UEC747: ANTENNA AND WAVE PROPAGATION

Jan-May 2021

Lecture 11: Antenna Parameters

Dr Rajesh Khanna, Professor ECE

and

Dr Amanpreet Kaur, Assistant Professor, ECE

Antenna Efficiency

Antenna Reference Terminals

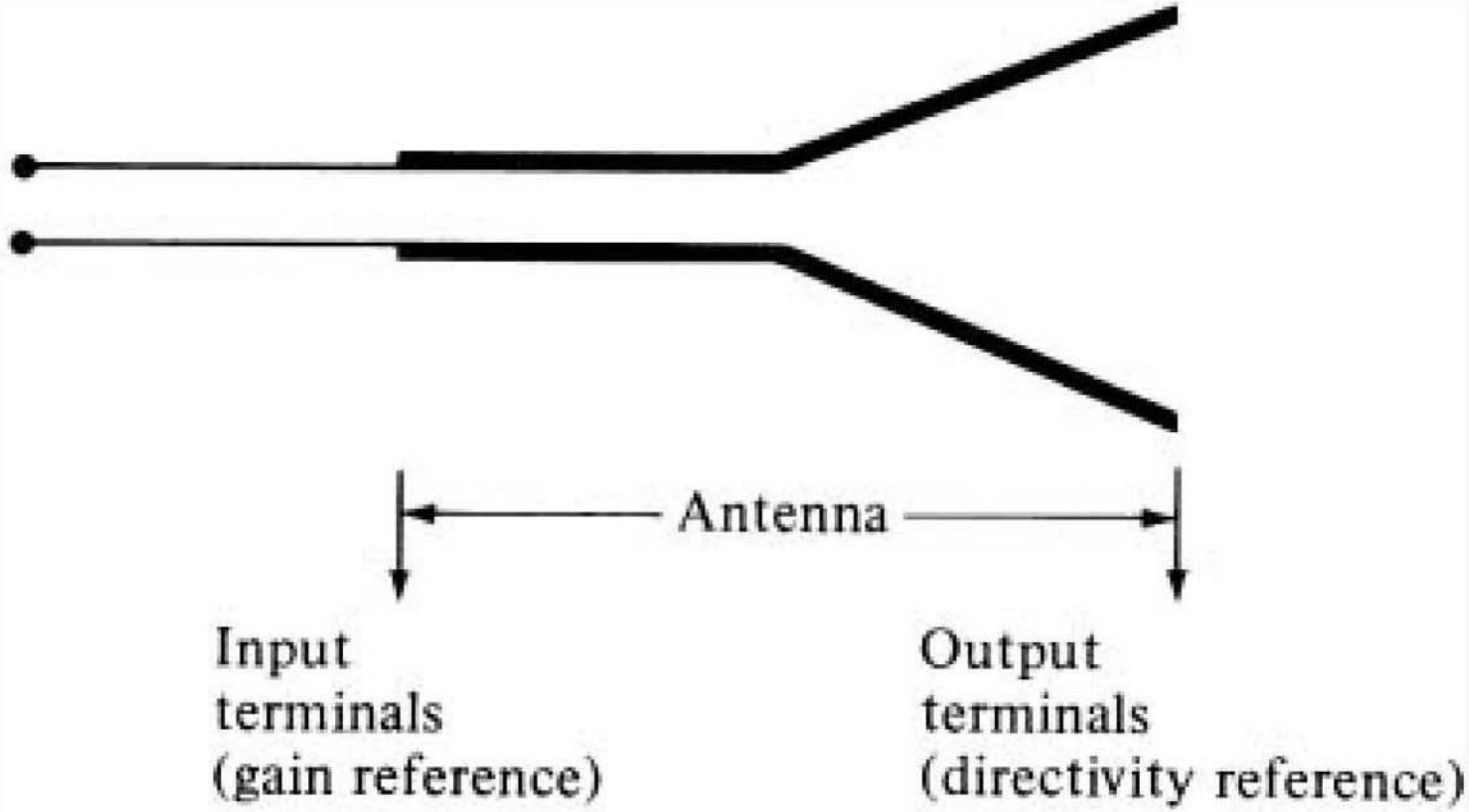


Fig. 2.22(a)

Antenna Efficiency

Associated with an antenna are a number of efficiencies and can be defined using Figure 2. Such losses may be due, referring to Figure 2(b), to

1. **reflections because of the mismatch between the transmission line and the antenna**
2. **I^2R losses (conduction and dielectric)**

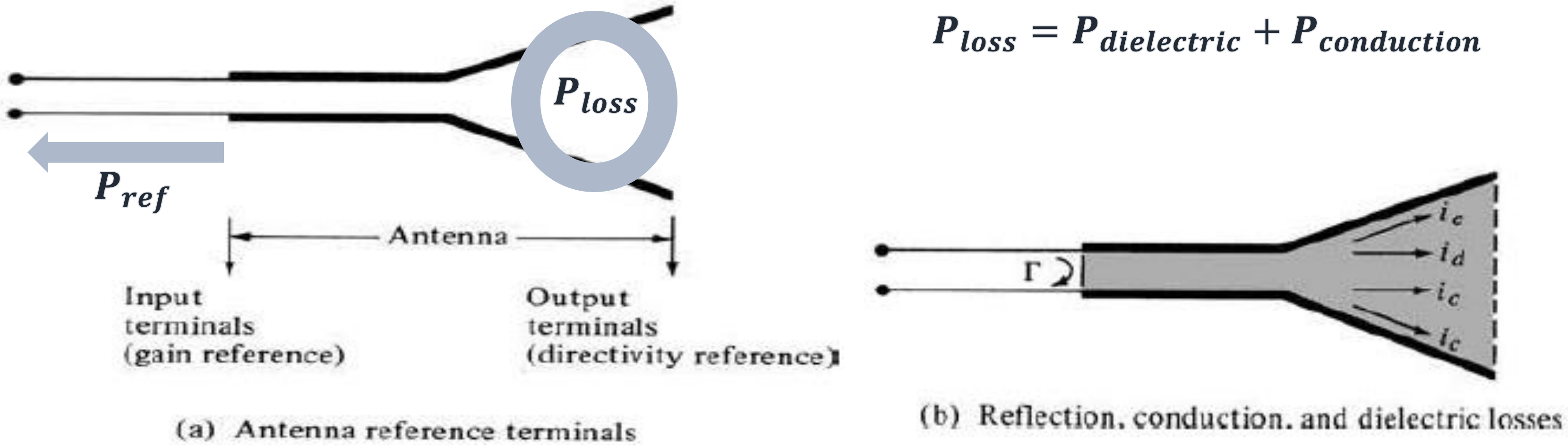


Figure 2. Reference terminals and losses of an antenna.

The total efficiency is used to take into account losses at the input terminals and within the structure of the antenna.

Such losses may be due to

1. reflection because of the mismatch between the transmission line and the antenna.
2. I^2R losses (conduction and dielectric)

In general, the overall efficiency can be written as $e_0 = e_r e_c e_d$

Where

e_0 = total efficiency (dimensionless)

e_r = reflection(mismatch) efficiency= $\frac{P_{in}}{P_s} = (1-|\Gamma|^2)$ (dimensionless)

e_c = conduction efficiency (dimensionless)

e_d = dielectric efficiency (dimensionless)

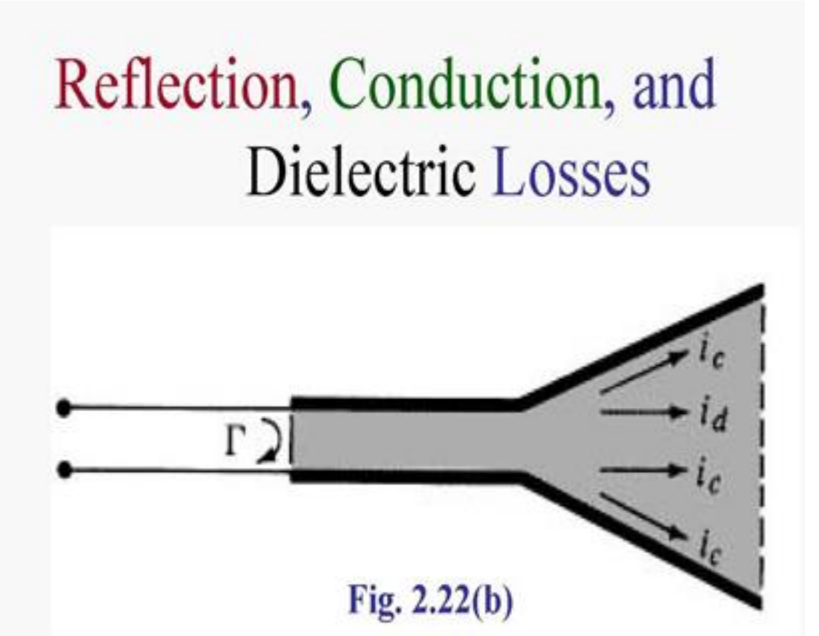


Fig. 2.22(b)

$\frac{1+|\Gamma|}{1-|\Gamma|}$ = voltage reflection coefficient at the input terminals of the antenna

VSWR = voltage standing wave ratio = $\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$, where Z_{in} = antenna input impedance

Z_o = characteristic impedance of the transmission line

Ususally e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write as (2-45)

$$e_0 = e_r e_{cd} = e_{cd}$$

Where $e_{cd} = e_c e_d$ =antenna radiation efficiency, which is used to relate the gain and directivity.

Antenna Efficiency e_o

$$e_o = e_r \boxed{e_c e_d} = e_r \boxed{e_{cd}} \qquad \text{(2-44)}$$

$$e_o = (1 - |\Gamma_{in}|^2) e_{cd} \qquad \text{(2-45)}$$

$$e_o = Total\ efficiency$$

$$e_r = Reflection\ efficiency$$

$$e_{cd} = Radiation\ efficiency$$

$$e_r = (1 - |\Gamma_{in}|^2)$$

$$\Gamma_{in} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$

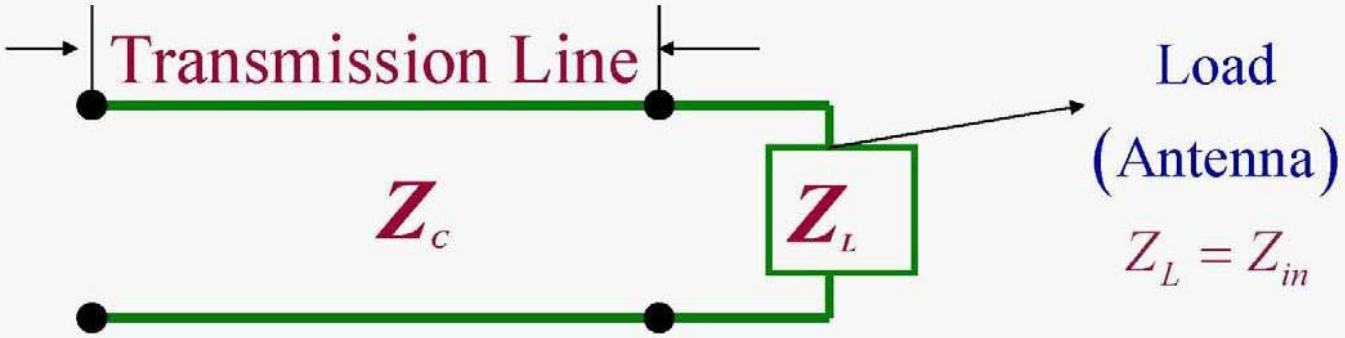
$$VSWR = \frac{1 + |\Gamma_{in}|}{1 - |\Gamma_{in}|}$$

$$\left|\Gamma_{in}\right| = \frac{VSWR - 1}{VSWR + 1}$$

Transmission Line and Load

Z_c = Characteristic Impedance of Line

Z_L = Load Impedance



$$\Gamma_{in} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$

$$P_{rad} = e_c e_d \, P_{in}$$

$$P_{rad} = e_{cd} \, P_{in}$$

1.14 Antenna radiation efficiency

• How can we calculate antenna radiation efficiency?
⇒It takes into account the reflection, conduction, and dielectric losses.

But, computing of conduction & dielectric losses of antenna is very difficult!

∴ conduction-dielectric efficiency(e_{cd}) as antenna radiation efficiency for convenience.

Definition: the ratio of the power delivered to the radiation resistance R_r to the power delivered to R_r ans R_L.

$$\therefore\ e_{cd} = \frac{P_r}{P_L + P_r} = \frac{R_r}{R_L + R_r} \text{ (dimensionless)}$$

2.9 Gain

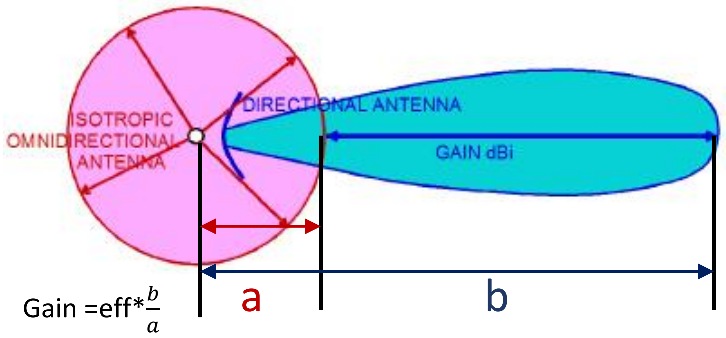
Gain of an antenna (in a given direction) is defined as “the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted(input) by antenna divided by 4 π

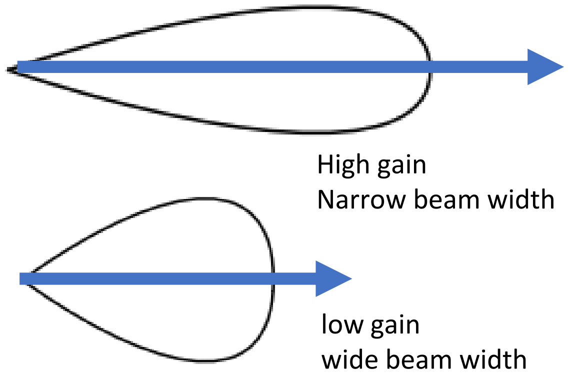
$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta,\phi)}{P_{in}} \text{ (dimensionless)} \qquad (2\text{-}46)$$

Relative gain is defined as “the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction. The power input must be the same for both antennas.

The reference antenna is usually a dipole, horn, or any other antenna whose gain can be calculated or it is known. In most case, however, the reference antenna is a lossless isotropic source.

$$G = \frac{4\pi U(\theta,\phi)}{P_{in}\text{(lossless isotropic source)}} \text{ (dimensionless)} \qquad (2\text{-}46a)$$





Antennas with a very high level of gain are very directive. Therefore high gain and narrow beam-width sometimes have to be balanced to provide the optimum performance for a given application.

Gain

According to the IEEE Standards, "gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches(losses).

In this edition of the book we define two gains

One, referred to as gain (G)

The other, referred to as absolute gain (G_{abs})

$$P_{rad} = e_{cd} P_{in} \tag{2-47}$$

$$G(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}\text{(lossless isotropic source)}} \text{ (dimensionless)}$$

$$= e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{rad}}\right] \tag{2-48}$$

Which is related to the directivity

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

$$D(\theta,\ \phi) = 4\pi \frac{F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta d\theta d\phi}$$

$$G(\theta,\phi) = e_{cd} D(\theta,\phi) \tag{2-49}$$

In a similar manner, the maximum value of the gain is related to the maximum directivity

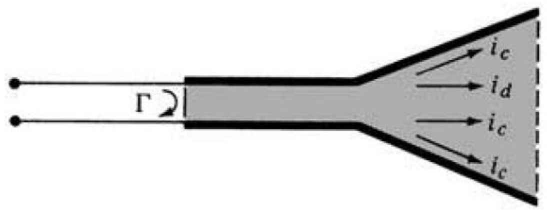
$$G_0 = G(\theta,\phi)|_{\max} = e_{cd} D(\theta,\phi)\,|\max = e_{cd} D_0 \tag{2-49a}$$

G takes into account the losses of the antenna element itself, but does not take into account the losses when the antenna element is connected to a transmission line.

Absolute Gain

When the antenna element is connected to a transmission line, connection losses(reflections/mismatch) are taken into account by introducing a reflection (mismatch) efficiency e_r is related to the reflection coefficient. $e_r = (1-|\Gamma|^2)$

Absolute gain takes into account the reflection/mismatch losses
(due to the connection of the antenna element to the transmission line.)



$$G_{abs}(\theta,\phi) = e_r G(\theta,\phi) = (1-|\Gamma|^2)G(\theta,\phi)$$
$$= e_r e_{cd} D(\theta,\phi) = e_o D(\theta,\phi)$$

$$e_r = (1-|\Gamma|^2) \quad \text{(2-49b)}$$

Where e_0 is the overall efficiency

Similarly, the maximum absolute gain G_{0abs} is related to the maximum directivity D_0

$$e_r = (1-|\Gamma|^2) \quad \text{(2-49c)}$$

If the antenna is matched to the transmission line, that is, the antenna input impedance Z_{in} is equal to the characteristic impedance Z_c of the line $(|\Gamma|=0)$ then the two gains are equal (G_{abs}=G)

2.9 Gain

Partial gain of an antenna for a given polarization in a given direction is defined as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically".

Total gain is the sum of the partial gains for any two orthogonal polarizations. For a spherical coordinate system,

$$G_0 = G_\theta + G_\phi \quad (2\text{-}50)$$

While the partial gains and are expressed as

$$G_\theta = \frac{4\pi U_\theta}{P_{in}}$$

$$G_\phi = \frac{4\pi U_\phi}{P_{in}} \quad (2\text{-}50a,\ 2\text{-}50b)$$

Where G_θ=radiation intensity in a given direction contained in field component
G_ϕ =radiation intensity in a given direction contained in field component
P_{in}=total input (accepted) power

In practice, whenever the term "gain" is used, it is usually refers to the maximum gain as defined by

$$G_0 = G(\theta,\phi)|_{\max} = e_{cd} D(\theta,\phi) | \max = e_{cd} D_0 \quad (2\text{-}49a)$$

$$\begin{aligned} G_{0abs} &= G_{abs}(\theta,\phi)|_{\max} = e_r G(\theta,\phi)|_{\max} = (1-|\Gamma|^2)G(\theta,\phi)|_{\max} \\ &= e_r e_{cd} D(\theta,\phi)|_{\max} = e_0 D(\theta,\phi)|_{\max} = e_0 D_0 \end{aligned} \quad (2\text{-}49c)$$

Usually the gain is given in terms of decibels instead of the dimensionless quantity of (2-49a)
The conversion formula is given by

$$G_0(\text{dB}) = 10\log_{10}[e_{cd} D_0 \ \text{(dimensionless)}] \quad (2\text{-}52)$$

Relation between gain and directivity

Directivity
&
Efficiency

→ Gain

$$\boxed{G(\theta,\phi) = e_{cd} D(\theta,\phi)} \qquad (2\text{-}49)$$

$$G_0 = G(\theta,\phi)|_{\max} = e_{cd} D(\theta,\phi)|_{\max} = e_{cd} D_0 \qquad (2\text{-}49a)$$

Absolute gain (reflections losses are included)

$$\begin{aligned} G_{abs}(\theta,\phi) &= e_r G(\theta,\phi) \boxed{=} (1-|\Gamma|^2) G(\theta,\phi) \\ &= e_r e_{cd} D(\theta,\phi) = e_o D(\theta,\phi) \end{aligned} \qquad (2\text{-}49c)$$

Partial Gain

$$\boxed{G_0 = G_\theta + G_\phi} \qquad (2\text{-}50)$$

$$G_\theta = \frac{4\pi U_\theta}{P_{in}} \qquad (2\text{-}50a)$$

$$G_\phi = \frac{4\pi U_\phi}{P_{in}} \qquad (2\text{-}50b)$$

dBi (dB isotropic) : the forward gain of an antenna compared with the hypothetical isotropic antenna, which uniformly distributes energy in all directions.

dBd (dB dipole) : the forward gain of a antenna compared with a half-wave dipole antenna. (0 dBd = 2.15 dBi)

Example 2.10

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3 \theta$$

Find the maximum absolute gain of this antenna.

Solution

Example 2.10 (Resonant, lossless λ/2 dipole):

$$Z_{in} = 73, \quad Z_c = 50, \quad \text{lossless} \Rightarrow e_{cd} = 1$$

$$U = B_0 \sin^3 \theta$$

Solution:

$$D_0 = 4\pi \frac{U_{\max}}{P_{rad}}, \quad G_0 = e_0 D_0$$

$$U_{\max} = B_0 \sin^3 \theta \big|_{\max} = B_0$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi$$

$$= B_0 \int_0^{2\pi} \underbrace{\left[\int_0^{\pi} \sin^4\theta \, d\theta \right]}_{3\pi/8} d\phi$$

$$P_{rad} = B_0 (2\pi)\left(\frac{3\pi}{8}\right) = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi\left[\frac{B_0}{B_0\left(3\pi^2/4\right)}\right] = \frac{16}{3\pi} = 1.697$$

$$G_0 = e_{cd} D_0 = (1)(1.697) = 1.967 = 2.297 \ \text{dB}$$

$$e_r = \left(1 - |\Gamma_{in}|^2\right)$$

$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_c}{Z_{in} + Z_c}\right| = \left|\frac{73-50}{73+50}\right| = 0.187$$

$$e_r = \left(1 - |0.187|^2\right) = 1 - 0.035 = 0.965$$

$$e_0 = e_r e_{cd} = 0.965(1) = 0.965$$

$$e_0(\text{dB}) = 10\log_{10}\left[0.965(1)\right] = \underbrace{10\log_{10}(0.965)}_{-0.155 \text{ dB}} + \underbrace{10\log_{10}(1)}_{0 \text{ dB}}$$

$$G_{abs} = e_o D_o = 0.965(1.697) = 1.638 = 2.142 \ \text{dB}$$

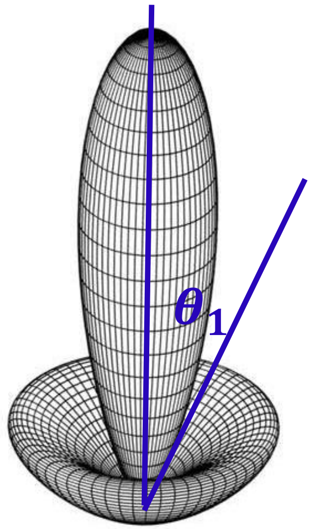
$$=(2.297-0.155) = 2.142 \ \text{dB}$$

1. Beam Efficiency
2. Bandwidth

Beam Efficiency

Another parameter that is frequently used to judge the quality of transmitting and receiving antennas is the beam efficiency.

$$\text{BE} = \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}} \text{(dimensionless)} \tag{2-53}$$

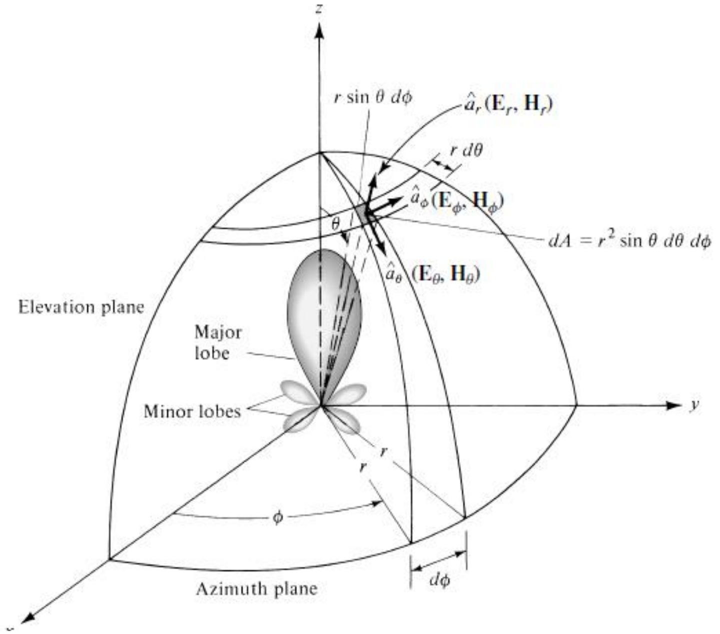


$$= \frac{\displaystyle\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\displaystyle\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi} \tag{2-54}$$

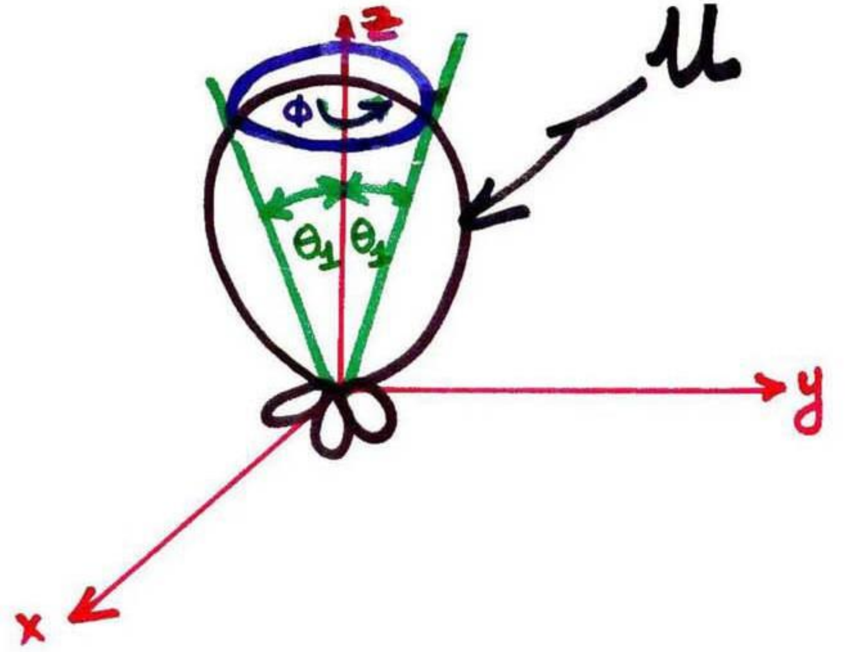
- If θ_1 is chosen as the angle where the first null or minimum occurs, then the beam efficiency will indicate the amount of power in the major lobe compared to the total power.

- Radiometry, astronomy, radar applications need

1. High beam efficiency is necessary(between the nulls or minimums)
2. Minor lobes must be minimized (where received signals through the minor lobes must be minimized.)



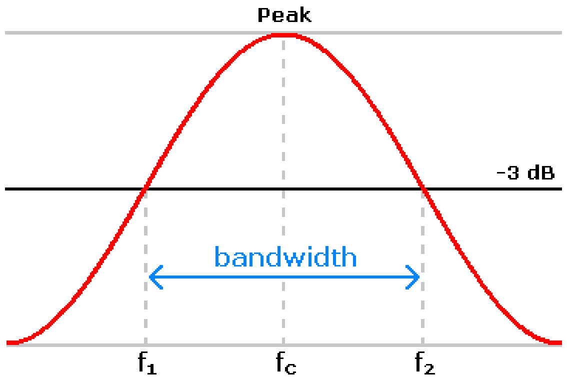
$$BE = \frac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi} \quad (2\text{-}54)$$

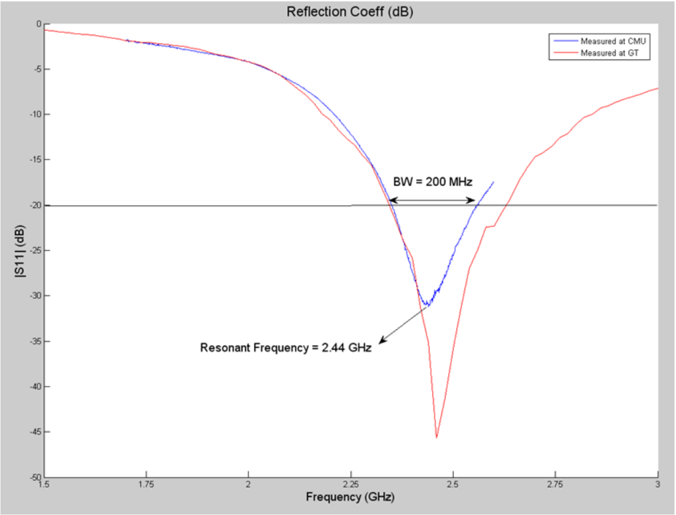


Bandwidth

The bandwidth of an antenna is defined as "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.

Usually there is a distinction made between pattern and input impedance variations. Accordingly pattern bandwidth and impedance bandwidth are used to emphasize this distinction.





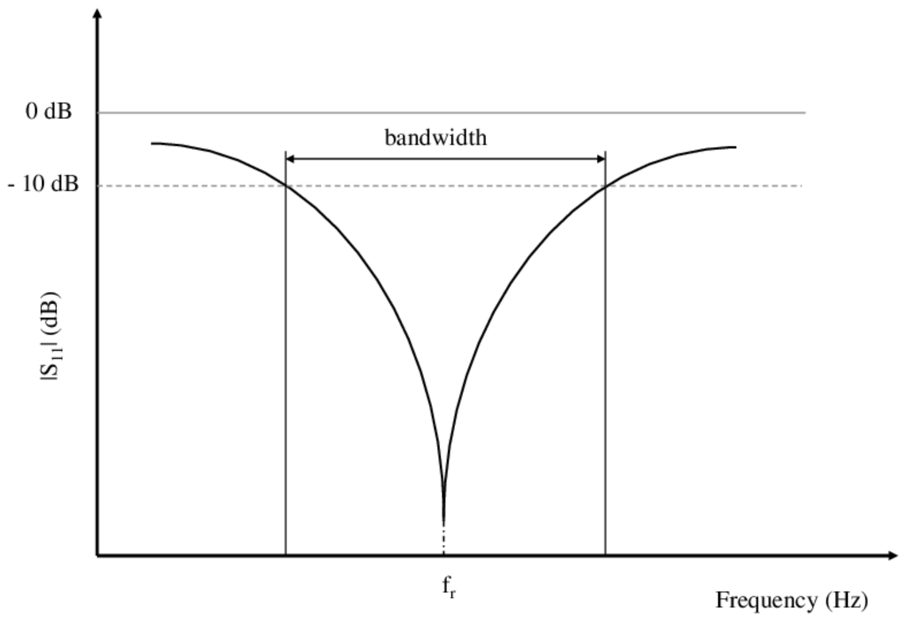
Accordingly pattern bandwidth and impedance bandwidth are used to emphasize this distinction.

Associated with pattern bandwidth are gain, side lobe level, beam-width, polarization, and beam direction while input impedance and radiation efficiency are related to impedance bandwidth.

$$Q = \frac{f_c}{f_2 - f_1}$$

Antennas with a high Q are narrowband, antennas with a low Q are wideband

2.11 Bandwidth



Bandwidth : the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard

Broad band antenna → ratio of the upper to lower frequencies (10:1 bandwidth)

Narrowband antenna → percentage of the frequency difference over the center frequency

I. Pattern Bandwidth

A. Directivity/Gain

B. Side lobe level

C. Beamwidth

D. Polarization

E. Beam direction

II. Impedance Bandwidth

A. Input impedance

B. Radiation efficiency

Elevation Plane Amplitude Patterns for a Thin Dipole with Sinusoidal Current Distribution (l = << λ, λ/4, λ/2, 3λ/4, λ)

HPBW

1. $\ell \le \dfrac{\lambda}{50}$: HPBW = 90°
2. $\ell \le \dfrac{\lambda}{2}$: HPBW = 74.93°
3. $\ell \le \lambda$: HPBW = 47.8°

$$\frac{\lambda}{50} \le \ell \le \lambda$$

$$90^o \ge \text{HPBW} \ge 47.8^o$$

$$\Delta(\text{HBPW}) = 42.2^o$$

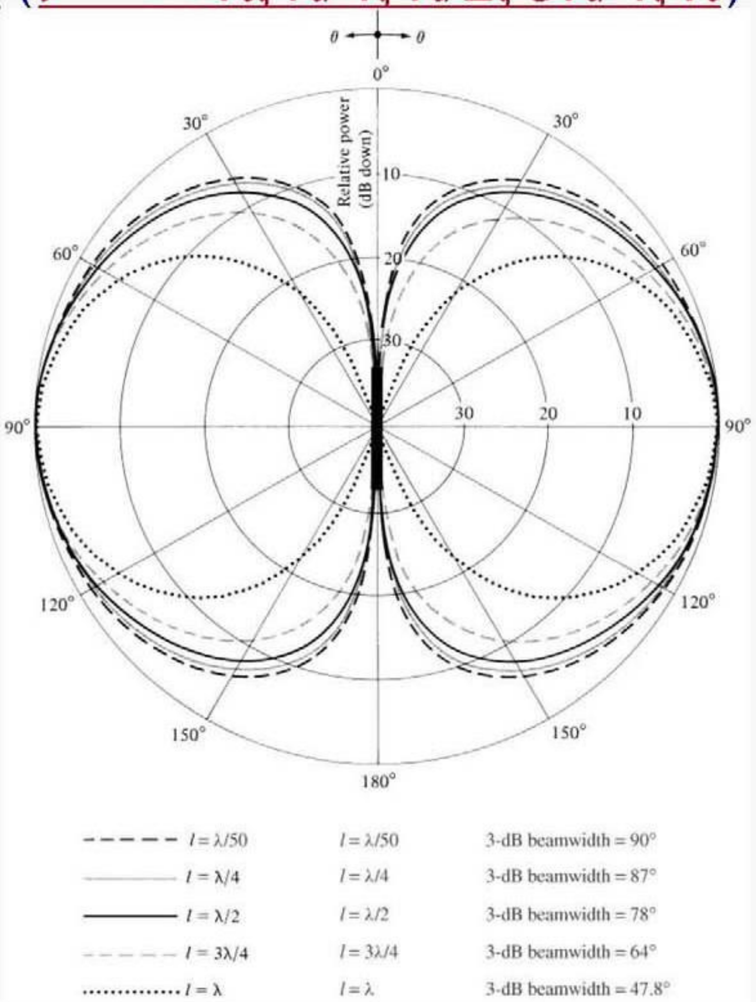


Fig. 4.6

Polarization

1.12 Polarization

- Definition of polarization of an antenna:

The polarization of the wave transmitted by the antenna. When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain.

- Definition of polarization of a radiated wave:

Property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector as observed along the direction of propagation.

- Definition of polarization of a wave received by an antenna:

Polarization of a plane wave, incident from a given direction and having a given power flux density, which results in maximum available power at the antenna terminals.

- Definition of co-polarization and cross-polarization:

Co-polarization: polarization is intended to radiate(receive)
Cross-polarization: polarization orthogonal to a specified polarization(usually co-polarization)

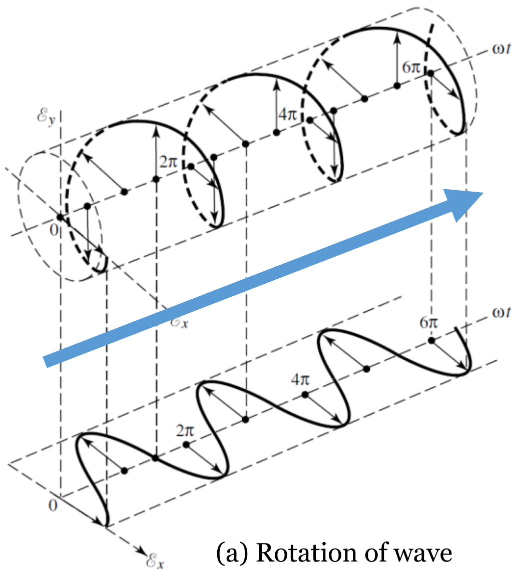
2.12 Polarization

Polarization of an antenna in a given direction is defined as "the polarization of the wave transmitted (radiated) by the antenna. (When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain)

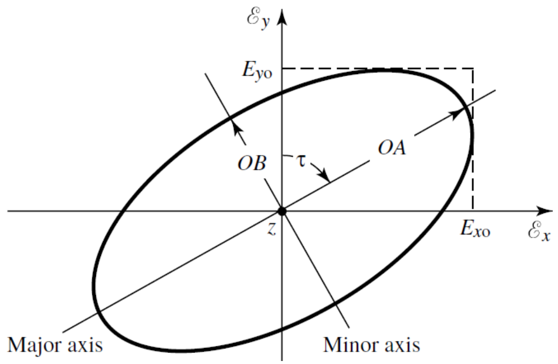
In practice, polarization of the radiated energy varies with the direction from the center of the antenna, so that different parts of the pattern may have different polarizations.

Polarization of a radiated wave is defined as "that property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector.

Specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, as observed along the direction of propagation.



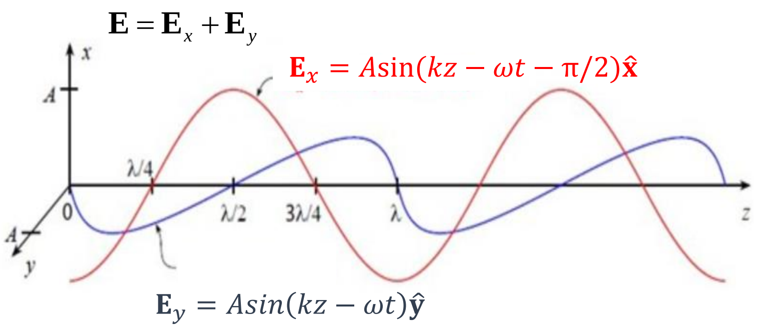
(a) Rotation of wave



(b) Polarization ellipse

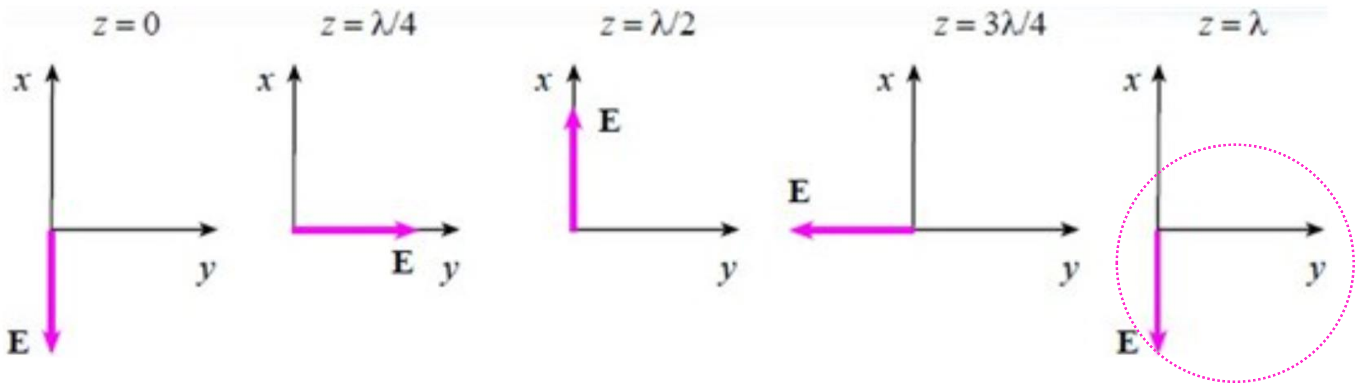
Figure 2.23 Rotation of a plane electromagnetic wave and its polarization ellipse at z=0 as a function of time

2.12 Polarization



Here is an example of the two waves $\mathbf{E}_x$ and $\mathbf{E}_y$ viewed in a "fixed time" picture (t = 0):

If we look down the propagation axis in the positive z direction, the vector **E** at various locations (and at t = 0) looks like:



We can see that the tip of **E** traces out a circle as we follow the wave along the z axis at a fixed time. Similarly, if we sit at a fixed position, the tip of **E** appears to trace out a circle as time evolves.

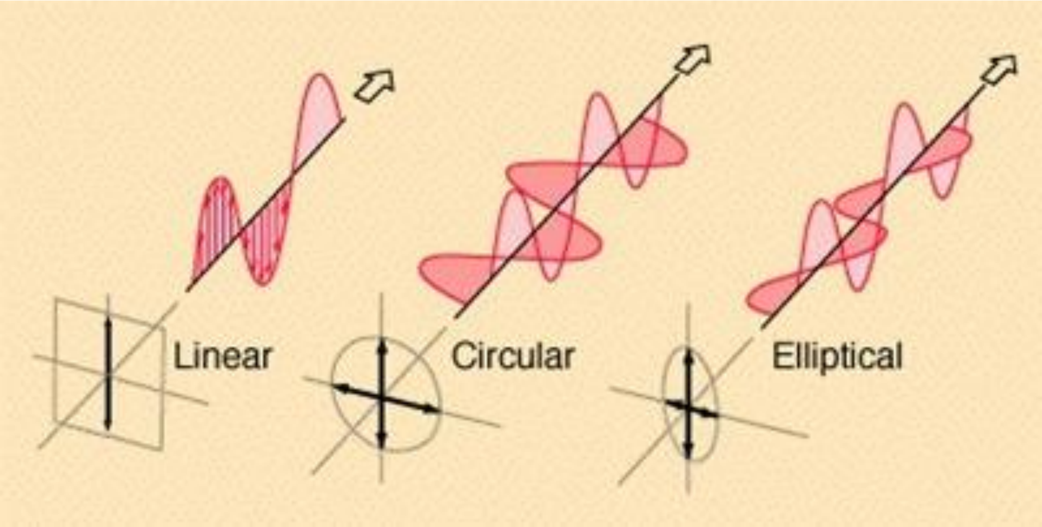
Polarization is the curve traced by the end point of the arrow (vector) representing the instantaneous electric field.

Polarization may be classified as

1. Elliptical

The figure that the electric field traces is an ellipse.

Linear and circular polarizations are special cases of elliptical



2. Linear

If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be linearly polarized.

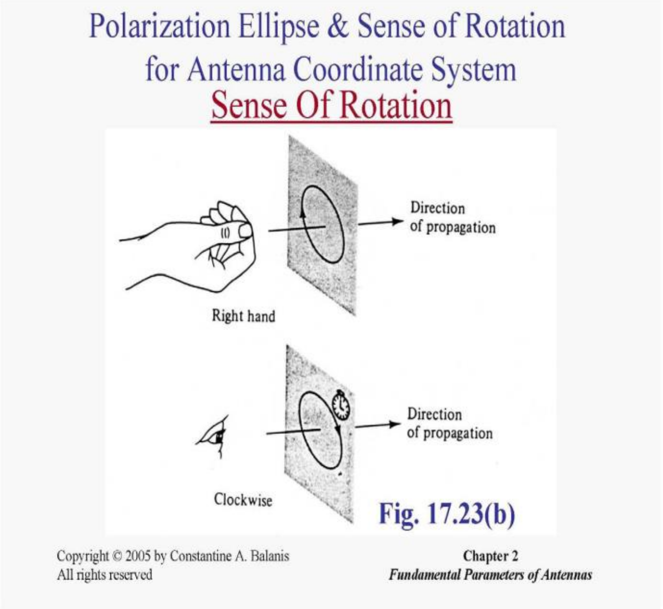
- Vertical polarization
- horizontal polarization

3. Circular

The figure of the electric field is traced in a

Clockwise (CW): the electric field vector is right-hand polarization

Counterclockwise (CCW):the electric field vector is left-hand polarization.



Co-polarization and cross polarization

At each point on the radiation sphere the polarization is usually resolved into a pair of orthogonal polarizations, the co-polarization and cross polarization.

Co-polarization represents the polarization the antenna is intended to radiate (receive) while cross-polarization represents the polarization orthogonal to the co-polarization.

2.12.1 Linear, Circular, and Elliptical Polarizations

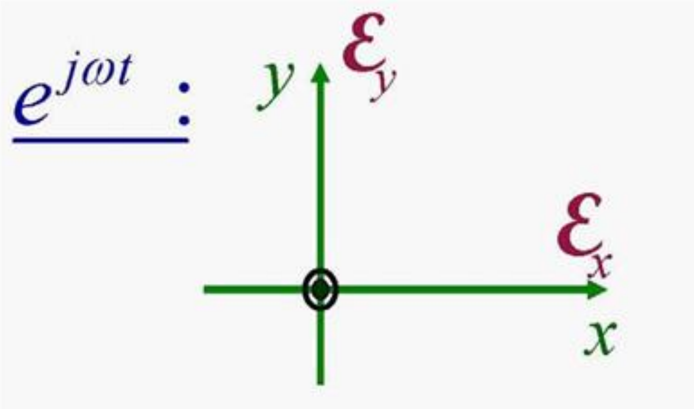
The instantaneous field of a plane wave, traveling in the negative z direction, can be written as

$$\varepsilon(z;t) = \hat{\mathbf{a}}_x \varepsilon_x(z;t) + \hat{\mathbf{a}}_y \varepsilon_y(z;t) \qquad \text{(2-55)}$$

and its components are related to their complex counterparts by

$$\varepsilon_x(z;t) = \mathrm{Re}[E_x^- e^{j(\omega t + kz)}] = \mathrm{Re}[E_{xo} e^{j(\omega t + kz + \phi_x)}] = E_{xo}\cos(\omega t + kz + \phi_x) \qquad \text{(2-56)}$$

$$\varepsilon_y(z;t) = \mathrm{Re}[E_y^- e^{j(\omega t + kz)}] = \mathrm{Re}[E_{yo} e^{j(\omega t + kz + \phi y}] = E_{yo}\cos(\omega t + kz + \phi_y) \qquad \text{(2-57)}$$



E_{xo} & E_{yo} : the maximum magnitudes of the x and y components
$\emptyset_x$ & $\emptyset_y$: the time-phase of the x and y components

To find the polarization, we consider E_{xo} & E_{yo} and $\emptyset_x$ & $\emptyset_y$.
To be specific, we think about the time-phase difference ($\Delta\emptyset = \emptyset_y - \emptyset_x$) between two components.

Linear, Circular, and Elliptical Polarizations

- Case A. Linear Polarization

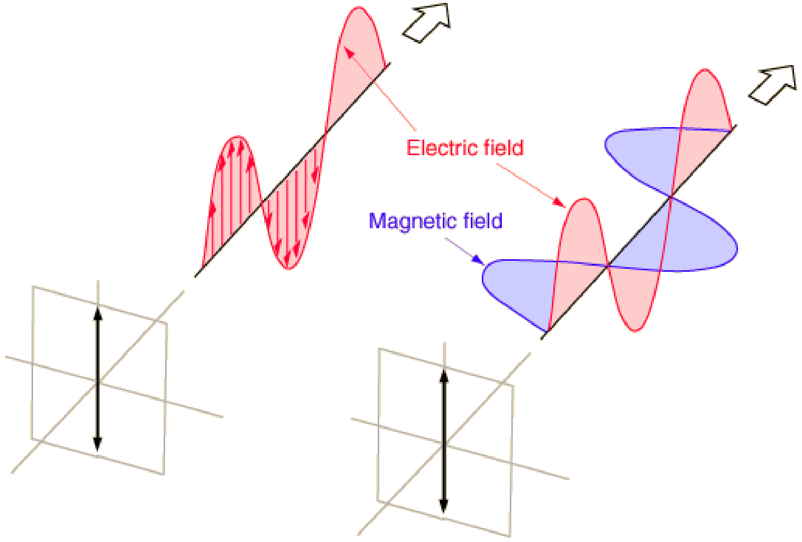
→Conditions: ①Only one component

②$\Delta\emptyset = \emptyset_y - \emptyset_x = \pm n\pi (n = 0,1,2,\ldots)$ between two orthogonal linear components

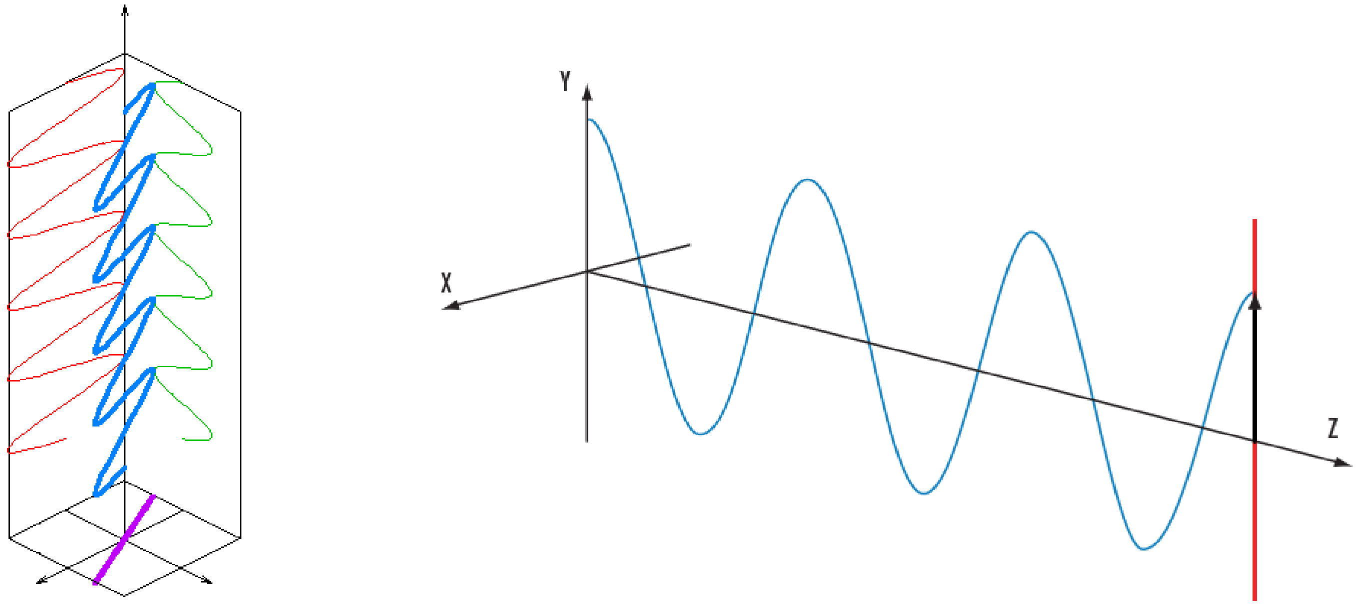
For the wave to have linear polarization, the time-phase difference between the two components must be

$$\Delta\emptyset = \emptyset_y - \emptyset_x = \pm n\pi (n = 0,1,2,\ldots)$$

∴When condition ① or ② is satisfied, we can see linear polarization regardless of E_{xo} & E_{yo}



<Condition ①>



<Condition ②>

1.12.1 Linear, Circular, and Elliptical Polarizations

• Case B. Circular Polarization   Circular polarization can be achieved only when

- the magnitudes of the two components are the same
- the time-phase difference between them is odd multiples of $\pi/2$.

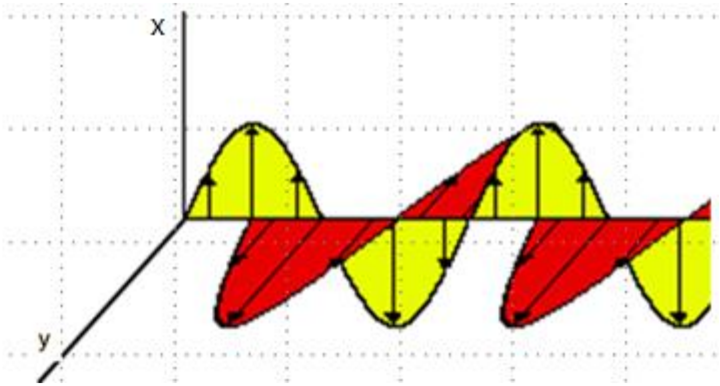
→Conditions: ①The field must have two orthogonal linear components.

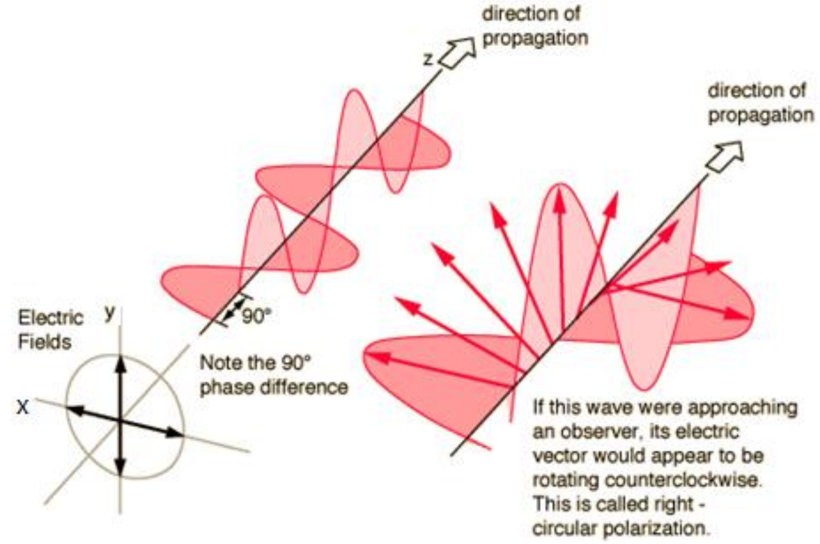
②Magnitudes of the two components are same → $E_{xo} = E_{yo}$

③$\Delta\emptyset = \emptyset_y - \emptyset_x = +\left(\frac{1}{2}+n\right)\pi, (n = 0,1,2,\ldots) \rightarrow$ CW (RHCP)

$= -\left(\frac{1}{2}+n\right)\pi, (n = 0,1,2,\ldots) \rightarrow$ CCW (LHCP)

∴All of the conditions must be satisfied for circular polarization





If the direction of wave propagation is reversed(i.e.,+z direction), the phase for CW and CCW rotation must be interchanged.

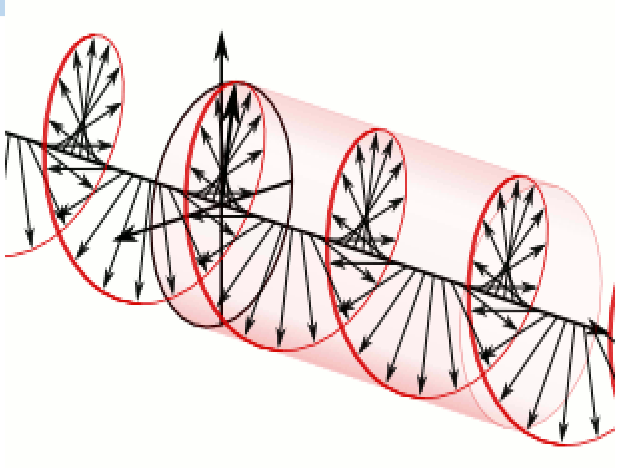


Figure 7. Circular Polarization.

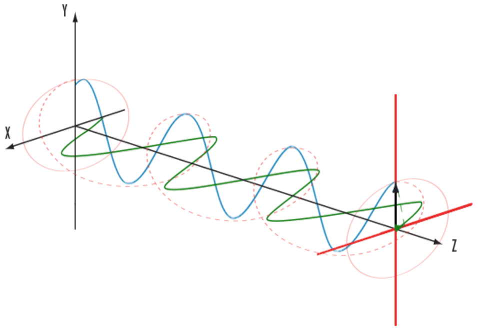
<CW circular polarization>

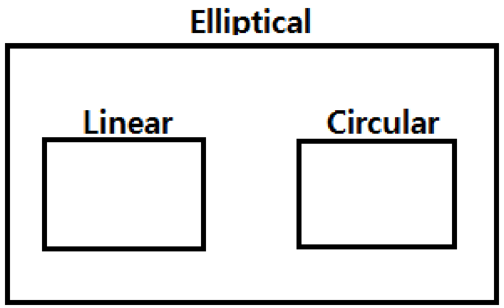
https://en.wikipedia.org/wiki/File:Circular.Polarization.Circularly.Polarized.Light_Left.Hand.Animation.305x190.255Colors.gif

C. Elliptical polarization

Elliptical polarization can be attained only when the time-phase difference between the two components is odd multiples of $\pi/2$ and their magnitudes are not the same or when the time-phase difference between the two components is not equal to multiples of $\pi/2$ (irrespective of their magnitudes). That is,

- the time-phase difference between the two components is odd multiples of $\pi/2$ and their magnitudes are not the same
- or, when the time-phase difference between the two components is not equal to multiples of $\pi/2$ (irrespective of their magnitudes). That is





<Linear and circular polarizations are special cases of elliptical

→Conditions: ①$E_{xo} \neq E_{yo}$ (Two orthogonal linear components)

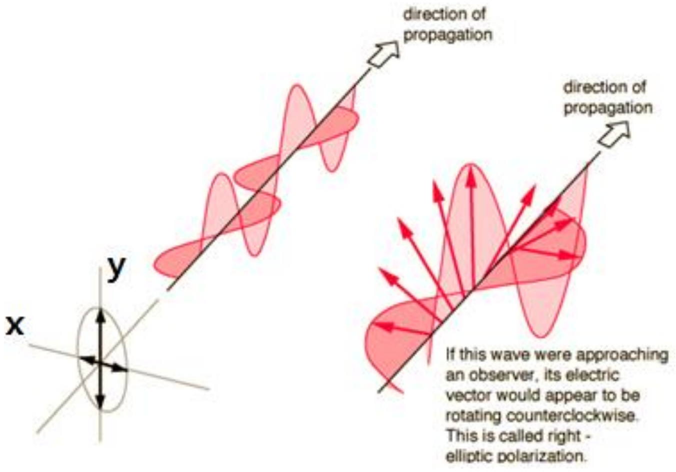
$$\Delta\emptyset = \emptyset_y - \emptyset_x = +\left(\frac{1}{2}+n\right)\pi, (n = 0,1,2,\ldots) \rightarrow \text{CW } \underline{\text{(RHEP)}}$$

$$\Delta\emptyset = \emptyset_y - \emptyset_x = -\left(\frac{1}{2}+n\right)\pi, (n = 0,1,2,\ldots) \rightarrow \text{CCW } \underline{\text{(LHEP)}}$$

②Regardless of E_{xo}, E_{yo} (Two orthogonal linear components)

$$\Delta\emptyset = \emptyset_y - \emptyset_x \neq \pm\frac{n}{2}\pi => 0 \rightarrow \text{CW } \underline{\text{(RHEP)}}$$

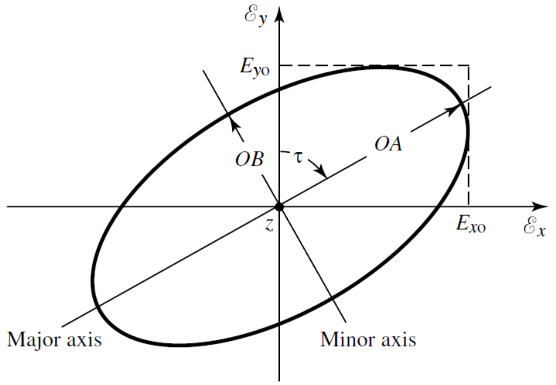
$$(n=0,1,2,\ldots) \quad < 0 \rightarrow \text{CCW } \underline{\text{(LHEP)}}$$



<CW elliptical polarization>

Elliptical Polarizations

For elliptical polarization, the curve traced at a given position as a function of time is a tilted ellipse, as shown in Figure 2.18(b). The ratio of the major axis to the minor axis is the axial ratio (AR),



Axial ratio

$$\mathrm{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, \quad 1 \le \mathrm{AR} \le \infty$$

Linear Polarization

Circular Polarization

$$OA = \left[\frac{1}{2}\{E_{xo}{}^2 + E_{yo}{}^2 + [E_{xo}{}^4 + E_{yo}{}^4 + 2E_{xo}{}^2 E_{yo}{}^2 \cos(2\Delta\phi)^{1/2}\}\right]^{1/2} \quad (2\text{-}65)$$

$$OB = \left[\frac{1}{2}\{E_{xo}{}^2 + E_{yo}{}^2 - [E_{xo}{}^4 + E_{yo}{}^4 + 2E_{xo}{}^2 E_{yo}{}^2 \cos(2\Delta\phi)^{1/2}\}\right]^{1/2} \quad (2\text{-}66)$$

(2-67)

The tilt of the ellipse, relative to the y axis, is represented by the angle τ given by

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{2E_{xo}E_{yo}}{E_{xo}{}^2 - E_{yo}{}^2}\cos(\Delta\phi)\right) \quad (2\text{-}68)$$

When the ellipse is aligned with the principal axes [τ=nπ/2, n=0,1,2,..], the major (minor) axis is equal to $E_{xo}(E_{yo})$ or $E_{yo}(E_{xo})$ and the axial ratio is equal to E_{xo}/E_{yo} or E_{yo}/E_{xo}.

SUMMARY

1.Linear Polarization

A time-harmonic wave is linearly polarized at a given point in space if the electric field (or magnetic field) vector at that point is always oriented along the same straight line at every instant of time. This is accomplished if the field vector (electric or magnetic) possesses:

a. Only one component, or
b. Two orthogonal linear components that are in time phase or 180° (or multiples of 180°) out of phase.

2.Circular Polarization

A time-harmonic wave is circularly polarized at a given point if the electric (or magnetic) field vector at that point traces a circle as a function of time. The necessary and sufficient conditions to accomplish this are:

a. The field must have two orthogonal linear components, and
b. The two components must have the same magnitude, and

c. The two components must have a time-phase difference of odd multiples of 90 ^{o}.

3. Elliptical Polarization

A wave is elliptically polarized if it is not linearly or circularly polarized. Although linear and circular polarizations are special cases of elliptical, usually in practice elliptical polarization refers to other than linear or circular. The necessary and sufficient conditions to accomplish this are if the field vector (electric or magnetic) possesses all of the following:

a. The field must have two orthogonal linear components, and

b. The two components can be of the same or different magnitude.

c. (1) If the two components are not of the same magnitude, the time-phase difference between the two components must not be 0^{o} or multiples of 180^{o} (because it will then be linear). (2) If the two components are of the same magnitude, the time-phase difference between the two components must not be odd multiples of 90 ^{o} (because it will then be circular).

To Determine Polarization of:

$$\underline{\mathcal{E}}(x, y, z; t) = \mathrm{Re}\left[\underline{E}(x, y, z)e^{j\omega t}\right]$$

1. Form $\underline{\mathcal{E}}(x, y, z; t)$
2. Plot $|\underline{\mathcal{E}}(x, y, z; t)|$ as a function of time
3. Plot Phase of $\underline{\mathcal{E}}(x, y, z; t)$ as a function of time

Example:

$$\underline{E} = \left(2\hat{a}_x + j2\hat{a}_y\right)e^{jkz} = \left(2\hat{a}_x + 2\hat{a}_y e^{j\pi/2}\right)e^{+jkz}$$

$$E_x = 2e^{+jkz}$$

$$E_y = 2e^{j\pi/2}e^{-jkz} = 2e^{j\left(\frac{\pi}{2}+kz\right)}$$

Solution:

1. $$\mathcal{E}_x = \mathrm{Re}\left[E_x e^{j\omega t}\right] = \mathrm{Re}\left[2e^{j(\omega t + kz)}\right]$$
$$= 2\cos(\omega t + kz)$$
$$\mathcal{E}_y = \mathrm{Re}\left[E_y e^{j\omega t}\right] = \mathrm{Re}\left[2e^{j\left(\omega t + \frac{\pi}{2} + kz\right)}\right]$$
$$= 2\cos\left(\omega t + \frac{\pi}{2} + kz\right) = -2\sin(\omega t + kz)$$

2. $$|\mathcal{E}| = \sqrt{\mathcal{E}_x^2 + \mathcal{E}_y^2}$$

$$= \sqrt{4\cos^2(\omega t + kz) + 4\sin^2(\omega t + kz)}$$

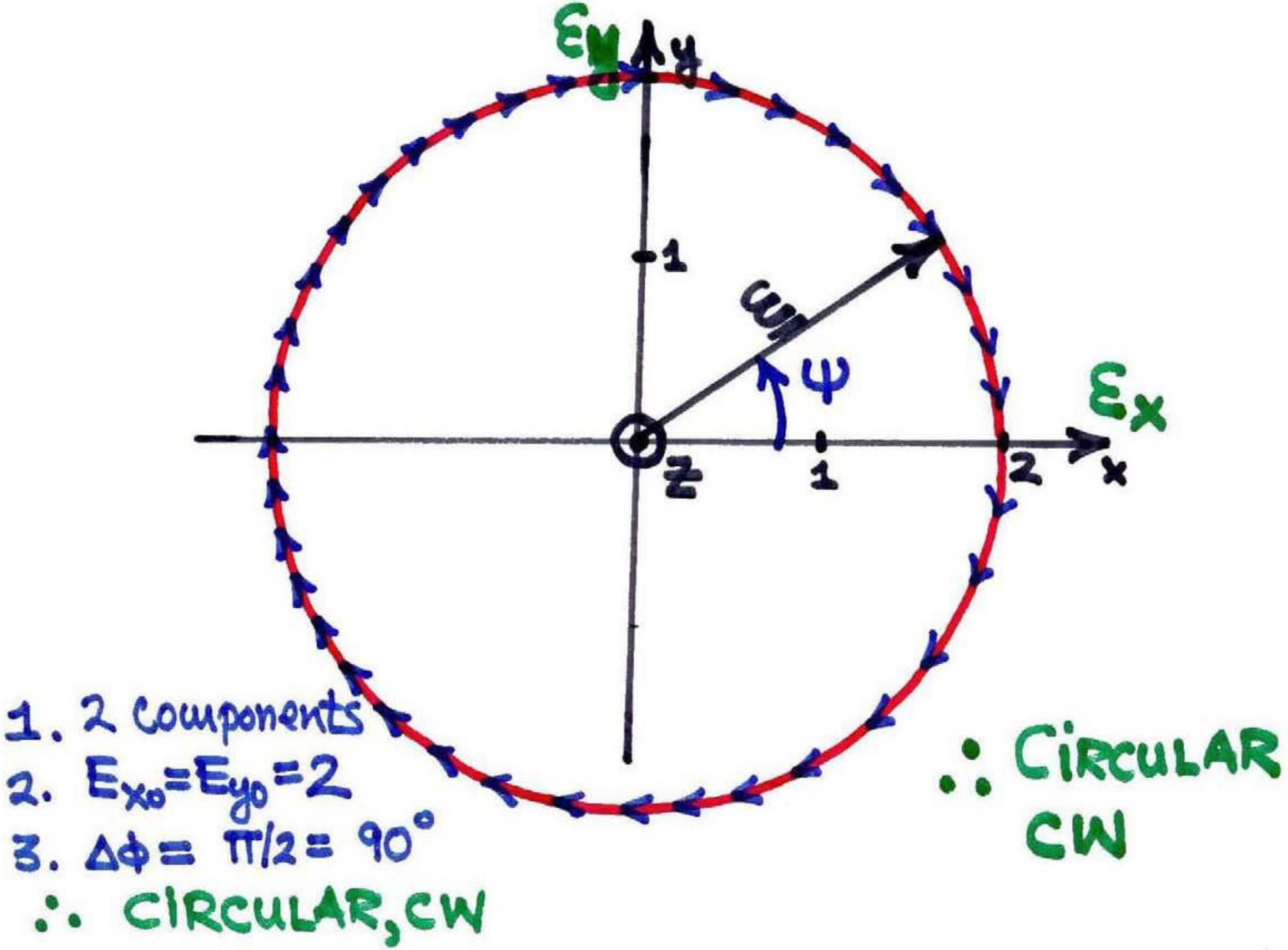
$$= 2\sqrt{\cos^2(\omega t + kz) + \sin^2(\omega t + kz)}$$

$$|\mathcal{E}| = 2$$

3. $$\psi = \tan^{-1}\left(\frac{\mathcal{E}_y}{\mathcal{E}_x}\right) = \tan^{-1}\left(-\frac{\sin(\omega t + kz)}{\cos(\omega t + kz)}\right)$$

$$= \tan^{-1}\left[-\tan(\omega t + kz)\right] = -(\omega t + kz)$$

$$\psi = -(\omega t + kz)\Big|_{z=0} = -\omega t$$



Ey
y
1
E
ψ
Ex
Z
1
2
x
1. 2 components
2. Exo=Eyo=2
3. Δφ= π/2= 90°
∴ CIRCULAR,CW
∴ CIRCULAR
CW

Example of Circularly Polarized Antennas

Crossed-Dipole on a Circular Ground Plane

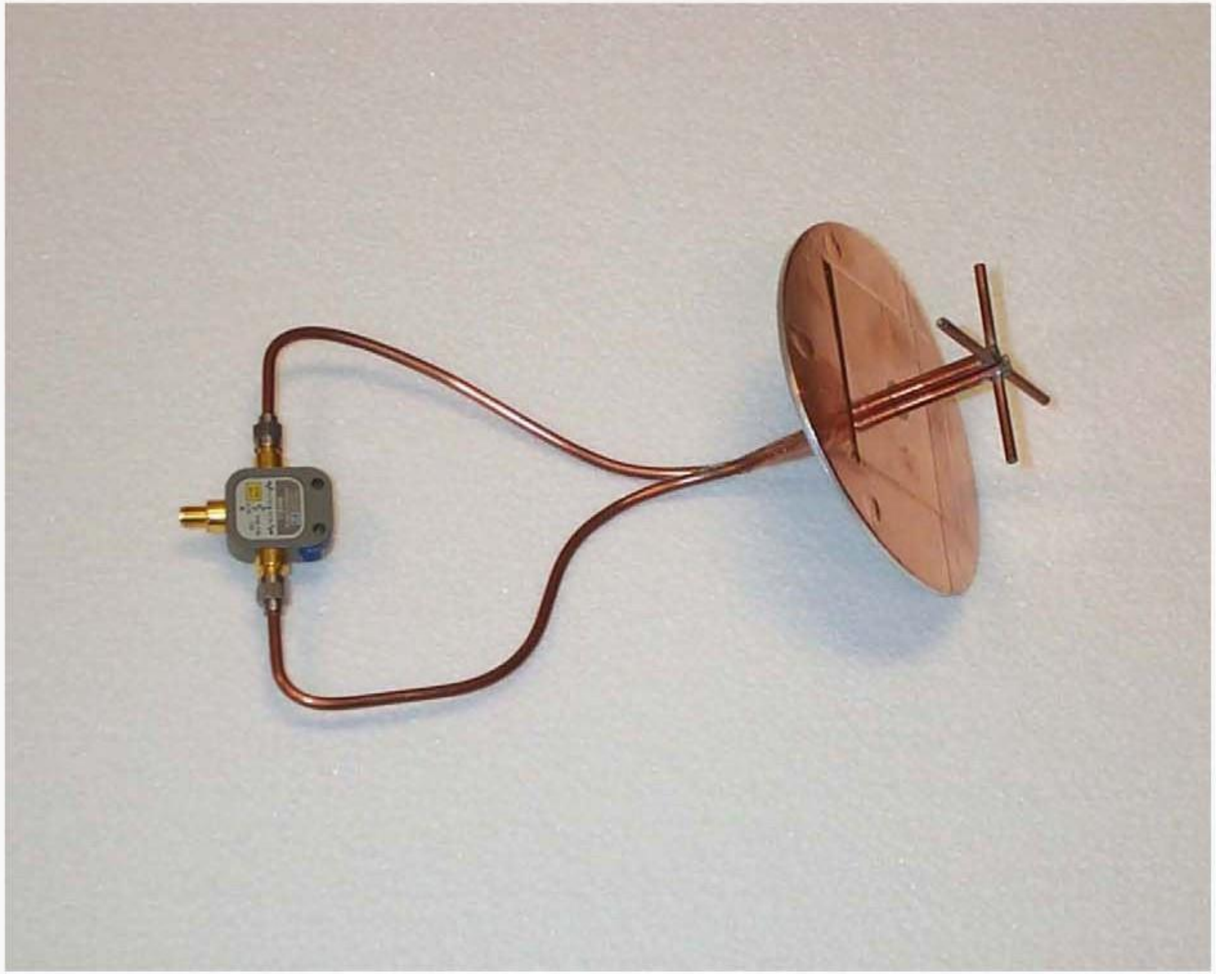


Fig. 2.26

Polarization Loss Factor (PLF)

Polarization Loss Factor and Efficiency

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as "polarization mismatch".
The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss.
The polarization loss can be taken into account by introducing a polarization loss factor(PLF).
Assuming that the electric field of the incoming wave can be written as

$$\mathbf{E}_i = \boldsymbol{\rho}_w E_i \qquad (2\text{-}69)$$

And the polarization of the electric field of the receiving antenna can be expressed as $\mathbf{E}_a = \boldsymbol{\rho}_a E_a$ (2-70)

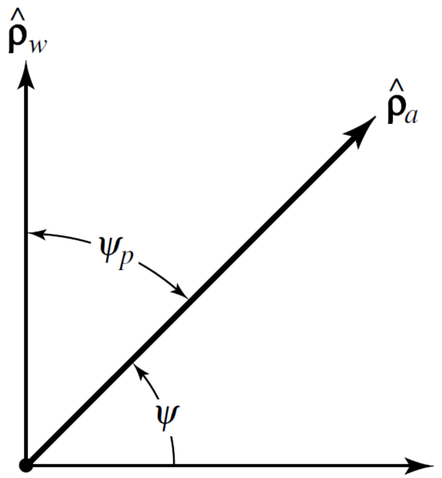


Figure 2.19 Polarization unit vectors of incident wave (ρw) and antenna (ρv)

Where $\boldsymbol{\rho}_w$ is the unit vector of the wave,
$\boldsymbol{\rho}_a$ is its unit vector(polarization vector)

PLF is defined, based on the polarization of the antenna in its transmitting mode

$$\text{PLF} = |\boldsymbol{\rho}_w \cdot \boldsymbol{\rho}_a|^2 = |\cos\Psi_p|^2 \quad \text{(dimensionless)} \qquad (2\text{-}71)$$

$$0 \le PLF \le 1$$
$$PLF(dB) = 10\log_{10}|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 20\log_{10}|\hat{\rho}_w \cdot \hat{\rho}_a|$$
$$-\infty \le PLF(dB) \le 0$$

If the antenna is polarization matched, its PLF =1 and the antenna will extract maximum power from the incoming wave.

Where ψp is the angle between the two unit polarization of the incoming wave of the vectors shown in Figure 2.19.

Using

A. Antenna Polarization in Transmitting Mode

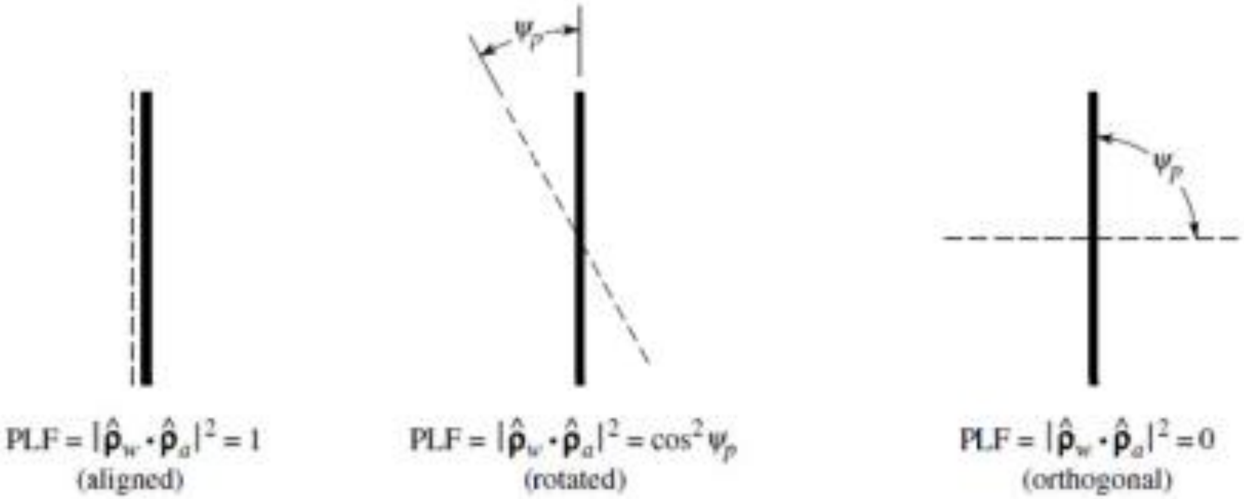
B. Incident Wave Polarization in Direction of Wave Travel

Correct Polarization Loss Factor

$$PLF = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = |\cos\psi_p|^2$$

Using this

$$\left.\begin{matrix}\text{CW-CW}\\ \text{CCW-CCW}\end{matrix}\right\}\text{Maximum} \qquad \left.\begin{matrix}\text{CW-CCW}\\ \text{CCW-CW}\end{matrix}\right\}\text{Minimum}$$



(b) PLF for transmitting and receiving linear wire antennas

Figure 2.25 Polarization loss factors (PLF) for aperture and linear wire antennas.

The polarization loss must always be taken into account in the link calculations design of a communication system because in some cases it may be a very critical factor.

Link calculations of communication systems for outer space explorations are very stringent because of limitations in spacecraft weight. In such cases, power is a limiting consideration. The design must properly take into account all loss factors to ensure a successful operation of the system.

Polarization Loss Factor for aperture

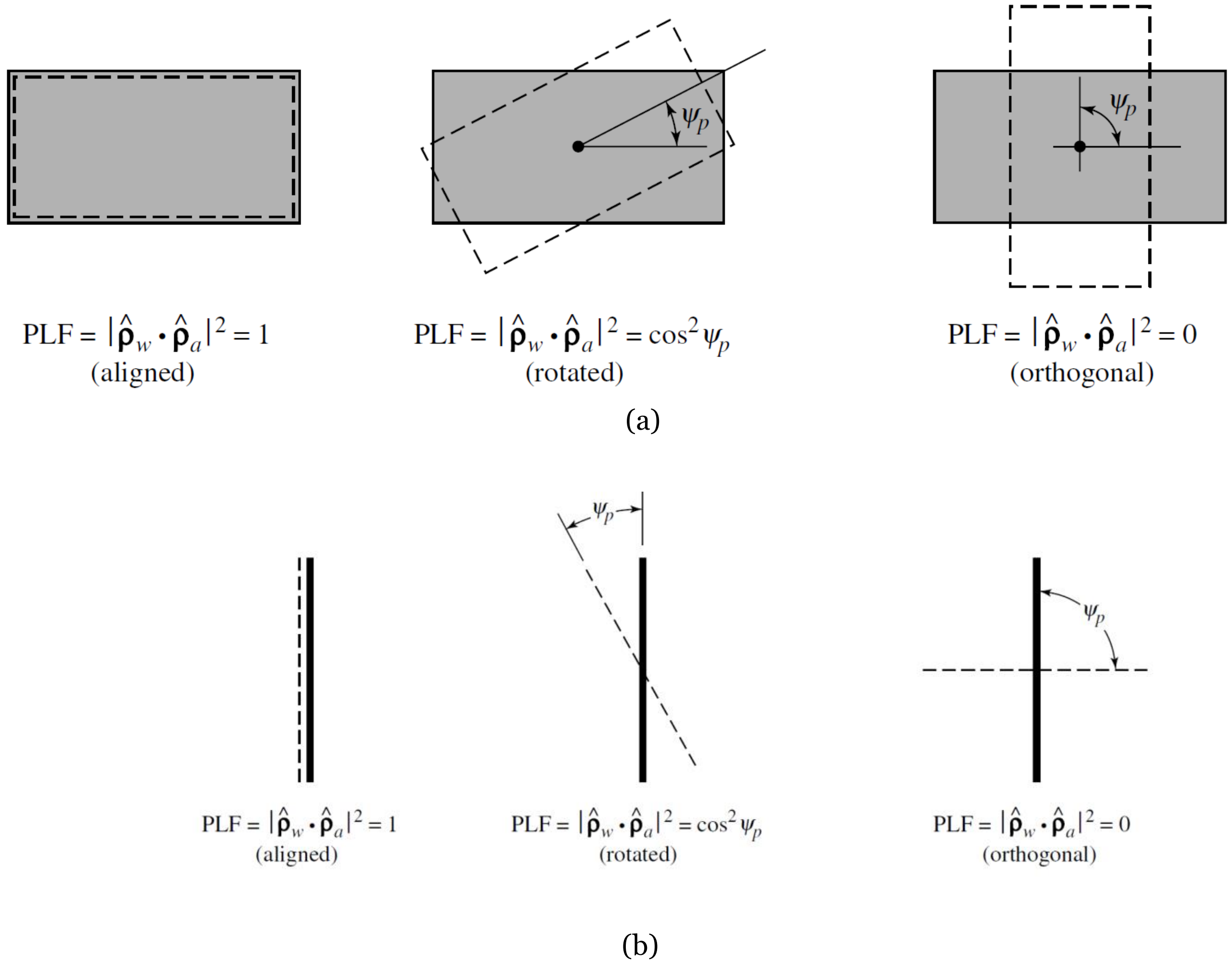


Figure2.25 Polarization loss factors (PLF) for aperture and wire antennas
(a) PLF for transmitting and receiving aperture antenna
(b) PLF for transmitting and receiving linear wire antennas

Another figure-of-merit describing the polarization characteristics of a wave and that of an antenna is the polarization efficiency.

Polarization efficiency is defined as:
"The ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation, whose polarization has been adjusted for a maximum received power."

$P_e = \dfrac{\lvert\vec{l}_e \cdot \vec{E}^{inc}\rvert^2}{\lvert\vec{l}_e\rvert^2 \cdot \lvert\vec{E}^{inc}\rvert^2}$ (2-71a)	Where $\vec{l}_e$ = vector effective length of the antenna $\vec{E}^{inc}$ = incident electric field

The vector effective length $\vec{l}_e$ is a vector that describes the polarization characteristics of the antenna. Both the PLF and P_e lead to the same answers.

Both the PLF and p_e lead to the same answer

Polarization Efficiency

(Polarization Mismatch or Loss Factor)

$$p_e = \frac{|\underline{\ell}_e \cdot \underline{E}^{inc}|^2}{|\underline{\ell}_e|^2 |\underline{E}^{inc}|^2} \quad \text{(2-71a)}$$

$\underline{\ell}_e$ = vector effective length of antenna

$\underline{E}^{inc}$ = incident electric field

Example 2.11

The electric field of a linearly polarized electromagnetic wave given by is incident upon a linearly

$$\mathbf{E}_i = \hat{\mathbf{a}}_x E_0(x, y) e^{-jkz}$$

polarized antenna whose electric-field polarization is expressed as $\mathbf{E}_a \square (\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y) E(r, \theta, \phi)$

Find the polarization loss factor(PLF).

Solution:

$$\underline{E}_a = (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi) = \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \right) \sqrt{2} E(r,\theta,\phi)$$

$$\hat{\rho}_a = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}, \qquad \hat{\rho}_w = \hat{a}_x$$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \hat{a}_x \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \right) \right|^2 = \frac{1}{2}$$

$$PLF = \frac{1}{2} = 10 \log_{10} \left(\frac{1}{2} \right) = -3 \text{ dB}$$

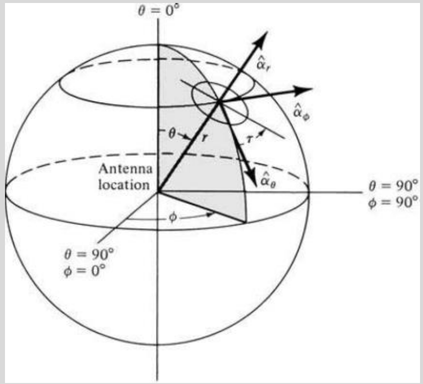
2.12.2 Polarization Loss Factor and Efficiency

Example 2.12

A right-hand (clockwise) circularly polarized wave radiated by an antenna, placed at some distance away from the origin of a spherical coordinate system, is traveling in the inward radial direction at an angle (θ, Φ) and it is impinging upon a right-hand circularly polarized receiving antenna placed at the origin (see Figures 2.1 and 17.23 for the geometry of the coordinate system). The polarization of the receiving antenna is defined in the transmitting mode, as desired by the definition of the IEEE. Assuming the polarization of the incident wave is represented by

$$\mathbf{E}_w = (\hat{\mathbf{a}}_\theta + j\hat{\mathbf{a}}_\phi)E(r,\theta,\phi)$$

Determine the polarization loss factor.



Solution

The polarization of the incident right-hand circularly polarized wave traveling along The –r radial direction is described by the unit vector

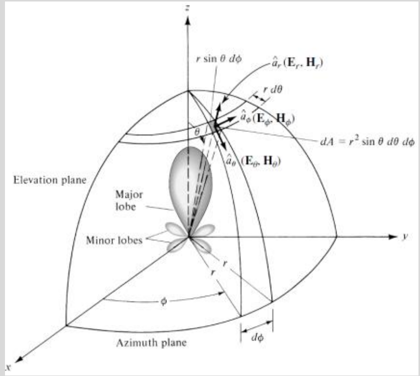
$$\hat{\boldsymbol{\rho}}_w = \left(\frac{\hat{\mathbf{a}}_\theta + j\hat{\mathbf{a}}_\phi}{\sqrt{2}}\right)$$

while that of the receiving antenna, in the transmitting mode, is represented by the unit vector

$$\hat{\boldsymbol{\rho}}_a = \left(\frac{\hat{\mathbf{a}}_\theta - j\hat{\mathbf{a}}_\phi}{\sqrt{2}}\right)$$

Therefore the polarization loss factor is

$$\mathrm{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = \frac{1}{4}|1+1|^2 = 1 = 0\mathrm{dB}$$



Since the polarization of the incoming wave matches (including the sense of rotation) the polarization of the receiving antenna, there should not be any losses. Obviously the answer matches the expectation.

Example 2.12:

$$\underline{E}_a = \left(\hat{a}_\theta - j\hat{a}_\phi\right) E(r,\theta,\phi)$$

Solution:

$$\underline{E}_a = \left(\hat{a}_\theta - j\hat{a}_\phi\right) E(r,\theta,\phi)$$

$$= \frac{\left(\hat{a}_\theta - j\hat{a}_\phi\right)}{\sqrt{2}} \sqrt{2} E(r,\theta,\phi)$$

$$\underline{E}_a = \hat{\rho}_a \sqrt{2} E(r,\theta,\phi), \quad \hat{\rho}_a = \frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}$$

CW-CW (Maximum)

$$\text{CW:}\quad \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}\right), \quad \text{CW:}\quad \hat{\rho}_w = \left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}}\right)$$

$$PLF = \left|\hat{\rho}_a \cdot \hat{\rho}_w\right|^2 = \left|\left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}\right)\left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}}\right)\right|^2$$

$$PLF = \left|\frac{1+1}{2}\right|^2 = 1 = 0 \ \text{dB}$$

Polarization Loss Factor and Efficiency

The polarization of an antenna in the receiving mode is related to that in the transmitting mode as follows

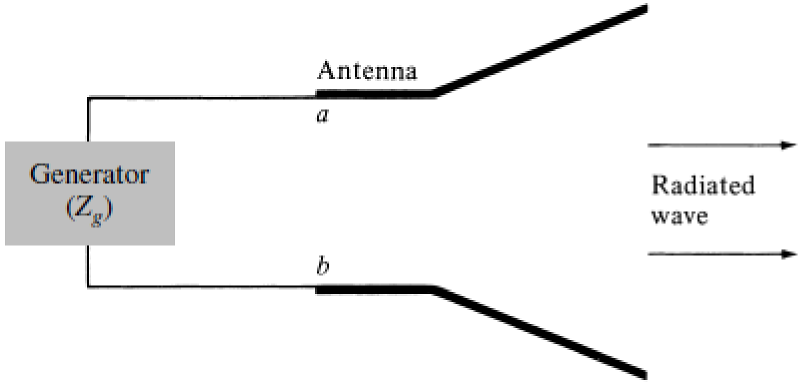
1. In the same plane of polarization, the polarization ellipses have the same axial ratio, the same sense of polarization (rotation) and the same spatial orientation.
2. Since their senses of polarization and spatial orientation are specified by viewing their polarization ellipses in the respective directions in which they are propagating, one should note that

    a. Although their senses of polarization are the same, they would appear to be opposite if both waves were viewed in the same direction.
    b. Their tilt angles are such that they are the negative of one another with respect to a common reference.

According to the IEEE Std 145-1983, the polarization of an antenna will almost always be defined in its transmitting mode.

2.13 Input impedance (in transmitting mode)

Definition : The impedance presented by an antenna at its terminals or **the ratio of the voltage to current at a pair of terminals** or **the ratio of the appropriate components of the electric to magnetic fields at a point.**



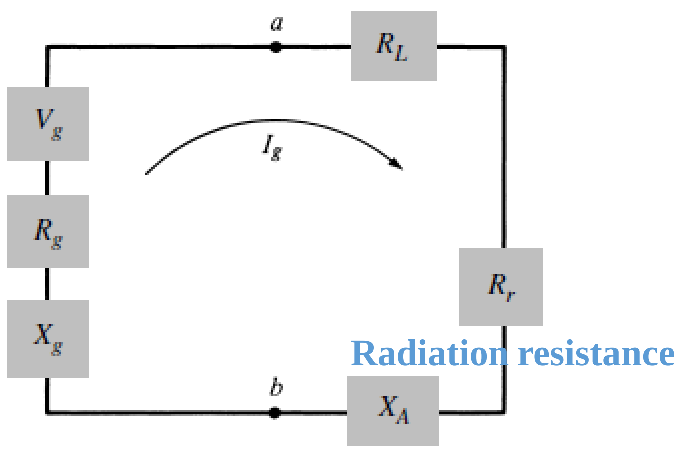


Figure 2.27 Transmitting antenna and its equivalent circuits.

$$Z_A = R_A + jX_A$$

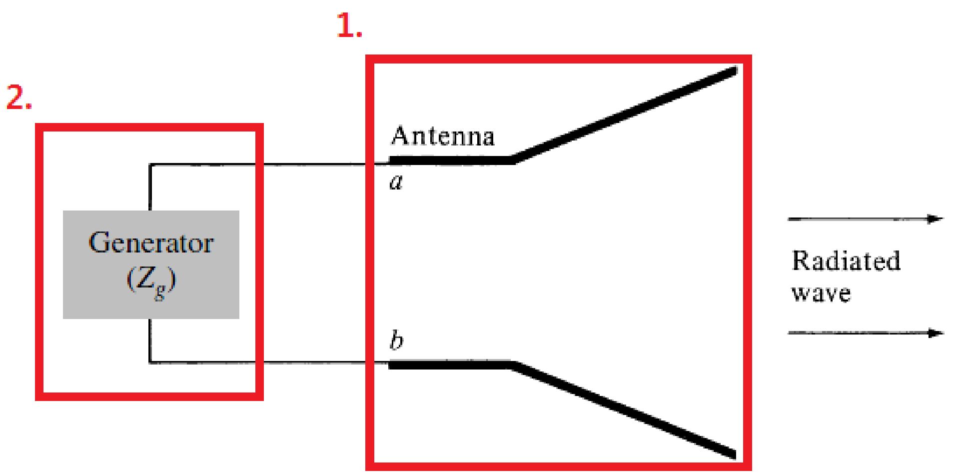
Z_A = antenna impedance at terminal a-b (ohms)
R_A = antenna resistance at terminal a-b (ohms)
X_A = antenna reactance at terminal a-b (ohms)

$$Z_A = R_A + jX_A$$

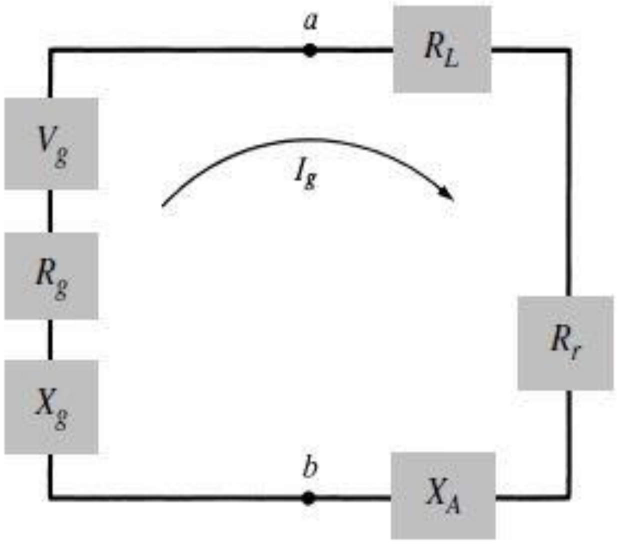
$$R_A = R_r + R_L$$

$$Z_g = R_g + jX_g$$

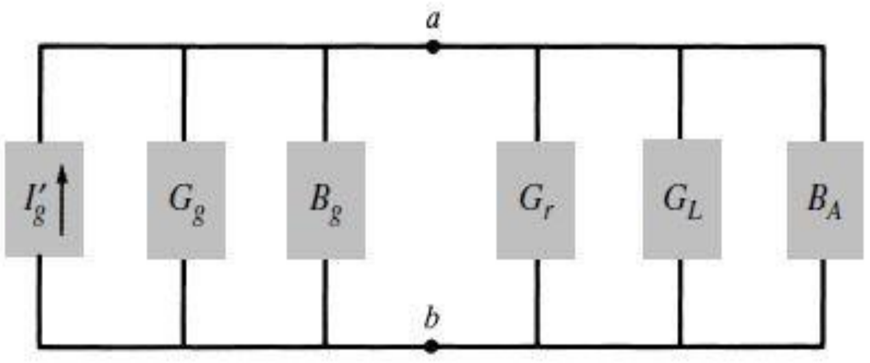
Z_g = Generator impedance (ohms)
R_g = Resistance of generator impedance (ohms)
X_g = Reactance of generator impedance (ohms)



(a) Antenna in transmitting mode

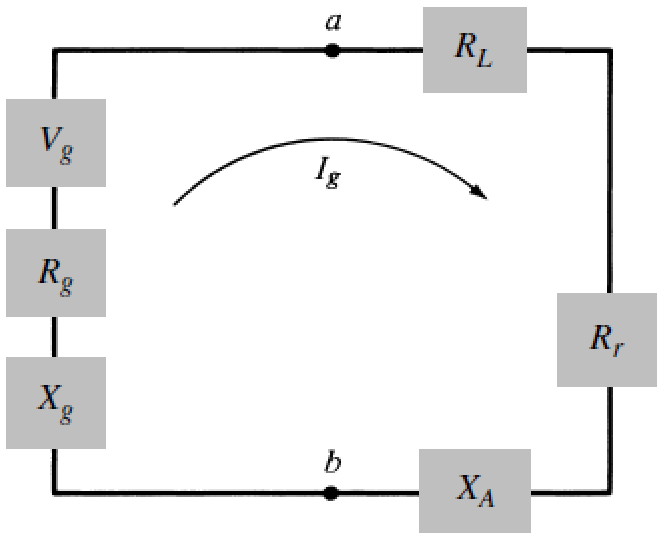


(b) Thevenin equivalent



(c) Norton equivalent

2.13 Input Impedance



(b) Thevenin equivalent

Figure 2.27 Transmitting antenna and its equivalent circuits.

$$Z_g = R_g + jX_g \qquad (2\text{-}74)$$

R_g = resistance of generator impedance (ohms)

X_g = reactance of generator impedance (ohms)

(Transmitting mode)

Find the amount of power delivered to R_r for radiation and amount of dissipated in R_L as heat.

$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)} \quad (A) \qquad (2\text{-}75)$$

$$|I_g| = \frac{|V_g|}{\left[(R_r + R_L + R_g)^2 + (X_A + X_g)^2\right]^{1/2}} \qquad (2\text{-}75a)$$

2.13 Input Impedance

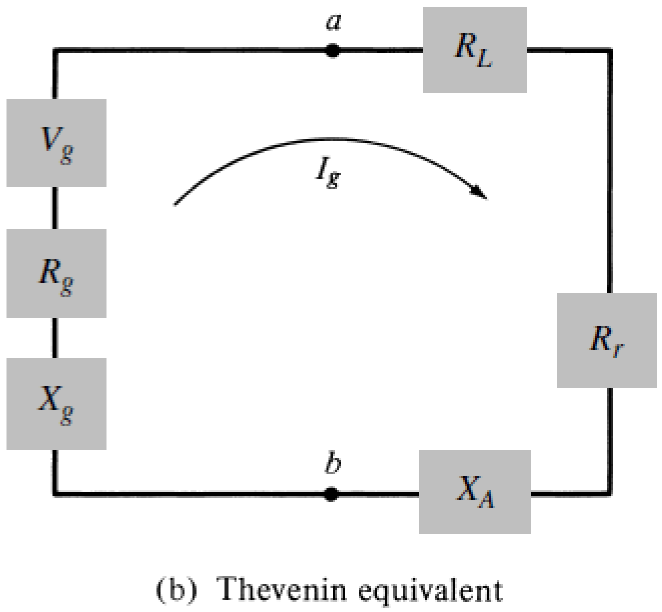


Figure 2.27 Transmitting antenna and its equivalent circuits.

the power delivered to the antenna for radiation

$$P_r = \frac{1}{2}\left|I_g\right|^2 R_r = \frac{\left|V_g\right|^2}{2}\left[\frac{R_r}{(R_r+R_L+R_g)^2+(X_A+X_g)^2}\right] \quad (W) \qquad (2\text{-}76)$$

dissipated as heat

$$P_L = \frac{1}{2}\left|I_g\right|^2 R_L = \frac{\left|V_g\right|^2}{2}\left[\frac{R_L}{(R_r+R_L+R_g)^2+(X_A+X_g)^2}\right] \quad (W) \qquad (2\text{-}77)$$

dissipated as heat on the internal resistance R_g of the generator

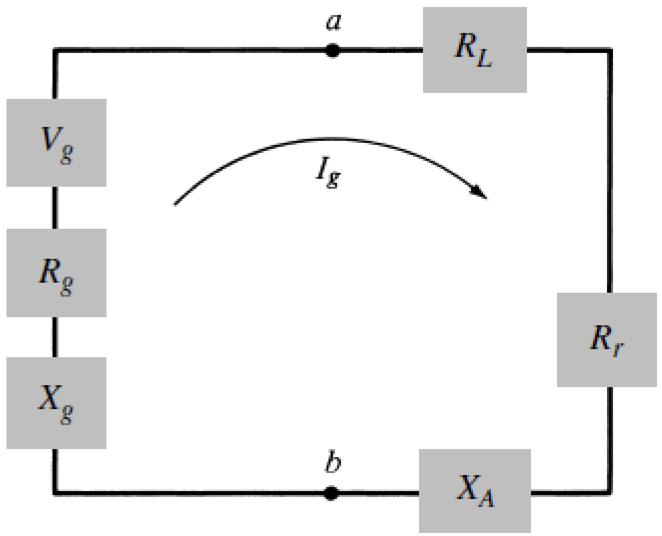
$$P_g = \frac{1}{2}\left|I_g\right|^2 R_g = \frac{\left|V_g\right|^2}{2}\left[\frac{R_g}{(R_r+R_L+R_g)^2+(X_A+X_g)^2}\right] \quad (W) \qquad (2\text{-}78)$$

Maximum power delivered to the antenna when conjugate matching.

$$R_r + R_L = R_g \qquad (2\text{-}79)$$

$$X_A = -X_g \qquad (2\text{-}80)$$

2.13 Input Impedance



(b) Thevenin equivalent

Figure 2.27 Transmitting antenna and its equivalent circuits.

(when conjugate matching, ($R_r + R_L = R_g$, $X_A = -X_g$))

$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right] \quad (W) \qquad (2\text{-}81)$$

$$P_L = \frac{|V_g|^2}{2}\left[\frac{R_L}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right] \quad (W) \qquad (2\text{-}82)$$

$$P_g = \frac{|V_g|^2}{2}\left[\frac{R_g}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_g}{(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{1}{R_r + R_L}\right] = \frac{|V_g|^2}{8R_g} \quad (W) \qquad (2\text{-}83)$$

$$P_g = P_r + P_L = \frac{|V_g|^2}{8}\left[\frac{R_g}{(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r + R_L}{(R_r + R_L)^2}\right] \qquad (2\text{-}84)$$

$$P_s = \frac{1}{2}V_g I_g^* = \frac{1}{2}V_g\left[\frac{V_g^*}{2(R_r + R_L)}\right] = \frac{|V_g|^2}{4}\left[\frac{1}{R_r + R_L}\right] \quad (W) \qquad (2\text{-}85)$$

1.13 Input impedance (in transmitting mode)

- Total supplied power(P_s) by generator during conjugate matching,

$$P_s = \frac{1}{2}V_g I_g^* = \frac{1}{2}V_g\left[\frac{V_g^*}{2(R_r+R_L)}\right] = \frac{|V_g|^2}{4}\left[\frac{1}{R_r+R_L}\right] \quad \text{(W)}$$

$\therefore\ P_s = P_g + (P_r + P_L)$ ➡ Conjugate matching & lossless transmission lines,

➡ $$\frac{1}{2}P_s = P_g \ or\ (P_r + P_L)$$

- If the antenna is lossless(R_L=0) and matched to lossless transmission line,

half of P_s is radiated by the antenna, and the other half is dissipated as heat in generator.

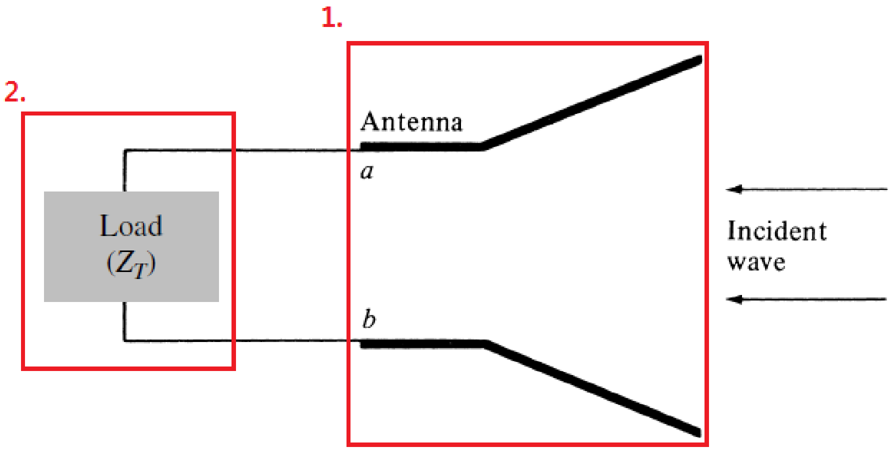
$$e_{cd} = \frac{R_r}{R_L+R_r} = 1$$

- If matched lossy transmission line,

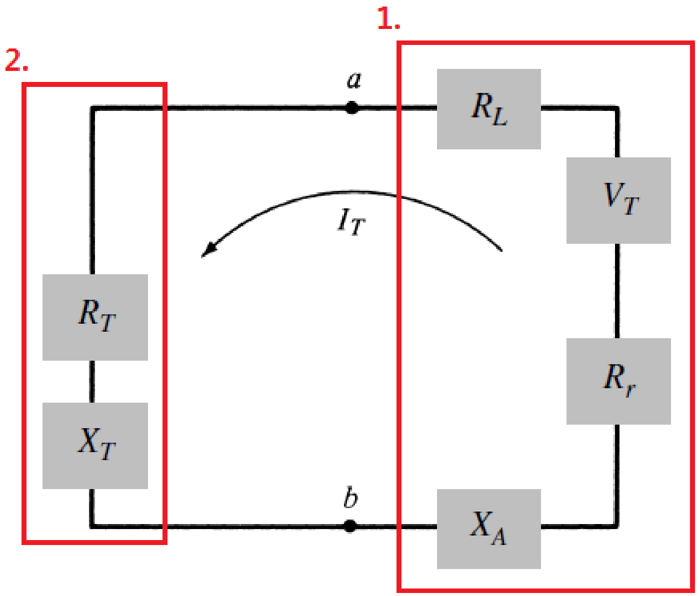
available radiating power will be reduced by losses of transmission line.

$$e_{cd} = \frac{R_r}{R_L+R_r} < 1$$

1.13 Input impedance (in receiving mode)



(a) Antenna in receiving mode



(b) Thevenin equivalent

1. Impedance of the Antenna :

$$Z_A = (R_r + R_L) + jX_A$$

R_r = re-radiation resistance of the antenna (Ω)
R_L = loss resistance of the antenna (Ω)
X_A = antenna reactance (Ω)

V_T = induced voltage by incident wave (V)

2. Load Impedance :

$$Z_T = R_T + jX_T$$

R_T = load resistance (Ω)
X_T = load reactance (Ω)

1.13 Input impedance (in receiving mode)

Analysis of equivalent circuit (receiving mode)

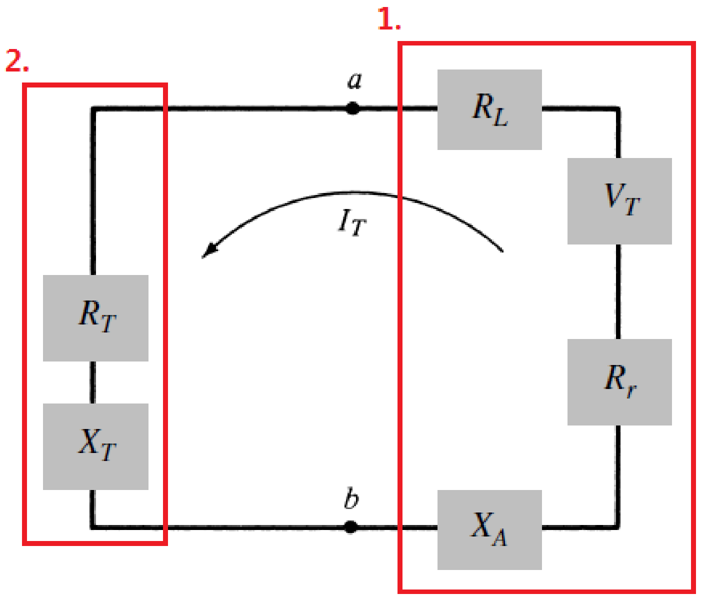
1. Find the current(I_T) developed within the loop

$$I_T = \frac{V_T}{Z_{total}} = \frac{V_T}{Z_A+Z_T} = \frac{V_T}{(R_r+R_L+R_T)+j(X_A+X_T)} \quad \text{(A)}$$

$$|I_T| = \frac{|V_T|}{\sqrt{(R_r+R_L+R_T)^2+(X_A+X_T)^2}}$$

V_T: peak induced voltage

2. Find the amount of maximum power(P_r, P_L, P_T) after conjugate matching



(b) Thevenin equivalent

①Maximum power delivered to R_T (load)

$$P_T = \frac{|V_T|^2}{8}\left[\frac{R_T}{(R_r+R_L)^2}\right] = \frac{|V_T|^2}{8}\left(\frac{1}{R_r+R_L}\right) = \frac{|V_T|^2}{8R_T} \quad \text{(W)}$$

②Maximum power delivered to R_r that is reradiated(or scattered)

$$P_r = \frac{|V_T|^2}{8}\left[\frac{R_r}{(R_r+R_L)^2}\right] \quad \text{(W)}$$

③Maximum power delivered to R_L that is dissipated as heat

$$P_L = \frac{|V_T|^2}{8}\left[\frac{R_L}{(R_r+R_L)^2}\right] \quad \text{(W)}$$

2.13 Input Impedance

(under conjugate matching, ($R_r + R_L = R_T, \ X_A = -X_T$))

$$P_c = \frac{|V_T|^2}{4}\left[\frac{1}{R_r + R_L}\right] \ (W) \qquad (2\text{-}89)$$

↓

50%

$$P_T = \frac{|V_T|^2}{8R_T} \ (W) \qquad (2\text{-}86)$$

$$P_r = \frac{|V_T|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right] \ (W) \qquad (2\text{-}87)$$

$$P_L = \frac{|V_T|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right] \ (W) \qquad (2\text{-}88)$$

50%

If the losses are zero ($R_L = 0$), then half of the captured power is delivered to the load and the other half is scattered.

The most that can be delivered to the load is only half of that captured and that is only under conjugate matching and lossless transmission line.

2.14 Antenna Radiation Efficiency

Definition : The antenna efficiency takes into account the reflection, conduction, and dielectric losses

The conduction and dielectric losses of an antenna are very difficult to compute.

→ lumped together to form the e_{cd} efficiency

The resistance R_L is used to represent the conduction-dielectric losses.

conduction-dielectric efficiency (e_{cd}) - the ratio of the power delivered to the radiation resistance R_r to the power delivered to R_r and R_L

$$P_r = \frac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right] \quad (2\text{-}76)$$

$$P_L = \frac{1}{2}|I_g|^2 R_L = \frac{|V_g|^2}{2}\left[\frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right] \quad (2\text{-}77)$$

$$\frac{P_r}{P_r + P_L} = e_{cd} = \left[\frac{R_r}{R_r + R_L}\right] \quad (2\text{-}90)$$

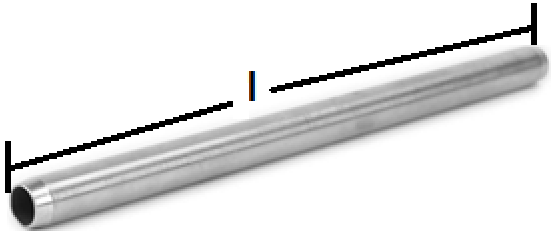
2.14 Antenna Radiation Efficiency

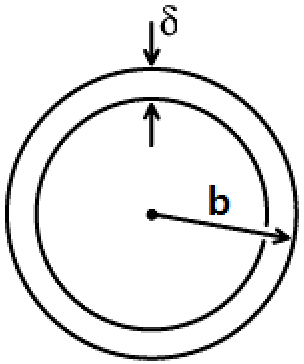
$$R_{dc} = \frac{l}{\sigma A} \quad \text{(ohms)} \qquad \text{(2-90a)}$$

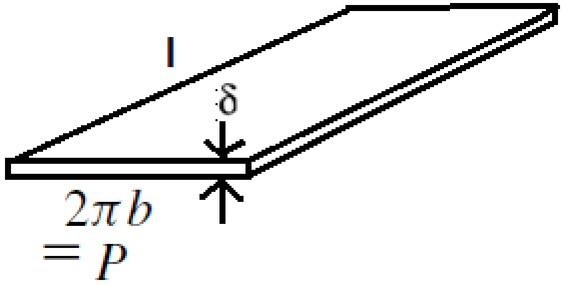
l = metal rod of length
A = uniform cross-sectional area
σ = conductivity of the metal

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$$

δ = skin depth of the metal







If the skin depth δ of the metal is very small compared to the smallest diagonal of the section of the rod, the current is confined to a thin layer near the conductor surface. Therefore the high-frequency resistance can be written, based on a uniform current distribution, as

$$\mathrm{R_{hf}} = \frac{l}{P} R_s = \frac{l}{P}\sqrt{\frac{\omega \mu_0}{2\sigma}} \quad \text{(ohms)} \qquad \text{(2-90b)}$$

P = the perimeter of the cross section of the rod
R_s = the conductor surface resistance
ω = the angular frequency
μ_0 = the permeability of free-space
σ = conductivity of the metal

2.15 Antenna Vector Effective Length and Equivalent Areas

An antenna in the receiving mode is used to capture(collect) electromagnetic waves and to extract power from them.
For each antenna, an equivalent length and a number of equivalent areas can be defined.
Equivalent quantities are used to describe the receiving characteristics of an antenna when a wave is incident upon the antenna.

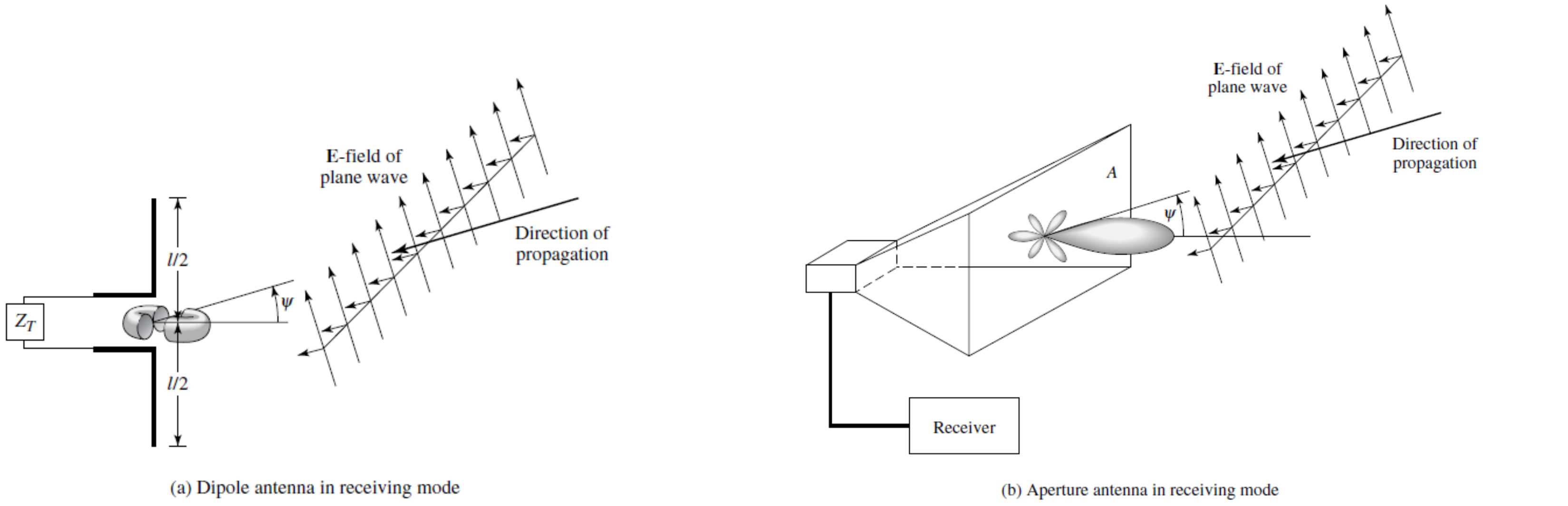


Figure 2.29 Uniform plane wave incident upon dipole and aperture antennas.

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.1 Vector Effective Length

The effective length of an antenna is a quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it.

$$\ell_e(\theta,\phi) = a_\theta l_\theta(\theta,\phi) + a_\phi l_\phi(\theta,\phi) \qquad \text{(2-91)}$$

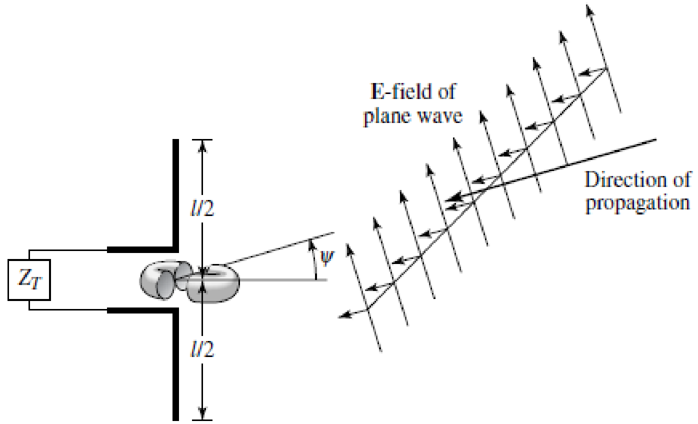


Figure 2.29 (a)Dipole antenna in receiving mode

It is a far-field quantity and it is related to the far-zone field $\boldsymbol{E}_a$ radiated by the antenna, with current $\boldsymbol{I}_{in}$ in its terminals.

$$\boldsymbol{E}_a = a_\theta E_\theta + a_\phi E_\phi = -j\eta \frac{k\boldsymbol{I}_{in}}{4\pi r} \ell_e e^{-jkr} \qquad \text{(2-92)}$$

$$E^i = E_0 e^{jkr}$$

E_0 = a constant complex vector

k = the propagation vector

r = radius vector in any direction defined by the angles θ and ϕ

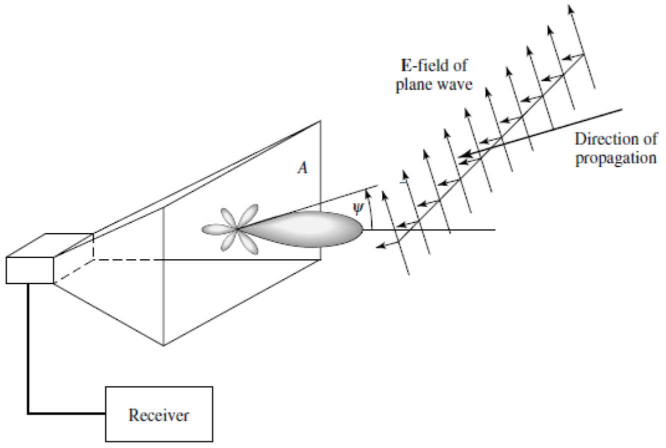
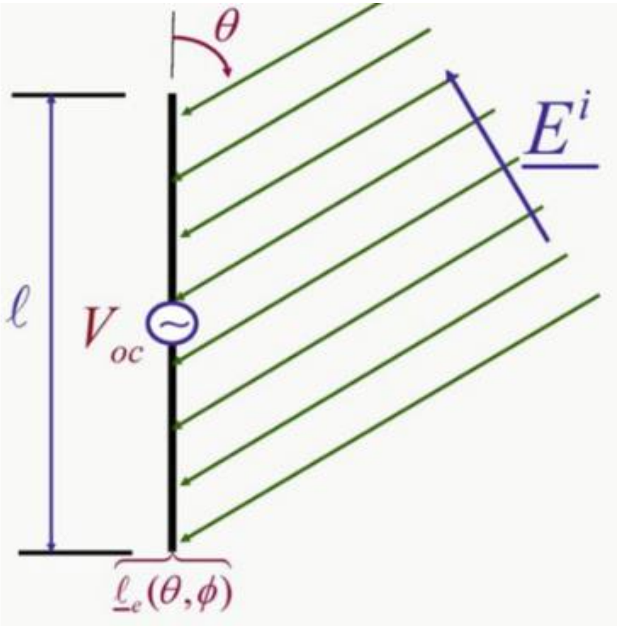


Figure 2.29 (b) Aperture antenna in receiving mode

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.1 Vector Effective Length

The effective length represents the antenna in its transmitting and receiving modes, and it is particularly useful in relating the open-circuit voltage V_{oc} of receiving antennas.



the effective length is a vector

$$V_{oc} = \int_c \boldsymbol{E}^i \cdot d\ell = \boldsymbol{E}^i \ell_e(\theta,\phi)$$

when taking the maximum value over θ, ϕ this becomes

$$V_{oc} = \boldsymbol{E}^i \cdot \ell_e \qquad (2\text{-}93)$$

V_{oc} = open-circuit voltage at antenna terminals
$\boldsymbol{E}^i$ = incident electric field
ℓ_e = vector effective length

For linear antennas

$$|\ell_e| \leq \ell_{physical}$$

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.1 Vector Effective Length

Effective length of a linearly polarized antenna receiving a plane wave in a given direction

- The ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization.
- It is used to determine the polarization efficiency of the antenna.

Ex) a small dipole of length $l < \lambda/10$ and with a triangular current distribution

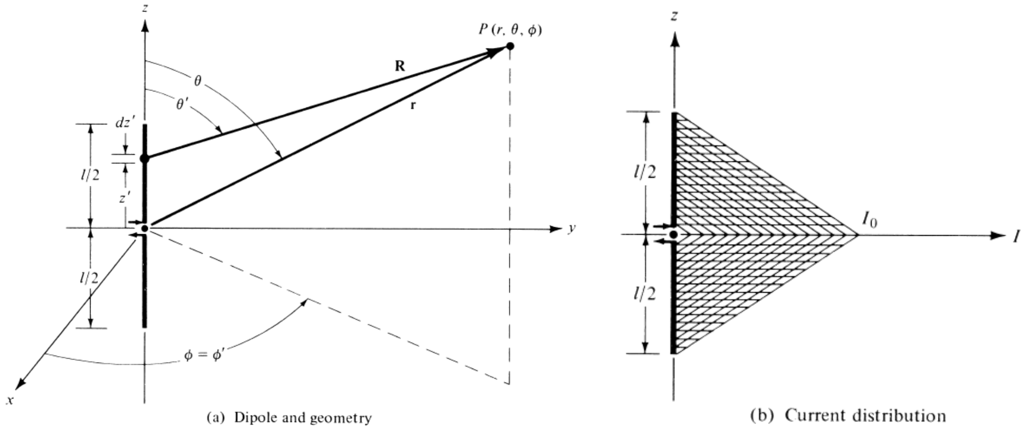


Figure 4.4 Geometrical arrangement of dipole and current distribution.

$$\boldsymbol{E}_a = a_\theta \, j\eta \frac{k\boldsymbol{I}_{in} l e^{-jkr}}{8\pi r} \sin\theta \qquad \text{(4-36a)}$$

$$\boldsymbol{E}_a = a_\theta E_\theta + a_\phi E_\phi = -j\eta \frac{k\boldsymbol{I}_{in}}{4\pi r} \ell_e e^{-jkr} \qquad \text{(2-92)}$$

$$\ell_e = -a_\theta \frac{l}{2} \sin\theta$$

- The antenna vector effective length is used to determine the polarization efficiency of the antenna.

Example 2.14

The far-zone field radiated by a small dipole of length $l < \lambda/10$ and with a triangular current distribution, as shown in Figure 4.4, is derived in Section 4.3 of Chapter 4 and it is given by (4-36a), or

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta \, j\eta \frac{kI_{in} l e^{-jkr}}{8\pi r} \sin\theta$$

Determine the vector effective length of the antenna.

Solution: According to (2-92), the vector effective length is

$$\ell_e = -\hat{\mathbf{a}}_\theta \frac{l}{2} \sin\theta$$

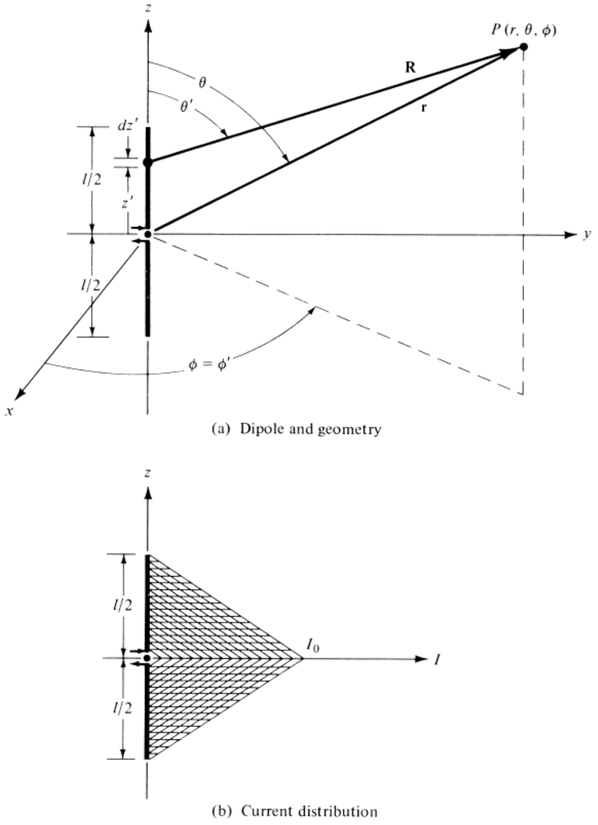


Figure 4.4 Geometrical arrangement of dipole and current distribution.

- This indicates, as it should, that the effective length is a function of the direction angle θ, and its maximum occurs when θ = 90◦.

- The effective length of the dipole to produce the same output open-circuit voltage is only half (50%) of its physical length if it were replaced by a thin conductor having a uniform current distribution.

2.15 Antenna Equivalent Areas

2.15 .2 Antenna Equivalent Areas

With each antenna, we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it.

Antenna effective area (aperture)

Definition :

the ratio of the **available power** at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna.

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T / 2}{W_i}$$

A_e = effective area (effective aperture)

P_T = power delivered to the load

W_i = power density of incident wave

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.2 Antenna Equivalent Areas

Equivalent area - the power capturing characteristics of the antenna when a wave impinges on it

Effective area (aperture) - the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

A_e = effective area (effective aperture) (m^2)

P_T = power delivered to the load (W)

W_i = power density of incident wave $(\mathrm{W/m}^2)$

Effective aperture - the area which when multiplied by the incident power density gives the power delivered to the load.

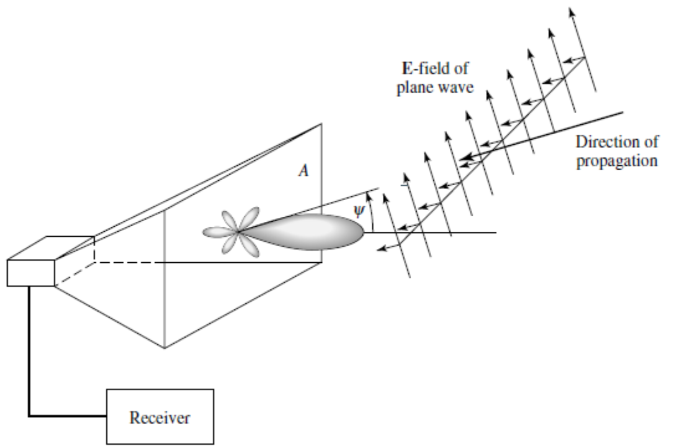


Figure 2.29 (b) Aperture antenna in receiving mode

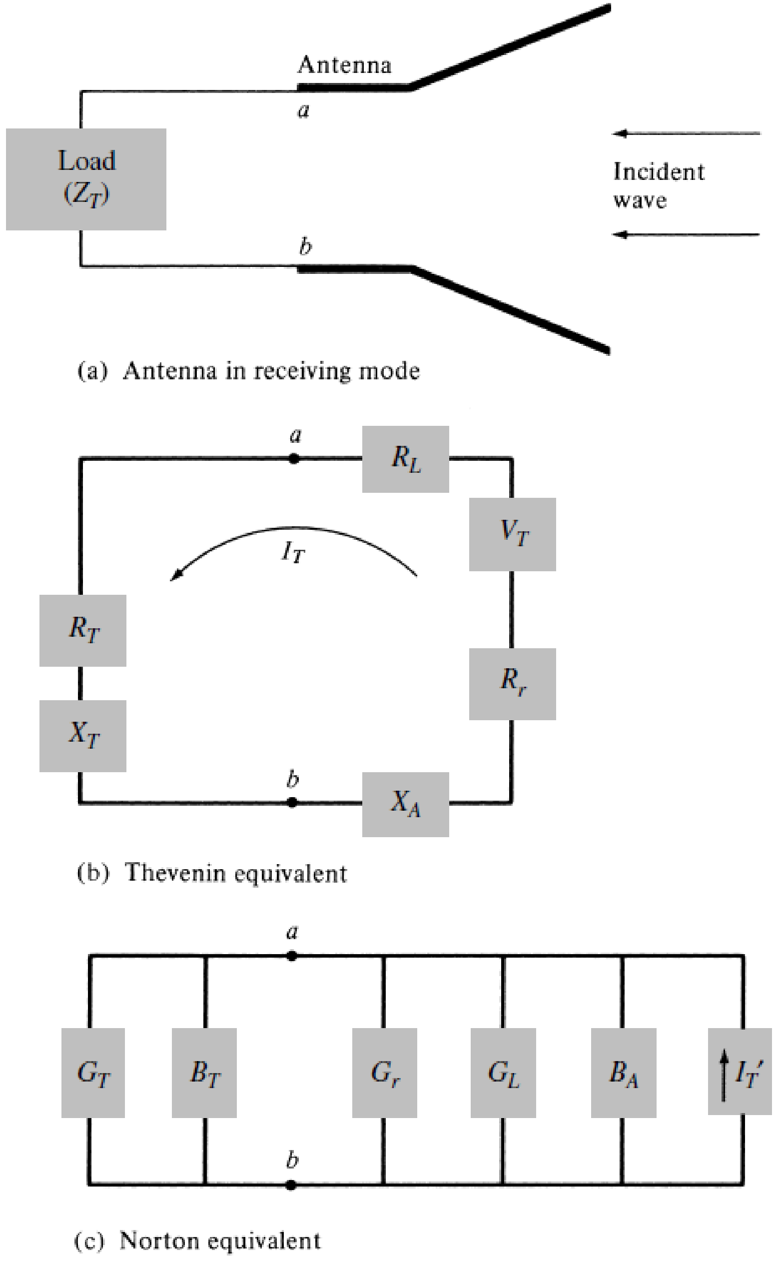


Figure 2.28 Antenna and its equivalent circuits in the receiving mode.

In this characteristic using the equivalent of Figure 2.28, equation (2-94) can be expressed as equation (2-95).

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right] \tag{2-95}$$

Under conditions of maximum power transfer (*conjugate matching*), $R_r + R_L = R_T$ and $X_A = -X_T$, the effective area of (2-95) reduces to the maximum effective aperture given by

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right]$$

- All of the power that is intercepted, collected, or captured by an antenna is not delivered to the load.

- In fact, under conjugate matching only half of the captured power is delivered to the load; the other half is scattered and dissipated as heat.

- Therefore to account for the scattered and dissipated power we need to define, in addition to the effective area, the *scattering, loss* and *capture* equivalent areas.

◆ **The scattering area**

The equivalent area when multiplied by the incident power density is equal to the scattered or reradiated power.

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right] \tag{2-97}$$

◆ **The loss area**

The equivalent area, which when multiplied by the incident power density leads to the power
dissipated as heat through R_L.

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_L + R_r)^2}\right] \qquad (2\text{-}98)$$

◆ **The capture area**

The equivalent area, which when multiplied by the incident power density leads to the total
power captured, collected, or intercepted by the antenna.

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T + R_r + R_L}{(R_L + R_r)^2}\right] \qquad (2\text{-}99)$$

When capture area multiplied by the incident power density, it leads to the captured power. In general, the total capture area is equal to the sum of the other three, or

$$\textit{Capture Area} = \textit{Effective Area} + \textit{Scattering Area} + \textit{Loss Area}$$

◆ The aperture efficiency

The ratio of the maximum effective area A_{em} of the antenna to its physical area A_p

$$\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}} \qquad (2\text{-}100)$$

($A_{em} \leq A_p$ or $0 \leq \varepsilon_{ap} \leq 1$).

- For a lossless antenna (R_L = 0) the maximum value of the scattering area is also equal to the physical area. Therefore even though the aperture efficiency is greater than 50%, for a lossless antenna under conjugate matching only half of the captured power is delivered to the load and the other half is scattered.

◆ Partial effective area

The ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction and with a specified polarization differing from the receiving polarization of the antenna.

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.2 Antenna Equivalent Areas

Partial effective area of an antenna *for a given polarization* in a given direction
- the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction and with a specified polarization differing from the receiving polarization of the antenna

The effective area of an antenna is not necessarily the same as the physical aperture.

uniform amplitude and phase field distributions maximum effective areas = physical areas

nonuniform field distributions effective areas $<$ physical areas

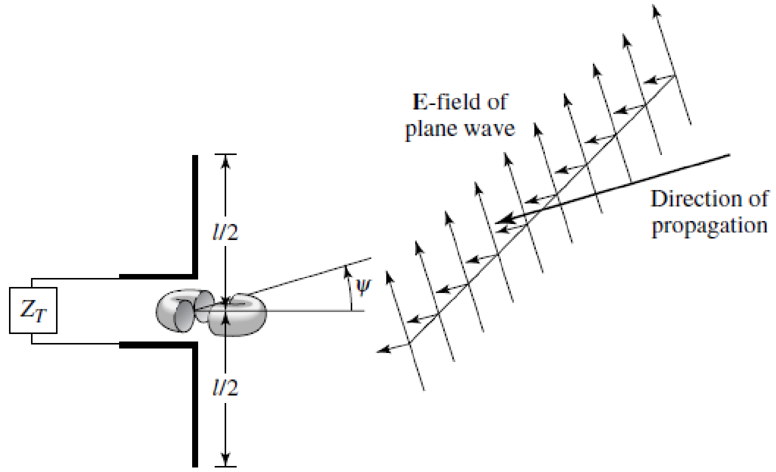
The maximum effective area of wire antennas is greater than the physical area.
(if taken as the area of a cross section of the wire when split lengthwise along its diameter)

the wire antenna can capture much more power than is intercepted by its physical size

2.15 Antenna Vector Effective Length and Equivalent Areas

2.15.2 Antenna Equivalent Areas

Ex) a very short lossless dipole ($l \ll \lambda$) $\quad R_r = 80(\pi l/\lambda)^2$, the incident field is linearly polarized along the axis of the dipole



(a) Dipole antenna in receiving mode

Figure 2.29 Uniform plane wave incident upon dipole antennas.

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right] \quad (\because R_L = 0) \quad \left(A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_r+R_L)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r+R_L}\right] \quad \text{(2-96)} \right)$$

$$V_T = El$$

V_T = induced voltage on the dipole

E = electric field of incident wave

l = length of dipole

$$W_i = \frac{E^2}{2\eta} \quad (\eta \approx 120\pi \text{ ohms for a free-space medium})$$

$$A_{em} = \frac{(El)^2}{8(E^2/2\eta)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

If $(R_T = R_r + R_L = 2R_r)$, the effective area is only one-half of the maximum effective area given above.

2.15 Antenna Vector Effective Length and Equivalent Areas

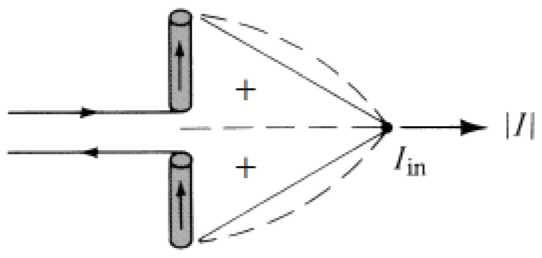
2.15.2 Antenna Equivalent Areas

$$A_{em} = 0.119\lambda^2 \qquad (l \ll \lambda,\ l \le \lambda/50)$$

assume $l = \lambda/50$

$$A_{em} = 0.119\lambda^2 = l w_e = (\lambda/50)\, w_e$$

$$w_e = 5.95\lambda$$



(a) $l << \lambda$

Figure 1.16 Current distribution on linear dipoles.

Typical physical diameters (widths) of wires used for dipoles

$$w_p = \lambda/300$$

$$w_e \approx 1785\, w_p$$

the wire antenna can capture much more power than is intercepted by its physical size

Example 2.15

A uniform plane wave is incident upon a very short lossless dipole ($l \ll \lambda$), as shown in Figure 2.29(a). Find the maximum effective area assuming that the radiation resistance of the dipole is $R_r = 80(\pi l/\lambda)^2$, and the incident field is linearly polarized along the axis of the dipole.

Solution: For $R_L = 0$, the maximum effective area of (2-96) reduces to

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right]$$

Since the dipole is very short, the induced current can be assumed to be constant and of uniform phase. The induced voltage is

$$V_T = El$$

where

$V_T =$ induced voltage on the dipole
$E =$ electric field of incident wave
$l =$ length of dipole

For a uniform plane wave, the incident power density can be written as

$$W_i = \frac{E^2}{2\eta}$$

where η is the intrinsic impedance of the medium ($\simeq 120\pi$ ohms for a free-space medium). Thus

$$A_{em} = \frac{(El)^2}{8(E^2/2\eta)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

Let us now examine the significance of the effective area.

From Example 2.15, the maximum effective area of a short dipole with $l << \lambda$ was equal to
A_{em} = 0.119λ^2.

Typical antennas that fall under this category are dipoles whose lengths are $l \le \lambda/50$.
For the purpose of demonstration, let us assume that $l = \lambda/50$.

Because A_{em} = 0.119λ^2 = $l\, w_e$ = $(\lambda/50)\, w_e$, the maximum effective electrical width of this dipole is w_e = 5.95λ.

Typical physical diameters (widths) of wires used
for dipoles may be about $w_p = \lambda/300$.

Thus the maximum effective width w_e is about 1,785 times larger than its physical width.

2.15 Maximum directivity and Maximum effective area

To derive the relationship between directivity and maximum effective area, the geometrical arrangement of Figure 2.30 is chosen.

If antenna 1 were isotropic, its radiated power density at a distance R would be

$$W_0 = \frac{P_t}{4\pi R^2} \tag{2-101}$$

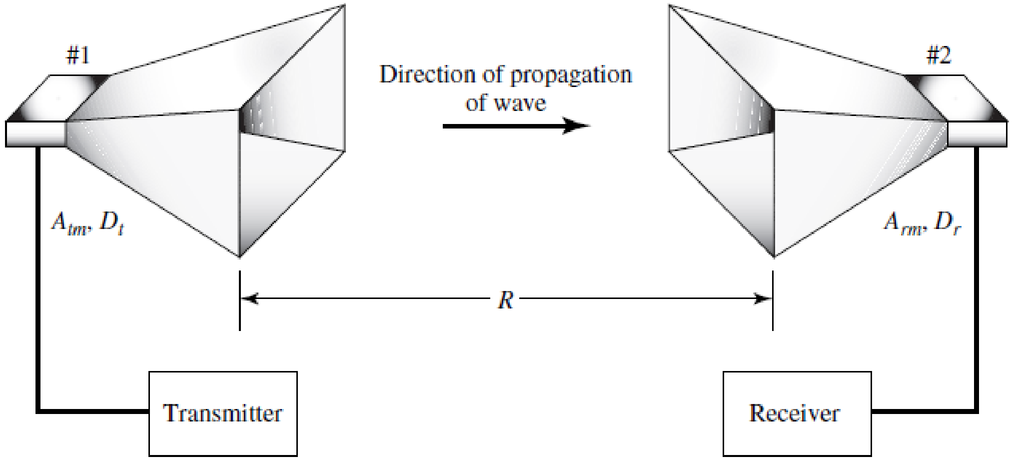


Figure 2.30 Two antennas separated by a distance R.

A_t : effective area of transmitter
A_r : effective area of receiver
D_t : directivities of transmitter
D_r : directivities of receiver
W_0 : power density of isotropic antenna at a distance R
W_t : power density of transmitter at a distance R
P_t : total radiated power
P_r : power received and transferred to the load

P_t is the total radiated power. Because of the directive properties of the antenna,
its actual density is

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2} \qquad (2\text{-}102)$$

The power collected (received) by the antenna and transferred to the load would be

$$P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2} \qquad (2\text{-}103)$$

If antenna 2 is used as a transmitter, 1 as a receiver, and the intervening medium is linear, passive, and isotropic, we can write that

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \qquad (2\text{-}104)$$

- **Equating (2-103) and (2-104) reduces to**

$$D_t A_r = \frac{P_r}{P_t}(4\pi R^2) \qquad (2\text{-}103a)$$

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \qquad (2\text{-}104)$$

$$\Rightarrow \quad \frac{D_t}{A_t} = \frac{D_r}{A_r} \qquad (2\text{-}105)$$

Increasing the directivity of an antenna increases its effective area in direct proportion.Thus, (2-105) can be written as

$$\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}} \tag{2-106}$$

If antenna 1 is isotropic, then D_{0t} = 1 and its maximum effective area can be expressed as

$$A_{tm} = \frac{A_{rm}}{D_{0r}} \tag{2-107}$$

- **Equation (2-107) states that the maximum effective area of an isotropic source is equal to the ratio of the maximum effective area to the maximum directivity of any other source.**

For example, let the other antenna be a very short *($l << \lambda$)* dipole whose effective area ($0.119\lambda^2$ from Example 2.15) and maximum directivity (1.5) are known. The maximum effective area of the isotropic source is then equal to

$$A_{tm} = \frac{A_{rm}}{D_{0r}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi} \tag{2-108}$$

$$A_{rm} = D_{0r}A_{tm} = D_{0r}\left(\frac{\lambda^2}{4\pi}\right) \tag{2-109}$$

In general then, the maximum effective aperture (A_{em}) of any antenna is related to its
maximum directivity (D_0) by

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_0} \tag{2-110}$$

Thus, when A_{em} is multiplied by the power density of the incident wave it leads to the maximum power that can be delivered to the load.
(This assumes that e_{cd}, e_r, PLF, p_e is unity.)

2.16 Maximum Directivity And Maximum Effective Area

Include losses

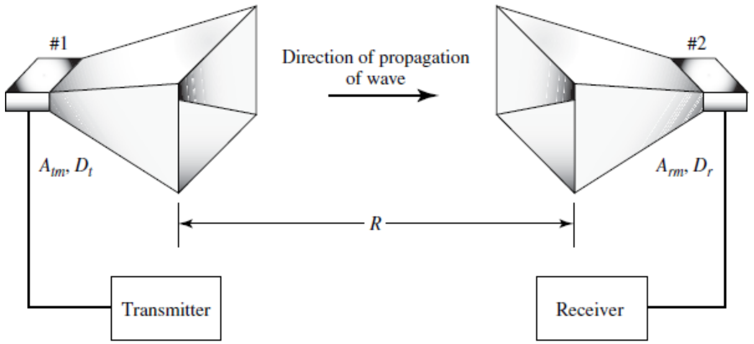


Figure 2.30 Two antennas separated by a distance R

If conduction-dielectric ,reflection and polarization losses are also included, then the maximum effective area

If there are losses associated with an antenna, it can be expressed

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right)D_0 \qquad (2\text{-}111)$$

$$A_{em} = e_0(\frac{\lambda^2}{4\pi})^2 D_0\left|\rho_w \cdot \rho_a\right|^2 = e_{cd}(1-|\Gamma|^2)(\frac{\lambda^2}{4\pi})^2 D_0\left|\rho_w \cdot \rho_a\right|^2$$

If reflection and polarization losses are also included, then the maximum effective area of (2-111) is represented by

$$\boxed{\begin{aligned} A_{em} &= e_0\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\rho}_w \cdot \hat{\rho}_a|^2 \\ &= e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\rho}_w \cdot \hat{\rho}_a|^2 \end{aligned}} \qquad (2\text{-}112)$$

2.17.1 FRIIS TRANSMISSION EQUATION

▪ Definition

- The Friis Transmission Equation relates the power received to the power transmitted between two antennas separated by a distance $R > 2D^2/\lambda$, where D is the largest dimension of either antenna.

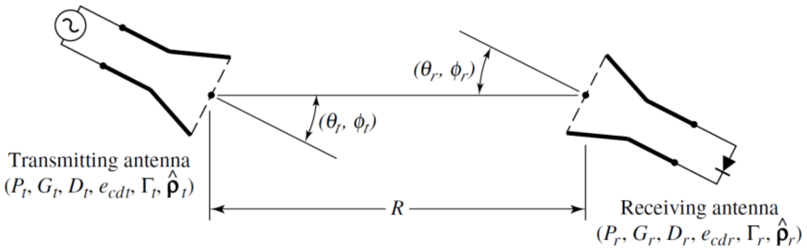


FIGURE 2.31
Geometrical orientation of transmitting and receiving antennas for Friss transmission equation.

▪ Derivation

- Referring to Figure 2.31, let us assume that the transmitting antenna is initially isotropic. If the input power at the terminals of the transmitting antenna is P_t, then its isotropic power density W_0 at distance R from the antenna is

$$W_0 = e_t \frac{P_t}{4\pi R^2} \tag{2-113}$$

- where e_t is the radiation efficiency of the transmitting antenna. For a nonisotropic transmitting antenna, the power density of (2-113) in the direction θ_t, ϕ_t can be written as

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2} \tag{2-114}$$

- where G_t is the gain and D_t is the directivity of the transmitting antenna in the direction θ_t, ϕ_t .

2.17.1 FRIIS TRANSMISSION EQUATION

- **Since the effective area A_r of the receiving antenna is related to its efficiency e_r and directivity D_r by**

$$A_r = e_r D_r(\theta_r, \phi_r)\left(\frac{\lambda^2}{4\pi}\right) \qquad (2\text{-}115)$$

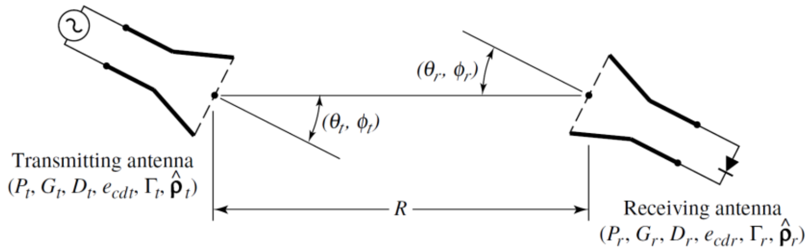


FIGURE 2.31
Geometrical orientation of transmitting and receiving antennas for Friss transmission equation.

- **the amount of power P_r collected by the receiving antenna can be written, using (2-114) and (2-115) as**

$$P_r = e_r D_r(\theta_r, \phi_r)\frac{\lambda^2}{4\pi}W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t}{(4\pi R)^2}|\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \qquad (2\text{-}116)$$

- **or the ratio of the received to the input power as**

$$\boxed{\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2}} \qquad (2\text{-}117)$$

- **The power received based on (2-117) assumes that the transmitting and receiving antennas are matched to their respective lines or loads (reflection efficiencies are unity) and the polarization of the receiving antenna is polarization-matched to the impinging wave (polarization loss factor and polarization efficiency are unity).**

2.17.1 FRIIS TRANSMISSION EQUATION

- If these two factors (reflection efficiencies and polarization efficiency) are also included, then the ratio of the received to the input power of (2-117) is represented by

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}\boxed{(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)}\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)\boxed{|\hat{\boldsymbol{\rho}}_t\cdot\hat{\boldsymbol{\rho}}_r|^2} \qquad (2\text{-}118)$$

- For reflection and polarization-matched antennas aligned for maximum directional radiation and reception, (2-118) reduces to

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t}G_{0r} \qquad (2\text{-}119)$$

- Equations (2-117), (2-118), or (2-119) are known as the ***Friis Transmission Equation***, and it relates the power P_r (delivered to the receiver load) to the input power of the transmitting antenna P_t.

- The term $\left(\frac{\lambda}{4\pi R}\right)^2$ is called the *free-space loss factor*, and it takes into account the losses due to the spherical spreading of the energy by the antenna.

2.17.1 Friis Transmission Equation.

Example 2.16

Two lossless X-band (8.2 -12.4 GHz) horn antennas are separated by a distance of 100λ.
The reflection coefficients at the terminals of the transmitting and receiving antenna are 0.1 and 0.2 respectively.
The maximum directivities of the transmitting and receiving antennas are 16dB and 20dB, respectively.
Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2W, and antenna are aligned for maximum radiation between them and are polarization-matched,
Find the power delivered to the load of the receiver.

Solutions:

$e_{cdt} = e_{cdr} = 1$ because antennas are lossless

$|\hat{\rho_t} \cdot \widehat{\rho_r}|^2 = 1$ because antennas are polarization matched

$$\left.\begin{array}{l} D_t = D_{0t} \\ D_r = D_{or} \end{array}\right\} \text{because antennas are aligned for maximum radiation between them}$$

$$D_{0r} = 20dB \ \rightarrow 100 \ (dimensionless)$$

$$D_{0t} = 16dB \ \rightarrow 39.81(dimensionless)$$

Using (2-118), we can write

$$P_r = [1-(0.1)^2][1-(0.2)^2]\left[\frac{\lambda}{4\pi(100\lambda)^2}\right](39.81)(100)(2) = 4.777mW$$

Radar Equation

2.17 .2 Radar Range Equation

- The **radar cross section** or **echo area** **(σ)** of a target which is defined as the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target.

$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s$$

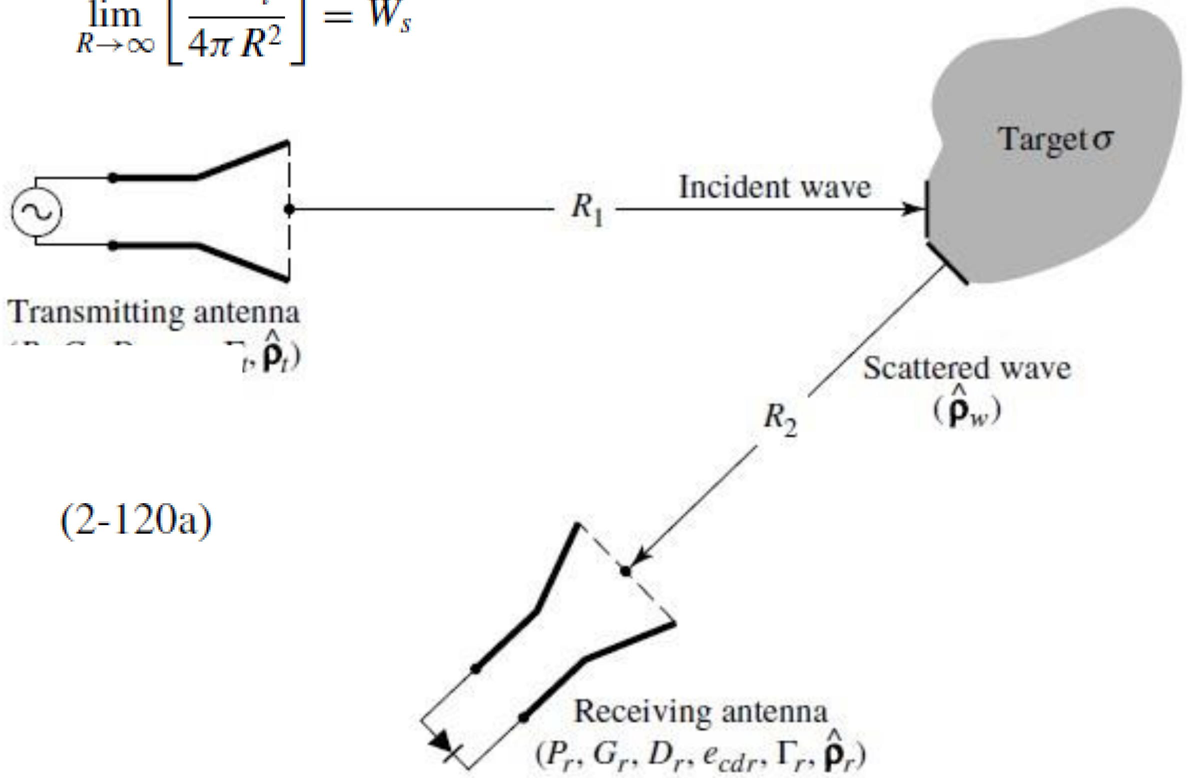


Figure 2.32 Geometrical arrangement of transmitter, target, and receiver for radar range equation.

- **In equation form**

$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s$$

- **or**

$$\boxed{\begin{aligned}\sigma &= \lim_{R\to\infty}\left[4\pi R^2\frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2\frac{|\mathbf{E}^s|^2}{|\mathbf{E}^i|^2}\right]\\ &= \lim_{R\to\infty}\left[4\pi R^2\frac{|\mathbf{H}^s|^2}{|\mathbf{H}^i|^2}\right]\end{aligned}}$$

(2-120a)

σ = radar cross section or echo area (m^2)

R =Observation distance from target (m)

W_i =incident power density (W/m^2)

W_s =scattered power density (W/m^2)

$E^i\ E^s$ =incident (scattered) electric field (V/m)

$H^i H^s$ =incident (scattered) magnetic field (A/m)

2.17.2 RADAR RANGE EQUATION

- **The amount of captured power (P_c) is obtained by multiplying the incident power density of (2-114) by the radar cross section σ,**

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t,\phi_t)}{4\pi R_1^2} = e_t \sigma \frac{P_t D_t(\theta_t,\phi_t)}{4\pi R_1^2} \qquad (2\text{-}121)$$

  - **The power captured by the target is reradiated isotropically, and the scattered power density can be written as**

$$W_S = \frac{P_C}{4\pi R_2^2} = e_{cdt}\sigma \frac{P_t D_t(\theta_t,\phi_t)}{(4\pi R_1 R_2)^2} \qquad (2\text{-}122)$$

- The amount of power delivered to the receiver load

$$P_r = A_r W_S = e_{cdr} e_{cdt} \sigma \frac{P_t D_t(\theta_{t,}\phi_t) D_r(\theta_{r,}\phi_r)_t}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2}\right)^2$$

Receiving effective area

$$A_r = e_r D_r(\theta_{r,}\phi_r)\left(\frac{\lambda^2}{4\pi}\right)$$

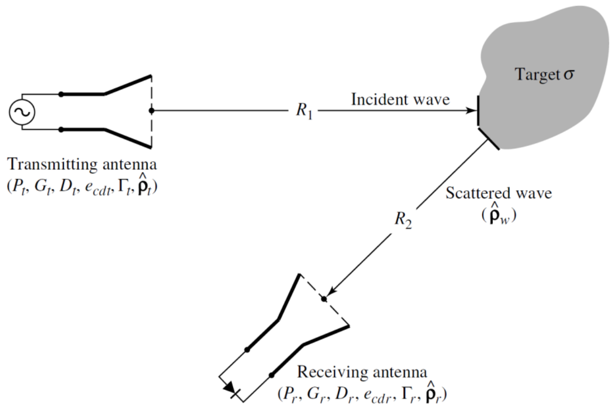


FIGURE 2.32
Geometrical arrangement of transmitter, target, and receiver for radar range equation.

2.17.2 RADAR RANGE EQUATION

- The amount of power delivered to the receiver load is given by

$$P_r = A_r W_s = e_{cdt} e_{cdr} \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \qquad (2\text{-}123)$$

- The ratio of the received power to the input power

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2} \qquad (2\text{-}124)$$

- If reflection losses (reflection efficiency) and polarization losses (polarization loss factor or polarization efficiency) are also included, then (2-124) must be expressed as

$$\boxed{\begin{aligned} \frac{P_r}{P_t} &= e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \\ &\quad \times \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_r|^2 \end{aligned}} \qquad (2\text{-}125)$$

$\hat{\boldsymbol{\rho}}_w$ = polarization unit vector of the scattered waves
$\hat{\boldsymbol{\rho}}_r$ = polarization unit vector of the receiving antenna

- For polarization-matched antennas aligned for maximum directional radiation and reception, (2-125) reduced to

$$\boxed{\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left[\frac{\lambda}{4\pi R_1 R_2} \right]^2} \qquad (2\text{-}126)$$

2.17.3 Antenna Radar Cross Section (RCS)

- For a target, there is **monostatic** or **backscattering** RCS when the transmitter and receiver of Figure 2.32 are at the same location, and a **bistatic** RCS when the transmitter and receiver are not at the same location.
- The RCS of a target can be controlled using primarily two basic methods: **shaping** and the use of **materials**.
- Both methods, shaping and materials, are used together in order to optimize the performance of a radar target.
- One of the "golden rules" to observe in order to achieve low RCS is to "round corners, avoid flat and concave surfaces, and use material treatment in flare spots."
- In general the electric field scattered by an antenna with a load impedance ZL can be expressed by

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t}\frac{Z_L}{Z_L + Z_A}\mathbf{E}^t \qquad (2\text{-}127)$$

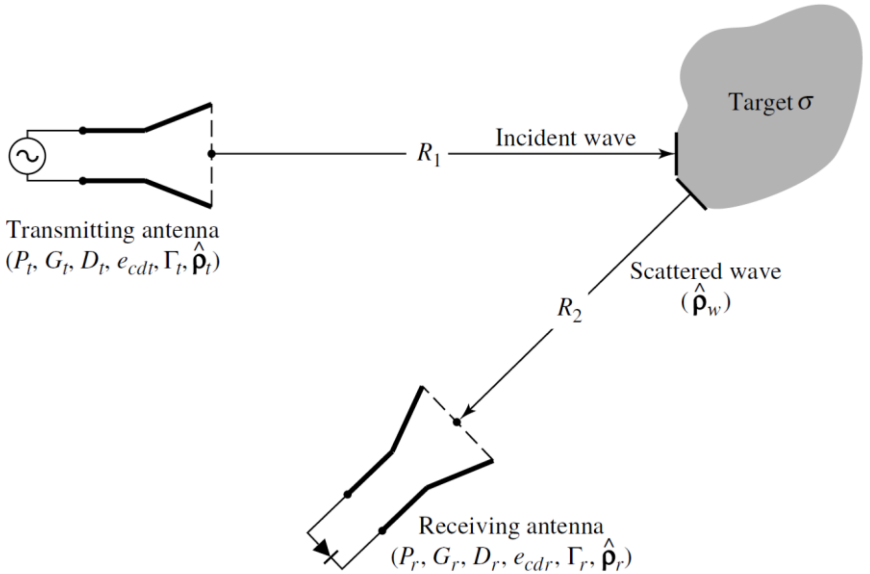


FIGURE 2.32
Geometrical arrangement of transmitter, target, and receiver for radar range equation.

2.17.3 Antenna Radar Cross Section (RCS)

$$\Gamma_A = \frac{Z_L - Z_A}{Z_L + Z_A} \qquad (2\text{-}128)$$

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t}\frac{1}{2}(1 + \Gamma_A)\mathbf{E}^t \qquad (2\text{-}129)$$

- Conjugate matching ($Z_L = Z^*_A$)

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_s}{I_t}\frac{\Gamma^* Z_A}{2R_A}\mathbf{E}^t \qquad (2\text{-}130)$$

$$\Gamma^* = \frac{Z_L - Z_A^*}{Z_L + Z_A^*} \qquad (2\text{-}130\text{a})$$

2.17.3 Antenna Radar Cross Section (RCS)

For the short-circuited case and the conjugate-matched transmitting (radiating) case, the product of their currents and antenna impedance are related by [34]

$$I_s Z_A = I_m^*(Z_A + Z_A^*) = 2R_A I_m^* \tag{2-131}$$

where I_m^* is the scattering current when the antenna is conjugate-matched ($Z_L = Z_A^*$). Substituting (2-131) into (2-130) for I_s reduces (2-130) to

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_m^*}{I_t}\Gamma^*\mathbf{E}^t \tag{2-132}$$

It can also be shown that if the antenna is matched with a load Z_A (instead of Z_A^*), then (2-132) can be written as

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A) - \frac{I_m}{I_t}\Gamma_A\mathbf{E}^t \tag{2-133}$$

2.17.3 Antenna Radar Cross Section (RCS)

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t}\frac{1}{2}(1 + \Gamma_A)\mathbf{E}^t \qquad (2\text{-}129)$$

- The field scattered by an antenna when it is loaded with an impedance ZL is equal to the field scattered by the antenna when it is short-circuited minus a term elated to the antenna reflection coefficient and the field transmitted by the antenna.

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_m^*}{I_t}\Gamma^*\mathbf{E}^t \qquad (2\text{-}132)$$

- The field scattered by an antenna when it is terminated with an impedance Z_L is equal to the field scattered by the antenna when it is conjugate-matched with an impedance Z^*_A minus the field transmitted times the conjugate reflection coefficient.

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A) - \frac{I_m}{I_t}\Gamma_A\mathbf{E}^t \qquad (2\text{-}133)$$

- The field scattered by the antenna when it is terminated with an impedance Z_L is equal to the field scattered by the antenna when it is matched with an impedance A_Z minus the field transmitted times the reflection coefficient weighted by the two currents.

2.17.3 Antenna Radar Cross Section (RCS)

$$\boxed{\sigma = |\sqrt{\sigma^s} - (1+\Gamma_A)\sqrt{\sigma^a}e^{j\phi_r}|^2 \qquad \text{(2-134)}}$$

$$\sigma_{\text{short}} = \sigma^s \tag{2-135}$$

$$\sigma_{\text{open}} = |\sqrt{\sigma^s} - 2\sqrt{\sigma^a}e^{j\phi_r}|^2 = \sigma_{\text{residual}} \tag{2-136}$$

$$\sigma_{\text{match}} = |\sqrt{\sigma^s} - \sqrt{\sigma^a}e^{j\phi_r}|^2 \tag{2-137}$$

$$|\sigma^s - \sigma^a| \leq \sigma \leq |\sigma^s + \sigma^a| \tag{2-138}$$

2.17.3 Antenna Radar Cross Section (RCS)

- The minimum value occurring when the two RCSs are in phase while the maximum occurs when they are out of phase. To produce a zero RCS, (2-134) must vanish.

$$Re(\Gamma_A) = -1 + \cos\phi_r \sqrt{\sigma^s/\sigma^a} \quad \text{(2-139a)}$$

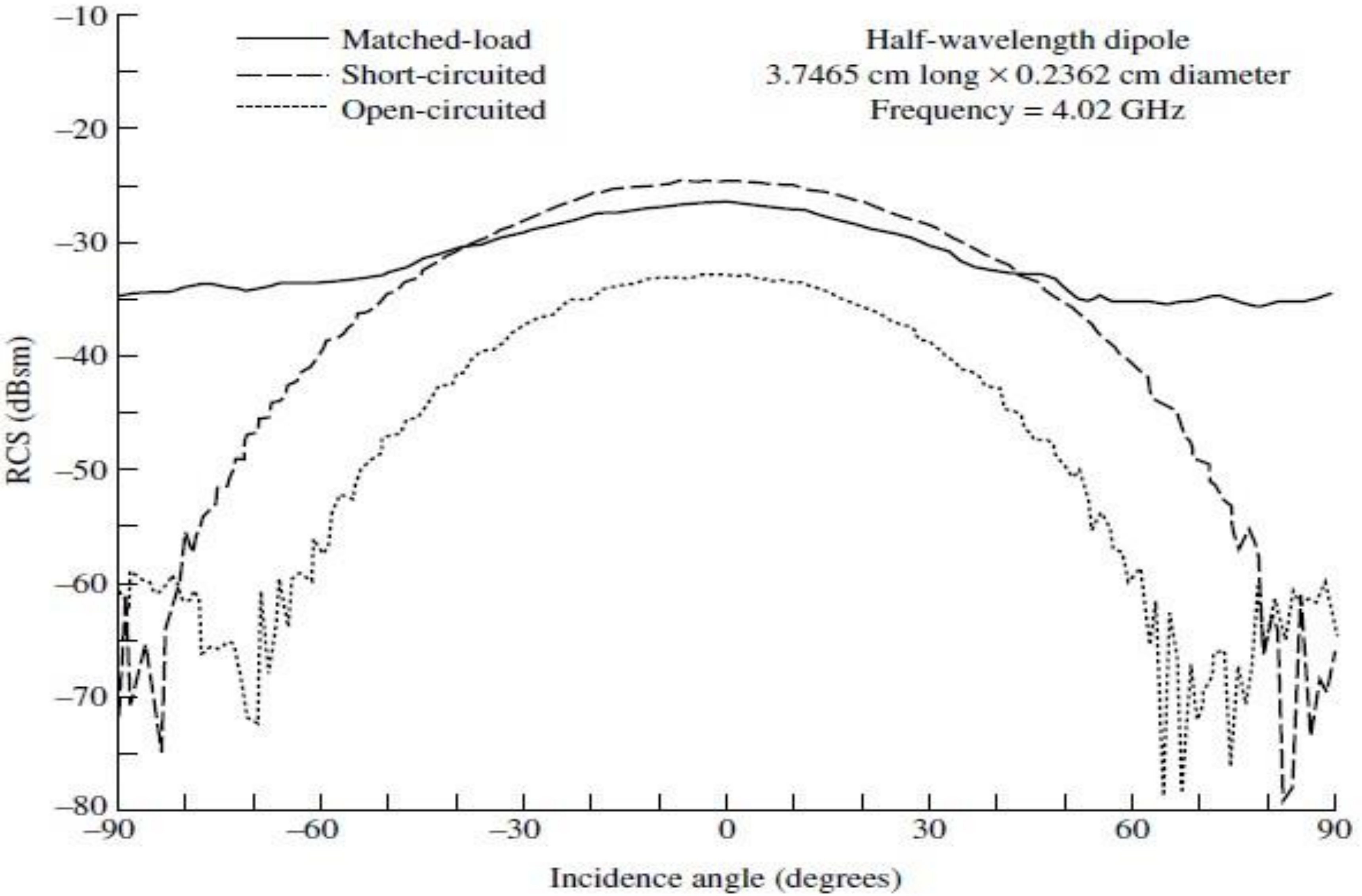
$$Im(\Gamma_A) = -\sin\phi_r \sqrt{\sigma^s/\sigma^a} \quad \text{(2-139b)}$$

- Assuming positive values of resistances, the real value of Γ_A cannot be greater than unity. Therefore there are some cases where the RCS cannot be reduces to zero by choosing Z_L. Because Z_A can be complex, there is no limit on the imaginary part of Γ_A.

$$\sigma = \frac{\lambda_0^2}{4\pi} G_0^2 |A - \Gamma^*|^2 \quad \text{(2-140)}$$

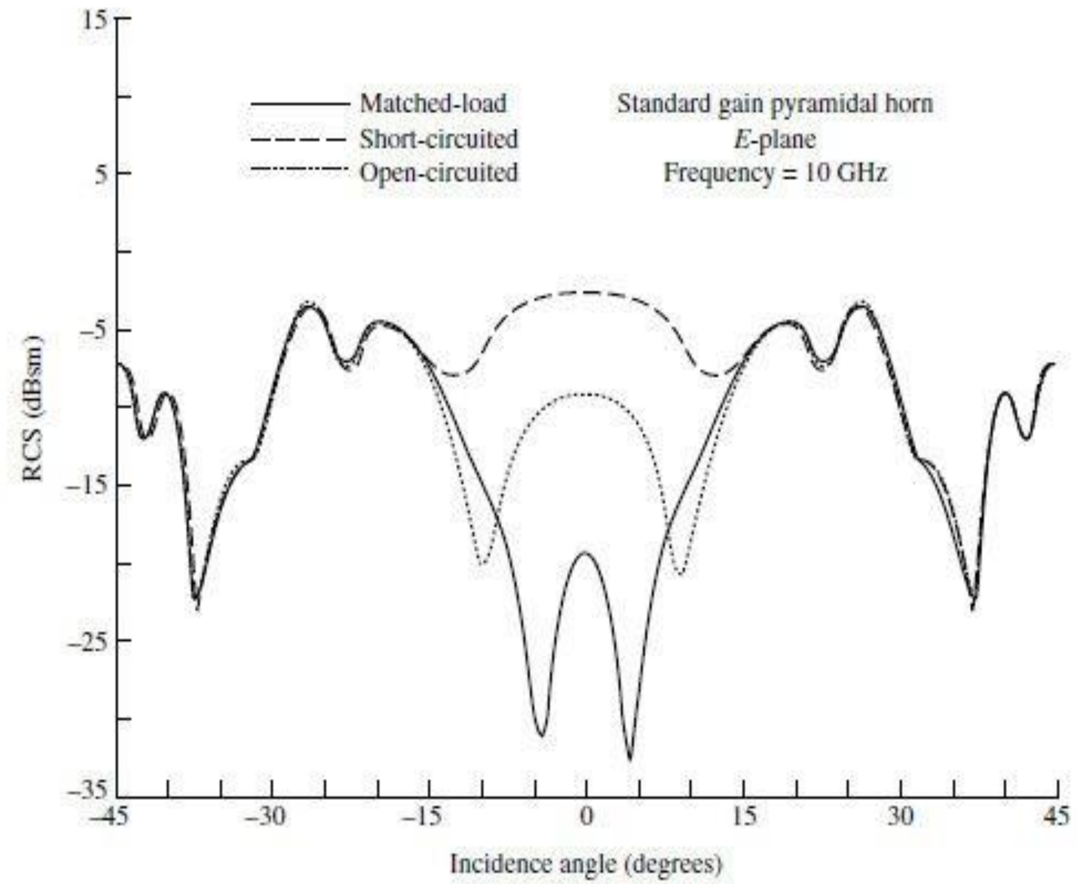
- If the antennas are identical ($G_{0r} = G_{0t} = G_0$) and are polarization-matched ($P_r = P_t = 1$), the total radar cross section of the antenna for backscattering can be written as

2.17.3 Antenna Radar Cross Section (RCS)

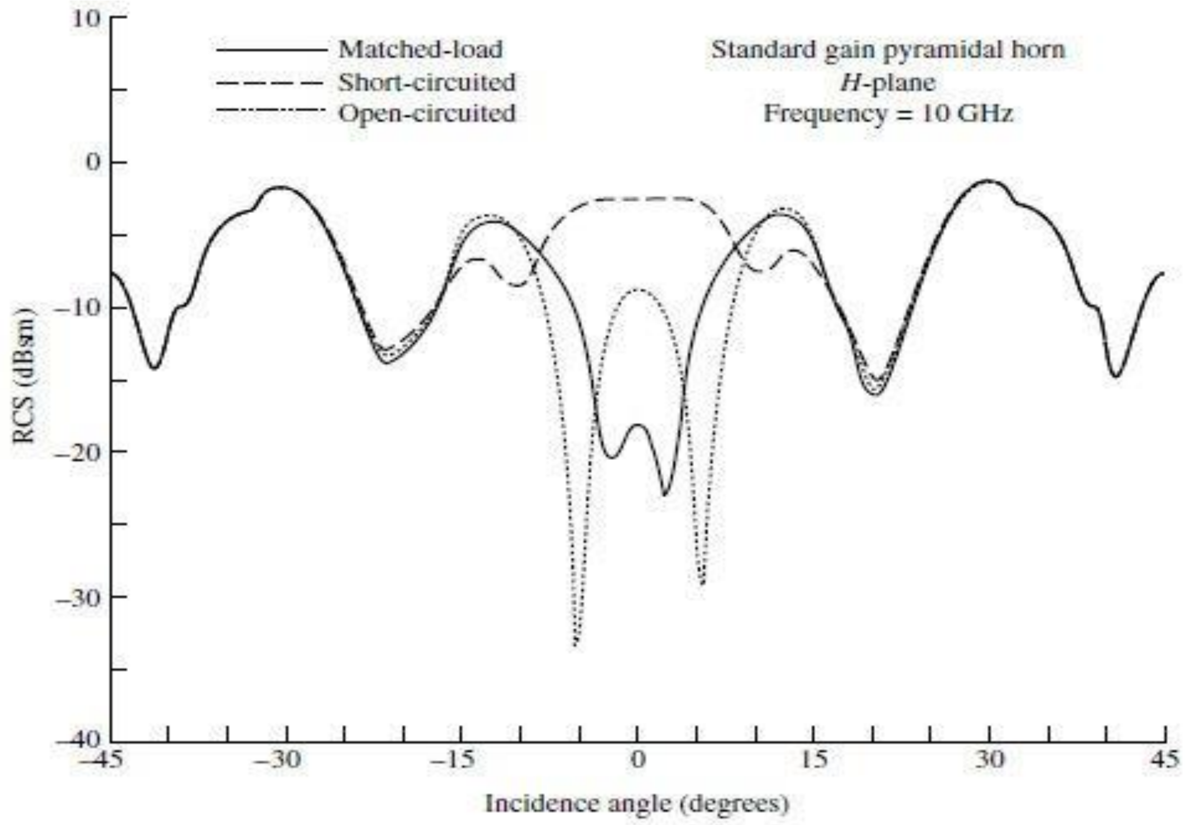


E-plane

2.17.3 Antenna Radar Cross Section (RCS)



(a) *E*-plane ($\sigma_{\theta\theta}$)



(b) *H*-plane ($\sigma_{\phi\phi}$)

2.17.3 ANTENNA RADAR CROSS SECTION

Radar Cross Section (RCS)

- The radar cross section, usually referred RCS, is a far-field parameter, which is used to characterize the scattering properties of a radar target.
- Monostatic (or backscattering) RCS : the transmitter and receiver are at the same location.
- Bistatic RCS : the transmitter and receiver are not at the same location.
- RCS is may depend more on its shape than physical size.

Variables of RCS

- polarization of the incident wave
- angle of incidence
- angle of observation
- geometry of the target
- electrical properties of the target
- frequency of operation

Units of RCS

- meters squared (m^2)
- decibels per squared meter (dBsm)
- RCS per suared wavelength in decibels (RCS/λ^2 in dB)

2.17.3 ANTENNA RADAR CROSS SECTION

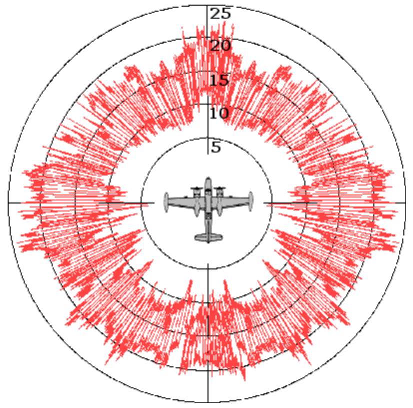
▪ RCS of Some Typical Targets

- **The RCS of a target can be controlled using primarily two basic methods : shaping and the use of materials.**

TABLE 2.2
RCS of some typical targets

TABLE 2.2 RCS of Some Typical Targets

Object	Typical RCSs [22] RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60



Typical RCS diagram (A-26 Invader)

For achieve low RCS
- Round corners
- Avoid flat and concave surfaces
- Use material treatment in flare spots

- **One of the "golden rules" to observe in order to achieve low RCS is to "round corners, avoid flat and concave surfaces, and use material treatment in flare spots."**

2.17.3 Antenna Radar Cross Section

“The RCS of a target can be controlled using primarily two basic methods: shaping and the use of materials.”

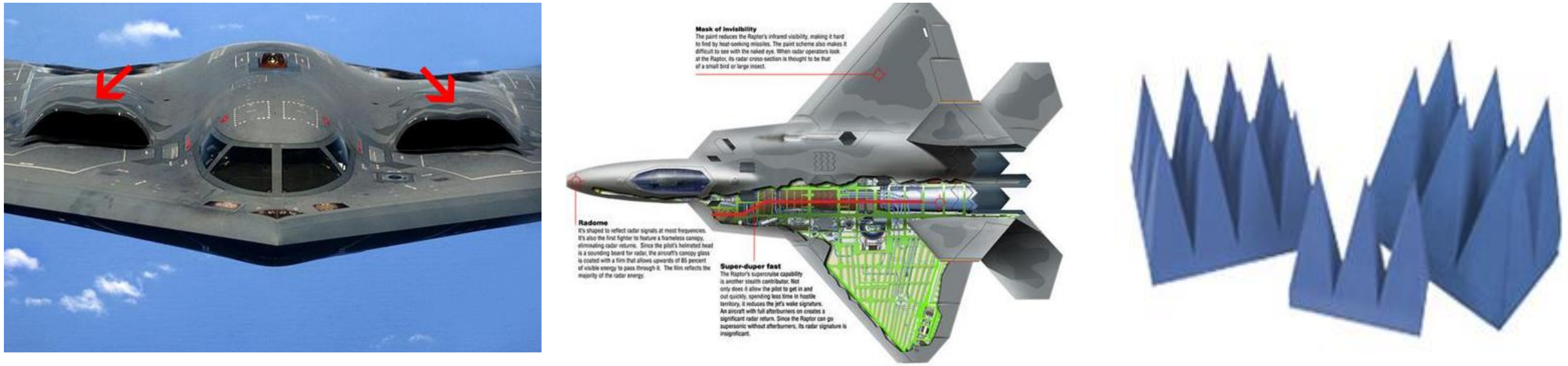


Figure 2.32. Examples of the shaping method or using the materials.

One of the “golden rules” to observe in order to achieve low RCS is to “round corners, avoid flat and concave surfaces, and use material treatment in flare spots.”

2.17.3 Antenna Radar Cross Section

There are many methods of analysis to predict the RCS of a target.

-full wave method,

-others are designated as asymptotic method either low- frequency or high-frequency.

-some are considered as numerical method

-some targets, because of complexity, are often simplified and are decomposed into a number of basic shapes

Antennas individually are radar targets which many exhibit large RCS

Figure 2.33. Examples with antennas mounted on the surface.

Therefore in designing low-observable targets, the antenna type, location and contributions become an important consideration of the overall design

1.2 ANTENNA TEMPERATURE

- Every object with a physical temperature above absolute zero (0 K=−273◦C) radiates energy.
  - This radiated energy is all the electromagnetic waves (radio wave, infra wave)[1].

- Some of the better natural emitters of energy at microwave frequencies :
  (1) The ground with equivalent temperature of about 300 K
  (2) The sky with equivalent temperature of about 5 K when looking toward zenith
  (3) The sky with equivalent temperature of about 100–150 K toward the horizon.

- *Examples*[3]
  - Let examine a directional antenna that look to the horizon.
  - Half of radiation pattern "feels" the 300°K of the ground, the other feels the 0°K of the clear sky.
  - The noise temperature is total equivalent temperature that antenna feels at its terminals, which in this case 150°K.

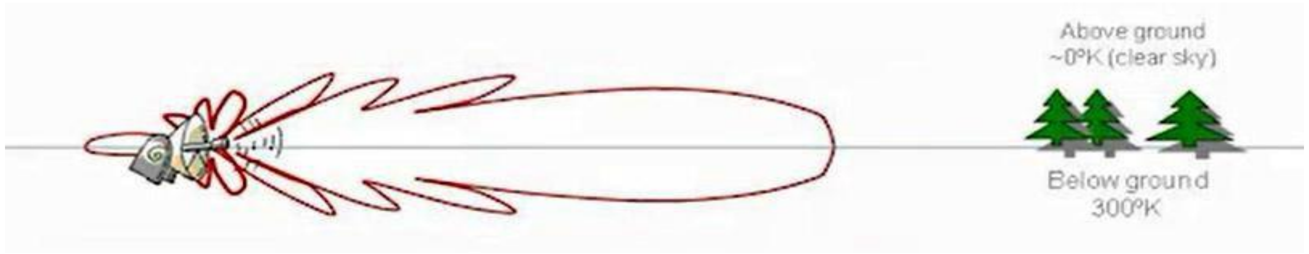


Figure 1. 24-2 Easy example of Calculation of antenna noise temperature.

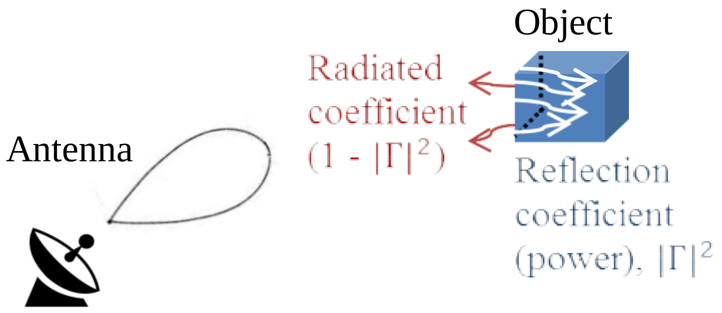
- The amount of energy radiated is usually represented by an equivalent temperature T_B, better known as **brightness temperature**.
  - Usually the emissivity ϵ is a function of the frequency of operation, polarization of the emitted energy, and molecular structure of the object.

$$T_B(\theta,\phi) = \epsilon(\theta,\phi)T_m = (1 - |\Gamma|^2)T_m$$

The range of emissivity ε
: $0 \le \varepsilon \le 1$

where

T_B = brightness temperature (equivalent temperature; K)
ϵ = emissivity (dimensionless)
T_m = molecular (physical) temperature (K)
$\Gamma(\theta,\phi)$ = reflection coefficient of the surface for the polarization of the wave



- The brightness temperature emitted by the different sources is intercepted by antennas[2], and it appears at their terminals as an antenna temperature.
- To define the environment (and hence give the full definition of antenna temperature), we will introduce a temperature distribution - this is the temperature in every direction away from the antenna in spherical coordinates.
- For instance, the night sky is roughly 4 Kelvin; the value of the temperature pattern in the direction of the Earth's ground is the physical temperature of the Earth's ground.
- This temperature distribution will be written as $T_B(\theta,\phi)$.
- Hence, an antenna's temperature will vary depending on whether it is directional and pointed into space or staring into the sun.

For an antenna with a radiation pattern given by $G(\theta,\phi)$, the noise temperature is mathematically defined as:

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\boxed{G(\theta,\phi)\sin\theta\, d\theta\, d\phi}}{\boxed{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi}} \qquad (1.80)$$

where

T_A = antenna temperature (effective noise temperature of the antenna radiation resistance; K)

$G(\theta,\phi)$ = gain (power) pattern of the antenna

This states that the temperature surrounding the antenna is integrated over the entire sphere, and weighted by the antenna's radiation pattern. Hence, an isotropic antenna would have a noise temperature that is the average of all temperatures around the antenna; for a perfectly directional antenna (with a pencil beam), the antenna temperature will only depend on the temperature in which the antenna is "looking".

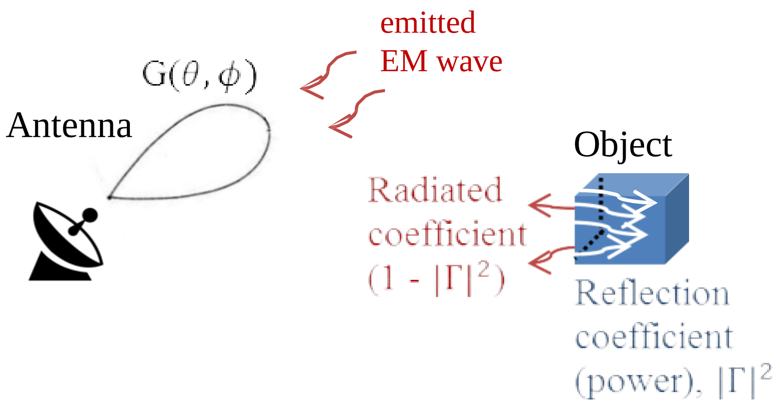
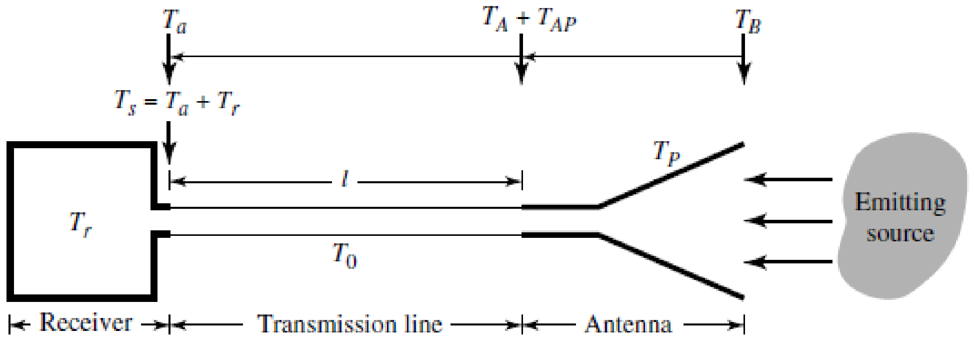


Figure 1. 24-1 Definition of antenna noise temperature.

2. ANTENNA TEMPERATURE

- The temperature appearing at the terminals of an antenna is that given by Eq. (1.79), after it is weighted by the gain pattern of the antenna.

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\boxed{G(\theta,\phi)\sin\theta\, d\theta\, d\phi}}{\boxed{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi}} \qquad (1.80)$$



Antenna Temperature (T_A) is a parameter that describes how much noise an antenna produces in a given environment. This temperature is not the physical temperature of the antenna. Moreover, an antenna does not have an intrinsic "antenna temperature" associated with it; rather the temperature depends on its gain pattern and the thermal environment that it is placed in. Antenna temperature is also sometimes referred to as **Antenna Noise Temperature**.

The noise power received from an antenna at temperature T_A can be expressed in terms of the bandwidth (*B*) the antenna (and its receiver) are operating over: $P_{TA} = KT_AB$

In the above, *K* is Boltzmann's constant (1.38 * 10^-23 [Joules/Kelvin = J/K]).

2. ANTENNA TEMPERATURE

- Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by

$$P_r = kT_A \Delta f \tag{1.81}$$

where

P_r = antenna noise power (W)
k = Boltzmann's constant (1.38×10^{-23} J/K)
T_A = antenna temperature (K)
Δf = bandwidth (Hz)

Assuming no losses or other contributions
between the antenna and the receiver

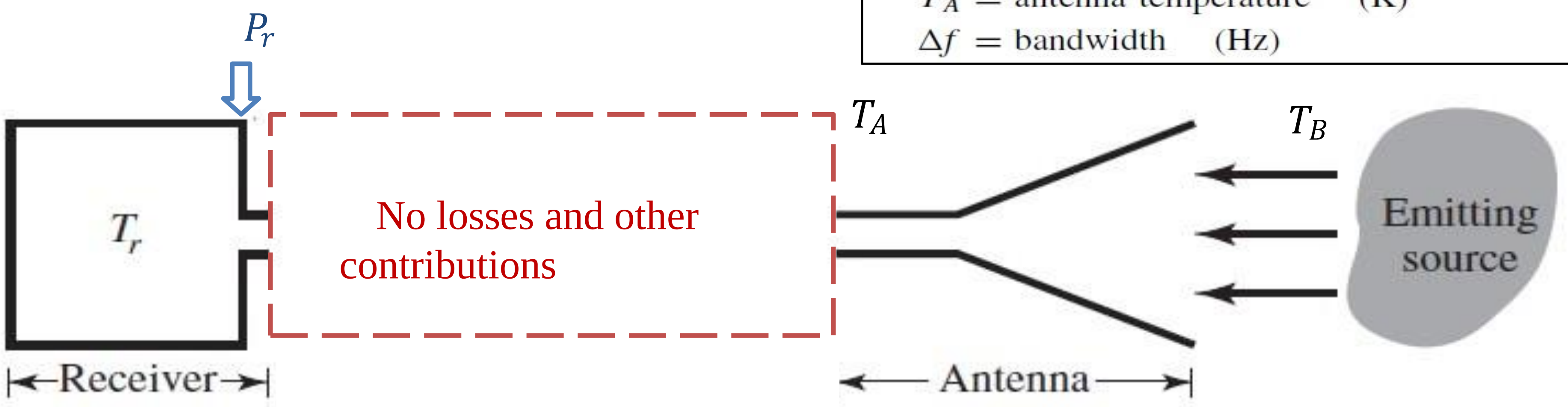


Figure 1. 24-2 System noise power calculation without losses and other contributions between the antenna and the receiver.

The receiver also has a temperature associated with it (T_R), and the total system temperature (antenna plus receiver) has a combined temperature given by $T_{sys} = T_A + T_R$
This temperature can be used in the above equation to find the total noise power of the system.

2. ANTENNA TEMPERATURE

- If the antenna and transmission line are maintained at certain physical temperatures (T_P), and the transmission line(l, T_0) is lossy (α), the antenna temperature T_A as seen by the receiver must be modified to include the other contributions and the line losses.
  - T_P: Antenna physical temperature
  - T_{AP}: Antenna temperature at the antenna terminals due to physical temperature
  - T_0: Physical temperature of the transmission line

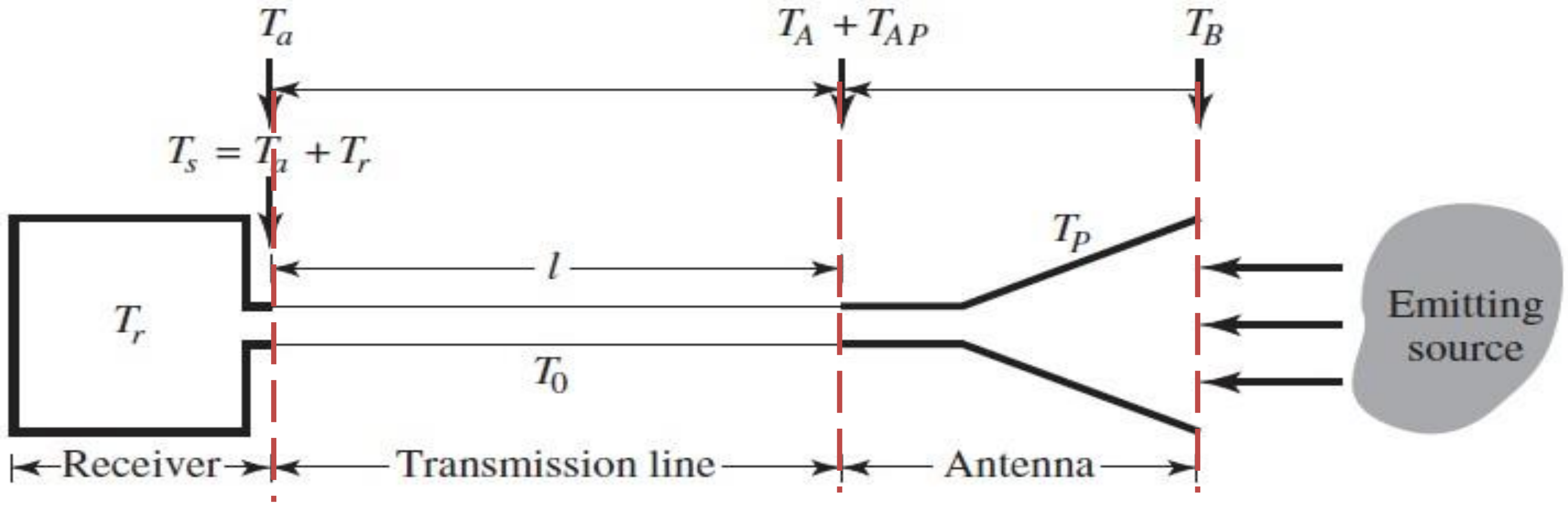


Figure 1. 24 Antenna, transmission line, and receiver arrangement for system noise power calculation.

2. ANTENNA TEMPERATURE

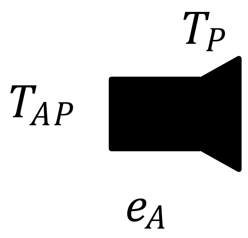
- Modified the effective antenna temperature at the receiver terminals is given by

(1.82)

Increase of l : $T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0(1 - e^{-2\alpha l})$

where

$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_p \qquad (1.82a)$$



- $e_A = \frac{T_P}{T_{AP} + T_P}$
- $T_{AP} = \frac{T_P}{e_A} - T_P$

T_a	= antenna temperature at the receiver terminals	(K)
T_A	= antenna noise temperature at the antenna terminals (Eq. (1.80))	(K)
T_{AP}	= antenna temperature at the antenna terminals due to physical temperature (Eq. (1.82a))	(K)
T_p	= antenna physical temperature	(K)
α	= attenuation coefficient of transmission line	(Np/m)
e_A	= thermal efficiency of antenna	(dimensionless)
l	= length of transmission line	(m)
T_0	= physical temperature of the transmission line	(K)

2. ANTENNA TEMPERATURE

- The antenna noise power of Eq. (1.81) must also be modified and written as

$$P_r = kT_a\,\Delta f \tag{1.83}$$

- *Ta* is the antenna temperature at the receiver input.

- If the receiver itself has a certain noise temperature *Tr* (due to thermal noise in the receiver components), the *system noise power at the receiver terminals* is given by

$$P_s = k(T_a + T_r)\,\Delta f = kT_s\,\Delta f$$

where

P_s = system noise power (at receiver terminals)
T_a = antenna noise temperature (at receiver terminals)
T_r = receiver noise temperature (at receiver terminals)
$T_s = T_a + T_r$ = effective system noise temperature (at receiver terminals)

Additional explanation

- **Differences of density and intensity**
- **Definition and Detail of dB**
- **Why infinitesimal area of sphere is $\boldsymbol{r^2 sin\theta d\theta d\varphi}$?**

- **Differences of density and intensity**
  - Purpose : find the total radiated power
  - Difference : W/surface, W/unit solid angle

Density

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}$$

where

$\mathscr{W}$ = instantaneous Poynting vector (W/m^2)
$\mathscr{E}$ = instantaneous electric-field intensity (V/m)
$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m)

$$P_{rad} = P_{av} = \oiint_S \mathbf{W}_{rad} \cdot d\mathbf{s} = \oiint_S \mathbf{W}_{av} \cdot \hat{\mathbf{n}}\, da$$
$$= \frac{1}{2} \oiint_S \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s}$$

Intensity

$$U = r^2 W_{rad}$$

where steradian

U = radiation intensity (W/unit solid angle)
W_{rad} = radiation density (W/m^2)

$$P_{rad} = \oiint_\Omega U\, d\Omega = \int_0^{2\pi} \int_0^{\pi} U \sin\theta\, d\theta\, d\phi$$

where d=element of solid angle=$\sin\theta\, d\theta\, d\varphi$.

- **Why infinitesimal area of sphere is $r^2 sin\theta d\theta d\varphi$?**

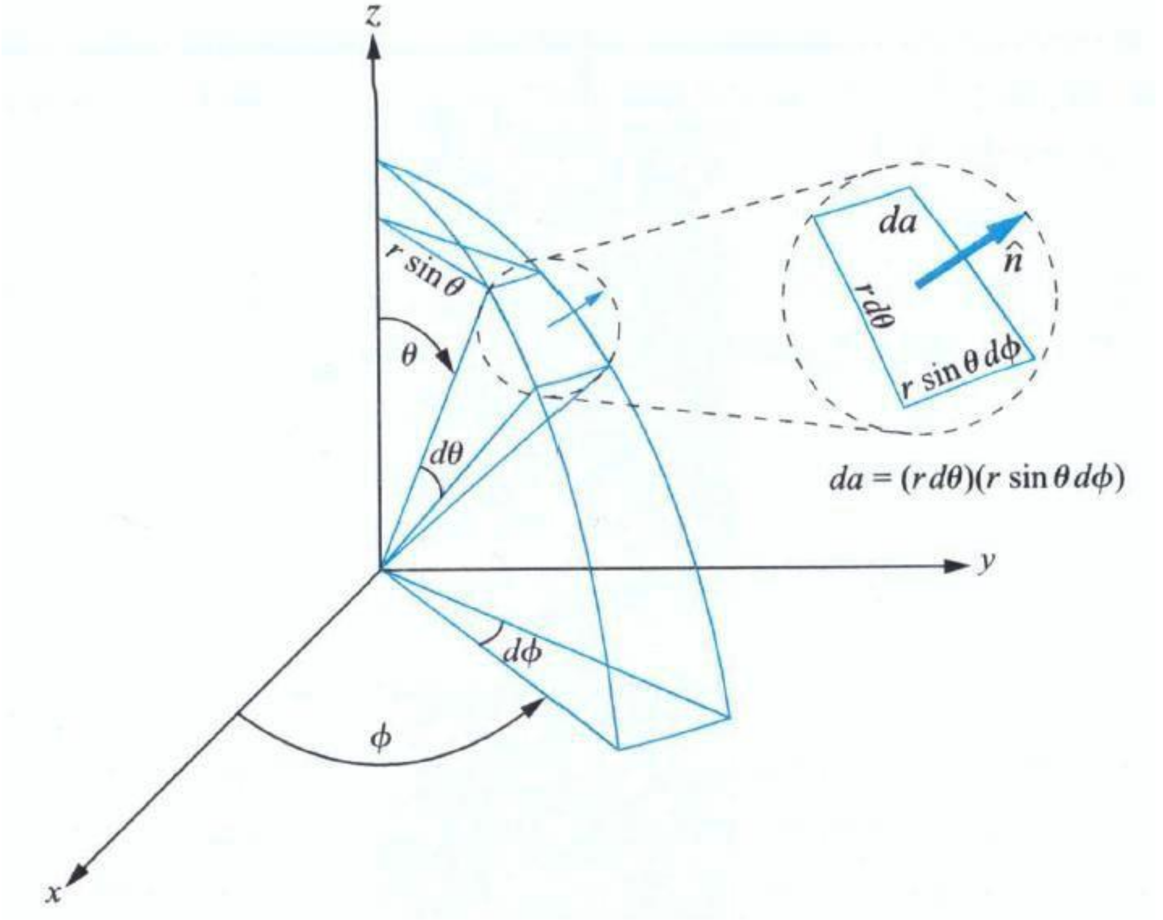


Figure 2. Detail of infinitesimal area of sphere.

- Definition and Detail of dB
  - The **decibel** (**dB**) is a logarithmic unit used to express the ratio of two values of a physical quantity, often power.
  - One of these values is often a standard reference value, in which case the decibel is used to express the level of the other value relative to this reference.

$$\text{dB} = 10\log\left(\frac{P_2}{P_1}\right)$$

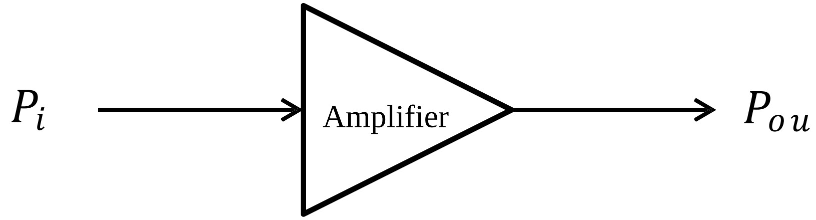
$$= 20\log\left(\frac{V_2}{V_1}\right), \text{ when } R_1 = R_2$$

$$= 10\log\left(\frac{I_2}{I_1}\right), \text{ when } R_1 = R_2$$

whereas P_1 is reference value.

- **Properties of dB**
  - The logarithmic scale nature of the decibel means that a very large range of ratios can be represented by a convenient number.
  - Level values in decibels can be added instead of multiplying the underlying power values, which means that the overall gain of a multi-component system, such as a series of amplifier stages, can be calculated by summing the gains in decibels of the individual components, rather than multiply the amplification factors; that is, log(A × B × C) = log(A) + log(B) + log(C)

Example) P_{in}= 713[uW], P_{out}= 450[mW], Gain [dB]=?



$$\text{Gain [dB]} = 10\log\left(\frac{450[mW]}{713[uW]}\right) = 28\text{dB}$$

- **Definitions of dBm, dBW**
  - **dBm** (sometimes **$\mathbf{dB_{mW}}$** or decibel-milliwatts) is an abbreviation for the power ratio in decibels (dB) of the measured power referenced to one milliwatt (mW).

    dBm = $10\log(\frac{P}{1mW})$

  - Compare dBW, which is referenced to one watt (1000 mW).

    dBW = $10\log(\frac{P}{1W})$

Example) 425mW => dBm?

sol) $10\log(\frac{425mW}{1mW})$= 26.28 dBm

- Etc) dBuV, dBuV/m

Thank you