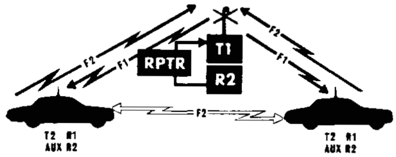

Fig. 11. Mobile relay system with standby communications using PSLM.

dilemma created by present methods. PSLM allows both agencies to monitor the other's transmissions while always assuring that each agency receives their own channel. Another typical application would be in a mobile relay system as shown in Fig. 11. The priority channel is the repeater transmitter frequency. Mobile relay operation is normal unless the relay station fails; then, mobile-to-mobile contact is retained automatically. Operator selection of priority allows a choice of either direct mobile or relay reception in weak signal areas.

PSLM adds a new concept to the tools which are available to the system designer. Its versatility will introduce the use of monitoring where it has not, before now, been satisfactory. Also, it will allow the expansion of present radio systems without adding the limitations of past monitoring methods.

Conclusion

An explanation of system problems which provoked the design of priority search-lock monitor has been presented. Also, the circuitry required to solve the problems encountered in the application of sampling techniques to multifrequency reception has been explained. Excellent results have been obtained from field tests performed in various large mobile systems. The use of this new receiver design will allow the design of improved communications systems.

Acknowledgment

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A Study of Automatic Car Following

JAMES G. BENDER AND ROBERT E. FENTON, MEMBER, IEEE

***Abstract*—Virtually all proposed vehicle control systems for highway automation must include a steady-state car-following mode. This mode was intensively investigated for various situations where headway and relative velocity inputs were used. In addition, a fundamental relationship between the flow capacity of an automated highway and the small-signal longitudinal response of a vehicle was investigated.**

The predictions obtained from mathematical models of various car-following modes were compared to those from full-scale tests. It was concluded that a linear mathematical representation of the longitudinal dynamics was valid, and thus it could be used for predictive purposes. Furthermore, it was verified that flow capacity on an automated highway is sharply limited by certain vehicle characteristics if headway feedback is used for vehicle control.

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The authors are with the Communication and Control Systems Laboratory, Department of Electrical Engineering, Ohio State University, Columbus, Ohio.

I. Introduction

An EXAMINATION of traffic conditions today, i.e., congested roadways, a large number of accidents and fatalities, and extremely powerful automobiles, indicates the need for improvement in our highway system. Unfortunately, these conditions will be much worse in 1980, for it is predicted that the total number of vehicles registered will be 62 percent greater than in 1960, and 75 percent more vehicle miles will be traveled [1]. If one would look further ahead to the turn of the century, he would see vast sprawling supercities with populations characterized by adequate incomes, longer life spans, and increased amounts of leisure time. One predictable result is greatly increased travel. The resulting traffic situation could be chaotic, unless some radical changes are instituted beforehand.

One promising approach for a partial solution to some of these problems is highway automation. This approach has been examined by a number of investigators [2]–[8], for in addition to the retention of the individual transportation unit, it appears that considerable improvements in highway capacity and safety can be achieved, as well as a considerable reduction in driver effort.

There are an extremely large number of possible systems for achieving such automation, and great care must be exercised so that an optimum or near-optimum one is chosen. However, there are certain functions which must be performed by virtually all such systems, and these can be profitably studied without regard to the overall system configuration. One of these, automatic steady-state car following, is examined here in detail, and certain fundamental limitations on system performance are obtained.

II. Car-Following System Requirements

The following restrictions must be imposed on the steady-state car-following mode of an automatic vehicle control system.

1) The average separation between adjacent vehicles must not be excessive; the smaller the spacing at a fixed velocity, the greater the capacity of the system.

2) The individual vehicle must be stable relative to the position of the lead car.

3) Disturbances propagated to the rear must be attenuated or at least limited in amplitude (this is termed asymptotic stability).

4) The control system must not be required to respond so as to exceed the response capabilities of the vehicle.

5) To insure the comfort of the occupants, the normal acceleration and deceleration of a vehicle must not exceed $0.1g$.

These requirements are next used to obtain a fundamental limitation on vehicle flow rates on an automated highway.

III. Theory

Consider a line of identical vehicles that are attempting to follow one another in a stable manner. For simplicity, only the first two vehicles in the line are considered; however, the results obtained also apply to the ith and the $(i+1)$th vehicles. It is assumed that the vehicles are initially in a steady-state situation with each vehicle traveling at a constant speed of V_s and separated from its two closest neighbors, front and rear, by a constant uniform headway h_0.

The small-signal car-following model under investigation is one in which the incremental acceleration pv_2 of a controlled vehicle is equal to a linear combination of relative velocity v and a headway error term. Thus

$$pv_2 = k_1 v + k_2(h - k_3 v_1 - k_4 v_2), \quad p \equiv d/dt \tag{1}$$

where v_1 is the incremental speed change of the lead car, h is the incremental change in headway between the lead and controlled vehicles, and k_1, k_2, k_3, and k_4 are constants. Generally, either k_3 or k_4 is zero. Note that the controlled vehicle only responds to changes in state between itself and the nearest lead vehicle.

If an incremental change in the lead-car speed occurs, the resulting steady-state change in spacing Δh_0 is obtained from (1) by setting $p = 0$ and recognizing that $v = 0$ in steady state. The result is

$$\Delta h_0 = (k_3 + k_4)\Delta V_s \tag{2}$$

where ΔV_s is the steady-state value of the speed change. It is convenient to define k as

$$k \equiv k_3 + k_4$$

so that

$$\Delta h_0 = k\Delta V_s.$$

The new steady-state headway h_{ss} is

$$h_{ss} = h_0 + k\Delta V_s.$$

In order to avoid small headways for decreases in $V_s(\Delta V_s < 0)$, one must have

$$h_0 \geq kV_s$$

or

$$h_{ss} \geq k(V_s + \Delta V_s). \tag{3}$$

Note that the factor k can be interpreted as the time headway if the equality is satisfied. This factor, together with the stream speed V_s, determines the vehicle flow rate.

The basic flow equation is

$$p = \lambda V_s$$

where p equals flow (cars/lane/time), λ equals concentration (cars/lane/distance), and V_s equals stream speed (mi/h). For the case under consideration

$$\lambda = [5280/(L + 1.465kV_s)] \text{ cars/lane/mile}$$

where L is the vehicle length in feet;

$$p = \frac{5280V_s}{L + 1.465kV_s}$$

$$= \frac{5280}{L/V_s + 1.465k} \text{ cars/lane/hour.}$$

Thus k must be kept small if one is to achieve high flow rates.

The system function corresponding to (1) is

$$\frac{V_2(s)}{V_1(s)} = \frac{(k_1 - k_2k_3)[s + k_2/(k_1 - k_2k_3)]}{s^2 + (k_1 + k_2k_4)s + k_2}, \quad s = \sigma + j\omega \tag{4}$$

where $V_2(s)$ equals the Laplace transform of $v_2(t)$ and $V_1(s)$ equals the Laplace transform of $v_1(t)$. If this system is to be locally stable, it is necessary that the coefficients of the denominator polynomial be positive, i.e.,

$$k_2 > 0 \tag{5}$$

$$k_1 + k_2k_4 > 0. \tag{6}$$

If the system is also to be asymptotically stable, it is necessary that [9]–[10]

$$|V_2(s)/V_1(s)|_{s=j\omega} \leq 1, \quad \text{for all } \omega. \tag{7}$$

The vehicle response to small changes in command velocity can be described in terms of an effective vehicle time constant τ. Here, τ is defined as the time required for the vehicle to reach 0.632 of its final velocity value for a command step change in velocity.

If one suitably combines (3)–(7) and utilizes the effective time constant concept, he can show that a necessary condition for asymptotic stability is [11]

$$k \geq 0.787\tau. \tag{8}$$

A stronger condition is

$$k \geq \tau \tag{9}$$

however, there are circumstances when a value intermediate between 0.787τ and τ can be used.

In practice, one encounters vehicle time constants ranging from 8 to 40 seconds; however, if control compensation is used, the effective time constant can be reduced to 4 seconds or less, as shown in a succeeding section. However, even for $\tau = 4$, the corresponding necessary condition is

$$k \geq 3.148$$

and one has a relatively large value of required intervehicular spacing, e.g., 323 feet at a stream speed of 70 mi/h (102 ft/s). If one is to have both asymptotic stability and a small intervehicular spacing ($k < 1$), it would be necessary to reduce τ to 1 second or less. This is, of course, based on the premise that headway information is used in conjunction with relative velocity in the car-following mode of control. This conclusion was verified by several experimentally obtained car-following models, which are discussed in succeeding sections.

IV. Experimental Apparatus

The specially instrumented vehicle shown in Fig. 1 was used to obtain experimental data for several of the control configurations. The three main control functions, braking, acceleration, and steering, were accomplished using electrohydraulic control systems, with the closed-loop system for control of the throttle valve shown in Fig. 2. The input signal was compared with a signal from a potentiometer coupled to the system output—the position of the throttle valve. The resulting error signal was applied to an electrohydraulic control valve, which in turn controlled a hydraulic actuator connected to the throttle valve.

The braking control system was similar to that shown in Fig. 2. However, the hydraulic actuator was connected so that it engaged the brake-pedal assembly. Thus the original brakes were intact and available for emergency use. The steering control system was also similar to that shown in Fig. 2, the principal difference being that the output was front-wheel angular position, rather than throttle-valve position.

A hybrid computer consisting of 20 solid-state operational amplifiers and 15 potentiometers was installed over the back seat. The computing elements were used for system compensation and data collection. All data collected were recorded on a four-channel FM magnetic tape recorder which was mounted under the computer.

Fig. 1. Instrumented test vehicle.

The longitudinal dynamics of the vehicle for the small-signal case were represented by

$$v_2/T_v = K/(Tp + 1)$$

where T_v equals the incremental change in throttle-valve position from a steady-state position T_{v0}, T equals the uncompensated vehicle time constant, and K equals the steady-state gain from throttle valve to output speed. It should be noted that the dynamics of the electrohydraulic throttle-valve control system were negligible in comparison with vehicle dynamics and hence, were not considered.

In practice, both K and T tended to vary with road conditions, wind direction, temperature, etc. The adverse effects of such variations were partially overcome by using an internal feedback loop around the vehicle, as shown in Fig. 3. The remaining blocks in this figure correspond to system compensation. This block diagram can be simplified to the one shown in Fig. 4 where

$$T_c = T/(1 + K\delta)$$

and

$$K_c = K/(1 + K\delta).$$

If the system compensation is properly chosen (i.e., the pole at $-(1 + K\delta)/T$ is cancelled by the compensator zero), the resulting function is

$$v_2/e = K_d/p$$

where K_d equals K_aK_c, K_a equals the constant in the compensation network, and e equals the input signal to the compensated vehicle dynamics.

The resultant block diagram for the small-signal car-following situation is shown in Fig. 5. Note that the net input to the compensated vehicle dynamics K_d/p is

$$(1/K_d)[k_1v + (k_2v/p) - k_2k_3v_1 - k_2k_4v_2],$$

and thus one is led directly to the basic system car-following equation.

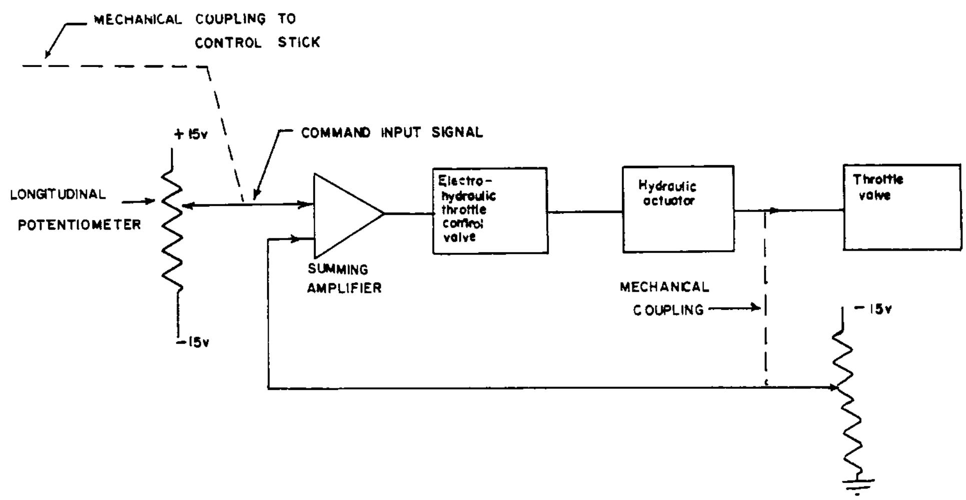

Fig. 2. Closed-loop system for control of throttle valve.

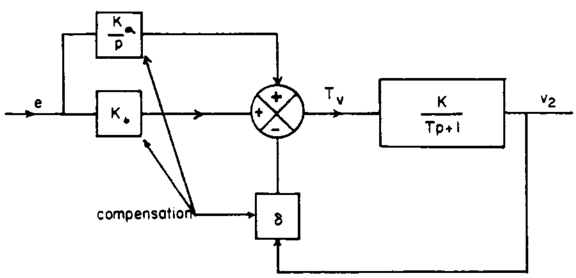

Fig. 3. Modification of vehicle longitudinal dynamics.

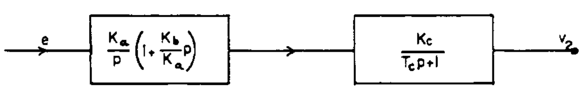

Fig. 4. Reduced block diagram.

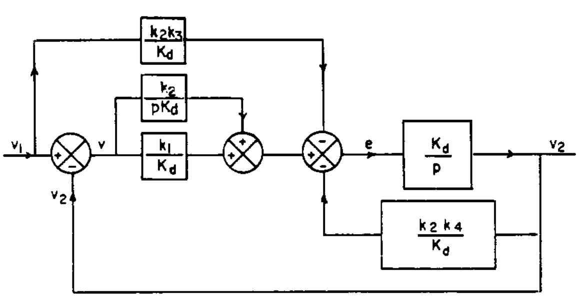

Fig. 5. Longitudinal control system.

The incremental velocity and headway, v_1 and h respectively, must be available for full-scale testing. However, instead of using a lead vehicle in the testing program, it was decided to use a "phantom car" since the vehicle control system could not distinguish between an actual lead car and a phantom lead car. This phantom car was represented by an appropriate signal fed into the onboard computer. Thus, the required measures of v_1 and h were available. (It should be noted that some of the tests described herein were duplicated using an actual lead car together with a mechanical yo-yo for obtaining v_1 and h. The results were similar to those obtained from the phantom car tests.) No satisfactory practical method has yet been developed for continuously measuring v_1 and h, and this lack certainly represents one of the most pressing problems in highway automation.

V. Experimental Description

The experimental work was done on a 5-mile unopened section of typical interstate highway (Interstate Route 270 near Columbus, Ohio). Six different car-following situations were examined, and a car-following model $[G(j\omega)]$ of the form

$$G(j\omega) = [V_2(s)/V_1(s)]_{s=j\omega} \tag{10}$$

was obtained for each case. This is simply a small-signal frequency-response model.

The experimental procedure was the same for each case. First, a steady-state car-following situation was established with both the phantom lead car and the controlled vehicle traveling at the same constant speed. A sinusoidal component with a peak-to-peak magnitude of 4 ft/s was then added to the lead-car speed, and the resulting sinusoidal response of the controlled car was obtained. Tests were conducted at lead-car average speeds of 40 and 60 mi/h, and data were collected at various frequencies between 0.06 and 1.0 rad/s. This frequency band was chosen as it encompassed the critical frequencies for steady-state longitudinal control. The six cases studied are defined in the following sections.

Case 1: The parameters chosen for this case are $k_1 = 0.25$, $k_2 = 0.125$, $k_3 = 0$, and $k_4 = 1$. The corresponding system differential equation is

$$pv_2 = 0.25v + 0.125(h - v_2) \tag{11}$$

so that the instantaneous headway deviation is a function of v_2. The effective system time constant and the required time headway are determined from this equation to be 2.5 and 1.0, respectively. Thus the condition posed by (8) is not satisfied, and the system is, at least in theory, asymptotically unstable.

Case 2: Here, $k_1 = 0.25$, $k_2 = 0.0625$, $k_3 = 0$, and $k_4 = 4.0$; and the corresponding system equation is

$$pv_2 = 0.25v + 0.0625(h - 4v_2). \tag{12}$$

Here again the instantaneous headway deviation is a function of v_2, and both τ and k are 4. Thus (8) is satisfied; however, it should be noted that excessive intervehicular spacing results, e.g., 352 feet at 60 mi/h.

Case 3: The values chosen for the k_i are $k_1 = 0.5$, $k_2 = 0.125$, $k_3 = 1$, and $k_4 = 0$; and the resulting system equation is

$$pv_2 = 0.5v + 0.125(h - v_1). \tag{13}$$

In this case the headway control signal is a function of the incremental change in lead-car speed. The corresponding values of τ and k are 2 and 1, respectively. Thus, one would predict from (8) that this system is asymptotically unstable.

Case 4: This case is similar to that of Case 2 except that the instantaneous headway deviation is a function of v_1 rather than v_2. The k_i are $k_1 = 0.5$, $k_2 = 0.0625$, $k_3 = 4$, and $k_4 = 0$; and

$$pv_2 = 0.5v + 0.0625(h - 4v_1). \tag{14}$$

It is easily determined from this equation that $\tau = 4$ and $k = 4$. Thus the system is asymptotically stable; however, the required steady-state headway is excessive.

Case 5: The effects of a reduction in the effective vehicle time constant are considered here where $k_1 = 1$, $k_2 = 0.5$, $k_3 = 0$, and $k_4 = 1$; and

$$pv_2 = v + 0.5(h - v_2). \tag{15}$$

The condition posed by (8) is satisfied, for the corresponding values of τ and k are both 1. This choice of parameters results in both asymptotic stability and possible high traffic flow rates; however, it also leads to some practical difficulties. In order to insure system compatibility with existing traffic, the vehicle system time constant τ should be chosen such that it may be realized by the majority of automobiles in use on the highways. Currently, it appears doubtful that this requirement can be satisfied. Furthermore, the vehicle would be highly responsive to changes in lead-vehicle speed, possibly resulting in passenger discomfort and poor gas mileage.

Case 6: The velocity controller is defined by the following equation:

$$pv_2 = 0.25v \tag{16}$$

where $k_1 = 0.25$, and $k_2 = k_3 = k_4 = 0$. Here, no headway information is used, and the acceleration of the controlled vehicle is dependent only on relative velocity. The resulting system is asymptotically stable, as one can see from the transfer function equivalent of (16):

$$V_2(j\omega)/V_1(j\omega) = k_1/(j\omega + k_1).$$

The steady-state headway is arbitrary and can be specified by factors other than those discussed here.

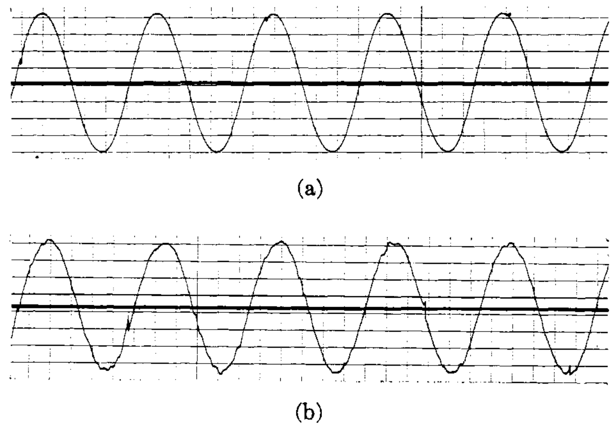

Fig. 6. Velocity time histories of lead and following cars; time scale, 0.5 cm/s. (a) ΔV_1, velocity variation of phantom vehicle; $\Delta V_1 = 2.0 \sin(1.14t)$ ft/s. (b) ΔV_2, response of controlled vehicle due to ΔV_1.

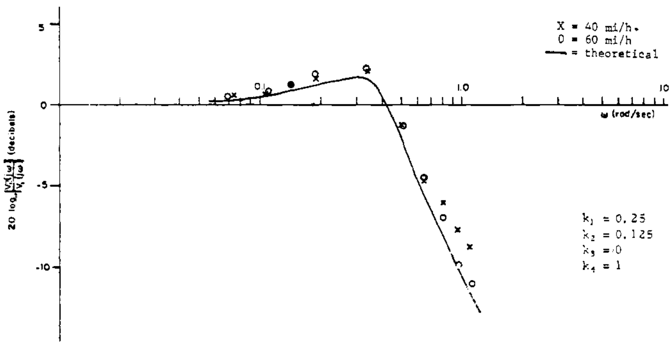

Fig. 7. Describing function, linear mode controller with headway information ($k = 1$, $\tau = 2.5$).

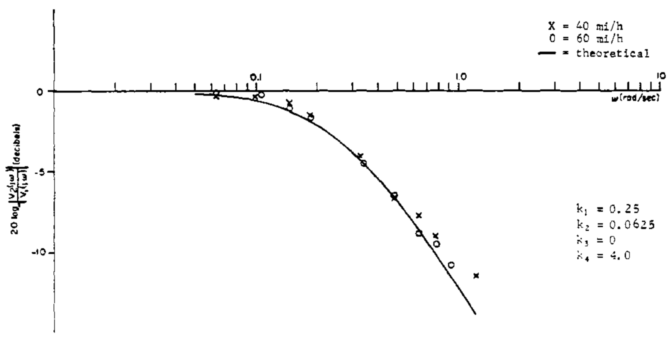

Fig. 8. Describing function, linear mode controller with headway information ($k = 4$, $\tau = 4$).

VI. Experimental Results

The data collected consisted of tape recordings of the sinusoidal components of both the lead-car velocity and the controlled-vehicle velocity. A time trace of some typical sinusoidal components is shown in Fig. 6. The data were reduced on an analog computer and two car-following models were obtained for each case, one corresponding to an average lead-car speed of 40 mi/h, and the other to an average speed of 60 mi/h.

The models obtained for Case 1 are shown in Fig. 7;

$$20 \log_{10} | V_2(j\omega)/V_1(j\omega) |$$

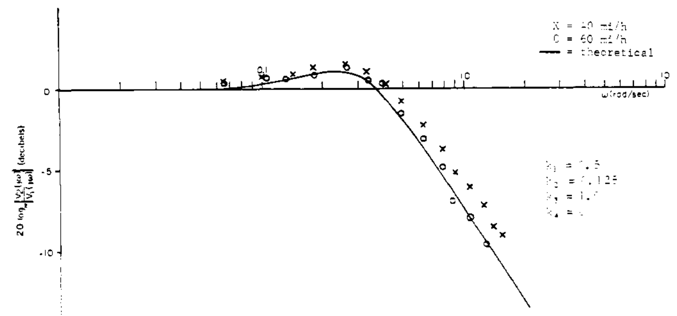

Fig. 9. Describing function, linear mode controller with headway information ($k = 1$, $\tau = 2$).

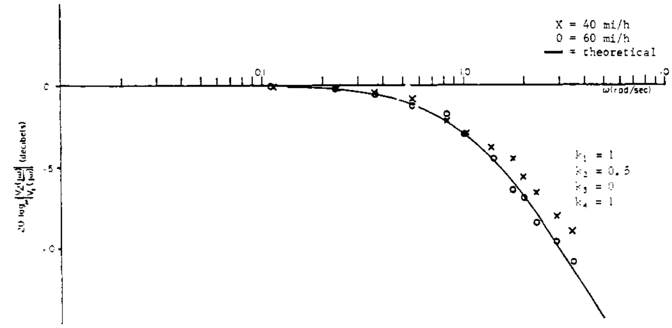

Fig. 11. Describing function, linear mode controller with headway information ($k = 1$, $\tau = 1$).

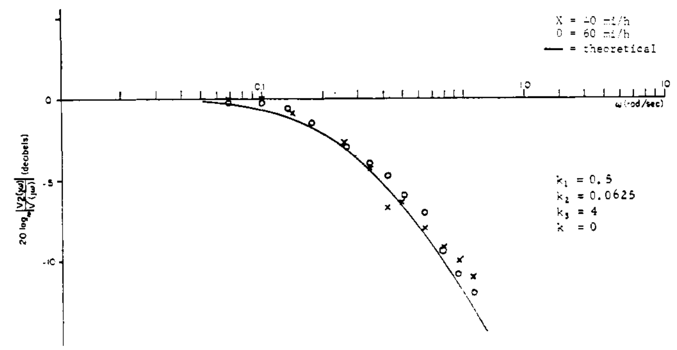

Fig. 10. Describing function, linear mode controller with headway information ($k = 4$, $\tau = 4$).

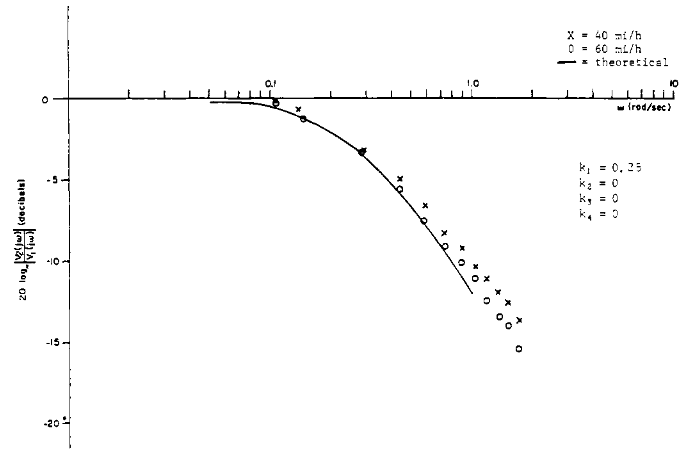

Fig. 12. Velocity controller.

is plotted versus ω for each speed. The solid curve was obtained from the system differential equation (11), and it represents the predicted performance of the system. The "circle" data points correspond to data collected at an average speed of 60 mi/h, and the "X" data points correspond to data obtained at 40 mi/h.

The magnitude of both the theoretical curve and the one formed by joining the data points is greater than 0 dB which indicates that, as expected, the system is asymptotically unstable. (The condition posed in (7) is equivalent to $20 \log_{10} | V_2(j\omega)/V_1(j\omega) | \leq 0$, for all ω).

Note the excellent agreement between the theoretical curve and the data, a clear indication of the validity of the small-signal model. It is also interesting to note the close correspondence of the data at 40 and 60 mi/h, a strong indication that the small-signal model is valid at other average speeds within this range.

The corresponding curves for Case 2 are shown in Fig. 8, from which it can be noted that the system is asymptotically stable as predicted. One should again note both the excellent agreement between the theoretical curve and those obtained by connecting the appropriate data points, and the close correspondence of the data at 40 and 60 mi/h. The curves obtained for Case 3 are shown in Fig. 9, from which both the close agreement between theory and experiment can be noted together with the recognition that the system is asymptotically unstable. In Case 4, the value of k was suitably increased, and, as shown in Fig. 10, the resulting system is asymptotically stable. The interesting and predictable effect of decreasing τ is shown in Fig. 11, which corresponds to Case 5. Here $k = 1$, as in Cases 1 and 3; however, by reducing τ to unity, an asymptotically stable system has been obtained. Finally, the results corresponding to the velocity controller (Case 6) are shown in Fig. 12. Note that the system is asymptotically stable as predicted.

VII. Conclusions

It has been shown that one must be quite careful in the choice of a linear controller for automatic car following, especially if the use of headway feedback is contemplated. If one wishes to utilize such feedback and also to achieve high traffic densities ($k \leq 1$), he must use a vehicle system with a small time constant, i.e., a vehicle system which responds very quickly to small command changes in speed. If the relationship between the vehicle response and the time headway is not satisfied, the resulting system is asymptotically unstable.

An examination of the Bode plots in the previous section clearly shows that the theoretical curves are extremely good representations of the system functions, as in all cases the theoretical curve and the data points are almost superimposed. Further, essentially the same data were obtained at both average testing speeds, thus indicating that the small-signal model is valid at other average speeds within the range from 40–60 mi/h. In light of these results, it appears that the small-signal mathematical model (1) can be used with considerable confidence for predicting system performance.

Thus, if one wishes to achieve high traffic densities, one must use either a high performance vehicle or a velocity controller. This assumes, of course, that one wishes to use a linear controller for automatic car following. It may also be possible to achieve high densities and an asymptotically stable system by using a nonlinear controller. This point is currently under investigation.

The use of a velocity controller yields the advantages of an arbitrary separation between vehicles and relative simplicity of implementation. The separation can be fixed by other factors. For these reasons, a velocity controller has been selected for use in the automatic longitudinal control system now being tested at Ohio State University [3].

Acknowledgment

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Fig. 1. Instrumented test vehicle.