Problem_Set_1

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R Programing exercises

1. Calculate the square root of 729

```
b = sqrt(729)
print(b)

## [1] 27
```

2. Create a new variable a with value 1947.0

```
a = as.integer(1947.0)
print(a)

## [1] 1947
```

3. Create a vector b contaning number form 1 to 6 and find out it's class.

b is a numeric variable.

```
b = seq(1, 6, by = 1)
b.1 <- class(b)
print(c(b, b.1))

## [1] "1"       "2"       "3"       "4"       "5"       "6"       "numeric"
```

4. Create a vector c containg following mixed elements

```
c = c(1, "a", 2, "b")
print(c)

## [1] "1" "a" "2" "b"
```

(a) Find out its class. It is a character variable

```
class(c)

## [1] "character"
```

(b) Get the length of the vector. The length is four

```
length(c) # Figuring out the length of c

## [1] 4
```

(c) Get the 2nd and 3rd elements, which is "a" and "2".

```
print(c[2]) # Printing the 2nd element

## [1] "a"

print(c[3]) # Printing the 3rd element

## [1] "2"
```

5. Create a vector d containing following elements c(1, 2, NA, 4, 5, 6, NA, NA, NA, 10)

Remove missing values from d

```
d = c(1, 2, NA, 4, 5, 6, NA, NA, NA, 10)
d = as.numeric(na.omit(d)) # Removing NA valuse and converting the vector to a numeric one
print(d)

## [1]  1  2  4  5  6 10
```

6. Create a vector of values of e^x cos 3 at x = 3, 3.1, 3.2, ... 6

```
x = seq(3, 6, by = 0.1)
x.1 = exp(x) * cos(x) # cosine is reading in x as a radian unit
print(x.1)

##  [1] -19.884531 -22.178753 -24.490697 -26.773182 -28.969238 -31.011186
##  [7] -32.819775 -34.303360 -35.357194 -35.862834 -35.687732 -34.685042
## [13] -32.693695 -29.538816 -25.032529 -18.975233 -11.157417  -1.362099
## [19]  10.632038  25.046705  42.099201  61.996630  84.929067 111.061586
## [25] 140.525075 173.405776 209.733494 249.468441 292.486707 338.564378
## [31] 387.360340

#x.2 = exp(x) * cos(x/180) # this funtions will read it in as degrees
#print(x.2)
```

7. Calculate $\sum_{i=10}^{100}(i^3 + 4 * i^2)$

```
s = seq(10, 100, by = 1) # Creating the sequence
s.1 <- s^3 + 4 * s^2 # transforming the sequence
s.2 = sum(s.1) # Calculating the sum of the transformed sequence
print(s.2)

## [1] 26852735
```

8. Execute the following line which create two vectors of random integers that are chosen with replacement from the integers 0, 1, ...999. Both vectors have length 250.

```
x <- sample(0:999, size = 250, replace = TRUE)
y <- sample(0:999, size = 250, replace = TRUE)
```

(a) Pick out the values in Y which are > 600

```
y.1 <- subset(y, y>600) # use the subset comand to pull out elements of a vector
print(y.1)

##  [1] 847 720 635 811 995 690 773 977 614 963 950 654 977 838 685 764 690
## [18] 889 836 948 833 969 982 791 847 879 641 975 685 815 876 865 770 819
## [35] 934 642 690 988 835 618 850 731 933 931 622 844 645 817 726 840 734
## [52] 726 868 830 650 913 863 662 912 806 983 626 775 716 631 901 786 755
## [69] 932 686 722 755 997 853 619 738 745 841 769 607 632 921 781 983 985
## [86] 969 707 914 614 805 787 794 708 607 835 793 801 820
```

(b) How many values in y are within 200 of the maximum value of the terms in y?

```
y.2 <- subset(y, y >= max(y)-200)
print(length(y.2))

## [1] 51
```

(c) Create the vector e

```
e <- abs(x-mean(x))^(1/2)
print(e)

##   [1]  7.926159 14.656876 12.213763 18.133505 17.715078 15.039149 17.199535
##   [8] 22.004000 21.373441 16.426320 22.117324 20.659719 21.334854  7.669681
##  [15] 14.006284 14.323966 22.409284  9.788973 21.683542 21.767499 15.900440
##  [22] 12.199344 16.253492 16.577817 10.140217 11.697179  9.755819 12.335964
##  [29]  3.977939  4.563332 21.019610 13.993713 19.446748 16.678609 17.228581
##  [36] 17.170440 21.051746 10.008796 11.908988 16.088008 19.574882 18.225696
```

```
##  [43] 21.358277 21.606110 17.939454 10.778497 15.356562 21.591109  6.842806
##  [50] 16.426320 12.916037 16.284225 17.949262  3.343052 19.230809 19.282738
##  [57] 22.409284 19.778170 13.957937  4.709140 13.644633 19.004631  6.230891
##  [64] 21.867236 16.191850 16.698024 18.470950 15.529842  7.336484 13.813906
##  [71] 12.090327 19.421020 21.775583 20.562685 14.860148 14.939076  7.268012
##  [78]  5.493269 16.056899 21.544744 17.743280 16.757804 16.376080 11.626521
##  [85] 21.521524  9.686279 19.489074 17.640181 11.481463 21.683542 19.004631
##  [92] 11.409470 21.287931 10.206664 21.090851 12.497040 16.067856 18.088007
##  [99] 17.257578  9.530163 17.753197 21.381674 19.945526 17.601818  7.669681
## [106] 15.204473 19.360372 20.900335  6.721309 17.034553 10.808515  4.563332
## [113] 15.453932  9.686279 22.386960 17.411950  7.012560  9.958715 14.939076
## [120] 17.459210 14.416102  7.153740 11.217130 14.450744 18.133505  9.174748
## [127]  8.990217 18.105911 16.160941 15.368019  9.496104 21.981265 13.447081
## [134]  8.707698 20.029378 22.207566 18.686252 12.616497 18.846114 19.794545
## [141] 12.576804 16.191850 20.659719 18.889786 17.024218 18.632659 19.021672
## [148] 21.067131 12.158289 19.954348 18.461419 15.721832 13.970540 19.100366
## [155]  8.650087 20.079243 14.416102 20.812881 20.683907 19.308651  3.489413
## [162] 19.702183  4.452415  7.268012 16.376080 14.554175 18.270851 11.409470
## [169] 11.437832 19.213953 20.416268 20.302118  5.729223 19.727544 19.565889
## [176] 21.790457 16.847077  8.650087 18.578913 21.698479 21.973256 17.024218
## [183] 12.034284 10.352584 14.393610 19.161837  4.144394 16.757804 14.183653
## [190] 15.335449 13.970540 20.948126 19.995600  9.941026 16.916737 21.138212
## [197] 12.172756 16.426320  7.129095  8.010992  5.179189 17.995110  4.379041
## [204] 13.970540 18.596344 12.954690 11.053325 19.844999 21.767499 20.900335
## [211] 15.900440 18.596344  8.954105 12.536985 19.021672 15.039149 13.348258
## [218] 17.315427 12.890927 10.591317 17.209765  8.256876 16.426320 13.460163
## [225] 17.286527  1.350555  9.371446 16.222700 18.942650 21.466812  6.097212
## [232] 21.279662 21.373441 13.826641 12.482948  8.954105 19.437695 16.528037
## [239] 21.890089  3.438604 21.051746  6.259073 16.222700  9.857789 17.640181
## [246] 18.605806  7.988992  6.570845 15.039149 15.900440
```

(d) Create another vector $(y_2 - x_1, y_3 - x_2, \ldots y_n - x_{n-1})$

```
n=2:length(y)
d <- (y[n]-x[n-1]) # It worked!
print(d)
```

```
##   [1]  417 -229 -409  -78  541  -84  614 -943  294  772 -292  451 -557 -119
##  [15]   84 -600 -555 -231   14  538  374  619  193  327  170 -180   66  332
##  [29]   43 -226  384 -105 -601   67  489  566 -846 -440  339  -82 -804   64
##  [43] -582  810  -74 -225 -155  -11  -89  610  227  211  167  287  409  449
##  [57] -148   -5  343 -457  296 -169  361 -508 -742  314 -757 -368  437 -117
##  [71] -633 -582 -535  140  593 -240 -321 -155   93 -920  -40  558   58  306
##  [85] -314 -118  316 -395  112 -683 -687 -603 -256 -414  238 -504  237 -353
##  [99]  640   59 -321 -836  490    5  132 -106  389  389 -515   67 -238 -137
## [113]  477  534 -705 -552 -534 -539 -676  743  315 -270  255 -139   41  -11
## [127]  432  -55 -272  -84 -536 -624    9 -213 -868 -605 -701  165 -122  -95
## [141]  189  -21   68  590  192 -389  458   54  523 -818  409   90 -428  522
## [155] -151   17   64 -918  227  743  157   31  333 -124 -588  306  824 -423
## [169]  264 -325  237 -130   40 -418  606   23    6 -120 -207  879  776   19
## [183]  -90  155  232 -358   70  474   28   27  309  -50  213  459 -363  527
## [197]  -22  515 -307 -423   47  576  329   81  -73  127 -312  -80  588  576
## [211]  681  634   -4  333  -78  266 -664  776 -365  102 -350  353 -104  -60
## [225]  303  314 -370 -213  653  333  264  212  394 -268  203  135  492 -305
## [239] -137   48 -680 -245 -262  203 -655  -38  391 -437 -255
```

9. In this exercise, we will consider a quadratic equations of the form $(y = \beta_0 + \beta_1 x + \beta_2 x^2)$. Create a vector of coefficients for a quadratic equations.

```
coeffs <- sample(-20:20, size = 3, replace = TRUE)
```

(a) Determine the length of the object coeffs.

```
print(length(coeffs))
```

```
## [1] 3
```

(b) Create 200 values of x from a regularly spaced vector between -3 and 3

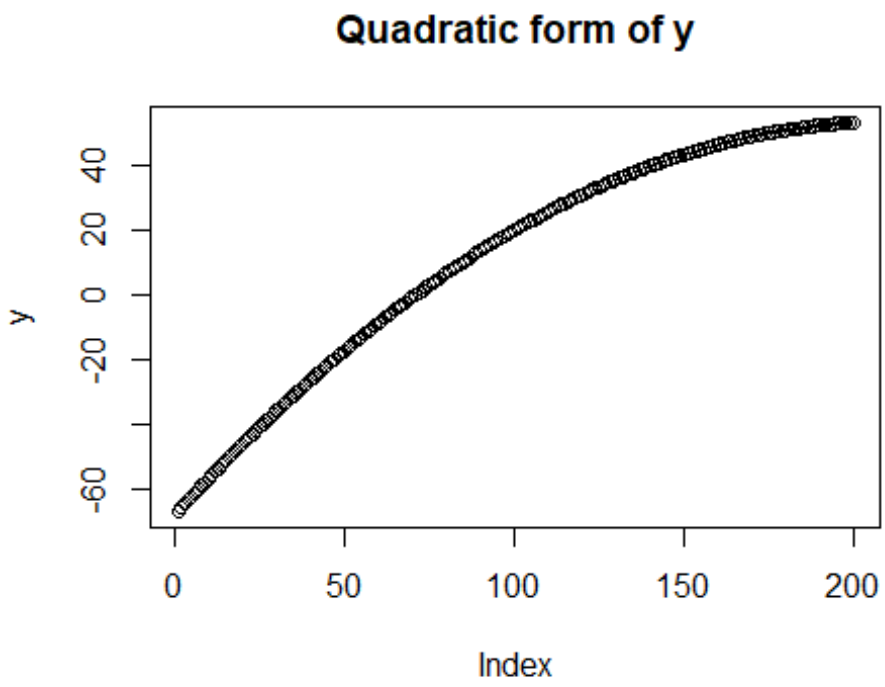
```
x <- seq(from = -3, to = 3, length.out = 200)
```

(c) Now obtain the value of the quadratic function (y) at each value of x

```
y = coeffs[1] + coeffs[2] * x + coeffs[3] * x^2
```

(d) Construct the plot

```
plot(y, main = "Quadratic form of y")
```



10. Without using R, determine the result of the following computation

$$x < -c(1,2,3)$$

$$x[1]/x[2]^3 - 1 + 2 * x[3] - x[2-1] = 1/2^3 - 1 + 2 * 3 - 1 = 4.125$$

11. Create the following matrix with 15 rows

```
A = matrix(c(rep(c(10, -5, 10), times = 15)), nrow = 15, byrow = TRUE)
print(A)
```

```
##       [,1] [,2] [,3]
##  [1,]   10   -5   10
##  [2,]   10   -5   10
##  [3,]   10   -5   10
##  [4,]   10   -5   10
##  [5,]   10   -5   10
##  [6,]   10   -5   10
##  [7,]   10   -5   10
##  [8,]   10   -5   10
##  [9,]   10   -5   10
## [10,]   10   -5   10
## [11,]   10   -5   10
## [12,]   10   -5   10
## [13,]   10   -5   10
## [14,]   10   -5   10
## [15,]   10   -5   10
```

```r
A.1 = A # Copy the matrix
A.1[,3] = A.1[,1] + A.1[,2] # rewrite the 3 column as a sum of the first two
print(A.1)
```

```
##       [,1] [,2] [,3]
##  [1,]   10   -5    5
##  [2,]   10   -5    5
##  [3,]   10   -5    5
##  [4,]   10   -5    5
##  [5,]   10   -5    5
##  [6,]   10   -5    5
##  [7,]   10   -5    5
##  [8,]   10   -5    5
##  [9,]   10   -5    5
## [10,]   10   -5    5
## [11,]   10   -5    5
## [12,]   10   -5    5
## [13,]   10   -5    5
## [14,]   10   -5    5
## [15,]   10   -5    5
```

12. Create a function that given two number will return he sum of those two number

```r
add <- function(a,b){
  c = a + b
  return(c)
}

add(5,10)
```

```
## [1] 15
```

13. Create a function that given a vector and an integer will return how many times the integer appears inside the vector

```r
count <- function(x, int){
  y <- vector()
  for(i in 1:length(x)){
    ifelse(x[i] == int, y[i] <- 1, y[i] <- 0)
  }
  z = sum(y)
  return(z)
}

# testing the function

x <- c(4,5, 6, 6, 7, 8)
count(x = x, int = 6) # The argument should return a 2 given vector x
```

```
## [1] 2
```

14. Create a function that given an integer vector $(z_1, z_2, \ldots, z_n)$ will return $(z_1, z_1^2, \ldots, z_n^n)$

```r
zsquared <- function(x){
  z <- numeric(length(x))
  for(i in 1:length(x)){
    z[i] <- x[i]^i
  }
  return(z)
}

# Testing the function

x <- c(2, 2, 3)
zsquared(x) # should return a 2, 4, and 27 given vector x
```

```
## [1]  2  4 27
```

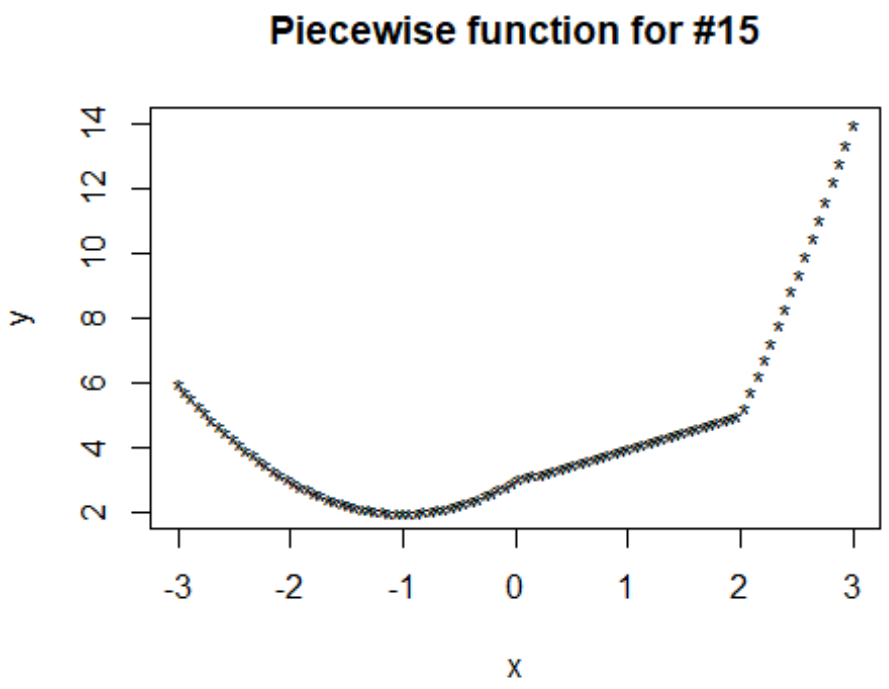
15. Create a piecewise function

```
piecewise <- function(x){
  y <- numeric(length(x))
  for(i in 1:length(x)){
    if(x[i] < 0 ){
      y[i] = (x[i]^2 + 2 * x[i] + 3)
    }
    else if(x[i] >= 0 & x[i] < 2 ){
      y[i] = (x[i] + 3)
    }
    else if( 2 <= x[i]){
      y[i] = (x[i]^2 + 4 * x[i] - 7)
    }
  }
  return(y)
}

# Testing the piecewise function

x <-seq(-3, 3, length = 100)
y <- piecewise(x)

plot(x, y, main = "Piecewise function for #15", pch = "*", col = 617, bg = 456)
```



Theory

Problem 1:

Show that for Y_t, $E(Y_t^2) = \sigma^2$ with $E(Y_t) = \mu_t = 0$ and $Var(Y_t) = \gamma_Y(0) = \sigma^2$

$$Var(Y_t) = \sigma^2 = E[(Y_t - \mu_t)^2] = E[Y_t^2 - 2Y_t\mu + \mu_t^2]$$

$$\sigma^2 = E[Y_t^2] - 2\mu_t E[Y_t] + E[\mu_t^2] = E[Y_t^2] - 2\mu_t * 0 + 0$$

$$\sigma^2 = E[Y_t^2]$$

Problem 2:

Show that the autocovariance function can be written as $\gamma_y(s,t) = E(Y_s = \mu_s)(Y_t - \mu_t) = E(Y_sY_t) - \mu_s\mu_t$ where $E(Y_t) = \mu_t$ and $E(Y_s) = \mu_s$

$$\gamma_y(s,t) = E(Y_s - \mu_s)(Y_t - \mu_t) = E(Y_sY_t - Y_s\mu_t - Y_t\mu_s + \mu_t\mu_s)$$

$$\gamma_y(s,t) = E(Y_sY_t) - E(Y_s\mu_t) - E(Y_t\mu_s) + E(\mu_t\mu_s) = E(Y_sY_t) - \mu_t E(Y_s) - \mu_s E(Y_t) + \mu_t\mu_s$$

$$\gamma_y(s,t) = E(Y_sY_t) - \mu_t\mu_s - \mu_s\mu_t + \mu_t\mu_s = E(Y_sY_t) - 2\mu_t\mu_s + \mu_t\mu_s = E(Y_sY_t) - \mu_t\mu_s$$

$$\gamma_y(s,t) = E(Y_sY_t) - \mu_t\mu_s$$

Problem 3:

time, t	Y_t	Y_{t-1}	Y_{t-3}	$\hat{\mu}_Y(t)$	$Y_t - \hat{\mu}_Y(t)$	$(Y_t - \hat{\mu}_Y(t))^2$	$Y_{t-1} - \hat{\mu}_Y(t)$	$(Y_{t-1} - \hat{\mu}_Y(t))^2$	$(Y_t - \hat{\mu}_Y(t))(Y_{t-1} - \hat{\mu}_Y(t))$
Jan-49	112.00			126.67	-14.67	215.11			
Feb-49	118.00	112.00		126.67	-8.67	75.11	-14.67	215.11	127.11
Mar-49	132.00	118.00	112.00	126.67	5.33	28.44	-8.67	75.11	-46.22
Apr-49	129.00	132.00	118.00	126.67	2.33	5.44	5.33	28.44	12.44
May-49	121.00	129.00	132.00	126.67	-5.67	32.11	2.33	5.44	-13.22
Jun-49	135.00	121.00	129.00	126.67	8.33	69.44	-5.67	32.11	-47.22
Jul-49	148.00	135.00	121.00	126.67	21.33	455.11	8.33	69.44	177.78
Aug-49	148.00	148.00	135.00	126.67	21.33	455.11	21.33	455.11	455.11
Sep-49	136.00	148.00	148.00	126.67	9.33	87.11	21.33	455.11	199.11
Oct-49	119.00	136.00	148.00	126.67	-7.67	58.78	9.33	87.11	-71.56
Nov-49	104.00	119.00	136.00	126.67	-22.67	513.78	-7.67	58.78	173.78
Dec-49	118.00	104.00	119.00	126.67	-8.67	75.11	-22.67	513.78	196.44
sum	1520.00	1402.00	1298.00	1520.00	0.00	2070.67	8.67	1995.56	1163.56

Sample Variance: = 2070.67/(12-1) = 188.24

Sample autocovariance: (1/11)*1163.56 = 105.78

Sample Autocorrelation: 1163.56/ sqrt(2070.67*1995.56) = 0.57