Practical String Stability of Platoon of Adaptive Cruise Control Vehicles

Lingyun Xiao and Feng Gao

***Abstract*—In this paper, the practical string stability of both homogeneous and heterogeneous platoons of adaptive cruise control (ACC) vehicles, which apply the constant time headway spacing policy, is investigated by considering the parasitic time delays and lags of the actuators and sensors when building the vehicle longitudinal dynamics model. The proposed control law based on the sliding-mode controller can guarantee both homogeneous and heterogeneous string stability, if the control parameters and system parameters meet certain requirements. The analysis of the negative effect of the parasitic time delays and lags on the string stability indicates that the negative effect of the time delays is larger than that of the time lags. This paper provides a practical means to evaluate the ACC systems applying the sliding-mode controller and provides a reasonable proposal to design the ACC controller from the perspective of the practical string stability.**

***Index Terms*—Adaptive cruise control (ACC) system, advanced driver assistant systems (ADASs), constant time headway (CTH), heterogeneous platoon, homogeneous platoon, practical string stability.**

I. Motivation

THE ADAPTIVE cruise control (ACC) system is the first commercial implementation of advanced driver assistant systems (ADASs) on a wide range of passenger vehicles and is also the first logic step in a progressive path leading toward an automated highway system [1], [2]. The first generation ACC-equipped vehicles have been available in the market in Japan since 1995, in Europe since 1998, and in North America since 2000 [3]. Until now, most automakers around the world have made the ACC system available in their luxury vehicles with a tendency to extend this feature from high-end vehicles to mid-range vehicles [3], [4].

The design of a controller for an ACC system requires the specification of a spacing policy, i.e., a rule that specifies the desired following distance of the controlled vehicle as a function of its speed, and the synthesis of a control system to regulate the speed and following distance of a vehicle in accordance with the spacing policy employed by the vehicle [5]–[7]. The constant time headway (CTH) spacing policy is commonly employed by ACC systems that have been equipped in high-end vehicles. Generally, these ACC systems provide three options of the value of the CTH, i.e., 1, 1.5, and 2 s, to the drivers [3]. The ACC system has two modes of operation: user-set speed control mode and user-set time headway control mode [8]. In the absence of a preceding vehicle, the ACC-equipped vehicle travels at user-set speed, which is much like the operation with the conventional cruise control system. If the preceding vehicle is close or traveling slowly, then the ACC system switches from user-set speed control mode to user-set time headway control mode. In this mode, the ACC system controls both the throttle and the brake to maintain a desired spacing, which depends on the user-set time headway and the velocity of the controlled vehicle.

String stability is an important concern in the design of ACC systems and in the selection of CTH. A precise definition of string stability was provided by Chu [9]. Intuitively, the term "string stability" indicates the uniform boundedness of all the states of the interconnected system for all time, if the initial states of the interconnected system are uniformly bounded [6], [10]. This property ensures that any perturbation of the velocity or position of the leading vehicle will not result in amplified fluctuations to the following vehicle's velocity and position [7]. If spacing errors and velocity errors amplify as they propagate upstream (this is the case when string instabilities are present), it not only is likely to provide poor ride quality but could also result in collisions [11]. The relationship between string stability and spacing policies has been a topic of significant interest [7], [12]–[15].

Most physical systems often involve parasitic time delays and lags, which may significantly impact the stability of systems in some circumstances. In particular, the ACC system involves the parasitic time delays and lags in the engine, actuators, sensors, driveline, and radars [8], [11], [16]. However, most of the previous vehicle longitudinal dynamics model is simplified without considering the time delays and lags [5]–[7], [13], [14], [17]. For the sake of practical design and implementation, the negative effect of the parasitic time delays and lags on the string stability should be taken into account. Then, some issues naturally arise, as follows.

1) How do the parasitic time delays and lags negatively impact on the string stability of a platoon of ACC-equipped vehicles?
2) What are the string stable conditions if the string stability can be guaranteed when the parasitic time delays and lags are under consideration?

Manuscript received May 29, 2008; revised August 27, 2009, June 21, 2010, and February 23, 2011; accepted April 10, 2011. Date of publication May 31, 2011; date of current version December 5, 2011. This work was supported in part by the Doctoral Foundation of the Ministry of Education, China, under Grant 20070006011 and in part by the China Scholarship Council. The Associate Editor for this paper was A. Eskandarian.

L. Xiao is with the Defective Product Administrative Center, General Administration of Quality Supervision, Inspection, and Quarantine of the People's Republic of China, Beijing 100088, China (e-mail: xiaoly@dpac.gov.cn).

F. Gao is with the School of Transportation Science and Engineering, Beihang University, Beijing 100191, China (e-mail:gaof@buaa.edu.cn).

Digital Object Identifier 10.1109/TITS.2011.2143407

3) For the parasitic lumped time delay and lag, which one is the main negative effect on string stability?

Darbha [12] has proposed that the relationship between the CTH and the parasitic time lags is given by the simple form $h > 2\tau$ to guarantee string stability. Huang and Ren [18] have proposed a proportional–derivative (PD) controller to obtain a sufficient condition of string stability under consideration of parasitic time delays and lags of actuators, but the condition did not state the clear relationship between the CTH and the parasitic time delays and lags. Yanakiev and Kanellakopoulos [19] have designed two nonlinear autonomous controllers in the presence of large delays for the automated commercial heavy vehicles (CHVs) but without a detailed analysis of the string stability and string stable condition. However, earlier research works do not focus on the issue that is dealt with in this paper.

Because the homogeneous platoon that is composed of the same ACC-equipped vehicles does not exist in a real traffic system, heterogeneous string stability is another very important issue for the design and implementation of ACC systems. In this paper, the term "heterogeneous platoon" indicates that the ACC-equipped vehicles in the platoon are provided with different control parameters and are subject to different parasitic time delays and lags with different values of CTH chosen by drivers. Shaw [20] and Shaw and Hedrick [21], [22] have designed a controller to guarantee heterogeneous string stability for the constant distance spacing policy. Lestas and Vinnicombe [23] have analyzed the heterogeneous string stability for the symmetric bidirectional information flow when the constant distance spacing policy is applied. However, the earlier analysis of heterogeneous string stability is conducted without consideration of the parasitic time delays and lags, as well as without consideration of the CTH spacing policy.

The organization of this paper is described as follows. In Section II, we obtain the longitudinal vehicle dynamics model with the consideration of parasitic time delays and lags. In Section III, we propose the sliding model control law and then obtain the related velocity and spacing error dynamics model of the two successive ACC-equipped vehicles in the heterogeneous platoon. In Section IV, we analyze the negative effect of parasitic time delays and lags on the homogeneous string stability and obtain the related homogeneous string stable conditions. In Section V, we define and analyze the heterogeneous string stability and obtain the related heterogeneous string stable conditions. In Section VI, we provide some corroborating numerical simulations and then conclude thereafter in Section VII.

II. Vehicle Longitudinal Dynamics Model

In this section, we first obtain the vehicle longitudinal dynamics model without consideration of parasitic time delays and lags by applying the input–output (I/O) linearization technique to simplify the complexity. Then, we point out where the parasitic time delays and lags occur and obtain the more practical vehicle longitudinal dynamics model.

A. Model Without Parasitic Time Delays and Lags

A model for the motion of a vehicle in the longitudinal direction must take into account the powertrain, longitudinal tire forces, aerodynamic drag forces, rolling resistance forces, and gravitational forces. The powertrain is composed of the internal combustion engine, the torque converter, the transmission, and the tires [8], [24]. To arrive at a proper model for the control law design, one should make some reasonable assumptions such as the following [12], [13], [16], [25], [26]:

1) The tire slip is negligible.
2) A vehicle may be treated as consisting of rigid bodies.
3) The ideal gas law holds in the intake manifold, and the temperature of the intake manifold is constant.
4) The characteristics of the engine may be quantified by an empirical static "engine map."
5) The torque converter is locked.

Based on the first assumption, we obtain $\upsilon = Rrw_e$, where υ denotes the velocity of the vehicle, R denotes the gear ratio of the transmission, r denotes the tire radius, and w_e denotes the engine speed. Based on the other assumptions, we obtain $\dot{w}_e = T_{\mathrm{net}} - c_a R^3 r^3 w_e^2 - R(rF_f + T_{\mathrm{br}})/J_e$, where T_{net} denotes the net engine torque that depends on the throttle angle and velocity of vehicle, J_e denotes the effective rotational inertia of the engine when the vehicle mass and the tire inertias are referred to the engine side, c_a denotes the drag coefficient, F_f denotes the force due to rolling resistance, and T_{br} denotes the brake torque at the tires.

When one is considering a platoon of $N+1$ automated vehicles, it is natural to index them from 0 to N. Let $x_0(t)$, $\upsilon_0(t)$, and $a_0(t)$ denote the position, velocity, and acceleration of the leading vehicle, respectively, and $x_i(t)$, $\upsilon_i(t)$, and $a_i(t)$ $(1 \le i \le N)$ denote the position, velocity, and acceleration of the ith following vehicle in the platoon in an inertial frame, respectively. Then

$$\begin{aligned}\dot{x}_i(t) &= \upsilon_i(t) = (Rrw_e)_i, \dot{\upsilon}_i(t) = a_i(t) = F_{\mathrm{net}_i}/M_i = (Rr\dot{w}_e)_i \\ &= \left(\frac{Rr}{J_e}\left[T_{\mathrm{net}} - c_a R^3 r^3 w_e^2 - R(rF_f + T_{\mathrm{br}})\right]\right)_i \end{aligned} \tag{1}$$

where F_{net_i} denotes the net force in the longitudinal direction of the ith vehicle, and M_i denotes the effective mass of the ith vehicle. Hence, we obtain

$$(T_{\mathrm{net}})_i = (c_a R^3 r^3 w_e^2 + RrF_f)_i + \left(\frac{J_e}{Rr}\right)_i \dot{\upsilon}_i \tag{2}$$

when the braking maneuver is deactivated, and

$$(T_{\mathrm{br}})_i = -\left(\frac{c_a R^3 r^3 w_e^2 + RrF_f}{R}\right)_i - \left(\frac{J_e}{R^2 r}\right)_i \dot{\upsilon}_i \tag{3}$$

when the braking maneuver is activated.

For convenience of analysis, the feedback structure is developed using the I/O linearization technique, which is best suited for the problem considering the nonlinearities in the engine model [12], [13]. Then, choose

$$(T_{\mathrm{nd}})_i = (c_a R^3 r^3 w_e^2 + RrF_f)_i + \left(\frac{J_e}{Rr}\right)_i u_i \tag{4}$$

when $\alpha(t) > \alpha_0$, where α_0 denotes the minimum allowable throttle angle, and then, braking should not be applied. In (4), $(T_{\mathrm{nd}})_i$ denotes the desired net engine torque, and u_i denotes the control law that is chosen to make the closed-loop system satisfy certain performance objectives such as $(T_{\mathrm{nd}})_i = (T_{\mathrm{net}})_i$. Combining (2) and (4), we obtain

$$\dot{v}(t) = u_i(t) \tag{5}$$

which means that the acceleration of a vehicle can be commanded or controlled at will during speeding maneuver or speed keeping maneuver. If $\alpha(t) \leq \alpha_0$, then braking should occur, in which case, the desired brake torque T_{bd} is given by

$$(T_{\mathrm{bd}})_i = -\left(\frac{c_a R^3 r^3 w_e^2 + RrF_f}{R}\right)_i - \left(\frac{J_e}{R^2 r}\right)_i u_i. \tag{6}$$

Combining (3) and (6), we still obtain (5), which means that the deceleration of a vehicle also can be commanded or controlled at will during the braking maneuver.

In essence, one may deal with the following vehicle longitudinal dynamics model on which to base the controller:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t) \tag{7}$$

where

$$X_i(t) = \begin{bmatrix} x_i(t) \\ v_i(t) \end{bmatrix} \quad A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } U_i(t) = u_i(t).$$

To reiterate, this vehicle longitudinal dynamics model indicates that the acceleration or deceleration of a vehicle can be commanded or controlled at will. This model has been frequently used to analyze the string stability, such as [6], [10], and [26]–[29].

B. Model With Parasitic Time Delays and Lags

The parasitic delays and lags are the inevitable nature properties of the actuators and sensors in the mechanical and control systems. During the analysis of the stability of a platoon of ACC-equipped vehicles, they usually come from several sources [8], [11], [16], [30]:

1) the pure time delay and lag in the engine response;
2) the pure time delay and lag of the throttle actuator;
3) the pure time delay and lag of the brake actuator;
4) the pure time delay and lag due to the radar filter;
5) the bandwidth of low-pass filters used for other sensors such as engine manifold pressure sensor, wheel speed sensor etc.

Since the presence of the parasitic time delays and lags of the actuators and sensors, the commanded acceleration may not be readily available immediately. In other words, the net engine torque T_{net} could not approach the desired net engine torque T_{nd} immediately when the control signal is sent by the ACC controller. The variable time delays and lags have cumulative effect on the control or the output [18], [19]; hence, the different parasitic delays and lags are combined into one "lumped" delay and one "lumped" lag, which are represented by symbols Δ and τ, respectively.

The time lag is modeled as a first-order system [18], [19], such as

$$\tau_i(\dot{T}_{\mathrm{net}})_i + (T_{\mathrm{net}})_i = (T_{\mathrm{nd}})_i \tag{8}$$

when the braking maneuver is deactivated, and

$$\tau_i(\dot{T}_{\mathrm{br}})_i + (T_{\mathrm{br}})_i = (T_{\mathrm{bd}})_i \tag{9}$$

when the braking maneuver is activated. Moreover, due to the pure information delay or sensing delay, the ACC controller can access just the delayed information but not the instantaneous information to yield the desired acceleration or deceleration. It means that the expression of the control law $u_i(t)$ is replaced with $u_i(t-\Delta_i)$ [18], [19]. Then, choosing

$$(T_{\mathrm{nd}})_i = \left(c_a R^3 r^3 w_e(w_e + 2\dot{w}_e) + RrF_f\right)_i + \left(\frac{J_e}{Rr}\right)_i u_i(t-\Delta_i) \tag{10}$$

$$(T_{\mathrm{bd}})_i = -\left(\frac{c_a R^3 r^3 w_e(w_e + 2\dot{w}_e) + RrF_f}{R}\right)_i - \left(\frac{J_e}{R^2 r}\right)_i u_i(t-\Delta_i) \tag{11}$$

we obtain

$$\tau_i \ddot{v}_i(t) + \dot{v}_i(t) = u_i(t-\Delta_i) \tag{12}$$

which indicates that if one commands the acceleration or deceleration of the ith vehicle at time t, the ith vehicle has a delay of Δ_i seconds and suffers a lag of τ_i seconds to respond to the commanding signal. One may treat Δ_i and τ_i as the "lumped" parasitic time delay and lag, respectively, from the command (throttle input and brake pedal) to the torque available at the tires.

Hence, we may consider the following longitudinal model of a vehicle for analyzing the practical string stability:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t-\Delta_i) \tag{13}$$

where

$$X_i = \begin{bmatrix} x_i \\ v_i \\ a_i \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}$$
$$B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix} \text{ and } U_i(t-\Delta_i) = u_i(t-\Delta_i).$$

III. Control Law and Spacing Error Dynamics

In this section, the ACC control law is proposed based on the sliding model control technique at first, and then, the related velocity and spacing error dynamics are derived from the proposed control law, the definition of the spacing policy, and the vehicle longitudinal dynamics model with consideration of the parasitic delays and lags.

A. ACC Control Law

For the development of a control law, we will assume that there is a leading vehicle, which is indexed by 0, that performs maneuvers, and the following vehicles are controlled to maintain a desired following distance. The desired following distance $D_i(t)$ of the ith $(1 \leq i \leq N)$ following vehicle is defined as $D_i(t) = h_i \upsilon_i(t) + D_{\min}$, where h_i denotes the CTH of the ith following vehicle, $\upsilon_i(t)$ denotes the velocity of the ith following vehicle, and $D_{\min}$ denotes the minimum distance of the two successive following vehicles when the platoon of vehicles stands still.

Let $\xi_i(t)$ denote the real spacing (real following distance) between the ith vehicle and the $(i-1)$th vehicle, l_{i-1} denote the length of the $(i-1)$th vehicle, and $\delta_i(t)$ denote the spacing error of the ith vehicle, which is the deviation between the real spacing and the desired spacing. Then

$$\xi_i(t) = x_{i-1}(t) - x_i(t) - l_{i-1} \tag{14}$$

$$\delta_i(t) = \xi_i(t) - h_i \upsilon_i(t) - D_{\min}. \tag{15}$$

Furthermore, it is clear that

$$\xi_i(t-\Delta_i) = x_{i-1}(t-\Delta_i) - x_i(t-\Delta_i) - l_{i-1} \tag{16}$$

$$\delta_i(t-\Delta_i) = \xi_i(t-\Delta_i) - h_i \upsilon_i(t-\Delta_i) - D_{\min}. \tag{17}$$

The purpose of the controller is to asymptotically drive the spacing error $S_i \equiv \delta_i(t) := \xi_i(t) - h_i \upsilon_i(t) - D_{\min}$ to zero in time, that is, $\dot{S}_i = -\lambda_i S_i$, where $\lambda_i > 0$ denotes the control gain. Clearly, $\dot{\upsilon}_i(t) = (1/h_i)(\dot{\xi}_i(t) + \lambda_i \delta_i(t))$, where $\dot{\xi}_i(t)$ is derived from differentiating both sides of (14) and denotes the relative velocity between the ith vehicle and the $(i-1)$th vehicle, that is

$$\dot{\xi}_i(t) = \upsilon_{i-1}(t) - \upsilon_i(t). \tag{18}$$

If the control law $u_i(t)$ is proposed as

$$u_i(t) = \frac{1}{h_i}\left(\dot{\xi}_i(t) + \lambda_i \delta_i(t)\right) \tag{19}$$

then it can apparently make $\dot{S}_i + \lambda_i S_i = 0$ for $\lambda_i > 0$. Due to the presence of the parasitic time delays of the actuators and sensors, only the delayed information, such as the velocity of the ith vehicle and the relative position and velocity between the ith vehicle and the $(i-1)$th vehicle, has been applied; then, the control law is replaced with

$$u_i(t-\Delta_i) = \frac{1}{h_i}\left(\dot{\xi}_i(t-\Delta_i) + \lambda_i \delta_i(t-\Delta_i)\right) \tag{20}$$

where $\dot{\xi}_i(t-\Delta_i) = \upsilon_{i-1}(t-\Delta_i) - \upsilon_i(t-\Delta_i)$. The control law (20) can also take the form of $u_i(t-\Delta_i) = (1/h_i)(\dot{\delta}_i(t-\Delta_i) + \lambda_i \delta_i(t-\Delta_i)) + \dot{\upsilon}_i(t-\Delta_i)$, which could be considered as a PD control law plus the acceleration information of the ith vehicle.

B. Velocity and Spacing Error Dynamics Models

Based on the vehicle longitudinal dynamics model (13)

$$\tau_i \dot{a}_i(t) + a_i(t) = \frac{1}{h_i}\left(\dot{\xi}_i(t-\Delta_i) + \lambda_i \delta_i(t-\Delta_i)\right). \tag{21}$$

Differentiating both sides of (21), we obtain

$$\begin{aligned}
&\dot{\upsilon}_{i-1}(t-\Delta_i) + \lambda_i \upsilon_{i-1}(t-\Delta_i) \\
&\quad = h_i \tau_i \dddot{\upsilon}_i(t) + h_i \ddot{\upsilon}_i(t) + (1 + h_i \lambda_i)\dot{\upsilon}_i(t-\Delta_i) \\
&\qquad + \lambda_i \upsilon_i(t-\Delta_i).
\end{aligned} \tag{22}$$

With no loss of generality, we assume that $\upsilon_i(0) = \upsilon_0(0) = 0$, $a_i(0) = a_0(0) = 0$ and that $\delta_i(0) = 0$ $(1 \leq i \leq N)$ hold at the initial state [17]. Then, taking the Laplace transformation on both sides of (22), we obtain

$$\begin{aligned}
G_i(s) &= \frac{\upsilon_i(s)}{\upsilon_{i-1}(s)} \\
&= \frac{(s+\lambda_i)e^{-\Delta_i s}}{h_i \tau_i s^3 + h_i s^2 + (1 + h_i \lambda_i) s e^{-\Delta_i s} + \lambda_i e^{-\Delta_i s}}
\end{aligned} \tag{23}$$

which describes the velocity dynamics model of two successive vehicles in the platoon.

Differentiating both sides of (15) and taking Laplace transformation on both sides of the differentiated equation, the relationship between the spacing error and the velocity of the ith vehicle is given by $s\delta_i(s) = (1 - (1 + h_i s)G_i(s))\upsilon_{i-1}(s)$. Similarly, the relationship between the spacing error and the velocity of the $(i-1)$th vehicle is given by $s\delta_{i-1}(s) = ((1/G_{i-1}(s)) - (1 + h_{i-1}s))\upsilon_{i-1}(s)$. Then, we obtain the spacing error dynamics model $H_i(s)$ of the successive two vehicles in the platoon as (24), shown at the bottom of the page, where

$$M_i(s) = \frac{(\tau_i s + 1 - e^{-\Delta_i s})(s + \lambda_{i-1})e^{-\Delta_{i-1}s}}{(\tau_{i-1}s + 1 - e^{-\Delta_{i-1}s})(s+\lambda_i)e^{-\Delta_i s}} \tag{25}$$

$$\begin{aligned}
H_i(s) &= \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{G_{i-1}(s)\left(1 - (1 + h_i s)G_i(s)\right)}{1 - (1 + h_{i-1}s)G_{i-1}(s)} \\
&= \frac{(h_i \tau_i s + h_i - h_i e^{-\Delta_i s})(s + \lambda_{i-1})e^{-\Delta_{i-1}s}}{(h_{i-1}\tau_{i-1}s + h_{i-1} - h_{i-1}e^{-\Delta_{i-1}s})(h_i \tau_i s^3 + h_i s^2 + (1 + h_i \lambda_i)s e^{-\Delta_i s} + \lambda_i e^{-\Delta_i s})} = \frac{h_i}{h_{i-1}} M_i(s) G_i(s)
\end{aligned} \tag{24}$$

which describes the parameters of two successive vehicles in the platoon.

IV. Analysis of Homogeneous Platoon

In this section, we focus on the analysis of the string stability of a homogeneous platoon, which is composed of identical ACC-equipped vehicles. First, we propose and prove the homogeneous string stability theorem. Then, we analyze the negative effect of the parasitic time delays and lags on the string stability.

A. Homogeneous String Stable Theorem

Intuitively, the homogeneous platoon implies that $\Delta_{i-1} = \Delta_i$, $\tau_{i-1} = \tau_i$, $\lambda_{i-1} = \lambda_i$ and $h_{i-1} = h_i$. Clearly, $M_i(s) = 1$ in the homogeneous platoon. Furthermore, we obtain

$$H_i(s) = G_i(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{\upsilon_i(s)}{\upsilon_{i-1}(s)} = \frac{(s+\lambda_i)e^{-\Delta_i s}}{h_i\tau_i s^3 + h_i s^2 + (1+h_i\lambda_i)se^{-\Delta_i s} + \lambda_i e^{-\Delta_i s}}. \quad (26)$$

As mentioned earlier, string stability implies that spacing errors do not amplify as they propagate upstream from one vehicle to another vehicle [10], [12], [17], that is

$$|H_i(jw)| < 1, \qquad \forall \quad w > 0 \quad (27)$$

where $H_i(jw)$ is derived from (26) by substituting $s = jw$. Due to $H_i(s) = G_i(s)$ for the homogeneous platoon, the condition that leads to $|H_i(jw)| < 1$ can also lead to $|G_i(jw)| < 1$. Before we proceed to present the homogeneous string stable theorem and its proof, we assume that $h_i > \Delta_i$ and $h_i > \tau_i$, because the traffic accident could apparently be caused during a sudden braking maneuver if $h_i \le \Delta_i$ and if $h_i \le \tau_i$.

Theorem 1 (Homogeneous String Stable Theorem): The condition $|H_i(jw)| < 1$ is satisfied for $\forall\, w > 0$ if the condition

$$h_i > 2 \quad (\Delta_i + \tau_i)$$

holds, where any control gain λ_i is chosen, where

$$\lambda_i \in \left(0,\ \frac{h_i - 2(\Delta_i+\tau_i)}{2\left(h_i(\Delta_i+\tau_i) - \Delta_i\tau_i\right)}\right].$$

Proof: $|H_i(jw)|$ can be expressed as $|H_i(jw)| = |\delta_i(jw)/\delta_{i-1}(jw)| = \sqrt{a/(a+b)}$, where

$$a = w^2 + \lambda_i^2 \quad (28)$$

$$b = \left(2h_i\lambda_i\left(1-\cos(\Delta_i w)\right) + h_i^2\lambda_i^2\right)w^2 - \left(\left(2h_i(1+h_i\lambda_i) - 2h_i\tau_i\lambda_i\right)\sin(\Delta_i w)\right)w^3 + \left(h_i^2 - 2h_i\tau_i(1+h_i\lambda_i)\cos(\Delta_i w)\right)w^4 + h_i^2\tau_i^2 w^6. \quad (29)$$

Apparently, if $b > 0$ holds for $\forall\, w > 0$, $\sqrt{a/(a+b)} < 1$ will be obtained, and then, $|H_i(jw)| < 1$ for $\forall\, w > 0$ follows.

Taking into account the fact that $1 - \cos(\Delta_i w) \ge 0$, $\sin(\Delta_i w) \le \Delta_i w$ and $-\cos(\Delta_i w) \ge -1$ for $\forall \; w > 0$, we obtain

$$b \ge h_i^2\lambda_i^2 w^2 + h_i^2\tau_i^2 w^6 + \left(\left(h_i^2 - 2h_i\tau_i - 2h_i^2\tau_i\lambda_i\right) - \left(2h_i + 2h_i^2\lambda_i - 2h_i\tau_i\lambda_i\right)\Delta_i\right)w^4 \quad (30)$$

for $\forall\, w > 0$.

Clearly, $h_i^2\lambda_i^2 w^2 + h_i^2\tau_i^2 w^6 > 0$ holds for $\forall\; w > 0$. Hence, if the condition

$$h_i - 2(\Delta_i + \tau_i) \;\ge\; 2\lambda_i\left(h_i(\Delta_i+\tau_i) - \Delta_i\tau_i\right) \quad (31)$$

holds, $b > 0$ for $\forall\; w > 0$ is then obtained. Due to $h_i(\Delta_i + \tau_i) - \Delta_i\tau_i = h_i\Delta_i + (h_i - \Delta_i)\tau_i > 0$, we obtain

$$\lambda_i \le \frac{h_i - 2(\Delta_i+\tau_i)}{2\left(h_i(\Delta_i+\tau_i) - \Delta_i\tau_i\right)}. \quad (32)$$

Notice that the control gain $\lambda_i > 0$; it means that

$$h_i \;>\; 2\left(\Delta_i \;+\; \tau_i\right). \quad (33)$$

Hence, if (32) and (33) are simultaneously achieved, the homogeneous string stability could be guaranteed. In other words, the condition $|H_i(jw)| < 1$ is satisfied for $\forall\, w > 0$ if the condition $h_i > 2(\Delta_i + \tau_i)$ holds with choosing any control gain λ_i, where $\lambda_i \in (0, (h_i - 2(\Delta_i+\tau_i)/2(h_i(\Delta_i+\tau_i) - \Delta_i\tau_i))]$.

Here, we complete the proof of the theorem. □

Theorem 1 has revealed the lower bound of the CTH (h), which is combined by the parasitic time delays (Δ) and lags (τ). For a specific individual ACC-equipped vehicle, it is possible that the value of the parasitic time lag is negligible compared to the parasitic time delay, or the value of the parasitic time delay is negligible compared to the parasitic time lag. To simplify the design and implementation, it is reasonable to only consider the larger negative effect on the string stability. Then, the simple lower bounds of the CTH are given by the following lemma.

Lemma 1:

1) If the parasitic time delays are negligible, the string stability can be guaranteed if holding $h_i > 2\tau_i$ and $\lambda_i \in (0, (h_i - 2\tau_i/2h_i\tau_i)]$.
2) If the parasitic time lags are negligible, the string stability can be guaranteed if holding $h_i > 2\Delta_i$ and $\lambda_i \in (0, (h_i - 2\Delta_i/2h_i\Delta_i)]$.

Proof: The proof of parts 1 and 2 of Lemma 1 are easily derived from the proof of Theorem 1 with $\Delta_i = 0$ and $\tau_i = 0$, respectively. □

B. Analysis of Negative Effect

In this paper, the negative effect that results from parasitic time delays and lags is either a reduction in a platoon's stability margin or its instability. The worse string stable performance means that the spacing error is amplified with the growing of the parasitic time delays and lags. For instance, if the string stability is analyzed without consideration of the parasitic time delays and lags, which is called as the ideal scenario, then

b is reduced as $b = h_i^2\lambda_i^2 w^2 + h_i^2 w^4$ from (29), with $\Delta_i = 0$ and $\tau_i = 0$. Clearly, $b > 0$ is obtained for $\forall\ w > 0$, no matter what the values of the CTH (h_i) and control gain (λ_i) are. It means that the proposed control law (19) can guarantee string stability with an arbitrary value of CTH in this ideal scenario. If one chooses an inappropriate CTH, the string stability is probably not available. Fancher *et al.* [31] and Bareket *et al.* [32] have demonstrated that string instability can occur during the operational experiments involving testing of strings with four to eight ACC-equipped vehicles in a platoon due to the presence of parasitic time delays and lags of actuators and sensors.

At the first glance of Theorem 1 and Lemma 1, the parasitic time delays and the parasitic time lags have the same negative effect on the string stability when the CTH is specified. The following theorem shows that the parasitic time delays and the parasitic time lags have different negative effects on the string stability and points out whose negative effect is larger than the other ones.

Before we proceed to provide the theorem and its proof, we assume a specific scenario, in which the ith vehicle in the platoon is running with only the parasitic time delays or parasitic time lags. If the spacing error that is caused by the parasitic time delays is larger than the one that is caused by the parasitic time lags, the parasitic time delays take the larger negative effect and *vice versa*. The spacing errors have most of their energy at the region of low frequencies, which is also called the key region of the string stability [5], [33]–[35]. Hence, a way to figure out which one has the larger negative effect between the parasitic time delays and parasitic time lags naturally arises by comparing the magnitude of the value of $|H_i(jw)|_{\Delta_i}$ and the value of $|H_i(jw)|_{\tau_i}$ at the low frequencies, where $|H_i(jw)|_{\Delta_i}$ denotes $|H_i(jw)|$ with $\tau_i = 0$, and $|H_i(jw)|_{\tau_i}$ denotes $|H_i(jw)|$ with $\Delta_i = 0$. If $|H_i(jw)|_{\Delta_i} > |H_i(jw)|_{\tau_i}$ holds, the parasitic time delays take the larger negative effect on the string stability and *vice versa*.

Theorem 2 (Negative Effect Theorem): The parasitic time delays take the larger negative effect on the string stability if the "lumped" parasitic time delay Δ_i and the "lumped" parasitic time lag τ_i take the same values, that is, $\Delta_i = \tau_i$.

Proof: $|H_i(jw)|_{\Delta_i}$ can be expressed as $|H_i(jw)|_{\Delta_i} = \sqrt{a_{\Delta_i}/(a_{\Delta_i} + b_{\Delta_i})}$ with $a_{\Delta_i} = w^2 + \lambda_i^2$, and

$$b_{\Delta_i} = h_i w^2 \left(2\lambda_i \left(1 - \cos(\Delta_i w)\right) + h_i\lambda_i^2 - 2(1 + h_i\lambda_i)\sin(\Delta_i w)w + h_i w^2\right) \quad (34)$$

which is derived from (29) with $\tau_i = 0$. Similarly, we can obtain $a_{\tau_i} = w^2 + \lambda_i^2$, and

$$b_{\tau_i} = h_i w^2 \left(h_i\lambda_i^2 + (h_i - 2\tau_i(1 + h_i\lambda_i))\, w^2 + h_i\tau_i^2 w^4\right) \quad (35)$$

which is derived from (29) with $\Delta_i = 0$.

It is clear that $a_{\Delta_i} = a_{\tau_i}$; hence, we can compare the magnitude of $|H_i(jw)|_{\Delta_i}$ with the magnitude of $|H_i(jw)|_{\tau_i}$ at the low frequencies by comparing the magnitude of b_{Δ_i} with the magnitude of b_{τ_i} at the low frequencies. If $b_{\Delta_i} < b_{\tau_i}$ holds, then $|H_i(jw)|_{\Delta_i} > |H_i(jw)|_{\tau_i}$ naturally follows, and *vice versa*.

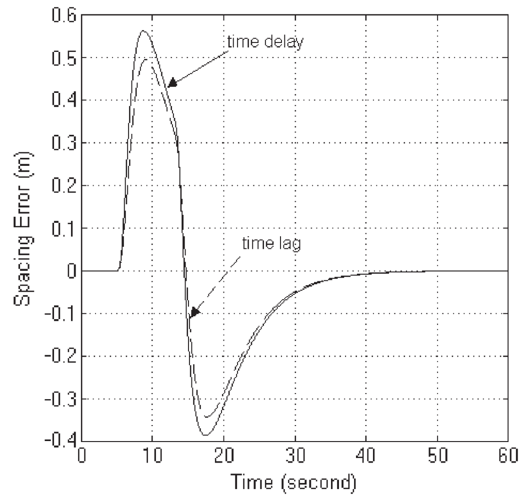


Fig. 1. Comparing the negative effect between the parasitic time delays and parasitic time lags on string stability.

Notice that $\cos(\Delta_i w) = 1$ and $\sin(\Delta_i w) = \Delta_i w$ could be obtained at the low frequencies; then, (34) is replaced with

$$b_{\Delta_i} = h_i w^2 \left(h_i\lambda_i^2 + (h_i - 2\Delta_i(1 + h_i\lambda_i))\, w^2\right). \quad (36)$$

Then, it is easy to obtain $b_{\tau_i} > b_{\Delta_i}$ by comparing (35) with (36) when $\tau_i = \Delta_i$. It means that $|H_i(jw)|_{\Delta_i} > |H_i(jw)|_{\tau_i}$ is obtained at the low frequencies. Finally, we conclude that the parasitic time delays take the larger negative effect on the string stability if $\Delta_i = \tau_i$.

Here, we complete the proof of the theorem. ■

To verify Theorem 2, a simulation has been conducted to compare the spacing error δ_{Δ_i} that is caused by the parasitic time delays with the one δ_{τ_i} that is cause by the parasitic time lags. Fig. 1 illustrates that $\delta_{\Delta_i} > \delta_{\tau_i}$ holds when $\Delta_i = \tau_i = 0.4$ s, $h_i = 1$ s, and $\lambda_i = 0.15$.

Remark 1: Naturally, we can obtain a criterion to evaluate real ACC systems that come from different automakers and their suppliers with the perspective of practical string stability, which is given as follows.

1) Compare $2(\Delta + \tau)$ with $h_{\min}$ for every ACC-equipped vehicle, where $h_{\min}$ denotes the minimum value of h that could be chosen by the drivers. If $2(\Delta + \tau) \geq h_{\min}$ for an individual vehicle, it means that this vehicle has bad string performance.
2) Compare the magnitude of $2(\Delta + \tau)$ for different ACC-equipped vehicles. The larger the value of $2(\Delta + \tau)$, the worse the string performance.
3) Compare the magnitude of Δ for different ACC-equipped vehicles. The larger the value of Δ, the worse the string performance.

V. Analysis of Heterogeneous Platoon

With the improvement of the market penetration of ACC-equipped vehicles, it is reasonable to expect that a platoon that is composed of ACC-equipped vehicles appears in the real highway traffic. However, since ACC systems are designed and implemented by the different automakers and their suppliers, the homogeneous platoon, which is composed of the identical ACC-equipped vehicles, is not easy to obtain. Hence, the heterogeneous platoon, which is composed of the different ACC-equipped vehicles, should be taken into account to analyze the string stability, and its related string stable conditions should be obtained for the more practical scenario.

For the heterogeneous platoon, we notice that the velocity dynamics model $G_i(s)$ and the spacing error dynamics model $H_i(s)$ take the different forms, that is, $G_i(s) \neq H_i(s)$. This means that the bounded velocities of all the following vehicles do not indicate that the spacing errors of all the following vehicles are bounded, and *vice versa*. In other words, if the heterogeneous string stability is obtained, the velocities and spacing errors of all following vehicles should be bounded simultaneously.

In fact, the desired distances between two successive vehicles in the heterogeneous platoon are varied with the different values of the CTH selected by drivers. As mentioned earlier, the communication delay resulted from the data loss and outside environmental disturbances during the data transferring. Naturally, the larger the vehicle spacing, the higher the probability of data loss and outside environmental disturbances. Hence, the larger the value of the CTH that is selected, the larger the value of the desired distance that is obtained, and then, the larger the value of the communication time delay that is presented, and finally, the larger the spacing error that is yielded, that is, $h_i > h_{i-1} \Rightarrow D_i > D_{i-1} \Rightarrow \Delta_i > \Delta_{i-1} \Rightarrow \delta_i > \delta_{i-1}$. Similarly, $h_i < h_{i-1} \Rightarrow D_i < D_{i-1} \Rightarrow \Delta_i < \Delta_{i-1} \Rightarrow \delta_i < \delta_{i-1}$. Hence, if $h_i > h_{i-1}$, the heterogeneous platoon is running safely, even though $\delta_i > \delta_{i-1}$ due to $D_i > D_{i-1}$, but if $h_i < h_{i-1}$, the heterogeneous platoon is running with potential risk, even though $\delta_i < \delta_{i-1}$ due to $D_i < D_{i-1}$.

According to the earlier analysis, the definition of the string stability for the homogeneous platoon $|H_i(jw)| < 1$ is no longer suitable. Therefore, the more practical condition of the string stability for the heterogeneous platoon is described as follows. The string stability of a heterogeneous platoon of ACC-equipped vehicles applying the CTH spacing policy is obtained if the conditions $|G_i(jw)| < 1$ and $|H_i(jw)| < h_i/h_{i-1}$ for $\forall\, w > 0$ hold simultaneously.

Strong evidence suggests that the conditions $|G_i(jw)| < 1$ and $|H_i(jw)| < h_i/h_{i-1}$ are guaranteed for $\forall\, w > 0$, if

$$h_i > 2(\Delta_i + \tau_i)$$

holds with choosing proper control gain λ_i, where

$$\lambda_i \in \left(0, \frac{h_i - 2(\Delta_i + \tau_i)}{2\left(h_i(\Delta_i + \tau_i) - \Delta_i \tau_i\right)}\right].$$

Following the proof of Theorem 1, it is easy to obtain the following condition:

$$h_i > 2(\Delta_i + \tau_i), \quad \lambda_i \in \left(0, \frac{h_i - 2(\Delta_i + \tau_i)}{2\left(h_i(\Delta_i + \tau_i) - \Delta_i \tau_i\right)}\right] \tag{37}$$

which yields $|G_i(jw)| < 1$ for $\forall\, w > 0$.

The expression of $|H_i(jw)|$ is given as $|H_i(jw)| = (h_i/h_{i-1})|M_i(jw)||G_i(jw)|$, which is derived from the spacing error dynamics model $H_i(s)$ (24) by substituting $s = jw$. Clearly, if we can obtain $|M_i(jw)||G_i(jw)| < 1$, then $|H_i(jw)| < h_i/h_{i-1}$ is followed.

The expression of $|M_i(jw)|$ is given as

$$\begin{aligned} &|M_i(jw)| \\ &\quad = \sqrt{\frac{\tau_i^2 w^2 + 2\tau_i w \sin(\Delta_i w) + 2 - 2\cos(\Delta_i w)}{\tau_{i-1}^2 w^2 + 2\tau_{i-1} w \sin(\Delta_{i-1} w) + 2 - 2\cos(\Delta_{i-1} w)}} \\ &\qquad \times \sqrt{\frac{\lambda_{i-1}^2 + w^2}{\lambda_i^2 + w^2}} \end{aligned} \tag{38}$$

which is derived from (25) by substituting $s = jw$. It is well known that the worst case occurs at the low frequencies. In other words, the maximum magnitude of $|M_i(jw)||G_i(jw)|$ occurs at the low frequencies. If the worst case occurs at the other frequencies, because the values of the system parameters such as τ and Δ are limited in the real situation and because of the structure of $|M_i(jw)|$, the magnitude of $|M_i(jw)|$ is bounded if proper choosing of the values of the successive control gains $(\lambda_i, \lambda_{i-1})$.

At the low frequencies, the value of w is far smaller than the values of the parameters, we could assume that $\sin(\Delta w) = \Delta w$, $\cos(\Delta w) = 1$, and $w^2 \to 0$. Then, $|M_i(jw)|$ could be simplified as $|M_i(jw)| = \sqrt{(\tau_i^2 + 2\tau_i\Delta_i)\lambda_{i-1}^2/(\tau_{i-1}^2 + 2\tau_{i-1}\Delta_{i-1})\lambda_i^2}$. Clearly, if

$$\frac{\lambda_i}{\lambda_{i-1}} \geq \sqrt{\frac{\tau_i^2 + 2\tau_i\Delta_i}{\tau_{i-1}^2 + 2\tau_{i-1}\Delta_{i-1}}} \tag{39}$$

holds, $|M_i(jw)| \leq 1$ is obtained. Then, if the following condition holds (37), $|G_i(jw)| < 1$ could be obtained; then, $|M_i(jw)||G_i(jw)| < 1$ naturally follows. Hence, if the following condition holds $h_i > 2(\Delta_i + \tau_i)$ with choosing proper control gain λ_i, where $\lambda_i \in (0, (h_i - 2(\Delta_i + \tau_i)/2(h_i(\Delta_i + \tau_i) - \Delta_i\tau_i))]$, $|M_i(jw)||G_i(jw)| < 1$ could be obtained; then, $|H_i(jw)| < h_i/h_{i-1}$ is guaranteed. Clearly, $|G_i(jw)| < 1$ could also be guaranteed.

At the high frequencies, the value of w is far larger than the values of the parameters; then, $|M_i(jw)|$ could be simplified as $|M_i(jw)| = \tau_i/\tau_{i-1}$ whose value is limited due to the "lumped" parasitic time lag being limited in practice. However, $|G_i(jw)| \to 0$ because $b_i \to +\infty$ at the high frequencies due to existing the term $h_i^2\tau_i^2 w^6$. Hence, $|M_i(jw)||G_i(jw)| < 1$ could be obtained whatever the values of the parameters are. Then, at the middle range of frequencies, the values of the parameters are close to the value of w; it is clear that if $\tau_i^2 w^2 + 2\tau_i w \sin(\Delta_i w) + 2 - 2\cos(\Delta_i w) \geq (\tau_{i-1}^2 w^2 + 2\tau_{i-1} w \sin(\Delta_{i-1} w) + 2 - 2\cos(\Delta_{i-1} w)$, we could properly choose the control gains λ_i and λ_{i-1} with the relationship $\lambda_i \geq \lambda_{i-1}$ to obtain $|M_i(jw)| \leq 1$. Furthermore, if the (37) holds, $|G_i(jw)| < 1$ could be obtained; then, $|M_i(jw)||G_i(jw)| < 1$ naturally follows.

As mentioned earlier, the spacing errors have most of their energy at the region of low frequencies. Hence, we could apply inequality (39) as the standard to select the successive two control gains λ_i and λ_{i-1} in practice. Notice that if $h_i \geq h_{i-1}$,

TABLE I
SPECIFIC PARAMETER FOR THE HOMOGENEOUS PLATOON

Parameter	Case1	Case2	Case3
h_i	$1s$	$1s$	$1s$
Δ_i	$0.2s$	$0.3s$	$0.3s$
τ_i	$0.2s$	$0.2s$	$0.3s$
λ_i	0.15	0.15	0.15
Condition	$h_i > 2(\Delta_i + \tau_i)$	$h_i = 2(\Delta_i + \tau_i)$	$h_i < 2(\Delta_i + \tau_i)$

then $\Delta_i \geq \Delta_{i-1}$ holds, and then, $\lambda_i \geq \lambda_{i-1}$ should be provided, and *vice versa*. Hence, a reasonable proposal to design an ACC system for an individual ACC-equipped vehicle is offered such that the different values of the control gain λ are provided for different CTH h values, and the larger value of λ is selected for the larger h. For example, $\lambda_2 > \lambda_{1.5} > \lambda_1$ should be held, where λ_2 denotes the control gain that is provided for the CTH $h = 2$ s, $\lambda_{1.5}$ denotes the control gain that is provided for the CTH $h = 1.5$ s, and λ_1 denotes the control gain that is provided for the CTH $h = 1$ s.

VI. COMPARATIVE SIMULATIONS

To corroborate the results of the theorems that demonstrate the practical string stable conditions with consideration of the parasitic time delays and lags for both the homogeneous and heterogeneous platoons, several numerical comparative simulations have been conducted. In the simulations, we assume that, without loss of the generality, the leading vehicle initially travels at a steady-state velocity of $v_0 = 0$ m/s, $a_0 = 0$ m/s^2, and $D_{\min} = 5$ m; then, the leading vehicle begins to accelerate to another steady-state velocity $v_f = 40$ m/s and $a_f = 0$ m/s^2 at $t = 20$ s.

To verify the homogeneous string stable theorem (see Theorem 1), a homogeneous platoon, which is composed of 16 identical ACC-equipped vehicles and indexed from 0 to 15, is applied. Three cases, i.e., string stability, critical string stability, and string instability, have been simulated by considering three groups of specific parameters (see Table I). Fig. 2 illustrates case 1 in Table I, which represents the string stable condition. Fig. 2 demonstrates excellent tracking in velocity, and it also shows the spacing errors of the vehicles in the platoon smoothly decrease upstream. If the initial intervehicle spacing is the same for all vehicles, this ensures that if the first following vehicle does not collide with the leading vehicle, then there will not be a collision upstream between the ith and the $(i+1)$th vehicle for every $i > 1$. Fig. 3 illustrates case 2 in Table I, which represents the critical string stable condition. Fig. 3 demonstrates unstable tracking velocity and maintaining spacing. In fact, spacing errors amplify as one goes from the preceding vehicle to the following vehicle. Fig. 4 illustrates case 3 in Table I, which represents the string unstable condition. Fig. 4 demonstrates worse instability in tracking velocity, as well as in maintaining spacing. Figs. 2–4 serve to remind us that the condition $h_i > 2(\Delta_i + \tau_i)$ is tight if any control gain $\lambda_i \in (0, (h_i - 2(\Delta_i + \tau_i)/2(h_i(\Delta_i + \tau_i) - \Delta_i\tau_i))]$, for which one can obtain stable tracking velocity and maintain spacing.

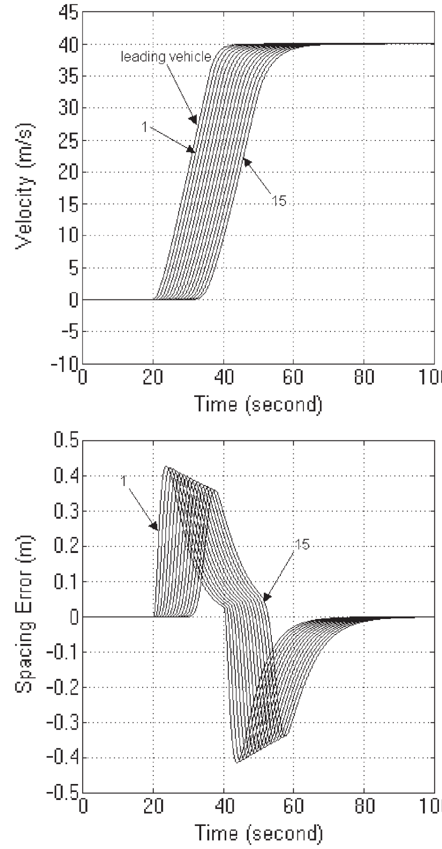


Fig. 2. String stable performance of the homogeneous platoon.

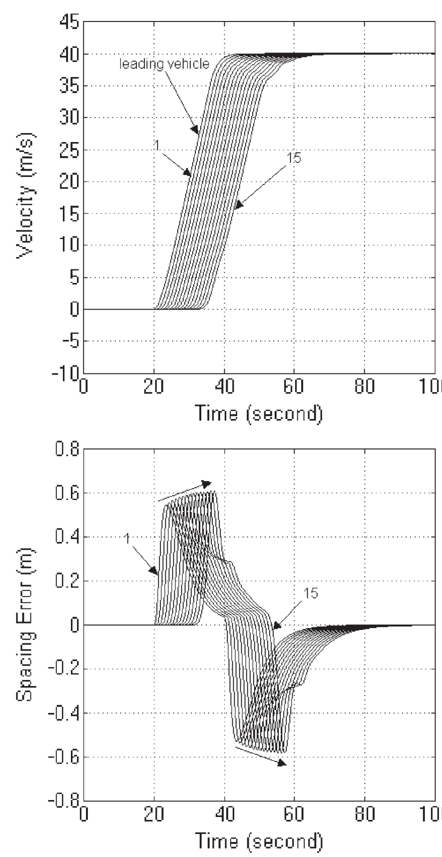


Fig. 3. Critical string stable performance of the homogeneous platoon

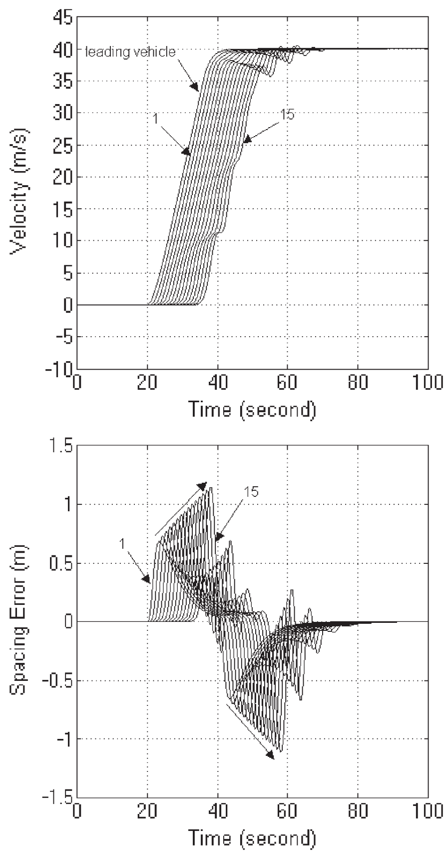


Fig. 4. String unstable performance of the homogeneous platoon.

TABLE II
SPECIFIC PARAMETER FOR THE HETEROGENEOUS PLATOON (STABLE)

Vehicle	$h_i(s)$	λ_i	$\tau_i(s)$	$\Delta_i(s)$	$\delta_i(m)$
1	1	0.15	0.2	0.2	0.4732
2	2	0.35	0.2	0.4	0.9166
3	2	0.35	0.2	0.4	0.8251
4	1	0.15	0.2	0.2	0.4012
5	1.5	0.25	0.2	0.3	0.5806
6	1.5	0.25	0.2	0.3	0.5577
7	2	0.35	0.2	0.4	0.6785
8	1.5	0.25	0.2	0.3	0.4980
9	1	0.15	0.2	0.2	0.3233
10	1	0.15	0.2	0.2	0.3209

To verify the analysis of the heterogeneous string stability, a heterogeneous platoon, which is composed of 11 different ACC-equipped vehicles and indexed from 0 to 10, is applied. For the sake of clarity, the detailed parameters of the whole following vehicles (1–10) are listed in Tables II and III. Three cases, i.e., $h_i > h_{i-1}$, $h_i = h_{i-1}$, and $h_i < h_{i-1}$ with $\lambda_i > \lambda_{i-1}$, $\lambda_i = \lambda_{i-1}$, and $\lambda_i < \lambda_{i-1}$, have been simulated to show the possible following scenarios. Fig. 5 illustrates that the heterogeneous string stability could be obtained if the following condition holds $h_i > 2(\Delta_i + \tau_i)$ with choosing the proper control gain $\lambda_i \in (0, (h_i - 2(\Delta_i + \tau_i)/2(h_i(\Delta_i + \tau_i) - \Delta_i\tau_i))]$. Fig. 5 demonstrates excellent tracking in velocity, and it also shows the spacing errors of successive following vehicles in the platoon smoothly evolve with the relationship $\delta_i/\delta_{i-1} < h_i/h_{i-1}$. The detailed values of the spacing errors of all of the whole following vehicles are listed in Table II, which clearly demonstrates that $\delta_i/\delta_{i-1} < h_i/h_{i-1}$ can be obtained during the simulations. Fig. 6 illustrates the case in which the condition $h_i \leq 2(\Delta_i + \tau_i)$ holds, and it demonstrates that the heterogeneous string stability could not be obtained. The detailed values of the spacing errors of all of the following vehicles are listed in Table III, which clearly demonstrates that $\delta_i/\delta_{i-1} < h_i/h_{i-1}$ cannot be obtained during the simulations. Clearly, Fig. 6 shows the worse velocity tracking and spacing maintaining compared with Fig. 5.

TABLE III
SPECIFIC PARAMETER FOR THE HETEROGENEOUS PLATOON (UNSTABLE)

Vehicle	$h_i(s)$	λ_i	$\tau_i(s)$	$\Delta_i(s)$	$\delta_i(m)$
1	1	0.15	0.3	0.3	0.7396
2	2	0.35	0.6	0.5	2.2253
3	2	0.35	0.6	0.5	2.3618
4	1	0.15	0.3	0.3	1.2928
5	1.5	0.25	0.4	0.4	1.6688
6	1.5	0.25	0.4	0.4	1.6839
7	2	0.35	0.6	0.5	2.7300
8	1.5	0.25	0.4	0.4	1.7445
9	1	0.15	0.3	0.3	1.0618
10	1	0.15	0.3	0.3	1.0845

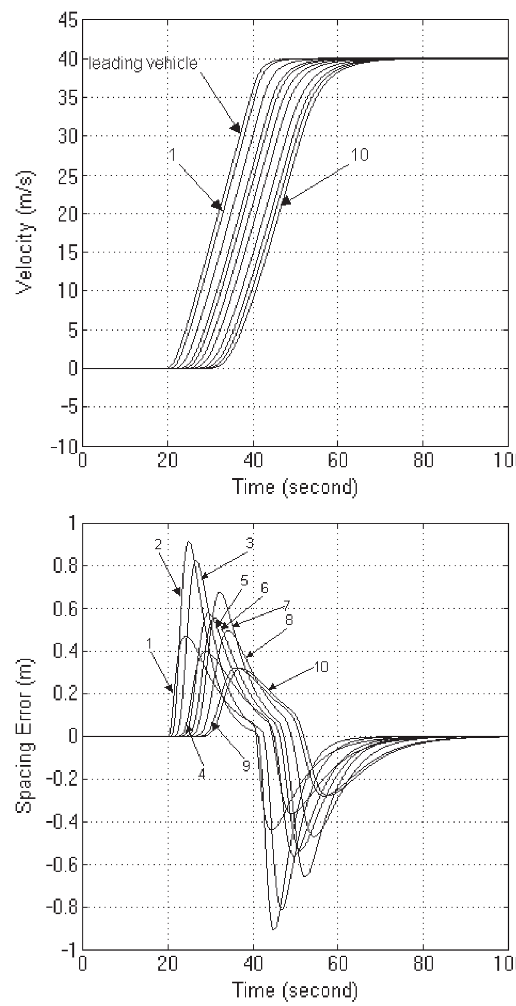


Fig. 5. String stable performance of the heterogeneous platoon.

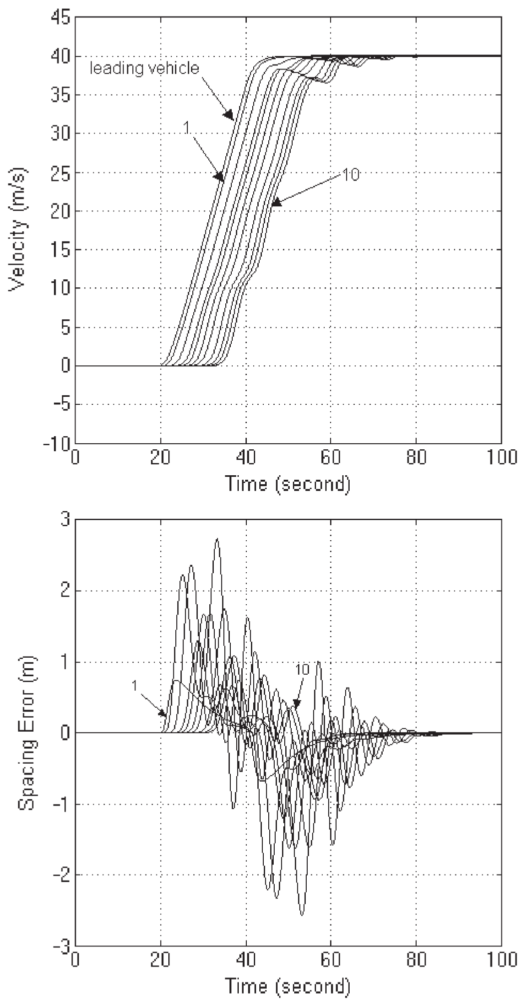


Fig. 6. String unstable performance of the heterogeneous platoon.

VII. Conclusion

This paper has extended the results, which have appeared in the literature, related to the string stable analysis and the string stable control of a platoon of ACC-equipped vehicles by considering the more practical scenarios such as the parasitic time delays and lags existing in the real actuators and sensors of ACC-equipped vehicles and the heterogeneous platoon existing in real traffic systems. It has clearly revealed that the proposed control law, which is based on the sliding model technique, can guarantee the string stability of both the homogeneous and the heterogeneous platoon. The related sufficient string stable conditions demonstrate that the CTH of any individual ACC-equipped vehicle is just constrained by the parasitic time delays and lags of the actuators and sensors of itself for both the homogeneous and the heterogeneous platoon, but the control gain of any individual ACC-equipped vehicle is constrained by itself and its immediate preceding ACC-equipped vehicle for the heterogeneous platoon.

The analysis of the negative effect on the string stability demonstrates that the negative effect of the "lumped" parasitic time delay is larger than that of the "lumped" parasitic time lag. It further proved that not considering the parasitic time delays in analyzing the string stability will lead to impractical results. In addition, based on the analysis of the practical string stability, a means to evaluate the performance of the current different ACC systems and a proposal to design the ACC systems have been offered with the perspective of the practical string stability.

Acknowledgment

The authors would like to thank Prof. S. Darbha of the Department of Mechanical Engineering, Texas A&M University, College Station, and the anonymous reviewers, who provided insightful and constructive advices and comments.

References

[1] L. Xiao and F. Gao, "A comprehensive review of the development of adaptive cruise control (ACC) systems," *Vehicle Syst. Dyn.*, vol. 48, no. 10, pp. 1167–1192, 2010.
[2] J. VanderWerf, S. Shladove, M. Mille, and N. Kourjanskaia, "Effects of adaptive cruise control systems on highway traffic flow capacity," *Transp. Res. Rec.*, vol. 1800, pp. 78–84, 2002, Paper No. 02-3665.
[3] R. Bishop, *Intelligent Vehicle Technology and Trends*. Boston, MA: Artech House, 2005.
[4] A. Vahidi and A. Eskandarian, "Research advances in intelligent collision avoidance and adaptive cruise control," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 3, pp. 143–152, Sep. 2003.
[5] P. Ioannou and C. Chien, "Autonomous intelligent cruise control," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 657–672, Nov. 1993.
[6] S. Darbha and K. Rajagopal, "A review of constant time headway policy for automatic vehicle following," in *Proc. IEEE Intell. Transp. Syst. Conf.*, Oakland, CA, Aug. 2001, pp. 65–69.
[7] J. Zhou and H. Peng, "Range policy of adaptive cruise control vehicle for improved flow stability and string stability," *IEEE Trans. Intell. Transp. Syst.*, vol. 6, no. 2, pp. 229–237, Jun. 2005.
[8] R. Rajamani, *Vehicles Dynamics and Control*. New York: Springer-Verlag, 2006.
[9] K. Chu, "Optimal decentralized regulation for a string of coupled systems," *IEEE Trans. Autom. Control*, vol. AC-19, no. 3, pp. 243–246, Jun. 1974.
[10] S. Darbha and J. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 349–357, Mar. 1996.
[11] R. Rajamani and S. Shladover, "An experimental comparative study of autonomous and co-operative vehicle-follower control systems," *Transp. Res. Part C*, vol. 9, no. 1, pp. 15–31, Feb. 2001.
[12] S. Darbha, "String stability of interconnected systems: An application to platooning in automated highway systems," Ph.D. dissertation, Univ. California, Berkeley, 1994.
[13] S. Darbha, J. Hedrick, C. Chien, and P. Ioannou, "A comparison of spacing and headway control strategy for automatically controlled vehicles," *Vehicle Syst. Dyn.*, vol. 23, no. 8, pp. 597–625, 1994.
[14] K. Santhanakrishnan and R. Rajamani, "On spacing policies for highway vehicle automation," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 4, pp. 198–204, Dec. 2003.
[15] D. Yanakiev and I. Kanellakopoulos, "Variable time headway for string stability of automated heavy-duty vehicles," in *Proc. 34th IEEE Conf. Decision Control*, New Orleans, LA, 1995, pp. 4077–4081.
[16] J. Moskwa and J. Hedrick, "Modeling and validation of automotive engines for control algorithm development," *Trans. ASME, J. Dyn. Syst. Meas. Control*, vol. 114, no. 2, pp. 278–285, Jun. 1992.
[17] P. Cook, "Stable control of vehicle convoys for safety and comfort," *IEEE Trans. Autom. Control*, vol. 52, no. 3, pp. 526–531, Mar. 2007.
[18] S. Huang and W. Ren, "Autonomous intelligent cruise control with actuator delays," *J. Intell. Robot. Syst.*, vol. 23, no. 1, pp. 27–43, Sep. 1998.
[19] D. Yanakiev and I. Kanellakopoulos, "Longitudinal control of automated CHVs with significant actuator delays," *IEEE Trans. Veh. Technol.*, vol. 50, no. 5, pp. 1289–1297, Sep. 2001.
[20] E. Shaw, "Heterogeneous string stability for vehicle formation control," Ph.D. dissertation, Univ. California, Berkeley, 2005.
[21] E. Shaw and J. Hedrick, "String stability analysis for heterogeneous vehicle strings," in *Proc. Amer. Control Conf.*, New York, Jul. 2007, pp. 3118–3125.
[22] E. Shaw and J. Hedrick, "Controller design for string stable heterogeneous vehicle strings," in *Proc. 46th IEEE Conf. Decision Control*, New Orleans, LA, Dec. 2007, pp. 2868–2875.

[23] I. Letas and G. Vinnicombe, "Scalability in heterogeneous vehicle platoons," in *Proc. Amer. Control Conf.*, New York, Jul. 2007, pp. 4678–4683.

[24] Y. Zhang, E. Kosmatopoulos, P. Ioannou, and C. Chien, "Autonomous intelligent cruise control using front and back information for tight vehicle following maneuvers," *IEEE Trans. Veh. Technol.*, vol. 48, no. 1, pp. 319–328, Jan. 1999.

[25] S. Mahal, "Effects of communication delays on string stability in an AHS environment," M.S. thesis, Univ. California, Berkeley, 2000.

[26] X. Liu, S. Mahal, A. Goldsmith, and J. Hedrick, "Effects of communication delays on string stability in vehicle platoons," in *Proc. IEEE Intell. Transp. Syst. Conf.*, Oakland, CA, Aug. 2001, pp. 625–630.

[27] S. Yadlapalli, S. Darbha, and K. Rajagopal, "Information flow and its relation to stability of the motion of vehicles in a rigid formation," *IEEE Trans. Autom. Control*, vol. 51, no. 8, pp. 1315–1319, Aug. 2006.

[28] P. Cook, "Conditions for string stability," *Syst. Control Lett.*, vol. 54, no. 10, pp. 991–998, Oct. 2005.

[29] S. Sheikholeslam and C. Desoer, "Control of interconnected nonlinear dynamical systems: The platoon problem," *IEEE Trans. Autom. Control*, vol. 37, no. 6, pp. 806–810, Jun. 1992.

[30] X. Lu and K. Hedrick, "Practical string stability for longitudinal control of automated vehicles," in *Proc. IAVSD Conf.*, Kanagawa, Japan, Aug. 2003.

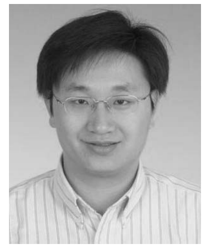
[31] P. Fancher, H. Peng, Z. Bareket, C. Assaf, and R. Ervin, "Evaluating the influences of adaptive cruise control systems on the longitudinal dynamics of strings of highway vehicles," in *Proc. IAVSD Conf.*, Copenhagen, Denmark, Aug. 2001.

[32] Z. Bareket, P. Fancher, H. Peng, K. Lee, and C. Assaf, "Methodology for assessing adaptive cruise control behavior," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 3, pp. 123–131, Sep. 2003.

[33] S. Huang and W. Ren, "Design of vehicle following control systems with actuator delays," *Int. J. Syst. Sci.*, vol. 28, no. 2, pp. 145–151, 1997.

[34] S. Sheikholeslam, "Control of class of interconnected nonlinear dynamical systems: The platoon problem," Ph.D. dissertation, Univ. California, Berkeley, 1991.

[35] P. Seiler, A. Pant, and K. Hedrick, "Disturbance propagation in vehicle strings," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1835–1841, Oct. 2004.



Lingyun Xiao was born in 1980. He received the Bachelors degree from Hunan University, Changsha, China, in 2003 and the Ph.D. degree in automotive engineering from Beihang University, Beijing, China, in 2010.

He was a visiting Ph.D. Candidate with the Department of Mechanical Engineering, Texas A&M University, College Station, from 2007 to 2009. He is currently an Assistant Research Fellow with the Defective Product Administrative Center, General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China, Beijing. His research interests include intelligent transportation systems, intelligent vehicles, intelligent control, traffic flow analysis, and time delay control.

Dr. Xiao is currently a reviewer for the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, the *Journal of Central South University of Technology*, and the Proceedings of several international conferences.

Feng Gao was born in 1955. He received the Ph.D. degree in automotive engineering from Jilin University, Changchun, China, in 1991.

He is currently a Professor with the School of Transportation Science and Engineering, Beihang University, Beijing, China, where he is also the Director of the Intelligent Vehicle Lab. His main research interests include intelligent vehicles, intelligent transportation systems, modern vehicle design methods, nonregular moving machines, and deep space explorer vehicles.

Dr. Gao is a Council Member of the Society of Automotive Engineers of China.