

Revenue Generated:  $5000 \times 500 \Rightarrow 2.5M$

Loss:  $200k \times 15 \Rightarrow 3M$

STA 119

PAL SI Session

Wednesday March 5<sup>th</sup>

$\frac{2.5M}{200k}$

$12.5$

# Worksheet: Poisson and Binomial Distributions

## Question Set 1

$$\lambda = 15 = 0.003 \times 5000 = 15$$

A health insurance company issued 5,000 policies (\$200,000 payout value) this year. The probability of a claim being made for each policy is 0.003. The company charges \$500 for each policy. Use the Poisson approximation to determine the following.

a) P(The company breaks even)

cdf

$$P(X = 13) \Rightarrow \pi(13) - \pi(12) =$$

b) P(Company Profits \$100,000 or more)

$$\Rightarrow P(X \leq 12) = \pi(12) =$$

c) P(Company Loses \$250,000 or more)

$$\Rightarrow P(X \geq 15) = 1 - \pi(14) =$$

d) What value of  $\lambda$  would be used if the company charged \$550 for each policy?

$$\mu = n \times p$$

$\lambda = \text{same}$

$$\lambda = n \times p$$

e) What value of  $\lambda$  would be used if the probability of a claim was 0.006?

$$\Rightarrow \lambda_{\text{new}} = \frac{0.006}{0.003} \times 15 = 30$$

$$\lambda_{\text{new}} = 5000 \times 0.006 = 30$$

**Question Set 2**

A car insurance company sold 12,000 policies (<sup>500</sup>\$50,000 payout value) this year. The probability of an accident resulting in a claim for each policy is 0.002. The company charges \$400 for each policy. Use the Poisson approximation to determine the following.

$$\lambda = 12000 \times 0.002$$

$$\Rightarrow 24$$

 $\Rightarrow$ 

$$\frac{4.8M}{500K} = 9.6$$

$$\approx 9.6$$

consider 10

Revenue:  $12000 \times 400$   
 $\Rightarrow 4.8M$

a) P(The company breaks even)

$$P(X=10) = \pi(10) - \pi(9)$$

b) P(Company Profits \$300,000 or more)

$$\Rightarrow P(X \leq 9) = \pi(9)$$

$$9 \rightarrow +500K$$

$$10 \rightarrow 0$$

$$11 \rightarrow -500K$$

$$12 \rightarrow -1M$$

c) P(Company Loses \$500,000 or more)

$$\Rightarrow P(X \geq 11) = 1 - \pi(10)$$

d) What value of  $\lambda$  would be used if the company charged \$450 for each policy?

 $\Rightarrow$ 

$$24$$

e) What value of  $\lambda$  would be used if the probability of an accident was 0.003?

 $\Rightarrow$ 

$$36$$

**Problem Context:**

A particular illness occurs very rarely, with a rate of 0.00005 in the general population. Consider the following towns and populations to answer the questions using the Poisson approximation.

- a) A town has 400,000 residents. What is the expected number of residents with the illness?

$$\Rightarrow \lambda = 400K \times 0.00005 \Rightarrow \underline{\underline{20}}$$

- b) Determine the probability that exactly 7 residents in this town have the illness.

$$P(X=7) = \pi(7) - \pi(6) = \underline{\hspace{2cm}}$$

- c) Suppose the town reports 10 residents with the illness. [Is this number higher than expected?]  $\rightarrow$  No

$$\Rightarrow P(X=10) = \pi(10) - \pi(9) = \underline{\hspace{2cm}} \quad \text{How rare is it to occur}$$

- d) If 10 residents have the illness, should public health officials be alarmed?  $\rightarrow$  No

- e) Calculate the probability that 12 or more residents in the town have the illness.

$$\Rightarrow \underline{\underline{P(X \geq 12)}} = 1 - \pi(11) = \underline{\hspace{2cm}}$$

- f) If 15 residents are diagnosed with the illness, should the town be concerned?

No

[0, 12] [13, 18]

- g) For this town, create a 97% to 3% split for the number of residents expected to have the illness. What is the upper limit of this split?

[0, 28] [29, ∞]

- h) The town reports 18 residents with the illness. Is this an unusual occurrence? Use probability to support your answer.

$$\Rightarrow \underline{\underline{P(X=18)}} = \underline{\hspace{2cm}}$$

- a) A city of 150,000 residents has an illness rate of 0.00008. What is the expected number of residents with the illness?

$$\Rightarrow 150k \times 0.00008 = \underline{12}$$

- b) Calculate the probability that exactly 12 residents have the illness in this city.

$$\Rightarrow P(X=12) = \pi(12) - \pi(11)$$

- c) If 15 residents are diagnosed with the illness, is this higher than expected?

Yes

- d) Should the city be alarmed if 15 residents are diagnosed with the illness? Provide reasoning.

Yes

- e) Calculate the probability that 17 or more residents have the illness.

$$P(X \geq 17) = 1 - \pi(16)$$

- f) The city reports 20 residents with the illness. Is this cause for concern? Why or why not?

Yes ,

- ~~g) For this city, determine the 99% to 1% split for the expected number of residents with the illness.~~

$$[0, 21] \quad [22, \infty]$$

- h) The city has 22 reported cases of the illness. Is this situation rare? Provide your reasoning using probability.

Yes