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Bounded Stability and Eventual String Stability of a Large Platoon of Vehicles Using Non-Identical Controllers

Maziar E. Khatir   Edward J. Davison

Abstract—**In this paper the notions of eventual string stability and bounded stability are introduced, and applied to the "constant spacing distance" control problem (in contrast with the "headway control" problem) of a platoon of identical vehicles. In particular, it is assumed that vehicles in the platoon, are controlled by decentralized non-identical controllers, and that it is desired to achieve asymptotic error regulation in the spacing of the vehicles, independent of any constant velocity which the lead vehicle may take, such that the control signals and states of the vehicles in the platoon remain bounded, and such that there exists vehicle $\overline{N}$ by which the system is string stable for all vehicles $n \geq \overline{N}+1$. These properties of a system are called "bounded stability" and "eventual string stability" respectively. A new decentralized controller is proposed which has the property that if the decentralized controller parameters are tuned appropriately, that all the inputs and states of the platoon will remain bounded with respect to the number of vehicles in the platoon, while ensuring asymptotic spacing regulation and eventual string stability of the spacing distance. A number of examples are included which illustrate the type of results which can be obtained for the case of a platoon of size 1000 vehicles.**

I. Introduction

STRING stability of a platoon of vehicles with respect to the so called "Spacing Control" strategy using decentralized control has been an active area of research since the early 90's; earlier works in the 60's and 70's used centralized controllers to control such platoons of vehicles. In the early 90's the PATH project in California provided a vast substrate for research in this area, and the topic dealing with "string stability" [1], [4] was, of great interest. The term "slinky effect" which refers to the amplification of the peak of the error from vehicle to vehicle, is directly related to string stability. In [1] and [2], it was shown that for identical vehicles, it is impossible to achieve string stability for spacing distance error using decentralized identical controllers. It was observed later in [5] that non-identical controllers with linearly increasing gains, can cause unbounded control when the number of vehicles "N" in the platoon, increases.

In this paper it is shown that a platoon of identical vehicles, with non-identical controllers, using fully decentralized control with no communication existing between the vehicles, can possess bounded control signals and states as the number of vehicles N in the platoon increases, when the lead vehicle undergoes a step change of speed. In section 2, a model of the platoon is given and the proposed decentralized controller is introduced. It is then shown that on using this controller, the infinity-norm of the transfer function from the spacing distance error of the first vehicle to the spacing distance error of the n^{th} vehicle of the platoon remains bounded when the controller coefficients increase linearly with respect to n, under certain slope conditions, and that the same type of result holds for velocities and control signals of the vehicles. In the velocity case, the slope condition is more restrictive than for the distance case. Conditions for the system to possess eventual string stability are then obtained. In section 4, various numerical examples are given to confirm the theoretical results obtained. It is to be noted that time-delay effects are not included in this development; they could be considered by using the same type of approach as carried out in [3].

II. Preliminary Results

Given N+1 identical vehicles traveling in a straight line, let the position of the lead vehicle from a given reference be denoted by y_0, and let the position of the next N vehicles be denoted by y_1, y_2, …, y_N respectively. Let the separation distance of the i^{th} vehicle to the i-1^{th} vehicle be denoted by $d_i = y_{i-1} - y_i$, i= 1, 2, …, N. Let the velocity of the lead vehicle be denoted by v_0^{ref} and the velocities of the remaining vehicles be denoted by $v_i = dy_i/dt$, i= 1, 2, …, N respectively. Let the force applied to the i^{th} vehicle, which has position y_i, be denoted by u_i, i= 1, 2, …, N where u_i and v_i are related by a first order differential equation $m\dot{v}_i + bv_i = u_i$. It is desired to solve the "spacing control" problem for this system, in which the separation distances of the vehicles are asymptotically regulated to desired distances independent of the constant velocity of the lead vehicle. It will be assumed that this platoon of vehicles is controlled by a decentralized controller, consisting of non-identical 3-term controller (PID) [2], given by:

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Maziar E. Khatir is a Ph.D. candidate in the Systems Control Group, Department of Electrical and Computer Engineering, University of Toronto (email: mkhatir@control.toronto.edu)

Edward J. Davison is a University Professor in the Systems Control Group, Department of Electrical and Computer Engineering, University of Toronto (email: ted@control.toronto.edu)

$$u_i(s) = (P_i + \frac{I_i}{s} + D_i s)e_i(s) \quad , i= 1 , 2,\ldots, N \tag{2.1}$$

where $e_i(s) = (d_i(s) - d_i^{ref}(s))$, where $d_i^{ref}(s)$ denotes the reference spacing set point of the i^{th} vehicle, and e_i denotes the error in the separation distance of the i^{th} vehicle with respect to the vehicle in front of it. Thus each vehicle requires only the measurement of the relative distance of itself with its immediate neighbour, and no communication is assumed to exist between vehicles. The controller gains P_i and D_i here are assumed to have a linearly increasing profile with respect to i as follows:

$$\begin{cases} I_i = I_0 \\ P_i = P_0 + \alpha\, i \\ D_i = D_0 + \beta\, i \end{cases} \tag{2.2}$$

where $I_0 > 0$, $P_0 > 0$, $D_0 > 0$, $\alpha > 0$, $\beta > 0$ are constant.
It will be shown that under certain conditions, that this controller possesses some desired features, e.g. the distances and velocities of the vehicles of the resultant closed loop system are bounded and are non-increasing with respect to the number of vehicles, N of the platoon.

2.1 *Development*

The transfer function from one agent to the next one can be obtained as follows from the above vehicle model and controller:

$$\frac{d_i(s)}{d_{i-1}(s)} = L_i(s) = \frac{(D_{i-1}s^2 + P_{i-1}s + I_{i-1})}{s^2(ms+b) + (D_i s^2 + P_i s + I_i)} \tag{2.3a}$$

$$\frac{v_i(s)}{v_{i-1}(s)} = G_i(s) = \frac{(D_i s^2 + P_i s + I_i)}{s^2(ms+b) + (D_i s^2 + P_i s + I_i)} \tag{2.3b}$$

and the transfer function from the leader to the n^{th} vehicle can be obtained as follows:

$$\frac{d_n(s)}{d_1(s)} = \prod_{i=2}^{n} L_i(s) = \prod_{i=2}^{n} \frac{(D_{i-1}s^2 + P_{i-1}s + I_{i-1})}{s^2(ms+b) + (D_i s^2 + P_i s + I_i)} := M_n(s) \tag{2.4a}$$

$$\frac{v_n(s)}{v_0(s)} = \prod_{i=1}^{n} G_i(s) = \prod_{i=1}^{n} \frac{(D_i s^2 + P_i s + I_i)}{s^2(ms+b) + (D_i s^2 + P_i s + I_i)} := H_n(s) \tag{2.4b}$$

substituting (2.2) into (2.3) results in

$$\left|M_n(j\omega)\right|^2 = \prod_{i=2}^{n} \left|L_i(j\omega)\right|^2 = \prod_{i=2}^{n} \frac{i^2 + A_d i + B_d}{i^2 + Ci + D} \quad ;\omega > 0 \tag{2.5a}$$

$$\left|H_n(j\omega)\right|^2 = \prod_{i=1}^{n} \left|G_i(j\omega)\right|^2 = \prod_{i=1}^{n} \frac{i^2 + A_v i + B_v}{i^2 + Ci + D} \quad ;\omega > 0 \tag{2.5b}$$

where for $\omega>0$:

$$A_d = \frac{(2D_0\beta - 2\beta^2)\omega^4 + (2P_0\alpha - 2I_0\beta - 2\alpha^2)\omega^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6a}$$

$$B_d = \frac{(D_0 - \beta)^2\omega^4 + (2I_0\beta - 2I_0D_0 + P_0^2 - 2P_0\alpha)\omega^2 + I_0^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6b}$$

$$A_v = \frac{(2D_0\beta)\omega^4 + (2P_0\alpha - 2I_0\beta)\omega^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6c}$$

$$B_v = \frac{D_0^2\omega^4 + (-2I_0D_0 + P_0^2)\omega^2 + I_0^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6d}$$

$$C = \frac{(2D_0\beta + 2b\beta - 2m\alpha)\omega^4 + (2P_0\alpha - 2I_0\beta)\omega^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6e}$$

$$D = \frac{m^2\omega^6 + ((D_0 + b)^2 - 2mP_0)\omega^4 + (-2I_0D_0 - 2I_0b + P_0^2)\omega^2 + I_0^2}{\beta^2\omega^4 + \alpha^2\omega^2} \tag{2.6f}$$

It will now be shown that convergence of these series (2.5a) and (2.5b) [6] depends only on the values of A_d, A_v, and C, assuming that B_d, B_v, and D are bounded.

III. Main Results

***Definition*:** Given a platoon of i = 0, 1, 2, …, N vehicles traveling in a straight line, which are controlled by either a centralized or decentralized controller, let the transfer function from the leader to the n^{th} vehicle for the spacing distance and velocity be given by $d_n(s)/d_1(s) := M_n(s)$ and $v_n(s)/v_0(s) := H_n(s)$ respectively; then if $\lim_{n\to\infty}\left|M_n(j\omega)\right| < \infty$ and $\lim_{n\to\infty}\left|H_n(j\omega)\right| < \infty$, the system is said to have a bounded spacing distance and bounded velocity respectively, or to be bounded stable.
The following result is a property of the Gamma function defined by:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$$

***Lemma* 3.1:** Assume that α and $\tilde{\alpha}$ are complex conjugate scalars; then:

$$\Gamma(\alpha)\Gamma(\tilde{\alpha}) = \left|\Gamma(\alpha)\right|^2 = \left|\Gamma(\tilde{\alpha})\right|^2$$

3.1 *Bounded Spacing Distance*

The following results are obtained:

***Theorem* 3.1:** Let $\omega > 0$ be fixed, and consider $M_n(j\omega)$ given by (2.5a) and $A_d(\omega)$, $B_d(\omega)$, $C(\omega)$, $D(\omega)$ given in

(2.6); then

(i) $\lim_{n\to\infty} |M_n(j\omega)|$ exists if and only if $A_d(\omega) \le C(\omega)$.

(ii) Assume $A_d(\omega) < C(\omega)$, then:

$$|M_n(j\omega)|^2 = \frac{\Gamma(2-y_1)\Gamma(2-y_2)}{\Gamma(2-x_1)\Gamma(2-x_2)} \cdot \frac{\Gamma(n+1-x_1)\Gamma(n+1-x_2)}{\Gamma(n+1-y_1)\Gamma(n+1-y_2)} \quad (3.1)$$

where x_1 and x_2 are the roots of (3.2a), and y_1 and y_2 are the roots of (3.2b), given as follows:

$$x^2 + A_d(\omega)\, x + B_d(\omega) = 0 \quad (3.2a)$$

$$y^2 + C(\omega)\, y + D(\omega) = 0 \quad (3.2b)$$

and $\Gamma(.)$ is the Gamma function.

(iii) Assume $A_d(\omega) = C(\omega)$, then:

$$\lim_{n\to\infty} |M_n(j\omega)|^2 = \frac{\Gamma(2-y_1)}{\Gamma(2-x_1)} \frac{\Gamma(2-y_2)}{\Gamma(2-x_2)} \quad (3.3)$$

(iv) Assume $A_d(\omega) < C(\omega)$, then:

$$\lim_{n\to\infty} |M_n(j\omega)|^2 = 0$$

***Proof*:** The proof is lengthly but directly follows from [6] on noting that the expression (2.5a) corresponds to equations (89.2.1) and (89.5.7) of [6].

***Remark* 3.1:** It follows from lemma 3.1 that if the roots of (3.2a) and (3.2b) are complex conjugate, $|M_n(j\omega)|^2$ described in (3.1) is real and is given by:

$$|M_n(j\omega)|^2 = \frac{|\Gamma(2-y_1)|^2}{|\Gamma(2-x_1)|^2} \cdot \frac{|\Gamma(n+1-x_1)|^2}{|\Gamma(n+1-y_1)|^2} \quad (3.4)$$

***Remark* 3.2:** Assume that $\omega>0$; then it can be noted that the condition $A_d(\omega) \le C(\omega)$ holds if and only if :

$$2b\beta\omega^4 - 2m\alpha\omega^4 \ge -2\beta^2\omega^4 - 2\alpha^2\omega^2 \quad (3.5)$$

is true or if and only if:

$$2b\beta - 2m\alpha \ge -2\beta^2 - 2\alpha^2\omega^{-2} \quad (3.6)$$

is true, which implies that (3.6) holds for any $\omega>0$ if:

$$2b\beta - 2m\alpha \ge -2\beta^2 \quad (3.7)$$

or if:

$$\beta \ge \sqrt{b^2/4 + m\alpha} - b/2 \quad (3.8)$$

The following result is now obtained:
Given a platoon of vehicles described by:

$$\begin{pmatrix} \dot{y}_i \\ \dot{v}_i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -b/m \end{pmatrix} \begin{pmatrix} y_i \\ v_i \end{pmatrix} + \begin{pmatrix} 0 \\ 1/m \end{pmatrix} u_i \qquad i = 1, 2, \ldots, N \quad (3.9)$$
$$d_i = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} y_i \\ v_i \end{pmatrix} + y_{i-1}$$

where y_0 is the position of the lead vehicle, then the following result can be directly obtained by direct computation of the closed loop characteristic equation and by applying the Routh test.

***Lemma* 3.2:** Consider the platoon of vehicles described by (3.9) and consider the decentralized controller

$$u_i(s) = (P_i + \frac{I_i}{s} + D_i s)(d_i(s) - d_i^{ref}(s)) \;\; i = 1, 2, \ldots, N \quad (3.10)$$

then there always exists positive constants I_0, P_0, D_0 such that $I_i = I_0$, $P_i = P_0$, $D_i = D_0$, $i = 1, 2, \ldots, N$, will stabilize the resultant closed loop system and thereby solve the spacing control problem for (3.9).

***Theorem* 3.2:** Consider a platoon of vehicles described by (3.9) and consider a decentralized controller described by (3.10) where:

$$I_i = I_0 \quad ; I_0 > 0 \quad (3.11a)$$
$$P_i = P_0 + \alpha i \quad ; P_0 > 0,\ \alpha > 0 \quad (3.11b)$$
$$D_i = P_0 + \beta i \quad ; D_0 > 0,\ \beta > 0 \quad (3.11c)$$

where I_0, P_0, D_0 have been chosen, from lemma 3.2, to stabilize the system, and assume that the following condition holds:

$$\beta \ge \sqrt{b^2/4 + m\alpha} - b/2 ;$$

then this implies that the closed loop system has the following properties:

- **(i)** The resultant closed loop system is asymptotically stable
- **(ii)** The resultant closed loop system is bounded stable in terms of spacing error between the vehicles
- **(iii)** Asymptotic tracking occurs in the closed loop system for the spacing error, i.e. $\lim_{t\to\infty}(d_i(t) - d_i^{ref}) = 0$, for all constant initial conditions and reference spacing set points $v_i(0)$, $y_i(0)$, d_i^{ref}, $i = 1, 2, \ldots, N$, and for all constant lead vehicle velocity v_0.

3.2 *Bounded Velocity*

The following results are obtained:

***Theorem* 3.3:** Let $\omega>0$ be fixed, and consider $H_n(j\omega)$ given by (2.5b) and $A_v(\omega)$, $B_v(\omega)$, $C(\omega)$, $D(\omega)$ given in (2.6); then:

(i) $\lim_{n\to\infty} |H_n(j\omega)|$ exists if and only if $A_v(\omega) \le C(\omega)$.

(ii) Assume $A_v(\omega) < C(\omega)$, then:

$$\left|H_n(j\omega)\right|^2 = \frac{\Gamma(1-y_1)\Gamma(1-y_2)}{\Gamma(1-z_1)\Gamma(1-z_2)} \cdot \frac{\Gamma(n+1-z_1)\Gamma(n+1-z_2)}{\Gamma(n+1-y_1)\Gamma(n+1-y_2)} \quad (3.12)$$

where y_1 and y_2 are the roots of (3.2b), and z_1 and z_2 are the roots of (3.13) given as follows:

$$z^2 + A_v(\omega)\, z + B_v(\omega) = 0 \quad (3.13)$$

(iii) If $A_v(\omega) = C(\omega)$, then:

$$\lim_{n\to\infty} \left|H_n(j\omega)\right|^2 = \frac{\Gamma(1-y_1)}{\Gamma(1-z_1)} \frac{\Gamma(1-y_2)}{\Gamma(1-z_2)} \quad (3.14)$$

(iv) If $A_v(\omega) < C(\omega)$, then:

$$\lim_{n\to\infty} \left|H_n(j\omega)\right|^2 = 0 \quad (3.15)$$

Proof: The proof directly follows from the proof of theorem 3.1.

***Remark* 3.3:** If the roots of (3.13) and/or (3.2b) are complex conjugate, then it follows from lemma 3.1 that $|H_n(j\omega)|^2$ is real and is given by:

$$\left|H_n(j\omega)\right|^2 = \frac{\left|\Gamma(1-y_1)\right|^2}{\left|\Gamma(1-z_1)\right|^2} \cdot \frac{\left|\Gamma(n+1-z_1)\right|^2}{\left|\Gamma(n+1-y_1)\right|^2} \quad (3.16)$$

***Remark* 3.4:** Assume that $\omega > 0$; then the condition $A_v(\omega) \leq C(\omega)$ holds if and only if :

$$2b\beta\omega^4 - 2m\alpha\omega^4 \geq 0 \quad (3.17)$$

which implies that (3.17) holds for any $\omega > 0$ if and only if:

$$\beta \geq {m\alpha}/{b} \quad (3.18)$$

***Theorem* 3.4:** Consider a platoon of vehicles described by (3.9) and consider a decentralized controller described by (3.10) and (3.11), where I_0, P_0, D_0 have been chosen, from lemma 3.2, to stabilize the system and assume (3.18) holds; then this implies that the closed loop system has the following properties:

- **(i)** The resultant closed loop system (3.9), (3.10), (3.11) is asymptotically stable
- **(ii)** The closed loop system is bounded stable in terms of velocity of the vehicles
- **(iii)** Asymptotic tracking occurs in the closed loop system for the spacing error as given in result (iii) of theorem 3.2

3.3 *Eventual String Stability*

Definition: Given a platoon of i=0, 1, 2, …, N vehicles traveling in a straight line, which are controlled by either a centralized or decentralized controller, assume that there exists a vehicle $\overline{N} < N$ by which the resultant closed loop system is string stable for all vehicles $i \geq \overline{N}+1$. Then such a controlled system is said to have "eventual string stability". The following result is obtained:

***Theorem* 3.5:** Consider a platoon of vehicles described by (3.9), and consider the decentralized controller (3.10), (3.11), where I_0, P_0, D_0 are chosen from lemma 3.2 to stabilize the system, and assume that (3.8) holds; then this implies that the closed loop system has the property that it is eventual string stable.

Proof: The result will be true if $\exists\ \overline{N} > 0$ so that the following two conditions hold for all $n \geq \overline{N}+1$:

(i) $$\left|L_n(0)\right| = 1 \quad (3.19)$$

(ii) $$\frac{d\left|L_n(j\omega)\right|^2}{d\omega} < 0 \quad (3.20)$$

where

$$\left|L_n(j\omega)\right|^2 = \frac{n^2 + A_d(\omega)n + B_d(\omega)}{n^2 + C(\omega)n + D(\omega)} \quad (3.21)$$

It can be easily verified that: $\lim_{\omega\to 0}\left|L_n(j\omega)\right|^2 = 1$, and on noticing that:

$$\frac{d\left|L_n(j\omega)\right|^2}{d\omega} = \frac{m_3(\omega)n^3 + m_2(\omega)n^2 + m_1(\omega)n + m_0(\omega)}{(n^2 + C(\omega)n + D(\omega))^2} \quad (3.22)$$

where

$$m_3(\omega) = \left(\frac{dA_d}{d\omega} - \frac{dC}{d\omega}\right) = -4(2b\beta - 2m\alpha + 2\beta^2)\omega^3 - 4\alpha^2\omega \quad (3.23)$$

and where the denominator of (3.22) is always positive, it can be verified that $m_3(\omega) < 0, \quad \forall\omega > 0,$ if condition (3.8) holds. Now since m_3 is the coefficient of the highest power of the polynomial of the numerator of (3.22), this implies that there exists $\overline{N} > 0$ such that if $n > \overline{N}$, properties (i) , (ii) will both hold, which proves the theorem

The following result summarizes the previous results:

***Theorem* 3.6:** Given a platoon of vehicles described by (3.9) then there always exists a decentralized controller (3.10), (3.11) which solves the spacing control problem for (3.9), such that the resultant closed loop system is asymptotically stable, is bounded stable with respect to both spacing error and velocity of the vehicles, and is eventual string stable.

Proof: Consider using the decentralized controller (2.1), (2.2) to control the platoon (3.9); then the result directly follows from lemma 3.2, on nothing that condition (3.18) implies that condition (3.8) is satisfied, and on noting that the conditions of theorem 3.3, 3.4, 3.5 are all satisfied if the controller parameter (2.2) β satisfies the condition:

$$\beta \geq {}^{m\alpha}\!/_{b}$$

IV. Numerical Examples

In the following examples, a platoon of N=1000 vehicles is considered. In all of the examples, it is assumed that the model of the platoon is given by (3.9), where m=0.1, and b=1 respectively denote the mass and damping ratio of the vehicle, and the input signal corresponds to the force applied to the vehicle. It is also assumed that the decentralized controller given by (2.1), (2.2) is applied to the system with controller parameters I_0=1, P_0=5, D_0=1, α =0.2, and where β is chosen to satisfy (3.8) and (3.18). The following different simulation results are now obtained for the resultant closed loop using the decentralized controller (2.1), (2.2).

***Example* 1:** In this example, both conditions (3.8) and (3.18) are strictly satisfied using α= 0.2, β= 0.2, and the results obtained in figure 4.1 show the decrease of the peaks of the magnitude of the transfer functions $H_n(s)$ and $M_n(s)$ and the peaks of time responses of velocities and spacing distances $v_i(t)$ and $d_i(t)$.

***Example* 2:** In this example neither conditions (3.8) or (3.18) are satisfied using α= 0.2, β= 0.0039, and the results obtained in figure 4.2 show that although the closed loop system is stable, the peaks of the magnitude of $H_n(j\omega)$ and $M_n(j\omega)$ and $v_i(t)$ and $d_i(t)$ diverge, and the spacing error and velocity approach infinity as $N \to \infty$.

***Example* 3:** In this example, condition (3.18) is only marginally satisfied using α= 0.2, β= 0.02, and the results obtained in figure 4.3 show that the peaks of the magnitude of $H_n(j\omega)$ asymptotically approach a non-zero constant as n becomes large as is shown in figure (4.5).

***Example* 4:** In this example, condition (3.18) is violated and (3.8) is only marginally satisfied using α= 0.2, β= 0.0196, and the results obtained in figure 4.4 are similar to the results of example (4.3); however on increasing n, divergence of the peaks now appears.

***Example* 5:** In this example it is shown that if condition (3.18) is only marginally satisfied using α= 0.2, β= 0.02, the magnitude of the transfer function $H_n(s)$ approaches the limit given by (3.14), as shown in figure 4.5.

***Example* 6:** In this example it is shown that if condition (3.18) is strictly satisfied using α= 0.1, β= 0.1, then (3.15) will be satisfied as n approaches infinity. Results given in figure 4.6 show the magnitude of the transfer function $H_n(s)$ when n approaches 10^9.

V. Conclusion

In this paper it is shown for a platoon of vehicles described by (3.9) moving in a straight line, that there exists a non-identical decentralized controller, so that the resultant system has *bounded stability* and *eventual string stability* properties, which solves the "spacing control" problem for a platoon of vehicles. This implies that all inputs and states of the vehicles remain bounded as the number of vehicles in the platoon approaches infinity, and that there exists $\overline{N} > 0$ by which system has string stability for all vehicles $n \geq \overline{N} + 1$.

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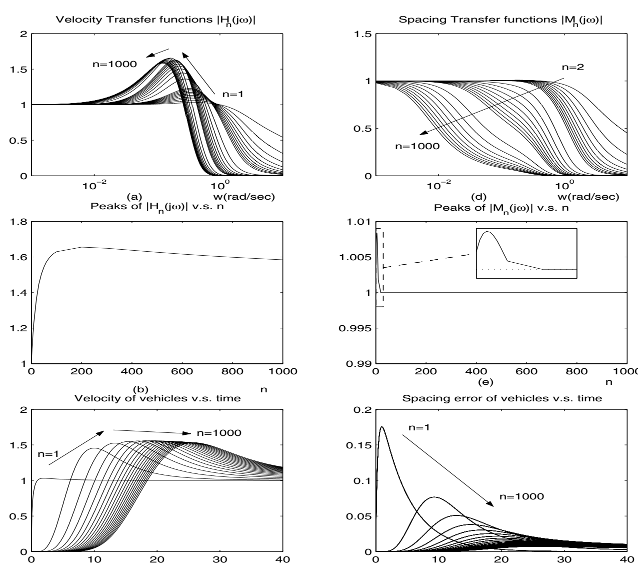

Figure 4.1 Simulation of closed loop system obtained with a N=1000 vehicle platoon using decentralized controller (3.10) with :

$$\beta > {}^{m\alpha}\!/_{b}\,,\ \beta > \sqrt{{}^{b^2}\!/_{4} + m\alpha} - {}^{b}\!/_{2}\quad (\alpha = 0.2, \beta = 0.2)$$

fig(a) and fig(d) show the magnitude of the transfer functions $H_n(j\omega)$ and $M_n(j\omega)$ respectively. fig(b) and fig(e) show the peaks of $H_n(j\omega)$ and $M_n(j\omega)$in terms of n and fig(c) and fig(f) show the time response of the velocity and spacing distance error of vehicles

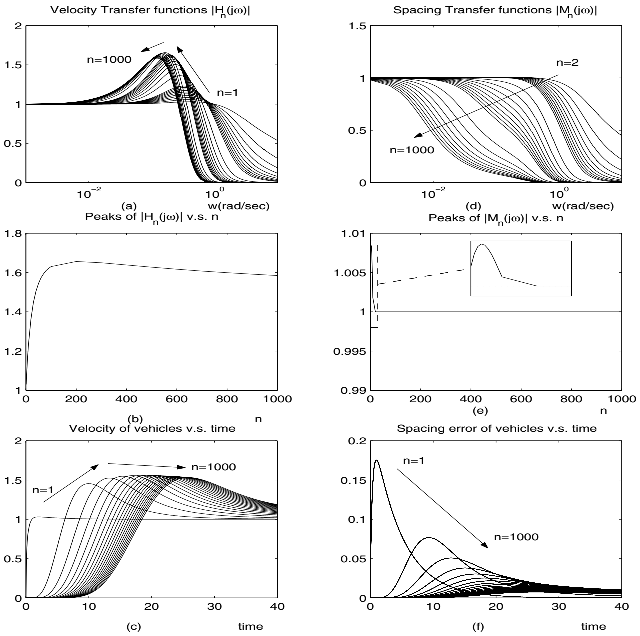

Figure 4.2 Simulation of closed loop system obtained with a N=1000 vehicle platoon using decentralized controller (3.10) with :

$$\beta < {}^{m\alpha}\!/_{b}\,,\ \beta < \sqrt{{}^{b^2}\!/_{4} + m\alpha} - {}^{b}\!/_{2}\quad (\alpha = 0.2, \beta = 0.0039)$$

fig(a) and fig(d) show the magnitude of the transfer functions $H_n(j\omega)$ and $M_n(j\omega)$ respectively. fig(b) and fig(e) show the peaks of $H_n(j\omega)$ and $M_n(j\omega)$in terms of n and fig(c) and fig(f) show the time response of the velocity and spacing distance error of vehicles

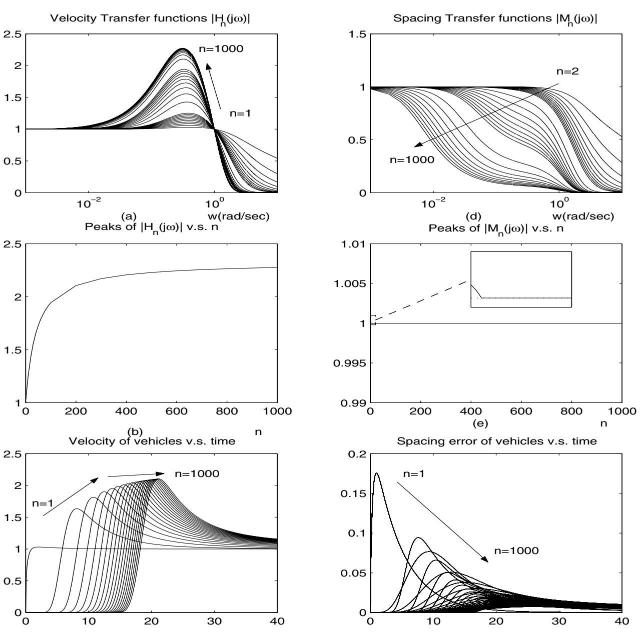

Figure 4.3 Simulation of closed loop system obtained with a N=1000 vehicle platoon using decentralized controller (3.10) with :

$$\beta = {}^{m\alpha}\!/_{b}\,,\ \beta > \sqrt{{}^{b^2}\!/_{4} + m\alpha} - {}^{b}\!/_{2}\quad (\alpha = 0.2, \beta = 0.02)$$

fig(a) and fig(d) show the magnitude of the transfer functions $H_n(j\omega)$ and $M_n(j\omega)$ respectively. fig(b) and fig(e) show the peaks of $H_n(j\omega)$ and $M_n(j\omega)$in terms of n and fig(c) and fig(f) show the time response of the velocity and spacing distance error of vehicles

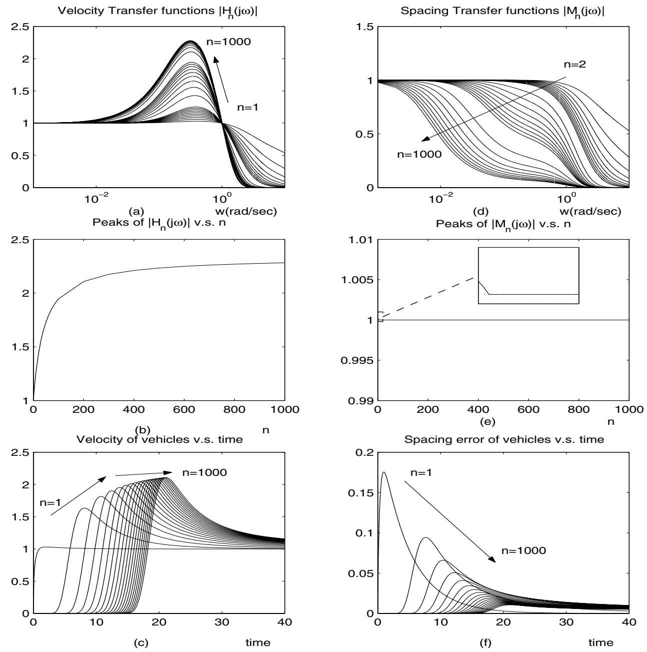

Figure 4.4 Simulation of closed loop system obtained with a N=1000 vehicle platoon using decentralized controller (3.10) with:

$$\beta < {}^{m\alpha}\!/_{b}\,,\ \beta = \sqrt{{}^{b^2}\!/_{4} + m\alpha} - {}^{b}\!/_{2}\quad (\alpha = 0.2, \beta = 0.0196)$$

fig(a) and fig(d) show the magnitude of the transfer functions $H_n(j\omega)$ and $M_n(j\omega)$ respectively. fig(b) and fig(e) show the peaks of $H_n(j\omega)$ and $M_n(j\omega)$in terms of n and fig(c) and fig(f) show the time response of the velocity and spacing distance error of vehicles

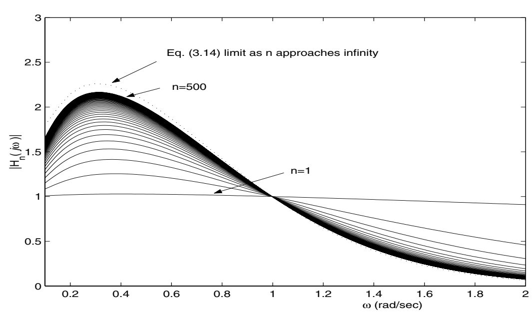

Figure 4.5 In this figure the condition given by (3.18) is only marginally satisfied using $\alpha = 0.2$, $\beta = 0.02$, and the results obtained show that the magnitude of $H_n(j\omega)$ asymptotically approaches the dotted line which represents the gamma function (3.15) as n approaches infinity.

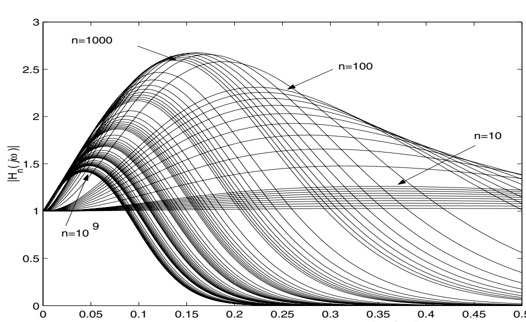

Figure 4.6 In this figure the condition given by (3.18) is strictly satisfied and the results obtained show that the magnitude of $H_n(j\omega)$ approaches zero as n approaches infinity.