String Stability of a Leader Following Formation Control with Dynamic Weights

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***Abstract*—In this work, we study the string stability properties of a leader following formation control architecture that uses non homogeneous weights for the leader and predecessor vehicle states. The architecture was presented recently [1] and it was shown to achieve constant inter-vehicle spacings (with no transient) for almost every vehicle pair when there are no disturbances at the followers. We expand the analysis of this interconnection by obtaining a condition on the design parameters that ensures the string stability of the interconnection to disturbances at any follower. Numerical simulations illustrate our results.**

I. Introduction

Control of autonomous vehicles has received an important amount of attention in recent decades [2], [3], [4], [5], [6], [7]. The importance of this topic is large, given the possible benefits in applications such as automated highway systems [8]. In particular, the simple case of a 1-D platoon of linear vehicles, has motivated several works considering diverse alternatives to achieve coordinated movement of a string of vehicles (see [9], [10] and the references therein).

One simple control strategy is to equip every vehicle in the formation with a controller that stabilizes its position in closed loop. Then, the position of its predecessor is used as a reference in addition to a desired inter-vehicle spacing. With the use of integral feedback control, this approach obtains tight formations in steady state, when the lead vehicle travels at a constant speed. Other approaches consider using the states of the lead vehicle and/or the states of other members of the formation (see for example [11], [12], [13], [14], [15] and the references therein).

Among the important properties of a formation control architecture we have the stability of the full interconnected system. An important work related to this was presented in [16], where the authors study the effect of the information flow in vehicle formations. The eigenvalues of the graph Laplacian matrix play an important role when determining stability. Other key aspects in the control architecture are those of performance and safety. In works such as [9], [17], [18] it can be seen how in interconnections that are stable, disturbances may be amplified along the string of vehicles, producing issues in the formation performance or even collisions (string instability).

The goal of achieving a tight formation using these types of architectures is motivated by the possibility of reducing fuel consumption, by means of decreasing drag in real applications (See for example [19] and the references therein). It was shown in [17], that the most reasonable way to achieve string stability and a tight formation in steady state, which is commonly known as using a constant spacing policy, is to broadcast the leader state to every follower. This is the main motivation for the study of leader following architectures. If a tight formation is not required, a constant time headway spacing policy, where the inter vehicle spacings increase with the speed of the platoon, can be used in order to achieve string stability [10].

The present work deals with a leader following unidirectional control architecture where the first vehicle moves independently; in such events a follower does not detect or manoeuvre in response to disturbances that affect members behind it. Moreover, the main goal of the architecture is to achieve constant inter-vehicle spacings (with no transients) whenever possible. This is the most attractive quality of the interconnection. We build from the results presented in [1] where a design procedure for a control architecture achieving this tightness property was obtained. The main contribution of this paper is to provide conditions on the design parameters of the architecture that will ensure string stability. This findings enable the design of a degree of safety and performance while achieving a train-like behaviour in nominal operation of the platoon. Moreover, the results are given in terms of arbitrary models and controllers, enabling the analysis of string stability properties of interconnected systems different from vehicles.

The paper is presented as follows. Section II provides notation and describes the leader following formation control scheme. Section III reviews the results on which the current work is based, mainly the architecture dynamics, connecting disturbances at the followers with inter-vehicle spacings. In Section IV we perform the string stability analysis and derive a condition on the design parameters for string stability. This is the main contribution of this work. Some numerical examples and comments on the main results are shown in section V. Section VI contains some conclusions and lines of future work.

II. Framework and Problem Formulation

A. Notation

The notation used in this paper follows much of the standard literature of systems and control. Lower-case letters are used for real scalar signals, $x : \mathbb{R} \to \mathbb{R}$ with specific values of the signal denoted by $x(t)$. Upper-case letters are used for scalar complex-valued Laplace transforms of signals and transfer functions, $X : \mathbb{C} \to \mathbb{C}$ with specific values denoted by $X(s)$. For the sake of brevity in the notation, where there is no confusion, the argument (s) will be omitted. Vectors will

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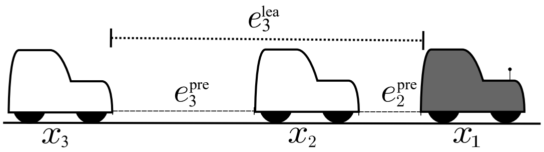

Fig. 1. Small vehicle platoon. x_i: position of the i-th vehicle. $e_i^{pre} = x_{i-1} - x_i$: inter-vehicle spacing between the $(i-1)$-th and i-th. $e_i^{lea} = x_1 - x_i$: spacing between the i-th and 1-st. vehicles.

be denoted as $\underline{x}(t) \in \mathbb{R}^n$ and $\underline{X} \in \mathbb{C}^n$, while $\underline{x}(t)^\top$ and $\underline{X}^\top$ denote their transposes. The imaginary unit is denoted by j, with $j^2 = -1$. Boldface will be used for matrices $\boldsymbol{G} \in \mathbb{C}^{n \times m}$ and the (i,k)-th entry of $\boldsymbol{G}$ is denoted by $G_{i,k}$. The magnitude of X when $s = j\omega$, $\omega \in \mathbb{R}$, is denoted by $|X|$ and its magnitude peak over all possible values of ω is denoted as $||X||_\infty := \sup_\omega |X(j\omega)|$.

B. Vehicle models and control strategy

As presented in [1], we consider a platoon of $N \in \mathbb{N}$ vehicles, with positions $x_i(t), 1 \leq i \leq N$ modeled by linear time invariant systems (See Fig. 1). In the frequency domain, the model of each member of the platoon is given by (omitting initial conditions)

$$X_i = H_i(U_i + D_i) \quad \text{for } 1 \leq i \leq N, \tag{1}$$

where X_i denotes the Laplace transform of $x_i(t)$, U_i is the control action and D_i is an input disturbance. U_i and D_i are both acting on the i-th member. The transfer functions H_i are rational, have a single pole at the origin and are strictly proper (i.e. the denominator has a strictly higher degree than the numerator). Now, we denote the separation errors, for $i > 1$, as

$$e_i^{pre}(t) = x_{i-1}(t) - x_i(t) - \varepsilon_i, \tag{2}$$

$$e_i^{lea}(t) = x_1(t) - x_i(t) - \sum_{j=2}^{i} \varepsilon_i, \tag{3}$$

which can be seen in Fig. 1, and using the Laplace transform,

$$E_i^{pre} = X_{i-1} - X_i - \frac{\varepsilon_i}{s}, \tag{4}$$

$$E_i^{lea} = X_1 - X_i - \sum_{j=2}^{i} \frac{\varepsilon_i}{s}, \tag{5}$$

where the $\varepsilon_i > 0$ are desired inter-vehicle spacing constants. The error E_i^{pre} is the inter-vehicle spacing between the i-th member of the platoon and its predecessor, while the E_i^{lea} is the spacing between the i-th member and the leader.

Remark: The requirement of every follower possessing their relative distance to the leader is quite strong in a practical setting. In that sense, the following results are of theoretical value. The use of the leader velocity is a more reasonable selection (see for example [20]) and is the subject of ongoing research.

For simplicity in the exposition we will assume that ε_i, and the initial conditions $x_i(0)$ and $\dot{x}_i(0)$ for $i = 1, \ldots, N$ are compatible, that is, such that $e_i(0) = 0$ for $i = 1, \ldots, N$. In this way, we only study the effect of the disturbances D_i on the separation errors E_i^{pre}. In particular, and without loss of generality, we will set $\varepsilon_i = 0$ for all i and set every initial condition to zero. Moreover, we define the usual complementary sensitivity functions as

$$T = \frac{HC}{1 + HC}, \quad S = 1 - T, \tag{6}$$

Now we define the control actions

$$\begin{aligned} U_2 &= C_2 E_2^{pre} \\ U_i &= C_i(\eta_i E_i^{pre} + (1 - \eta_i) E_i^{lea}), \end{aligned} \tag{7}$$

where U_i is the output of the compensator C_i at the i-th car and η_i are stable transfer functions to be designed for $i > 2$. With these selections, every member of the string aims to track the position of the leader, while maintaining a safe distance with respect to its immediate predecessor.

In typical leader following settings the weights η_i are such that $\eta_i = \eta$ for all i with $\eta \in (0, 1)$. The standard reason for this selection is to achieve a bounded propagation of disturbances along the string (string stability, see for example [9], [20]).

According to the results presented in [1], for the homogeneous case, where every vehicle has the same dynamics and local controller ($H_i = H$ and $C_i = C$) the selections

$$\eta_k = \frac{\eta_3}{1 + \eta_3 T}, \quad k \geq 4, \tag{8}$$

yield dynamics that achieve almost a train like motion for disturbances at the leader (that is $E_i \equiv 0$ for $i > 3$, when $D_i \equiv 0$ for $i > 1$), which is the main property of this interconnection.

For movements of the leader, these selections allow a response of the vehicle platoon that resembles the movement of a train (with the exception of the first two followers). It must be noted that the selection of η_3 can be any stable transfer function and it must be selected in order that $\eta_3/(1 + \eta_3 T)$ is also stable.

The selection of the sequence $\{\eta_k\}$ was made with the aim of obtaining $E_k^{pre} = 0$ for all s whenever possible for disturbances (or movement) at the leader. In the heterogeneous case, the selections

$$\eta_k = 1 - \frac{\tilde{T}}{H_k C_k (1 - \tilde{T})}, \quad k \geq 4, \tag{9}$$

with

$$\tilde{T} = T_3(1 - \eta_3 + \eta_3 T_2), \quad T_k = \frac{H_k C_k}{1 + H_k C_k}. \tag{10}$$

achieve the same behaviour for disturbances at the leader.

These selections may not provide a satisfactory response of the vehicle string for disturbances at the followers. The main contribution of this work is to assess the string stability of the interconnection described above for the homogeneous case. The heterogeneous case is the subject of ongoing research.

III. Dynamics of the Interconnected System

The interconnection defined by the control signals in (7) can be described by the transfer function from the disturbances $\underline{D} = [D_1 \ \cdots \ D_N]^\top$ to the positions $\underline{X} = [X_1 \ \cdots \ X_N]^\top$:

$$\underline{X} = (\boldsymbol{I} - HC\boldsymbol{G})^{-1} H\underline{D}, \tag{11}$$

where $\boldsymbol{I}$ is the $N \times N$ identity matrix and $\boldsymbol{G}$ is the matrix:

$$\boldsymbol{G} = \begin{bmatrix} 0 & & & & \\ 1 & -1 & & & \\ 1-\eta_3 & \eta_3 & -1 & & \\ \vdots & & \ddots & \ddots & \\ 1-\eta_N & & \dots & \eta_N & -1 \end{bmatrix}. \tag{12}$$

The term $G_{1,1} = 0$ implies that the first vehicle moves independently and its position is given by

$$X_1 = HD_1. \tag{13}$$

Although we have homogeneous vehicles, we consider η_i to be dynamic and arbitrary for $i > 2$. Then, we have

$$\underline{X} = \begin{bmatrix} S & & & & \\ -T & 1 & & & \\ (\eta_3 - 1)T & -\eta_3 T & 1 & & \\ \vdots & & \ddots & \ddots & \\ (\eta_N - 1)T & & & -\eta_N T & 1 \end{bmatrix}^{-1} SH\underline{D}. \tag{14}$$

Since the matrix to be inverted is lower triangular, we can easily express the dynamics of the vehicle positions.

The following lemma, taken from [21] gives expressions for the transfer functions from disturbances D_i to spacing errors E_i^{pre}.

***Lemma** 1:* The spacing errors defined by

$$\begin{bmatrix} E_2^{pre} \\ \vdots \\ E_N^{pre} \end{bmatrix} = \begin{bmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & -1 \end{bmatrix} \underline{\mathrm{X}}, \tag{15}$$

are given by

$$\underline{\mathrm{E}} = \begin{bmatrix} 1 \\ \mathcal{T}_2 \\ \vdots \\ \mathcal{T}_{N\text{-}1} \end{bmatrix} SHD_1 + \begin{bmatrix} -1 & & & \\ \mathcal{S}_{2,1} & -1 & & \\ \vdots & \ddots & -1 & \\ \mathcal{S}_{N,1} & \dots & \mathcal{S}_{N\text{-}1,N\text{-}2} & -1 \end{bmatrix} SH \begin{bmatrix} D_2 \\ \vdots \\ D_N \end{bmatrix}. \tag{16}$$

where $\mathcal{T}_k$ satisfies

$$\mathcal{T}_n = \eta_{n+1}T + (\eta_{n+1}T - 1)\sum_{i=3}^{n} T^{n-i+1}\prod_{j=i}^{n}\eta_j, \tag{17}$$

and $\mathcal{S}_{k,n}$ is given by

$$\mathcal{S}_{n,k} = (1 - \eta_{n+1}T)\left(\prod_{i=k+2}^{n}\eta_i\right)T^{n-k-1}, \tag{18}$$

for $k \le n-2$, and $\mathcal{S}_{n,k} = 1 - \eta_{n+1}T$ for $k = n-1$.

Proof: This follows in a straightforward fashion after the computation of $(\boldsymbol{I} - HC\boldsymbol{G})^{-1}$. ∎

Note that if $\eta_3 = \eta_4 = \ldots = \eta_N = \tilde{\eta}$, then $\mathcal{T}_k = (\tilde{\eta}T)^{k-1}$. This is in agreement with results presented in [9].

IV. String Stability Analysis

In this section we present the main contributions of the paper. In particular, we provide conditions on the design parameters that ensure string stability of the interconnection. String stability will be considered as in the following definition [20]:

***Definition** 1:* Let $\{F_n\}$ be a sequence of stable transfer functions. The sequence will be called *string stable* if there exists $c \in \mathbb{R}$, independent of $n \in \mathbb{N}$ such that $||F_n||_\infty \le c$ for all n. It will be called *string unstable* otherwise.

We let $F_{i,k}$ be the transfer function from the k-th disturbance to the i-th predecessor error, that is

$$E_i^{pre} = F_{i,k}D_k. \tag{19}$$

Moreover, we let $\{\mathcal{F}_n\}(k)$ be the sequence of transfer functions generated for a fixed k, that is

$$\{\mathcal{F}_n\}(k) = \{F_{1,k}, F_{2,k}, \ldots\}. \tag{20}$$

By construction of the interconnection, $F_{i,1} = 0$ for $i > 3$, and by the previous definition, the sequence $\{\mathcal{F}_n\}(1)$ would be string stable.

Using Lemma 1, we have that for $k > 1$

$$F_{n,k} = \begin{cases} 0 & k > n, \\ -SH & k = n, \\ SH(1 - \eta_{n+1}T) & k = n-1, \\ SH(1 - \eta_{n+1}T)\left(\prod_{i=k+2}^{n}\eta_i\right)T^{n-k-1} & k \le n-2, \end{cases} \tag{21}$$

with

$$\eta_i = \frac{\eta_3}{1 + \eta_3 T} \quad i \ge 4. \tag{22}$$

The following result gives a condition for the string stability of the sequences $\{\mathcal{F}_n\}(k)$ for $k \ge 2$.

***Theorem** 1:* Consider the sequence of transfer functions $\{\mathcal{F}_n\}(k)$ given by (20). If

$$\left|\frac{\eta_3 T}{1 + \eta_3 T}\right| \le 1, \text{ for all } \omega, \tag{23}$$

then $\{\mathcal{F}_n\}(k)$ is string stable. Otherwise, the sequence is string unstable.

Proof: Replacing the proposed η_i in the last case of (21) yields the product $(\eta_k T)^n$ for $k \ge 4$. This in turn translates into studying the magnitude

$$|\eta_k T| = \left|\frac{\eta_3 T}{1 + \eta_3 T}\right|. \tag{24}$$

According to Definition 1 the sequence is string stable whenever we can find $c > 0$ such that $||F_{n,k}||_\infty < c$ for all n. This will occur if

$$\left|\frac{\eta_3 T}{1+\eta_3 T}\right| \le 1, \forall\omega.$$

On the contrary, if

$$\left|\frac{\eta_3 T}{1+\eta_3 T}\right| > 1,$$

$||F_{n,k}||_\infty$ cannot be bounded independently of n since $|\eta_k T|^n$ would have and ω where it would grow with n. Therefore the sequence would be string unstable. ■

Theorem 1 gives a condition on the design parameters η_3 and C that ensures the string stability of the interconnection. The derivation of more explicit expressions for the critical value of η_3 in terms of the vehicle models and controllers is the subject ongoing research. We can mention that a converse of Rouche's Theorem [22] might be involved in the general case.

V. Numerical Examples and Discussion

We consider a homogeneous string with parameters (see [9])

$$H = \frac{1}{s(0.1s+1)}, \quad C = \frac{2s+1}{s(0.05s+1)}. \tag{25}$$

If we consider $\eta_3 = 0.5$ we have that

$$T = \frac{400s+200}{s^4+30s^3+200s^2+400s+200}, \tag{26}$$

$$\eta_k = \frac{1}{2+T} = \frac{0.5s^4+15s^3+100s^2+200s+100}{s^4+30s^3+200s^2+600s+300}, \tag{27}$$

for $k > 3$, which are stable for the particular selection of C and H. For a step input to the lead vehicle, $d_1(t) = \mu(t-1)$ we obtain the transient response shown in Fig. 2. All the inter-vehicle spacings $e_i^{pre}(t)$ are 0 for $i > 3$ as desired and imposed in the derivation of the sequence η_k.

In Fig. 3 we can see the time response of the platoon to a disturbance at the second vehicle $d_2(t) = \mu(t-1)$. It can be seen that $e_2^{pre}(t)$ is negative. This corresponds to the second vehicle getting closer to its predecessor as a consequence of the disturbance $d_2(t)$. The other inter-vehicle spacings along the string have magnitude peaks that are smaller than the peak for $|e_2^{pre}(t)|$. This suggests that the sequences of transfer functions from disturbances at the followers to other members of the platoon are string stable. In particular we can compute

$$\left\|\frac{0.5T}{1+0.5T}\right\|_\infty = 0.3897, \tag{28}$$

which agrees with Theorem 1.

If we consider $\eta = 5$, we have that

$$\left\|\frac{5T}{1+5T}\right\|_\infty = 2.1356, \tag{29}$$

and we should expect string instability. In Fig. 4 we have the time response for a step disturbance $d_2(t) = \mu(t-1)$ at the second member. It can be noted that the disturbances get amplified along the string, confirming the string instability

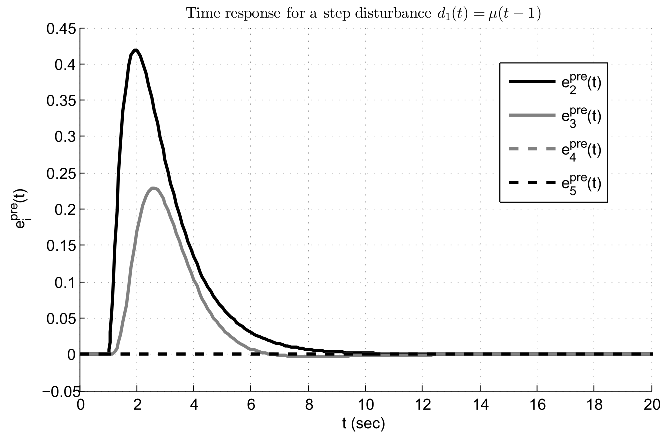

Fig. 2. Time response of the inter -vehicle spacings with a step disturbance at the leader $d_1(t) = \mu(t-1)$ for a platoon with non-homogeneous weights η_k. $\eta_3 = 0.5$

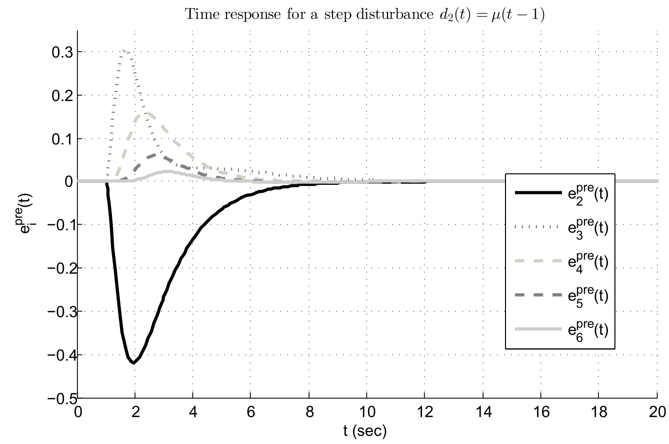

Fig. 3. Time response of the inter-vehicle spacings with a step disturbance at the second member $d_2(t) = \mu(t-1)$ for a platoon with non-homogeneous weights η_k. $\eta_3 = 0.5$

of the interconnection for the choice $\eta_3 = 5$. Fig. 5 shows the value of $\left\|\frac{\eta T}{1+\eta T}\right\|_\infty$ as a function of η for the given values of the vehicle model and controller. It can be seen that there is a full interval of η (even negative values) where the magnitude of $|\frac{\eta T}{1+\eta T}|$ is less or equal than 1.

VI. Conclusions and Future Works

In this work we obtained a condition for the string stability of a unidirectional strategy for the formation control of a string of vehicles. The main property of this architecture is that vehicles can use filtered measurements of the leader and their immediate predecessor in order to obtain a quasi train-like behaviour in nominal operation.

The condition for the string stability of the interconnection is given in terms of the design parameters. It is also straightforward to evaluate for a particular design choice. However, the derivation of more detailed expressions connecting the

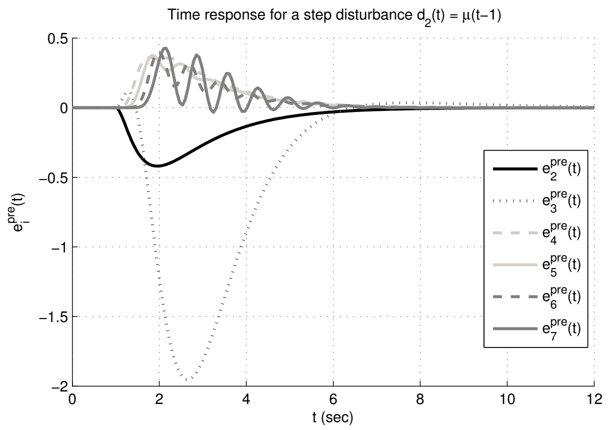

Fig. 4. Time response of the inter-vehicle spacings with a step disturbance at the second member $d_2(t) = \mu(t-1)$ for a platoon with non-homogeneous weights η_k. $\eta_3 = 5$

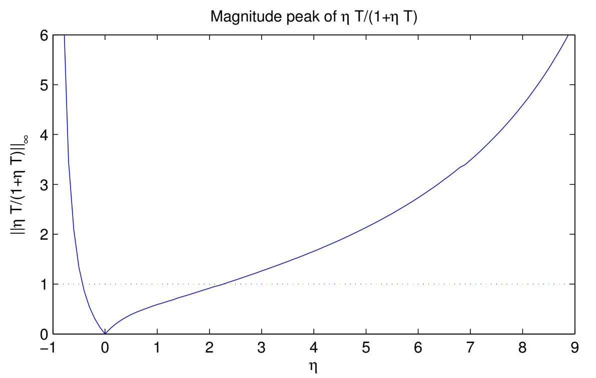

Fig. 5. Magnitude peak of $\eta T/(1+\eta T)$ as a function of η.

parameters and the data of the problem is the subject of ongoing research.

A noteworthy feature of the proposed control strategy is that the vehicles do not need to know all the characteristics of every predecessor but only of the first few vehicles. This allows for reduced coordination requirements and the possibility for new vehicles to merge in the platoon in a simplified fashion.

One immediate line of work is to consider the broadcast of the leader velocity instead of its position, in order to remove the strong coordination requirement of every follower knowing its position. Other lines of work aim to study disruptions of the leader state that must be transmitted to the followers and to consider the effect of parameter uncertainty on the stability properties.

Acknowledgment

This work was supported by the grant FONDECYT 3160738, CONICYT Chile. The author would also like to thank the support of the Advanced Center for Electrical and Electronic Engineering, AC3E, Basal Project FB0008.

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