Understanding the Hansen-Woodyard End Fire Array

An analysis of the Hansen-Woodyard End-Fire Array in comparison to an ordinary array.

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```
clear;
```

Antenna Arrays

Array Factor

The array factor is a formula that is used to represent a N-element array. The model uses the assumption that each element in the array are identical nodes.

$$AF_N = \frac{1}{N}\frac{sin(\psi N/2)}{sin(\frac{\psi}{2})}$$

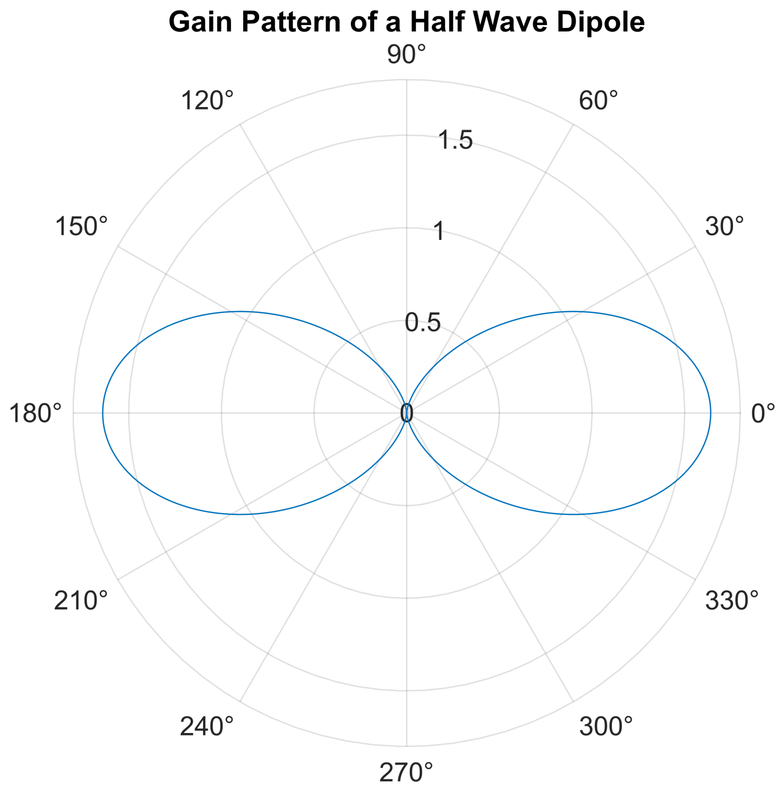
The product of the array factor with an antenna model of the designer's choice (for example, the gain of a dipole) is an N-element array.

Parameters:

```
f = 2*10^9;                  % Frequency
c = 3*10^8;                  % Speed of Light
lambda = c/f;                % Wavelenght
k = 2*pi/lambda;             % Wave Number
theta = 0:.001:2*pi;       % Angle (theta)
```

Antenna Models

```
%G = (1.67.*sin(theta).^3)
theta_rotate = theta+pi./2;
G = 1.64.*(cos(cos(theta_rotate).*pi./2).^2)./(sin(theta_rotate).^2); %Half-Wave
Dipole
%G = (3./2).*sin(theta).^2;                             %
figure
polarplot(theta, G)
hold on
title('Gain Pattern of a Half Wave Dipole');
hold off
```



Ordinary End-Fire Arrays

An ordinary end-fire array is characterized by an N-element array of equally spaced antennas. Each antenna are identical and of equally magnitude. The ordinary end-fire array can only be modified by the number of elements in the array and the distance between each element. The spacing determines if the antenna will point in both directions, or in only one direction.

```
N = 10;                                                   % Number of elements in the
array
d = [lambda/2;lambda/4;lambda/6;lambda];      % Distance between each antenna
element
k = 2.*pi./lambda;                                        % Wave number
```

The phase shift for an ordinary end fire array is equal to $\beta = -kd$ for $\theta = 0$ and $\beta = kd°$ for $\theta = 180°$. It is not influenced by the number of elements.

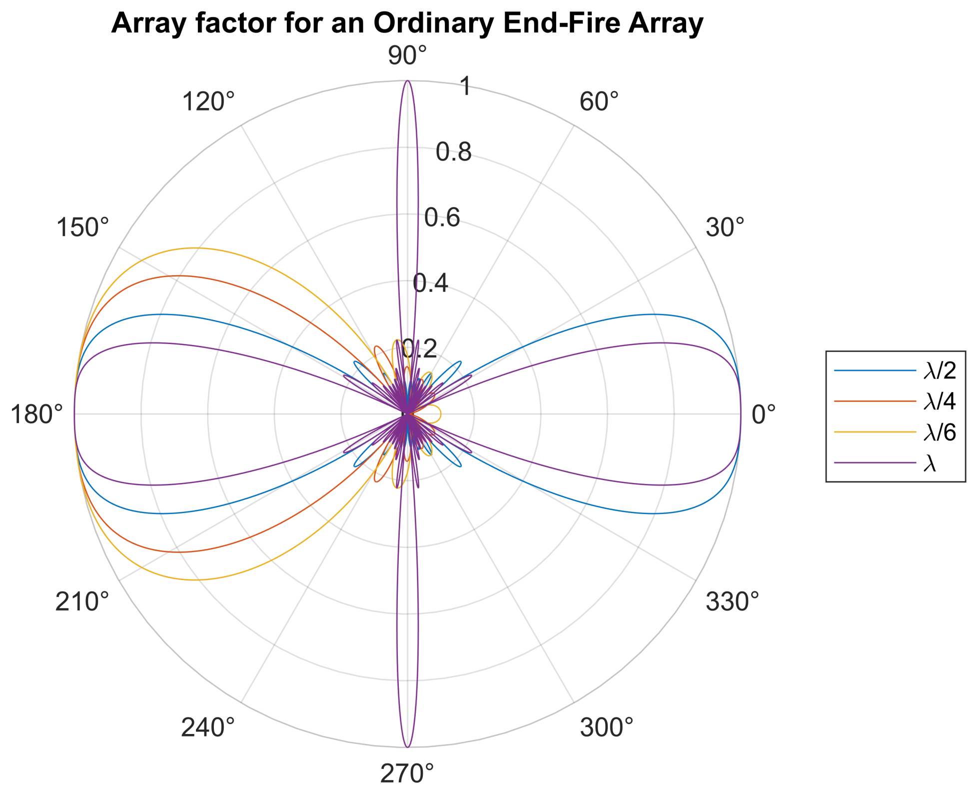
```
beta = k.*d;                                              % Phase Shift
psi = (k*d.*cos(theta) + beta);
AF_n_ordinary = abs((sin(psi.*N./2) ./ sin(.5.*psi) ));
AF_n_ordinary_norm = (AF_n_ordinary./N);      % Array Factor
```

You can visibly see in this plot how the distance between antenna elements affects the overall pattern of the array factor. For the antenna to have major lobes pointing at both ends, the space between the elements should be $d = \lambda/2, \lambda, n\lambda$ where $n = 1, 2, 3, \ldots$.

However, the further the distance between the elements, the more high intensity lobes will appear.

Ideally, a proper ordinary end-fire array should have spacing at most $\lambda/2$

```
figure
polarplot(theta,AF_n_ordinary_norm);
hold on
title('Array factor for an Ordinary End-Fire Array');
legend('\lambda/2','\lambda/4','\lambda/6','\lambda');
hold off
```



Hanson-Woodyard End-Fire Array

The Hansen-Woodyard Array is a modified form of the Ordinary End-fire array. It features a progressive phase shift that is influenced by the number of elements.

$\beta = -(kd + \frac{2.92}{N})$ for $\theta = 0°$

$\beta = +(kd + \frac{2.92}{N})$ for $\theta = 180°$

The distance between elements is also influenced by the number of elements - the smaller amount of elements, the closer together. The higher number, the closer to $d \approx \lambda/4$.

$d = (\frac{N-1}{N})\frac{\lambda}{4} \approx \frac{\lambda}{4}$

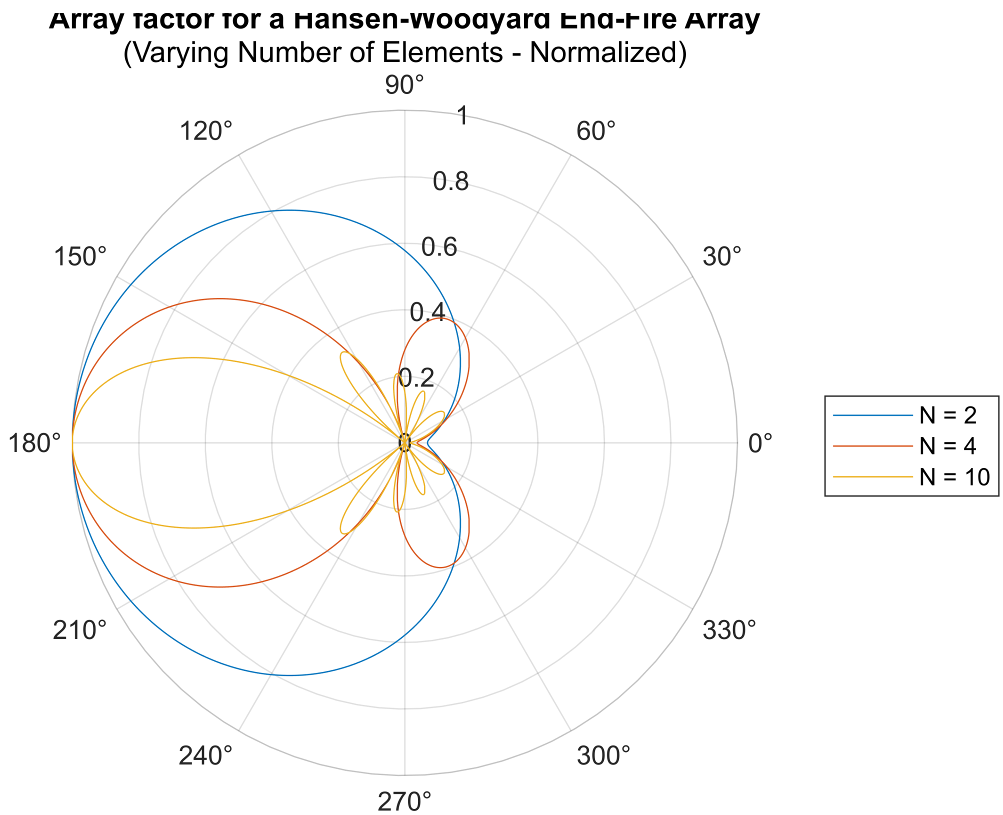
This design is useful for large N. We can see this with the by using the model for small N.

Parameters:

```
f = 2*10^9;                    % Frequency
c = 3*10^8;                    % Speed of Light
lambda = c/f;                  % Wavelenght
k = 2*pi/lambda;               % Wave Number
theta = 0:.00001:2*pi;         % Angle (theta)

N = [2;4;10];
d = ((N-1)./N).*(lambda./4);           % Distance between each antenna element
beta = k.*d + 2.94./N;                 % Phase Shift
psi = k.*d.*cos(theta) + beta;         % Relative Phase
AF_N = abs((sin(psi.*N./2) ./ sin(.5.*psi) ));
Max = [max(AF_N(1,:)); max(AF_N(2,:)); max(AF_N(3,:))];
AF_HW = abs(AF_N)./Max;
```

The directivity is low. The main beams are so wide that an ordinary end-fire array would be preferred over the Hansen-Woodyard.

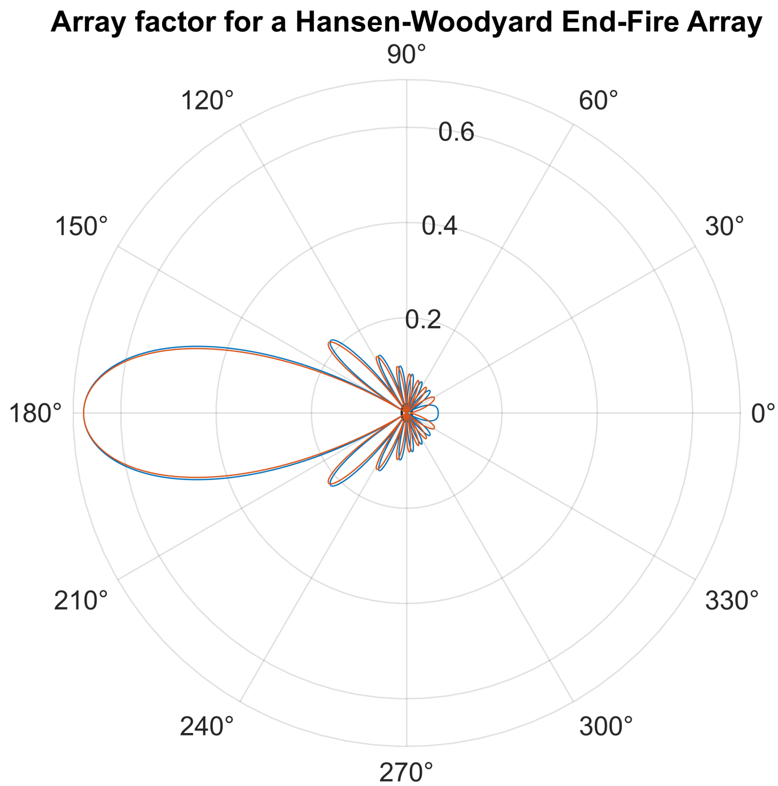
```
figure(3)
polarplot(theta,AF_HW);
hold on
title('Array factor for a Hansen-Woodyard End-Fire Array','(Varying Number of
Elements - Normalized)')
legend('N = 2','N = 4','N = 10');
hold off
```



For this analysis, I found that making $N \geq 10$ allows for a much fairer comparison between the two array designs, so onward I will be using 10-element arrays.

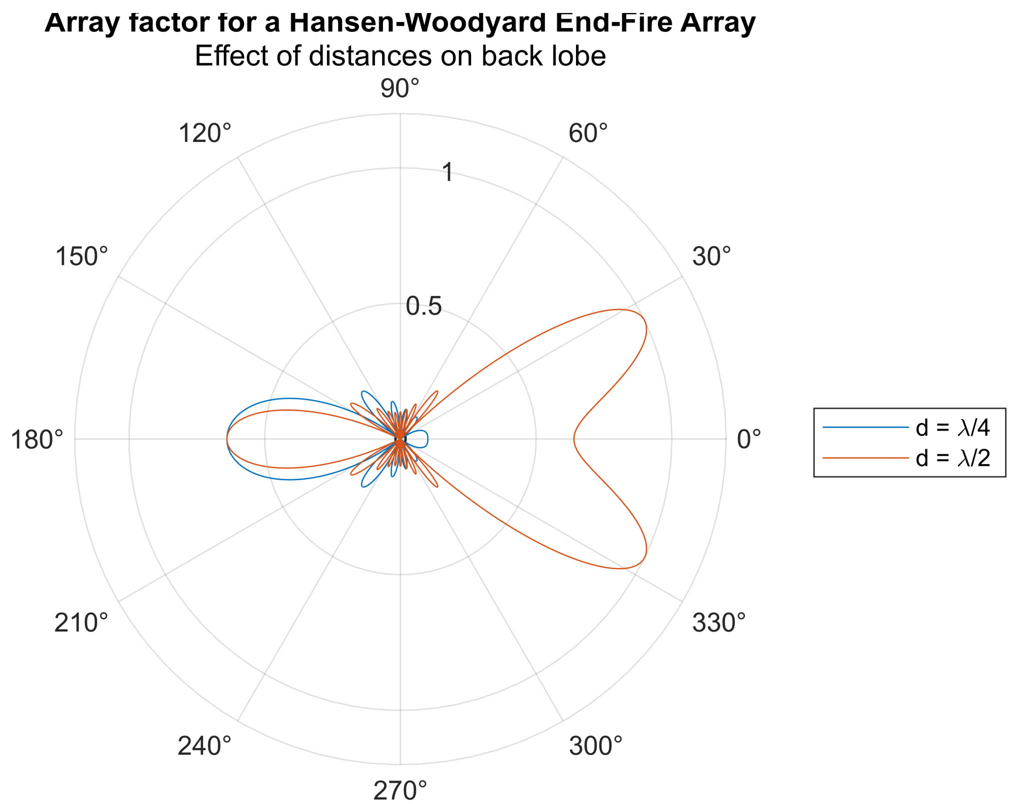
Comparing the distance formula and the proposed $\lambda/4$ between each element.

```
N = 15;
d = [((N-1)./N).*(lambda./4);lambda/4];                    % Distance between each
antenna element
beta = k.*d + 2.94./N;                         % Phase Shift
psi = k*d.*cos(theta) + beta;
AF_num = sin(psi.*N./2);
AF_den = sin(.5.*psi)*N;
AF_N = abs(AF_num./AF_den);

figure(4)
polarplot(theta,AF_N);
hold on
title('Array factor for a Hansen-Woodyard End-Fire Array')
hold off
```



Effect of the distance between elements and the backlobe

```
N = 10;
d = [lambda./4;lambda/2];                              % Distance between each antenna element
beta = (k.*d + pi./N);                                    % Phase Shift
psi = k*d.*cos(theta) + beta;
AF_num = sin(psi.*N./2);
AF_den = sin(.5.*psi);
AF_N = abs(AF_num./AF_den);
AF_N_norm = AF_N.*(1/N);

figure(5)
polarplot(theta,AF_N_norm);
hold on
title('Array factor for a Hansen-Woodyard End-Fire Array','Effect of distances on
back lobe')
legend('d = \lambda/4','d = \lambda/2');
hold off
```



Comparing the Hansen-Woodyard End-fire array with the Ordinary End-fire array

```
theta = -pi:.00001:pi;
N = 10;
d = lambda/4;                   % Distance between each antenna element
beta_HW = (k.*d + pi./N);       % Phase Shift - HW
beta = k.*d;                    % Phase Shift - Ordinary
psi_ordinary = k*d.*cos(theta) + beta;
psi_HW = k*d.*cos(theta) + beta_HW
```

```
psi_HW = 1×628319
    0.3142    0.3142    0.3142    0.3142    0.3142    0.3142    0.3142    0.3142 ···
```

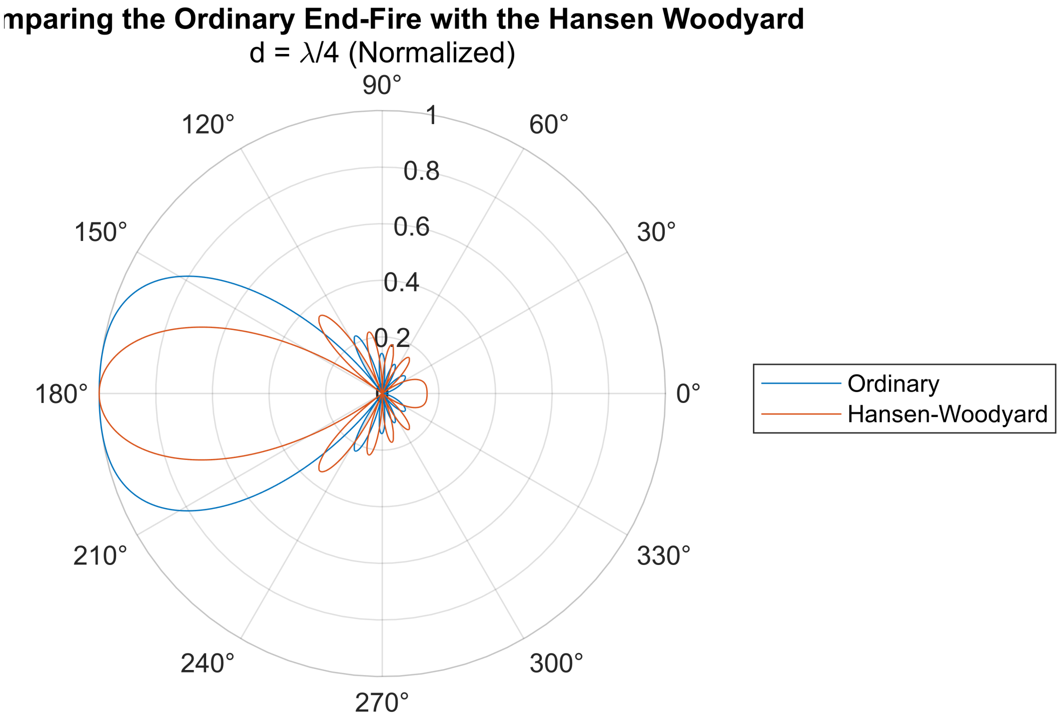
```
psi = [psi_ordinary; psi_HW];
AF_num = sin(psi.*N./2);
AF_den = sin(.5.*psi);
AF_N = abs(AF_num./AF_den);
AF_N_norm = AF_N(1,:).*(1/N);
AF_HW_Norm = AF_N(2,:)./(max(AF_N(2,:)))
```

```
AF_HW_Norm = 1×628319
    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000 ···
```

```

Null_first_index = find(AF_HW_Norm<.05,1);
Null_first_index(1);
MainLobeBW = rad2deg(2*pi + 2*(theta(Null_first_index)))
```

```
MainLobeBW = 71.3734
```

```
polarplot(theta,AF_N_norm);
hold on
polarplot(theta,AF_HW_Norm);
title('Comparing the Ordinary End-Fire with the Hansen Woodyard','d = \lambda/4
(Normalized)')
legend('Ordinary','Hansen-Woodyard');
hold off
```

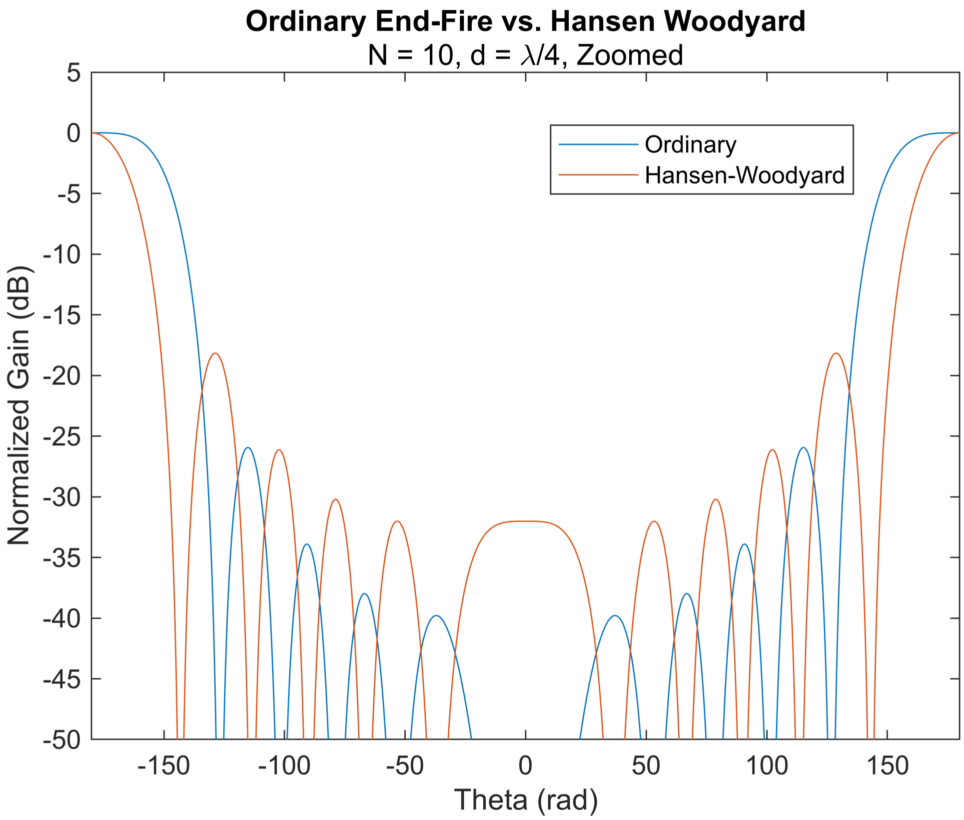


What I find odd is using the given formula for the nromalized array factor, I do not get magnitude of 1. It goes against the concept of normalization.

Radiation Intensity of the Antenna Arrays

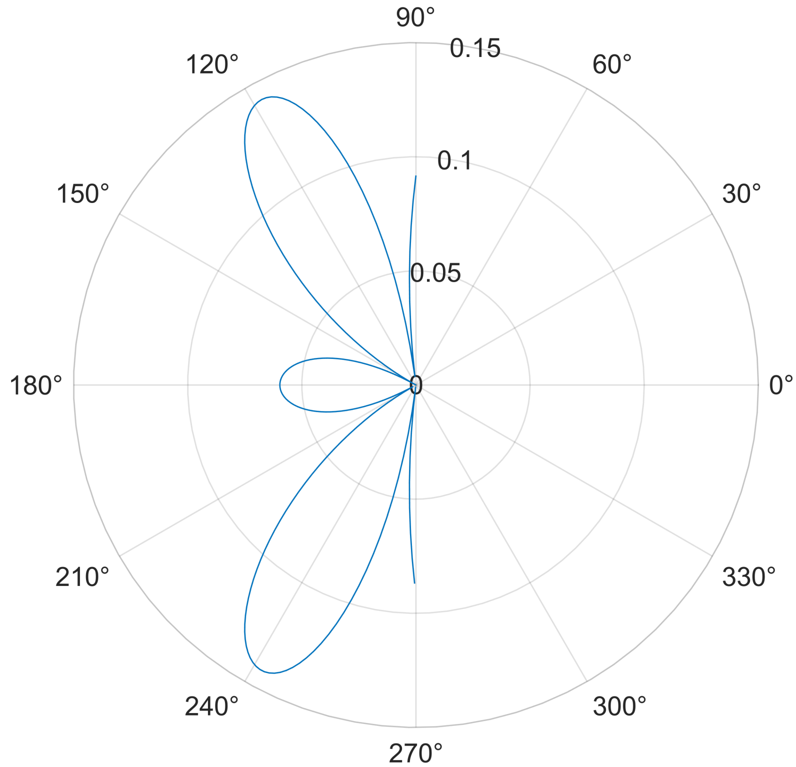
```
figure
plot(rad2deg(theta),mag2db(AF_N_norm.^2))
hold on
plot(rad2deg(theta), mag2db((AF_HW_Norm).^2))
xlim([-180 180])
ylim([-50 5])
title('Ordinary End-Fire vs. Hansen Woodyard','N = 10, d = \lambda/4')
legend('Ordinary','Hansen-Woodyard');
legend("Position", [0.55,0.8,0.25,0.0])
```

```
xlabel('Theta (rad)');
ylabel('Normalized Gain (dB)')
hold off
```

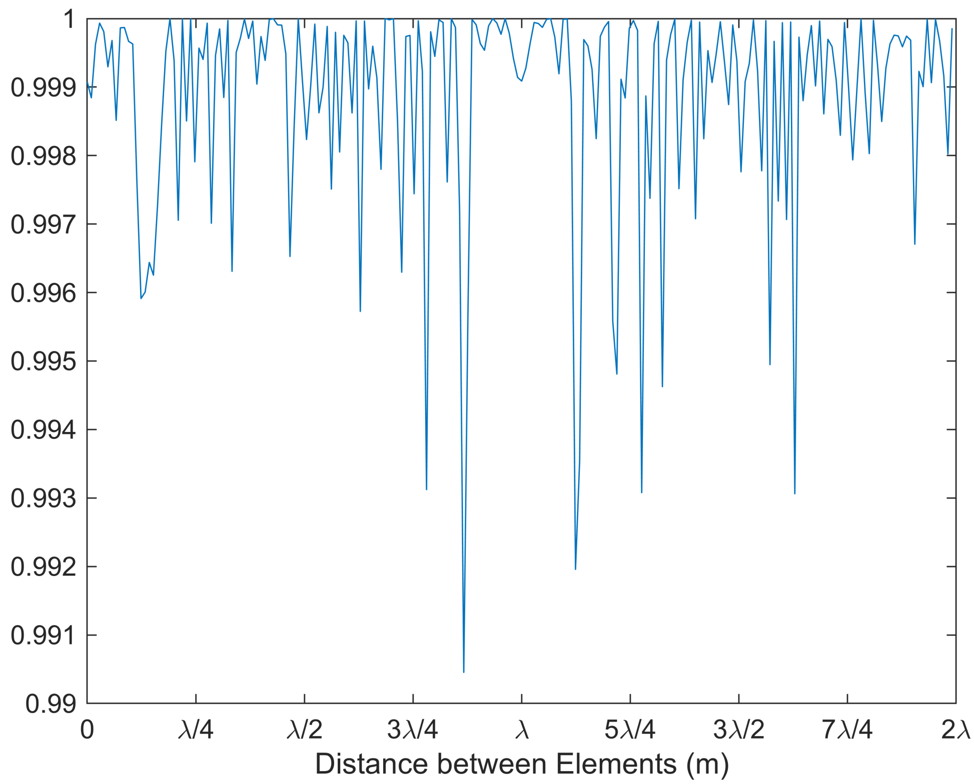


Effect of distance between each element and the back lobe

```
theta = pi/2:lambda/10:3*pi/2;      % Angle (theta)
N = 10;
a = size(theta,2);
% d = [2*lambda;lambda;lambda/2;lambda/4;lambda/6];                  % Distance
between each antenna element
d = 0:2*lambda/a:2*lambda;
d = d(1:end-1).';
beta = -(k.*d + pi./N);                          % Phase Shift
psi = k.*d.*cos(theta) + beta;
AF_num = sin(psi.*N./2);
AF_den = sin(psi./2);
AF_N = abs(AF_num./(AF_den.*N));
peak = max(AF_N);

figure
polarplot(theta, AF_N(15,:))
```




```
figure
plot(d,peak)
hold on
xlabel('Distance between Elements (m)')
set(gca,'Xtick',0:lambda/4:2*lambda,'XTickLabel',{'0','\lambda/4','\lambda/
2','3\lambda/4','\lambda','5\lambda/4','3\lambda/2','7\lambda/4','2\lambda'});
hold off
```

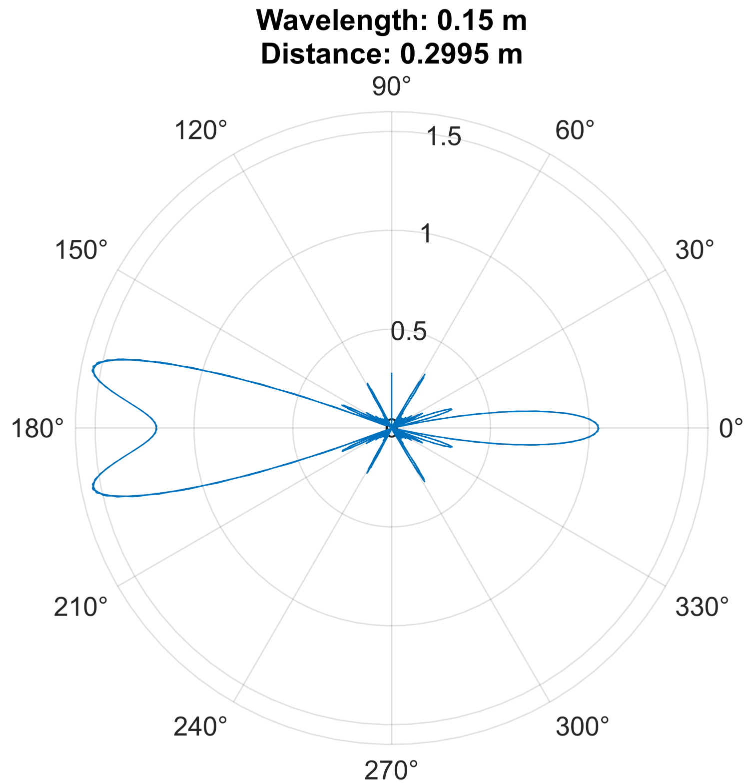



```
theta = pi/2:lambda/5:3*2*pi;      % Angle (theta)
N = 10;
a = size(theta,2);
G = 1.64.*(cos(cos(theta+pi/2).*pi./2).^2)./(sin(theta+pi/2).^2); %Half-Wave Dipole

% d = [2*lambda;lambda;lambda/2;lambda/4;lambda/6];                  % Distance
between each antenna element
d = 0:2*lambda/a:2*lambda;
d = d(1:end-1).';
beta = -(k.*d + pi./N);                            % Phase Shift
psi = k.*d.*cos(theta) + beta;
AF_num = sin(psi.*N./2);
AF_den = sin(.5.*psi);
AF_N = abs(AF_num./AF_den)./N;
Pattern = G.*AF_N;
peak = max(Pattern);

N = [1,2,3,4,5]
```

```
N = 1×5
     1     2     3     4     5
```

```
dipole_pos = d.*N;
Y = ones(1,5);

for i=1:size(theta,2)
```

```
    polarplot(theta,Pattern(i,:))
        h = title({sprintf('Wavelength: %.2f m',lambda);sprintf('Distance: %.4f m',d(i))}); % title is 13 chars in both cases
    drawnow
    pause(.1)
end
```



Optimizing the ordinary end-fire array

We want to maximize the directivity of our antenna, which means we want to minimize the beamwidth of our main lobe. Decreasing the half power beamwidth (HPBW)

```
lambda = 0.15;
N = 10;
learning_rate = 0.1
k = 2.* pi/lambda
d = .05
input_d = lambda/10:lambda/100:.05;
```

The HPBW approximation for an ordinary endfire is applicable when $\frac{\pi d}{\lambda} << 1$. This means that the distance between our elements are a limited range.

```
Qualify = pi.*input_d./lambda
plot(input_d,Qualify)
```

```
hold on
yline(1,'r')
xlim([.015,.075])
hold off

HPBW = 2.*acos(1 - (1.39.* lambda./(pi.*N.*input_d)))
plot(input_d, rad2deg(HPBW))
hold on
xlabel('Distance between elements (m)')
ylabel('HPBW (degrees)');
title('Approximated Half Power Beamwidth (HPBW)','as distance between elements is
varied');
hold off
```

When using a simple approach as this, the smallest we can get our beamwidth, if we are being generous with what counts as what is much less than 1, is around 60 degrees.

```
beta = k.*d;                                                % Phase Shift
psi = (k*d.*cos(theta) + beta);
AF_n_ordinary = abs((sin(psi.*N./2) ./ sin(.5.*psi) ));
AF_n_ordinary_norm = (AF_n_ordinary./N);      % Array Factor
figure(3)
polarplot(theta,AF_n_ordinary_norm);
hold on
title('Array factor for an Ordinary End-Fire Array','d = 0.05 m');
hold off
```

```
syms d_change
HPBW_sym = 2.*acos(1 - (1.39.* lambda./(pi.*N.*d_change)))
HPBW_p1 = diff(HPBW_sym)
HPBW = subs(HPBW_p1,d_change,input_d)
```

```
clear;
f = 2*10^9;                   % Frequency
N =10
```

```
N = 10
```

```
c = 3*10^8;                   % Speed of Light
lambda = c/f;                 % Wavelenght
k = 2*pi/lambda;              % Wave Number
theta = 0:.01:2*pi;        % Angle (theta)
mx = lambda/2
```

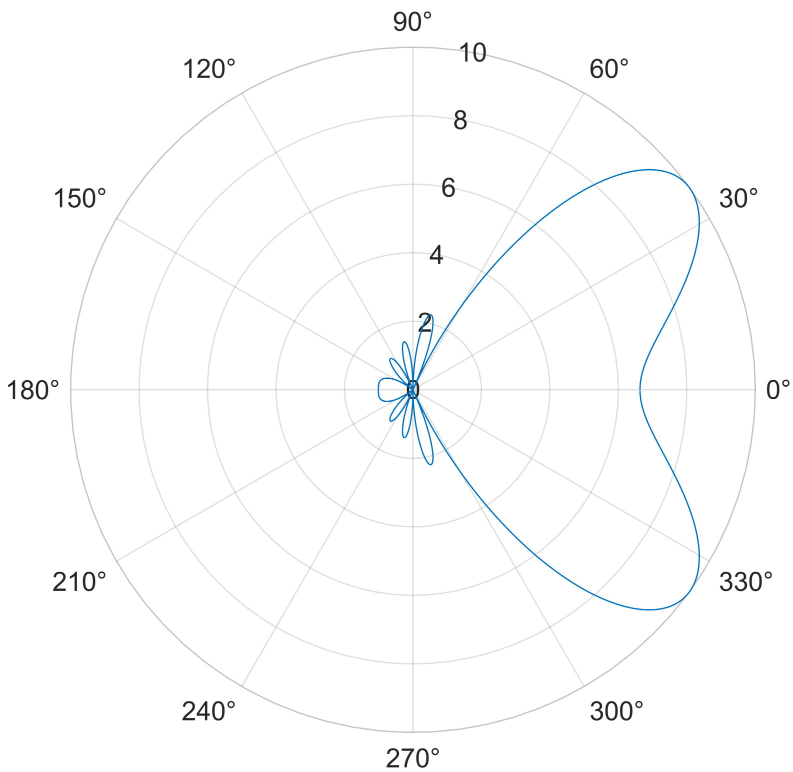
```
mx = 0.0750
```

```
% x =  0:mx/length(theta):mx;
% x = x(1:end-1).';
```

```
x = lambda/4
```

```
x = 0.0375
```

```
y =  0:2*pi/length(theta):2*pi;
y = y(1:end-1).';
AF = sin((k.*x.*cos(theta) + y).*N./2)./ sin(.5.*(k.*x.*cos(theta) + +y));

for i=1:size(theta,2)
    polarplot(theta,abs(AF(i,:)))
    drawnow
end
```




```
AF = sin((k.*x.*cos(0) + y).*N./2)./ sin(.5.*(k.*x.*cos(0 ) + +y));

plot(y,AF)
```