UEC747: ANTENNA AND WAVE PROPAGATION

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Chapter 2: Antenna Parameters

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- To describe the performance of an antenna, definitions of various parameters are necessary.

Chapter 2

Fundamental Parameters of Antennas

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2.2 Radiation Pattern
2.3 Radiation Power Density
2.4 Radiation Intensity
2.5 Beamwidth
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Chapter 2 (cont'd)

Fundamental Parameters of Antennas

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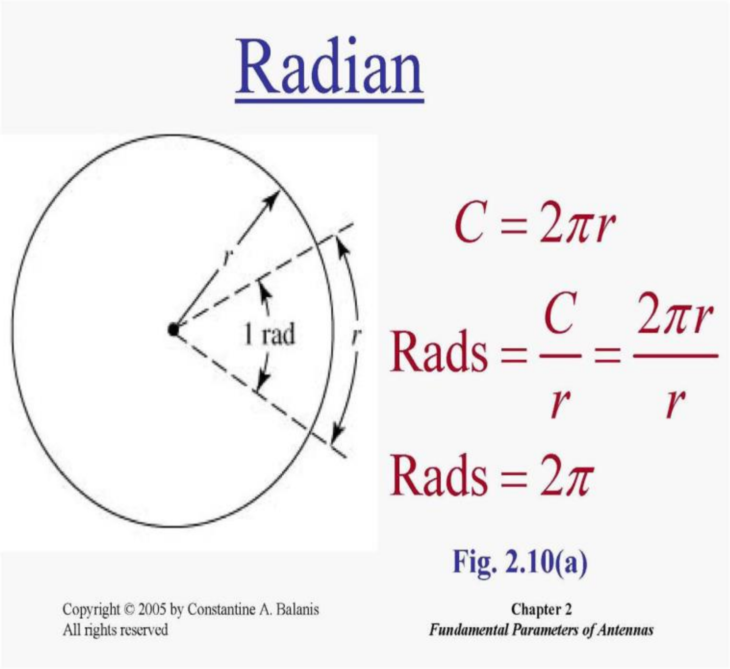
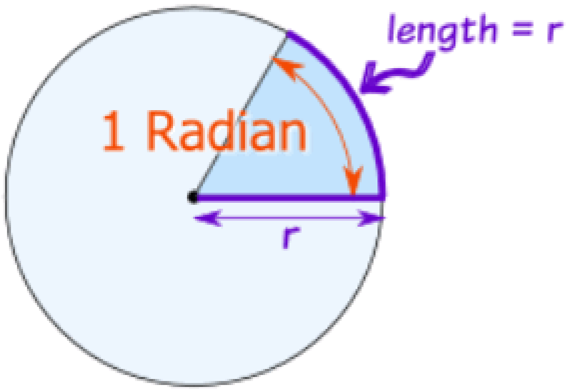
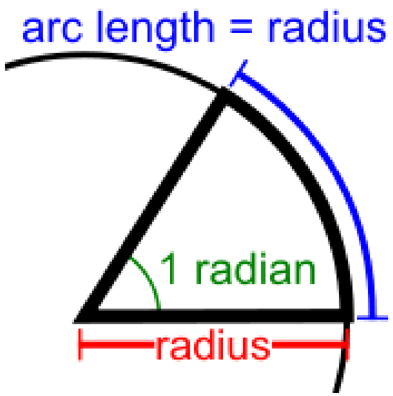
Angle

Radian - The plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r.



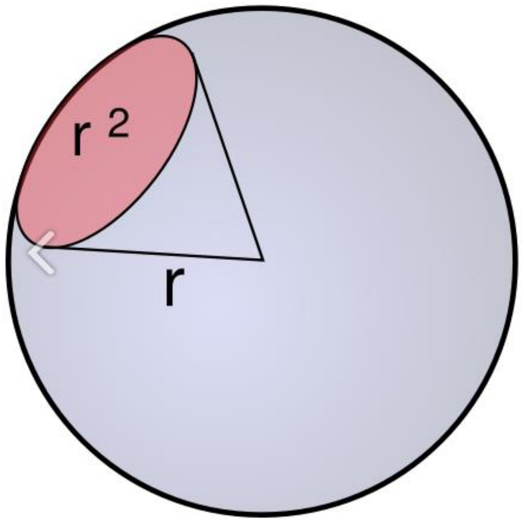




A full circle - $2\pi\, rad\ \ (2\pi r / r)$

Solid Angle

Steradian - The solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area equal to that of square with each side of length r.

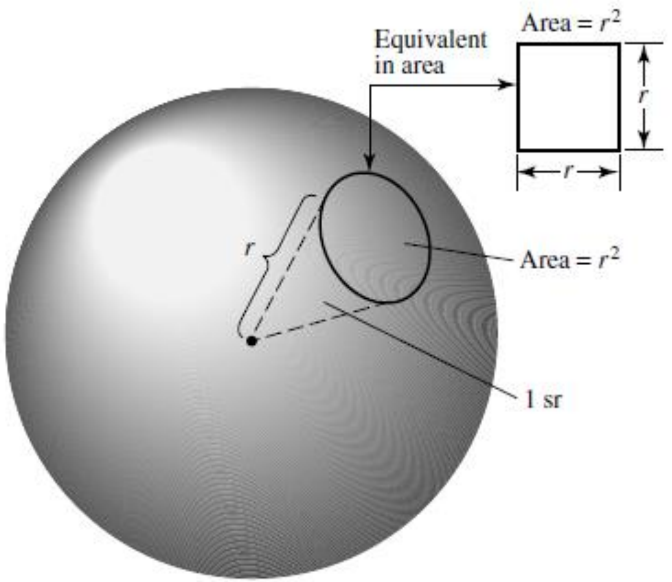


The solid angle $d\Omega$ is defined as

$$d\Omega = dA / r^2$$

The measure of a solid angle is a steradian.

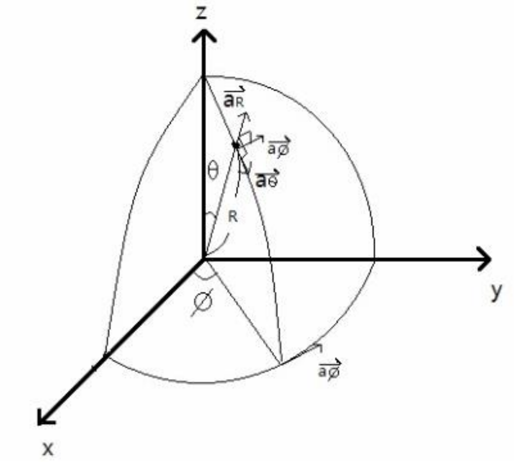
One steradian is defined as the solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area r^2.

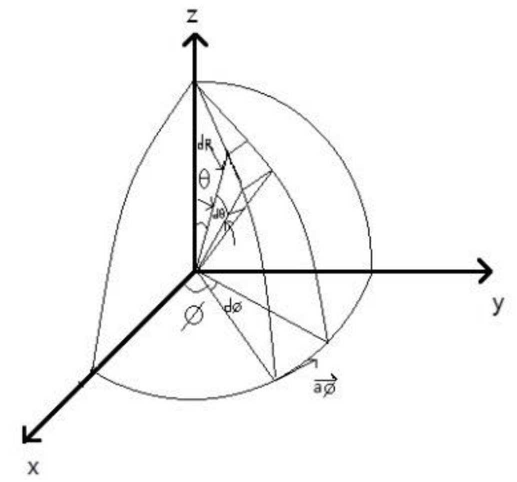


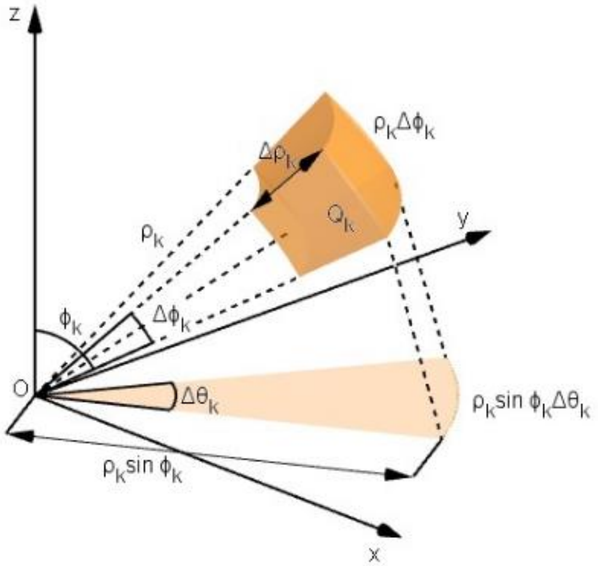
$$A = 4\pi r^2$$

A closed circle - $4\pi\ sr\ (4\pi r^2 / r^2)$

Radian and Steradian







	distance	area
$r, r+dr$	dr	$r dr d\theta$
$\theta, \theta + d\theta$	$r d\theta$	$r \sin\theta dr d\phi$
$\phi, \phi + d\phi$	$r \sin\theta d\phi$	$\boxed{r^2 \sin\theta d\theta d\phi}$

The infinitesimal area dA on the surface of a sphere is given by

$$dA = r^2 \sin\theta \, d\theta \, d\phi \quad (m^2)$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi \quad (sr)$$

Example 2.1

For a sphere of radius r, find the solid angle (in square radians or steradians) of a spherical cap on the surface sphere over the north-pole region defined by spherical angles of $0 \le \theta \le 30^o$, $0 \le \phi \le 360^o$. Do this

Solution:

a. Using (2-2), we can write that

$$\Omega_A = \int_0^{360^o} \int_0^{30^o} d\Omega = \int_0^{2\pi} \int_0^{\pi/6} sin\theta d\theta d\phi = 0.83566$$

b. $\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2|_{\Delta\Theta_1=\Delta\Theta_2} = \frac{\pi}{3} \cdot \frac{\pi}{3} = 1.09662$

The approximate beam solid angle is about 31.23% in error.

1. Radiation Power Density
2. Radiation Intensity

2.2 Radiation Power Density

Electromagnetic waves traveling in free space carry power and energy. The power associated with em waves is instantaneous power given by Poynting vector. It is given by

$$\mathrm{W}\,(\mathrm{r},\theta,\phi) = \mathrm{E} \times \mathrm{H}$$

Instantaneous Poynting vector is a power density and is used to describe the power associated with an electromagnetic wave

$$\vec{W} = \vec{E} \times \vec{H}$$

$\vec{W}$ - instantaneous Poynting vector (W/m^2)
$\vec{E}$ - instantaneous electric-field intensity (V/m)
$\vec{H}$ - instantaneous magnetic-field intensity (A/m)

Poynting vector is a power density,

The total power, crossing a closed surface, can be obtained by integrating the normal component of the Poynting vector over the entire surface

$$P = \oiint_s \vec{W} \cdot d\vec{s} = \oiint_s \vec{W} \cdot n\, da$$

P - instantaneous total power (W)
n - unit vector normal to the surface
da - infinitesimal area of the closed surface (m^2)

2.2 Average Power Density

For time varying fields, average power density is needed, which is obtained by integrating the instantaneous Poynting vector over one period and dividing by the period. For the form

Time-harmonic variations of the form $e^{j\omega t}$

$$\vec{E}(x,y,z;t) = \text{Re}[E(x,y,z)e^{j\omega t}]$$

$$\vec{H}(x,y,z;t) = \text{Re}[H(x,y,z)e^{j\omega t}]$$

using, $\text{Re}[Ee^{j\omega t}] = \frac{1}{2}[Ee^{j\omega t} + E^* e^{-j\omega t}]$

$$\vec{W} = \vec{E} \times \vec{H} = \frac{1}{2}\text{Re}[E \times H^*] + \frac{1}{2}\text{Re}[E \times He^{j2\omega t}]$$

$$W_{av}(x,y,z) = [W(x,y,z;t)]_{av} = \frac{1}{2}\text{Re}[E \times H^*] \quad (W/m^2)$$

$\frac{1}{2}\text{Re}[E \times H^*]$ - average (real) power density

$\frac{1}{2}\text{Im}[E \times H^*]$ - reactive (stored) power density

The 1/2 factor appears in (2-7) and (2-8) because the E and H fields represent peak values and it should be omitted from RMS values

It will be shown that the power density associated with the electromagnetic fields of an antenna in its far-field region is predominately real and will be referred to as radiation density.

Spatial Variations in Power Density

$$\left[\underline{\boldsymbol{\mathcal{W}}}(x,y,z;t)\right]_{ave} = \underline{W}(x,y,z)\Big|_{ave} = \frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]$$

$$\boxed{\underline{W}\Big|_{ave} = \frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]} \qquad (2\text{-}8)$$

It is assumed that both $\underline{E}$ and $\underline{H}$ represent peak amplitude values (not RMS: Root Mean Square). If $\underline{E}$ and $\underline{H}$ were to represent RMS values, then the one-half (1/2) must be omitted. Measuring instruments typically measure RMS values. Equation (2-8) is analogous to Ohm's Law in circuits

$$P = \frac{1}{2}VI^*$$

where V and I present peak (not RMS) values.

Based upon the definition of

the average power radiated power can be written as $W_{av}(x,y,z) = [W(x,y,z;t)]_{av} = \frac{1}{2}\mathrm{Re}[E \times H^*] \quad (W/m^2)$

$$\mathcal{P} = \oiint_S \underline{\mathcal{W}} \cdot d\underline{s} = \int_0^{2\pi}\int_0^{\pi} \underline{\mathcal{W}} \cdot \hat{a}_r r^2 \sin\theta d\theta d\phi$$

$$\mathcal{P} = \int_0^{2\pi}\int_0^{\pi} (\underline{\mathcal{E}} \times \underline{\mathcal{H}}) \cdot \hat{a}_r r^2 \sin\theta d\theta d\phi$$

$$P = \oiint_S \underline{W}_{ave} \cdot d\underline{s} = \int_0^{2\pi}\int_0^{\pi} \underline{W}_{ave} \cdot \hat{a}_r r^2 \sin\theta d\theta d\phi$$

$$\boxed{P_{rad}} = \boxed{P_{ave} = \int_0^{2\pi}\int_0^{\pi} \left(\frac{1}{2}\mathrm{Re}\left[\underline{E} \times \underline{\mathrm{H}}^*\right]\right) \cdot \hat{a}_r r^2 \sin\theta d\theta d\phi} \quad (2\text{-}9)$$

The power pattern of the antenna is just a measure, as a function of direction, of the average power density radiated by the antenna.

2.2 Radiation Power Density

☺ isotropic source

$$P_{rad} = \oiint_S W_0 \cdot ds = \int_0^{2\pi}\int_0^{\pi} [a_r W_0(r)] \cdot [a_r r^2 \sin\theta \, d\theta \, d\phi] = 4\pi r^2 W_0$$

$$W_0 = a_r W_0 = a_r \left(\frac{P_{rad}}{4\pi r^2} \right) \quad (W / m^2)$$

Example

The radial component of the radiated power density of an antenna is

$$\vec{W}_{rad} = \hat{a}_r A_0 \frac{sin\theta}{r^2} \quad (\text{W/m}^2)$$

A_0 is the peak value of the power density, θ is the spherical coordinate, and $\hat{a}_r$ is the radial unit vector. Determine the total radiated power.

SOLUTION

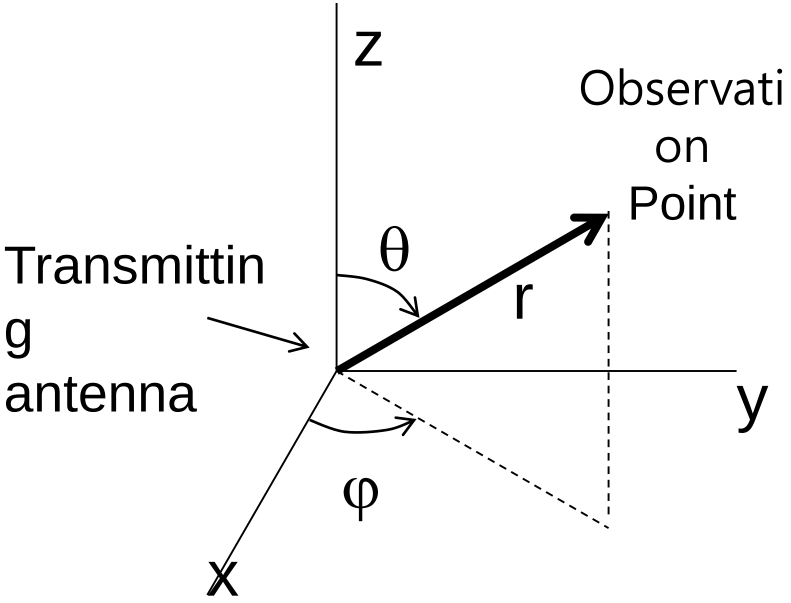
For a closed surface, a sphere of radius r is chosen. To find the total-radiated power, the radial component of the power density is integrated over its surface.

$$P_{rad} = \oiint_S \vec{W}_{rad} \cdot d\vec{s} = \int_0^{2\pi} \int_0^{\pi} (\hat{a}_r A_0 \frac{sin\theta}{r^2}) \cdot (\hat{a}_r r^2 sin\theta d\theta d\phi)$$

$$= \pi^2 A_0 \qquad (W)$$

Radiation Intensity $U(\theta, \phi)$

Radiation intensity in a given direction is defined as "the power radiated from an antenna per unit solid angle." The radiation intensity is function of angle θ and ϕ meaning that it is always defined in particular direction.



$$U(\theta, \phi) = \frac{P_r}{d\Omega}$$

P_r is power radiated and $d\Omega$ is solid angle. Radiation intensity is measure of the ability of an antenna to concentrate radiated power in a particular direction. It does not depend on distance It is far field parameter measured in far field. It has unit of Watts/solid angle.

An isotopic antenna is one which radiates equally in all directions. i.e. $U(\theta.\phi) =$ Constant

$$U_{iso}(\theta, \phi) = \frac{Pr}{4\pi}$$

Total Power Radiated by an isotropic antenna Pr = $U_{iso}(\theta, \phi) \times 4\pi$

- The radiation intensity is a far-field parameter, and it can be obtained by multiplying the radiation density by the square of the distance.

$$U = r^2 W_{rad}$$

U - radiation intensity (W/unit solid angle)

W_{rad} - radiation density (W/m^2)

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left|E(r,\theta,\phi)\right|^2 \simeq \frac{r^2}{2\eta}\left[\left|E_\theta(r,\theta,\phi)\right|^2 + \left|E_\phi(r,\theta,\phi)\right|^2\right]$$

$$\simeq \frac{1}{2\eta}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right]$$

$E(r,\theta,\phi)$ - far-zone electric-field intensity of the antenna $= E^\circ(\theta,\phi)\dfrac{e^{-jkr}}{r}$

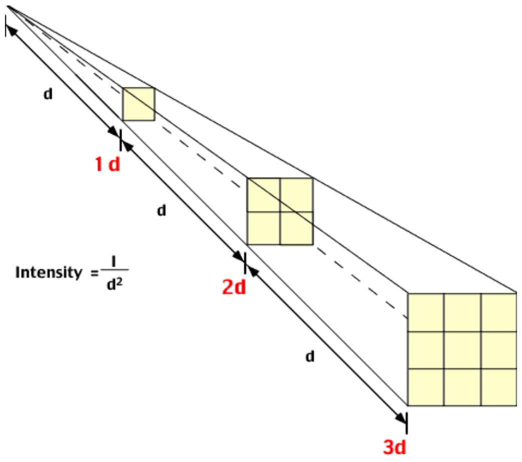
E_θ, E_ϕ - far-zone electric-field components of the antenna

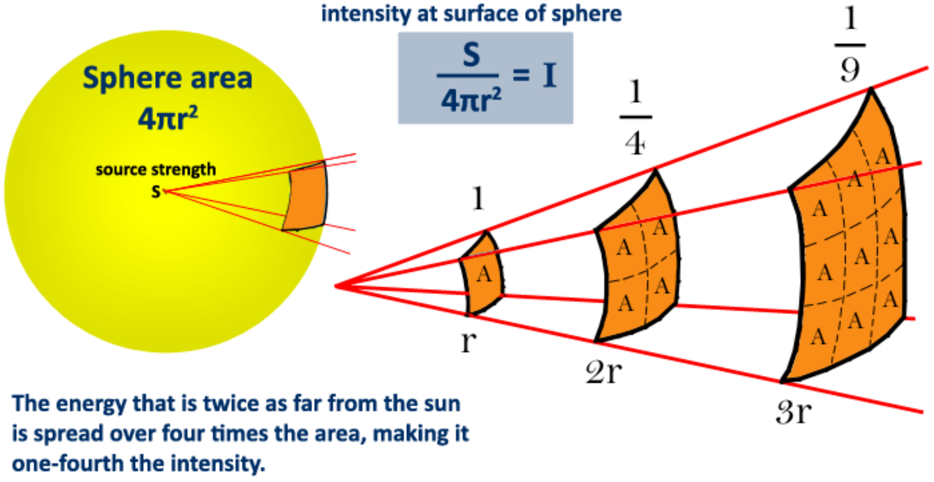
η - intrinsic impedance of the medium

2.1 Radiation Intensity

The total power is obtained by integrating the radiation intensity, over the entire solid angle of 4π. Thus

$$P_{rad} = \oiint_{\Omega} U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \qquad (d\Omega = element\ of\ solid\ angle = \sin\theta\, d\theta\, d\phi)$$





☺ isotropic source

$$P_{rad} = \oiint_{\Omega} U_0 d\Omega = U_0 \oiint_{\Omega} d\Omega = 4\pi U_0$$

$$U_0 = \frac{P_{rad}}{4\pi}$$

$$U = \frac{Power}{Unit\ Solid\ Angle} = \frac{Power}{Unit\ Area / r^2}$$

because

$$\boxed{d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta\, d\theta\, d\phi}{r^2} = \sin\theta\, d\theta\, d\phi}$$

$$U = r^2 \frac{Power}{Unit\ Area} = r^2 W_{ave} = r^2 W_{rad}$$

$$\boxed{U = r^2 W_{rad} \;\Rightarrow\; W_{rad} = \frac{U}{r^2}}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \hat{a}_r W_{rad} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} W_{rad} r^2 \sin\theta \, d\theta \, d\phi$$

<u>Since</u> $W_{rad} = \dfrac{U}{r^2}$

$$\boxed{P_{rad} = P_{ave} = \int_0^{2\pi}\int_0^{\pi} U \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}}$$

Example 2.3

$$U = r^2 W_{rad} = r^2 \left(A_0 \frac{\sin\theta}{r^2} \right) = A_0 \sin\theta$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \qquad (2\text{-}13)$$

$$= A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\, d\phi = A_0 \pi^2$$

$$P_{rad} = A_0 \pi^2$$

Cont'd:

$$\underline{W}_{ave} = \underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin^2\theta}{r^2}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} A_0 \sin^3\theta d\theta d\phi = 2\pi A_0 \int_0^{\pi} \sin^3\theta d\theta$$

$$P_{rad} = 2\pi A_0 \left(\frac{4}{3}\right) = \frac{8\pi}{3} A_0$$

Isotropic Source

$$W_o = \frac{P}{4\pi r^2}\left(W / m^2 \right)$$

$$U_o = \frac{P}{4\pi}\left(W / Sr \right)$$

$$\underline{W}_{ro} = \hat{a}_r W_{ro}(r)$$

$$P_{rad} = P_{ave} = \oint\!\!\!\oint_S \underline{W}_{ro} \cdot d\underline{s} = \int_0^{2\pi}\int_0^{\pi} \hat{a}_r W_{ro} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} W_{ro}(r) r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = W_{ro}(r) r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = W_{ro}(r) r^2 (2)(2\pi)$$

$$= 4\pi r^2 W_{ro}(r)$$

$$\boxed{W_{ro}(r) = \frac{P_{rad}}{4\pi r^2}}$$

Thank you