UEC747: ANTENNA AND WAVE PROPAGATION

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Lecture 6: Review of vector Analysis

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Module Objective:

On the completion of this module students should understand the Vector Analysis

Review of Vector Analysis

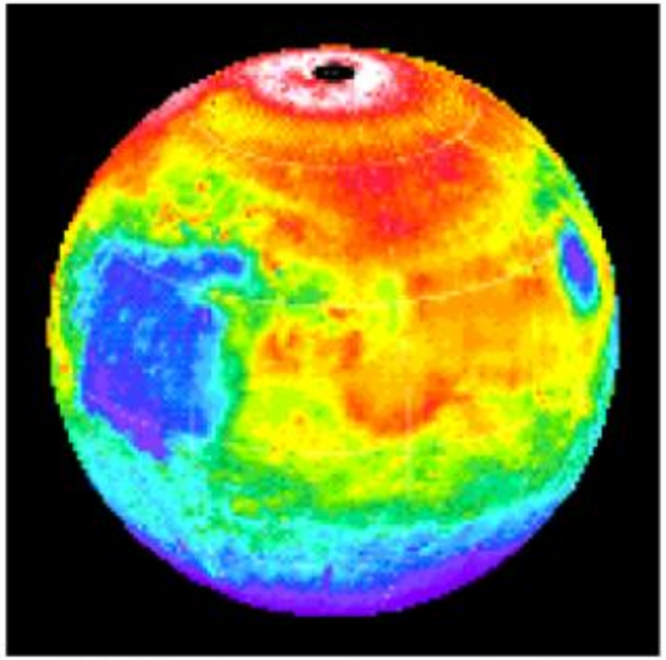
Vector analysis is a mathematical tool with which electromagnetic (EM) concepts are most conveniently expressed and best comprehended.

Scalar and Vector Fields

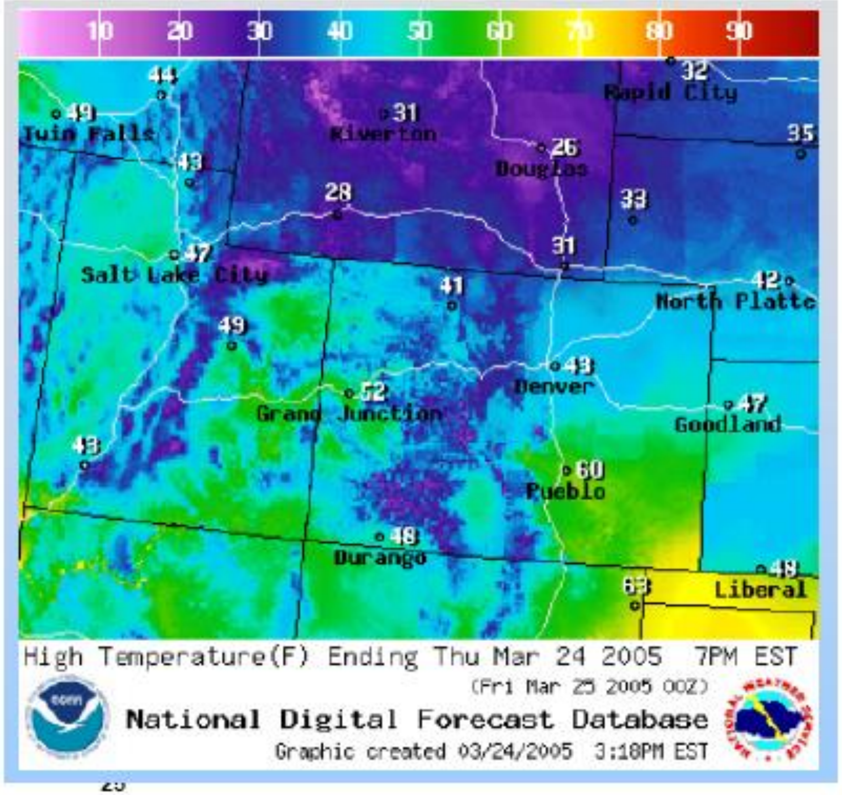
- **A scalar field is a function that gives us a single value of some variable for every point in space.**
  - **Examples: voltage, current, energy, temperature**
- A quantity that has only magnitude but no direction.
- Example: time, mass, distance, temperature and population are scalars.
- Scalar is represented by a letter. e.g., A, B

Example of a Scalar Field

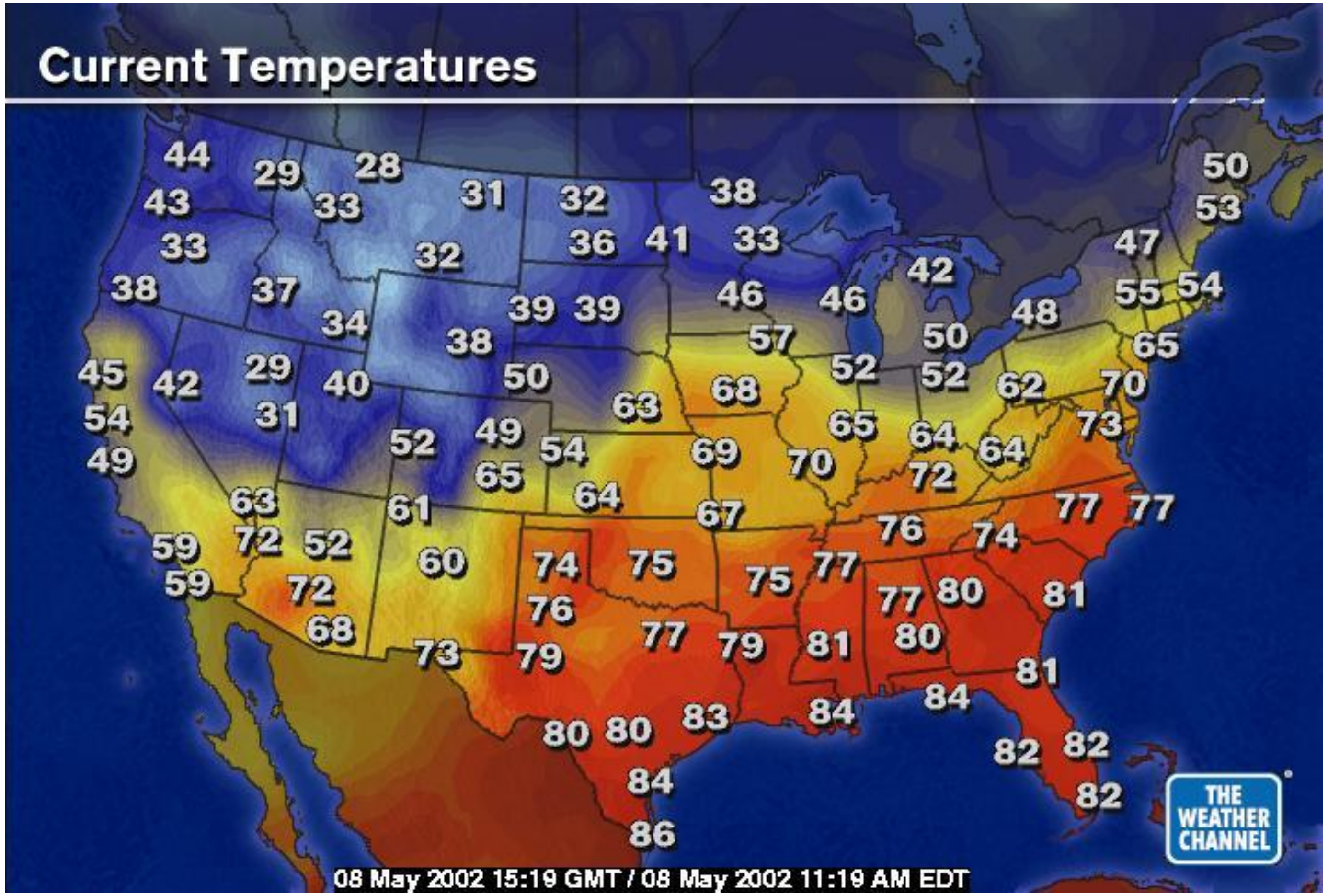
Temperature: Every location has associated value (number with units)



Nighttime temperature map for Mars

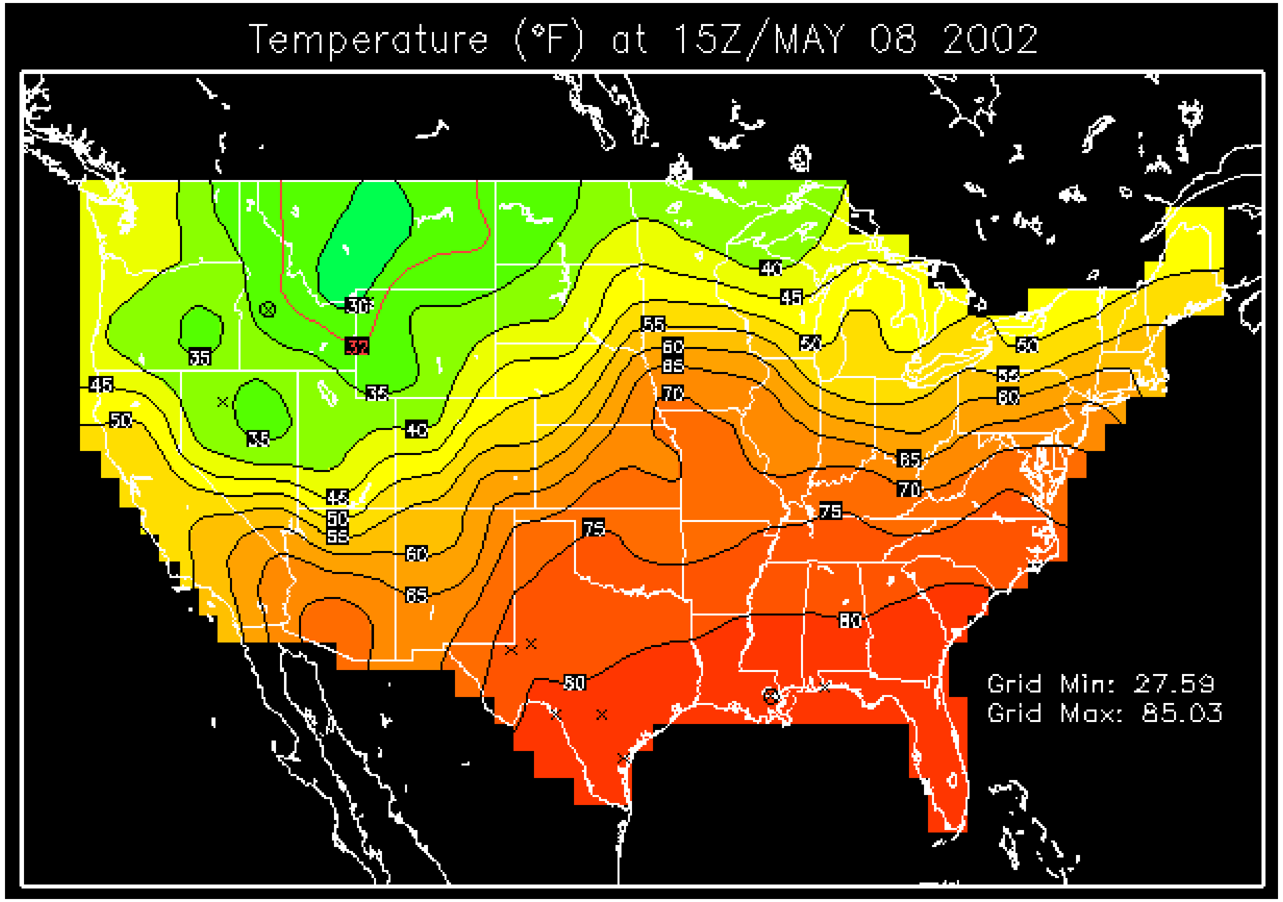


Scalar Fields



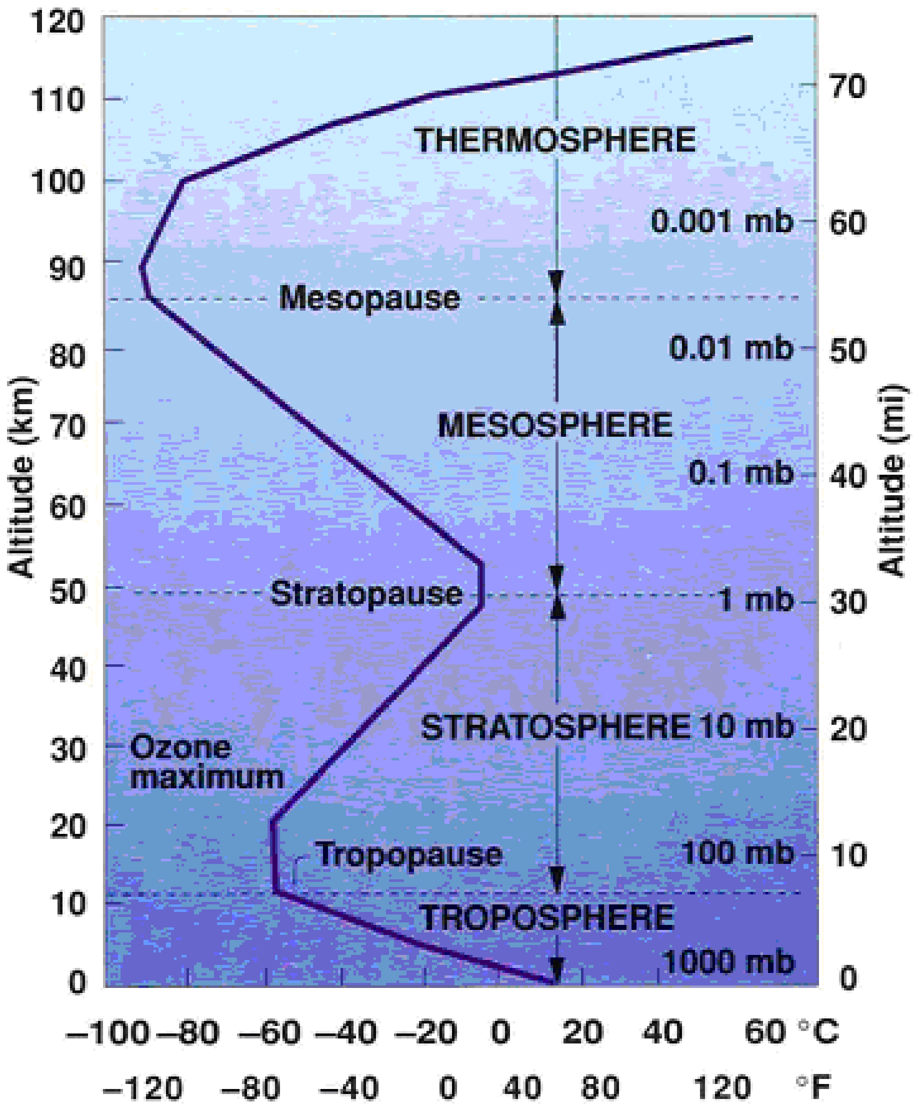
e.g. Temperature: Every location has associated value (number with units)

Scalar Fields - Contours



- Colors represent surface temperature
- Contour lines show constant temperatures

Fields are 3D



- $T = T(x,y,z)$
- Hard to visualize

→ Work in 2D

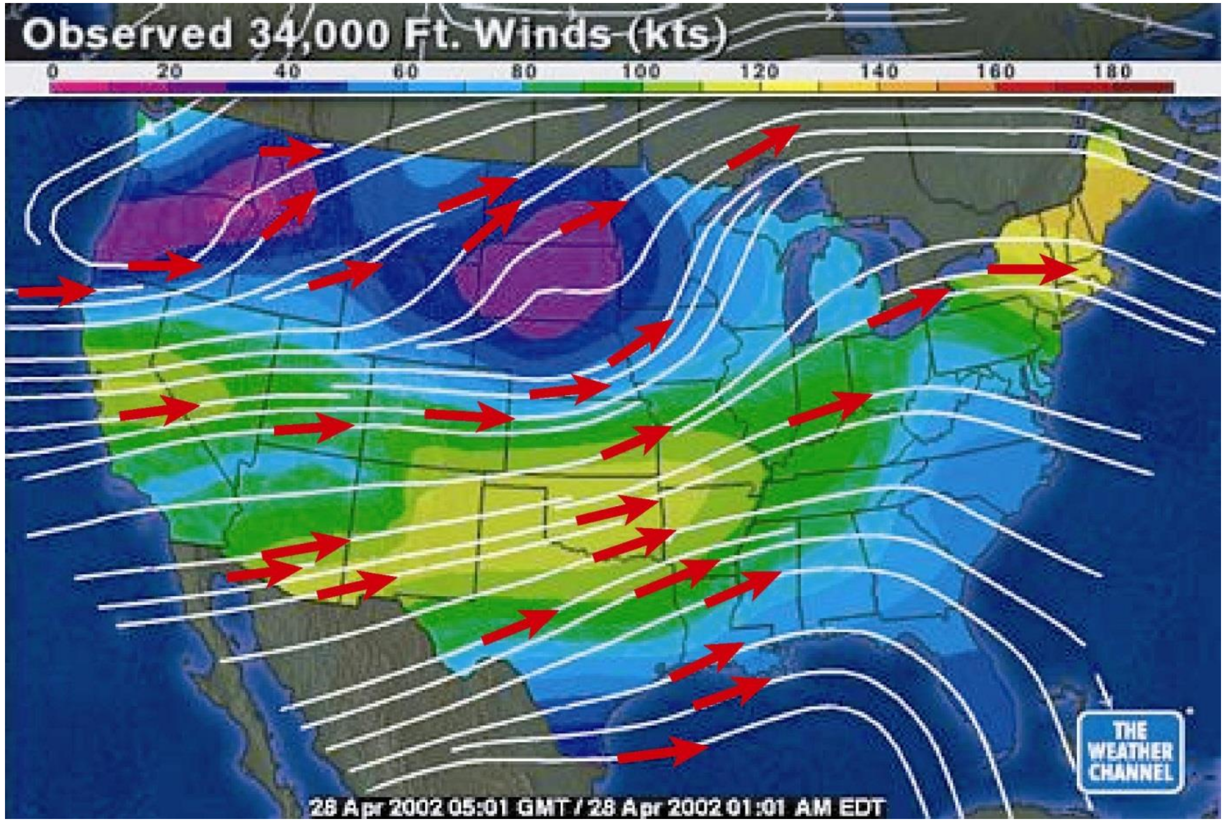
Vector

A vector is a quantity which has both a magnitude and a direction in space.

Examples: velocity, momentum, acceleration and force

Vector Fields

Vector (magnitude, direction) at every point in space

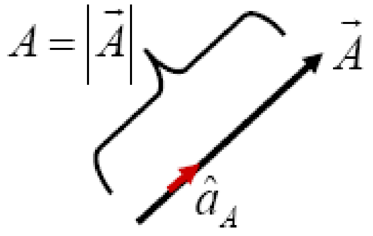


Example: Velocity vector field - jet stream

Vector Fields Explained

Vector has both magnitude and direction in space.

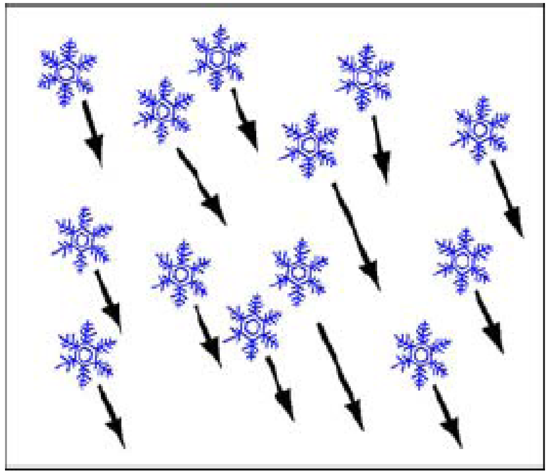
$$\vec{A} = \hat{a}_A A$$



$A = |\vec{A}|$ Magnitude of the vector $\vec{A}$

$\hat{a}_A = \dfrac{\vec{A}}{|\vec{A}|}$ Unit vector in the direction of $\vec{A}$

Falling Snowflakes

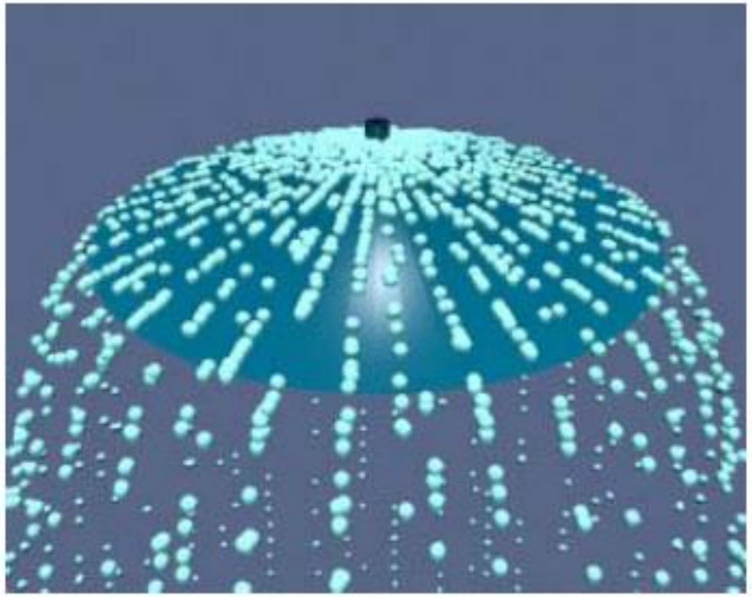


Velocity vector

Examples of Vector Fields

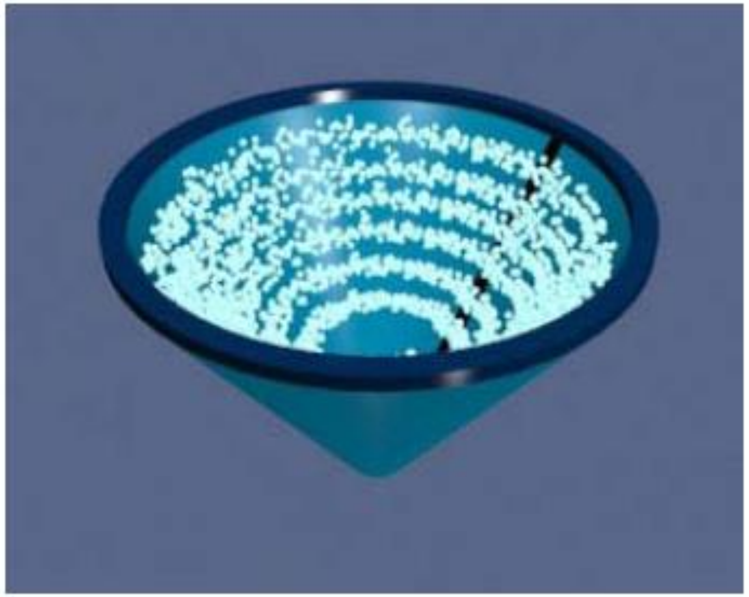
Fluid flow field
(Fluid is represented by a finite number of particles.)

Fluid flow associated with a source (or "faucet")



The vector velocities of the particles are all directed outwards from the center of the cone

A circulating flow of particles

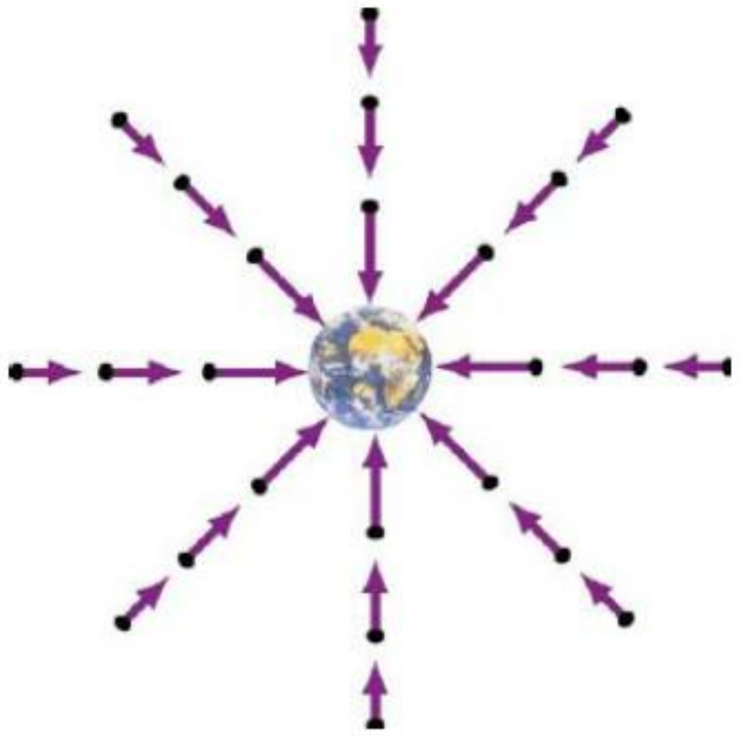


The vector velocities of the particles as seen from above are directed counterclockwise about the center of the cone

Examples of Vector Fields

Gravitational Field

The gravitational field describes the interaction between a massive object and the Earth.



The gravitational field points toward to the center of the Earth.

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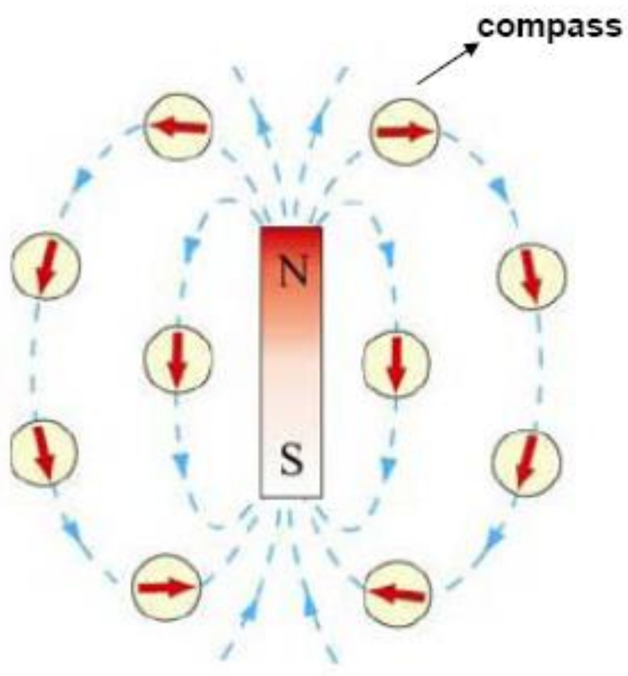
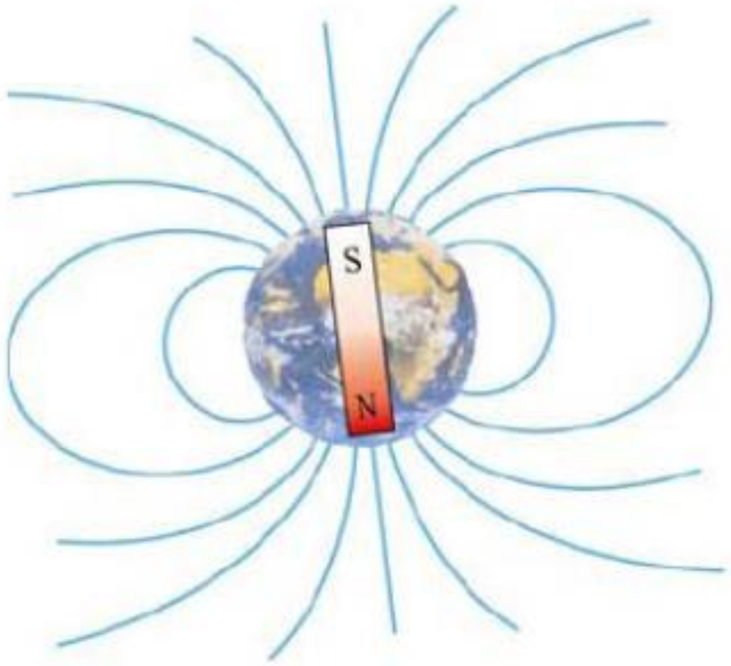
Examples of Vector Fields

Magnetic Field of a bar magnet



The needle aligns itself along the direction of magnetic field.



The Earth's magnetic field behaves as if there were a bar magnet in it.

Vector Quantity

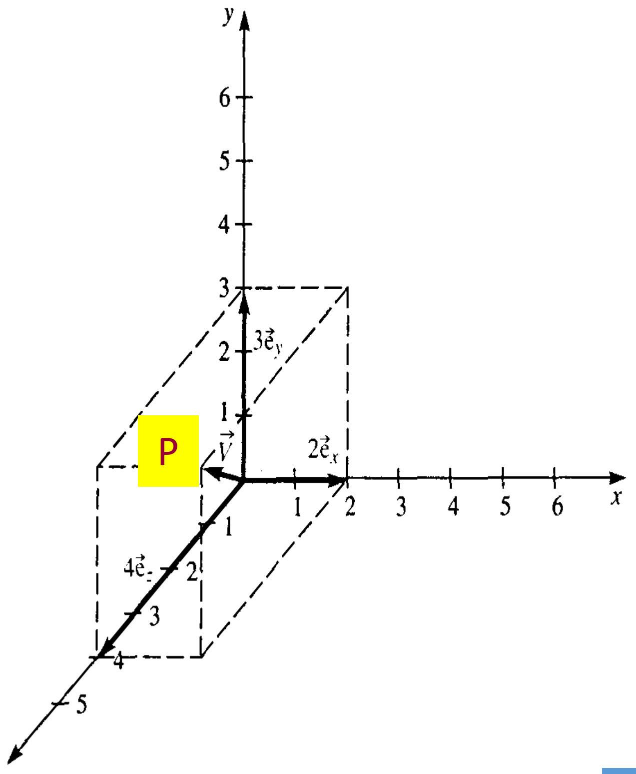
- A quantity that has both magnitude and direction.
- Example: Velocity, force, displacement and electric field intensity.
- Vector is represent by a letter such as A, B, or
- It can also be written as $\bar{A} = A a_A$

Where a_A is a **unit vector**

A vector $\bar{A}$ in Cartesian (or rectangular) coordinates may be represented as:

$$(A_x, A_y, A_z) \quad \text{or} \quad A_x a_x + A_y a_y + A_z a_z$$

where A_x, A_y, and A_z are called the **components** of $\bar{A}$ in the x, y, and z directions, respectively; a_x, a_y and a_z are **unit vectors** in the x, y and z directions, respectively.



Suppose a certain vector $\overline{V}$ is given by

$$\overline{V} = 2a_x + 3a_y + 4a_z$$

The magnitude or absolute value of the vector $\overline{V}$ is:

$$\left|\overline{V}\right| = \sqrt{2^2 + 3^2 + 4^2} = 5.385$$

(from the Pythagorean theorem)

Unit Vector

- Lets begin with vector A. Say we **divide** this vector by its **magnitude** (a scalar value). We create a new vector, which we will denote as a_A:

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|}$$

Q: How is vector a_A related to vector A**?**

A: Since we divided A by a scalar value, the vector a_A has the **same direction** as vector A.

- But, the **magnitude** of a_A is:

$$\hat{a}_A = \frac{|\vec{A}|}{|\vec{A}|} = 1$$

The vector a_A has a magnitude equal to **one**! We call such a vector a **unit vector**.

- A unit vector is essentially a **description of direction** only, as its magnitude is always **unit valued** (i.e., equal to one). Therefore:

- $|A|$ is a scalar value that describes the **magnitude** of vector A.
- a_A is a vector that describes the **direction** of A**.**

Example 1: Unit Vector

- Specify the unit vector extending from the origin towards the point

$$G(2,-2,-1)$$

Solution to Example 1

- Construct the vector extending from origin to point G

$$\mathbf{G} = 2\hat{x} - 2\hat{y} - \hat{z}$$

- Find the magnitude of

$$\vec{\mathbf{G}}$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

Solution to Example 1

- So, unit vector is

$$\mathrm{a}_G = \frac{\vec{\mathrm{G}}}{|\mathrm{G}|} = \frac{2}{3}\hat{x} - \frac{2}{3}\hat{y} - \frac{1}{3}\hat{z}$$

$$= 0.667\hat{x} - 0.667\hat{y} - 0.333\hat{z}$$

- The vector quantity can be of one dimensional or n-dimensional. Hence it is necessary to understand the coordinate systems in which these vector quantities are represented.

The Radius Vector

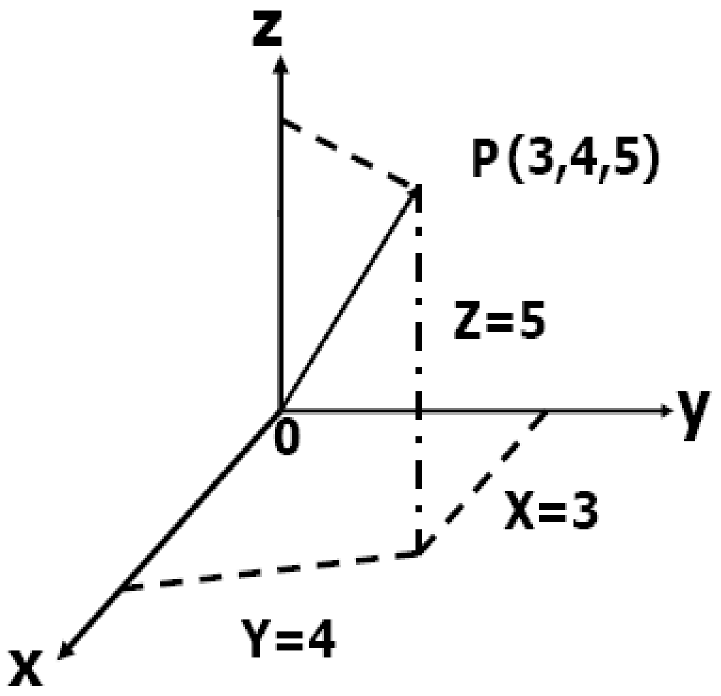
A point P in Cartesian coordinates may be represented by specifying (x, y, z). **The radius vector (or position vector) of point P is defined as the directed distance from the origin O to P**; that is,

$$\bar{r} = xa_x + ya_y + za_z$$

The **unit vector** in the direction of $\bar{r} = xa_x + ya_y + za_z$ is

$$a_r = \frac{xa_x + ya_y + za_z}{\sqrt{x^2 + y^2 + z^2}} = \frac{\bar{r}}{|\bar{r}|}$$

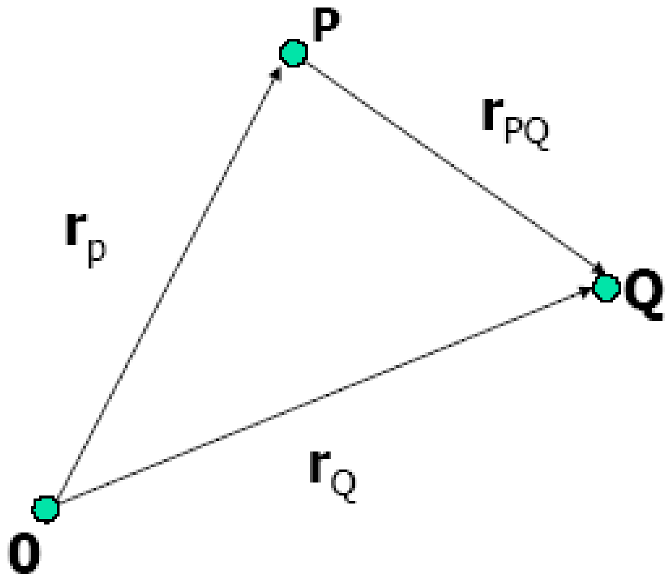
Position and Distance Vectors



$$\vec{\mathbf{r}}_P = 3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$$

Position and Distance Vectors

- If we have two position vectors, $\vec{\mathbf{r}}_P$ and $\vec{\mathbf{r}}_Q$, the third vector or ***distance vector*** can be defined as $\vec{\mathbf{r}}_{PQ}$:-



$$\vec{\mathbf{r}}_{PQ} = \vec{\mathbf{r}}_Q - \vec{\mathbf{r}}_P$$

Example 3: Position Vectors

Point **P** and **Q** are located at $(0,2,4)$ and $(-3,1,5)$. Calculate:

(a) The position vector **P**

(b) The distance vector from **P** to **Q**

(c) The distance between **P** and **Q**

(d) A vector parallel to $\overrightarrow{PQ}$ with magnitude of 10

Solution to Example 3

(a) $\vec{\mathbf{r}}_P = 0\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z = 2\mathbf{a}_y + 4\mathbf{a}_z$

(b) $\vec{\mathbf{r}}_{PQ} = \vec{\mathbf{r}}_Q - \vec{\mathbf{r}}_P$

$$= (-3,1,5) - (0,2,4)$$

$$= -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$$

(c) Since $\vec{\mathbf{r}}_{PQ}$ is a distance vector, the distance between **P** and **Q** is the magnitude of this distance vector.

Solution to Example 3

Distance, *d*

$$d = \left|\mathbf{r}_{PQ}\right| = \sqrt{(-3)^2 + (-1)^2 + (1)^2} = 3.317$$

(d) Let the required vector be $\vec{\mathbf{A}}$ then

$$\vec{\mathbf{A}} = A\mathbf{a}_A$$

Where $A = 10$ is the magnitude of $\vec{\mathbf{A}}$

Solution to Example 3

Since $\vec{\mathbf{A}}$ is parallel to $\vec{PQ}$, it must have the same unit vector as $\vec{\mathbf{r}}_{PQ}$ or $\vec{\mathbf{r}}_{QP}$

$$\therefore \; a_A = \frac{\pm \vec{\mathbf{r}}_{PQ}}{\left|\mathbf{r}_{PQ}\right|} = \frac{\pm\left(-3,-1,1\right)}{3.317}$$

So, $\vec{\mathbf{A}} = 10\,\dfrac{\pm\left(-3,-1,1\right)}{3.317}$

THANKS