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String Stability of Heterogeneous Leader-Following Vehicle Platoons Based on Constant Spacing Policy*

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Abstract— **This paper is concerned with a leader-follower problem for a heterogeneous vehicle platoon subject to external bounded unknown acceleration disturbances. Distributed controller based on sliding mode control (SMC) approach are designed for the second-order follower-vehicles under the common assumption that the initial spacing and velocity errors are zero. The constant spacing policy known to have high traffic density and thus have high traffic flow is applied to design distributed controller. In addition, adaptive compensation technique is applied to compensate the time-varying effect of external disturbances. It is worth mentioning that the upper and lower bounds of the disturbances are not required to be known in advance. Furthermore, with the help of an explicitly constructed Lyapunov function, it is proved that the string stability of the vehicle platoon can be guaranteed. At the same time, the reduction of the chattering in sliding mode is achieved by introducing continuous function in control. Finally, a numerical example is given for illustration.**

I. INTRODUCTION

It has been known that a platoon composed of a leader followed by vehicles in a straight line can significantly improve the efficiency of existing road systems [1]. Since the number of vehicles and vehicular traffic have significantly growth in the worldwide, platoon control has become a topic of considerable interest and has motivated much research in these fields (such as [2], [3], [4], [5], [6], [7], [8], [9] and references therein). The main concern in a platoon is to maintain a desired safety spacing from its leading vehicle and avoid collision [10], [11], [12], [13]. In order to maintain a safety spacing, the constant-spacing policy known to have high traffic density and thus have high traffic flow is often used in most existing works [2], [3], [4], [5], [6], [7], [8], [9]. Moreover, in order to avoid collisions, string stability should be assured, i.e the spacing errors should not be amplified in the platoon [14], [12], [13]. Therefore, how to guarantee string stability of a vehicle platoon has been a task of major practical interest as well as theoretical significance.

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On the other hand, vehicle platoon inevitably suffers from external disturbances such as vehicle acceleration disturbances, wind gust, parameters uncertainties and intermediate uncertainties induced by networks [15] in high maneuver and high noise environment. It should be pointed out that disturbances acting on one vehicle may affect other vehicles and even amplify spacing errors along the string, namely string instability [15], [16] since the vehicle platoon is an interconnected coupled system. Therefore, how to attenuate the influence of exogenous disturbance to the vehicle platoon is important and challenging in both theory and practice. Moreover, it is well known that although SMC approach is theoretically excellent in terms of robustness and disturbance rejection capabilities [17], the main drawback of the SMC approach is mostly related to the so-called chattering effect caused by the high-frequency discontinuous control input [18]. Therefore, particular attention need be paid to alleviate the chattering phenomena caused by SMC approach.

The motivation of this work is to design an adaptive distributed controller for a heterogeneous vehicle platoon where a leader to be followed moves in a straight line. The vehicle models considered here are second-order and subject to external bounded unknown acceleration disturbances. Combining SMC approach with adaptive compensation technique, a new adaptive distributed controller is designed based on constant spacing policy. It is worth mentioning that the upper and lower bounds of the disturbances are not required to be known in advance. In addition, based on Lyapunov theory, it is proved that the finite time stability of each vehicle and the string stability of the whole vehicle platoon can be guaranteed. Furthermore, effective methods are proposed to reduce the chattering phenomenon caused by using SMC approach. Finally, a numerical example is proposed to show the effectiveness of the proposed control strategy.

Notations: 1) $\|\cdot\|$ stands for the Euclidean norm of a vector; 2) The symbol of $|\cdot|$ represents the absolute value of real numbers; 3) The notation $sgn(\cdot)$ is the sign function, i.e.,

$$sgn(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0. \end{cases}$$

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

Consider a vehicle platoon consisting of one leader and N followers. Assume that the leader is active, that is, its state keeps changing throughout the entire process, with dynamics described as follows:

$$\dot{x}_0(t) = v_0(t) \tag{1}$$

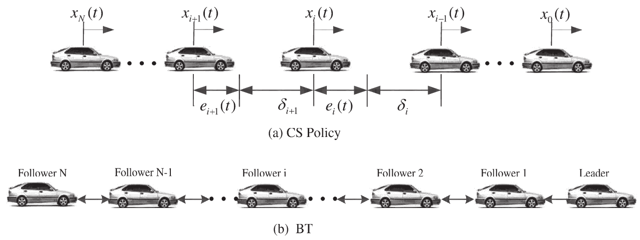

Fig. 1. (a) CS Policy; (b) Bidirectional Topology (BT)

where $x_0(t)$ is the position, $v_0(t)$ is the velocity. The velocity $v_0(t)$ is a known function of time.

The dynamics of follower-vehicle i in a platoon is modeled as follows:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= \frac{1}{m_i}u_i(t) + w_i(t) \end{aligned} \tag{2}$$

where $u_i(t)$ denotes the control input, m_i is the mass of the ith vehicle assumed to be constant, $w_i(t)$ denotes the external disturbances, respectively. As usual, we assume that the external disturbances of all followers have unknown bounds $\underline{w}_i$ and $\overline{w}_i$.

The configuration of the car-following control system is shown in Fig. 1. The safety spacing between the ith and $(i-1)$th vehicles is defined as δ_i. From the configuration, the spacing error $e_i(t)$ can be written as:

$$e_i(t) = x_{i-1}(t) - x_i(t) - \delta_i, \text{ for } i = 1, \cdots, N. \tag{3}$$

where δ_i is the desired constant inter-vehicular spacing. The above spacing policy is known as constant spacing (CS) policy, where the desired inter-vehicle spacing is independent of the velocity of the vehicle. Let $d_i(t) \triangleq x_{i-1}(t) - x_i(t)$ denotes the real spacing between two consecutive vehicles. By using the above CS policy, the information of the adjacent vehicles can be acquired by onboard sensors, the bidirectional topology is employed to the vehicle platoon.

The control objective of this paper is to design a distributed control law for the leader-following vehicle platoon by combining adaptive compensation technique with sliding mode control (SMC) approach to maintain a safety spacing (i.e., $d_i(t) \to \delta_i$), achieve velocity consensus (i.e., $v_i(t) \to v_0(t)$), and guarantee the finite time stability of each vehicle system and string stability of the whole vehicle platoon.

Before giving the main results, the following two important lemmas which will be useful in the derivation of our main results are introduced.

Lemma 1: (Barbalat Lemma [19]) If $\psi(t) : \mathbf{R} \to \mathbf{R}$ is a uniformly continuous function for $t \geq 0$ and if the limit of the integral

$$\lim_{t\to\infty} \int_0^t \psi(\tau) d\tau$$

exists and is finite, then

$$\lim_{t\to\infty} \psi(t) = 0.$$

Lemma 2: [2], [3], [7] Origin $e_i(0) = 0$, for $(i = 1, 2, \cdots, N)$ in Fig. 1, the whole vehicle platoon with a leader (1) and N followers (2) is string stabile in the strong sense if error propagation transfer function $G_i(s) := \frac{E_{i+1}(s)}{E_i(s)}$ satisfies $\|G_i(s)\| \leq 1$ for all $i = 1, \cdots, N$ (i.e., $\|e_N(t)\| \leq \|e_{N-1}(t)\| \leq \cdots, \leq \|e_1(t)\|$), where $E_i(s)$ denotes Laplace transform of $e_i(t)$.

III. Distributed Adaptive Sliding Mode Control (SMC) Strategy

In this section, based on CS policy, the distributed control strategy by combining adaptive compensation technique with SMC approach is proposed for the vehicle platoon under the assumption that the initial spacing and velocity errors are zero.

Recalling the main objective mentioned before, the control strategy is proposed to make $e_i(t)$ converge to zero in a finite time and to guarantee string stability. Then, a sliding surface is defined by

$$s_i(t) = \dot{e}_i(t) + \lambda e_i(t) \tag{4}$$

where λ is a positive design parameter. Furthermore, in order to guarantee string stability, similar to [7], a new coupled sliding surface is defined as follows

$$\begin{aligned} S_i(t) &= q s_i(t) - s_{i+1}(t), i = 1, \cdots, N-1 \\ S_i(t) &= q s_i(t), i = N \end{aligned} \tag{5}$$

where the fact that $s_{N+1}(t)$ does not exist for the last vehicle (i.e., $i = N$) is used in the above definition, and $q > 0$ is a weighting constant scalar. Based on the above definition, the following relationship between $S_i(t)$ and $s_i(t)$ can be obtained:

$$S(t) = Q s(t) \tag{6}$$

where

$$Q = \begin{bmatrix} q & -1 & \cdots & 0 & 0 \\ 0 & q & -1 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & q & -1 \\ 0 & 0 & \cdots & 0 & q \end{bmatrix}$$

$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_N(t) \end{bmatrix}, S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_N(t) \end{bmatrix}.$$

Recalling the fact $q > 0$, one can obtain that Q is invertible. Therefore, one can conclude that if $S_i(t)$ becomes to zero, $s_i(t)$ also becomes zero at the same time, and vise versa.

Then, a sufficient condition for guaranteeing the finite time stability of each vehicle and the string stability of the whole vehicle platoon systems is obtained in the following theorem.

Theorem 1: Consider the leader-following vehicle platoon (2) with a leader (1). For the case of zero initial spacing and velocity errors, the finite-time stability of each vehicle system can be achieved, if the adaptive SMC law is designed as:

$$\begin{aligned} u_i(t) &= -m_i[(1-\mu_i(t))\hat{\overline{w}}_i(t)+\mu_i(t)\hat{\underline{w}}_i(t)]+\tfrac{m_i}{q+1}A_i(t) \\ &\quad +\tfrac{km_i}{q+1}sgn(S_i(t)), \text{for } i=1,2,\cdots,N-1; \\ u_N(t) &= -m_N[(1-\mu_N(t))\hat{\overline{w}}_N(t)+\mu_N(t)\hat{\underline{w}}_N(t)] \\ &\quad +\tfrac{m_N}{q}A_N(t)+\tfrac{km_N}{q}sgn(S_N(t)) \end{aligned} \tag{7}$$

with

$$\begin{aligned} A_i(t) &= q\ddot{x}_{i-1}(t)+\ddot{x}_{i+1}(t)+\lambda(q\dot{e}_i(t)-\dot{e}_{i+1}(t)) \\ &\qquad \text{for } i=1,2,\cdots,N-1 \\ A_N(t) &= q\ddot{x}_{N-1}(t)+q\lambda\dot{e}_N(t) \end{aligned} \tag{8}$$

where λ, k and q are positive constants, and $\hat{\overline{w}}_i(t)$ and $\hat{\underline{w}}_i(t)$ are the estimates of the upper bound $\overline{w}_i$ and the lower bound $\underline{w}_i$ of the disturbance $w_i(t)$, respectively, which are updated according to the adaptive laws:

$$\begin{aligned} &\dot{\hat{\overline{w}}}_i(t)=-\eta_i(q+1)S_i(t),\ \dot{\hat{\underline{w}}}_i(t)=-\eta_i(q+1)S_i(t) \\ &\qquad \text{for } i=1,2,\cdots,N-1 \\ &\dot{\hat{\overline{w}}}_i(t)=-\eta_i qS_i(t),\ \dot{\hat{\underline{w}}}_i(t)=-\eta_i qS_i(t), \text{for } i=N \end{aligned} \tag{9}$$

and $\mu_i(t)$ is defined as

$$\mu_i(t)=\begin{cases} 1, S_i(t)>0 \\ 0, S_i(t)\le 0 \end{cases} \tag{10}$$

The parameters $\eta_i>0, i=1,2,\cdots,N$ are the adaptive law gains to be designed. Moreover, $\|G_i(s)\|=\|\frac{E_{i+1}(s)}{E_i(s)}\|\le 1$ holds if $0<q\le 1$, and then, the string stability of the whole leader-following vehicle platoon can be guaranteed.

Proof: Taking the time derivative of $S_i(t)$ along (4) and (5) gives

$$\begin{aligned} \dot{S}_i(t) &= q\dot{s}_i(t)-\dot{s}_{i+1}(t) \\ &= q(\ddot{e}_i(t)+\lambda\dot{e}_i(t))-(\ddot{e}_{i+1}(t)+\lambda\dot{e}_{i+1}(t)) \\ &= q(\ddot{x}_{i-1}(t)-\tfrac{1}{m_i}u_i(t)-w_i(t)+\lambda\dot{e}_i(t)) \\ &\quad -(\tfrac{1}{m_i}u_i(t)+w_i(t)-\ddot{x}_{i+1}(t)+\lambda\dot{e}_{i+1}(t)) \\ &= -\tfrac{q+1}{m_i}(u_i(t)+m_iw_i(t))+A_i(t) \end{aligned} \tag{11}$$

where $A_i(t)=q\ddot{x}_{i-1}(t)+\ddot{x}_{i+1}(t)+\lambda(q\dot{e}_i(t)-\dot{e}_{i+1}(t))$ for $i=1,2,\cdots,N-1$. When $i=N$, using the definition of (5), one can further obtain that

$$\begin{aligned} \dot{S}_N(t) &= q\dot{s}_N(t) \\ &= q(\ddot{e}_N(t)+\lambda\dot{e}_N(t)) \\ &= -\tfrac{q}{m_N}(u_N(t)+m_Nw_N(t))+A_N \end{aligned} \tag{12}$$

where $A_N=q\ddot{x}_{N-1}(t)+q\lambda\dot{e}_N(t)$.

First of all, consider a positive definite Lyapunov function candidate for vehicle i as follows:

$$V_i(t)=\tfrac{1}{2}S_i^2(t)+\tfrac{1-\mu_i}{2\eta_i}\tilde{\overline{w}}_i^2(t)+\tfrac{\mu_i}{2\eta_i}\tilde{\underline{w}}_i^2(t) \tag{13}$$

where

$$\begin{aligned} \tilde{\overline{w}}_i(t) &= \hat{\overline{w}}_i(t)-\overline{w}_i \\ \tilde{\underline{w}}_i(t) &= \hat{\underline{w}}_i(t)-\underline{w}_i. \end{aligned}$$

Because $\overline{w}_i$ and $\underline{w}_i$ are unknown constants, the above estimation errors can be rewritten as

$$\dot{\tilde{\overline{w}}}_i(t)=\dot{\hat{\overline{w}}}_i(t), \dot{\tilde{\underline{w}}}_i(t)=\dot{\hat{\underline{w}}}_i(t).$$

Then, the time derivative of $V_i(t)$ in (13) for $i=1,2,\cdots,N$ can be described as:

$$\dot{V}_i(t)=S_i(t)\dot{S}_i(t)+\tfrac{1-\mu_i}{\eta_i}\tilde{\overline{w}}_i(t)\dot{\hat{\overline{w}}}_i(t)+\tfrac{\mu_i}{\eta_i}\tilde{\underline{w}}_i(t)\dot{\hat{\underline{w}}}_i(t). \tag{14}$$

Substituting (11), (12) into (14), respectively, we have

$$\begin{aligned} \dot{V}_i(t) &= S_i(t)((q+1)[(1-\bar{\mu}_i(t))\hat{\overline{w}}_i(t)+\bar{\mu}_i(t)\hat{\underline{w}}_i(t)] \\ &\quad -ksgn(S_i(t))-(q+1)w_i(t)) \\ &\quad +\tfrac{1-\mu_i}{\eta_i}\tilde{\overline{w}}_i(t)\dot{\hat{\overline{w}}}_i(t)+\tfrac{\mu_i}{\eta_i}\tilde{\underline{w}}_i(t)\dot{\hat{\underline{w}}}_i(t) \\ &\le (q+1)[(1-\mu_i)\tilde{\overline{w}}_i(t)+\mu_i\tilde{\underline{w}}_i(t)]S_i(t)-k|S_i(t)| \\ &\quad +\tfrac{1-\mu_i}{\eta_i}\tilde{\overline{w}}_i(t)\dot{\hat{\overline{w}}}_i(t)+\tfrac{\mu_i}{\eta_i}\tilde{\underline{w}}_i(t)\dot{\hat{\underline{w}}}_i(t) \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{V}_N(t) &= S_N(t)(q[(1-\bar{\mu}_N(t))\hat{\overline{w}}_N(t)+\bar{\mu}_N(t)\hat{\underline{w}}_N(t)] \\ &\quad -ksgn(S_N(t))-qw_N(t)) \\ &\quad +\tfrac{1-\mu_N}{\eta_N}\tilde{\overline{w}}_N(t)\dot{\hat{\overline{w}}}_N(t)+\tfrac{\mu_N}{\eta_N}\tilde{\underline{w}}_N(t)\dot{\hat{\underline{w}}}_N(t) \\ &\le q[(1-\mu_N)\tilde{\overline{w}}_N(t)+\mu_N\tilde{\underline{w}}_N(t)]S_N(t)-k|S_N(t)| \\ &\quad +\tfrac{1-\mu_N}{\eta_N}\tilde{\overline{w}}_N(t)\dot{\hat{\overline{w}}}_N(t)+\tfrac{\mu_N}{\eta_N}\tilde{\underline{w}}_N(t)\dot{\hat{\underline{w}}}_N(t) \end{aligned} \tag{16}$$

where $A_i(t)$ is defined in (8), and the following fact obtained from (10) is used.

$$\begin{aligned} &-(q+1)w_i(t)S_i(t)\le-(q+1)[(1-\mu_i)\overline{w}_i+\mu_i\underline{w}_i]S_i(t) \\ &\quad -qw_N(t)S_N(t)\le-q[(1-\mu_N)\overline{w}_N+\mu_N\underline{w}_N]S_N(t). \end{aligned}$$

Recalling the adaptation laws (9), we can conclude (15) and (16) as

$$\dot{V}_i(t)\le-k|S_i(t)|, \text{for } i=1,2,\cdots,N. \tag{17}$$

Now, a global Lyapunov function $V(t)$ is constructed as follows:

$$V(t)=\sum_{i=1}^{N}V_i(t).$$

Then, it follows from (17) that

$$\begin{aligned} \dot{V}(t) &\le \sum_{i=1}^{N}(-k|S_i(t)|) \\ &= -K|S(t)|\le 0 \end{aligned} \tag{18}$$

where

$$\begin{aligned} K &= [k,k,\cdots,k]_{1\times N}>0 \\ |S(t)| &= [|S_1(t)|,|S_2(t)|,\cdots,|S_N(t)|]^T_{1\times N}. \end{aligned}$$

Since $V(t)$ is bounded for all times, consequently, $\tilde{\overline{w}}_i(t)$ and $\tilde{\underline{w}}_i(t)$ are also bounded.

Then, $\dot{V}(t)$ becomes

$$\dot{V}(t)\le-K|S(t)|=-\psi(t)\le 0, \tag{19}$$

where $\psi(t)=K|S(t)|\ge 0$. Integrating (19) from zero to t yields

$$V(0)-V(t)\ge\int_0^t\psi(\tau)d\tau. \tag{20}$$

Using the fact $\dot{V}(t)\le 0$ in (18), $V(0)-V(t)\ge 0$ is positive and bounded, hence $\lim_{t\to\infty}\int_0^t\psi(\tau)d\tau$ exists and is bounded. Then, according to the Lemma 1, it can be concluded that

$$\lim_{t\to\infty}\psi(t)=\lim_{t\to\infty}K|S(t)|=0. \tag{21}$$

Since K is positive, (21) implies $\lim_{t\to\infty} S(t) = 0$. Then, according (4) and (6), $s_i(t)$ and the spacing error $e_i(t)$ can converge to zero in a finite time.

Finally, the string stability of the whole vehicle platoon system is proved. Since $S_i(t) = qs_i(t) - s_{i+1}(t)$ converges to zero in a finite time, we can get the relationship

$$q(\dot{e}_i(t) + \lambda e_i(t)d) = \dot{e}_{i+1}(t) + \lambda e_{i+1}(t). \tag{22}$$

Since $e_i(0) = 0$ and $\dot{e}_i(t)|_{t=0} = 0$, taking Laplace transform of (22), we can get

$$q(sE_i(s) + \lambda E_i(s)) = sE_{i+1}(s) + \lambda E_{i+1}(s).$$

Then, $G_i(s) = \frac{E_{i+1}(s)}{E_i(s)} = q$. Therefore, to meet the string stability requirement, according to Lemma 2, let $0 < |q| \leq 1$ to guarantee $\|G_i(s)\| \leq 1$. Then, string stability can be guaranteed. The proof is completed. ■

Remark 1: It is worth mentioning that the upper and lower bounds of external disturbances in the protocol (7) are adaptively updated, which is different from that in [7], [20].

Remark 2: The sign function is a high frequency function, which may cause chattering phenomenon in practical implementation. To reduce the chattering phenomenon, a sigmoid-like function $\frac{S_i(t)}{\|S_i(t)\|+\sigma}$ [21], [22] replacing of sign function is used, where σ is a small positive constant. On the other hand, note that $\mu_i(t)$ in (10) is an indicator function, which is a switching function and may also cause chattering phenomenon. To attenuate this chattering phenomenon, a smooth continuous function is introduced as follows.

$$\bar{\mu}_i(t) = \frac{1}{1+e^{-a(S_i(t)-b)}}$$

Fig. 2 shows the effect of different values of the parameters a and b to $\bar{\mu}_i(t)$. Observer that from Fig. 2, (10) is most likely to be satisfied by choosing bigger value for a and smaller value for b. Therefore, $\bar{\mu}_i(t)$ can be seen as a smooth approximation of μ_i by choosing bigger value for a and smaller value for b. Then, $u_i(t)$ can be rewritten as

$$\begin{aligned} u_i(t) &= -m_i[(1-\bar{\mu}_i(t))\hat{\overline{w}}_i(t) + \bar{\mu}_i(t)\hat{\underline{w}}_i(t)] + \frac{m_i}{q+1}A_i(t) \\ &\quad + \frac{km_i}{q+1}\frac{S_i(t)}{\|S_i(t)\|+\sigma}, \text{for } i = 1,2,\cdots,N-1; \\ u_N(t) &= -m_N[(1-\bar{\mu}_N(t))\hat{\overline{w}}_N(t) + \bar{\mu}_N(t)\hat{\underline{w}}_N(t)] \\ &\quad + \frac{m_N}{q}A_N(t) + \frac{km_N}{q}\frac{S_N(t)}{\|S_N(t)\|+\sigma} \end{aligned} \tag{23}$$

where

$$\bar{\mu}_i(t) = \frac{1}{1+e^{-a(S_i(t)-b)}}$$

and $A_i(t)$ $(i = 1,2,\cdots,N)$ is as defined in (8).

IV. Numerical Examples

In this section, the proposed control strategy is applied to a leader-following vehicle platoon with six vehicle followers and a leader. The initial position of the leader is set as $x_0(0) = 20m$, $v_0(0) = 1m/s$ and the desired trajectory is given by

$$v_0(t) = \begin{cases} 1\, m/s, & 0 \leq t < 2s, \\ 0.5t\, m/s, & 2s \leq t < 6s, \\ 3\, m/s, & otherwise. \end{cases}$$

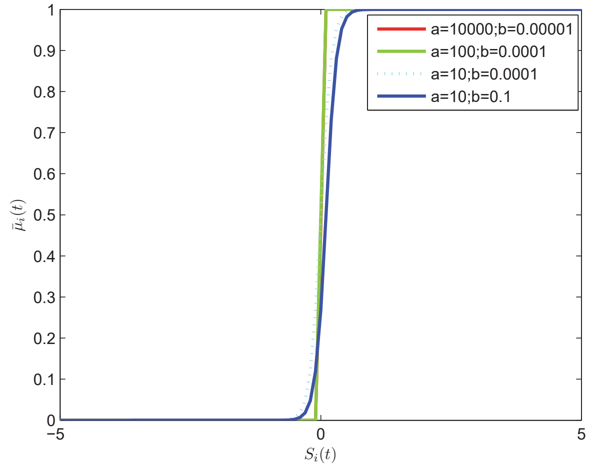

Fig. 2. Graph of $\bar{\mu}_i(t)$ for different values of a and b.

Obviously, the required string velocity is $3m/s$ to be achieved with a desired spacing $\delta_i = 1m$ for all $i = 1,2,\cdots,6$. Then, the final desired spacing and velocity are $1m$ and $3m/s$, respectively. The disturbances $w_i(t)$ are assumed as $w_i(t) = 1.5\sin(3t)e^{\frac{-(t-5-0.2i)^2}{4}}, i = 1,2,\cdots,6$, which are shown in Fig. 3. In numerical simulations, let $m_i = 1kg$ $(i = 1,2,\cdots,6)$, and all controller's parameters are chosen as $k = 3$, $q = 0.9$, $\lambda = 0.2$, $\eta_i = 0.01$, $\zeta_i = 10$, $\sigma = 0.3$, $a = 10$ and $b = 0.0001$. The initial states of $\hat{\overline{w}}_i(0)$ and $\hat{\underline{w}}(0)$ are chosen as $\hat{\overline{w}}_i(0) = 1.5$ and $\hat{\underline{w}}_i(0) = -1.5$, respectively. The initial spacing and velocity errors are set to zero by choosing $x(0) = [19,18,17,16,15,14]$ and $v(0) = [1,1,1,1,1,1]$. The simulations results are shown in Fig. 4. From the simulation results of Fig. 4, it shows that the proposed control law (23) works well. Fig. 4(a) shows that the spacing errors $e_i(t)$ converge to zero in a finite time and the string stability of the whole vehicle platoon is guaranteed since the amplitude of the pacing error decreases through the string of vehicles (i.e., $\|e_6(t)\| \leq \|e_5(t)\| \leq \cdots, \leq \|e_1(t)\|$). Fig. 4(b) shows the collision can be avoided. It can be seen from Fig. 4(c) that the spacings of all consecutive vehicles converge to a constant spacing $1m$ and the velocities of the vehicles follow the trajectory of the leader as shown in Fig. 4(d). The control input $u_i(t)$ and sliding surface $S_i(t)$ are shown in Figs. 4(e) and (f), respectively. Fig. 4(f) shows that the sliding modes $S_i(t)$ reach the surface $S_i(t) = 0$ in a finite time and the chattering phenomenon is almost eliminated fully. The adaptive updated states $\hat{\overline{w}}_i(t)$ and $\hat{\underline{w}}(t)$ are presented in Figs. 4(g) and (h), which show the estimates converge to constants. The above simulation results illustrate the effectiveness of the proposed control strategy (23).

V. Conclusion

This paper presents a new distributed control strategy by combining adaptive compensation technique with SMC approach to achieve the finite time stability of each vehicle and the string stability of the whole vehicle platoon subject to external disturbances. The adaptive control law does not require the knowledge of maximum and minimum values of the external disturbances in advance. CS policy known as high traffic density is applied to guarantee the string stability of the leader-following vehicle platoon. In addition, the chattering phenomenon caused by SMC approach is reduced by introducing continuous approximations. However, the drawback of the proposed control strategy is that the initial spacing and velocity errors are assumed to be zero. Future work will be directed to remove this assumption.

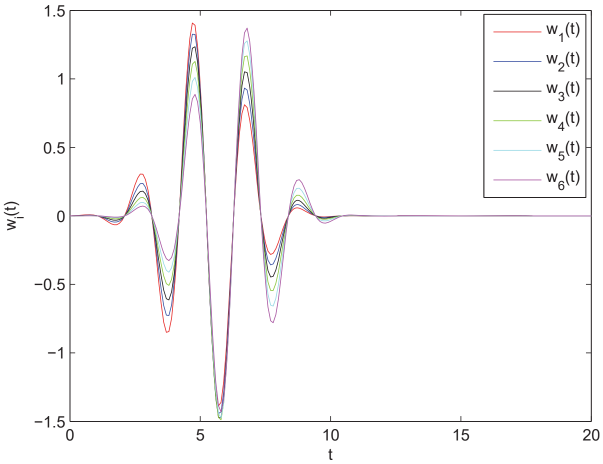

Fig. 3. The disturbance $w_i(t)$.

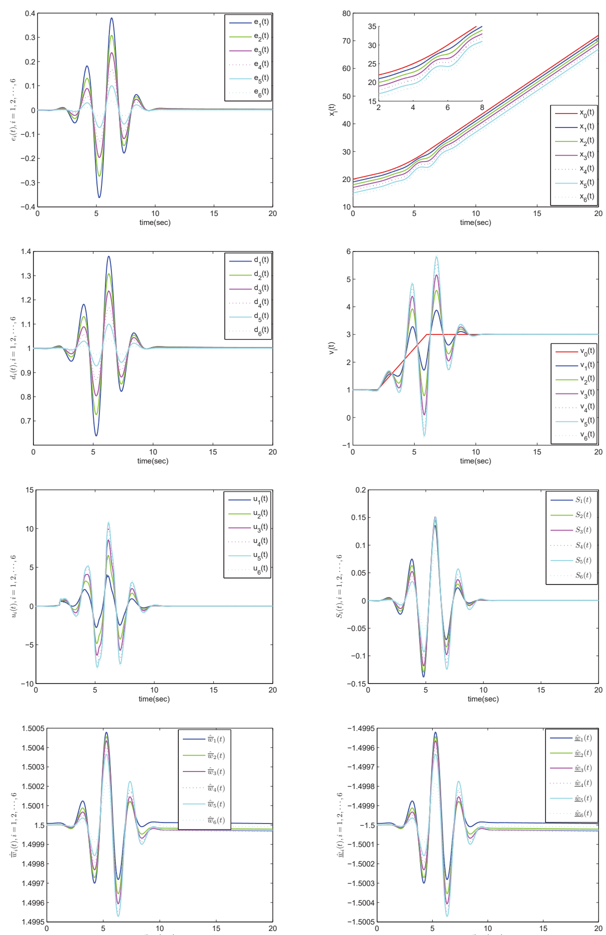

Fig. 4. (a) Spacing error $e_i(t)$; (b) Position $x_i(t)$ (c) Distance $d_i(t)$; (d) Velocity $v_i(t)$; (e) Control input $u_i(t)$; (f) Sliding Surface $s_i(t)$; (g) Upper bound estimation $\hat{\bar{w}}_i(t)$; (h) Lower bound estimation $\hat{\underline{w}}_i(t)$ by using (23).

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