Broadside and End fire Antenna Arrays

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Broadside Array

- The maximum radiation of an array directed normal to the axis of the array [broadside; $\theta_0 = 90$]
- To optimize the design, the maxima of the single element and of the array factor should both be directed toward $\theta_0 = 90°$.
- The requirements that allow the array factor to "radiate" efficiently broadside will be developed.
- The first maximum of the array factor occurs when

$$\psi = kd\cos\theta + \beta = 0$$

- Since it is desired to have the first maximum directed toward $\theta_0 = 90°$, *then*

$$\boxed{\psi = kd\cos\theta + \beta|_{\theta=90°} = \beta = 0}$$

- Thus to have the maximum of the array factor of a uniform linear array directed broadside to the axis of the array
- All the elements have the same phase excitation.
- In addition to the same amplitude excitation.
- To ensure that there are no principal maxima in other directions, which are referred to as *grating lobes, the separation between the elements* should not be equal to multiples of a wavelength

$$\psi = kd\cos\theta + \beta\Big|_{\substack{d=n\lambda \\ \beta=0 \\ n=1,2,3,\ldots}} = 2\pi n\cos\theta\Big|_{\theta=0^\circ,180^\circ} = \pm 2n\pi$$

- Thus for a uniform array with $\beta = 0$ *and* $d = n\lambda$, *in addition to having the* maxima of the array factor directed broadside ($\theta_0 = 90^\circ$) *to the axis of the array*, there are additional maxima directed along the axis ($\theta_0 = 0^\circ$, 180°) *of the array (end fire* radiation).

- *To avoid any grating lobe, the largest spacing between the elements should be less than one wavelength* ($d_{max} < \lambda$).

Example

- **The three-dimensional array factor of a 10-element (*N = 10*) uniform array with *β = 0 and d = λ/4***

Nulls: 66, 113
36, 143

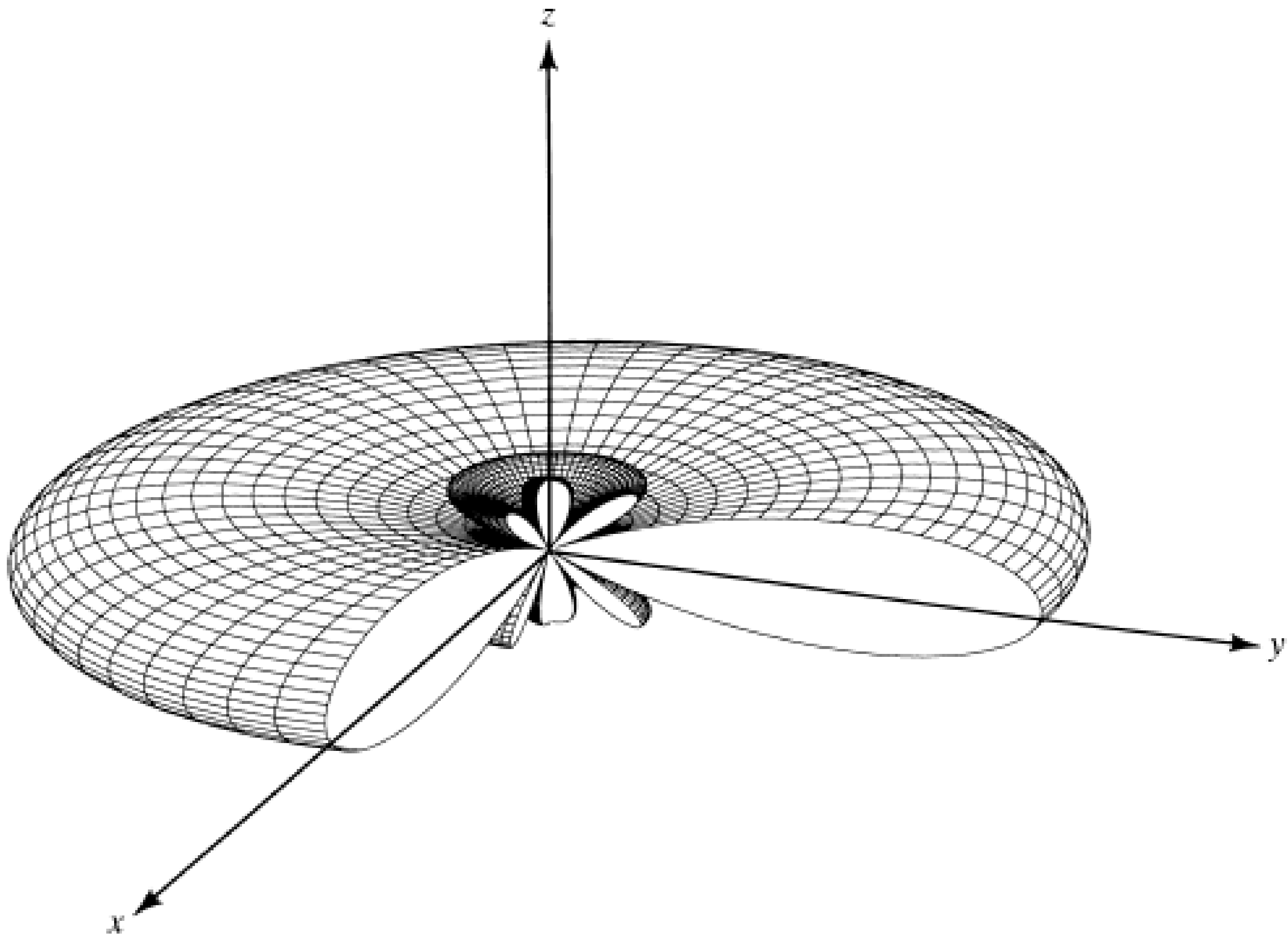
Major lobe max.:
90, 270

Minor lobe max:
53, 126,0 , 180

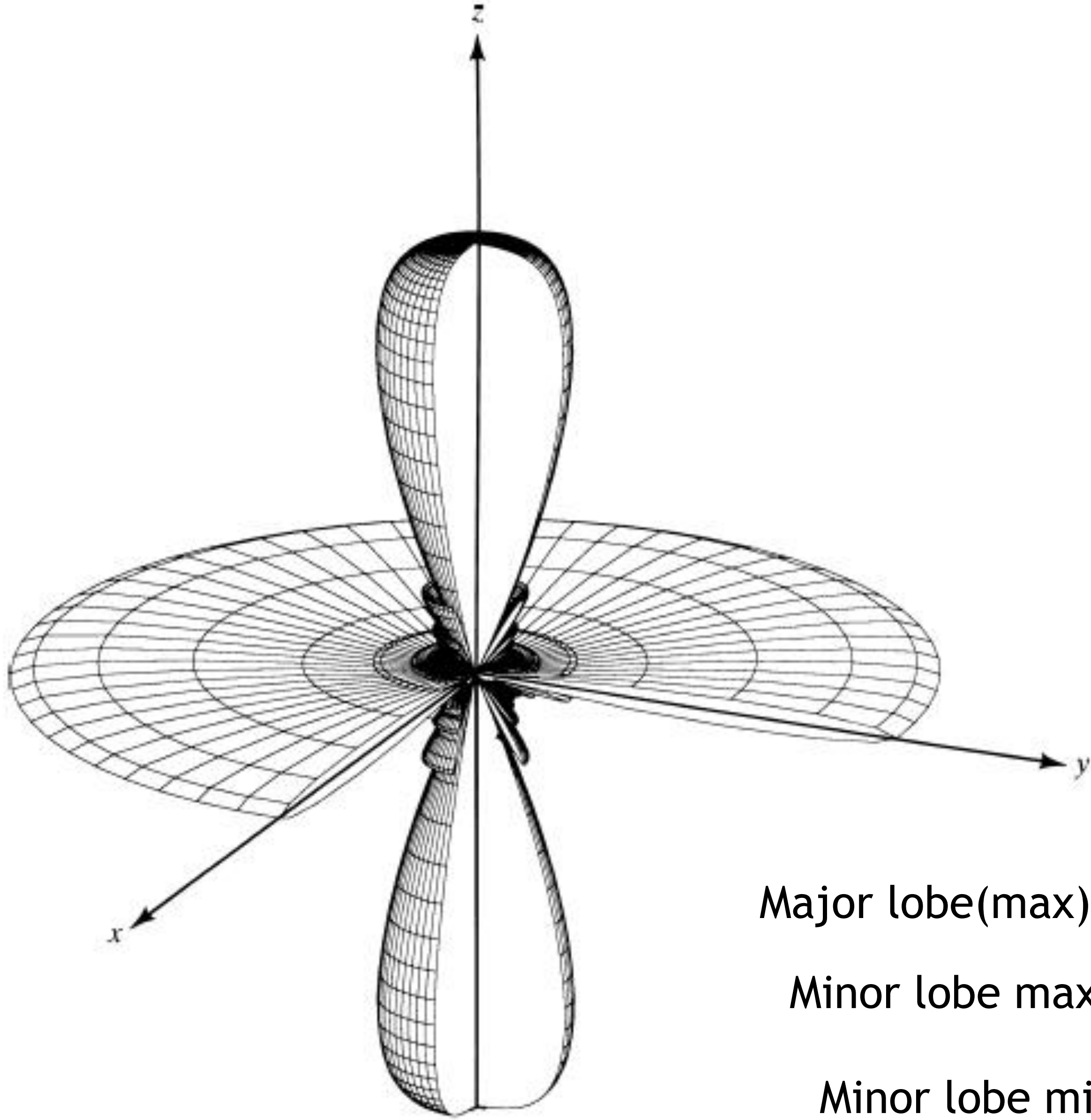
Half power points:
69.5, 110

TABLE 6.1 Nulls, Maxima, Half-Power Points, and Minor Lobe Maxima for Uniform Amplitude Broadside Arrays

NULLS	$\theta_n = \cos^{-1}\left(\pm \frac{n}{N}\frac{\lambda}{d}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(\pm \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(\pm \frac{1.391\lambda}{\pi N d}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[\pm \frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$ $s = 1, 2, 3, \ldots$ $\pi d/\lambda \ll 1$



(a) Broadside ($\beta = 0$, $d = \lambda/4$)



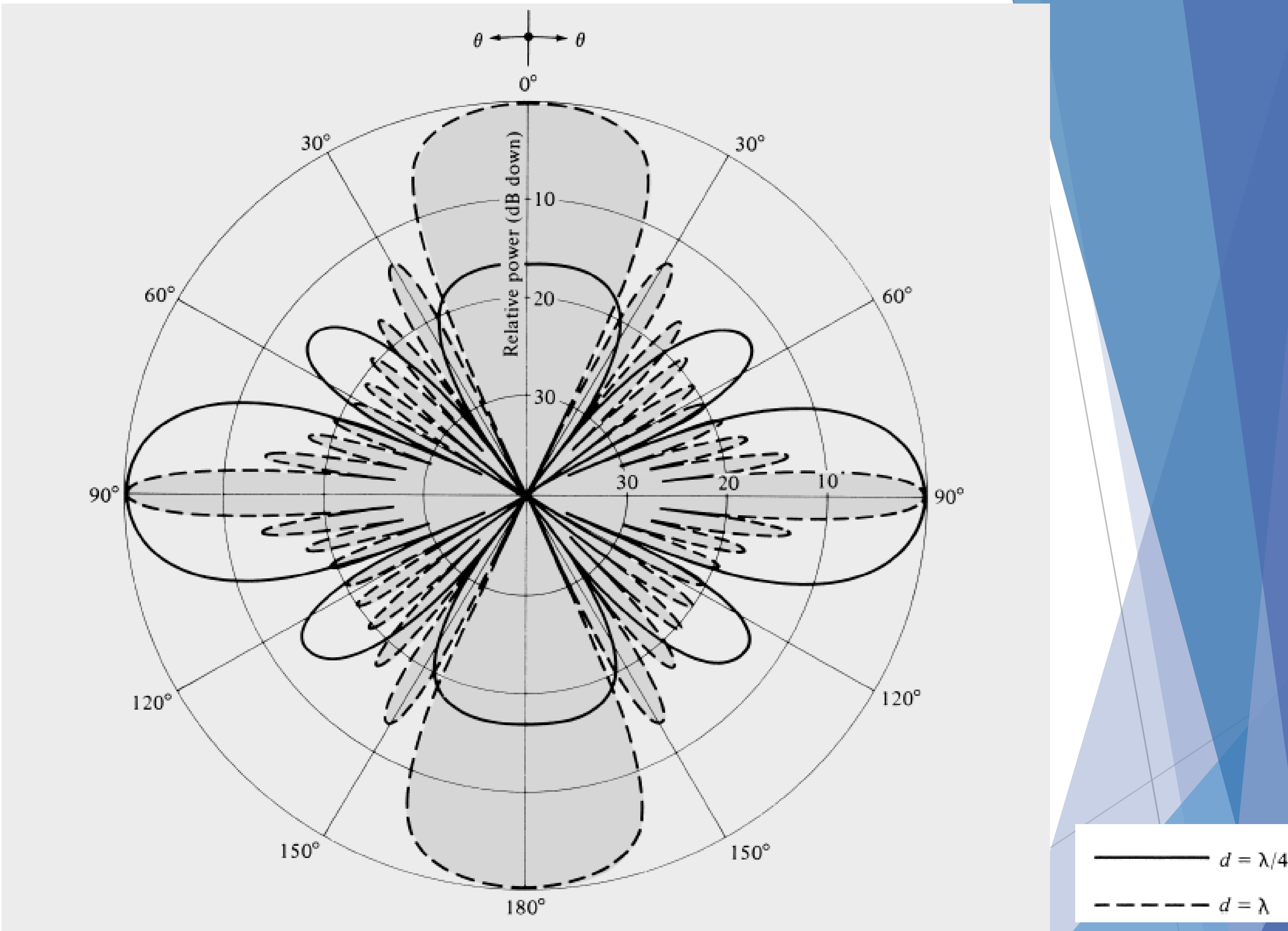
(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

Major lobe(max) :90, 270, 0, 180

Minor lobe max:20 points

Minor lobe minima:18 points

Half power points: 87, 92



Array factor patterns of a 10-element uniform amplitude broadside array($N = 10$, $\beta = 0$).

Ordinary End-Fire Array

- **Radiates Toward Only One Direction (Either *Θ_0* = *0◦ Or 180◦)***
- **To Direct The First Maximum Toward *Θ_0* = *0* ◦**

$$\boxed{\psi = kd\cos\theta + \beta|_{\theta=0^\circ} = kd + \beta = 0 \Rightarrow \beta = -kd}$$

- **If The First Maximum Is Desired Toward *Θ_0* = *180◦,then***

$$\boxed{\psi = kd\cos\theta + \beta|_{\theta=180^\circ} = -kd + \beta = 0 \Rightarrow \beta = kd}$$

- **If The Element Separation Is *D = Λ/2, End-fire Radiation Exists Simultaneously In* Both Directions (*Θ_0 = 0◦ And Θ_0 = 180◦)***

- *If the element spacing is a multiple of a* wavelength ($d = n\lambda$, $n = 1, 2, 3, \ldots$), *then in addition to having end-fire radiation in* both directions, there also exist maxima in the broadside directions

- *To have only one end-fire maximum and to avoid any grating lobes, the maximum spacing between the elements should be less than dmax* $< \lambda/2$.

Eg: The three-dimensional radiation patterns of a 10-element (*N = 10) array with d = λ/4, β = +kd along theta = 180 degree*

Minor lobe minima:
78, 101
53, 126
0

Major lobe maxima:
180

Half power points:
145

Minor lobe maxima:
113, 90, 270,36,66

TABLE 6.3 Nulls, Maxima, Half-Power Points, and Minor Lobe Maxima for Uniform Amplitude Ordinary End-Fire Arrays

NULLS	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$ $s = 1, 2, 3, \ldots$ $\pi d/\lambda \ll 1$

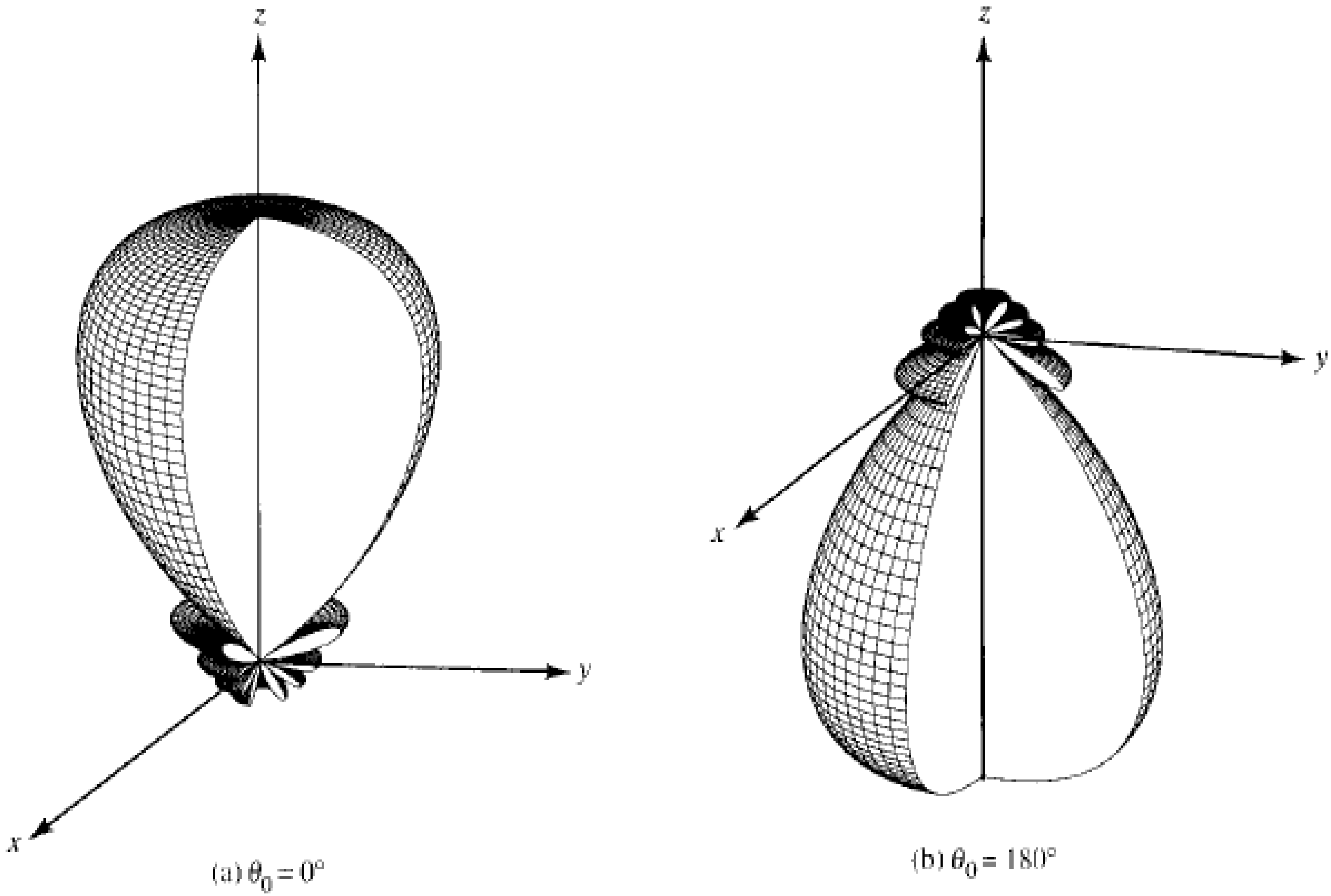


Figure 6.8 Three-dimensional amplitude patterns for end-fire arrays toward $\theta_0 = 0^\circ$ and 180° $(N = 10,\ d = \lambda/4)$.

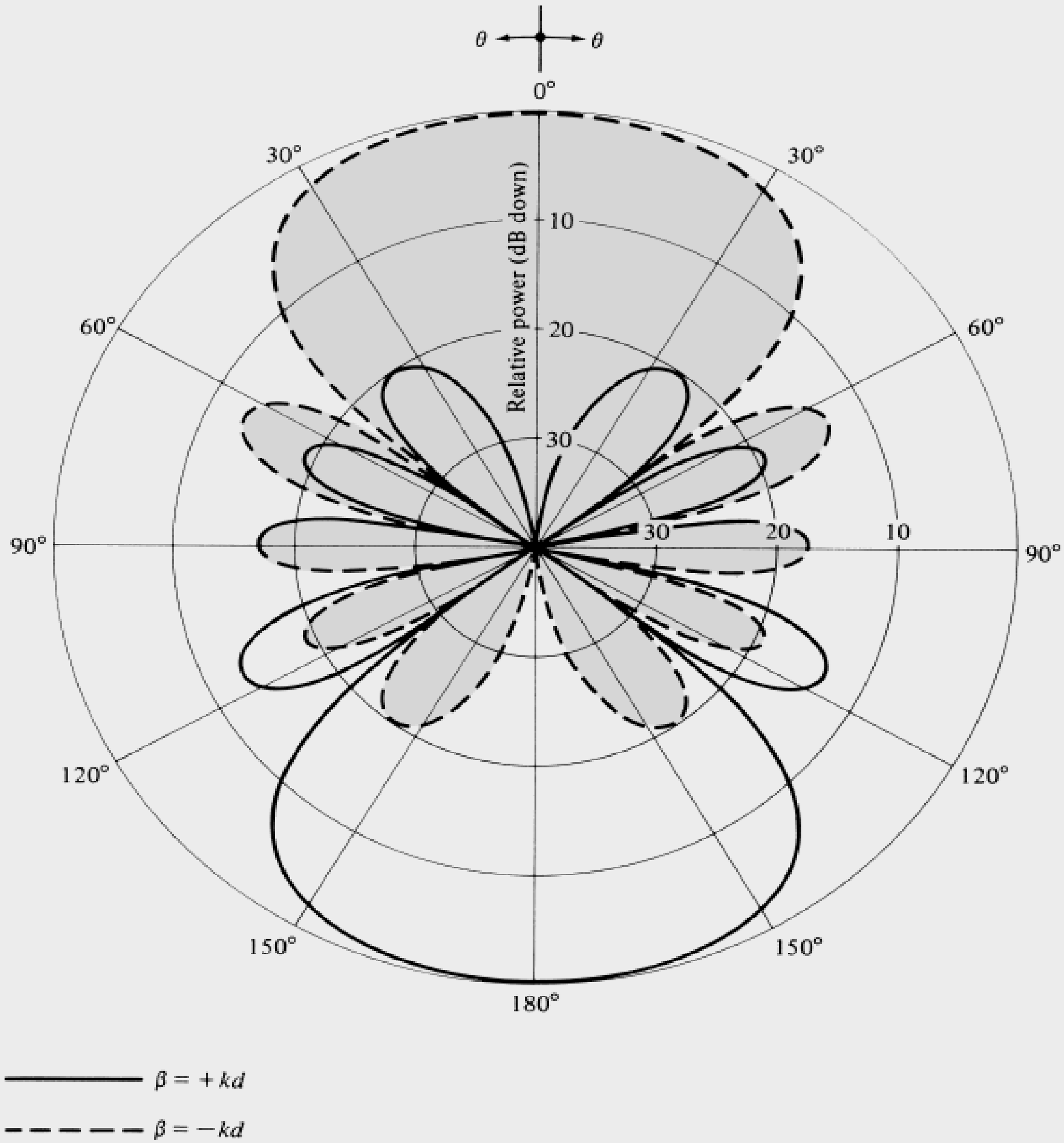


Figure 6.9 Array factor patterns of a 10-element uniform amplitude end-fire array ($N = 10, d = \lambda/4$).

Amplitude difference between major lobe and 1st minor lobe

The maximum of the first minor lobe of (iv) occurs *approximately when*

$$\frac{N}{2}\psi = \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_s} \simeq \pm\left(\frac{3\pi}{2}\right)$$

$$\theta_s = \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm \frac{3\pi}{N}\right]\right\}$$

At this point, the magnitude of (iv) *reduces to*

$$(\mathrm{AF})_n \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]_{\substack{\theta=\theta_s \\ s=1}} = \frac{2}{3\pi} = 0.212$$

$$(\mathrm{AF})_n = 20\log_{10}\left(\frac{2}{3\pi}\right) = -13.46\ \mathrm{dB}$$

Thus the maximum of the first minor lobe of the array factor of (iv) is 13.46 dB down from the maximum at the major lobe.

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$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right]$$

ı the elements:

$$(\mathrm{AF})_n \simeq \left[\frac{\sin\left(\frac{N}{2}kd\cos\theta\right)}{\left(\frac{N}{2}kd\cos\theta\right)}\right]$$

$$U(\theta) = [(\mathrm{AF})_n]^2 = \left[\frac{\sin\left(\frac{N}{2}kd\cos\theta\right)}{\frac{N}{2}kd\cos\theta}\right]^2 = \left[\frac{\sin(Z)}{Z}\right]^2$$

$$Z = \frac{N}{2}kd\cos\theta$$

The directivity of the array factor can be evaluated using

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{U_{\max}}{U_0}$$

$$U_0 = \frac{P_{\text{rad}}}{4\pi} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta)\sin\theta\, d\theta\, d\phi$$

$$U_0 = \frac{1}{4\pi}P_{\text{rad}} = \frac{1}{2}\int_0^{\pi}\left[\frac{\sin(Z)}{Z}\right]^2 \sin\theta\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi}\left[\frac{\sin\left(\frac{N}{2}kd\cos\theta\right)}{\frac{N}{2}kd\cos\theta}\right]^2 \sin\theta\, d\theta$$

By making a change of variable, that is,

$$Z = \frac{N}{2}kd\cos\theta$$

$$dZ = -\frac{N}{2}kd\sin\theta\, d\theta$$

$$U_0 = -\frac{1}{Nkd}\int_{+Nkd/2}^{-Nkd/2}\left[\frac{\sin Z}{Z}\right]^2 dZ = \frac{1}{Nkd}\int_{-Nkd/2}^{+Nkd/2}\left[\frac{\sin Z}{Z}\right]^2 dZ$$

For a large array ($Nkd/2 \to$ large), (6-41) can be approximated by extending the limits to infinity. That is,

$$U_0 = \frac{1}{Nkd}\int_{-Nkd/2}^{+Nkd/2}\left[\frac{\sin Z}{Z}\right]^2 dZ \simeq \frac{1}{Nkd}\int_{-\infty}^{+\infty}\left[\frac{\sin Z}{Z}\right]^2 dZ$$

$$\int_{-\infty}^{+\infty} \left[\frac{\sin(Z)}{Z}\right]^2 dZ = \pi$$

$$U_0 \simeq \frac{\pi}{Nkd}$$

$$D_0 = \frac{U_{\max}}{U_0} \simeq \frac{Nkd}{\pi} = 2N\left(\frac{d}{\lambda}\right)$$

Directivity : **Ordinary End-Fire Array**

For an end-fire array, with the maximum radiation in the $\theta_0 = 0^\circ$ direction, the array factor is given by

$$(\mathrm{AF})_n = \left[\frac{\sin\left[\frac{N}{2}kd(\cos\theta - 1)\right]}{N\sin\left[\frac{1}{2}kd(\cos\theta - 1)\right]} \right]$$

$$(\mathrm{AF})_n \simeq \left[\frac{\sin\left[\frac{N}{2}kd(\cos\theta - 1)\right]}{\left[\frac{N}{2}kd(\cos\theta - 1)\right]} \right]$$

The corresponding radiation intensity can be written as

$$U(\theta) = [(\mathrm{AF})_n]^2 = \left[\frac{\sin\left[\frac{N}{2}kd(\cos\theta - 1)\right]}{\frac{N}{2}kd(\cos\theta - 1)} \right]^2 = \left[\frac{\sin(\mathrm{Z})}{Z}\right]^2$$

$$\mathrm{Z} = \frac{N}{2}kd(\cos\theta - 1)$$

$$U_0 = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \left[\frac{\sin\left[\frac{N}{2}kd(\cos\theta - 1)\right]}{\frac{N}{2}kd(\cos\theta - 1)} \right]^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{2}\int_0^{\pi}\left[\frac{\sin\left[\frac{N}{2}kd(\cos\theta - 1)\right]}{\frac{N}{2}kd(\cos\theta - 1)}\right]^2 \sin\theta\, d\theta$$

$$Z = \frac{N}{2}kd(\cos\theta - 1)$$

$$dZ = -\frac{N}{2}kd\sin\theta\, d\theta$$

$$U_0 = -\frac{1}{Nkd}\int_0^{-Nkd}\left[\frac{\sin(Z)}{Z}\right]^2 dZ = \frac{1}{Nkd}\int_0^{Nkd}\left[\frac{\sin(Z)}{Z}\right]^2 dZ$$

For a large array (*Nkd* $\rightarrow$ *large), eqn. can be approximated by extending the* limits to infinity.

$$U_0 = \frac{1}{Nkd} \int_0^{Nkd} \left[\frac{\sin(Z)}{Z} \right]^2 dZ \simeq \frac{1}{Nkd} \int_0^{\infty} \left[\frac{\sin(Z)}{Z} \right]^2 dZ$$

$$U_0 \simeq \frac{\pi}{2Nkd}$$

$$D_0 = \frac{U_{\max}}{U_0} \simeq \frac{2Nkd}{\pi} = 4N\left(\frac{d}{\lambda}\right)$$