CISC 7200X – Analysis of Algorithms

Realizing Degree Sequences Programming Project – Testing

Testing your program:

- Below there are six tasks each containing several (already sorted) input sequences to test your program.
- Do as many tasks as you can (not necessarily in their order) before the video meeting.
- For the first four problems you are also asked if you can construct the output graph without running your program.
- Do the same tasks for each variant of the HH algorithm that you implemented.

Remarks:

- If a sequence is not realizable, your program should announce this.
- If the sequence is realizable, illustrate the graph that realizes it according to your program. This graph must be the output of the HH algorithm that was taught in class.
- Call the vertices $A, B, C, \ldots$ (the longest sequence has less than 27 vertices) for which

  $$d(A) \geq d(B) \geq d(C) \geq \cdots$$

  for the sequence $(A, B, C, \ldots)$.
- Show the graph produced by your program if it has an interface that illustrates the output graph. Otherwise, show the adjacency matrix or the adjacency lists that your program outputs and illustrate the graph yourself.
- In the illustrations, the name of the vertices should appear inside the circles that represent these vertices.

Tasks:

1. Run your program on 2-regular geraphs. For $n \geq 3$, the input sequence is $(2, 2, \ldots, 2)$ in which 2 appears n times.

   (a) What is the output of the algorithm for $n = 6, 7, 8, 9, 10, 11, 12$?

   (b) Can you find the answer for a general n without running the algorithm?

2. Run your program on 3-regular geraphs. For $n \geq 4$, the input sequence is $(3, 3, \ldots, 3)$ in which 3 appears n times for an even n.

   (a) What is the output of the algorithm for $n = 4, 6, 8, 10, 12, 14, 16$?

   (b) Can you find the answer for a general n without running the algorithm?

3. For an even $n = 2k$, run your program on the following sequence:

   $$(k, k, k-1, k-1, \ldots, 2, 2, 1, 1)$$

   in which each of the numbers $1, 2, \ldots, k$ appears exactly twice.

   (a) What is the output of the algorithm for $n = 2, 4, 6, 8, 10, 12$?

   (b) Can you find the answer for a general n without running the algorithm?

4. For $n \geq 2$ and $1 \leq i \leq n-1$, let

   $$S(n, i) = (n-1, n-2, \ldots, i+1, i, i, i-1, \ldots, 1)$$

   That is, $S(n, i)$ is a sequence of length n that contains all the numbers $1, 2, \ldots, n-1$ while the number i is the only one that appears twice in $S(n, i)$. Run your program on $S(n, i)$ sequences

   (a) What is the output of the algorithm for $n = 6$ and $i = 1, 2, 3, 4, 5$?

   (b) What is the output of the algorithm for $n = 7$ and $i = 1, 2, 3, 4, 5, 6$?

   (c) What is the output of the algorithm for $n = 8$ and $i = 1, 2, 3, 4, 5, 6, 7$?

   (d) What is the output of the algorithm for $n = 9$ and $i = 1, 2, 3, 4, 5, 6, 7, 8$?

   (e) Can you find the answer for general n and i without running the algorithm?

5. Run your program on the following sequences:

   (a) $(3, 3, 3, 3, 3, 3, 3, 3)$

   (b) $(3, 3, 3, 3, 3, 3, 2, 2)$

   (c) $(3, 3, 3, 3, 2, 2, 2, 2)$

   (d) $(3, 3, 2, 2, 2, 2, 2, 2)$

   (e) $(2, 2, 2, 2, 2, 2, 2, 2)$

6. Run your program on the following sequences:

   (a) $(2, 2, 1, 1)$

   (b) $(3, 3, 3, 2, 2, 2, 1, 1, 1)$

   (c) $(4, 4, 4, 4, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1, 1, 1)$