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A NEW PRINCIPLE IN DIRECTIONAL ANTENNA DESIGN*

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Summary—*It is shown that in certain types of directional antenna arrays the gain can be increased by arranging so that waves going from the array elements in the direction of maximum transmission are* not *strictly in phase at large distances. Three examples are given, an end-fire array and two antennas designed to radiate, as far as possible, only in a horizontal plane. In the case of the end-fire array it is shown that readjustment of any existing antenna according to the ideas proposed here will increase the gain by about 1.8.*

The other two examples correspond to the kind of directivity generally desired in a broadcast antenna. One of these consists of short antennas placed in concentric rings. A typical array of this type containing 22 short antennas with the radius of the outer ring equal to 1.39λ has a gain of 2.31 as compared with 1.56 for a vertical half-wave antenna. The other example of a horizontally radiating array consists of a single ring of short antennas. An example of this type is calculated which has a gain of 2.0 with a total of 23 antennas placed in a circle with a radius of 1.43λ. These figures are not given as the best that can be done, but only as examples.

I. Introduction

IN DESIGNING a directional antenna array the aim is to arrange matters such that the total power radiated is a minimum while maintaining the intensity radiated in some direction or directions constant. Moreover, there are other requirements not to be formulated so definitely which may perhaps be included in the statement that the antenna and associated apparatus must not cost too much. Thus, for example, we must avoid designs that are too complicated or that gain their directivity at the expense of too large a decrease in radiation resistance.

This problem might be formulated as a problem in calculus of variations, though it would be enormously complicated if the economic factors were taken in with any pretense of exactitude and, even with simplifying assumptions as to the economic factors, the problem is still intractable in the sense that we cannot hope for an exact solution.

Nevertheless, the point of view is a good one and while not pretending to find the *best* design, i.e. the one corresponding to an absolute minimum, we will show that it is possible to improve considerably on present designs.

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II. End-fire Array

The end-fire array is chosen as the first example to be discussed as it involves less mathematics of an unfamiliar kind than the other examples to follow.

We start by assuming that the economic questions can be approximately handled by considering that arrays of equal length cost the same. This assumption is of course subject to verification when the array design is completed.

We make this assumption definite by agreeing to make the array a fixed length, say $2z_0$, and moreover we will for convenience take $2z_0 \gg \lambda$. Then the problem is to find that current distribution which will radiate the least power while keeping the field at some distant point in line with the array constant. Also, for mathematical convenience, we shall assume that we have a continuous line of antennas all so short as to act as dipoles. Then we will show later that practical approximations to such a continuous distribution give sensibly the same results provided the elements of the array are spaced about $\lambda/3$ or less.

As remarked above, an exact solution of the variation problem is beyond the writers' ability. Instead we shall assume some current distribution which depends on one or more parameters and find what choice of parameters gives the greatest gain. Of course, we can never be sure that by so doing we will find the *best* current distribution and in fact we are not so sanguine as to hope for such a thing. On the other hand, a good choice of trial functions suggested by a good understanding of the problem may lead to a considerable improvement over present designs.

As a trial current distribution we will take the dipoles on the axis to have moments proportional to $e^{ik'z}$ with the best value of k' to be determined. The ordinary end-fire array corresponds to $k'=k=2\pi/\lambda$.

Then by standard methods and disregarding constant factors it is easily found that the field at some large fixed distance in θ direction ($\theta=0$ being in the line of the array) is

$$E \sim \frac{\sin (k \cos \theta - k')z_0}{(k \cos \theta - k')z_0} \cdot \tag{1}$$

Setting up the integral for the power radiated and imposing the condition that at $\theta=0$ the field must be constant we find that we want to minimize the expression

$$\left(\frac{k - k'}{\sin (k - k')z_0}\right)^2 \int_0^{\pi} \left(\frac{\sin (k \cos \theta - k')z_0}{k \cos \theta - k'}\right)^2 \sin \theta d\theta \tag{2}$$

as a function of k'. The integrations are easily done in terms of Si integrals and we find that (2) is given by

$$\frac{1}{kz_0}\left(\frac{u}{\sin u}\right)^2\left(\frac{\pi}{2}+\frac{\cos 2u-1}{2u}+Si(2u)\right) \tag{3}$$

with

$$u = (k - k')z_0. \tag{4}$$

The function (3) multiplied by kz_0 is plotted in Fig. 1 and it is seen

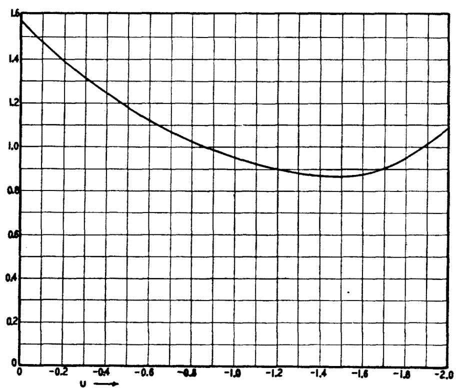


Fig. 1—Curve showing the variation with u of the power radiated from an end-fire array producing a constant signal in the desired direction. The value $u=0$ corresponds to the conventional design.

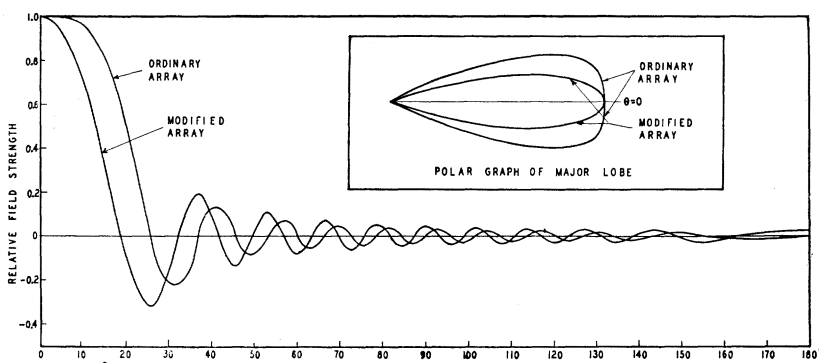


Fig. 2—Graphs showing field strength as a function of angle for normal and modified end-fire arrays, both being 10λ long. To avoid confusion the minor lobes have been omitted from the polar graph.

that the optimum value of k' is specified by $u=-1.47$ and that this value gives a power requirement of 0.55 times that corresponding to $k'=k$ which is the usual practice. The value $u=-1.47$ corresponds approximately to having the number of half waves of current along the array about one greater than the number of times a half wavelength is contained in the array length. In Fig. 2 we have plotted the field strength as a function of angle for an antenna 10 wavelengths long both for $u=0$ and $u=-1.47$.

The merit of an array is often measured by a number called the gain, which is the ratio of the power radiated by a dipole to the power radiated by the array in question when both are producing the same signal in the desired direction. For the end-fire array under discussion (3) gives

$$\text{gain} \cong 4.83 \frac{2z_0}{\lambda} \cdot \qquad (5)$$

This expression is only strictly valid for $1 \ll 2z_0/\lambda$ but will be found to be satisfactory in most practical cases.

The above figures are for a continuous distribution of dipoles. However, if, as has been assumed, the array is several wavelengths long, replacing the dipoles by antennas of height $\lambda/4$ or less will make no noticeable change. Finally it is well known that if the array elements are not separated by more than about $\lambda/3$ the effect is much the same as if a continuous distribution of antennas were used.

The only disadvantage of this array that has occurred to us is that the radiation in the preferred direction is not quite as strong as that from a conventional end-fire array with the same current in the antennas. To get the same field strength the array here proposed would need a current about 1.48 times that of the conventional array.

III. An Array with Strong Vertical Directivity and No Horizontal Directivity

Arrays of this type are easily devised provided one allows the use of radiating elements one wavelength or more high. In the region of broadcast wavelengths, however, this involves rather expensive structures and one wonders whether an array with satisfactory directional properties can be made without the use of radiators extending a considerable distance from the ground. In fact, at least one such array has already been suggested.[1] In this section we show how to devise a more or less infinite number more. Moreover one group of these arrays will be discussed in some detail in this section; another group will be studied in the following section.

For the purpose of inventing such arrays it is convenient to make use of a theory of antenna radiation as developed by one of the au-

[1] F. E. Terman, "Radio Engineering," second edition, p. 678, McGraw-Hill Book Co.

thors.[2] Using various equations in the references cited, and disregarding constant factors, it is easily found that at a large distance from the antenna the field is proportional to[3]

$$E_\theta \sim \sum_n i^{-(n+1)} e^{in\phi} \frac{1}{\sin\theta} \int \bar{A}_{3,n} \cdot \boldsymbol{i} d\tau \tag{6}$$

with

$$A_{3,n} = e^{i(kz\cos\theta + n\phi)} \left[\boldsymbol{k}_\rho i \cos\theta \frac{d}{d\rho} J_n(k\rho \sin\theta) - \boldsymbol{k}_\phi \frac{n\cos\theta}{\rho} J_n(k\rho\sin\theta) + \boldsymbol{k}_z k \sin^2\theta J_n(k\rho\sin\theta) \right]. \tag{7}$$

The notation is as follows:

ρ, ϕ, z are cylindrical co-ordinates with origin at the surface of the ground and with the z axis perpendicular thereto.

$\boldsymbol{k}_\rho$, $\boldsymbol{k}_\phi$, $\boldsymbol{k}_z$ are unit vectors in the ρ, ϕ, and z directions.

θ is the angle between the vertical and the line from the origin to the point at which the field is desired.

$d\tau = \rho d\rho d\phi dz$

$\boldsymbol{i}$ is the current density

$i = \sqrt{-1}$

Note: **Bold-face** type is used for vectors.

Only the vertically polarized component has been retained in (6).

Now let us suppose that all array elements are vertical. Then the ρ and ϕ components of A do not matter. Also, as promised above, let us take the height of the antenna to be small: thus $kz \ll 1$.

$$E_\theta \sim \sum_n i^{-(n+1)} \sin\theta\, e^{in\phi} \int \boldsymbol{i}_z(\rho, \phi) J_n(k\rho\sin\theta) e^{-in\phi} \rho d\rho d\phi. \tag{8}$$

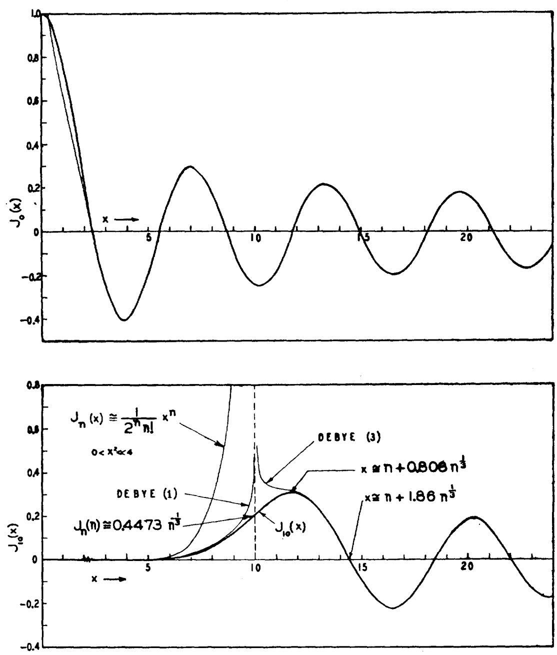
The first thing to observe is that if uniform horizontal coverage is desired, i.e., the time average of the field is to be independent of ϕ, then all but one of the terms in (8) must be substantially zero. If we

[2] W. W. Hansen and J. G. Beckerley, "Concerning new methods of calculating radiation resistance, either with or without ground," Proc. I.R.E., vol. 24, pp. 1594–1621; December, (1936);
W. W. Hansen, "Directional characteristics of any antenna over a plane earth," *Physics*, vol. 7, p. 460; December, (1936).
W. W. Hansen, "A new type of expansion in radiation problems," *Phys. Rev.*, vol. 47, p. 139; January 15, (1935).
W. W. Hansen and J. G. Beckerley, "Radiation from an antenna over a plane earth of arbitrary characteristics," *Physics*, vol. 7, p. 220; June, (1936).
W. W. Hansen, "Transformations useful in certain antenna calculations," *Jour. App. Phys.*, vol. 8, p. 282; April, (1937).

[3] This follows from equation (18) of the second reference and (12) of the third.

wish the nonzero term to be the nth one we can accomplish this by having i be $e^{in\phi}$ times some function of ρ. It will be supposed for the moment that all terms but one have been suppressed in this way. Later, we shall have to investigate how well this assumption can be fulfilled when instead of a continuous distribution of antennas we have some practical approximation thereto.

(a)



(b)

Fig. 3

(a) The heavy line is a plot of $J_0(x)$ as a function of x. The light line shows the approximation $J_0(x) \cong \sqrt{2/\pi x}\cos(x - \pi/4)$ which is good when x is large.

(b) Plot of $J_{10}(x)$ as a function of x. Also plotted are various approximations for $J_n(x)$. The two curves labeled Debye (1) and Debye (3) are graphs of the functions

$$(1) \qquad J_n\left(\frac{n}{\cosh\tau}\right) \cong \sqrt{\frac{1}{2\pi n \tanh\tau}}\, e^{n(\tanh\tau - \tau)}, \quad \sqrt{n^2 - x^2} \gg 1$$

$$(3) \quad J_n\left(\frac{n}{\cos\tau}\right) \cong \sqrt{\frac{2}{\tau n \tan\tau}} \cos\left(n\tan\tau - n\tau - \frac{\pi}{4}\right), \; \sqrt{x^2 - n^2} \gg 1.$$

There exists a third Debye expansion valid when $x \cong n$ but this has been omitted. Also not shown is the approximation used in Fig. 3(a); namely,

$$J_n(x) \cong \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{2n+1}{4}\pi\right), \frac{n^2}{2} \ll x.$$

This has been omitted because it is not a useful approximation to $J_{10}(x)$ when x is below 100 or so.

Having saved just one term of (8) and so made $\boldsymbol{E}$ independent of ϕ we now inquire how we can make $\boldsymbol{E}$ finite at $\theta=\pi/2$ and nearly zero elsewhere. For progress in this direction a knowledge of the general behavior of Bessel functions is needed. To refresh the reader's memory, Figs. 3(a) and 3(b) give plots of $J_0(x)$ and $J_{10}(x)$ together with various approximation formulas. One method is now plain. If we choose to use the nth term of (8) we can make $\boldsymbol{i}$ as a function of ρ vary about like $J_n(k\rho)$. Then when $\theta=\pi/2$, sin $\theta=1$, the integrand in (8) will always be positive. But when θ differs greatly from $\pi/2$ the current and $J_n(k\rho \sin \theta)$ will have sometimes the same sign and sometimes the opposite and more or less complete cancellation will result. This idea may be rendered more precise by noting that

$$\int_0^\infty xJ_n(\alpha x)J_n(x)dx = \tfrac{1}{2}\delta(1-\alpha) \tag{9}$$

where $\delta(1-\alpha)$ is a function of $1-\alpha$, that is, zero when $\alpha\neq 1$ and infinite at $\alpha=1$ in such a way that the integral of $\delta(1-\alpha)d\alpha$ is unity.

Thus if we had $\boldsymbol{i}\sim \boldsymbol{k}_z e^{in\phi}k\rho J_n(k\rho)$ and this current distribution extended to $k\rho\rightarrow\infty$ we would get $\boldsymbol{E}$ finite at $\theta=\pi/2$ and zero elsewhere. Such an infinite antenna is not possible but one might reasonably expect that cutting off the current at some finite radius would give the same general result and this is in fact true.

Now it would be desirable to investigate the gains obtainable as a function both of the radius of the circle occupied by the antennas and of the order of the Bessel function used. However, we have not succeeded in solving this problem analytically and a numerical attack seems hardly worth while. We have therefore been satisfied with an analytic investigation of the gain as a function of antenna size for J_0 and certain special results for higher-order Bessel functions.

We consider then a current distribution consisting of dipoles oriented in the z direction and with strength $\rho J_0(k\rho)$ up to some radius and zero elsewhere. Then it is readily found that if the radius of the array is considerably larger than λ, the field near $\theta=\pi/2$ is given approximately by

$$E_\theta \sim \tan\theta\,\frac{\sin\left((1-\sin\theta)(k\rho_0-\pi/4)\right)}{\cos\theta} \tag{10}$$

where ρ_0 is the radius of the array and is chosen subject to the restriction that $k\rho_0=(m-1/4)\pi$, with m an integer. Using this approximate expression for E we can easily find the gain,

$$\text{gain} \cong 0.707\sqrt{m-1/2}. \tag{11}$$

Remembering our experience with the end-fire array we surmise that distributing the dipoles according to $\rho J_0(k'\rho)$ might be better. This is true and it is found by some numerical work that if k' is determined by

$$\frac{k' - k}{k}\left(k\rho_0 - \frac{\pi}{4}\right) = 1.8 \tag{12}$$

then the gain is

$$\text{gain} \cong 1.17\sqrt{m - 1/2}. \tag{13}$$

How many antennas will be needed to approximate the assumed distribution satisfactorily is the next, and somewhat tedious, question. Going in the ρ direction one ring of antennas per loop of the Bessel function is plainly sufficient. How many antennas are to be in each ring is a harder question. In a general way there must be a large enough

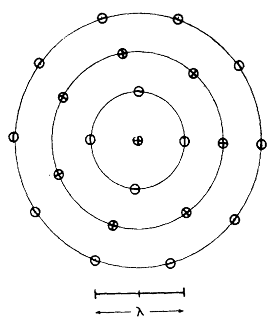


Fig. 4—Top view of multiple-ring array with $n=0$, $m=4$. Circles are at $k\rho$ values 0, 3.29, 6.02, 8.73 with relative currents per antenna 1, 0.50, 0.41, and 0.36. Gain $=2.31$. Circles represent array elements, those with a + inside have the current flowing 180 degrees out of phase with that in the antennas labeled −.

number to keep higher-order J_n from appearing with appreciable coefficients in the expression for the field. To a very rough approximation this will be avoided if the number of antennas per ring exceeds $k\rho_0$. However, because of the rapid decrease of high-order Bessel functions on going toward $k\rho=0$ it is not necessary that the inner rings have as many antennas as the outer. Any real solution of this problem will involve a knowledge of just how much horizontal directivity can be tolerated and will also require some cut-and-try work. As an example of the general form such a solution would take we have prepared Fig. 4 which shows a possible array with $m=4$ which gives for the coefficients of the first few J_n the following: 1.00, 0, 0, 0, 0.17, 0, 0, 0.19, 0, 0, 0.17, gives a gain of 2.31, and does not use an impossible number of antennas.[4] By a

[4] It must be remembered that comparatively short antennas may be used. We had thought, for example, of using wooden poles like those used for power distribution which should be quite cheap by comparison with, say, a half-wave unguyed tower.

slight increase in the number of antennas the coefficients of the higher J can be greatly reduced.

It will be observed in the above case that the number of array elements in any ring is about equal to the $k\rho$ value of the ring plus one or two. This will be true in general and in consequence the total number of elements in such an array will, in the limit of large m, be a quadratic function of m. Thus the number of elements rises rapidly with m (like m^2) while the gain increases with $\sqrt{m-1/2}$.

IV. Second Type of Array to Radiate Uniformly in a Horizontal Direction

Instead of having the array extend over a number of waves of the Bessel function as in the preceding section, it is possible to get the desired directivity by putting a ring of antennas at a $k\rho$ value smaller than that corresponding to the first maximum of the Bessel function chosen.

To see how well this will work let us choose a current distribution purely for analytic convenience; namely, let there be a continuous ring of dipoles of strength $e^{in\phi}$ at a radius such that $(k\rho)^2 \ll 4$. Then J_n in the integral of (8) may be sufficiently well approximated by x^n and we find $E_\theta \sim \sin^{n+1}\theta \, e^{in\phi}$ and from this the gain is found

$$\text{gain} = \frac{\int_0^{\pi/2} \sin^3 \theta d\theta}{\int_0^{\pi/2} \sin^{2n+3} \theta d\theta} = \frac{2}{3} \cdot \frac{1 \cdot 3 \cdot 5 \cdot \cdot \cdot (2n+3)}{2 \cdot 4 \cdot \cdot \cdot (2n+2)} \tag{14}$$

and for large values of n this is well approximated as

$$\text{gain} \cong \frac{4}{3\sqrt{\pi}} \sqrt{n+3/2} = 0.752\sqrt{n+3/2}. \tag{15}$$

For example for $n=1$ the approximation is good to five per cent and it gets better with increasing n. Besides this purely mathematical approximation we may recall that (15) depends on the assumption of a continuous ring of short antennas at radius $(k\rho)^2 \ll 4$. In Fig. 5 we have plotted the gain as a function of n and in Fig. 6 we have shown a directional pattern for $n=5$. It may be noticed that unlike any previous array known to the authors, this array produces a field that is a monotonically decreasing function of angle away from the optimum direction. In other words there are no minor lobes.

We may note in passing that this array and the two previously discussed have one feature in common; namely, that the antennas are not so placed and so phased as to make the effects add as well as possible in the preferred direction.

We consider now how such a current distribution is to be approximated in practice and how the performance of such a practical realization will compare with the results above.

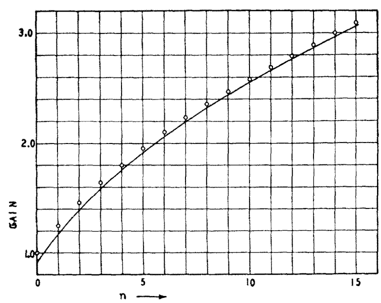


Fig. 5—Gain of a single ring of antennas, very near to the origin, as a function of n. Points are exact values, the curve is computed from the approximate equation (15).

The radius chosen for the ring will be some practical compromise between the following two conflicting requirements. First, the smaller the radius the more rapidly the Bessel function will decrease with decreasing argument and so the better the directivity. Second, the larger the radius the larger the radiation, for a given current in the elements, and so the less important the ground losses.

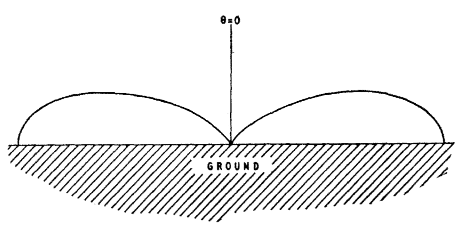


Fig. 6—Field produced by a single-ring array $n=5$, $k\rho \ll 2$.

To aid in selecting a good compromise we have prepared Figs. 7, 8 and 9.

Fig. 7 shows, for the special case $n=10$, how the directional pattern is changed by changing the radius of the ring.

From curves like those of Fig. 7 one could find, by numerical integration, the exact value of gain for any desired values of n and $k\rho$. This would be a lot of work, however, and so we have used an approximate method which works well within a limited range. We assumed that, for $0<k\rho<n$, J_n could be approximated by a single power of $k\rho$, with the power chosen to give the best fit at a $k\rho$ value corresponding to the antenna position. Then we used the approximate formula (15) and so constructed Fig. 8, which shows gain as a function of $k\rho$ for a

number of values of n. We may note that both approximations are wrong in the same direction so that the curves of Fig. 8 always underestimate the gain.

More exact values of the gain may be wanted or one might want the gain in the region above $k\rho = n$. If so, numerical integration can be used.

Another method that is perhaps easier is to work the problem in spherical co-ordinates using (4) and (5) of the first paper cited in footnote 2. It will be found that the gain comes out as a ratio of two series

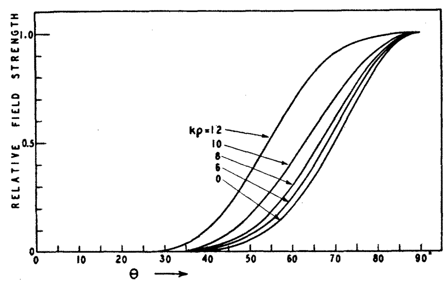


Fig. 7—Fields produced by single-ring arrays with $n = 10$, $k\rho = 12$, 10, 8, 6, 0.

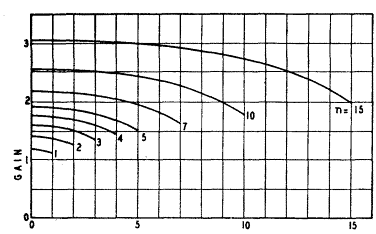


Fig. 8—Approximate values of the gain for single-ring arrays for various values of n and $k\rho$. The curves always underestimate the gain and the error becomes larger with increasing $k\rho/n$.

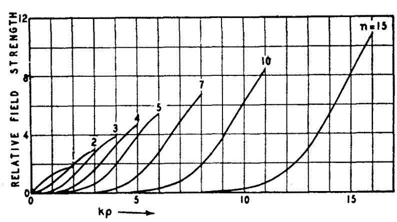


Fig. 9—Field at $\theta = \pi/2$ produced by a ring of $3n$ radiators measured in units of the field that would be produced by an isolated radiator.

which converge very rapidly. Unfortunately the individual terms are some bother to compute as they involve $P_l^n(\theta = \pi/2)$, $\partial P_l^m/\partial\theta$, and $J_{l+1/2}$, functions which are not too well tabulated, especially for the high values of l and n likely to be needed in the present application. The following formulas, however, eliminate part of this trouble.

$$P_l^n(\theta = \pi/2) = \frac{(l+n)!}{2^l\left(\dfrac{l-n}{2}\right)!\left(\dfrac{l+n}{2}\right)!}(-1)^{(l-n)/2}$$

$$l - n \text{ even,}$$

$$= 0, \qquad l - n \text{ odd}$$

$$\frac{\partial}{\partial\theta} P_l^n\left(\theta = \frac{\pi}{2}\right) = \frac{(l+n)!}{2^{l-1}\left(\frac{l-n-1}{2}\right)!\left(\frac{l+n-1}{2}\right)!n!} (-1)^{(l-n+1)/2} \tag{16}$$

$$l - n \text{ odd} \tag{17}$$

$$= 0, \qquad l - n \text{ even.}$$

However, we have not thought it worth while at present to extend the curves of Fig. 8 into the region where calculation by one or other of the above methods is needed.

Together with Fig. 8 we need Fig. 9, which gives data from which the relative effectiveness of the array as a radiator can be found. These curves assume $3n$ radiators[5], short compared to $\lambda/4$, and give the field strength produced by the array, divided by the field strength that would be produced by a single radiator acting alone. For example if we use $n=5$ and put a ring of 15 short antennas at $k\rho=5$ ($\rho=0.795\lambda$) the field at a large distance at angle $\theta=\pi/2$ will be 3.91 times that produced by a single antenna working alone.

We now consider how many antennas must be in the ring for any given n. We first note that a ring of, say, m dipoles may be considered as built up of a sum of continuous dipole distributions, the proper sum being the summation over l of

$$e^{in\phi} \sum_{-\infty}^{+\infty} e^{ilm\phi}. \tag{18}$$

Then by reference to (8) we see that the ratio of the coefficient of the desired term in $e^{in\phi}$ to the first unwanted term which is $e^{i(n-m)\phi}$ is just $J_n/J_{|n-m|}$. Thus m must be bigger than $2n$. Just how much depends on how much variation of field with ϕ can be tolerated and is to be determined when this and the other general features of the array design are known. A value $m=3n$ is always more than enough and something of the order of magnitude $2n+3$ is perhaps the minimum tolerable. For example the antenna mentioned above with $n=5$, $k\rho=5$ gives a ratio of maximum field strength to minimum of 3.02 for 11 antennas in the ring, 1.51 for 12, 1.15 for 13, 1.04 for 14, and 1.01 for 15.

Thus the total number of elements in a single-ring J_n array is a more or less linear function of n, while the gain goes with $\sqrt{n+3/2}$. Numerical trial shows that with about 20 array elements the multiple-ring J_0 antenna gets better gain than the single-ring J_n antenna. It follows that if the total number of antennas is the important factor economi-

[5] See later where it is shown that $3n$ radiators is enough to give very uniform horizontal coverage. If fewer antennas are used the field can be found by direct proportion.

cally the J_0 antenna will be best for low gains and the J_n for high. The transition point depends on the uniformity of coverage in the ϕ direction and other things.

V. Further Possibilities

It is plain that there are an infinite number of possible arrays with vertical directivity not here explored. For example one might distribute dipoles with density $\rho J_2(k\rho)e^{i2\phi}$ and let ρ go to values large enough so that J_2 will go through several maxima and minima, say three. In fact the antenna suggested by Terman[1] is of approximately this type.

In addition to these the present methods might well be applied to the design of arrays with different directivity characteristics than those here considered. For example, it would be easy to make an array that would direct the radiation along the ground like the last two described and would also confine the radiation between specified values of the angle ϕ. Such an antenna might be useful, for example, to broadcast stations near a seacoast. Another array on which we have done some figuring involves two groups of antennas with arrangements made for disconnecting one group when desired. In the daytime one group only is used and the design is such that the radiation is confined to angles close to the ground. At night both groups of antennas are used and, in addition to the ground wave, a sky wave is radiated. Moreover, there is a zero of intensity between the radiation going upward and that going along the ground, and one may hope that, if the radiation pattern is chosen correctly, the radiation directed upward will be reflected from the Heaviside layer and add service area without coming down where the ground wave is of comparable intensity and spoiling service by introducing fading.