Problem_Set_1

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R Programing exercises

1. Calculate the square root of 729

```
b = sqrt(729)
print(b)

## [1] 27
```

2. Create a new variable a with value 1947.0

```
a = as.integer(1947.0)
print(a)

## [1] 1947
```

3. Create a vector b contaning number form 1 to 6 and find out it's class.

b is a numeric variable.

```
b = seq(1, 6, by = 1)
b.1 <- class(b)
print(c(b, b.1))

## [1] "1"       "2"       "3"       "4"       "5"       "6"       "numeric"
```

4. Create a vector c containg following mixed elements

```
c = c(1, "a", 2, "b")
print(c)

## [1] "1" "a" "2" "b"
```

(a) Find out its class. It is a character variable

```
class(c)

## [1] "character"
```

(b) Get the length of the vector. The length is four

```
length(c) # Figuring out the length of c

## [1] 4
```

(c) Get the 2nd and 3rd elements, which is "a" and "2".

```
print(c[2]) # Printing the 2nd element

## [1] "a"

print(c[3]) # Printing the 3rd element

## [1] "2"
```

5. Create a vector d containing following elements c(1, 2, NA, 4, 5, 6, NA, NA, NA, 10)

Remove missing values from d

```
d = c(1, 2, NA, 4, 5, 6, NA, NA, NA, 10)
d = as.numeric(na.omit(d)) # Removing NA valuse and converting the vector to a numeric one
print(d)

## [1]  1  2  4  5  6 10
```

6. Create a vector of values of e^x cos 3 at x = 3, 3.1, 3.2, ... 6

```
x = seq(3, 6, by = 0.1)
x.1 = exp(x) * cos(x) # cosine is reading in x as a radian unit
print(x.1)

##  [1] -19.884531 -22.178753 -24.490697 -26.773182 -28.969238 -31.011186
##  [7] -32.819775 -34.303360 -35.357194 -35.862834 -35.687732 -34.685042
## [13] -32.693695 -29.538816 -25.032529 -18.975233 -11.157417  -1.362099
## [19]  10.632038  25.046705  42.099201  61.996630  84.929067 111.061586
## [25] 140.525075 173.405776 209.733494 249.468441 292.486707 338.564378
## [31] 387.360340

#x.2 = exp(x) * cos(x/180) # this funtions will read it in as degrees
#print(x.2)
```

7. Calculate $\sum_{i=10}^{100}(i^3 + 4 * i^2)$

```
s = seq(10, 100, by = 1) # Creating the sequence
s.1 <- s^3 + 4 * s^2 # transforming the sequence
s.2 = sum(s.1) # Calculating the sum of the transformed sequence
print(s.2)

## [1] 26852735
```

8. Execute the following line which create two vectors of random integers that are chosen with replacement from the integers 0, 1, ...999. Both vectors have length 250.

```
x <- sample(0:999, size = 250, replace = TRUE)
y <- sample(0:999, size = 250, replace = TRUE)
```

(a) Pick out the values in Y which are > 600

```
y.1 <- subset(y, y>600) # use the subset comand to pull out elements of a vector
print(y.1)

##  [1] 939 646 844 850 783 933 670 755 797 857 791 911 632 975 906 757 605
## [18] 705 625 748 906 805 978 780 832 829 998 823 603 759 716 819 764 740
## [35] 820 821 839 705 742 825 957 923 771 671 852 724 751 871 801 661 835
## [52] 793 964 783 699 805 660 768 658 946 938 947 744 898 927 788 817 744
## [69] 997 866 689 919 685 686 857 979 890 822 724 931 718 930 766 755 940
## [86] 907 936 654 655 618 662 807 854 987
```

(b) How many values in y are within 200 of the maximum value of the terms in y?

```
y.2 <- subset(y, y >= max(y)-200)
print(length(y.2))

## [1] 49
```

(c) Create the vector e

```
e <- abs(x-mean(x))^(1/2)
print(e)

##   [1] 17.807976  8.069944 18.442451 20.442016 15.070634 14.598767 20.177314
##   [8]  8.492585 22.200811 18.360937 15.520180  1.695877 14.692719 17.744746
##  [15] 10.398269  9.226267  9.226267 20.955286 12.850058 16.518959  8.238689
##  [22] 18.624607  2.622213 20.096666 20.319350 20.121531 10.764572 20.448081
##  [29] 12.323798 15.783662 16.465843 12.654011 12.762288 21.139442 13.670260
##  [36] 20.417541 15.807467  7.132741  7.491595  9.893230 18.189997 15.520180
```

```
##  [43] 22.402589 21.705391 11.624285 11.537937 18.944023 12.732792 21.020847
##  [50]  7.008852 17.919933 21.491301 12.693463 12.414669 13.071955 10.529767
##  [57] 12.364303 14.903825 21.980082 15.293005 16.907868  4.015470  6.846605
##  [64] 19.927970  4.886307 20.325452 12.879286 16.766753 18.678223 18.597742
##  [71] 19.496769 15.592434 18.224270 18.031195 19.055813  6.717440 20.949368
##  [78]  7.866130 14.487098  6.092947 14.243455 21.957322 15.775804  5.086846
##  [85] 10.240898 20.594271 17.230090 20.251518 19.023039 10.914394 15.846892
##  [92]  9.480295 17.836031 19.541648 20.220682 13.752236  9.893230  5.488534
##  [99]  4.257229  8.767896 10.577145 14.004428 19.206145 15.037420 19.802121
## [106] 15.656436 17.975650 19.284087  5.841575  3.335266 14.031251 14.252158
## [113] 22.093528 19.522397  2.263625  8.551257  6.846605 21.092084  3.622706
## [120] 15.037420 10.289606 13.224069 11.624285 16.151656 17.432039  8.418789
## [127] 10.253000 19.522397  9.048536  7.491595 16.465843 10.204117 17.940903
## [134] 13.851931 11.308227 21.427179 12.444919 18.516911 16.404999 20.733451
## [141] 22.223321  5.396666 12.564872 10.301650 16.996353 20.374592 18.107347
## [148]  2.263625  4.886307  6.072561 17.638707 12.879286 10.301650 20.877644
## [155] 19.669164 18.604408 20.399118 15.712288 17.857099 16.759356 20.618535
## [162] 16.244261 12.129468  9.440551 18.791594  9.740431 13.337016  8.595115
## [169] 11.005635 17.410457  7.881878 16.275012 13.788546 13.743217 14.794458
## [176] 18.711601 12.693463 19.821100 17.150044 22.469446  3.142610 21.286709
## [183] 13.670260  2.622213 19.368118 15.839066 11.440105 10.386337 15.775804
## [190] 13.337016 11.185884 10.588862 15.909871 17.638707 22.200811 16.282629
## [197] 19.490408 21.768693 21.468023  5.733760 17.779876 13.670260 21.002952
## [204] 10.093364 21.092084 21.263208  8.477971 20.399118 10.857071  5.578889
## [211] 19.770584 22.138744 19.003263 18.244890 13.642727  8.192924 14.555961
## [218] 14.426503 21.537781  5.578889 19.921747 12.534911 19.258141 19.871487
## [225]  8.881216  9.006886 11.581192 14.348380 18.496594 15.495935 16.282629
## [232] 21.636173 10.301650 19.776855 13.715830 19.342285  6.566887  8.653092
## [239] 13.308794 21.121648 21.216126 16.282629 16.855978 11.319187  9.006886
## [246] 21.928885  8.433505  9.387438 12.364303 16.937414
```

(d) Create another vector $(y_2 - x_1, y_3 - x_2, \ldots y_n - x_{n-1})$

```
n=2:length(y)
d <- (y[n]-x[n-1]) # It worked!
print(d)
```

```
##   [1]  -81   77  786 -790 -198  366  307  423 -667  405 -364  -43 -150   42
##  [15]  398 -169  525  486  342  -11 -381 -471  297  -40 -115 -683 -448  109
##  [29] -230  204  689  299 -437   35  226 -704   14   61  -89  114 -199 -723
##  [43] -247  -12  146 -121 -717  575 -424  361  387   23  -87  441 -315 -404
##  [57] -383   31  548  105 -367 -323  289  902 -320  255  164  391 -503  -80
##  [71]  216 -131  322  291  421 -292 -459  -40 -210   39 -403  705   77  245
##  [85] -206  302  -50  737  -34  465  -97 -199  530 -358 -160  521  366  460
##  [99]  296  101 -417  555 -445  231  623  503   35 -485 -233 -358  181   63
## [113]   39  343  313   21 -114 -835  104  284   62  167  435 -713  167  219
## [127] -136  100  124  368  438  379 -157  261 -434  123 -205 -664  714 -785
## [141]  -40  133   93   -5 -562  331 -360  -84 -363 -498  228  239 -248 -454
## [155] -736  780  440 -209  -24 -347   70   60   51   49  308  156 -444  -91
## [169]  625 -118  -66 -241  563    7 -180  776   73 -201 -199 -312   34  389
## [183]  177 -462  861  146  198 -268 -283   53  563  -89 -148  536 -959  157
## [197]   57 -702 -188 -394 -108 -133  207  160 -572  486 -272  252 -500 -387
## [211] -508  435  808 -301  600 -179 -208  651 -523  192 -826  319 -588 -745
## [225]   46 -366   48 -669  511 -234 -137  108    1  308  215   60  357 -211
## [239] -129  182   39  626  778  225 -232 -426  164  118 -607
```

9. In this exercise, we will consider a quadratic equations of the form $(y = \beta_0 + \beta_1 x + \beta_2 x^2)$. Create a vector of coefficients for a quadratic equations.

```
coeffs <- sample(-20:20, size = 3, replace = TRUE)
```

(a) Determine the length of the object coeffs.

```
print(length(coeffs))
```

```
## [1] 3
```

(b) Create 200 values of x from a regularly spaced vector between -3 and 3

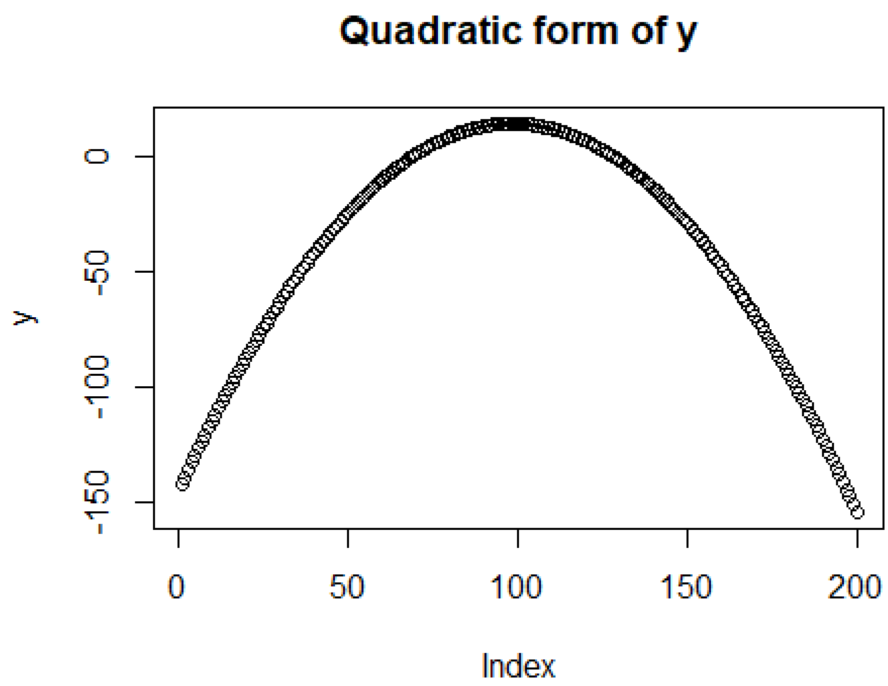
```
x <- seq(from = -3, to = 3, length.out = 200)
```

(c) Now obtain the value of the quadratic function (y) at each value of x

```
y = coeffs[1] + coeffs[2] * x + coeffs[3] * x^2
```

(d) Construct the plot

```
plot(y, main = "Quadratic form of y")
```



10. Without using R, determine the result of the following computation

$$x < -c(1,2,3)$$

$$x[1]/x[2]^3 - 1 + 2 * x[3] - x[2-1] = 1/2^3 - 1 * 3 - 1 = 4.125$$

11. Create the following matrix with 15 rows

```
A = matrix(c(rep(c(10, -5, 10), times = 15)), nrow = 15, byrow = TRUE)
print(A)

##       [,1] [,2] [,3]
##  [1,]   10   -5   10
##  [2,]   10   -5   10
##  [3,]   10   -5   10
##  [4,]   10   -5   10
##  [5,]   10   -5   10
##  [6,]   10   -5   10
##  [7,]   10   -5   10
##  [8,]   10   -5   10
##  [9,]   10   -5   10
## [10,]   10   -5   10
## [11,]   10   -5   10
## [12,]   10   -5   10
## [13,]   10   -5   10
## [14,]   10   -5   10
## [15,]   10   -5   10
```

```r
A.1 = A # Copy the matrix
A.1[,3] = A.1[,1] + A.1[,2] # rewrite the 3rd column as a sum of the first two
print(A.1)

##       [,1] [,2] [,3]
##  [1,]   10   -5    5
##  [2,]   10   -5    5
##  [3,]   10   -5    5
##  [4,]   10   -5    5
##  [5,]   10   -5    5
##  [6,]   10   -5    5
##  [7,]   10   -5    5
##  [8,]   10   -5    5
##  [9,]   10   -5    5
## [10,]   10   -5    5
## [11,]   10   -5    5
## [12,]   10   -5    5
## [13,]   10   -5    5
## [14,]   10   -5    5
## [15,]   10   -5    5
```

12. Create a function that given two number will return he sum of those two number

```r
add <- function(a,b){
  c = a + b
  return(c)
}

add(5,10)

## [1] 15
```

13. Create a function that given a vector and an integer will return how many times the integer appears inside the vector

```r
count <- function(x, int){
  y <- vector()
  for(i in 1:length(x)){
    ifelse(x[i] == int, y[i] <- 1, y[i] <- 0)
  }
  z = sum(y)
  return(z)
}

# testing the function

x <- c(4,5, 6, 6, 7, 8)
count(x = x, int = 6) # The argument should return a 2 given vector x

## [1] 2
```

14. Create a function that given an integer vector $(z_1, z_2, \ldots, z_n)$ will return $(z_1, z_1^2, \ldots, z_n^n)$

```r
zsquared <- function(x){
  z <- numeric(length(x))
  for(i in 1:length(x)){
    z[i] <- x[i]^2
  }
  return(z)
}

# Testing the function

x <- c(2, 2)
zsquared(x) # should return two 4s given vector x

## [1] 4 4
```

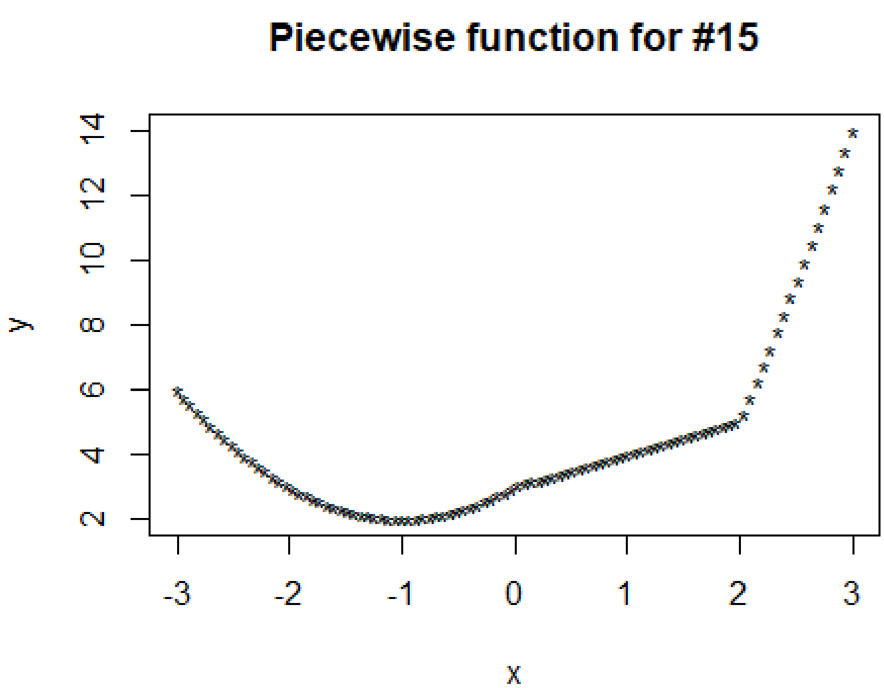
15. Create a piecewise function

```
piecewise <- function(x){
  y <- numeric(length(x))
  for(i in 1:length(x)){
    if(x[i] < 0 ){
      y[i] = (x[i]^2 + 2 * x[i] + 3)
    }
    else if(x[i] >= 0 & x[i] < 2 ){
      y[i] = (x[i] + 3)
    }
    else if( 2 <= x[i]){
      y[i] = (x[i]^2 + 4 * x[i] - 7)
    }
  }
  return(y)
}

# Testing the piecewise function

x <-seq(-3, 3, length = 100)
y <- piecewise(x)

plot(x, y, main = "Piecewise function for #15", pch = "*", col = 617, bg = 456)
```



Theory

Problem 1:

Show that for Y_t, $E(Y_t^2) = \sigma^2$ with $E(Y_t) = \mu_t = 0$ and $Var(Y_t) = \gamma_Y(0) = \sigma^2$

$$Var(Y_t) = \sigma^2 = E[(Y_t - \mu_t)^2] = E[Y_t^2 - 2Y_t\mu + \mu_t^2]$$

$$\sigma^2 = E[Y_t^2] - 2\mu_t E[Y_t] + E[\mu_t^2] = E[Y_t^2] - 2\mu_t * 0 + 0$$

$$\sigma^2 = E[Y_t^2]$$

Problem 2:

Show that the autocovariance function can be written as $\gamma_y(s,t) = E(Y_s = \mu_s)(Y_t - \mu_t) = E(Y_sY_t) - \mu_s\mu_t$ where $E(Y_t) = \mu_t$ and $E(Y_s) = \mu_s$

$$\gamma_y(s,t) = E(Y_s - \mu_s)(Y_t - \mu_t) = E(Y_sY_t - Y_s\mu_t - Y_t\mu_s + \mu_t\mu_s)$$

$$\gamma_y(s,t) = E(Y_sY_t) - E(Y_s\mu_t) - E(Y_t\mu_s) + E(\mu_t\mu_s) = E(Y_sY_t) - \mu_t E(Y_s) - \mu_s E(Y_t) + \mu_t\mu_s$$

$$\gamma_y(s,t) = E(Y_sY_t) - \mu_t\mu_s - \mu_s\mu_t + \mu_t\mu_s = E(Y_sY_t) - 2\mu_t\mu_s + \mu_t\mu_s = E(Y_sY_t) - \mu_t\mu_s$$

$$\gamma_y(s,t) = E(Y_sY_t) - \mu_t\mu_s$$

Problem 3:

time, t	Y_t	Y_{t-1}	Y_{t-3}	$\hat{\mu}_Y(t)$	$Y_t - \hat{\mu}_Y(t)$	$(Y_t - \hat{\mu}_Y(t))^2$	$Y_{t-1} - \hat{\mu}_Y(t)$	$(Y_{t-1} - \hat{\mu}_Y(t))^2$	$(Y_t - \hat{\mu}_Y(t))(Y_{t-1} - \hat{\mu}_Y(t))$
Jan-49	112.00			126.67	-14.67	215.11			
Feb-49	118.00	112.00		126.67	-8.67	75.11	-14.67	215.11	127.11
Mar-49	132.00	118.00	112.00	126.67	5.33	28.44	-8.67	75.11	-46.22
Apr-49	129.00	132.00	118.00	126.67	2.33	5.44	5.33	28.44	12.44
May-49	121.00	129.00	132.00	126.67	-5.67	32.11	2.33	5.44	-13.22
Jun-49	135.00	121.00	129.00	126.67	8.33	69.44	-5.67	32.11	-47.22
Jul-49	148.00	135.00	121.00	126.67	21.33	455.11	8.33	69.44	177.78
Aug-49	148.00	148.00	135.00	126.67	21.33	455.11	21.33	455.11	455.11
Sep-49	136.00	148.00	148.00	126.67	9.33	87.11	21.33	455.11	199.11
Oct-49	119.00	136.00	148.00	126.67	-7.67	58.78	9.33	87.11	-71.56
Nov-49	104.00	119.00	136.00	126.67	-22.67	513.78	-7.67	58.78	173.78
Dec-49	118.00	104.00	119.00	126.67	-8.67	75.11	-22.67	513.78	196.44
sum	1520.00	1402.00	1298.00	1520.00	0.00	2070.67	8.67	1995.56	1163.56

Sample Variance: = 2070.67/(12-1) = 188.24

Sample autocovariance: (1/11)*1163.56 = 105.78

Sample Autocorrelation: 1163.56/ sqrt(2070.67*1995.56) = 0.57