Equalization techniques in MIMO systems - An Analysis

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***Abstract*— One of the important techniques, which can combat and exploit the frequency-selectivity of the wireless channel in providing high data rate wireless systems, lies in the hand of equalization. The wireless systems that is evolved the last few decades necessitates in the design and analysis of equalization techniques. The current trend as well as the future generation of wireless systems is supposed to possess very high spectral efficiency, presence of rapidly time varying channels due to high mobility and the availability of partial or no channel state information at the transmitter and/or receiver which prescribes MIMO systems. When the data is transmitted at high bit rates, over mobile radio channels, the channel impulse response can extend over many symbol periods, which lead to Inter-symbol interference (ISI). This paper analyses the performances of equalization techniques by considering 2 transmit and 2 receive antenna (2×2 MIMO) over a flat fading Rayleigh multipath channel and provides a bit error rate analysis of the same.**

***Index Terms*— Zero Forcing, Minimum Mean Square Error, ML equalization, MIMO, BER**

I. INTRODUCTION

The recent developments in coding and signal processing drags the new origin in equalization strategies for both single and multi-carrier based wireless communication systems. The higher data rates that are required by data demanding applications are provided by Multiple Input Multiple Output (MIMO) systems.[1] The requirement for high data rates results in significant inter-symbol interference for single carrier based systems, which emphasis the use of suitable equalization strategies. There are several ways to satisfy the above said condition, which includes iterative (turbo) techniques using soft-input soft-output (SISO) signal processing algorithms. Even though turbo equalization is a computationally efficient, the complexity of the SISO algorithms used in every iteration becomes very large when the data rates and the number of transmit antennas increase. On the other hand designing broadband MIMO systems, this problem becomes more and more important as the channel impulse response is characterized by a large number of parameters. Hence, there is a need for the development of novel practical, low complexity equalization techniques and for understanding their potentials and limitations when used in wireless communication systems characterized by very high data rates, high mobility and the presence of multiple antennas.

The higher data rates that are required by data demanding applications are provided by Multiple Input Multiple Output (MIMO) systems. The standard method for achieving spatial diversity to combat fading without expanding the bandwidth of the transmitted signal is using multiple antennas at the receiver. Spatial diversity can also be achieved by employing multiple antennas at the transmitter side. Multiple transmitting antennas can also be used to create multiple spatial channels and thus provide the capability to increase the data rate. In particular with N transmitting antennas and M receiving antennas, it is possible to achieve an M-fold increase in the data rate and simultaneously provide *Mth* order reception diversity to combat fading for each of the N transmitted signals. As there is no orthogonal structure imposed on the signals at the N transmitting antennas, there will be Inter Channel Interference (ICI) among the spatial channels. The data is encoded and interleaved. The interleaver is necessary in the case of encoded data in order to ensure independent fading of the coded bits or symbols. A block of N symbols is converted from serial to parallel, and each symbol is fed to one of the identical modulators, where each modulator is connected to a spatially separated antenna. Thus, the N symbols are transmitted in parallel and received on M spatially separated receiving antennas where $M > N$. [2].

Another important issue to consider is estimation and tracking of time-varying wireless channels. Given the growing trend in designing broadband MIMO systems, this problem becomes more and more important as the channel impulse response is characterized by a large number of parameters. Also, the time-varying nature of the channel results in significant inter-carrier interference (ICI) which becomes poor factor for multi-carrier based systems. Hence, it is crucial to develop appropriate signal processing techniques to combat the effects of ICI at the receiver when multicarrier systems are used. Efficient design of precoding techniques at the transmitter and equalization techniques at the receiver can significantly enhance the performance of wireless systems. To mitigate the issues in equalization along with consideration of high peak-to-average power ratio of multicarrier systems, frequency diversity single carrier systems with frequency domain equalization and the associated channel estimation problems are focussed in many research activities. Present trend has grown towards the use of partial channel state information at the transmitter and jointly optimizing the

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transmitter/receiver in the presence of such channel state information with the futuristic considerations. Efficient design of precoding techniques at the transmitter and equalization techniques at the receiver can significantly enhance the performance of wireless systems. The focus of this paper is on the development of such novel practical, low complexity equalization techniques and understanding their potential and limitations when used in wireless communication systems characterized by very high data rates, high mobility and the presence of multiple antennas. Here an adaptive modulation technique for the same with base of BPSK scheme is simulated using QAM modulation scheme and compared their results. Further our work in this article is organized as follow. Section II describes the system model adopted. Section III gives the study and description of the equalization techniques. In Section IV, the simulation environment and results are described and Section V we concludes the paper.

II. MIMO SYSTEM MODEL

A. Mimo Space Time Coding Model

The MIMO-STC system implemented in this paper is based on Tarokh and Alamouti scheme.[4]. The binary information is fed to a BPSK modulator, which generates a constellation symbol. The Space Time Code encoder takes two constellation symbols 0 s and 1 s and transmits them from the two transmit antennas based on the table below:

TABEL I
TWO BRANCH TRANSMIT DIVERSITY SCHEME

	antenna0	antenna1
time t	S_0	S_1
time t+T	$-S_1^*$	S_0^*

The receiver obtains the signal back from its original data stream. The receiver is assumed to have the perfect knowledge of the channel to enable comparison of the several schemes

The overall system can be divided into three main modules: Transmitter, Channel. Receiver.

Transmitter Module:

The basic operation of the transmitter section of the system for a given SNR is as follows:

Step 1:Binary data is randomly selected and placed into packets

Step 2:The packets are converted into DPSK symbols (i.e. ±1)

Step 3:Symbols are then encoded and up sampled

Step 4:The signal is finally modulated and transmitted out of two separate antennas (in case of Multiple Inputs)

Channel Module:

The channel is assumed to be flat fading Rayleigh multipath channel.

Receiver Module:

The received signal may be expressed as

$$r_0 = r(t) = h_0.s_0 + h_1.s_1 + n_0$$
$$r_1 = r(t+T) = -h_0.s_1^* + h_1.s_0^* + n_1 \quad (1)$$

Where,

r_0 & r_1 – Received signals at time t & (t+T)

n_0 & n_1 – Noise & Interference.

The signal estimate $\hat{s}_0$ & $\hat{s}_1$ are calculated

$$\hat{s}_0 = h_0^*.r_0 + h_1.r_1^*$$
$$\hat{s}_1 = h_1^*.r_0^* - h_0.r_1^* \quad (2)$$

The received signal is followed by the Maximum Likelihood Detector, which recovers the original transmitted signal with a combiner stage.

III. EQUALIZATION

When the signal is passed through the channel, distortion is introduced in terms of amplitude and delay, which results with Inter Symbol Interference (ISI). ISI caused by multipath in band limited time dispersive channel distorts the transmitted data, causing bit errors at the receiver. ISI has been recognized as the major drawback in high speed data transmission over wireless channels. Hence, Equalizers combat ISI [7]. An equalizer is implemented at the baseband or at IF in a receiver. And the basic receiving technique of any communication lies with noise signal performance. Hence to suppress the noise component present in any communication, one should go for equalization technique. Since the baseband complex envelope expression can be used to represent band pass waveforms, the channel response, demodulated signal and adaptive equalizer algorithms are usually simulated and implemented at the baseband [3]. If x(t) is the original information signal and c(t) is the combined complex baseband impulse response of the transmitter, channel and RF/IF sections of the receiver, signal received by equalizer can be expressed as

$$y(t) = x(t) \otimes c^*(t) + n_b(t) \quad (3)$$

where $c^*(t)$ is the complex conjugate of $c(t)$, $n_b(t)$ is baseband noise at the input and $\otimes$ is convolution operation. If the impulse response of the equalizer is $h_{eq}(t)$, then the output of equalizer is

$$\hat{d}(t) = x(t) \otimes c^*(t)h_{eq}(t) + n_b(t) \otimes h_{eq}(t)$$
$$= x(t) \otimes g(t) + n_b(t) \otimes h_{eq}(t) \quad (4)$$

where $g(t)$ is the combined response of the transmitter, channel, RF/IF sections of the receiver, and the equalizer at the receiver. The complex base band impulse response of the transversal filter equalizer is given by

$$h_{eq}(t) = \sum_n c_n \delta(t - nT) \quad (5)$$

where c_n are the complex filter coefficients of the equalizer. The desired output of the equalizer is $x(t)$,the original source data. Assume that $n_b(t)$. Then, in order to force $\hat{d}(t) = x(t)$ in equation, $g(t)$ must be equal to

$$g(t) = c^*(t) \otimes h_{eq}(t) = \delta(t) \quad (6)$$

If the receiver satisfies the above said equation, then it is said to have achieved equalization.

A. Zero Forcing Algorithms:

Here the zero forcing equalizer co-efficient c_n are chosen to force the samples of the combined channel equalizer impulse response to zero. By increasing the number of co-efficient without bound, an infinite length equalizer with zero ISI at receiver can be achieved. When each of the delay elements provide a time delay equal to the symbol duration T, the frequency response of the channel is said to be periodic.

This Zero forcing algorithm was first designed by Lucky for wired Communication. ZF equalizer neglects the effect of noise & thus it is used mostly for wired communication.

In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1}\ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (7)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1}\ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (8)$$

where y_i represents received signal with noise n_i and i representing number of antennas. For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (9)$$

$$y = \boldsymbol{H}x + n \quad (10)$$

To solve for x, we know that we need to find a matrix $\boldsymbol{W}$ which satisfies $\boldsymbol{WH} = \boldsymbol{I}$. The Zero Forcing (ZF) linear detector for meeting this constant is given by,

$$\boldsymbol{W} = (\boldsymbol{H^H H})^{-1}\boldsymbol{H^H} \quad (11)$$

which is known as the pseudo inverse matrix for a general mxn matrix. Here the off-diagonal terms in the matrix $\boldsymbol{H^H H}$ are not zero . Hence, the zero forcing equalizer tries to null out the interfering terms when performing the equalization. But in performing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best choice of equalizer. However, it is simple and reasonably easy to implement [5]. For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\left(\frac{E_b}{N_o}\right)}{\left(\frac{E_b}{N_o}\right)+1}}\right) \quad (12)$$

B. Minimum Mean Square Error (MMSE) equalizer:

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient which minimizes the criterion,

$$E\left\{[\boldsymbol{W}_y - x][\boldsymbol{W}_y - x]^H\right\}$$

$$\boldsymbol{W} = [\boldsymbol{H}^H\boldsymbol{H} + \boldsymbol{N_o I}]^{-1}\boldsymbol{H}^H \quad (13)$$

When noise term is zero, MMSE equalizer reduces to ZF equalizer.

C. Zero Forcing with Successive Interference Cancellation:

The Zero Forcing (ZF) equalization approach can be extended to obtain an estimate of the two transmitted symbols, $\hat{x}_1$ and $\hat{x}_2$ expressed by,

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (14)$$

Here one of the estimated symbols say $\hat{x}_2$ is subtracted from the received vector y_1 and y_2 given by,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 & -h_{1,2}\,\hat{x}_2 \\ y_2 & -h_{2,2}\hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1}\,\hat{x}_1 & +n_1 \\ h_{2,1}\hat{x}_1 & +n_2 \end{bmatrix} \quad (17)$$

The equation obtained is same as for receive diversity case. Maximal Ratio Combining (MRC) can be applied for achieving optimal way of combining the information from multiple copies of the received symbols in receive diversity.

D. Successive Interference Cancellation with optimal ordering:

In an ideal Successive Interference Cancellation technique, the receiver arbitrarily takes one of the estimated symbols and subtract its effect from the received symbol y_1 and y_2. Since here estimation is done arbitrarily, prediction is impossible, hence it could be more useful, if the receiver is able to choose $\hat{x}_1$ or $\hat{x}_2$based on an optimal approach. [5]. To make optimal decision, let us find the transmit symbol, which is received with higher power at the receiver. The received power at transmitting and receiving antennas corresponding to the transmitted symbol x_1 is given by,

$$P_{x_1} = |h_{1,1}|^2 + |h_{2,1}|^2 \quad (15)$$

The received power at transmitting and receiving antennas corresponding to the transmitted symbol x_1 is given by,

$$P_{x_2} = |h_{1,2}|^2 + |h_{2,2}|^2 \quad (16)$$

If $P_{x_1} > P_{x_2}$, then the receiver decides to remove the effect of $\hat{x}_1$ from the received vector y_1 and y_2 and re-estimate $\hat{x}_2$.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_1, 1\,\widehat{x_1} \\ y_2 - h_1, 1\,\widehat{x_1} \end{bmatrix} = \begin{bmatrix} h_1, 2x_2 + n_1 \\ h_2, 2x_2 + n_2 \end{bmatrix} \quad (17)$$

$$r = hx_2 + n \quad (18)$$

Maximal Ratio Combining approach is used at the receiver to find an optimal solution, when the information is combination of multiple copies of received symbols in receive diversity. The equalized symbol is for x_1 and x_2,

$$\bar{\bar{x}}_1 = \bar{\bar{x}}_2 = \frac{\boldsymbol{h^H r}}{\boldsymbol{h^H r}} \quad (19)$$

Performing Successive Interference Cancellation with optimal ordering ensures that the reliability of the first decoded symbol with lower probability of error than the other symbols.

E. Minimum Mean Square Error (MMSE) equalizer with Successive Interference Cancellation:

The above said concept of Successive Interference Cancellation is extended to the MMSE equalization by adapting shift keying modulation in flat fading Rayleigh multipath channel. The MMSE approach tries to find a co-efficient $\boldsymbol{W}$ which minimizes the criterion,

$$E\left\{[\boldsymbol{W}_y - x][\boldsymbol{W}_y - x]^H\right\} \quad (20)$$

Solving the above results with estimated value for $\boldsymbol{W}$ given by

$$\boldsymbol{W} = [\boldsymbol{H}^H\boldsymbol{H} + N_0\boldsymbol{I}]^{-1}\boldsymbol{H}^H \quad (21)$$

Hence the receiver obtain an estimate of the two transmitted symbols x_1 , x_2 as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (H^H H + N_0 I)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (22)$$

IV. SIMULATED RESULTS

In this section, computer simulated results is provided to illustrate the performance of Equalization in MIMO. The simulation parameter chosen is tabulated below.

TABEL II
SIMULATION PARAMETERS

Parameter	**Specifications**
Number of Transmit Antennas	2
Number of receive Antennas	2
Modulation Scheme	BPSK,QAM
Channel Model	Rayleigh,AWGN
Equalizer	ZF,ML,MMSE, MMSE-SIC

A. The BER performance of the ZF –MIMO Equalizer:

The Zero forcing equalizer is not much suitable. To say, it achieves diversity but doubles the data rate. It is clear from the following zero forcing equalization, the channel for symbol transmitted from each spatial antenna is similar to a 1×1 Rayleigh fading channel. Hence it is not much suitable equalizer though very much simpler than all other equalizers.

B. The BER performance of ML Detector:

The results for 2×2 MIMO with Maximum Likelihood (ML) equalization obtained is closely matching the 1X2 antenna of Maximum Ratio Combining (MRC) type. Of all the equalizer types Maximum Likelihood offers better BER. The ML equalizer is optimal since it minimizes the probability of a sequence error. Further, this equalizer requires knowledge of the channel characteristics and statistical distribution of noise corrupting the signal to compute the metrics for decision making.

C. The BER performance of MMSE–MIMO Equalizer:

The results for 2×2 MIMO MMSE equalization shows a 3 dB improvement when compared to zero forcing equalizer. When noise term is zero, MMSE equalizer reduces to ZF equalizer.

D. The BER performance of MMSE-SIC MIMO Equalizer:

The results shows that compared to simple Minimum Mean Square (MMSE) Equalization with Successive Interference Cancellation case, results in around 5.0dB of improvement for BER of 10^{-3} is achieved. Performing Successive Interference Cancellation with optimal ordering ensures that the reliability of the first decoded symbol with lower probability of error than the other symbols.

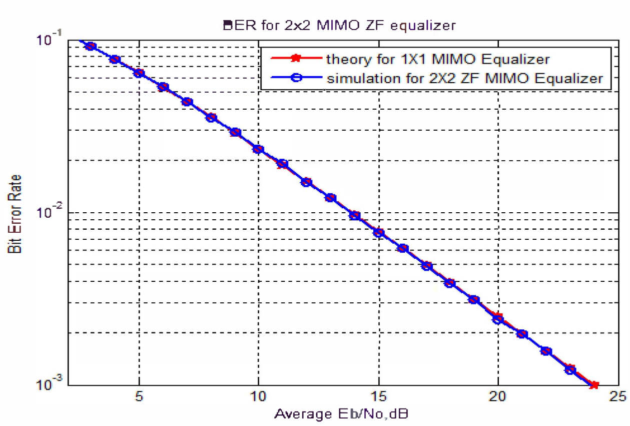

Fig.3: Performance of 2X2 ZF-MIMO Equalizer

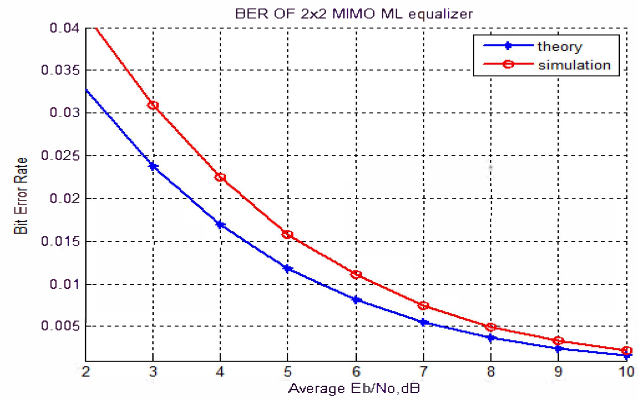

Fig.4: Performance of 2X2 MIMO ML Equalizer

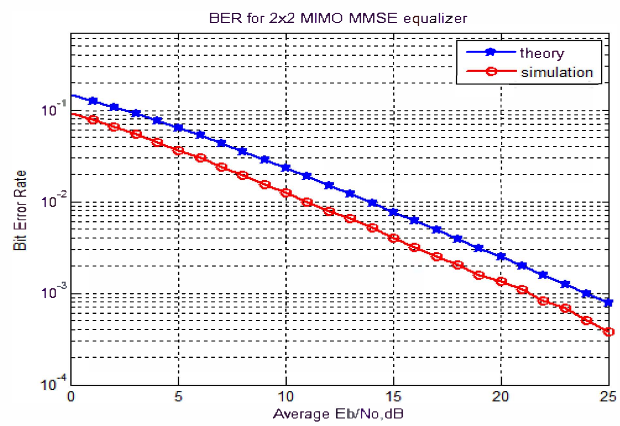

Fig.5: Performance of 2X2 MIMO MMSE Equalizer

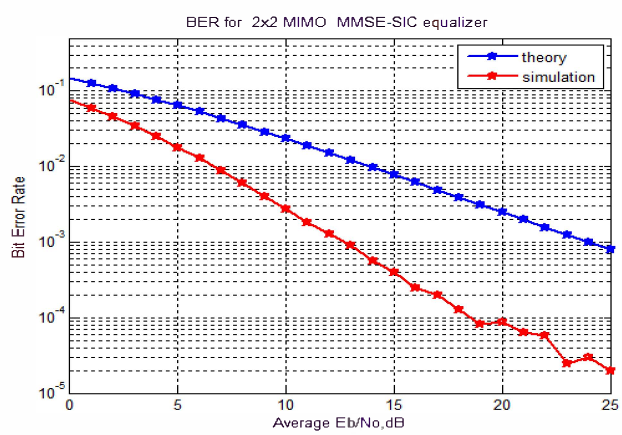

Fig.6: Performance of 2X2 MIMO MMSE-SIC Equalizer

E. The BER performance of different MIMO Equalizer:

The "Fig.6" above shows the BER performance comparison for equalizers. It has been shown that ML detection algorithm is good enough by providing ~5 dB of BER 10^{-2} when compared to MMSE equalizer algorithm and by ~2 dB of BER 10^{-2} when compared to MMSE-SIC equalizer algorithm.

TABEL III
SIMULATION PARAMETERS

Parameter	Specifications
Zero Forcing Algorithm	10 dB
ML Detection Algorithm	3 dB
MMSE Algorithm	6 dB
MMSE-SIC	4 dB

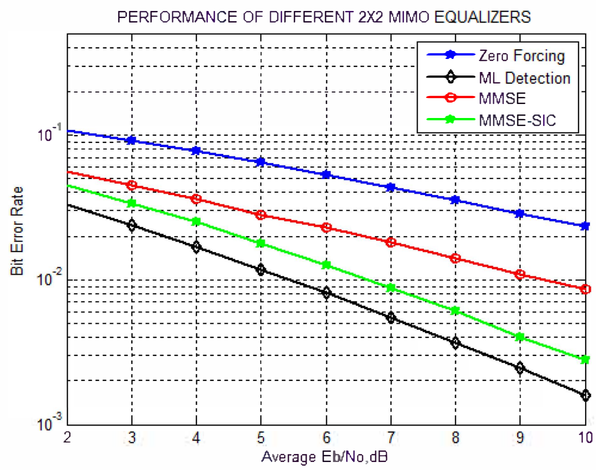

Fig.7: Performance of different MIMO Equalizers

V. CONCLUSION AND FUTURE WORK

In this paper, a combined approach for the processing of 2X2 MIMO equalizer is attempted. The output obtained is summarized in the table below at the BER of $10^{-1.85}$.

Here we considered the basic equalizer and other equalizer, whose performance is evaluated based on bit error rate. As it is known, equalization technique is the most important measure in designing high date rate wireless communication system, since it could combat the effect of ISI and provide high efficiency even in mobile fading channel. The graph performance for equalizer with two transmits and receive antennas shows that the MMSE equalization results with improvement in BER of around 10^{-3} . The tabulation summarizes that MMSE equalization algorithm has an improvement of ~3 dB at a bit error rate of $10^{-1.85}$. The performance of these different equalizers for two transmit and two receive antennas has been analyzed with simulations. Further it was observed that among the different equalizers, MMSE equalizer when combined with Successive Interference Canceller was able to provide unambiguous tracking after applying the temporal filter and enhance the signal quality.

The effectiveness of this paper work to provide maximum signal gain in the presence of several interference sources was shown using simulated data and analyzed which is better based on their performance.

The future work lies in applying the analysis for any kind of data and obtains its performance characteristics under various channel scenarios. Currently we have planned for histogram equalization technique [10] for image processing and the work is under process.

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