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Decentralized Controller Design for String of Moving Vehicles Based on Inclusion Principle*

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***Abstract* - A decomposition methodology to design decentralized feedback controllers for a linear system is proposed in this paper. The controller regulates the position and velocity of every vehicle in a densely packed string of high-speed moving vehicles. The linear model of the vehicle-string velocity and distance bias system can be of overlapping interconnected subsystem models constructed by pair-wise vehicles. Centralized controllers can provide better results but are not suitable for the system due to vehicle autonomies. By using the inclusion principle conditions, the overlapping interconnected structure of the system appears as decoupled. The general theoretical formulation and solution of the decentralized LQG control problem is then presented. Simulation studies are also considered for the case of a string of four vehicles based on the decentralized algorithm. The application results of the decentralized controller illustrate the effectiveness of the proposed methodology.**

Index Terms* – *Intelligent transportation system, overlapping decomposition, LQG control, inclusion principle

I. INTRODUCTION

Due to the congestion of vehicles in highways, automated highway systems have recently become a research topic in transportation, and as a result there has been a good deal of attention paid recently to the problem of controlling a platoon of identical vehicles along a straight line. Meanwhile, the decentralized control for string of moving vehicles has attracted considerable attention of researchers in the field of complex and large-scale systems [1, 2, 4]. Platoon of identical vehicles systems are strongly coupled systems, generally of large scale. They are a good application for the recent improvements in decentralized control theories. Moreover, decentralized overlapping multivariable control, allowing to highly decrease coupling for large-scale systems, is a innovative strategy for platoon of vehicles systems. In striving for high efficiency of decentralized control schemes, a suitable choice of subsystems is absolutely essential. More often than not, this choice is limited by the physical constraints of the plant, which implies that a compromise has to be made between the computational simplifications resulting from partitioning of the system, and the practicality of the subsequent control implementation. Platoon of identical vehicles systems can be partition by overlapping decompositions via the selection of appropriate decompositions when disjoint decompositions fail. Therefore, decomposition methodologies play the central role in solving problems in these systems.

A general mathematical framework for overlapping decompositions and decentralized control is the Inclusion Principle [3, 5]. A dynamic system with overlapping subsystems is expanded into a larger state space where the subsystems appear as disjoint. Then, the estimation and control laws are designed in the expanded space using standard methods for disjoint subsystems. Under the inclusion conditions, the laws can be contracted to the smaller space for implementation in the original system.

In this paper, a new strategy is proposed for decentralized state feedback design with overlapping information structure constraints. The method combines LQG control algorithm and the inclusion principle in a way that eliminates controllability problems that are inherent to standard decentralized control design in the expanded space. In this paper, section 2 describes the linear stochastic model of a vehicle with respect to the states relative to its immediate neighbour. In section 3, the Inclusion Principle is described and in section 4, a controller design procedure is given. In section 5, simulation results for a platoon consisting of 4 vehicles with a decentralized controller application are studied.

II. DESCRIPTION OF VEHICLES-STRING SYSTEM MODEL

We consider the error regulation problem of a string of high speed longitudinal forward moving vehicles [1]. The motion of each vehicle in the string is described by two states: position and velocity. When normal operating conditions are specified, the equations of motion for the string can be written in terms of the deviations from a desired separation distance between adjacent vehicles and the deviations from a desired nominal string velocity. Furthermore, since the individual inputs to the vehicles can be chosen to represent the deviations from the nominal forces applied to each individual vehicle at nominal string velocity, the equations of motion can be written in the following form [8].

$$\begin{aligned} \Delta\dot{v}_i &= -\Delta v_i + \Delta u_i \\ \Delta\dot{d}_{i,i+1} &= \Delta v_i - \Delta v_{i+1} \end{aligned} \qquad i = 1,2,\cdots N \tag{1}$$

System state-space model can be described as follow:

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$$S:\begin{cases}\dot{x}=Ax+Bu+F\xi \\ y=Cx+\eta\end{cases} \tag{2}$$

Where $x=\left(v_1,d_{1,2},v_2,d_{2,3},\cdots,d_{n-1,n},v_n\right)^T$ is the state vector with v_i as the normalized velocity deviation of the ith vehicle and $d_{i,i+1}$ as the normalized distance deviation between the ith and (i+1)th vehicle, and $u=\left(u_1,u_2,\cdots,u_n\right)^T$ is the input vector with each component u_i being the normalized incremental force applied to the ith vehicle. ξ and η are input white noise and output measurement white noise vector respectively. We assume that all states are available as outputs.

We shall consider the case of a platoon of N identical vehicles, where the ith vehicle and (i+1)th are interconnected. Consider adjacent two vehicles as a subsystem, the N-1 pairs subsystems constructed by the following:

$$\begin{cases}\begin{bmatrix}\dot{v}_i \\ \dot{d}_{i,i+1} \\ \dot{v}_{i+1}\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}v_i \\ d_{i,i=1} \\ v_{i+1}\end{bmatrix}+\begin{bmatrix}1 & 0 \\ 0 & 0 \\ 0 & 1\end{bmatrix}\left(\begin{bmatrix}u_i \\ u_{i+1}\end{bmatrix}+\begin{bmatrix}\xi_i \\ \xi_{i+1}\end{bmatrix}\right) \\ \begin{bmatrix}y_{vi} \\ y_{di} \\ y_{vi+1}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_i \\ d_{i,i=1} \\ v_{i+1}\end{bmatrix}+\begin{bmatrix}\eta_{vi} \\ \eta_{di} \\ \eta_{vi+1}\end{bmatrix}, i=1,2,\cdots,N-1\end{cases} \tag{3}$$

If consider states of velocity and distance, for example, system matrices for three subsystem of four vehicles can be described as follow:

$$A=\begin{bmatrix}a_{11} & a_{12} & & & & & & & \\ a_{21} & a_{22} & a_{23} & & & & & & \\ & a_{32} & a_{33} & a_{34} & & & & & \\ & & a_{43} & a_{44} & a_{45} & & & & \\ & & & a_{54} & a_{55} & & & & \\ & & & & & \ddots & & & \\ & & & & & & a_{2N-3,2N-3} & a_{2N-3,2N-2} & \\ & & & & & & a_{2N-2,2N-3} & a_{2N-2,2N-2} & a_{2N-2,2N-1} \\ & & & & & & & a_{2N-1,2N-2} & a_{2N-1,2N-1}\end{bmatrix} \tag{4}$$

For example, for a string of three vehicles, the system matrices A , B and C of (1) has the form

$$A=\begin{bmatrix}-1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1\end{bmatrix}, F=\begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix} \tag{5}$$

Where the overlapping subsystems are identified as substrings of two adjacent vehicles and indicated by the broken lines. It is assumed that the two vehicles in one subsystem are controlled together on the basis of the full state information, that is, their velocities and the distance between them.

III. INCLUSION PRINCIPLE

The Inclusion Principle is a general mathematical framework for overlapping decompositions and decentralized control [3].

Let us consider the two systems S and $\widetilde{S}$, which are described as

$$S:\begin{cases}\dot{x}=Ax+Bu+F\xi \\ y=Cx+\eta\end{cases}$$

And

$$\widetilde{S}:\begin{cases}\dot{\widetilde{x}}=\widetilde{A}\widetilde{x}+\widetilde{B}\widetilde{u}+\widetilde{F}\widetilde{\xi} \\ \widetilde{y}=\widetilde{C}\widetilde{x}+\widetilde{\eta}\end{cases} \tag{6}$$

Where $x(t)\in\Re^n, u(t)\in\Re^m, y(t)\in\Re^l$ are the state, input, output of the system S at time $t\in\Re_+$,and $\widetilde{x}(t)\in\Re^{\widetilde{n}}, \widetilde{u}(t)\in\Re^{\widetilde{m}}, \widetilde{y}(t)\in\Re^{\widetilde{l}}$ are those of $\widetilde{S}$ .The matrices A, B,C and $\widetilde{A},\widetilde{B},\widetilde{C}$ are constant and of appropriate dimensions. Our crucial assumption is that the dimensions of the state, input, and output vectors x, u, y of S are smaller than (or at most equal to) those of $\widetilde{x},\widetilde{u},\widetilde{y}$ of $\widetilde{S}$, respectively, that is, $n\le\widetilde{n}, m\le\widetilde{m}, l\le\widetilde{l}$ .By $x(t;x_0,u)$ and $y[x(t)]$ we denote the state behavior and corresponding output of S for a fixed input $u(t)$ and the initial state $x_0=x(0)$ .The similar notions $\widetilde{x}(t;\widetilde{x}_0,\widetilde{u})$ and $\widetilde{y}[\widetilde{x}(t)]$ are used to denote the behavior and output of $\widetilde{S}$.

The systems S and $\widetilde{S}$ are related by the transformations

$$\begin{aligned}&\widetilde{x}=Vx, x=U\widetilde{x}, \\ &\widetilde{u}=Ru, u=Q\widetilde{u}, \\ &\widetilde{y}=Ty, y=S\widetilde{y},\end{aligned} \tag{7}$$

Where V, R and T are constant matrices with proper dimensions and full column ranks; U, Q, R are constant matrices with proper dimensions and full row ranks, which satisfy the relations

$$UV=I_n, QR=I_m, ST=I_l \tag{8}$$

I_n, I_m, I_l are identity matrices of indicated dimensions.We have the following statement.

Definition 1. We say that the system $\widetilde{S}$ includes the system S, that is, S is included by $\widetilde{S}$, if there exists a quadruplet

(U,V,R,S) such that, for any initial state x_0 and any fixed input $u(t)$ of S, the choice

$$\begin{cases} \tilde{x}_0 = Vx_0 \\ \tilde{u}(t) = Ru(t) \end{cases} \tag{9}$$

Of the initial state $\tilde{x}_0$ and input $\tilde{u}(t)$ of $\tilde{S}$, implies

$$\begin{aligned} x(t;x_0,u) &= U\tilde{x}(t;\tilde{x}_0,\tilde{u}) \\ y[x(t)] &= S\tilde{y}[\tilde{x}(t)] \end{aligned} \tag{10}$$

for all $t \geq 0$

The condition of this definition implies that the system $\tilde{S}$ contains all the necessary information about the behavior of the system S. We can extract any property such as stability and optimality of S from $\tilde{S}$, which is the underlying idea of the Inclusion Principle [3].

A particular attention has been paid to restriction and aggregation, two special cases of inclusion; here shall focus our attention to one particular case of restriction.

Theorem 1: the system S is a restriction of $\tilde{S}$, if there exists full rank matrices (V,R,T),such that

$$\tilde{A}V = VA; \tilde{B}R = VB; \tilde{C}V = TC \tag{11}$$

When we consider overlapping decompositions of S, we are interested in generating expansions $\tilde{S}$ of S. If the pairs of matrices (U, V),(Q, R),and (S, T) are specified, the matrices $\tilde{A}, \tilde{B}, \tilde{C}$ can be expressed as

$$\tilde{A} = VAU + M_A, \tilde{B} = VBQ + M_B, \tilde{C} = TCU + M_C \tag{12}$$

Where M_A, M_B, and M_V are complementary matrices of appropriate dimensions. For $\tilde{S}$ to be an expansion of S, a proper choice of M_A, M_B, and M_V is required [2].

IV. DECENTRALIZED OVERLAPPING CONTROL

Overlapping in decentralized control gives extra degrees of freedom that improves performances compared to disjointed decomposition [2]

A. Overlapping Structural Decomposition

Firstly, the initial system model is decomposed with an appropriate input, output and state expansion that respects the inclusion principle, the system with coupling is expanded into a new space, called expanded space, where the subsystems are disjointed.

For instance, let us consider a linear system (2), which system matrices as (4). We regard the system as composed of N-1 overlapping subsystems. By choose expansion matrices

$$\begin{aligned} V &= blockdiag\left(I_2, \begin{bmatrix}1\\1\end{bmatrix}, 1, \begin{bmatrix}1\\1\end{bmatrix}, 1, \cdots, \begin{bmatrix}1\\1\end{bmatrix}, I_2\right), \\ U &= blockdiag\left(I_2, (0.5, 0.5), 1, \cdots, (0.5, 0.5), I_2\right) \end{aligned} \tag{13}$$

When satisfy constraint condition $\tilde{A}V = VA$, has expansion space system matrix $\tilde{A}$. Or when satisfy $M_A V = 0$, has complement matrix

$$M_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & -0.5 & 0.5 & 0 & 0 & & & \\ 0 & 0 & -0.5 & 0.5 & 0 & 0 & & & \\ 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 & 0 & -0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0.5 & 0 & 0 \\ & & & 0 & 0 & 0.5 & -0.5 & 0 & 0 \\ & & & 0 & 0 & -0.5 & 0.5 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & \ddots \end{bmatrix} \tag{14}$$

We obtain overlapping structural decomposition pair-wise subsystem. That is

$$\tilde{A} = blockdiag\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \begin{bmatrix} a_{33} & a_{34} & a_{35} \\ a_{43} & a_{44} & a_{45} \\ a_{53} & a_{54} & a_{55} \end{bmatrix}, \cdots \right) \tag{15}$$

System state vector $x = \left[x_1^T, x_2^T, x_3^T, \cdots, x_{2N-1}^T\right]^T$ is expanded $\tilde{x} = \left[x_1^T, x_2^T, x_3^T, x_3^T, \cdots, x_{2N-3}^T, x_{2N-3}^T, x_{2N-2}^T, x_{2N-1}^T\right]^T$, similarity system S is expanded into $\tilde{S}$ respecting the inclusion principle.

B. Decentralized LQG Control

For the decentralized LQG control of the platoon of vehicles, we treat the system as a pairs of subsystems. then formulate decentralized LQG control for the subsystems . Generally, diagonal subsystems have the law of optimal observer and feedback control as follow:

$$\begin{aligned} \dot{\hat{\tilde{x}}} &= \tilde{A}_D \hat{\tilde{x}} + \tilde{B}_D \tilde{u} + \tilde{L}_D(\tilde{y} - \tilde{C}_D \hat{\tilde{x}}) \\ \tilde{u} &= -\tilde{K}_D \hat{\tilde{x}} \end{aligned} \tag{16}$$

Where,

$$\tilde{A}_D = \begin{bmatrix} \tilde{A}_{11} & & & 0 \\ & \tilde{A}_{22} & & \\ & & \ddots & \\ 0 & & & \tilde{A}_{N-1N-1} \end{bmatrix},$$

$$\widetilde{B}_D = \begin{bmatrix} \widetilde{B}_{11} & & & 0 \\ & \widetilde{B}_{22} & & \\ & & \ddots & \\ 0 & & & \widetilde{B}_{N-1N-1} \end{bmatrix},$$

$$\widetilde{C}_D = \begin{bmatrix} \widetilde{C}_{11} & & & 0 \\ & \widetilde{C}_{22} & & \\ & & \ddots & \\ 0 & & & \widetilde{C}_{N-1N-1} \end{bmatrix} \qquad (17)$$

Where A_{ii}, B_{ii}, C_{ii} represent ith subsystem matrices.

Observer gain matrices and controller gain matrices are respectively

$$\widetilde{L}_D = diag\left[\widetilde{L}_1, \widetilde{L}_2, \cdots, \widetilde{L}_{2N-1}\right]$$
$$\widetilde{K}_D = diag\left[\widetilde{K}_1, \widetilde{K}_2, \cdots, \widetilde{K}_{2N-1}\right] \qquad (18)$$

For control implementation, according to Inclusion Principle [7], Observer and controller are contracted to

$$L_S = U(L_D + \Delta L)T, K_S = (K_D + \Delta K)V \qquad (19)$$

V. SYSTEM SIMULATION STUDY

To illustrate the inclusion strategy, we consider the error regulation problem of a string of high speed moving vehicles and propose to build a decentralized controller for a string of four vehicles, namely there are three subsystems, which is a representative situation for strings of any length. System matrices can be described as:

$$A = \begin{bmatrix} a_{11} & a_{12} & & & & & \\ a_{21} & a_{22} & a_{23} & & & & \\ & a_{32} & a_{33} & a_{34} & & & \\ & & a_{43} & a_{44} & a_{45} & & \\ & & & a_{54} & a_{55} & a_{56} & \\ & & & & a_{65} & a_{66} & a_{67} \\ & & & & & a_{76} & a_{77} \end{bmatrix}$$

By choosing the transformation matrices

$$U = \begin{bmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 0.5 & 0.5 & & & & & \\ & & & & 1 & & & & \\ & & & & & 0.5 & 0.5 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & 1 & 0 & \\ & 0 & 1 & \\ & & 1 & 0 \\ & & 0 & 1 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$V = T;$

We obtain the expanded system $\widetilde{S}$ with system matrices

$$\widetilde{A} = \begin{bmatrix} a_{11} & a_{12} & & & & & & & \\ a_{21} & a_{22} & a_{23} & & & & & & \\ & a_{32} & a_{33} & & a_{34} & & & & \\ & a_{32} & & a_{33} & a_{34} & & & & \\ & & & a_{43} & a_{44} & a_{45} & & & \\ & & & & a_{54} & a_{55} & & a_{56} & \\ & & & & a_{54} & & a_{55} & a_{56} & \\ & & & & & & a_{65} & a_{66} & a_{67} \\ & & & & & & & a_{76} & a_{77} \end{bmatrix}$$

That is

$$\widetilde{A} = \begin{bmatrix} -1 & 0 & 0 & & & & & & \\ 1 & 0 & -1 & & & & & \mathbf{0} & \\ 0 & 0 & -1 & & & & & & \\ & & & -1 & 0 & 0 & & & \\ & & & 1 & 0 & -1 & & & \\ & & & 0 & 0 & -1 & & & \\ & & & & & & -1 & 0 & 0 \\ & \mathbf{0} & & & & & 1 & 0 & -1 \\ & & & & & & 0 & 0 & -1 \end{bmatrix}$$

$$\widetilde{B} = \begin{bmatrix} 1 & 0 & & & & \\ 0 & 0 & & & & \\ 0 & 1 & & & & \\ & & 1 & 0 & & \\ & & 0 & 0 & & \\ & & 0 & 1 & & \\ & & & & 1 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 1 \end{bmatrix}$$

The overlapping subsystems now appear as disjoint and completely decoupled.

In order to respecting the distance deviation between vehicles, we choose optimal performance index weight matrix $W_7 = I_7, W_u = I_4$; set $R_\xi = 0.1 \times I_4, R_\eta = 0.1 \times I_7$,when first vehicle drive force deviation u_1 has disturbance as follow:

$$du_1 = \begin{cases} 1, & 0 \le t \le 10 \\ 0, & 10 < t \le 20 \\ -1, & 20 < t \le 30 \\ 0, & 30 < t \end{cases} \qquad (21)$$

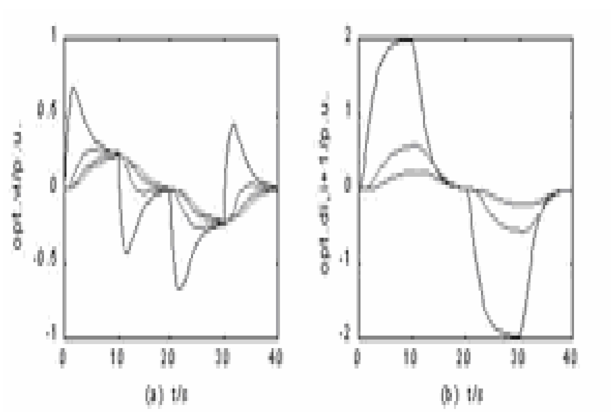
That is, firstly generate a positive unit (p.u.) drive force deviation disturbance, then lost; subsequently generate a negative unit(p.u.) drive force deviation disturbance, then lost; according to standard LQG algorithm, get solutions of optimal control gain matrix and optimal observer gain matrix.

$$K_D = \begin{bmatrix} 0.0738 & 0.1084 & 0.1061 & 0.2706 & 0.2855 & 0.8155 & 0.8796 \\ 0.1061 & 0.1622 & 0.2532 & 0.6533 & 1.0590 & 0.5449 & 0.2855 \\ 0.2855 & 0.5449 & 1.0590 & 0.6533 & 0.2532 & 0.1622 & 0.1061 \\ 0.8796 & 0.8155 & 0.2855 & 0.2706 & 0.1061 & 0.1084 & 0.0738 \end{bmatrix}$$

$$L_D = \begin{bmatrix} 0.0008 & 0.0048 & 0.0018 & 0.0194 & 0.0095 & 0.1830 & 0.4021 \\ 0.0048 & 0.0334 & 0.0146 & 0.1871 & 0.1636 & 0.7724 & 0.1830 \\ 0.0018 & 0.0146 & 0.0085 & 0.1684 & 0.3944 & 0.1636 & 0.0095 \\ 0.0194 & 0.1871 & 0.1684 & 0.7391 & 0.1684 & 0.1871 & 0.0194 \\ 0.0095 & 0.1636 & 0.3944 & 0.1684 & 0.0085 & 0.0146 & 0.0018 \\ 0.1830 & 0.7724 & 0.1636 & 0.1871 & 0.0146 & 0.0334 & 0.0048 \\ 0.4021 & 0.1830 & 0.0095 & 0.0194 & 0.0018 & 0.0048 & 0.0008 \end{bmatrix}$$

According to decentralized suboptimal LQG algorithm, we obtain L_s and K_s. Fig1. shows the simulation curves of centralized optimal LQG control and decentralized suboptimal LQG control. Where (a) and (b) represent response curves of each vehicle's velocity and distance following first vehicle drive force disturbance when adopt optimal control respectively. (c) and (d) represent response curves of suboptimal control.

Simulation results show that centralized control is of the same quality as decentralized control. Meanwhile, the controlled string of moving vehicles is stable under structural perturbations and controller's order is reduced distinctly.



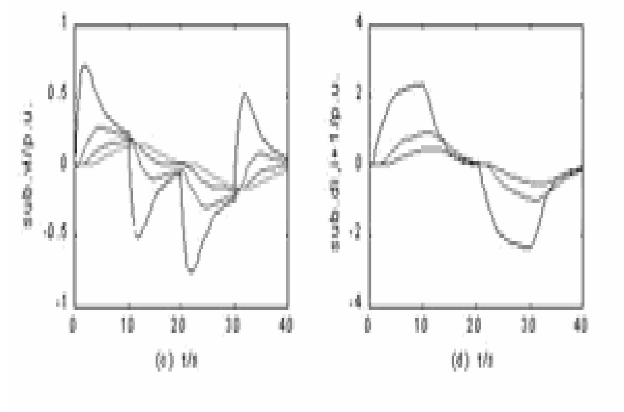


Fig1. Comparison of centralized optimal control and decentralized suboptimal control

7. CONCLUSIONS

A mathematical framework has been described for decentralized control design of vehicle-string system, which is composed of overlapping subsystems having certain parts of inputs, states, and outputs in common, that is Inclusion Principle. It has been show how to formulate control laws in the expanded spaces, which can be contracted for implementation in the original system. The controlled string of moving vehicles is stable under structural perturbations whereby the string can fall apart and get together again in various ways during operation.

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