Worksheet: Poisson and Binomial Distributions

Question Set 1

A health insurance company issued 5,000 policies ($200,000 payout value) this year. The probability of a claim being made for each policy is 0.003. The company charges $500 for each policy. Use the Poisson approximation to determine the following.

a) P(The company breaks even)

b) P(Company Profits $100,000 or more)

c) P(Company Loses $250,000 or more)

d) What value of λ would be used if the company charged $550 for each policy?

e) What value of λ would be used if the probability of a claim was 0.006?

Question Set 2

A car insurance company sold 12,000 policies ($500,000 payout value) this year. The probability of an accident resulting in a claim for each policy is 0.002. The company charges $400 for each policy. Use the Poisson approximation to determine the following.

a) P(The company breaks even)

b) P(Company Profits $300,000 or more)

c) P(Company Loses $500,000 or more)

d) What value of λ would be used if the company charged $450 for each policy?

e) What value of λ would be used if the probability of an accident was 0.003?

Problem Context:
A particular illness occurs very rarely, with a rate of 0.00005 in the general population. Consider the following towns and populations to answer the questions using the Poisson approximation.

a) A town has 400,000 residents. What is the expected number of residents with the illness?

$\Rightarrow \lambda = 400K \times 0.00005 \Rightarrow 20$

b) Determine the probability that exactly 7 residents in this town have the illness.

$P(X=7) = \pi(7) - \pi(6) =$

c) Suppose the town reports 10 residents with the illness. Is this number higher than expected? → No

$\Rightarrow P(X=10) = \pi(10) - \pi(9) =$ [ ] ← How rare is it to occur

d) If 10 residents have the illness, should public health officials be alarmed?

No

e) Calculate the probability that 12 or more residents in the town have the illness.

$\Rightarrow P(X \geq 12) = 1 - \pi(11) =$

f) If 15 residents are diagnosed with the illness, should the town be concerned?

No

$[0, 12] [13, 18]$

g) For this town, create a 97% to 3% split for the number of residents expected to have the illness. What is the upper limit of this split?

$[0, 28] [29, \infty]$

h) The town reports 18 residents with the illness. Is this an unusual occurrence? Use probability to support your answer.

$\Rightarrow P(X=18) =$

a) A city of 150,000 residents has an illness rate of 0.00008. What is the expected number of residents with the illness?

$\Rightarrow 150k \times 0.00008 = 12$

b) Calculate the probability that exactly 12 residents have the illness in this city.

$\Rightarrow P(X=12) = \pi(12) - \pi(11)$

c) If 15 residents are diagnosed with the illness, is this higher than expected?

Yes

d) Should the city be alarmed if 15 residents are diagnosed with the illness? Provide reasoning.

Yes

e) Calculate the probability that 17 or more residents have the illness.

$P(X \geq 17) = 1 - \pi(16)$

f) The city reports 20 residents with the illness. Is this cause for concern? Why or why not?

Yes ,

g) For this city, determine the 99% to 1% split for the expected number of residents with the illness.

$[0, 21] \; [22, \infty]$

h) The city has 22 reported cases of the illness. Is this situation rare? Provide your reasoning using probability.

Yes