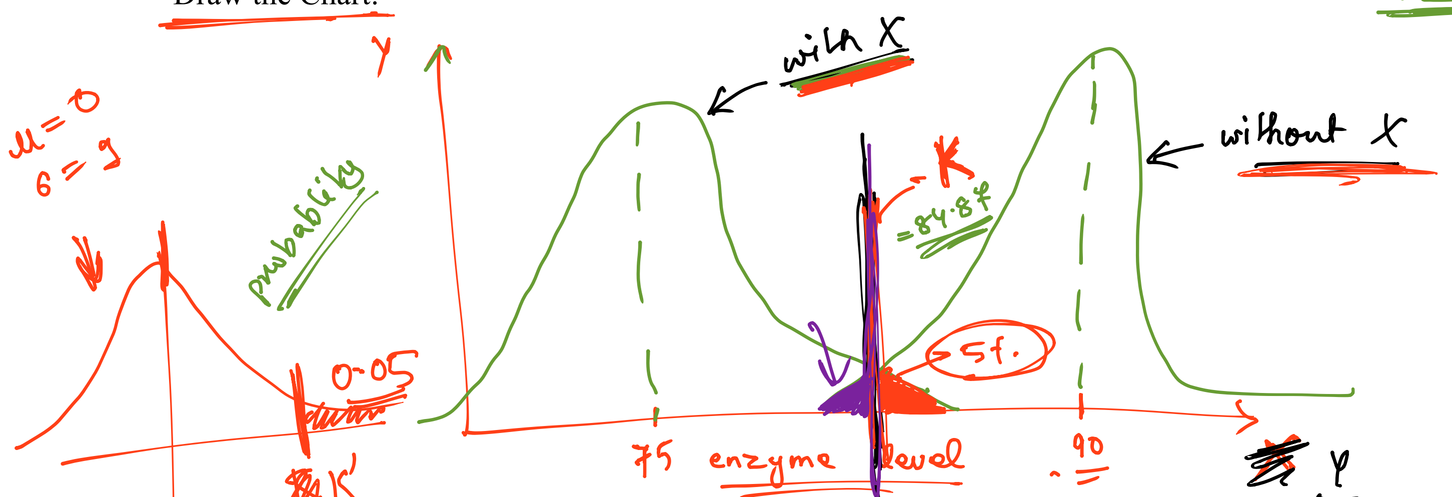
Example (Test Positive for Condition X):

For individuals with Condition X, the level of enzyme Y in the blood is normally distributed with a mean of 75 and a standard deviation of 6. For individuals without Condition X, enzyme Y levels are normally distributed with a mean of 90 and a standard deviation of 3.

$\Rightarrow 2 \rightarrow \boxed{\mu_A = 75 \quad \sigma_A = 6}$

$\rightarrow \mu_B = 90 \quad \sigma_B = 3$

Draw the Chart:



a) Intuitively, what values would make you think the patient has Condition X?

~~b)~~ At what enzyme Y level should the "Tested Positive for Condition X" threshold start so that only ~~0.005~~ of people with Condition X would test negative?

0.05

5%

$$P(X > K) = 0.05$$

$$P\left(Z > \frac{K-75}{6}\right) = 0.05$$

$$1 - \Phi\left(\frac{K-75}{6}\right) = 0.05$$

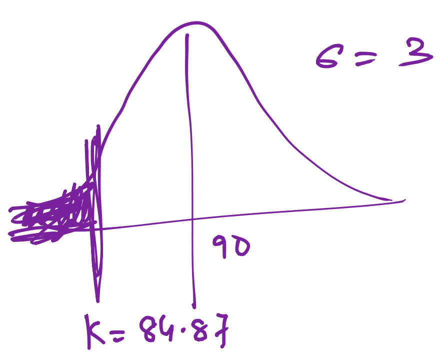
$$\Phi\left(\frac{K-75}{6}\right) = 0.95$$

$$\Rightarrow \frac{K-75}{6} = 1.645$$

$$\boxed{K = 75 + 1.645 \times 6}$$

$$K = 84.87$$

c) What would be the probability of a false positive (an individual without Condition X tests positive)?



$\sigma = 3$

$$P(X < K)$$

$$P(X < 84.87) = \Phi\left(\frac{84.87-90}{3}\right)$$

$$= \Phi(-1.79) = 97\%$$

< 84.87 → +ve have X

threshold = 84.87

d) A patient with Condition X has an enzyme Y level of 80. Will we properly diagnose that patient?

Yes

e) A patient with Condition X has an enzyme Y level of 85. Will we properly diagnose that patient?

NO

we state doesnot

~~f)~~ A healthy patient has an enzyme Y level of 80. Will we properly diagnose that patient?

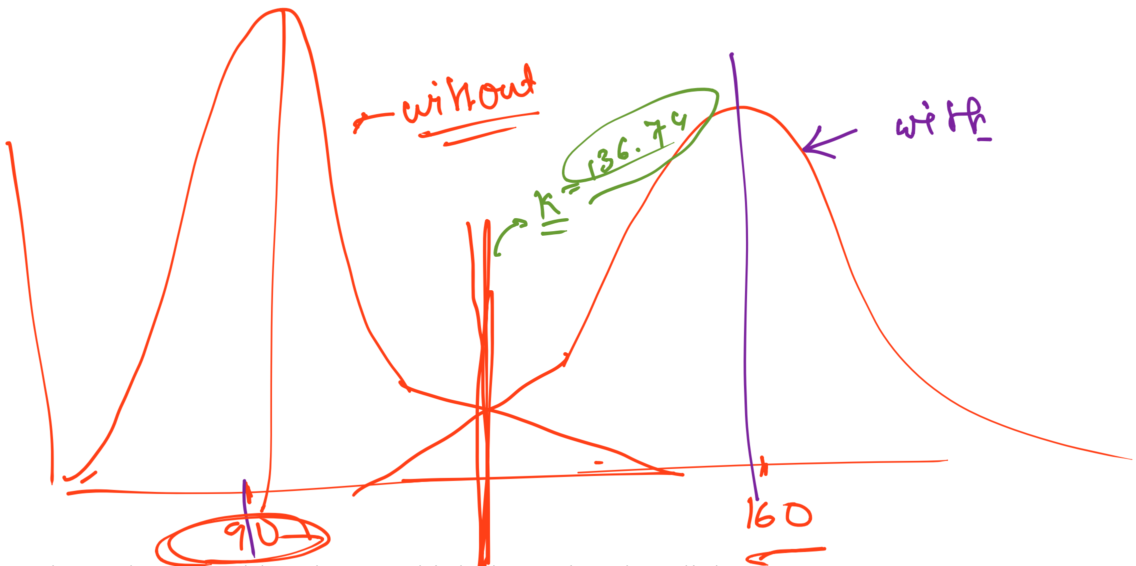
No

~~g)~~ A healthy patient has an enzyme Y level of 85. Will we properly diagnose that patient?

Yes

Example (Blood Sugar Levels): For individuals with diabetes, blood sugar levels (mg/dL) after fasting are normally distributed with a mean of 160 and a standard deviation of 10. For individuals without diabetes, the distribution has a mean of 90 and a standard deviation of 7.

Draw the Chart



a) Intuitively, what values would make you think the patient has diabetes?

higher

b) Where would you set the threshold for "Tested Positive for Diabetes" so that the probability of a diabetic patient testing negative is ~~0.001~~? 0.01

-2.32 → 0.102

-2.33 → 0.099

$$P(X<k) = \Phi\left(\frac{k-160}{10}\right) = 0.01$$

$$k = 160 + 10 \times (-2.326) \Rightarrow 136.74$$

c) What is the probability of a false positive (non-diabetic individual tests positive)?

2/3

$P(X>k)$

$$1-\Phi\left(\frac{k-90}{7}\right) = 1-\Phi\left(\frac{136.74-90}{7}\right) = 1-\Phi(6.68)$$

threshold → 136.74

~~d)~~ A diabetic patient has a fasting blood sugar level of 150. Will we properly diagnose that patient?

→ Yes

120 → then No

e) A diabetic patient has a fasting blood sugar level of 170. Will we properly diagnose that patient?

Yes

~~f)~~ A non-diabetic individual has a fasting blood sugar level of 150. Will we properly diagnose that patient?

→ No

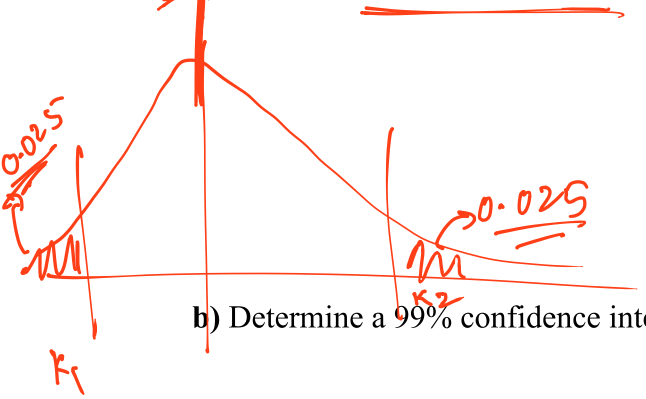
~~g)~~ A non-diabetic individual has a fasting blood sugar level of 170. Will we properly diagnose that patient?

⇒ No

1. Battery Life Testing

A company tested the lifespan of 10 randomly selected batteries, recording an average time to failure of 9.5 hours with a standard deviation of 1.4 hours.

a) Determine a 95% confidence interval for the population mean time to failure.



$[K_1, K_2]$

b) Determine a 99% confidence interval for the population mean time to failure.

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2. Water Bottle Volume Testing

A water bottle company checks that each bottle holds approximately 500 ml of water. A random sample of 12 bottles yields the following volumes (in ml): 498, 502, 499, 500, 497, 504, 501, 498, 500, 503, 497, 496.

a) Determine a 95% confidence interval for the population mean bottle volume.

b) Determine a 98% confidence interval for the population mean bottle volume.

1. Heart Rate Measurement

A cardiologist records the resting heart rate of 300 randomly selected male patients aged 20-30. The sample shows an average heart rate of 72.3 beats per minute, with a standard deviation of 5.1.

a) Determine a 95% confidence interval for the average heart rate in this age group.

b) Determine a 99% confidence interval for the average heart rate in this age group.

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2. Salary Survey

A survey aims to estimate the average annual salary of employees in a city. A random sample of 500 workers reveals a mean salary of $42,350 with a standard deviation of $5,300.

a) Determine a 95% confidence interval for the average salary of workers in this city.

b) Determine a 98% confidence interval for the average salary of workers in this city.