End-Fire Array Enhancement with Gradient-Based Numerical Optimization

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Abstract – **Endfire linear arrays comprising isotropic radiators are optimized for sidelobe reduction and additional suppression of the first sidelobes. Our approach utilizes the sequential-quadratic programming algorithm exploiting analytical derivatives of the array directivity with respect to the designable parameters, as well as initialization through a smart random search. Several test cases varying in the number of design variables and optimization constrains are considered. The initial design, the reference, is the Hansen-Woodyard (HW) design. It was possible to essentially nullify the first sidelobes, without compromising the directivity compared to the HW design and without shrinking the inter-element spacing. A comprehensive numerical study is provided.**

1 INTRODUCTION

An endfire linear array of isotropic radiators is a basic antenna array configuration [1]. The Hansen-Woodyard (HW) design with the progressive phase shift [2] improves the peak directivity of an ordinary, uniform, endfire array without destroying other array characteristics [3] even though the HW configuration exhibits the higher sidelobe level (SLL). Further enhancement of the endfire array directivity is possible; however, superdirective designs finally result in serious issues with bandwidth, efficiency, etc., as described in [1]. Therefore, the tasks of improving endfire array characteristics have been formulated and carried out as analytical, semi-analytical, and fully numerical optimization problems with constrains, e.g., see [1] and [4] for the state of the art outline.

When considering array synthesis as an optimization problem, design variables can be element spacing, phase shifts, current amplitudes, as well as their combinations [1], [4]. As the adjustment of amplitudes is problematic from the implementation point of view [1], spacings and phases are the only design variables considered in our study. While the array synthesis problem is generally not heavily constrained, a reasonable lower bound for elements spacing should be imposed [1] in order to avoid impractical designs [1].

In general, the array adjustment or synthesis is a challenging problem with many design variables and multiple local optima. Therefore, the use of population-based algorithms such as genetic algorithms (GA) [5] and [6] or particle swarm optimizers (PSO) [7], [8], and [9] has become popular for array optimization. The downside of these methods is their high computational cost. Typically, thousands and tens of thousands of objective function evaluations are necessary to produce a satisfactory design. Also, population-based techniques—as statistical search techniques—do not ensure repeatability of the results.

Here, we study endfire array synthesis utilizing gradient-based optimization for suppression of the first sidelobes and overall sidelobe reduction. In order to reduce the computational cost of the design process we exploit analytical derivatives of the array directivity with respect to all designable variables. Our approach also includes optional initialization through a smart random search. Cases different in the number of design variables and constrains are considered. The initial design, a reference, is the Hansen-Woodyard design. Depending on the number of design variables and optimization objectives/constrains, it is possible to significantly reduce the sidelobe level without compromising the directivity compared to the HW design without shrinking the inter-element spacing. Operation and performance of the proposed design methodology is illustrated through examples.

2 ARRAY OPTIMIZATION PROBLEM

The array factor of a linear array comprising N elements, depicted in Fig. 1, can be written as

$$F(\theta) = \sum_{n=1}^{N} i_n \exp\{j(\alpha_n + 2\pi z_n \cos(\theta))\} \quad (1)$$

where i_n are amplitudes (we consider all i_n=1, i.e., uniform amplitude excitation), z_n (in wavelengths) are locations, and α_n are phases of the nth element; also

$$z_n = \sum_{k=1}^{n} d_k \quad (2a)$$

$$\alpha_n = \sum_{k=1}^{n} \Delta\alpha_k \quad (2b)$$

where d_k and $\Delta\alpha_k$ are the spacing and phase shift, respectively. References d_1 and $\Delta\alpha_1$ are set to zeros.

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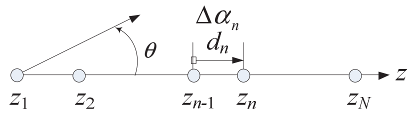

Fig. 1. A linear array.

Directivity of the array is [3]

$$D(\theta) = \frac{2|F(\theta)|^2}{\int_0^{\pi} |F(\theta)|^2 \sin\theta \, d\theta} \tag{3}$$

Starting from the HW design of the endfire array under study we are simultaneously targeting: to suppress the first sidelobe bellow a prescribed level, e.g., −35 dB; to keep all other minor lobes below a level of -15dB. These design specifications are formulated as $D(\theta) \le D_{\max}$ or for specific ranges of angle θ. Additionally we constrain the minimal element spacing, d_k to prevent impractical results.

3 OPTIMIZATION PROCEDURE

In this paper we exploit gradient search combined with smart random initialization. We demonstrate that this is both efficient and computationally cheap way of solving the array synthesis problem.

The main optimization engine is the standard sequential-quadratic programming (SQP) algorithm [10] implemented in Matlab *fminimax* [11] optimization routine. This routine is very convenient as the array synthesis problem can be formulated as a minimax optimization task with upper and lower specifications. Although the synthesis problem has, in general, many design variables, analytical derivatives can easily be calculated for the array factor (1) and array directivity (2). Using analytical derivatives greatly reduces the computational cost of optimization. Typical number of objective function evaluations does not exceed a few hundred instead of thousands or tens of thousands normally required by population-based methods such as GAs or PSO.

As the synthesis problem may be multimodal with many local minima, we also include an optional random search as an initial synthesis step. Our smart random search algorithm can be formulated as follows:

$$\boldsymbol{x}^{(i+1)} = \lambda^{(i)} \boldsymbol{x}^{rand} + (1 - \lambda^{(i)}) \boldsymbol{x}^{best} \tag{4}$$

where $\boldsymbol{x}^{rand}$ is a randomly generated point, $\boldsymbol{x}^{best}$ is the best design found so far, whereas $\lambda^{(i)}$ is a scalar coefficient that decreases towards the end of the search process. In particular, we have $\lambda^{(i)} = i/i_{\max}$, where $i_{\max}$ is the maximum allowed number of function calls for the random search stage. The procedure (4) biases the search towards the best design found so far and turns to a local search when i gets closer to $i_{\max}$. This implementation of a random search is more efficient than pure random search and nearly as efficient as most of population-based search techniques.

4 RESULTS

Linear arrays comprising of 10, 20, and 40 elements have been optimized using the procedure described in Section 2. In each case, the initial design was the HW design corresponding to the particular number of array elements [3]. For the element spacing, the two cases of the lower bound of 0.125 (one eights of the wavelength), and 0.167 are considered in the examples. Figures 2, 4 and 6 show the initial and optimized patterns for the first case, whereas Figs. 3, 5 and 7 show the patterns for the second case. Tables 1 and 2 indicate the values of peak directivity and SLL for all considered test cases as well as their comparison with the reference HW designs.

It is interesting that the results obtained with the lower bound of 0.167 as usually slightly better than those for 0.125 bound, even though the latter corresponds to a larger design space.

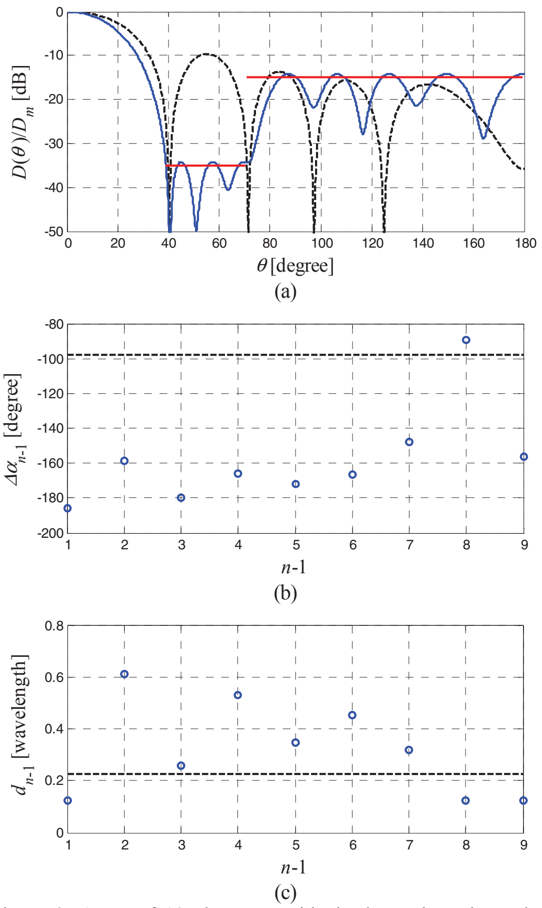

Figure 2. Array of 10 elements, with the lower bound on element spacing of 0.125, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (− −) is the reference. The lower bound on element spacing is 0.125.

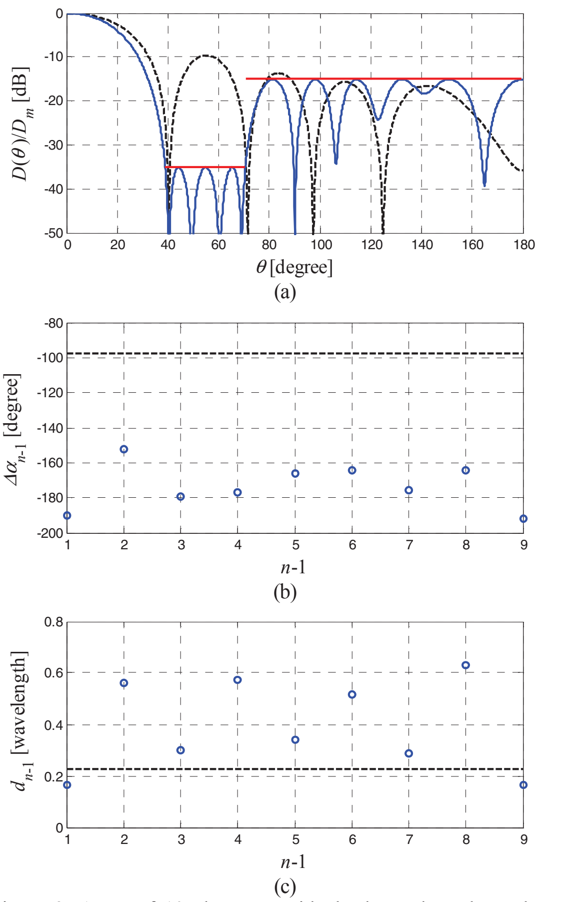

Figure 3. Array of 10 elements, with the lower bound on element spacing of 0.167, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (– –) is the reference.

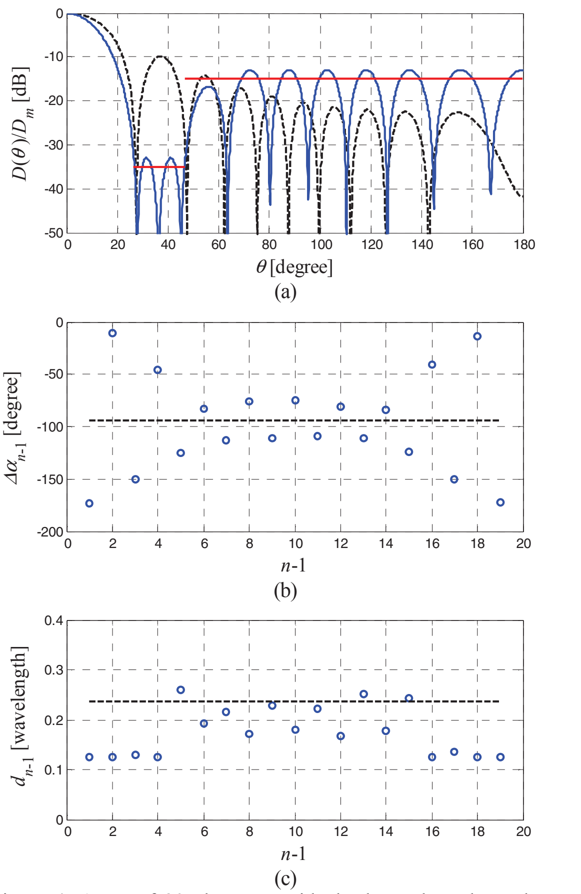

Figure 4. Array of 20 elements, with the lower bound on element spacing of 0.125, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (– –) is the reference.

Table 2. Array design results with the lower bound on element spacing of 0.167.

Number of Elements	Peak Directivity, D_m, [dBi]		SLL [dB]	
	Initial (HW)	*Optimized*	*Initial (HW)*	*Optimized#*
10	12.16	14.0	–9.66	–15.0
20	15.36	16.3	–9.86	–15.0
40	18.47	17.6	–9.91	–17.5

#SLL for first side lobs is around 20dB lower than indicated in the table (see Figs. 3, 5, and 7).

Table 1. Array design results with the lower bound on element spacing of 0.125.

Number of Elements	Peak Directivity, D_m, [dBi]		SLL [dB]	
	Initial (HW)	*Optimized*	*Initial (HW)*	*Optimized#*
10	12.16	14.0	–9.66	–14.5
20	15.36	15.1	–9.86	–13.0
40	18.47	18.6	–9.91	–17.5

#SLL for first side lobs is around 20dB lower than indicated in the table (see Figs. 2, 4, and 6).

5 CONCLUSION

Synthesis of end-fire linear arrays comprising of isotropic radiators is considered. The use of standard gradient-based algorithm working with analytical derivatives allows us to obtain optimum designs at a low computational cost. Design of several arrays consisting of 10 to 40 elements and optimized for sidelobe reduction and additional suppression of the first sidelobs is demonstrated.

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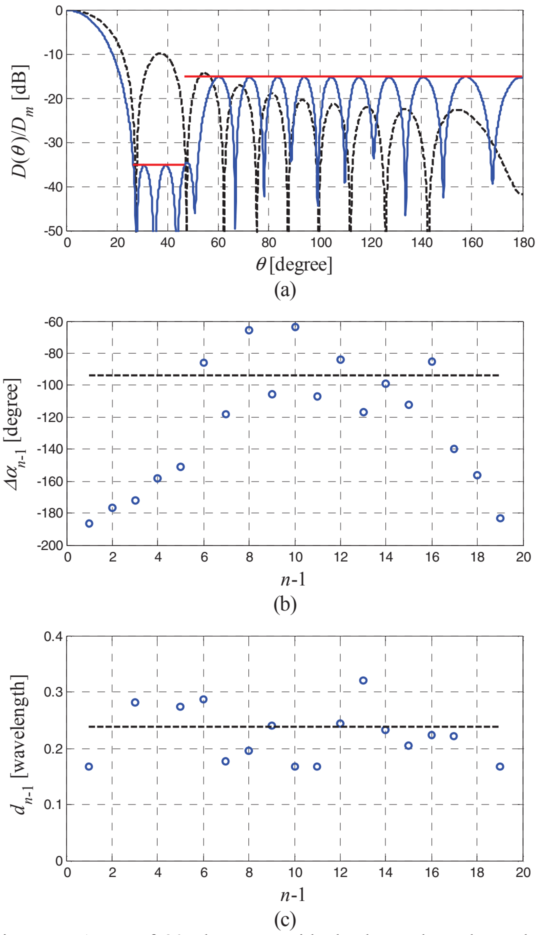

Figure 5. Array of 20 elements, with the lower bound on element spacing of 0.167, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (− −) is the reference.

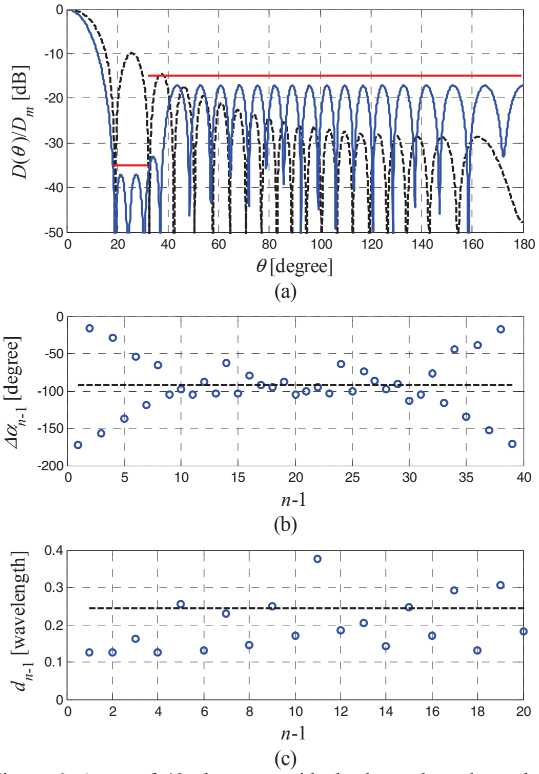

Figure 6. Array of 40 elements, with the lower bound on element spacing of 0.125, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (− −) is the reference.

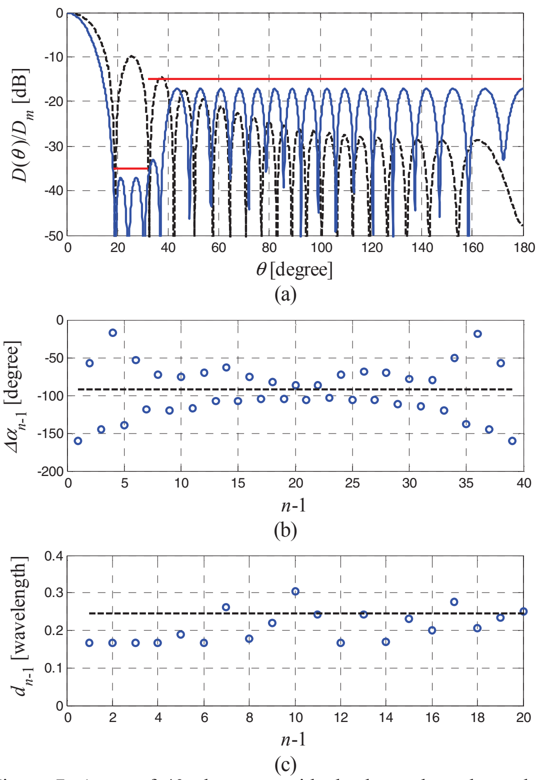

Figure 7. Array of 40 elements, with the lower bound on element spacing of 0.167, optimization results: (a) normalized pattern; (b) progressive phase shift; and (c) spacing where n stands for a particular element number. The HW array (− −) is the reference.