minimum distance $d_{i,\min}$ for all safe initial situations:

$$d_i(t_0) \geq d_{i,\min} \Rightarrow d_i(t) \geq d_{i,\min} \;\; \forall i \in S_n \;\; \forall t > t_0. \tag{5}$$

If safety allows, the string stability condition needs to be guaranteed, which will be defined in detail in Section IV.

To realize the control objective in a heterogeneous setting, it is assumed that the future accelerations, for a limited prediction horizon, of the leader are known.

## III. Model Predictive Control Design

This section describes a Model Predictive Controller (MPC) design for the control problem described in the previous section. The reason for proposing an MPC-based design is twofold. Firstly, the MPC design is beneficial for improving the robustness against packet loss (as shown in [8]). Secondly, the

MPC approach can directly accommodate constraints related to safety and actuator limitations. A hierarchy in the requirements on safety and actuator limits, on the one hand, and string stability on the other hand can be defined, where the former is considered more important. In this section, the structure of the control law and the closed-loop model description are given. Next, the prediction model, the objective function to be minimized and the constraints, jointly forming the optimization problem underlying the MPC design, are presented. Moreover, an explicit unconstrained control law is derived, which will be used for the string stability analysis in Section IV.

### A. Control Law Design

The control law is derived based on the formulation of error dynamics, as in [8], leading to the following time-derivative of the plant input $u_i$:

$$\dot{u}_i(t) = \frac{1}{h_i} q_i(t) - \frac{1}{h_i} u_i(t), \tag{6}$$

with $q_i(t)$ to be computed by the controller.

### B. Closed-Loop Model Description

Based on (4) and (6), a continuous state-space model is derived. In this section, its discrete representation based on exact discretization is presented. For each vehicle $i \in S_n$, the discrete state is denoted as $x_i(k) = [e_i(k)\Delta e_i(k) a_i(k) u_i(k - \phi_i^d) q_i(k - \phi_i^d) \dots q_i(k-1)]^\top \in \mathbb{R}^{(\phi_i^d+4)\times 1}$, in which $\Delta e_i(k)$ is the discrete-time equivalent of state $\dot{e}_i(t)$ and the actuator delay $\phi_i$ is approximated by the discrete-time delay $\phi_i^d$, defined as the closest integer multiple of the sampling time, $\phi_i^d = \left\lfloor \frac{\phi_i}{t_s} \right\rceil \in \mathbb{N}_0$. The corresponding system dynamics and matrices including the actuator delay are given by:

$$x_i(k+1) = A_i x_i(k) + B_i q_i(k) + E_i a_{i-1}(k), \tag{7}$$

with $A_i \in \mathbb{R}^{(\phi_i^d+4)\times(\phi_i^d+4)}$, $B_i \in \mathbb{R}^{(\phi_i^d+4)\times 1}$ and $E_i \in \mathbb{R}^{(\phi_i^d+4)\times 1}$, these system matrices defined as

$$A_i = \begin{bmatrix} A_{i,1} & B_{i,1} & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, E_i = \begin{bmatrix} E_{i,1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

The discretized matrices $A_{i,1}$, $B_{i,1}$ and $E_{i,1}$ following from exact discretization are defined as in (8) shown at the bottom of this page, and by

$$B_{i,1} = \begin{bmatrix} \tau_i^2 e^{-t_s/\tau_i} + \tau_i t_s - \tau_i^2 - t_s^2/2 \\ \tau_i - t_s - \tau_i e^{-t_s/\tau_i} \\ \frac{h_i - \tau_i - h_i e^{-t_s/h_i} + \tau_i e^{-t_s/\tau_i}}{h_i - \tau_i} \\ 1 - e^{-t_s/h_i} \end{bmatrix} \text{ and } E_{i,1} = \begin{bmatrix} \frac{t_s^2}{2} \\ t_s \\ 0 \\ 0 \end{bmatrix}.$$

### C. Prediction Model

The predicted future states of vehicle $i$ are denoted as $X_i(k) = [\hat{x}_i(k+1|k)^\top \hat{x}_i(k+2|k)^\top \dots \hat{x}_i(k+N|k)^\top]^\top \in \mathbb{R}^{(\phi_i^d+4)N\times 1}$ with $N \in \mathbb{N}$ the prediction horizon and $\hat{x}_i(k+j|k)$ the estimated states of vehicle $i$ at time $k+j$ evaluated at time $k$. The measured states at time $k$ are denoted as $\hat{x}_i(k|k) = x_i(k)$. It should be noted that the prediction horizon should be large enough to cover differences in actuator delays between heterogeneous vehicles. Also the choice of communication frequency influences $N$. To predict these future states, it is assumed that the predecessor shares a prediction vector of its future accelerations through wireless communication with the following vehicle, as an extension on the current ETSI message set, [24]. This vector of predicted accelerations, $\mathcal{A}_{i-1}(k-\theta)$, obtained from vehicle $i-1$, is defined as: $\mathcal{A}_{i-1}(k-\theta) = [\hat{a}_{i-1}(k-\theta|k-\theta)\hat{a}_{i-1}(k+1-\theta|k-\theta) \dots \hat{a}_{i-1}(k+N-1-\theta|k-\theta)]^\top \in \mathbb{R}^{N\times 1}$ with $\theta \in \mathbb{N}_0$ the discretized communication delay, which is assumed to be constant and equal to the average communication delay. Assuming packets to arrive uniformly during each communication time step, the average communication delay equals half of the communication time step. Further, $\hat{a}_{i-1}(k+j-\theta|k-\theta)$ is the predicted acceleration of vehicle $i-1$ at time $k+j-\theta$ evaluated at time $k-\theta$. Using this vector, the prediction model can be written in compact form as follows:

$$X_i(k) = \Phi_i x_i(k) + \Gamma_i \Delta U_i(k) + \Gamma_{d,i} \mathcal{A}_{i-1}(k-\theta), \tag{9}$$

in which $\Delta U_i(k) = [\Delta\hat{q}_i(k|k)\Delta\hat{q}_i(k+1|k) \dots \Delta\hat{q}_i(k+N-1|k)]^\top \in \mathbb{R}^{N\times 1}$, with $\Delta\hat{q}_i(k+j|k) = \hat{q}_i(k+j|k) - \hat{q}_i(k+j-1|k)$ the predicted rate of change of the control input at time $k+j$, evaluated at time $k$. Further, $\Phi_i \in \mathbb{R}^{(\phi_i^d+4)N\times(\phi_i^d+4)}$, $\Gamma_i \in \mathbb{R}^{(\phi_i^d+4)N\times N}$ and $\Gamma_{d,i} \in \mathbb{R}^{(\phi_i^d+4)N\times N}$ are defined as

$$\Phi_i = \begin{pmatrix} A_i \\ A_i^2 \\ \vdots \\ A_i^N \end{pmatrix}, \Gamma_i = \begin{pmatrix} B_i & 0 & \dots & 0 \\ A_i B_i & B_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_i^{N-1} B_i & A_i^{N-2} B_i & \dots & B_i \end{pmatrix}$$

$$A_{i,1} = \begin{bmatrix} 1 & t_s & h_i t_s - h_i\tau_i - \tau_i^2 e^{-t_s/\tau_i} - \tau_i t_s + \tau_i^2 + h_i\tau_i e^{-t_s/\tau_i} & h_i\tau_i - h_i t_s - h_i\tau_i e^{-t_s/\tau_i} \\ 0 & 1 & h_i - \tau_i - h_i e^{-t_s/\tau_i} + \tau_i e^{-t_s/\tau_i} & h_i e^{-t_s/\tau_i} - h_i \\ 0 & 0 & e^{-t_s/\tau_i} & \frac{h_i e^{-t_s/h_i} - h_i e^{-t_s/\tau_i}}{h_i - \tau_i} \\ 0 & 0 & 0 & e^{-t_s/h_i} \end{bmatrix} \tag{8}$$

$$\text{and } \Gamma_{d,i} = \begin{pmatrix} E_i & 0 & \dots & 0 \\ A_i E_i & E_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_i^{N-1} E_i & A_i^{N-2} E_i & \dots & E_i \end{pmatrix}.$$

Hence, the states can be predicted based on the obtained prediction vector of the predecessor's acceleration and the current states of the vehicle.

### D. Cost Function

The main objective is to achieve vehicle following, which is realized by minimization of the errors ($e_i$ and $\Delta e_i$). Further, fuel consumption and comfort are optimized by minimization of the absolute value of the control action, and by minimizing the rate of change of the control action. All these objectives are represented in the minimization of the cost function $J_i$, which depends on the current measured state $x_i(k)$ and predicted disturbance $\mathcal{A}_{i-1}(k-\theta)$:

$$\min_{\Delta U_i(k)} J_i(x_i(k), \Delta U_i(k), \mathcal{A}_{i-1}(k-\theta))$$

with

$$\begin{aligned} &J_i\Big(x_i(k), \Delta U_i(k), \mathcal{A}_{i-1}(k-\theta)\Big) \\ &= \hat{x}_i(k+N|k)^\top P_i \hat{x}_i(k+N|k) \\ &\quad + \sum_{j=0}^{N-1} \big(\hat{x}_i(k+j|k)^\top Q_i \hat{x}_i(k+j|k) \\ &\quad + \Delta\hat{q}_i(k+j|k)^\top R_{\Delta,i} \Delta\hat{q}_i(k+j|k)\big). \end{aligned} \tag{10}$$

and with the weighting matrix

$$Q_i = \mathrm{diag}\mathrm{w}_{1,\mathrm{i}}, \mathrm{w}_{2,\mathrm{i}}, 0 \dots, 0, \mathrm{R_i} \in \mathbb{R}^{(4+\phi_\mathrm{i}^\mathrm{d})\times(4+\phi_\mathrm{i}^\mathrm{d})},$$

in which $w_{1,i}$ and $w_{2,i}$ represent the weighting on error and the first time-derivative of error, respectively. Further, $R_i$ represents the weight on the control input $q_i$, $R_{\Delta,i}$ represents the weighting on rate of change of the input and the matrix $P_i$ is the terminal state penalty matrix.

### E. Explicit Controller Design for the Unconstrained Problem

Here, we consider the problem of the cost function optimization in (10) without the enforcement of constraints. In Section IV, we will show under which tuning conditions (on the weighting matrices in (10)) this explicit controller guarantees string stability in nominal operation.

We adopt the practically relevant perspective that in nominal operation (no emergency braking, leading to unsafe situations and actuator saturations), the constraints are not addressed. For such nominal operation, the proposed approach guarantees string stability, which is highly valuable in practice. In particular, locally around the nominal (constant velocity) solution (i.e., for small state perturbations and input perturbations), the actuator/safety limits are not addressed and string stability is indeed ensured. We argue that when safety limits are exceeded, then string stability is of less importance and the controller should just ensure safety while taking into account actuator constraints. E.g., when the safety constraint in (5), or any of the vehicle constraints, becomes active, string stability is no longer of concern and the goal of the MPC controller (with constraints taken into account) is primarily to ensure satisfaction of the (safety) constraints while optimizing the objective in (10). Hereto, (10) is written in compact form as follows:

$$\begin{aligned} J_i\Big(x_i(k), \Delta U_i(k), \mathcal{A}_{i-1}(k-\theta)\Big) &= x_i(k)^\top Q_i x_i(k) \\ &\quad + X_i(k)^\top \Omega_i X_i(k) + \Delta U_i(k)^\top \Psi_i \Delta U_i(k), \end{aligned}$$

with $\Omega_i = \mathrm{diag}\mathrm{Q_i}, \dots, \mathrm{Q_i}, \mathrm{P_i} \in \mathbb{R}^{(4+\phi_\mathrm{i}^\mathrm{d})\mathrm{N}\times(4+\phi_\mathrm{i}^\mathrm{d})\mathrm{N}}$, and $\Psi_i = \mathrm{diag}\mathrm{R}_{\Delta,\mathrm{i}}, \dots, \mathrm{R}_{\Delta,\mathrm{i}} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$. By using the prediction model in (9) and removing the constant parts, the cost function can be rewritten to the following final cost function $J_i^{\mathrm{fin}}$ (note that $\Omega_i$ and $\Psi_i$ are diagonal and $\Gamma_i$ is lower triangular, so the commutativity property holds):

$$\begin{aligned} &J_i^{\mathrm{fin}}\Big(x_i(k), \Delta U_i(k), \mathcal{A}_{i-1}(k-\theta)\Big) \\ &= \frac{1}{2}\Delta U_i^\top(k) G_i \Delta U_i(k) + \Delta U_i^\top(k) F_i x_i(k) \\ &\quad + \Delta U_i^\top(k) H_i \mathcal{A}_{i-1}(k-\theta), \end{aligned}$$

given the following definitions of $G_i \in \mathbb{R}^{N\times N}$, $F_i \in \mathbb{R}^{N\times(4+\phi_i^d)}$ and $H_i \in \mathbb{R}^{N\times N}$:

$$G_i := 2\big(\Psi_i + \Gamma_i^\top \Omega_i \Gamma_i\big), F_i := \big(2\Gamma_i^\top \Omega_i \Phi_i\big) \text{ and}$$

$$H_i := \big(2\Gamma_i^\top \Omega_i \Gamma_{d,i}\big).$$

By choosing $Q_i \succeq 0$, $R_i > 0$, $R_{\Delta,i} > 0$ and $P_i \succeq 0$ this cost function is convex, so there exists a unique global minimum, which can be used to obtain the unconstrained explicit control law. Since the matrix $G_i$ is invertible ($\Psi_i \succ 0$ because $R_{\Delta,i} > 0$, and $\Omega_i \succeq 0$ because $Q_i \succeq 0$ and $P_i \succeq 0$), the optimal input rate for the delayed (extended) system can be written as follows:

$$\Delta q_i^*(k) = K_i^{ff} \mathcal{A}_{i-1}(k-\theta) + K_i^{fb} x_i(k), \tag{11}$$

which consists of a feedforward part $K_i^{ff}\mathcal{A}_{i-1}(k-\theta)$, based on the predicted disturbance, and a feedback part $K_i^{fb} x_i(k)$, based on the system's current states, with $K_i^{ff} = -\big(1\ 0\ \dots\ 0\big) G_i^{-1} H_i$, and $K_i^{fb} = -\big(1\ 0\ \dots\ 0\big) G_i^{-1} F_i$. By adding this value to the previous input, the optimal input for the system is obtained as follows:

$$q_i^*(k) = q_i(k-1) + \Delta q_i^*(k). \tag{12}$$

### F. Constraints

The inter-vehicle distance and velocity constraints are implemented by applying dynamic constraints. All constraints are written into a compact form, such that these can be implemented in a Quadratic Programming (QP) optimization problem, as follows:

$$L_i \Delta U_i(k) \le c_i(k) + W_i x_i(k) - \mathcal{M}_i \Gamma_{d,i} \mathcal{A}_{i-1}(k-\theta), \tag{13}$$

with $L_i = \mathcal{M}_i \Gamma_i \in \mathbb{R}^{5N\times N}$, $W_i = -\mathcal{D}_i - \mathcal{M}_i \Phi_i \in \mathbb{R}^5 N \times (4+\phi_i^d)$, $\mathcal{M}_i \in \mathbb{R}^{5N\times(\phi_i^d+4)N}$, $M_{i,a}, M_{i,v} \in \mathbb{R}^{5\times N}$ and $c_i \in$

$\mathbb{R}^{5N\times 1}$:

$$\mathcal{M}_i = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ M_{i,a} & 0 & 0 & \dots & 0 \\ M_{i,v} & M_{i,a} & 0 & \dots & 0 \\ M_{i,v} & M_{i,v} & M_{i,a} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{i,v} & M_{i,v} & M_{i,v} & \dots & M_{i,a} \end{bmatrix}, \quad \mathcal{D}_i = \begin{bmatrix} M_{i,a} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and $c_i(k) = [b_i(k)^\top b_i(k)^\top \dots b_i(k)^\top]^\top$. In these matrices $M_{i,a}$ contains the part dependent on the predicted states and $M_{i,v}$ represents the Euler approximation of the change in velocity. Furthermore, $b_i(k)$ contains the constant terms and the measured velocities. These matrices are defined as follows:

$$M_{i,a} = \begin{bmatrix} 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 0 & \dots & 0 \\ -1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix},$$

$$M_{i,v} = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & -h_i t_s & 0 & \dots & 0 \\ 0 & 0 & -t_s & 0 & \dots & 0 \\ 0 & 0 & t_s & 0 & \dots & 0 \end{bmatrix}$$

and $b_i(k) = [a_{\max,i}, -a_{\min,i}, r + h_i v_i(k) - d_{i,\min}, v_i(k), v_{\max,i} - v_i(k)]^\top$. By updating the values in $b_i(k)$, at every time step, the constrained MPC problem can be implemented as a QP optimization problem by applying the mpcqpsolver of MATLAB [25], which yields fast computations.

## IV. Discrete-Time String Stability

Motivated by the considerations laid out at the beginning of Section III and in Section III-E, we now show how string stability of the unconstrained system can be guaranteed by design.

In case the constraints are met, it is desired to ensure string stable platooning. Based on the work in [6] and [26], two different types of string stability are considered here, namely $\mathcal{L}_2$ string stability and $\mathcal{L}_\infty$ string stability. In general, $\mathcal{L}_2$ string stability is less strict than $\mathcal{L}_\infty$ string stability. In [6], the conditions for $\mathcal{L}_2$ and $\mathcal{L}_\infty$ are presented for cascades of continuous-time systems. $\mathcal{L}_2$ and $\mathcal{L}_\infty$ string stability are system (platoon) properties which prevent amplification of the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norm of vehicle responses in upstream direction. In Sections IV-A and IV-B, the conditions for these types of string stability for a *discrete-time* closed-loop system representation are derived. In Sections IV-C and IV-D, the relation between string stability, the weighting parameter on the input ($R_i$) and the time gap $h_i$ is analyzed in detail, which supports the design of an MPC controller guaranteeing string stability by design (for the case without constraints).

The acceleration propagation along the string of vehicles is studied in this section. The output acceleration of vehicle $i$ equals $y_i(k) = C_i x_i(k)$ with

$$C_i = [0\ 0\ 1\ 0 \dots 0]^\top \in \mathbb{R}^{(\phi_i^d+4)\times 1}. \tag{14}$$

### A. $\ell_2$ String Stability

We introduce $y_i(z) = \mathcal{Z}\{y_i(k)\}$, where $z$ is the $\mathcal{Z}$-transform variable. Further, we define the discrete-time transfer function $\Gamma_i(z)$ of the closed-loop cascaded control system, such that $y_i(z) = \Gamma_i(z) y_{i-1}(z)$. This discrete-time transfer function $\Gamma_i(z)$ will be derived in detail for the closed-loop platooning system described in (7) with the optimal control law of (11) and (12) in Section IV-C.

A system is defined to be $\ell_2$ string stable if the energy represented by the $\ell_2$-norm of the output $y_i$ does not increase in upstream direction over a string of vehicles:

$$\sup_{y_{i-1}\neq 0} \frac{\|y_i(k)\|_{\ell_2}}{\|y_{i-1}(k)\|_{\ell_2}} \leq 1,$$

where the $\ell_2$-norm of a discrete-time signal $x(k)$ is defined as $||x||_{\ell_2} = (\sum_{k\in\mathbb{Z}} |x(k)|^2)^{1/2}$.

As also used in [27], discrete-time $\ell_2$ string stability is analyzed based on the magnitude of the discrete-time string stability transfer function $\Gamma_i(z)$.

A closed-loop control system is $\ell_2$ string stable if and only if the following property for the transfer function holds:

$$\|\Gamma_i(z)\|_{\mathcal{H}_\infty} \leq 1, \forall i \in \mathbb{N}, \tag{15}$$

where the $\mathcal{H}_\infty$-norm is defined as, [28]:

$$||\Gamma_i(z)||_{\mathcal{H}_\infty} := \sup_{y_{i-1}\neq 0} \frac{||\Gamma_i(z) y_{i-1}(z)||_{\ell_2}}{||y_{i-1}(z)||_{\ell_2}}.$$

The condition for $\ell_2$ string stability in (15) will be derived in two steps. First, we will show that $\sup_{y_{i-1}\neq 0} \frac{\|y_i(k)\|_{\ell_2}}{\|y_{i-1}(k)\|_{\ell_2}} \leq \|\Gamma_i(z)\|_{\mathcal{H}_\infty}$, and secondly that $\|\Gamma_i(z)\|_{\mathcal{H}_\infty}$ is a tight upper bound. Applying the definition of the squared $\ell_2$ vector norm to Parseval's Theorem [29] for discrete-time signals, gives:

$$\|y_i(k)\|_{\ell_2}^2 = \sum_{n=-\infty}^{\infty} |y_i(k)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y_i(e^{j\omega})|^2 d\omega,$$

with $Y_i$ the discrete-time Fourier transform of $y_i$ and $\omega$ the angular frequency in radians per sample. The output $y_i(k)$ is then bounded according to:

$$\begin{aligned} \|y_i(k)\|_{\ell_2}^2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y_i(e^{j\omega})|^2 d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\Gamma_i(e^{j\omega}) Y_{i-1}(e^{j\omega})|^2 d\omega \\ &\leq \frac{1}{2\pi}\int_{-\pi}^{\pi} (\|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty} |Y_{i-1}(e^{j\omega})|)^2 d\omega \\ &= \|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}^2 \|y_{i-1}(k)\|_{\ell_2}^2. \end{aligned}$$

If we can show that a signal $y_{i-1}(k)$ exists, such that the inequality becomes an equality, we have shown that $\|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}$ is the tightest upper bound of $\|y_i(k)\|_{\ell_2}/\|y_{i-1}(k)\|_{\ell_2}$. Let us

choose $y_{i-1}(t)$ such that:

$$|Y_{i-1}(e^{j\omega})|=\begin{cases}\sqrt{\pi/(2\epsilon)}, & \text{if } |\omega-\omega_0|<\epsilon \text{ or } |\omega+\omega_0|<\epsilon\\ 0, & \text{otherwise}\end{cases} \tag{16}$$

with $\epsilon$ a positive number approaching zero ($\epsilon \to 0^+$) and $\omega_0$ is chosen such that $|\Gamma_i(e^{j\omega_0})| = \|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}$, i.e., the transfer function $\Gamma_i$ has maximum gain at this frequency. Choosing $y_{i-1}(k)$ such that (16) holds implies that $y_{i-1}(k)$ has unit $\ell_2$ norm (Parseval's Theorem) i.e., $\|y_{i-1}(e^{j\omega})\|_{\ell_2}^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}|Y_{i-1}(e^{j\omega})|^2 d\omega = 1$. Then

$$\begin{aligned}\|y_i(k)\|_{\ell_2}^2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi}|Y_i(e^{j\omega})|^2 d\omega\\ &= \lim_{\epsilon\to 0^+}\frac{1}{2\pi}\left(\int_{-\omega_0-\epsilon}^{-\omega_0+\epsilon}|\Gamma_i(e^{j\omega})Y_{i-1}(e^{j\omega})|^2 d\omega\right.\\ &\qquad\left.+\int_{\omega_0-\epsilon}^{\omega_0+\epsilon}|\Gamma_i(e^{j\omega})Y_{i-1}(e^{j\omega})|^2 d\omega\right)\\ &= \|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}^2\frac{1}{2\pi}\int_{-\pi}^{\pi}|Y_{i-1}(e^{j\omega})|^2 d\omega\\ &= \|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}^2\end{aligned}$$

holds, which shows that $\|\Gamma_i(e^{j\omega})\|_{\mathcal{H}_\infty}$ is a tight upper bound for the proposed signal. Therefore, we know that it is the smallest possible upper-bound of $\frac{\|y_i(k)\|_{\ell_2}}{\|y_{i-1}(k)\|_{\ell_2}}$.

### B. $\ell_\infty$ *String Stability*

The closed-loop cascaded control system is defined to be $\ell_\infty$ string stable if the $\ell_\infty$-norm of the output signal decays or remains the same in upstream direction of the cascade of systems (string of vehicles in the scope of platooning):

$$\max_{y_{i-1}\neq 0}\frac{\|y_i(k)\|_{\ell_\infty}}{\|y_{i-1}(k)\|_{\ell_\infty}} \leq 1,$$

with the $\ell_\infty$-norm of a discrete-time signal $y_i(k)$ defined as $\|y_i(k)\|_{\ell_\infty} = \max_k |y_i(k)|$. The $\ell_\infty$ string stability property is analyzed based on the magnitude of the discrete-time impulse response of $\Gamma_i(z)$. Define $\gamma_i(k)$ as the discrete-time impulse response of the transfer function $\Gamma_i(z)$, such that the following discrete-time convolution holds:

$$\begin{aligned}y_i(k) &= \gamma_i(k) * y_{i-1}(k)\\ &= \sum_{m=-\infty}^{\infty}\gamma_i(m)y_{i-1}(k-m)\\ &= \sum_{m=-\infty}^{\infty}\gamma_i(k-m)y_{i-1}(m).\end{aligned}$$

The impulse response $\gamma_i$ can be determined numerically by applying a Kronecker delta as input to the state-space representation of the cascaded system. The closed-loop system is $\ell_\infty$ string stable if and only if

$$\sum_{k=0}^{\infty}|\gamma_i(k)| \leq 1, \forall i \in \mathbb{N}. \tag{17}$$

Numerically, this condition can be checked by replacing the infinite sum by a limited sum up to $k_{\text{end}}$, with $k_{\text{end}}$ sufficiently large such that the impulse response has converged to zero for $k > k_{\text{end}}$ (within some small bounds). The $\ell_\infty$ string stability condition in (17) is based on the Young's inequality for convolutions, [30], applied to a discrete-time system, which states that a system's output is bounded by its input as follows:

$$\|\gamma_i(k) * y_{i-1}(k)\|_{\ell_\infty} \leq \|\gamma_i(k)\|_{\ell_1}\|y_{i-1}(k)\|_{\ell_\infty},$$

with the $\ell_1$-norm defined as $\|y_i(k)\|_{\ell_1} = \sum_{k=0}^{\infty}|y_i(k)|$. This leads to the following inequality:

$$\frac{\|y_i(k)\|_{\ell_\infty}}{\|y_{i-1}(k)\|_{\ell_\infty}} \leq \|\gamma_i(k)\|_{\ell_1}.$$

From this we can conclude that if $\|\gamma_i(k)\|_{\ell_1} \leq 1$, then it also holds that $\frac{\|y_i(k)\|_{\ell_\infty}}{\|y_{i-1}(k)\|_{\ell_\infty}} \leq 1$. Now, if we can find a signal $y_{i-1}(k)$ such that $\|y_i(k)\|_{\ell_\infty} = \|\gamma_i(k)\|_{\ell_1}$, then we have shown that $\|\gamma_i(k)\|_{\ell_1}$ is the smallest possible upper bound of $\frac{\|y_i(k)\|_{\ell_\infty}}{\|y_{i-1}(k)\|_{\ell_\infty}}$. Define $y_{i-1}(k)$ such that

$$|y_{i-1}(m)| = \begin{cases}\left(\sum_{k=0}^{\infty}|\gamma_i(k)|\right)/\max_k|\gamma_i(k)|, & \text{if } m=0,\\ 0 & \text{otherwise.}\end{cases} \tag{18}$$

Then,

$$\begin{aligned}\|y_i(k)\|_{\ell_\infty} &= \max_k\left|\sum_{m=-\infty}^{\infty}\gamma_i(k-m)y_{i-1}(m)\right|\\ &= \max_k|\gamma_i(k)\left(\sum_{k^*=0}^{\infty}|\gamma_i(k^*)|\right)/\max_{k^*}|\gamma_i(k^*)||\\ &= \sum_{k=0}^{\infty}|\gamma_i(k)| = \|\gamma_i(k)\|_{\ell_1},\end{aligned}$$

which shows that $\|\gamma_i(k)\|_{\ell_1}$ is a tight upper bound.

Using these results, it is possible to analyze the string stability properties of the unconstrained MPC controlled heterogeneous platooning system, as detailed in the next sub-section.

### C. *String Stability of the Unconstrained System*

The derivation of the discrete-time transfer function from $a_{i-1}(k)$ to $a_i(k)$, (which equals the discrete-time transfer function from $v_{i-1}(k)$ to $v_i(k)$), will be studied in this section. This transfer function can be used to find weighting parameters for the objective function in (10) that (a priori) guarantee ($\ell_\infty$ and/or $\ell_2$) string stability of the unconstrained MPC controlled platooning system. Since all constraints as presented in (13) are considered to be inactive, the unconstrained explicit MPC control law is applied. By combining the closed-loop dynamics and the explicit control law, the following closed-loop dynamics are

obtained:

$$x_i(k+1) = A_i x_i(k) + B_i\big(K_i^{ff}\mathcal{A}_{i-1}(k-\theta) + K_i^{fb}x_i(k)\big) + E_i a_{i-1}(k).$$

The prediction vector $\mathcal{A}_{i-1}(k)$ is obtained by selecting the corresponding state from the prediction vector as follows:

$$\mathcal{A}_{pred,i-1}(k) = \begin{bmatrix} C_i & 0 & \dots & 0 & 0 \\ 0 & C_i & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & C_i & 0 \end{bmatrix} X_{i-1}(k)$$

and including the measured acceleration, measured at vehicle $i-1$, as the first element of this vector:

$$\mathcal{A}_{i-1}(k) = \begin{bmatrix} a_{i-1}(k) & \mathcal{A}_{pred,i-1}^{\top}(k) \end{bmatrix}^{\top}.$$

Now, the closed-loop dynamics of the system can be rewritten to:

$$\begin{aligned} x_i(k+1) &= \big(A_i + B_i K_i^{fb}\big)x_i(k) \\ &+ \big(z^{-\theta}B_i K_i^{ff} + E_i\begin{bmatrix}1 & 0 & \dots & 0\end{bmatrix}\big)\mathcal{A}_{i-1}(k), \\ y_i(k) &= C_i x_i(k), \end{aligned} \tag{19}$$

with $z^{-\theta}$ the discretized communication delay. Using the system dynamics in (19), and the defined output matrix in (14), a MISO transfer function in the $z$-domain can be obtained from $\mathcal{A}_{i-1}(k)$ to $a_i(k)$. To obtain a SISO transfer function from $a_{i-1}(k+N-1)$ to $a_i(k)$, it is assumed that the predictions are correct:

$$\forall i \in 1,\dots,S_p, \forall j \in 1,\dots,N, \forall k \in \mathbb{N} : \hat{x}_i(k+j|k) = x_i(k+j).$$

With this assumption in place, the prediction vector $\mathcal{A}_{i-1}(k)$, which is usually updated every discrete-time step, can be written as a combination of $a_{i-1}(k+N-1|k)$ and delays:

$$\mathcal{Z}\{\mathcal{A}_{i-1}(k)\} = \begin{bmatrix} z^{-N+1} \\ z^{-N+2} \\ \vdots \\ z^0 \end{bmatrix} \mathcal{Z}\{a_{i-1}(k+N-1|k)\}.$$

Now, the transfer function from $a_{i-1}(k+N-1)$ to $a_i(k)$ can be written as:

$$\begin{aligned} \Gamma_i^{\text{MPC}}(z) &:= \frac{\mathcal{Z}\{a_i(k)\}}{\mathcal{Z}\{a_{i-1}(k+N-1)\}} \\ &= C_i\big(zI - (A_i + B_i K_i^{fb})\big)^{-1}(z^{-\theta}B_i K_i^{ff} \\ &+ E_i\begin{bmatrix}1 & 0 \dots 0\end{bmatrix})\begin{bmatrix} z^{-N+1} \\ z^{-N+2} \\ \vdots \\ z^0 \end{bmatrix}. \end{aligned}$$

TABLE I
SYSTEM PARAMETERS FOR THE NUMERICAL ANALYSIS

| Parameter | Value [Unit] |
|---|---|
| $N$ | 30 [-] |
| $t_s$ | 0.01 [s] |
| $\theta$ | 2 [-] |
| $\phi_i$ | 0.2 [s] |
| $\tau_i$ | 0.1 [s] |
| $w_{1,i}$ | 0.4 [-] |
| $w_{2,i}$ | 0.4 [-] |
| $R_{\Delta,i}$ | $2 \times 10^{-4}$ [-] |

Below, we argue that if $\Gamma_i^{MPC}(z)$ fulfills the string stability conditions, the shifted version of the transfer function $\Gamma_i(z) = \frac{\mathcal{Z}\{a_i(k)\}}{\mathcal{Z}\{a_{i-1}(k)\}}$, also fulfills the posed string stability conditions. For $\ell_2$ string stability, it is straightforward that the transfer function $\Gamma_i(z)$ is string stable if the string stability property for the shifted transfer function, $\Gamma_i^{MPC}(z)$, holds. The magnitude of the transfer function does not change, because the shift operator $z^{(N-1)}$ has a magnitude of one at all frequencies. The $\ell_\infty$ string stability conditions can also be applied directly to the shifted transfer function. Because the system dynamics are time-invariant, a time delay in the transfer function simply results in a shifted impulse response with the same amplitude. Since the obtained transfer function is causal, the obtained $\ell_1$ norm is the same for the shifted and non-shifted transfer function: $\|\gamma_i(z)\|_{\ell_1} = \|\gamma_i^{MPC}(z)\|_{\ell_1}$.

### D. Numerical String Stability Analysis

Now the results of a numerical string stability analysis are presented, using the system parameters in Table I. Note that the vehicle-specific parameters in Table I reflect the parameters of a vehicle for which the MPC controller is designed to induce string stability. We care to stress that this can be done individually for any vehicle in the heterogeneous platoon, without sharing (confidential) knowledge on vehicle characteristics between the vehicles, under the assumptions that the required predicted acceleration vector is communicated. The influence of the headway time $h_i$ and the weighting parameter $R_i$ on the string stability properties is considered, since these are directly related to fuel consumption. $w_{1,i}$, $w_{2,i}$ and $R_{\Delta,i}$ are tuned (and fixed) to induce comfortable driving. Furthermore, the weighting on acceleration and intended acceleration are zero and the terminal cost is a zero matrix, $P_i = 0_{24,24}$.

Using the bisection method, for a range of values of $h_i$, the maximal value of $R_i$ is obtained, such that the system is $\ell_2$ or $\ell_\infty$ string stable. The results of this string stability analysis are displayed in Fig. 1. This figure shows three different colored areas; when choosing parameters in the green area, the system is both $\ell_\infty$ and $\ell_2$ string stable, in the orange area the system is $\ell_2$ string stable (and not $\ell_\infty$ string stable) and in the red area the system is string unstable (in both the $\ell_2$ and $\ell_\infty$ sense). This figure clearly shows that relatively high weighting $R_i$ results in loss of string stability. So, the controller parameters have to be chosen to make a trade-off between string stability and fuel

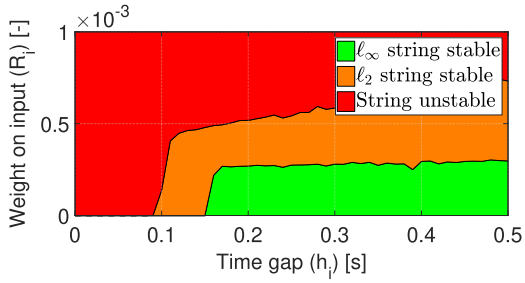


Fig. 1. Relation between weighting $R_i$ of the control input $q_i$, headway time $h_i$, and string stability for a communication rate of 25 Hz. The other parameters, used for this analysis, are given in Table I. Note that in the green area both $\ell_2$ and $\ell_\infty$ string stability is guaranteed.

TABLE II
SYSTEM PARAMETERS FOR THE SIMULATIONS AND EXPERIMENTS

| Parameter | Value [Unit] |
|---|---|
| $h_i$ | 0.3 [s] |
| $r_i$ | 10 [m] |
| $R$ | $2 \times 10^{-5}$ [-] |
| $d_{min,i}$ | 0.5 [m] |
| $v_{max,i}$ | 25 [m/s] |
| $a_{min,i}$ | -6 [m/s$^2$] |
| $a_{max,i}$ | 3 [m/s$^2$] |

TABLE III
COMPARISON OF MINIMUM STRING STABLE TIME GAPS

| Communication rate | Controller | $\ell_2$ | $\ell_\infty$ |
|---|---|---|---|
| 25 Hz | [26] | 0.16 | 0.5 |
| | MPC | 0.1 | 0.16 |
| 10 Hz | [26] | 0.25 | 0.67 |
| | MPC | 0.16 | 0.21 |

consumption, since a high weighting $R_i$ in the cost function penalizes fuel consumption.

The results in Fig. 1 can also be interpreted in another way. For the given parameters, minimal time gaps have been found, such that the system is $\ell_2$ string stable ($h_i = 0.1$ s for $R_i \approx 1.4 \times 10^{-4}$) and $\ell_\infty$ string stable ($h_i = 0.16$ s for $R_i \approx 2.2 \times 10^{-4}$). A comparison to the results of the PD controller in [26] is shown in Table III for two different communication rates (25 Hz and 10 Hz). Clearly, the proposed controller can achieve a significant improvement in minimal time gap, for which string stability is ensured. This improvement is mainly obtained because of the communication of predictions, whereas the PD controlled platoon only communicates the input at the current time step.

*Remark:* Choosing different system parameters (e.g., a higher actuator lag or a lower communication rate) will change the string stability results in Fig. 1; however, the analysis tools provided can readily be used for such parametric analysis.

## V. SIMULATION RESULTS

The goal of the simulation-based case study presented here is two-fold. Firstly, the behavior of the MPC controller with active constraints is verified. Secondly, string stability of a heterogeneous platoon is studied. A platoon of five vehicles is used, with one virtual reference vehicle (vehicle index 0). The virtual reference vehicle follows a precomputed acceleration profile, which is implemented as a (known) disturbance for the first vehicle ($i = 1$).

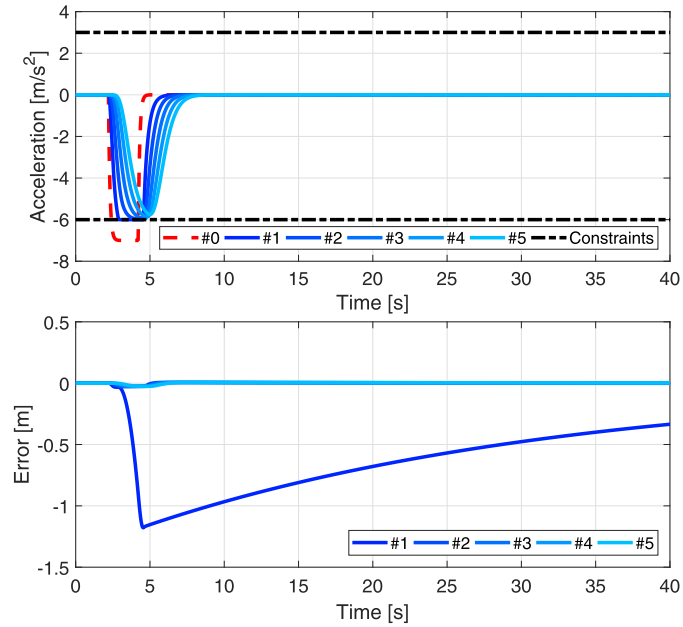


Fig. 2. Simulation of a five vehicle platoon with active acceleration constraints, with in the upper figure the accelerations and in the lower figure the inver-vehicle distance errors.

### A. Constrained Platooning

To show that the controller is satisfying the implemented constraints, a virtual reference is defined which exceeds the acceleration constraint. The controller and system parameters, resulting in an $\ell_\infty$ string stable system, are presented in Tables I and II. The time constant, $\tau_i$ has been chosen to equal 0.1 s, for all $i \in \{1, 2, 3, 4, 5\}$. The acceleration profiles of each vehicle in the platoon are displayed in the upper figure of Fig. 2. It can be concluded that all vehicles follow the reference vehicle, until the reference vehicle exceeds the acceleration constraints. When the virtual reference exceeds the constraint, the platoon has a maximum braking action of $-6$ m/s$^2$. After a couple of seconds, the virtual reference reduces its braking action to satisfy the given constraint and all vehicles start following the reference vehicle again. The inter-vehicle distance error of the platoon is depicted in the lower figure of Fig. 2. It clearly shows that the error of the first vehicle is increasing as expected, because it is unable to achieve the same braking action as its predecessor. After the braking action, the vehicle is reducing the error back to zero. Note that if the predictions are accurate, an increase in the prediction horizon will result in a smaller error (which is in this example almost $-1.2$ m), since this will allow for an earlier brake action to compensate for the acceleration limit.

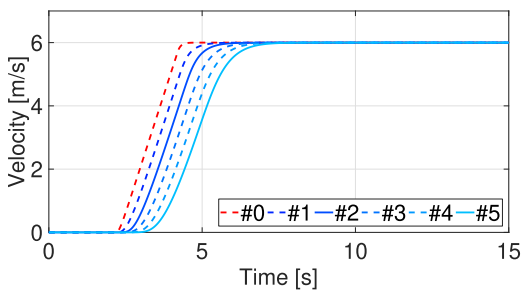


Fig. 3. Velocity of the virtual reference vehicle and a string of five vehicles for an $\ell_\infty$ string stable heterogeneous platoon.

### B. Heterogeneous Platooning

One of the main advantages of the obtained string stability property is that these hold for heterogeneous platoons as well. Therefore, simulations are conducted for a heterogeneous platoon. The time constant $\tau_i$ of vehicles 1, 3 and 4 are chosen different to the time constant of vehicles 2 and 5: $\tau_1 = \tau_3 = \tau_4 = 0.1$ s and $\tau_2 = \tau_5 = 0.2$ s. The other vehicle parameters are given in Tables I and II. The weighting $R_i$ is chosen to equal $2 \times 10^{-7}$ for all vehicles, such that the entire platoon is $\ell_\infty$ string stable (the numerical study as presented in Fig. 1 has also been repeated for $\tau_i = 0.2$ and the weighting parameters are chosen accordingly). The velocity response of each vehicle is shown in Fig. 3. No overshoot occurs, so it can be concluded that the unconstrained heterogeneous platoon indeed behaves $\ell_\infty$ string stable, in the velocity, if the constraints are inactive.

## VI. Experimental Results

The string stability properties of the obtained unconstrained MPC controller are tested in an experimental setting. Two types of experiments are performed. The first experiment compares the proposed controller to that of an existing control strategy to study the improvement in string stability properties for a homogeneous platoon. The second experiment focuses on the string stability properties of a heterogeneous platoon. For all experiments, a platoon of two Toyota Prii and a virtual reference vehicle are used. The Toyota Prii are equipped with CACC technology. More information on the used setup can be found in [26]. Each vehicle is equipped with wireless communication, operating according to the IEEE 802.11p-based ETSI ITS G5 standard, [31]. The message set is slightly extended (as explained in Section III). The communication frequency is chosen equal to 25 Hz, as also applied in the European i-Game project [32].

### A. String Stability Comparison to Previous Work

To verify the improvement with respect to string stability in previous work based on an MPC approach, the results are compared to the controller as designed in [8]. In [8], the buffer approach to compensate packet dropouts is integrated in the MPC design, and no confidential information is shared - covering two of the four main aspects which are integrated in this newly proposed controller. To show that this previous work did not ensure the string stability properties, the experimental results using this (existing) controller are presented in Fig. 4. As shown by the overshoot in velocity, see the zoomed inset of the response between 30–40 s, the platoon does not behave $\ell_\infty$ string stable. The controller as proposed in this paper with controller parameters as given in Tables I and II is implemented in the experimental setup and the same profile for the reference vehicle is repeated. The resulting velocities of the platoon are shown in Fig. 5. Since the overshoot is negligible with respect to the measurement noise, it can be concluded that the system indeed behaves $\ell_\infty$ string stable. Therefore, the newly defined controller outperforms the controller in [8] in terms of string stability properties.

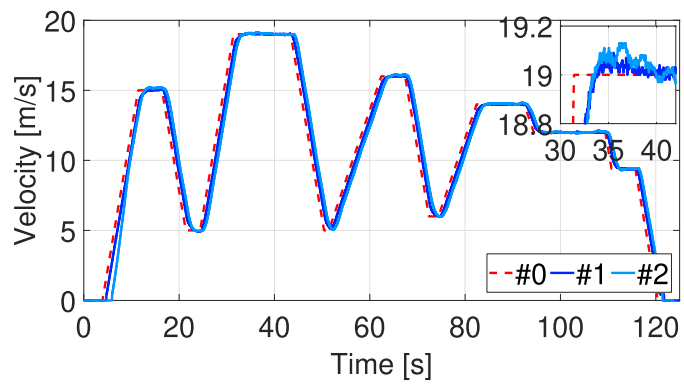


Fig. 4. Experimental results using the controller of [8].

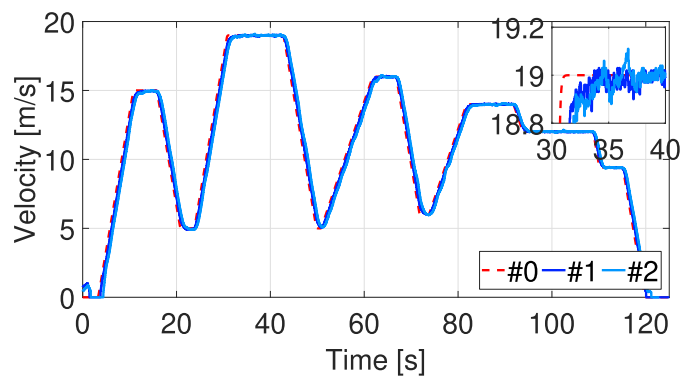


Fig. 5. Experimental results using an $\ell_\infty$ string stable unconstrained controller.

### B. String Stability for a Heterogeneous Platoon

Since two similar vehicles are used in the experimental setup, the vehicle with index 1 is adjusted to behave with time constant $\tau_2 = 0.2$, instead of $\tau_1 = 0.1$ s. In order to emulate such heterogeneity experimentally, the transfer function $L(s) = \frac{1+\tau_1 s}{1+\tau_2 s}$ is applied to the intended acceleration $u_1$, before it enters the driveline. The resulting velocity of the platoon is shown in Fig. 6. No overshoot occurs, so the behavior indeed appears to be $\ell_\infty$ string stable.

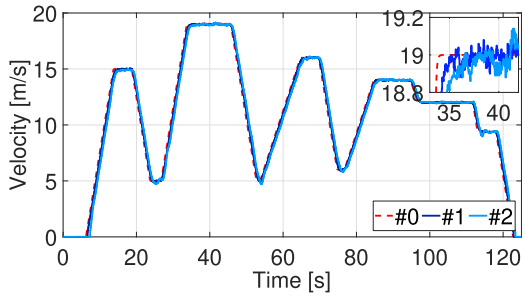


Fig. 6. Experimental results for a heterogeneous platoon.

## VII. Conclusion

In this paper, a novel model predictive control (MPC) strategy for heterogeneous platoons has been proposed with the following benefits. Firstly, conditions for both $\ell_2$ and $\ell_\infty$ string stability are derived and design rules for the MPC approach, guaranteeing string stability, are provided. Secondly, the approach is applicable to heterogeneous platoons without the need of sharing confidential information on vehicle characteristics. Finally, a simulation and experimental study illustrates the above benefits of the proposed approach. A comparison to an existing MPC-implementation as presented in [8] shows that the newly proposed MPC indeed shows string stable behavior, which was not the case for the MPC in [8].

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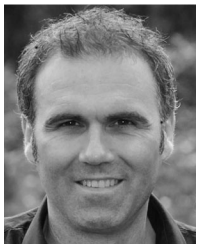

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