Proceedings of the 11th International IEEE
Conference on Intelligent Transportation Systems
Beijing, China, October 12-15, 2008

Stability of String of Adaptive Cruise Control Vehicles with Parasitic Delays and Lags

Lingyun Xiao
Department of Automobile Engineering
Beihang University
Beijing, 100191, China
Email: lingyunxiao@ae.buaa.edu.cn

Swaroop Darbha
Department of Mechanical Engineering
Texas A&M University
College Station, Texas 77843, USA
Email: dswaroop@tamu.edu

Feng Gao
Department of Automobile Engineering
Beihang University
Beijing 100191, China
Email: gaof@buaa.edu.cn

***Abstract*—In this paper, we examine the effect of parasitic delays and lags on the stability of a string of vehicles equipped with Adaptive Cruise Control (ACC) system which is employed a constant time headway (CTH) policy. The control law of the ACC system is based on a simpler model of the vehicle that does not ignore the parasitic delays and lags. The main result of this paper is that string stability can be guaranteed if the constant time headway h, is at least twice the sum of the parasitic delays Δ, and the parasitic lags τ, that is, $h > 2(\Delta+\tau)$. This result extends and generalizes the earlier results of Darbha [6] by considering parasitic delays and lags and provides a practical direction for ACC system design and implementation from the viewpoint of robustness to parasitic delays and lags.**

I. Motivation

String stability is an important concern in the design of ACC systems. A precise definition of string stability was provided by Chu [1]. Intuitively, the term "string stability" indicates that spacing errors do not amplify as they propagate upstream from one vehicle to another vehicle [2], [3]. This property ensures that any perturbation of the speed or position of the lead vehicle will not result in amplified fluctuations to the following vehicle's speed and position [4]. If spacing and velocity errors amplify as they propagate upstream (this is the case when string instabilities are present), it is not only likely to provide poor ride quality but also could result in collisions [5]. The relationship between string stability and spacing policies has been an topic of significant interest [4], [6], [7], [8], [9].

Often control algorithms are based on simplified models of a physical system for a variety of reasons. Firstly, the algorithms will be simple and easier to implement as the control law may only require the measurements of only a few states of the system. Sometimes, the simplified model on which a control law is based may be easy to obtain while refinements may be very expensive or difficult to obtain. A natural simplification of a model occurs if it has distinct time scales. Similarly, it is easier to neglect the parasitic delays and lags that are all-pervasive in the actuators and sensors. One may have a rough idea of the magnitudes of parasitic delays and lags in a system. The issue then is to use these bounds in the synthesis of a simple control law which does not explicitly require the exact values of parasitic delays and lags but guarantees string stability despite the presence. This is the central issue and motivation of this paper and we present a lower bound on the allowable constant time headway in an ACC system in the presence of parasitic delays and lags. In particular, we extend the result that Darbha [6] first proposed for the relationship between the constant time headway h, and parasitic lags τ, given by the simple form $h \geq 2\tau$ based on an analysis of string stability for the singularly perturbed system. We generalize the result that Huang and Ren [10] obtained complicated inequalities involving the control parameters in the presence of parasitic delays and lags. However, the earlier research did not focus on the issue that is dealt with in this paper.

The organization of this paper is as follows: In section II, we briefly point out where the parasitic lags and delays occur and obtain the related longitudinal vehicle model. We then present the control law employing a constant time headway in the presence of parasitic delays and lags. In section III, we present an analysis of how spacing errors propagate upstream from one vehicle to another and derive the main results. In section IV, we provide some corroborating numerical simulations and conclude thereafter in section V.

II. Vehicle Model and ACC Control Law

In this section, we demonstrate where parasitic delays and lags occur. We then describe the longitudinal vehicle model used in this paper. The rest of the section deals with the development of a control law when such parasitic delays and lags are considered.

A. Parasitic Delays and Lags

For the purposes of control of vehicles, we make assumptions which lead to unaccounted lags and delays in the system such as the following [11], [12]:

1) The pure time delay and lag in the engine response;
2) the bandwidth of the lower level multiple-sliding-surface controller that tracks acceleration;
3) the bandwidth of low-pass filters used for other sensors such as engine manifold pressure sensor, wheel speed sensor etc.;
4) the pure time delay and lag of the throttle actuator;
5) the pure time delay and lag of the brake actuator;
6) the time delay due to discrete sampling at 50Hz;

7) the 200ms delay and lag due to the radar filter.

In this paper, the different parasitic delays and lags are lumped into two "combined" delay and lag which represent by symbol Δ and τ respectively. For convenience, we introduce the following transfer function:

$$DL^*(s) = \frac{e^{-\Delta s}}{\tau s + 1}.$$

B. Longitudinal Vehicle Model

A model for the motion of a vehicle in the longitudinal direction must take into account the powertrain, longitudinal tire forces, aerodynamic drag forces, rolling resistance forces and gravitational forces. The powertrain consists of the internal combustion engine, the torque converter, the transmission and the wheels [11]. In order to arrive at a simple model, one should makes some strong assumptions [7], [12], [13].

When one is considering a string of vehicles, it is natural to index them. Let the subscript i be associated with the i^{th} vehicle in the string; the higher the subscript, the farther upstream the vehicle is in the string. Let $x_i(t), \upsilon_i(t), a_i(t)$ denote the position, velocity and acceleration of the i^{th} vehicle in the string in an inertial frame. Then,

$$\begin{aligned} \dot{x}_i(t) &= \upsilon_i(t), \\ \dot{\upsilon}_i(t) &= a_i(t) = F_{net}/M_i \\ &= k_1 T_{net}(\alpha_i(t), \upsilon_i(t)) - k_2 T_L(\upsilon_i(t)) \end{aligned} \tag{1}$$

where F_{net} denotes the net force in the longitudinal direction of the vehicle, M_i denotes the effective mass of the i^{th} vehicle (which depends on the gear employed), T_{net} denotes the net engine torque and T_L denotes the load torque and depends on the aerodynamic and rolling resistance and the brake torque applied at the wheels, $\alpha_i(t)$ denotes throttle angle of the i^{th} vehicle, k_1 and k_2 reflect terms that include gear ratios and moments of inertia to the vehicle's center of mass [6], [7], [13], [14].

The first three assumptions allow the net engine torque T_{net} to be expressed directly as a function of throttle angle $\alpha_i(t)$ and engine rotating speed ω_e. The next three assumptions allow one to relate the engine rotating speed to the vehicle's velocity by the gear ratio r_i^* as $\upsilon_i = r_i^* \omega_e$. If these assumptions hold an engine torque T_{net}, and the brake torque T_{brake}, can be produced so that the acceleration $u_i(t)$, of a vehicle can be commanded at will, that is,

$$u_i(t) = k_1 T_{net}(\alpha_i(t), \upsilon_i(t)) - k_2 T_L(\upsilon_i(t)). \tag{2}$$

In essence, one may deal with the following simple longitudinal vehicle model to base the controller on:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t) \tag{3}$$

where $X_i = \begin{bmatrix} x_i \\ \upsilon_i \end{bmatrix}$, $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $U_i(t) = u_i(t)$. To reiterate, this longitudinal vehicle model indicates that the acceleration of a vehicle can be commanded arbitrarily. This model has been frequently used to analyze the string stability, such as [2], [3], [13], [15], [16].

Due to the presence of delays in the powertrain and brake system and the fact that pressure evolution in the intake manifold or in the brake chamber is not instantaneous, the commanded acceleration may not be readily available. In essence, for the purposes of analyzing robustness of a control law, we may assume that the acceleration of a vehicle depends on the commanded acceleration as:

$$a_i(s) = DL^*(s) u_i(s) \quad \Rightarrow \quad \tau \dot{a}_i(t) + a_i(t) = u_i(t - \Delta).$$

The above model indicates that if one commands an acceleration of $u_i(t)$ at time t, the acceleration of the vehicle has a delay of Δ seconds and suffers a lag of τ seconds. One may treat Δ and τ as the lumped delay and lag respectively from the command (brake pedal and throttle input) to the torque available at the wheels.

Hence, we may consider the following model of a vehicle for analyzing the robustness of control laws:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t - \Delta) \tag{4}$$

where $X_i = \begin{bmatrix} x_i \\ \upsilon_i \\ a_i \end{bmatrix}$, $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}$ and $U_i(t - \Delta) = u_i(t - \Delta)$.

C. ACC Control Law

For the development of a control law, we will assume that there is a lead vehicle, indexed by 1, that performs maneuvers and the following vehicles are controlled to maintain a desired following distance. The desired following distance of any following vehicle is proportional to its longitudinal velocity and the constant of proportionality, h, (referred to as a time headway) is the same for all the following vehicles.

Let $\xi_i(t)$ denotes the spacing (following distance) between i^{th} vehicle and $(i-1)^{th}$ vehicle, l_{i-1} denotes the length of the $(i-1)^{th}$ vehicle, $\delta_i(t)$ denotes the spacing error of the i^{th} vehicle which is the deviation between the spacing and desired spacing, h denotes the constant time headway and $\upsilon_i(t)$ denotes the velocity of the i^{th} vehicle, $h\upsilon_i(t)$ denotes the desired spacing between i^{th} vehicle and $(i-1)^{th}$ vehicle. Then,

$$\xi_i(t) = x_{i-1}(t) - x_i(t) - l_{i-1}, \tag{5}$$

$$\delta_i(t) = \xi_i(t) - h\upsilon_i(t). \tag{6}$$

It is clear then that

$$\xi_i(t - \Delta) = x_{i-1}(t - \Delta) - x_i(t - \Delta) - l_{i-1}, \tag{7}$$

$$\delta_i(t - \Delta) = \xi_i(t - \Delta) - h\upsilon_i(t - \Delta). \tag{8}$$

The purpose of the controller is to drive the following error, S_i, to zero asymptotically in time:

$$S_i \equiv \delta_i(t) := \xi_i(t) - h\upsilon_i(t).$$

The purpose of the controller is served if

$$\dot{S}_i = -\lambda S_i$$

where $\lambda > 0$. This is in keeping with the design of sliding model controller. For the purpose of deriving the control law, the simplified model of a vehicle will be used, where the delays and lags are neglected. Clearly, then

$$a_i(t) = \frac{1}{h}(\dot{\xi}_i(t) + \lambda\delta_i(t))$$

where $\dot{\xi}_i(t)$ is derived by differentiating both sides of equation (5) and denotes the relative velocity between the i^{th} vehicle and the $(i-1)^{th}$ vehicle, that is,

$$\dot{\xi}_i(t) = \upsilon_{i-1}(t) - \upsilon_i(t). \quad (9)$$

Then a synthetic control law $u_i(t)$ is given by

$$u_i(t) = a_i(t) = \frac{1}{h}(\dot{\xi}_i(t) + \lambda\delta_i(t)) \quad (10)$$

which can make $\dot{S}_i + \lambda S_i = 0$ for $\lambda > 0$.

In the presence of parasitic delays and lags of actuators, $u_i(t) \neq a_{i_d}(t)$, rather,

$$\begin{aligned} a_i(s) &= DL^*(s)u_i(s), \\ \tau\dot{a}_i(t) + a_i(t) &= \frac{1}{h}(\dot{\xi}_i(t-\Delta) + \lambda\delta_i(t-\Delta)) \quad (11) \end{aligned}$$

where $\delta_i(t-\Delta)$ is derived from (8) and $\dot{\xi}_i(t-\Delta)$ is derived from the equation (9) with the time-delay consideration, that is,

$$\dot{\xi}_i(t-\Delta) = \upsilon_{i-1}(t-\Delta) - \upsilon_i(t-\Delta). \quad (12)$$

III. Analysis of String Stability

In this section, we calculate the spacing error dynamics and spacing error transform function between successive two ACC-vehicles based on the adopted longitudinal vehicle model and proposed control law. Then we present the string stable theorem and lemma and their proof.

A. Spacing Error Dynamics

We assume that the longitudinal motion of all vehicles is controlled by identical ACC control law. The longitudinal motion of the $(i)^{th}$ vehicle is controlled by equation (11), then the motion of the $(i-1)^{th}$ vehicle by

$$\tau\dot{a}_{i-1}(t) + a_{i-1}(t) = \frac{1}{h}(\dot{\xi}_{i-1}(t-\Delta) + \lambda\delta_{i-1}(t-\Delta)). \quad (13)$$

We then obtain a transfer function that relates the deviation of the velocity successive vehicles by equation (13) $-$ equation (11) :

$$\begin{aligned} & \tau(\dot{a}_{i-1}(t) - \dot{a}_i(t)) + (a_{i-1}(t) - a_i(t)) \\ = & \frac{1}{h}[\dot{\xi}_{i-1}(t-\Delta) - \dot{\xi}_i(t-\Delta) \\ & +\lambda(\delta_{i-1}(t-\Delta) - \delta_i(t-\Delta))]. \quad (14) \end{aligned}$$

From equation (9), we obtain the following equations related with the spacing between successive vehicles by differentiating once and twice respectively with respect to time (t) as follows:

$$\begin{aligned} \ddot{\xi}_i(t) &= a_{i-1}(t) - a_i(t), \quad (15) \\ \dddot{\xi}_i(t) &= \dot{a}_{i-1}(t) - \dot{a}_i(t). \quad (16) \end{aligned}$$

Then, the equation (14) combined with equation (15) and equation (16) yields

$$\begin{aligned} & \tau\dddot{\xi}_i(t) + \ddot{\xi}_i(t) \\ = & \frac{1}{h}[(\dot{\xi}_{i-1}(t-\Delta) - \dot{\xi}_i(t-\Delta)) \\ & + \lambda(\delta_{i-1}(t-\Delta) - \delta_i(t-\Delta))]. \quad (17) \end{aligned}$$

Using spacing error (6) of the i^{th} vehicle, the deviation between the spacing error of the i^{th} vehicle and the $(i-1)^{th}$ vehicle is given as

$$\begin{aligned} & \delta_{i-1}(t) - \delta_i(t) \\ = & (\xi_{i-1}(t) - \xi_i(t)) - h(\upsilon_{i-1}(t) - \upsilon_i(t)) \\ = & \xi_{i-1}(t) - \xi_i(t) - h\dot{\xi}_i(t) \quad (18) \end{aligned}$$

which is also dependent on the relationship between the spacing and spacing errors of the successive vehicles. Then we consider the time delay factor into (18) and differentiate both sides of (18) to obtain

$$\begin{aligned} & \dot{\delta}_{i-1}(t-\Delta) - \dot{\delta}_i(t-\Delta) \\ = & \dot{\xi}_{i-1}(t-\Delta) - \dot{\xi}_i(t-\Delta) - h\ddot{\xi}_i(t-\Delta). \quad (19) \end{aligned}$$

Using equation (17) and equation (19), we obtain

$$\begin{aligned} & \tau\dddot{\xi}_i(t) + \ddot{\xi}_i(t) - \ddot{\xi}_i(t-\Delta) \\ = & \frac{1}{h}[\dot{\delta}_{i-1}(t-\Delta) - \dot{\delta}_i(t-\Delta) \\ & +\lambda(\delta_{i-1}(t-\Delta) - \delta_i(t-\Delta))] \quad (20) \end{aligned}$$

which just depends on the relationship between the spacing and spacing errors of the successive vehicles.

For eliminating the terms of $\dot{a}_i(t)$ and $a_i(t)$ in equation (11) to get a equation which just depends on the spacing and spacing errors of the successive vehicles, we obtain

$$a_i(t) = \frac{1}{h}(\dot{\xi}_i(t) - \dot{\delta}_i(t)) \quad (21)$$

and

$$\dot{a}_i(t) = \frac{1}{h}(\ddot{\xi}_i(t) - \ddot{\delta}_i(t)) \quad (22)$$

which are derived from the equation (6). Substituting equation (21) and (22) into equation (11), then we obtain

$$\tau\ddot{\xi}_i(t) + \dot{\xi}_i(t) - \dot{\xi}_i(t-\Delta) = \ddot{\delta}_i(t) + \dot{\delta}_i(t) + \lambda\dot{\delta}_i(t-\Delta). \quad (23)$$

Differentiating both sides of equation (23), we obtain

$$\tau\dddot{\xi}_i(t) + \ddot{\xi}_i(t) - \ddot{\xi}_i(t-\Delta) = \dddot{\delta}_i(t) + \ddot{\delta}_i(t) + \lambda\ddot{\delta}_i(t-\Delta) \quad (24)$$

which has the same terms of the left side with the equation (20) . Then substituting the equation (20) into the equation (24) to eliminate the spacing terms, we obtain the spacing error dynamics model of the successive vehicles in a string as

$$\begin{aligned} & h\tau\dddot{\delta}_i(t) + h\ddot{\delta}_i(t) + (1+h\lambda)\dot{\delta}_i(t-\Delta) + \lambda\delta_i(t-\Delta) \\ = & \dot{\delta}_{i-1}(t-\Delta) + \lambda\delta_{i-1}(t-\Delta). \quad (25) \end{aligned}$$

Taking the Laplace transformation on both sides of the spacing error dynamics model(25), we obtain

$$\begin{aligned} G(s) &= \frac{\delta_i(s)}{\delta_{i-1}(s)} \\ &= \frac{(s+\lambda)e^{-\Delta s}}{h\tau s^3 + hs^2 + (1+h\lambda)se^{-\Delta s} + \lambda e^{-\Delta s}} \end{aligned} \quad (26)$$

which is called as spacing error propagation transfer function between any two successive ACC-vehicles in the presence of parasitic delays and lags.

B. String Stable Theorem

Intuitively, string stability implies uniform boundedness of all the states of the interconnected system for all time if the initial states of the interconnected system are uniformly bounded. A necessary condition for string stability [2], [6], [16] is that

$$\mid G(jw) \mid^2 \ \le\ 1, \quad \forall\, w > 0$$

where $G(jw)$ is derived from the spacing error transform function (26) by substituting $S = jw$.

Theorem 1:

$$\|G(jw)\| \le 1$$

if

$$h \ >\ 2(\Delta+\tau) \ \ and \ \ 0 < \lambda \le \frac{h - 2(\Delta+\tau)}{2((h-\tau)\Delta + h\tau)}$$

holds.

Proof: $|G(jw)|^2$ can be expressed as

$$|G(jw)|^2 \ =\ \mid \frac{\delta_i(jw)}{\delta_{i-1}(jw)} \mid^2 \ =\ \frac{a}{a+b}$$

where

$$a \ =\ w^2 + \lambda^2 \quad (27)$$

and

$$\begin{aligned} b \ =\ & (2h\lambda(1-\cos(\Delta w)) + h^2\lambda^2)w^2 \\ & -((2h + 2h^2\lambda - 2h\tau\lambda)\sin(\Delta w))w^3 \\ & +(h^2 - 2h\tau(1+h\lambda)\cos(\Delta w))w^4 \\ & +h^2\tau^2 w^6. \end{aligned} \quad (28)$$

If we can obtain $b \ge 0$ for $\forall\, w > 0$, and from equation (27) we can know $a \ge 0$ for $\forall\, w > 0$. Then we can obtain

$$a + b \ \ge\ a \ \Rightarrow\ \frac{a}{a+b} \ \le\ 1, \ \ \forall\, w > 0$$

which means

$$|G(jw)|^2 \ \le\ 1, \ \ \forall\, w > 0$$

can be obtained. Hence, if we can prove $b \ge 0$, the string stable theorem with consideration of parasitic delays and lags is proved.

Taking into account the fact that $1 - \cos(\Delta w) \ \ge\ 0$ for $\forall\, w > 0$, we obtain

$$(2h\lambda(1-\cos(\Delta w)) + h^2\lambda^2)w^2 \ge h^2\lambda^2 w^2 \quad (29)$$

and taking into account the fact that $\sin(\Delta w) \le \Delta w$ for $\forall\, w > 0$, we obtain

$$-(2h+2h^2\lambda-2h\tau\lambda)\sin(\Delta w)w^3 \ge -(2h+2h^2\lambda-2h\tau\lambda)\Delta w^4. \quad (30)$$

The inequality (30) holds because

$$2h + 2h^2\lambda - 2h\tau\lambda \ \ge\ 0$$

which stems from $h - \tau$ being non-negative. Taking into account the fact that $-\cos(\Delta w) \ \ge\ -1$ for $\forall\ w > 0$, we obtain

$$(h^2 - 2h\tau(1+h\lambda)\cos(\Delta w))w^4 \ge (h^2 - 2h\tau - 2h^2\tau\lambda)w^4. \quad (31)$$

Substituting (29), (30) and (31) into (28), we obtain

$$\begin{aligned} b \ \ge\ & h^2\lambda^2 w^2 + h^2\tau^2 w^6 \\ & +((h^2 - 2h\tau - 2h^2\tau\lambda) \\ & -(2h + 2h^2\lambda - 2h\tau\lambda)\Delta)w^4. \end{aligned} \quad (32)$$

From the inequality (32) and supposing h, Δ, τ and λ are positive values, we can obtain

$$h^2\lambda^2 w^2 \ +\ h^2\tau^2 w^6 \ \ge\ 0, \quad \forall\ \ w > 0.$$

Hence, if the condition

$$(h^2 - 2h\tau - 2h^2\tau\lambda) - (2h + 2h^2\lambda - 2h\tau\lambda)\Delta \ge 0 \quad (33)$$

is achieved, we can obtain $b \ge 0$ for $\forall\ w > 0$. We rewrite inequality (33) as

$$h - 2(\Delta+\tau) \ \ge\ 2\lambda((h-\tau)\Delta + h\tau). \quad (34)$$

Taking into account the fact that $h - \tau$ is non-negative value and λ, h, τ and Δ are positive values, we obtain $2\lambda((h-\tau)\Delta + h\tau) > 0$. In other words, we obtain

$$h - 2(\Delta+\tau) \ >\ 0, \quad \forall\, w > 0. \quad (35)$$

Then, we obtain

$$0 \ <\ \lambda \ \le\ \frac{h - 2(\Delta+\tau)}{2((h-\tau)\Delta + h\tau)} \quad (36)$$

from inequality (34). Hence, if the conditions of theorem, such as $h > 2(\Delta+\tau)$ and $0 < \lambda \le \frac{h-2(\Delta+\tau)}{2((h-\tau)\Delta+h\tau)}$ hold, the inequality (33) and $b \ge 0$ are obtained for $\forall\ w > 0$, then $\|G(jw)\| \le 1$ is obtained for $\forall\ w > 0$. This completes the proof. □

Remark 1: In practical design of ACC system, it is possible that one of the parasitic delay and lag is far smaller than the other one. For the sake of simplicity, the smaller one can be neglected during system design process. (1) If the parasitic delay is far smaller than the parasitic lag, the string stable conditions are $h > 2\tau$ and $\lambda \in (0, \frac{h-2\tau}{2h\tau}]$. This conditions have been proved by Darbha in [6]. (2) If the parasitic lag is far smaller than the parasitic delay, the string stable conditions are $h > 2\Delta$ and $\lambda \in (0, \frac{h-2\Delta}{2h\Delta}]$.

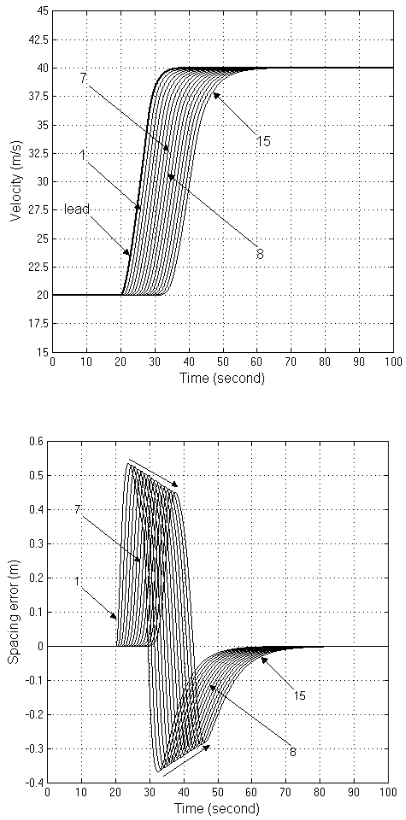

Fig. 1. String stable performance of fifteen ACC-vehicles

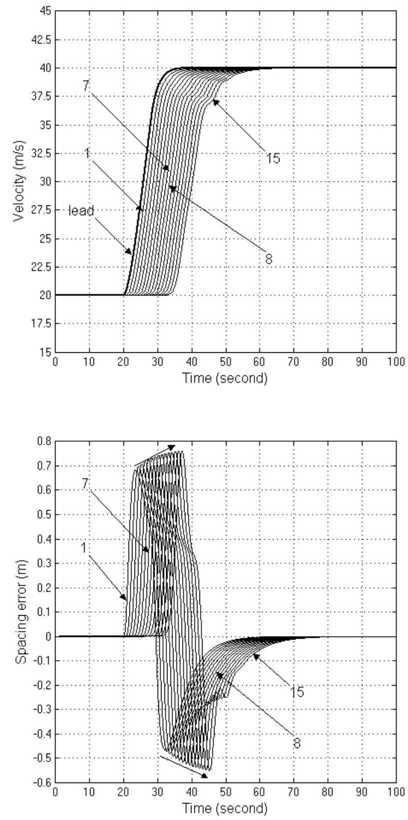

Fig. 2. Critical string stable performance of fifteen ACC-vehicles

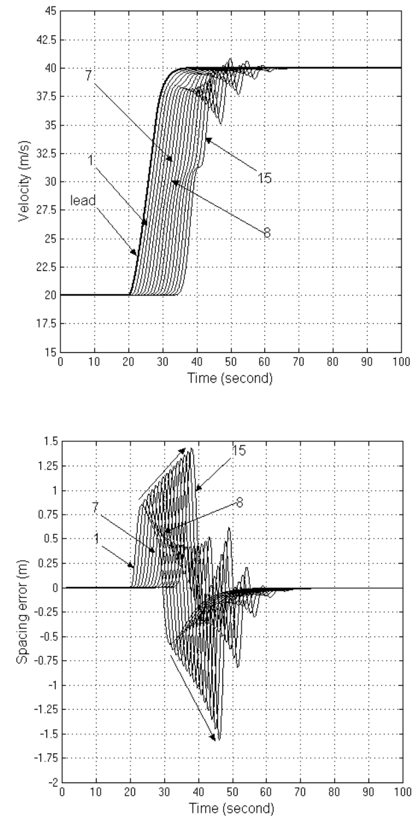

Fig. 3. String unstable performance of fifteen ACC-vehicles

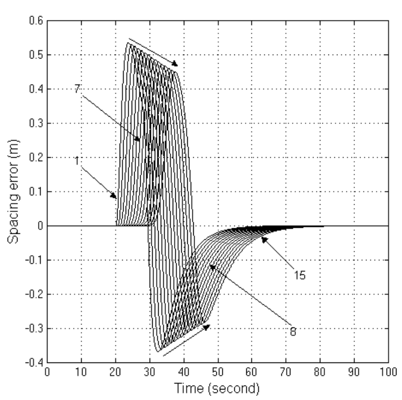

Fig. 4. String stable spacing error performance

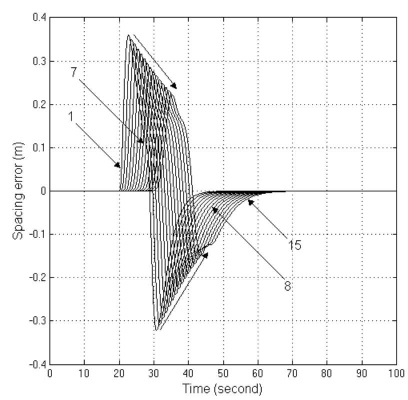

Fig. 5. Critical string stable spacing error performance

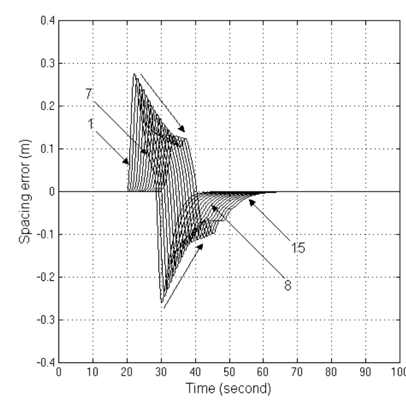

Fig. 6. String unstable spacing error performance

IV. Simulations

To corroborate the result given by the Theorem which states the string stable conditions with consideration of parasitic delays and lags, we conducted a serial of numerical simulations. In the simulations, the desired acceleration for the lead vehicle is given by:

$$a_{des}(t) = \begin{cases} 0, & \text{for} \quad t \geq 20s \\ 2m/s^2, & \text{for} \quad 20s < t < 30s \\ 0, & \text{for} \quad t \geq 30s \end{cases}$$

The initial velocity is $20m/s$ and the final desired velocity is $40m/s$. Two groups of specific parameters have been applied to demonstrate the conditions of Theorem. Table I presents the relationship between h and $2(\Delta + \tau)$ by three cases when control parameter $\lambda = 0.2$. Table II presents the relationship between λ and λ_{max} by three cases when system parameter $h = 1$, $\Delta = 0.2$ and $\tau = 0.2$. The both three cases have been simulated to demonstrate the stable, critical stable and unstable situation respectively.

Fig. 1 illustrates case1 in Table I which represents the string

TABLE I
SPECIFIC PARAMETER FOR STABILITY ANALYSIS1

Parameters[1]	Case1	Case2	Case3
h	$1s$	$1s$	$1s$
Δ	$0.2s$	$0.3s$	$0.3s$
τ	$0.2s$	$0.2s$	$0.3s$
Condition	$h > 2(\Delta + \tau)$	$h = 2(\Delta + \tau)$	$h < 2(\Delta + \tau)$

[1] $\lambda = 0.2$.

TABLE II
SPECIFIC PARAMETER FOR STABILITY ANALYSIS2

Parameter[2]	Case1	Case2	Case3
λ	0.2	0.5	0.8
Condition	$\lambda < \lambda_{max}$[3]	$\lambda = \lambda_{max}$[3]	$\lambda > \lambda_{max}$[3]

[2] $h = 1s$, $\Delta = 0.2s$ and $\tau = 0.2s$.
[3] $\lambda_{max} = (h - 2(\Delta + \tau))/2((h - \tau)\Delta + h\tau)$.

stable condition $h > 2(\Delta + \tau)$. Fig. 1 demonstrates excellent tracking in velocity and it also shows the spacing errors of the vehicles in the string smoothly decrease upstream. If the initial inter-vehicle spacing is same for all vehicles, this ensures that if the first following car does not collide with the lead car, then there will not be a collision upstream between the i^{th} and the $(i+1)^{th}$ vehicle for every $i > 1$. Fig. 2 illustrates case2 in Table I which represents the critical string stable condition $h = 2(\Delta + \tau)$. Fig. 2 demonstrates unstable tracking velocity and maintaining spacing. In fact, spacing errors amplify as one goes from vehicle to vehicle. Fig. 3 illustrates case3 in Table I which represents the string unstable condition of $h < 2(\Delta+\tau)$. Fig. 3 demonstrates worse instability in tracking velocity as well as in maintaining spacing. The Fig. 1 - Fig. 3 serve to remind that the condition $h > 2(\Delta + \tau)$ is tight if there is proper λ for which one can obtain stable tracking velocity and maintaining spacing. Fig. 4 illustrates case1 in Table II which represents the string stable condition $\lambda < \lambda_{max}$. Fig. 4 demonstrates the spacing errors of the vehicles in the string smoothly decrease upstream. Fig. 5 illustrates case2 in Table II which represents the critical string stable condition $\lambda = \lambda_{max}$. Fig. 5 demonstrates the disturbance of the smooth performance of string of vehicles appears. Fig. 6 illustrates case3 in Table II which represents the string unstable condition $\lambda > \lambda_{max}$. Fig. 6 demonstrates the bad smooth performance of string of vehicles. Fig. 4, Fig. 5 and Fig. 6 demonstrate that the condition $\lambda \leq \lambda_{max}$ would guarantee string stability. But this condition is weaker than the condition $h > 2(\Delta + \tau)$.

Remark 2: In practice, $\frac{h-2(\Delta+\tau)}{2((h-\tau)\Delta+h\tau)}$, $\frac{h-2\tau}{2h\tau}$ and $\frac{h-2\Delta}{2h\Delta}$ are very small values. Hence, for the sake of simplicity, the small values of control parameter λ can be selected to design and test the string stability of ACC system with the only condition $h > 2(\Delta + \tau)$, $h > 2\tau$ and $h > 2\Delta$.

V. CONCLUSION

In contrast to previous work [2], [6], [7], this paper introduces not only the parasitic lags but also parasitic delays in the analysis of string stability of string of ACC-vehicles. It further provides a sufficient condition that the time headway, h, to be maintained by an ACC equipped vehicle must be at least twice the sum of parasitic lags, τ, and parasitic delays, Δ, that is, $h > 2(\tau + \Delta)$. Simulations have been conducted to indicate that the sufficient condition is tight, i.e., if the time headway , h, is chosen to be less than twice the sum of parasitic lags, τ, and parasitic delays, Δ, that is, $h \leq 2(\tau+\Delta)$, string instability is observed for different values of the gain, λ.

ACKNOWLEDGMENT

The authors acknowledge the support of China Scholarship Council and the comments of Mr. Liu Rui, Department of Mathematics, Texas A&M University, College Station.

REFERENCES

[1] K. Chu, *"Decentralized control of high-speed vehicular strings"*, Transportation Sicence, Vol. 8, No. 3, pp. 361-384, Nov. 1974.

[2] S. Darbha and K. Rajagopal, *"A Review of Constant Time Headway Policy for Automatic Vehicle Following"*, 2001 IEEE ITS Conference Proceedings, Oakland, CA, pp. 65-69, Aug. 2001.

[3] S. Darbha and J. Hedrick, *"String Stability of Interconnected Systems," IEEE Transactions on Automatic Control"*, Vol. 41, No. 3, pp. 349-357, Mar. 1996.

[4] J. Zhou and H. Peng, *"Range Policy of Adaptive Cruise Control Vehicle for Improved Flow Stability and String Stability"*, IEEE Transactions on ITS, Vol. 6, No. 2, pp. 229-237, Jun. 2005.

[5] R. Rajamani and S. Shladover, *"An Experimental Comparative Study of Autonomous and Co-operative Vehicle-follower Control Systems"*, Transportation Research Part C 9, pp. 15-31, 2001.

[6] S. Darbha, *"String Stability of Interconnected Systems: An Application to Platooning in Automated Highway Systems"*, Ph.D. Dissertation, University of California, Berkeley, 1994.

[7] S. Darbha, J. Hedrick, C. Chien and P. Ioannou, *"A Comparison of Spacing and Headway Control Strategy for Automatically Controlled Vehicles"*, Vehicle System Dynamics, Vol. 23, No. 8, pp. 597-625, 1994.

[8] K. Santhanakrishnan and R. Rajamani, *"On Spacing Policies for Highway Vehicle Automation"*, IEEE Transactions on Intelligent Transportation Systems, Vol. 4, No. 4, pp. 198-204, Dec. 2003.

[9] D. Yanakiev and I. Kanellakopoulos, *"Variable Time Headway for String Stability of Automated Heavy-duty Vehicles"*, in Proc. 34th IEEE Conf. Decision and Control, New Orleans, LA, pp. 4077-4081, 1995.

[10] S. Huang and W. Ren, *"Autonoumous Intelligent Cruise Control with Actuator Delays"*, Joural of Intelligent and Robotic Systems, Vol. 23, pp. 27-43, 1998.

[11] R. Rajamani, *"Vehicles Dynamics and Control"*, New York, USA: Springer, 2006.

[12] J. Moskwa and J. Hedrick, *"Modeling and Validation of Automotive Engines for Control Algorithm Development"*, Journal of Dynamic Systems, Measurement, and Control, Vol. 114, pp. 278-285, Jun. 1992.

[13] X. Liu, S. Mahal, A. Goldsmith and J. Hedrick, *"Effects of Communication Delays on String Stability in vehicle platoons"*, 2001 IEEE ITS Conference Proceedings, Oakland CA, Aug. 2001.

[14] S. Mahal, *"Effects of Communication Delays on String Stability in an AHS Environment"*, M.S. Thesis, University of California, Berkeley, 2000.

[15] S. Yadlapalli, S. Darbha and K. Rajagopal *"Information Flow and Its Relation to Stability of the Motion of Vehicles in a Rigid Formation"*, IEEE Transactions on Automatic Control, VOL. 51, NO. 8, Aug. 2006.

[16] P. Cook, *"Conditions for String Stability"*, System and Control Letters, Vol. 54, pp. 991-998, 2005.

[17] S. Sheikholeslam and C. Desoer, *"Control of Interconnected Nonlinear Dynamical Systems: The Platoon Problem"*, IEEE Transaction on Automatic Control, Vol. 37, No. 6, pp.806-810, Jun. 1992.