# Error Rate for Optimal Follower Tone-Jamming

Amer A. Hassan, Wayne E. Stark, and John E. Hershey

***Abstract*— The performance of a countermeasure technique in the presence of the optimal follower multitone jammer is evaluated for frequency-hopped spread spectrum (FHSS) communications. It is shown that, with a certain probability, the optimal jammer will have dual tones in a frequency channel.**

## I. Introduction

A JAM RESISTANT approach was proposed in [1] in which a transmission technique enhances the strategy available to the communicator in the presence of a follower Gaussian partial-band jammer. This technique utilizes randomized decisions by the transmitter and the receiver to lure the follower jammer into helping the communicator's performance part of the time; the technique also allows the jammer an extra strategy not to follow the signal part of the time. The same technique was also evaluated in [2] for an uncoded slow frequency-hopping spread spectrum (FHSS) system in the presence of multitone follower jammer with the jammer restricted to one tone per frequency channel. In this paper, we demonstrate that the optimal tone jammer is one which is allowed to transmit either one tone or two tones in a frequency channel, depending on the mode of transmission. The system model is summarized in Section II and the performance analysis is given in Section III.

## II. System Model

Consider a slow FHSS communication system that uses orthogonal $M$-ary frequency-shift-keyed ($M$-FSK) data modulation. Let $q$ be the number of frequency channels available for hopping, such that $qM$ tones are available. A channel is, therefore, a group of $M$ tones. In slow frequency hopping more than one $M$-ary symbol are transmitted in a frequency hop.

For each signaling interval, the transmitter/receiver operates in one of two modes—*conventional* or *unconventional.* The *conventional* mode is selected by the transmitter and the receiver with pseudorandom probability $p_c$. In this case, the transmitter transmits one of $M$ tones within the corresponding channel (i.e., within one of $q$ channels). The receiver consists of a dehopper followed by $M$ noncoherent matched filters. The filter corresponding to the largest output is taken to be the transmitted symbol.

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The *unconventional* mode is selected with probability $1-p_c$. In this case, the transmitter randomly chooses one of the $M$ tones and transmits it in one of $M$ channels, where the $M$ channels are selected *pseudo-randomly.* Notice that the $M$ tones do not carry any information, but it is the presence or absence of energy in the $M$ channels that conveys information. The receiver looks at the whole of each of $M$ channels and consists of an energy detector which may be a compressive receiver or a bank of $M$ independent radiometers. The channel with the largest energy is chosen as the transmitted symbol.

The channel is described as follows. The main source of interference is a multiple tone follower jammer. This jammer consists of $Q$ tones ($Q > 0$) each with power large enough to dominate a communicator's tone. In [2], the jammer model is such that only one jamming tone can be transmitted in a channel. Now we evaluate the performance of a tone jammer who can inject multiple tones per channel if it chooses. Two modes of operation are also available to the jammer—*conventional* and *unconventional.*

The jammer chooses to operate in the *conventional* mode with probability $p_j$. In this case, the jammer injects a single tone in the transmitter's channel and transmits the other $Q-1$ tones in $N \leq Q-1$ distinct and randomly chosen channels. Each channel can have one or more tones. Recall that each of the $Q$ tones has power large enough to dominate a communicator's tone; the rest of the tones are randomly distributed to combat the unconventional mode of the communicator.

The *unconventional* mode is chosen with probability $1-p_j$. In this case, the jammer does not jam the transmitter's channel, but randomly distributes his $Q$ tones over the other $q-1$ distinct channels such that each channel has at most one interfering tone. The jammer is assumed to have knowledge of the parameter $M$.

## III. Performance Evaluation

The multitone jammer is power limited to $J$, and it transmits a tone with power just enough to exceed that of the transmitter's power. If the communicator's power at the receiver is $S$, we have

$$Q = \left\lfloor \frac{J}{S} \right\rfloor = \left\lfloor \frac{W_{ss}N_J}{R_bE_b} \right\rfloor = \left\lfloor \frac{Mq}{\log_2 M \dfrac{E_b}{N_J}} \right\rfloor \tag{1}$$

where $W_{ss}$ is the total spread spectrum bandwidth, $E_b$ is the transmitted energy-per-information bit, and $R_b$ is the bit rate in b/s. The effective noise power spectral density (PSD) is $N_J = J/W_{ss}$.

Now let $Z_c$ be a random variable that takes values in $\{0, 1\}$ such that $Z_c = 0$ indicates that the transmitter/receiver are in the conventional mode, and $Z_c = 1$ indicates the unconventional mode. Similarly

$$Z_J = \begin{cases} 0, & \text{if the jammer is in conventional mode,} \\ 1, & \text{if the jammer is in unconventional mode.} \end{cases}$$

Also, let $E_s$ be the event that an information symbol is in error, and $p_b$ be the bit error rate (BER). Then

$$\begin{aligned} P(E_s) &= \sum_{i,j} P(E_s, Z_c = i, Z_J = j) \\ &= \sum_{i,j} P(E_s | Z_c = i, Z_J = j) \\ &\quad \cdot P(Z_c = i) P(Z_J = j) \end{aligned} \tag{2}$$

where $P(Z_c = 0) = p_c$ and $P(Z_J = 0) = p_j$ and in the absence of thermal noise

$$\begin{aligned} P(E_s|Z_c = 0, Z_J = 0) &= 1 \\ P(E_s|Z_c = 0, Z_J = 1) &= 0. \end{aligned} \tag{3}$$

An *optimal* conventional jammer will follow the transmitter's tone by injecting one tone in the complementary channel *and* randomly distribute the remaining $Q - 1$ tones among $\lfloor (Q-1)/2 \rfloor$ channels with *exactly* two tones per channel. In this case, $P(E_s|Z_c = 1, Z_J = 0) \neq 0$, as was the case in [2]. Because the two tones, with finite probability, can coincide with one of the transmitter's unconventional channels, the error probability is nonzero. Notice that two tones are optimal and more tones will be a waste of the jammer's energy. Equivalently, the jammer can use one tone with slightly more than twice the power of the communicator's tone.

Let $p_h(Q) = P(E_s|Z_c = 1, Z_J = 1)$, then, $p_h$ is the probability that the jammer hits at least one of the $M - 1$ remaining signaling channels. This is given by

$$p_h(Q) = \begin{cases} 1 - \prod_{i=1}^{Q} \left(1 - \dfrac{M-1}{q-i}\right), & \text{if } Q \leq q - M + 1 \\ 1, & Q > q - M + 1. \end{cases}$$

The condition $Q \leq q - M + 1$ is equivalent to $E_b/N_J \geq Mq/\log_2 M(q - M + 1)$. Also

$$P(E_s|Z_c = 1, Z_J = 0) = p_h\left(\left\lfloor \frac{Q-1}{2} \right\rfloor\right). \tag{4}$$

Therefore, the BER is given by

$$\begin{aligned} p_b = \frac{M}{2(M-1)} \Big\{ & p_c p_j + (1-p_j)(1-p_c) p_h(Q) \\ & + (1-p_c) p_j p_h\left(\left\lfloor \frac{Q-1}{2} \right\rfloor\right)\Big\}. \end{aligned} \tag{5}$$

Let

$$\begin{aligned} p_b(p_j, p_c) = & \, p_j p_c \alpha + (1-p_j)(1-p_c)\beta \\ & + (1-p_c) p_j \gamma \end{aligned}$$

for obvious expressions of $\alpha$, $\beta$, and $\gamma$.

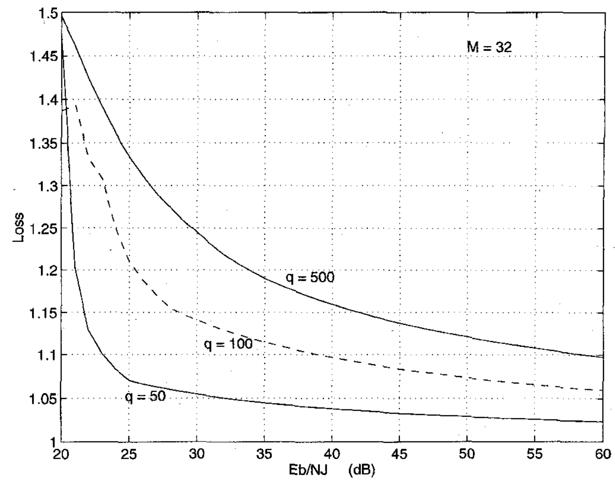


Fig. 1. Loss in $p_b$ for different bandwidth expansions.

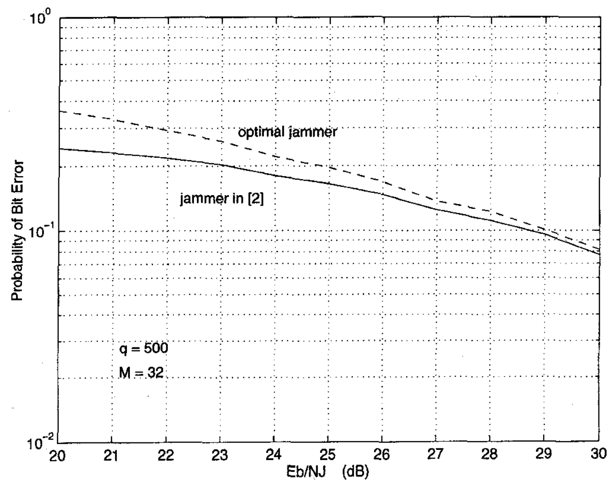


Fig. 2. Performance in the presence of an optimal jammer as compared with [2].

For $\gamma = 0$, it was shown in [1] that

$$\begin{aligned} \max_{p_j} \min_{p_c} p_b(p_j, p_c) &= \min_{p_c} \max_{p_j} p_b(p_j, p_c) \\ &= \frac{\alpha\beta}{\alpha + \beta} \\ &\equiv p_0. \end{aligned}$$

For the case $\gamma > 0$, the function $p_b(p_j, p_c)$ is asymmetric because of the term $(1 - p_c)p_j$. However, it can still be shown that

$$\begin{aligned} \max_{p_j} \min_{p_c} p_b(p_j, p_c) &= \min_{p_c} \max_{p_j} p_b(p_j, p_c) \\ &= \frac{\alpha\beta}{\alpha - \gamma + \beta} \geq p_0. \end{aligned}$$

An optimal jammer will cause degradation in the BER by $[1 - \gamma/(\alpha + \beta)]^{-1}$ as compared to the single tone (per frequency channel) jammer. Fig. 1 shows the loss in the probability of bit error as a function of $E_b/N_J$, and for different spread spectrum bandwidths. The loss increases as

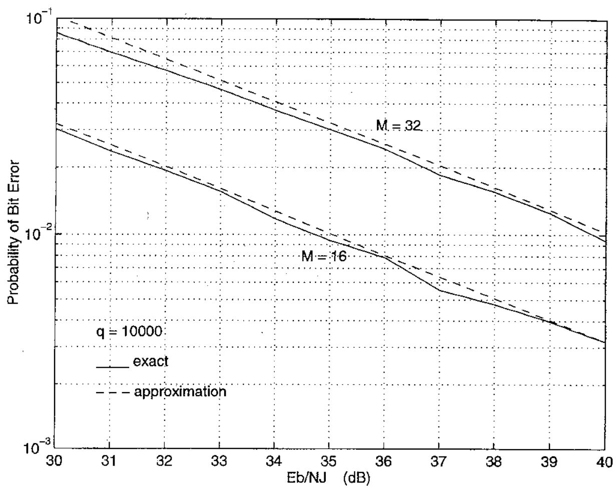


Fig. 3. Asymptotic approximation of error probability.

the spread spectrum bandwidth increases. Fig. 2 shows the relative performance of the optimal jammer as compared to the jammer in [2]. For instance, for $q = 500$ and at $p_b = 2 \times 10^{-1}$, the optimal jammer causes further degradation of 1.7 dB in $E_b/N_J$.

Next, we find an asymptotic expression for $p_b$ as $q$ becomes large and $E_b/N_J \gg M/\log_2 M$. Recall the following:

$$p_h = 1 - \prod_{i=1}^{aq}\left(1 - \frac{M-1}{q-i}\right)$$

where $a = M/(\log_2 M E_b/N_J)$ (as in (1), and assuming $aq$ is an integer).

Assume that $E_b/N_J$ is large. Then, $a < 1$ and the following inequality always hold

$$\left[1 - \frac{M-1}{q(1-a)}\right]^{aq} \le \prod_{i=1}^{aq}\left(1 - \frac{M-1}{q-i}\right) \le \left(1 - \frac{M-1}{q-1}\right)^{aq}$$

For asymptotically large $E_b/N_J$, $a \ll 1$. Using the approximation $(1-x)^n \approx 1 - nx$ for small $x$, the following holds for $q$ large

$$1 - aq\frac{M-1}{q(1-a)} \le \prod_{i=1}^{aq}\left(1 - \frac{M-1}{q-i}\right) \le 1 - aq\frac{M-1}{q-1}$$

$$1 - \frac{(M-1)M}{\log_2 M}\frac{1}{\frac{E_b}{N_J}} \le \prod_{i=1}^{aq}\left(1 - \frac{M-1}{q-1}\right) \le 1 - \frac{(M-1)M}{\log_2 M}\frac{1}{\frac{E_b}{N_J}}.$$

Thus

$$p_h(Q) \approx p_h\left(\left\lfloor\frac{Q-1}{2}\right\rfloor\right) \approx 1 - \frac{(M-1)M}{\log_2 M}\frac{1}{\frac{E_b}{N_J}}.$$

The optimal probability of bit error is

$$p_b = \frac{\alpha\beta}{\alpha+\beta} = \frac{M^2}{\log_2 M}\frac{1}{\frac{E_b}{N_J}}.$$

Figure 3 shows that the asymptotic approximation of $p_b$ for large $q$ and large $E_b/N_J$ is very accurate.

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# A Co-Channel Interference Cancellation Technique Using Orthogonal Convolutional Codes

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*Abstract*—**This paper proposes a new parallel co-channel interference cancellation technique which utilizes orthogonal convolutional codes. Co-channel interference (CCI) limits the performance of a spread spectrum multiple access communication link. Several CCI cancellation techniques have been proposed to remove this interference. Of particular interest are techniques which do not require the receiver to have knowledge of the cross-correlation between user sequences. These techniques reconstruct the CCI based on the initial decisions regarding the signals from the other users. However, these techniques leave residual interference after cancellation caused by errors in these initial decisions. To improve the initial decisions and reduce the residual interference, our proposed scheme utilizes the error correcting capability of orthogonal convolutional codes. This paper evaluates the performance of this scheme. We show that, given a processing gain of 128 for up to about 40 users, the performance of the proposed CCI canceller approaches the performance of a system without multi-user interference. We also show that the proposed CCI canceller offers an improvement in capacity by a factor of** $1.5 \sim 3$ **over that of a conventional canceller.**

## I. Introduction

SPREAD SPECTRUM techniques have recently received much attention in wireless communication applications such as the low-cost wireless local area network (LAN). This is in large part due to the fact that spread spectrum techniques have superior multi-access capability, anti-multipath fading capability, and anti-jamming capability [1].

There are three basic multiple access schemes for wireless communications: frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA) [2]. CDMA capacity is interference limited, unlike FDMA, and TDMA capacities, which are primarily bandwidth limited [3]. It has been suggested that CDMA could be used to increase the number of channels per unit bandwidth under certain conditions [2], [3]. The promise of an increase in capacity has made CDMA very attractive, for instance, in cellular communications where no additional spectrum will be allocated for digital cellular [2]. CDMA has also been considered for satellite and other wireless applications.

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To fully realize the capacity potential of CDMA, multi-user detection has been considered [4]. In [5], the optimum multi-user detector for asynchronous CDMA was derived and analyzed. It consists of a matched filter front end followed by a Viterbi algorithm. Although the optimum detector significantly outperforms the conventional single user detector, its computational complexity grows exponentially with the number of users.

Less complex suboptimum multi-user detectors have also been considered. In [6], sequential decoding is applied instead of the Viterbi algorithm. In [7]–[10], the decorrelating detector, which multiplies an inverse cross-correlation matrix with the matched filter outputs, has been investigated. In [11], the decorrelator is combined with a decision-feedback detector. Although the performance of these multi-user detectors is close to that of the optimum detector and they involve a reasonable amount of computation, these detectors must calculate the inverse cross-correlation matrix. In [12], the minimum mean-square error (MMSE) detector which calculates the inverse matrix adaptively, was proposed. This detector outperforms the decorrelating detector when background noise is the dominant factor limiting the performance. However, it requires a training sequence. In [13] and [14], tentative-decision based multi-user detectors have been investigated. These detectors have a multistage structure, where the first stage consists of a bank of conventional detectors. The second and third stages assume that the previous decisions are correct, calculate the co-channel interference (CCI) caused by undesired users' signals, and remove the result from the correlator output of the desired user's signal.

Unfortunately, these suboptimum techniques are not practical if long pseudo noise (PN) sequences are used as signature sequences to separate the users [15]. This is because these techniques assume that the receiver has (*a priori* or through training) knowledge of the sequence's cross-correlation. However, with long PN sequences, the cross-correlation varies from one symbol to another as different parts of the signature sequences are used for different symbols. If the receiver has to calculate the cross-correlation for each symbol, the computational complexity grows exponentially with the number of users.

Multi-user detection methods which do not require knowledge of the sequence's cross-correlation and whose complexity grows only linearly with the number of users have also been proposed [16], [17]. In [16], users are detected successively; in [17], they are detected simultaneously. In these methods, the receiver reconstructs other users' transmitted signals by using