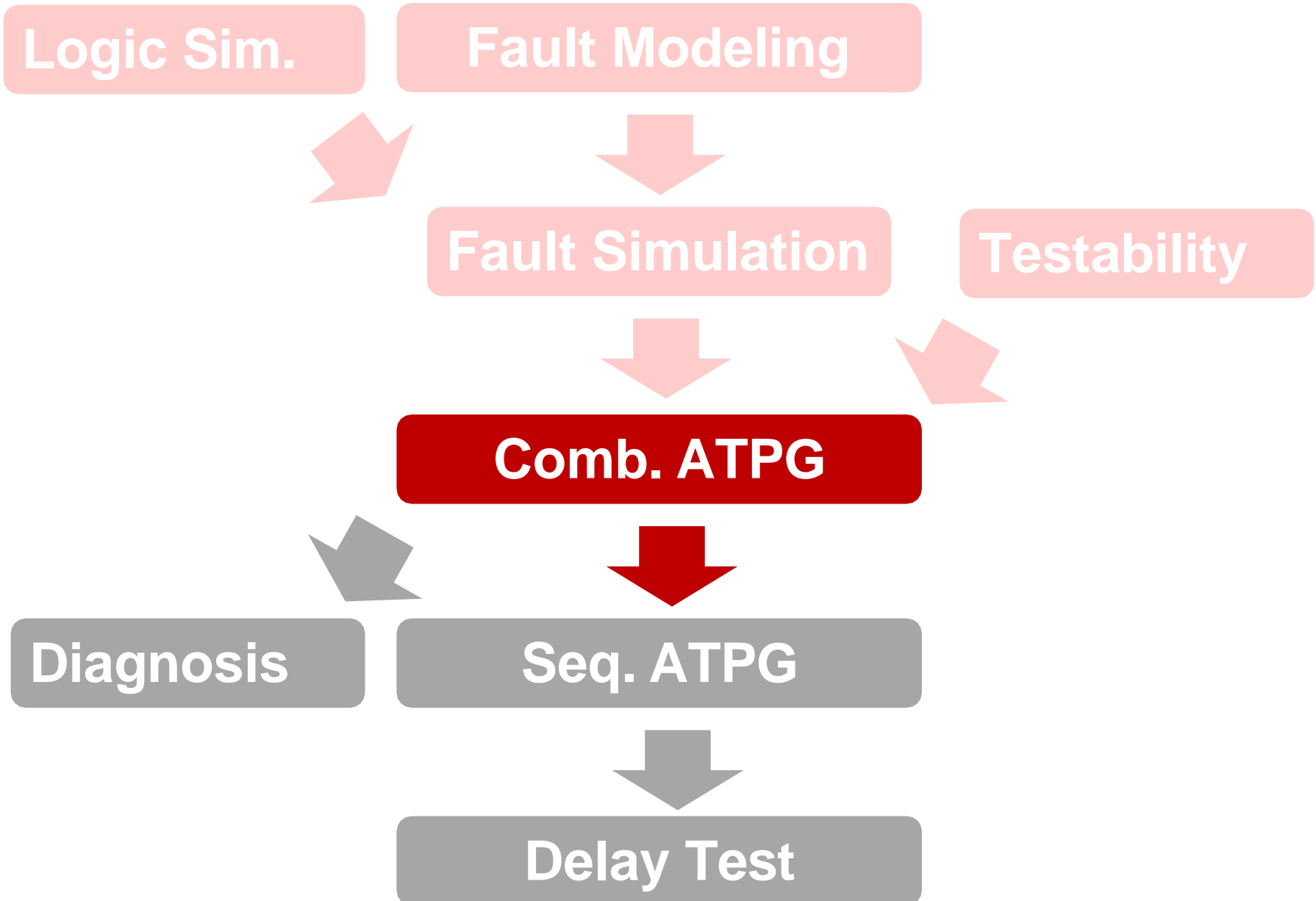


Combinational ATPG

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Assistant Professor,
DECE, TIET, Patiala

Course Roadmap (EDA Topics)



Why Am I Learning This?

- Automatic Test Pattern Generation (ATPG)
 - ◆ 1. Generate high quality test patterns
 - ◆ 2. Reduce Human efforts

“Testing a product is a learning process.”

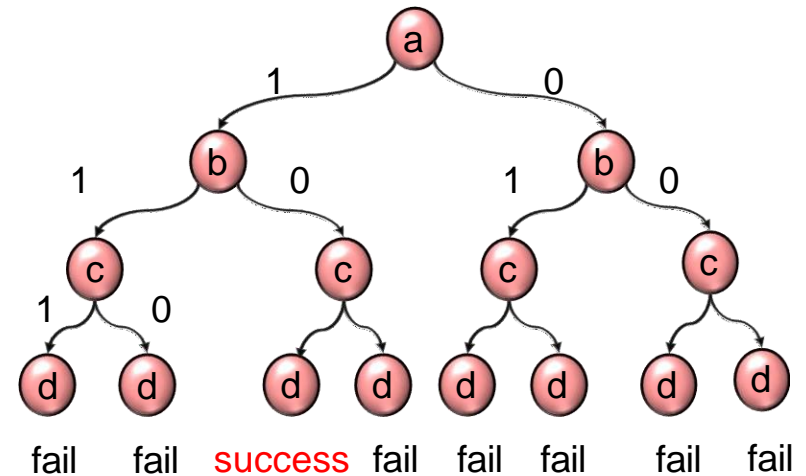
(Brian Marick)

Test Generation

Fault Models	Combinational Circuits (seq. ckt. w/ scan)	Sequential Circuits
No fault model	PET	Checking experiment
Single Stuck-at Fault Model	D PODEM FAN	Extended D 9-valued
Delay Fault Model	Path delay Transition delay	Launch on capture Launch on shift

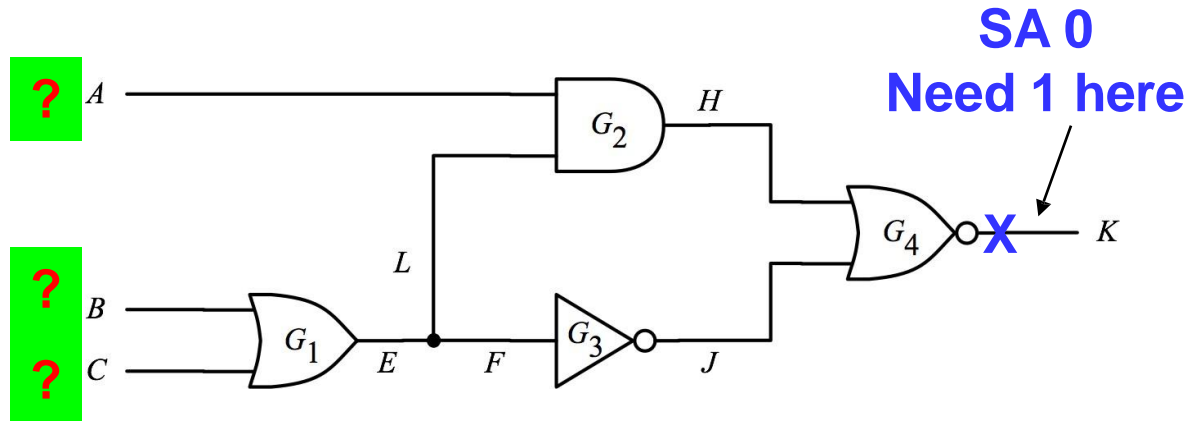
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
- Acceleration Techniques
- Concluding Remarks



Motivating Problem

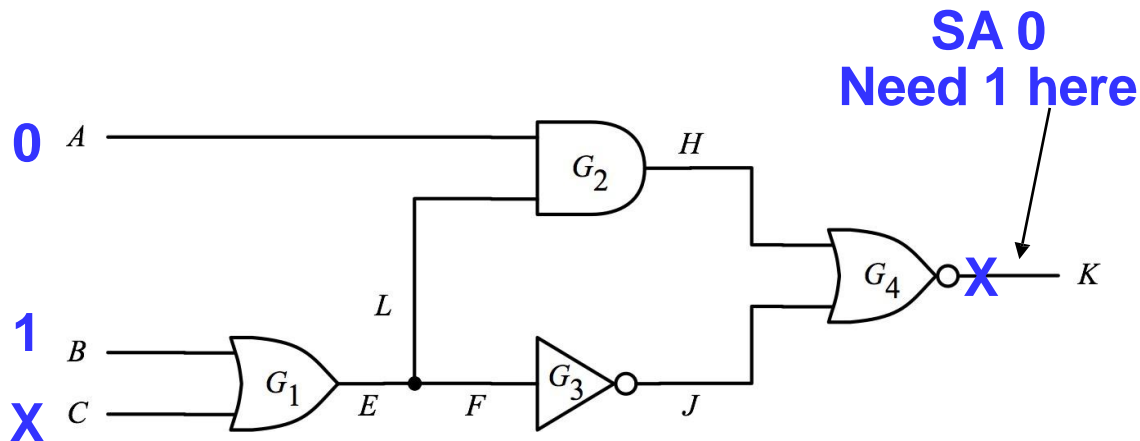
- Your manger asks you to generate a test pattern for output stuck-at zero fault. What is your answer? (Maybe more than one answer)



- Simulation is like *substitution*, easy
 - ♦ $f(x)=x^5+3x^4+2, x=3.2 \Rightarrow f(x)=?$
- Finding test pattern is like *finding roots*, very hard
 - ♦ $f(x)=x^5+3x^4+2, f(x)=0, \Rightarrow x=?$

What is Complexity of TPG?

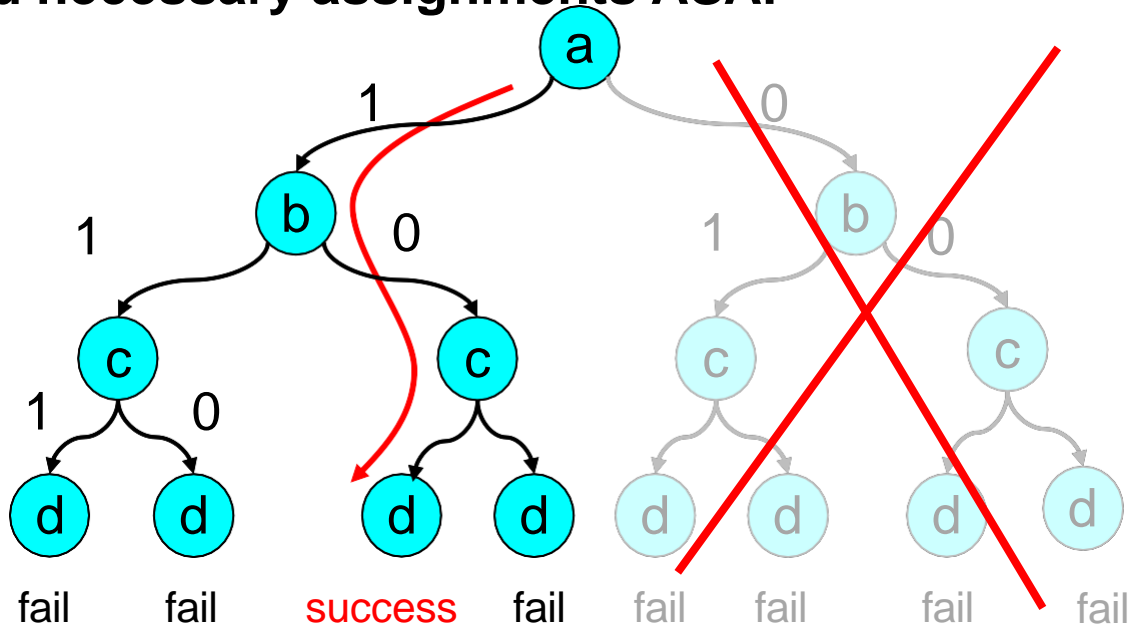
- **Test pattern generation TPG**
 - ♦ generate test patterns for a given fault model
- Generating a test pattern for a fault is **NP-complete**
 - ♦ Same as **satisfiability** problem
 - ♦ Worst case $2^{\text{number_PI}}$ assignments to try
- Need automatic tool, **ATPG (Automatic Test Pattern Generator)**



TPG is NP-complete

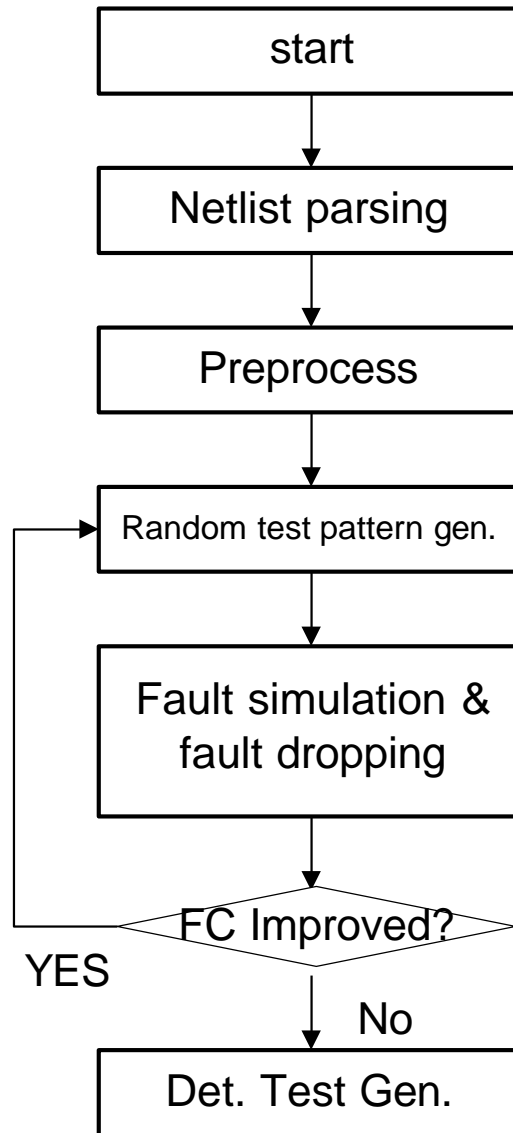
ATPG is Decision Problem

- Huge binary decision tree (**exponential size!**)
 - ♦ There could be *one, many, or even zero* answers
- Need smart heuristic to speed up
 - ♦ 1. Prune impossible sub-trees ASAP
 - ♦ 2. Find necessary assignments ASAP



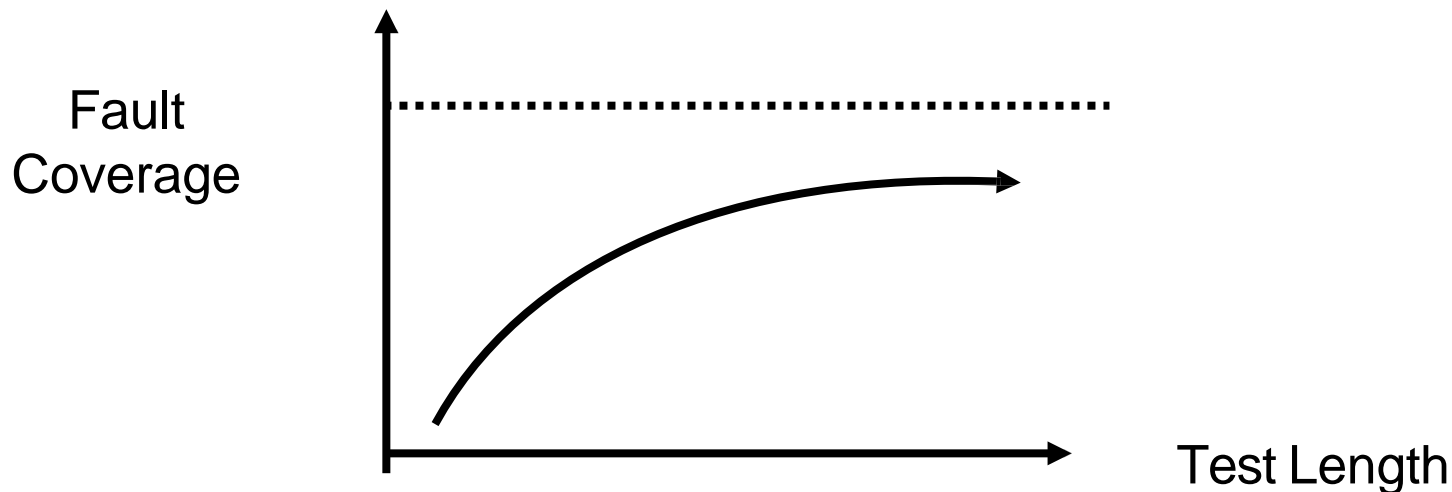
Search a Test Pattern in a Huge Tree

Random Test Pattern Gen.

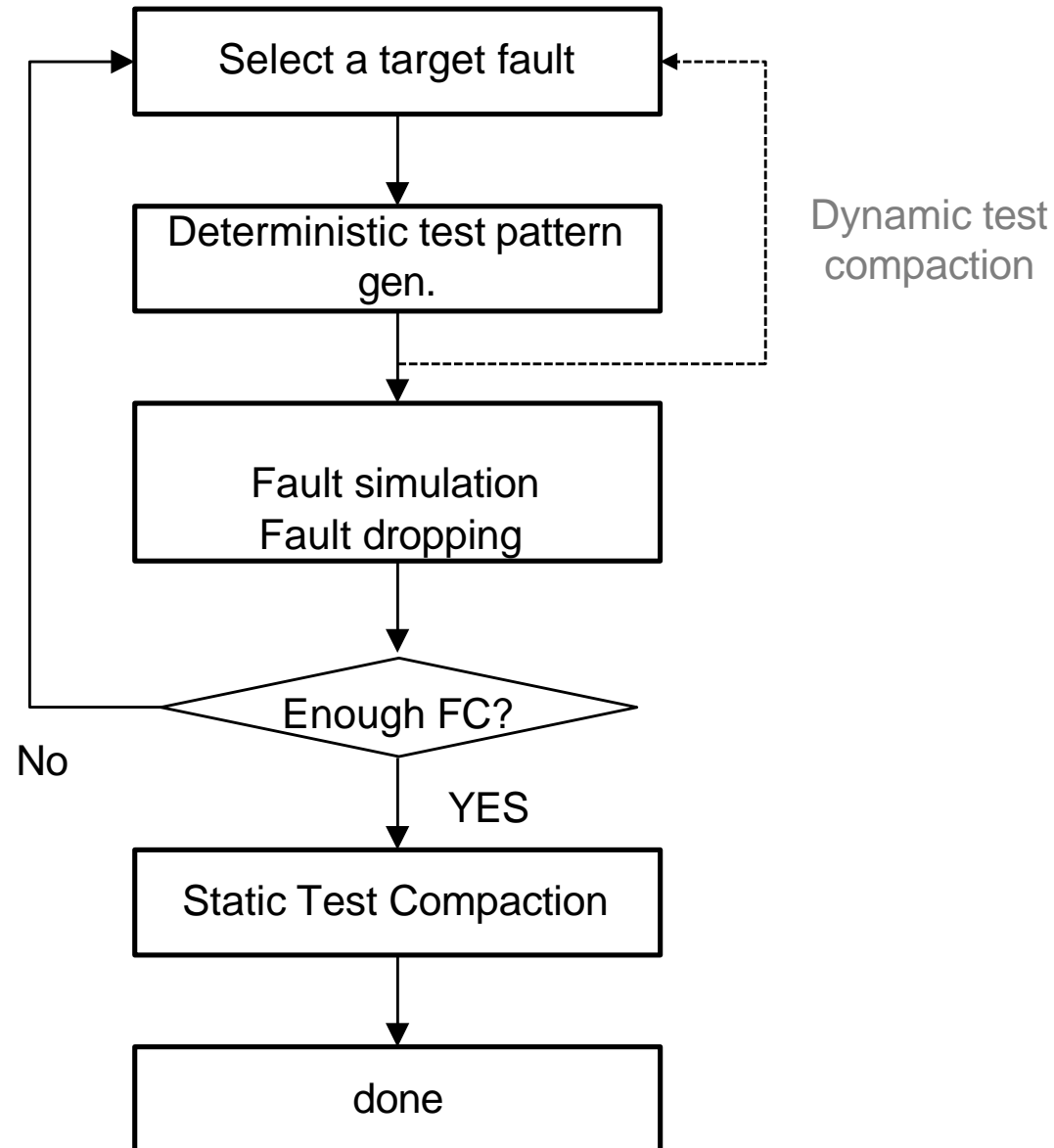


Random Test Pattern Generation

- Idea: there are many easy-to-detect faults
 - ◆ First generate random patterns and
 - ◆ then select patterns that defect undetected faults
- Problem
 - ◆ Fault coverage often saturates after easy faults are detected
 - * *Random pattern resistant faults* not easy to detect



Deterministic Test Pattern Gen.



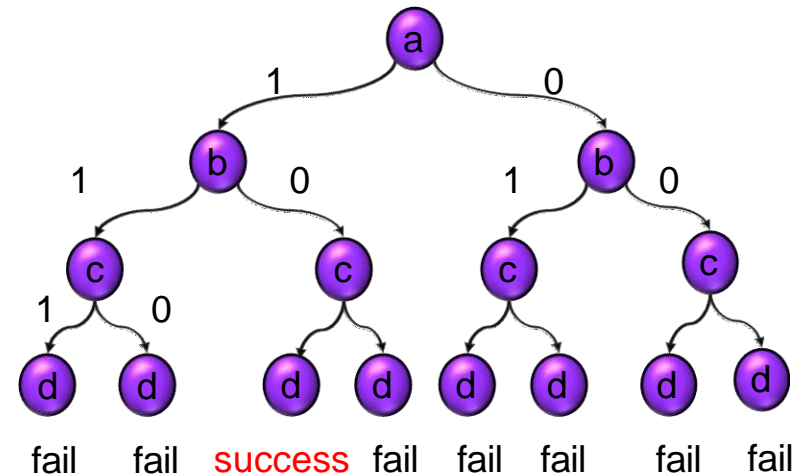
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation

- ◆ Boolean difference*
- ◆ Path sensitization**
- ◆ D-Algorithm (1965)**
- ◆ PODEM (1981)**
- ◆ FAN(1985) **
- ◆ SAT-based (1992)*

*Boolean-based methods
**path-based methods

- Acceleration Techniques
- Concluding Remarks



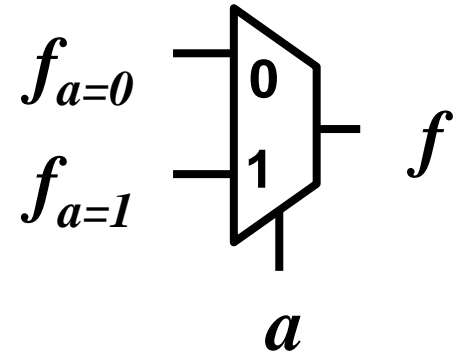
Decomposition

- Consider a circuit that realizes the function $f(a, b, c, \dots)$
- Fixed a to one:
 - ♦ *positive cofactor of f with respect to a*

$$f_{a=1} = f(a = 1, b, c, \dots)$$

- Fixed a to zero:
 - ♦ *negative cofactor of f with respect to a*

$$f_{a=0} = f(a = 0, b, c, \dots)$$

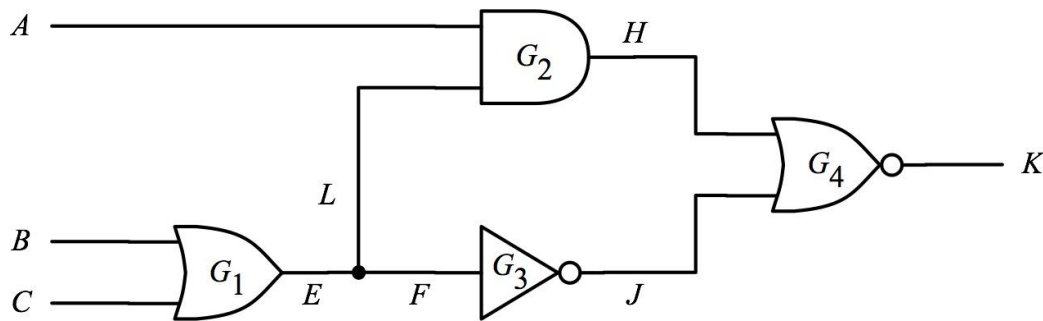


- *Shannon's Expansion* w.r.t. input a [Shannon 1948]

$$f = a f_{a=1} + a' f_{a=0}$$

Quiz

Q: Shannon's Expansion for output K w.r.t. input B
A:



Boolean Difference (1)

- To detect a stuck-at zero fault

- ♦ Good output (f) and faulty output ($f_{a=0}$) are different: $f \oplus f_{a=0} = 1$

- ♦ After Shannon Expansion: $[af_{a=1} + a'f_{a=0}] \oplus f_{a=0}$
 $= a[f_{a=1} \oplus f_{a=0}] + a'[f_{a=0} \oplus f_{a=0}]$
 $= a[f_{a=1} \oplus f_{a=0}] + 0$
 $= 1$

- ♦ Thus: $a[f_{a=0} \oplus f_{a=1}] = 1$

- To detect a stuck-at one fault

- ♦ Good output (f) and faulty output ($f_{a=1}$) are different: $f \oplus f_{a=1} = 1$

- ♦ After Shannon Expansion: $[af_{a=1} + a'f_{a=0}] \oplus f_{a=1} = 1$

- ♦ Thus: $a'[f_{a=0} \oplus f_{a=1}] = 1$

- Boolean difference of f w.r.t. a

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}]$$

Boolean Difference (2)

- Boolean Difference = 1 means $f_{a=0}$ and $f_{a=1}$ are different
 - ♦ which means: a is sensitized to output f

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 1$$

- Example: AND gate, when $b=1$, $f = A$

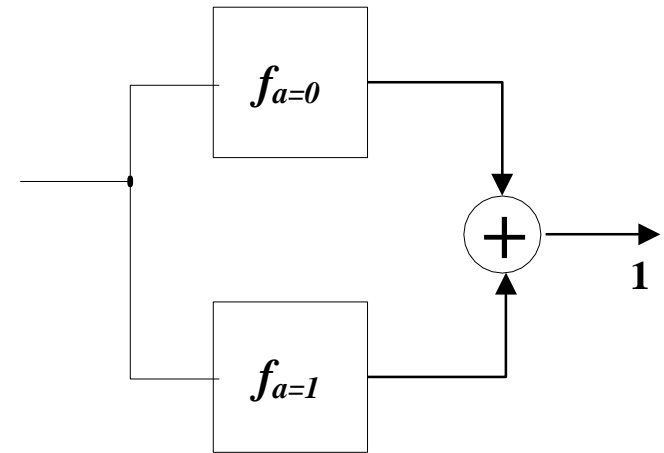
$$f = ab$$

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 0 \oplus b = b$$

- Example: OR gate, when $b=0$, $f = A$

$$f = a + b$$

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = b \oplus 1 = b'$$



BD=1: sensitization condition

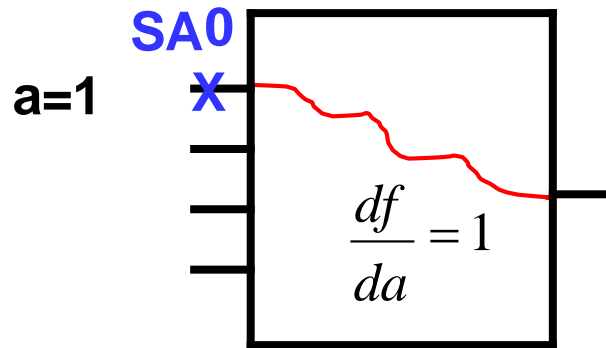
Boolean Difference (3)

- All patterns to detect a stuck-at-0 fault

- * $a = 1$: **fault excitation**

- * $BD = 1$: **sensitization** (aka **fault effect propagation**)

$$a \frac{df}{da} = 1$$



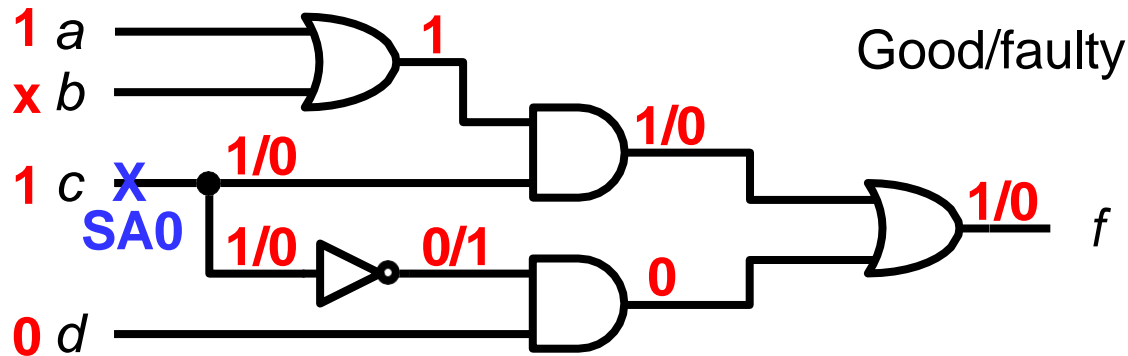
- All patterns to detect a stuck-at-1 fault

- * $a = 0$: **fault excitation**

- * $BD = 1$: **sensitization**

$$a' \frac{df}{da} = 1$$

Test Generation Example



- $f = (a + b)c + c'd$
- Set of all tests for c stuck-at-0 is $c \frac{df}{dc} = 1$

$$\frac{df}{dc} = f(a,b,0,d) \oplus f(a,b,1,d) = d \oplus (a + b) = ad' + bd' + a'b'd$$

$$c \frac{df}{dc} = acd' + bcd' + a'b'cd$$

- Set of all tests = {1x10, x110, 0011} (x = don't care inputs)
- One **fully specified test pattern**: e.g. 1110
- One **partially specified test pattern (aka. test cube)**: 1x10

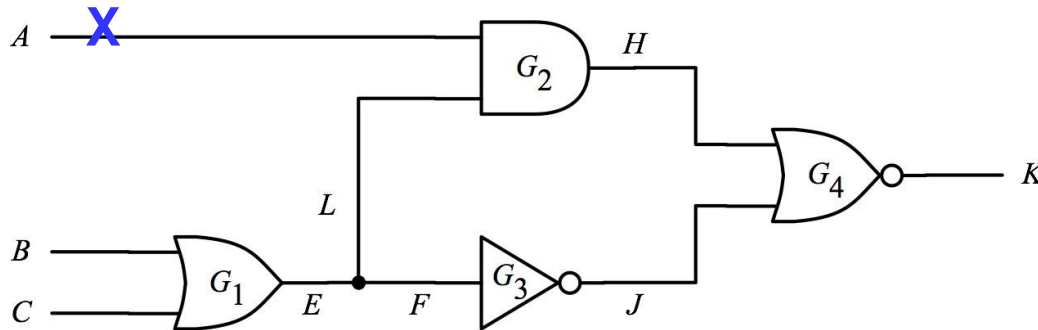
Quiz

(Cont'd) We already know $K = A'B'C + A'B$

Q1: Boolean difference $dK/dA = ?$

Q2: Use BD to find all test patterns for A stuck-at one fault.

SA 1



Internal Faults

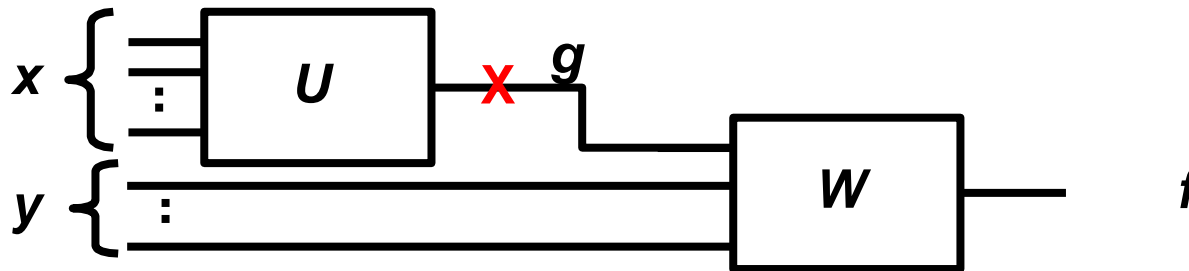
- Same approach also used for stuck-at faults internal faults
- Let g be internal signal of Boolean function f

$$f(x_1, x_2, \dots, y_1, y_2, \dots) = W(g, y_1, y_2, \dots)$$

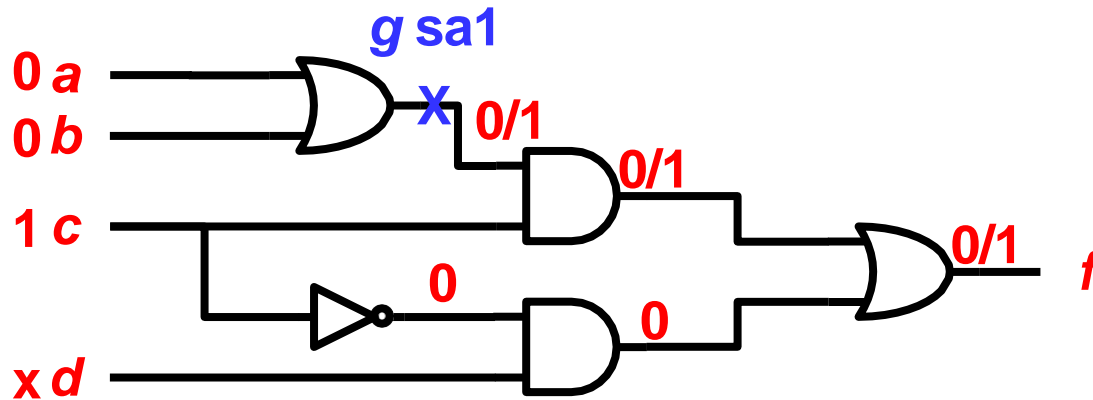
- Test sets for faults

♦ g SA0: $g \frac{dW}{dg} = 1$

♦ g SA1: $g' \frac{dW}{dg} = 1$



Example



$$f = (a + b)c + c'd \quad g = a + b$$

$$W = gc + c'd$$

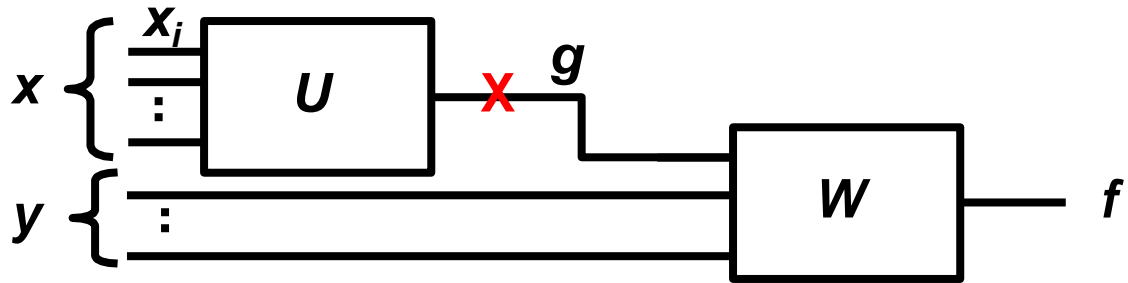
$$\frac{dW}{dg} = W_{g=0} \oplus W_{g=1} = c'd \oplus (c + c'd) = c$$

- Tests for g sa0
 $(a + b)c = ac + bc$
- Tests for g sa1
 $(a + b)'c = a'b'c$

Chain Rule

- Chain rule:

$$\frac{df}{dx_i} = \frac{df}{dg} \frac{dg}{dx_i}$$



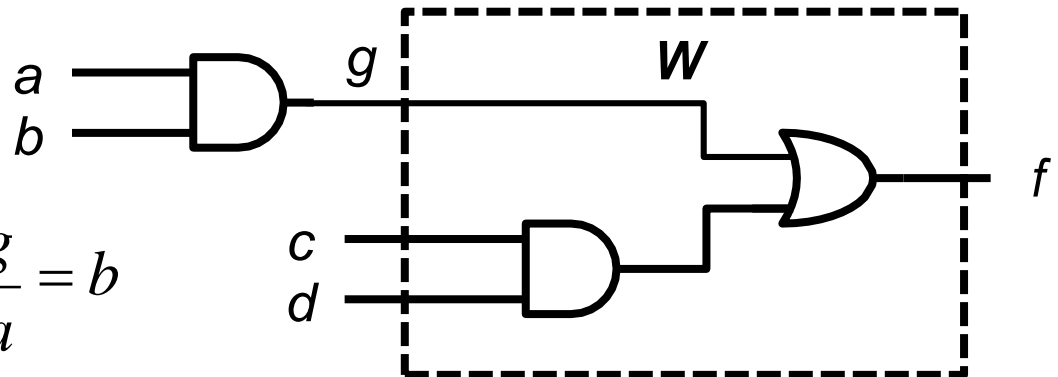
- Example

$$g = ab$$

$$W = g + cd$$

$$\frac{dW}{dg} = 1 \oplus cd = c' + d' \quad \frac{dg}{da} = b$$

$$\frac{df}{da} = \frac{dW}{dg} \frac{dg}{da} = (c' + d')b$$



Summary

- Deterministic Test Pattern Generation

- ◆ **BD=1 means sensitization condition**

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 1$$

- ◆ **Test patterns to detect a stuck-at-0 fault**

$$a \frac{df}{da} = 1$$

- ◆ **Test patterns to detect g stuck-at-0 fault**

$$g \frac{dW}{dg} = 1$$

- ◆ **Chain rule**

$$\frac{df}{dx_i} = \frac{df}{dg} \frac{dg}{dx_i}$$

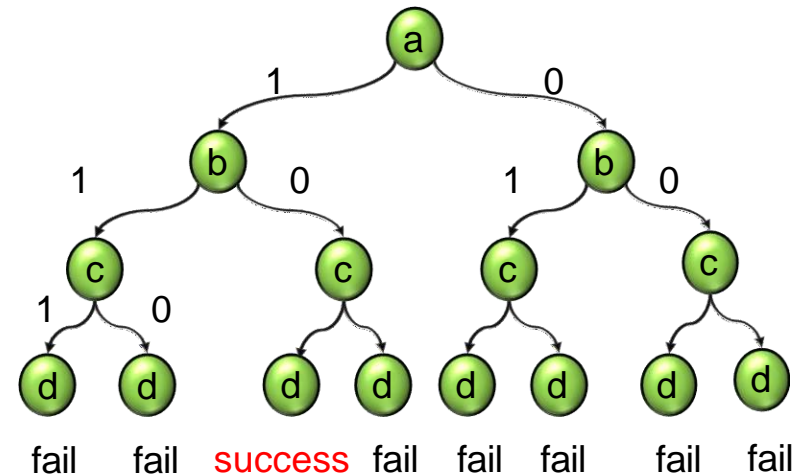
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
 - ◆ Boolean difference *
 - ◆ Path sensitization **
 - ◆ D-Algorithm**
 - ◆ PODEM**
 - ◆ FAN**
 - ◆ SAT-based *
- Acceleration Techniques
- Concluding Remarks

Two ATPG categories:

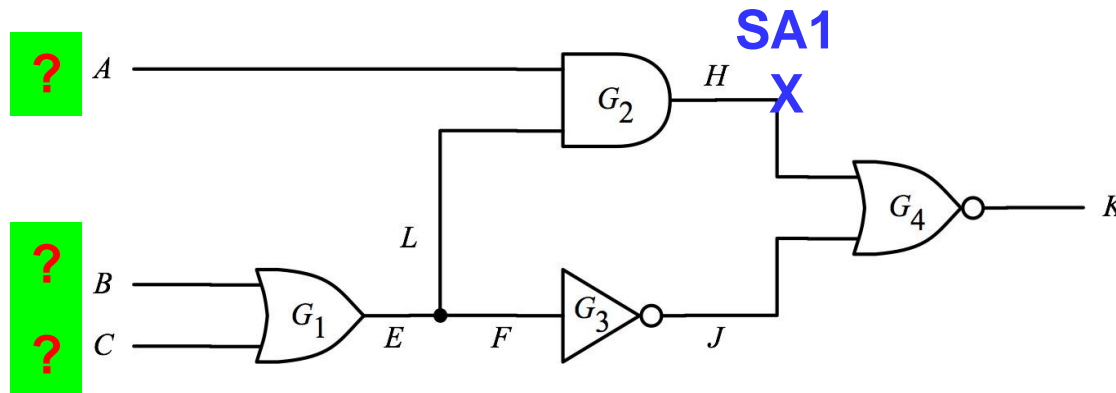
***Boolean-based methods**

****Path-based methods**



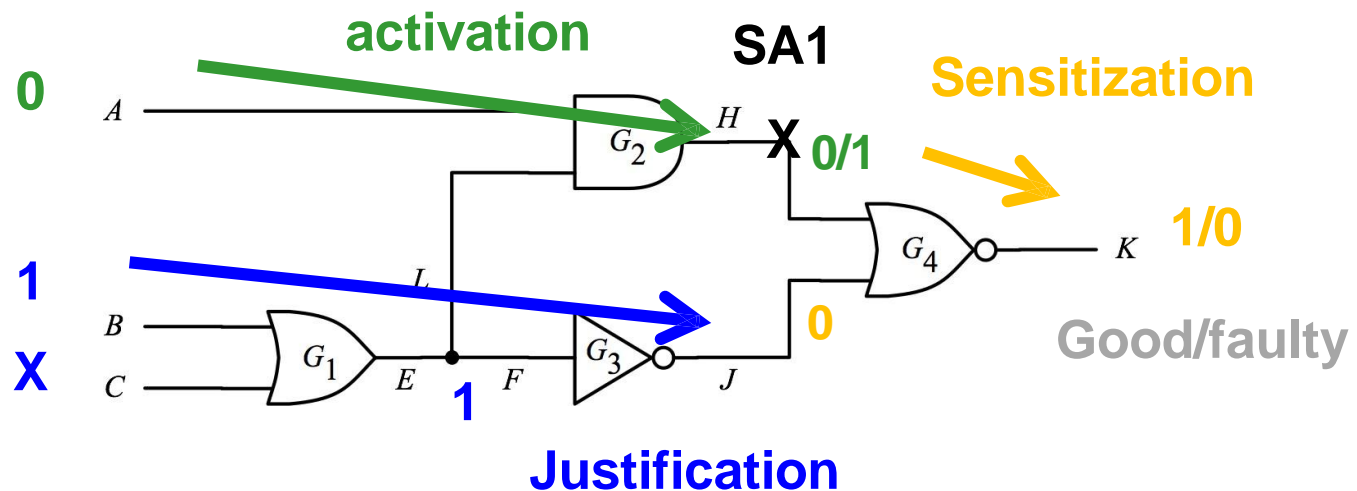
Motivating Problem

- We do not need to know Boolean expression
 - ♦ we can just find a test from circuit netlist



Let's Analyze What We Did

- **Fault activation:** Assign gate inputs to generate appropriate value at fault site (H)
 - ♦ $A=0$
- **Sensitization:** Assign side-inputs to non-controlling value to propagate fault effect forward
 - ♦ $J=0$
- **Justification:** Assign primary inputs to achieve desired values
 - ♦ $B=1$



Single Path Sensitization

- Single path sensitization (SPS) Algorithm:

- ① ***Fault activation*** (aka. ***Fault excitation***)

- Assign gate inputs to generate value at fault site

- Desired value opposite to the faulty value (e.g. 0 for SA1)

- ② ***Fault effect propagation***:

- Select **one single path** from fault site to an output

- Assign side inputs to sensitize fault effect along the path

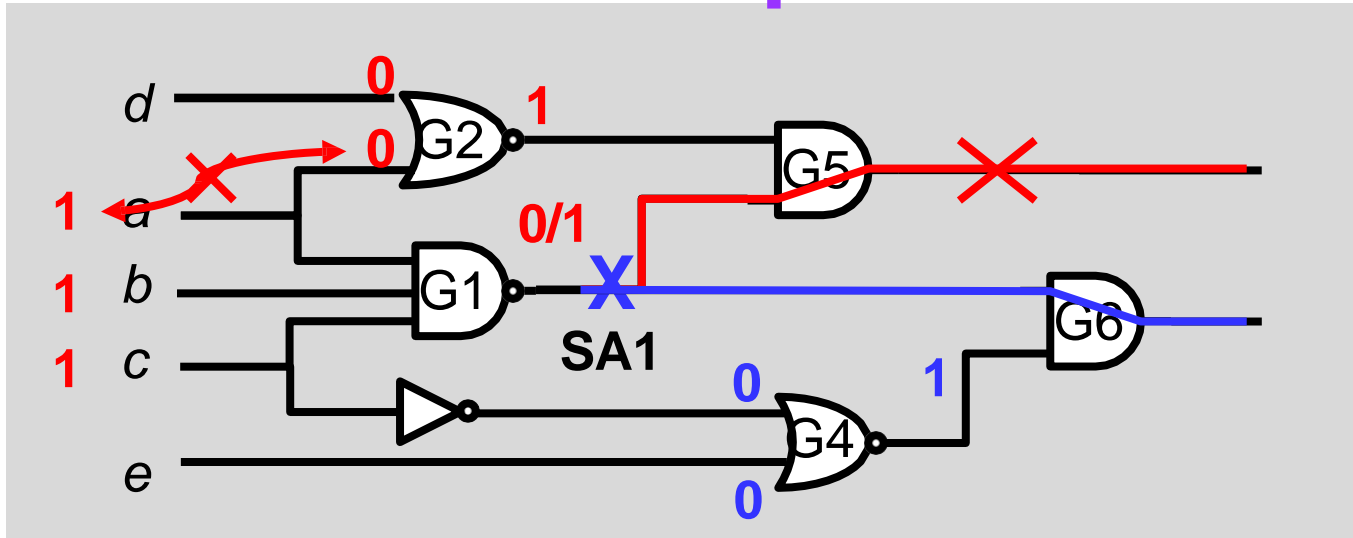
- ③ ***Justification***:

- Assign primary inputs to justify desired values assigned in

- ① & ②

- If justification fails, **backtrack**.

Example



Consider stuck-at-1 fault

① Fault activation

♦ $a = b = c = 1$

② Fault effect propagation: two propagation paths

♦ Choose path {G5}. Want $G2 = 1$

③ $a = d = 0 \rightarrow$ justification fails!

② Backtrack! Choose another path {G6}. Want $G4 = 1$

③ $c = 1, e = 0 \rightarrow$ justification succeeds

♦ test pattern: $abce'$ generated

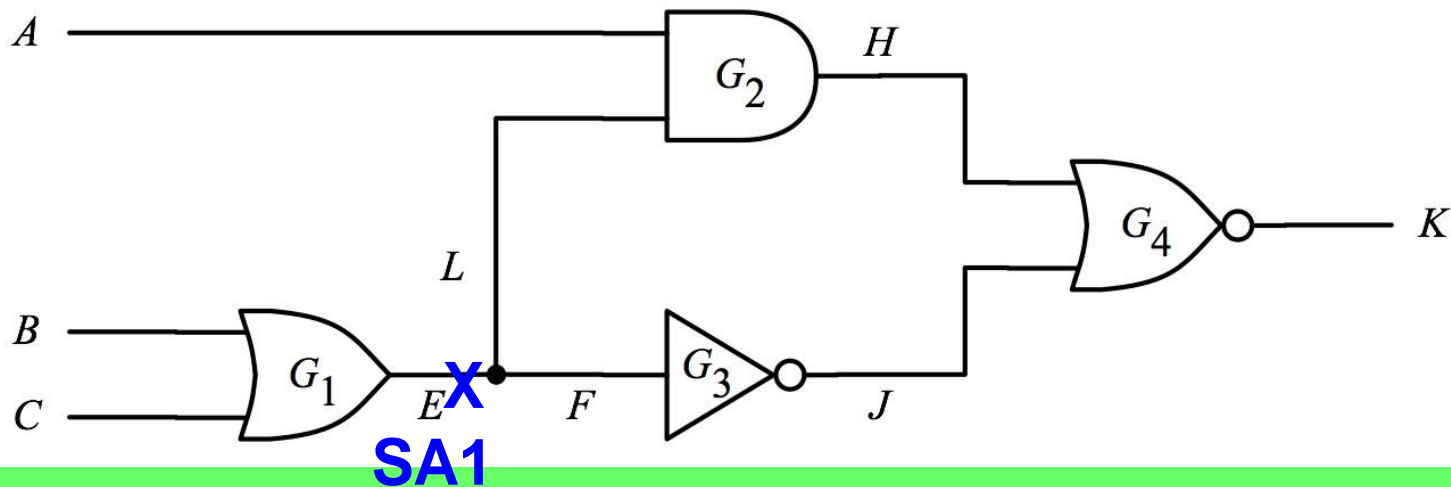
Quiz

Q1: Generate a test pattern for *E* SA1 fault. Choose path *ELHK*.

A:

Q2: (Cont'd) Backtrack to another path *EFJK*.

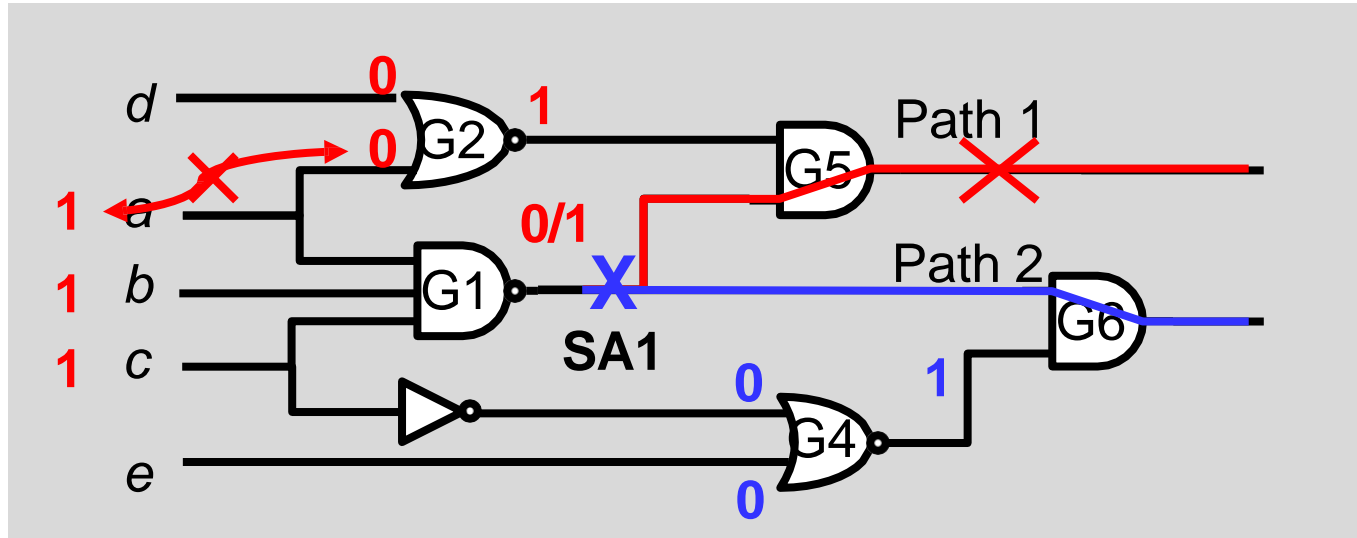
A:



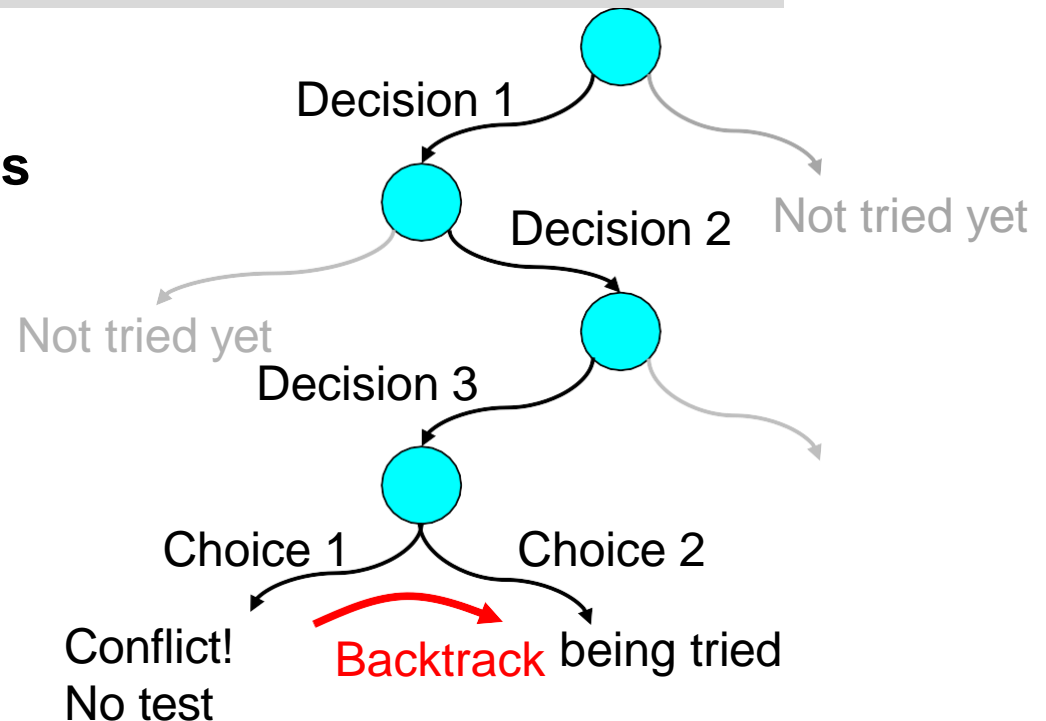
Important to Choose Correct Path

But it is difficult...

Backtrack



- When we made a mistake in decision tree, **conflict** occurs
 - ♦ Go back to a previous decision point
 - ♦ Change decision
 - ♦ Redo the rest



Pros and Cons

😊 Pros

- ◆ Easy to implement
- ◆ No Boolean equation needed

😞 Cons

- ◆ Q1: Too many paths to choose, which one is correct?
- ◆ Q2: Single-path sensitization not enough to detect all faults

- Single path sensitization (SPS) Algorithm:

- ① **Fault activation** (aka. **Fault excitation**)

Assign internal signals to generate value at fault site

Desired value opposite to the faulty value

- ② **Fault effect propagation:**

Select **one single path** from fault site to an output

Assign internal signals to sensitize fault effect along the path

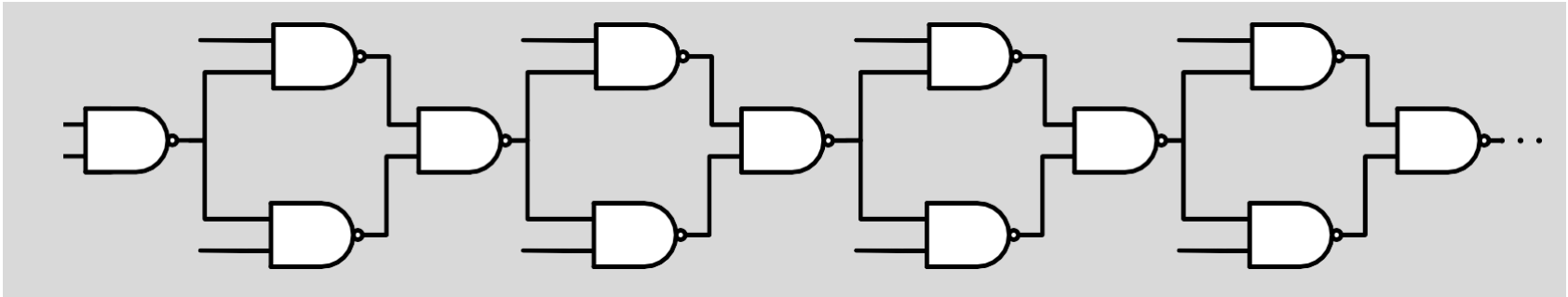
- ③ **Justification:**

Assign primary inputs to justify signal values assigned in ①&②

If justification fails, **backtrack**.

Q1: Too Many Paths !

- How to choose correct path?
 - ♦ No smart algorithm
 - ♦ Simple idea: exhaustively try all paths
- How many paths in a circuit?
 - ♦ Worst case example: $3n$ gates, 2^n paths! ($n = \#$ of stages)

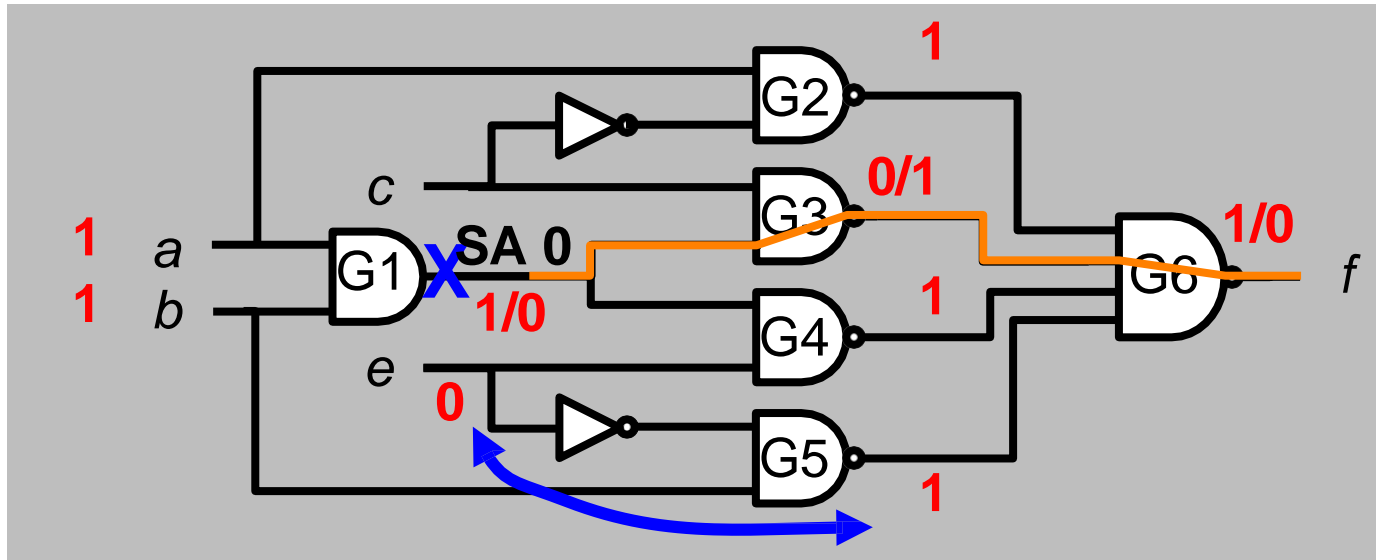


of Paths is Exponential to Circuit Size!
Impossible to Try All

Q2: Single Path Not Enough

- Example

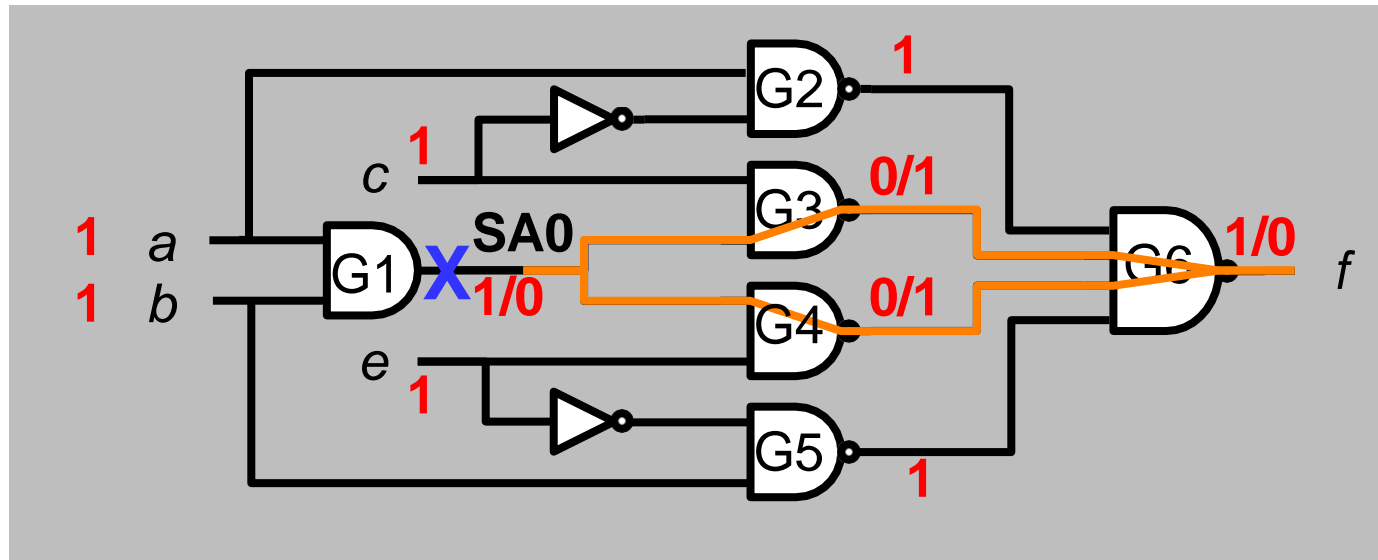
- ① Fault activation: $a = b = 1$
 - ② Fault propagation: Choose path **{G3-G6}**. want $G2 = G4 = G5 = 1$
 - ③ $G4 = 1 \rightarrow e = 0 \rightarrow G5 = 0 \rightarrow$ justification fails
 - ② Choose another path **{G4-G6}**. Justification also fails.
- ◆ SPS algorithm fails



This Fault Is Actually Testable. Why SPS Fail?

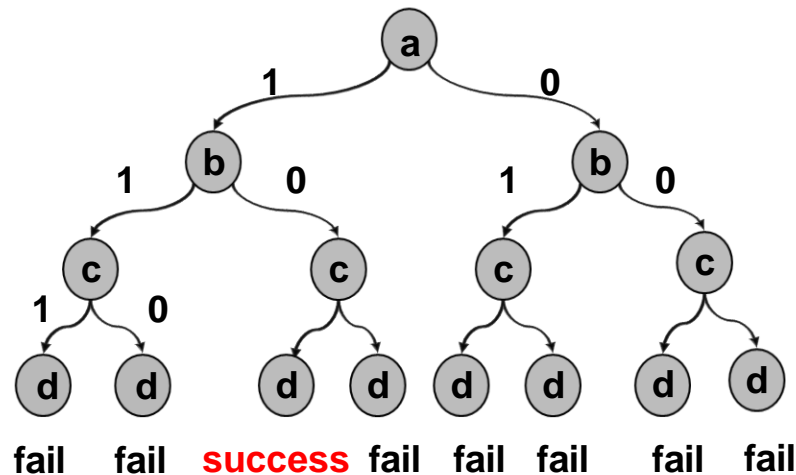
Solution: Multiple Path Sensitization

- This fault requires *multiple path sensitization*
- Both two paths {G3-G6} and {G4-G6} are sensitized
 - ♦ Error is propagated along both paths simultaneously
 - ♦ $G2 = G5 = 1$, $c = e = 1$
 - ♦ Test generated successfully
- FFT: can we extend 1-path to 2, 3, 4, 5...path sensitization?



Complete ATPG Algorithm

- A **complete** ATPG exhausts the whole input space (2^n)
 - ♦ If a test pattern exists, complete ATPG will find it **for sure**
- An **incomplete** ATPG does **NOT** exhaust the whole input space
 - ♦ ATPG may fail even though a test pattern **DOES** exist



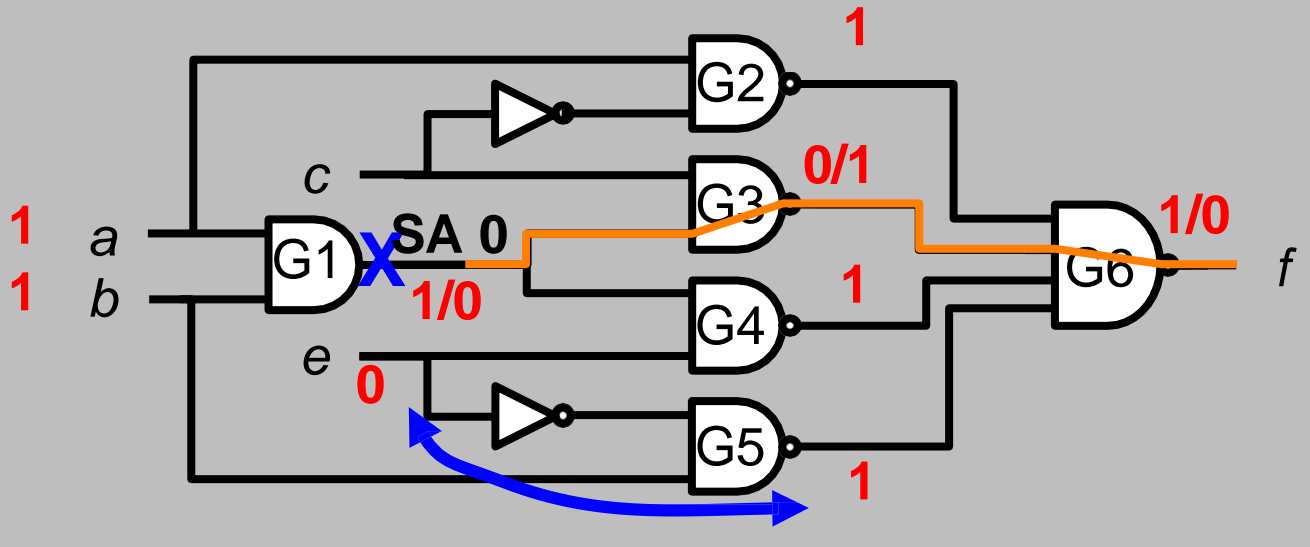
**Complete ATPG Guarantees to Find Solution
if it exists**

Quiz

Q: Is Single Path Sensitization a **complete ATPG algorithm**?

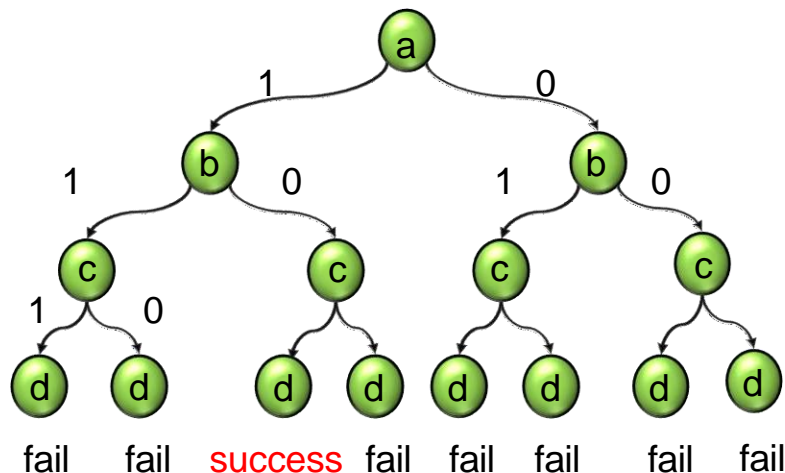
Hint: recall this example

- Single path sensitization (SPS) Algorithm:
 - ① **Fault activation** (aka. **Fault excitation**)
Assign internal signals to generate value at fault site
Desired value opposite to the faulty value
 - ② **Fault effect propagation**:
Select **one single path** from fault site to an output
Assign internal signals to sensitize fault effect along the path
 - ③ **Justification**:
Assign primary inputs to justify signal values assigned in ①&②
If justification fails, **backtrack**.



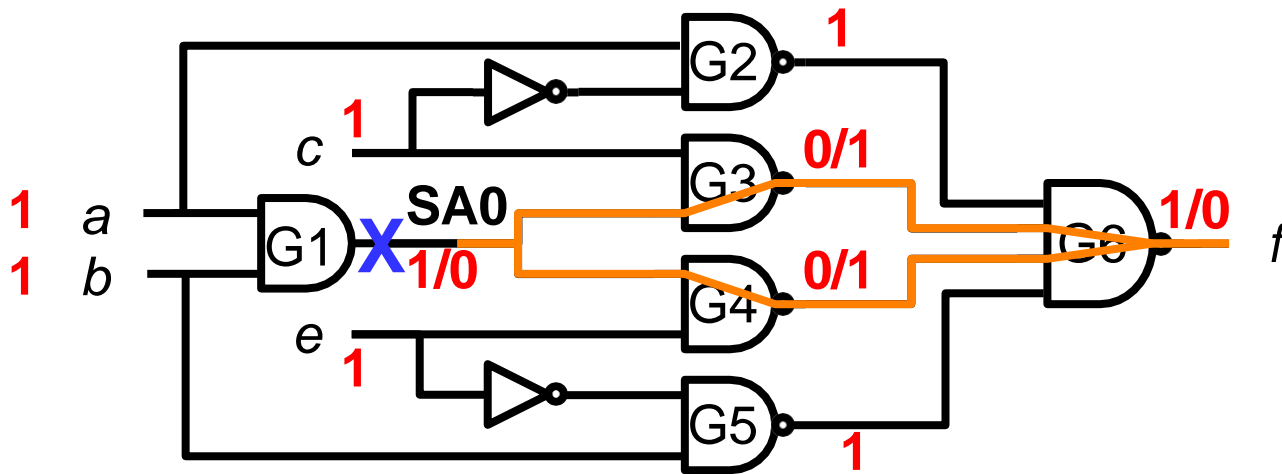
Summary

- **Single Path Sensitization Algorithm**
 - ① **Fault activation**
 - ② **Fault effect propagation**
 - ③ **Justification**
- **Complete ATPG**: guarantee to find solution if one exists
- **Disadvantages of SPS algorithm**
 - ◆ Number of paths is **exponential** to circuit size. Too many!
 - ◆ Single path sensitization is **incomplete** ATPG



FFT

- Q1: If SPS fails, can we extend 1-path to 2, 3, 4, 5... paths?
 - ♦ What is the problem for multiple path sensitization algorithm?
- Q2: Pros and Cons of complete ATPG algorithm?
 - ♦ in terms of run time and fault coverage



Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
 - ◆ Boolean difference *
 - ◆ Path sensitization **
 - ◆ D-Algorithm [Roth 1966] **
 - ◆ PODEM [Goel 1981]**
 - ◆ FAN [Fujiwara 1983]**
 - ◆ SAT-based [Larrabee 1992]*
- Acceleration Techniques
- Concluding Remarks

*Boolean-based methods

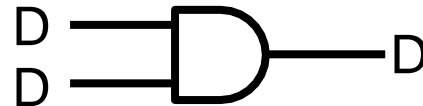
**path-based methods

D-Algorithm

- The D-algebra
- **An D-algorithm example**
- **Types of cubes**
- **Implication and Justification**
- **Flowchart of the D-algorithm**
- **Another example**
- **Problems with the D-Algorithm**

D-Algebra

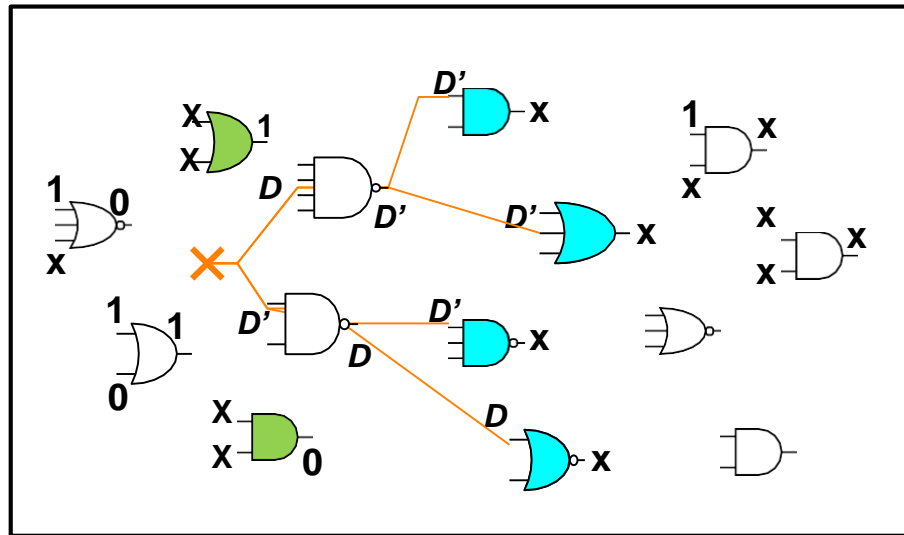
- **Five-valued logic: 1, 0, D, D', X**
 - ♦ Symbol D
 - ♦ $D = 1/0$
 - * 1 in fault-free circuit and 0 in the faulty circuit
 - ♦ $D' = \bar{D} = 0/1$
 - * 0 in fault-free circuit and 1 in the faulty circuit
 - ♦ x means “not yet specified” in ATPG



AND	0	1	D	\bar{D}	X
0	0	0	0	0	0
1	0	1	D	\bar{D}	X
D	0	D	D	0	X
\bar{D}	0	\bar{D}	0	\bar{D}	X
X	0	X	X	X	X

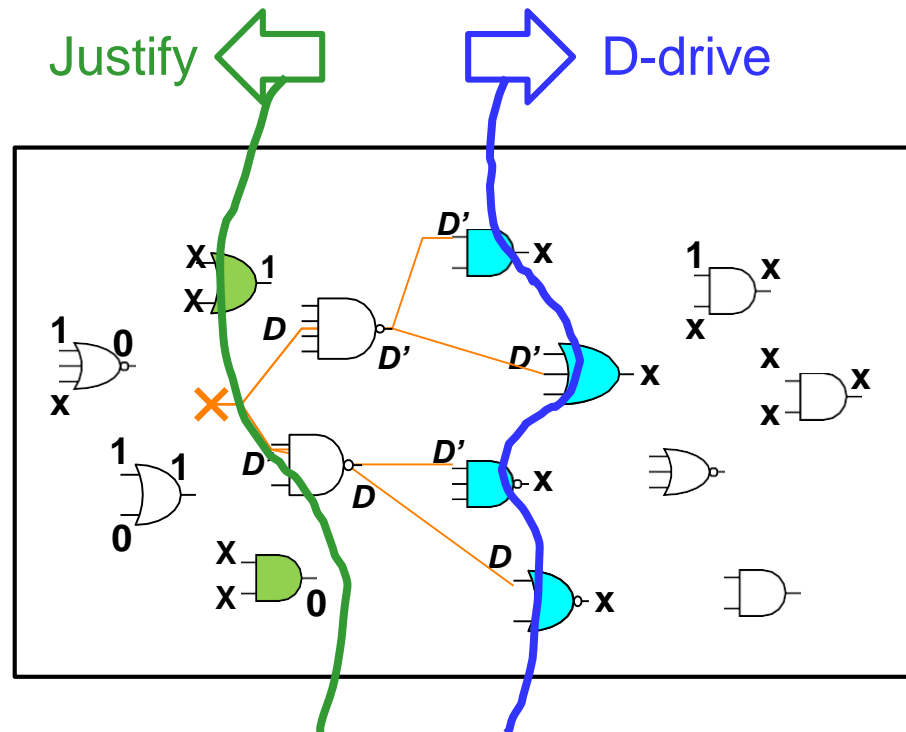
D-frontier , J-frontier

- **D-frontier** : a set of gates whose output value is currently x,
 - ♦ but have one or more D (or D') at their inputs
- **J-frontier**: a set of gates whose output value is assigned
 - ♦ But input values have not been decided yet
- Example
 - ♦ Blue gates are D-frontier; Green gates are J-frontier



Idea of D-Algorithm

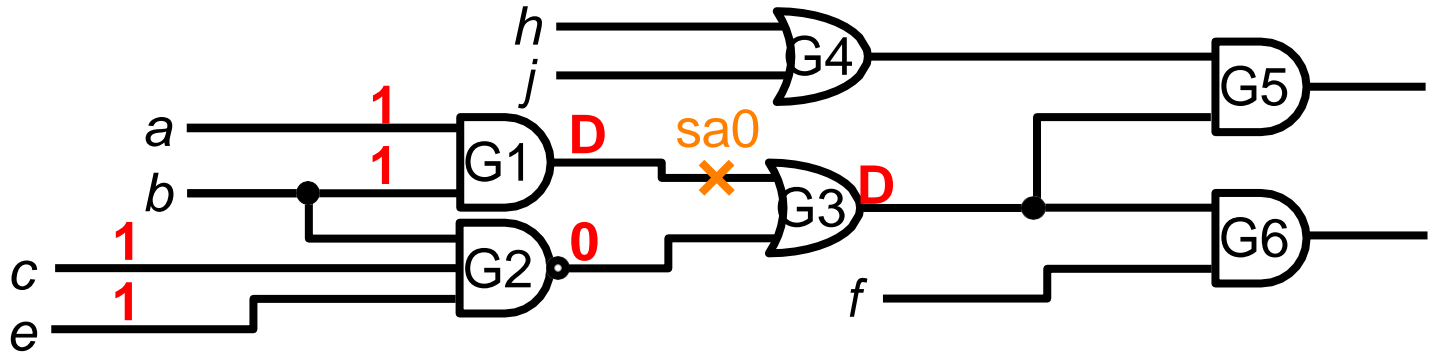
- 1. Create D-frontier (fault activation)
- 2. **Drive** D-frontier toward output (fault effect propagation)
- 3. **Justify** J-frontiers
- 4. **Backtrack** if any conflict occurs



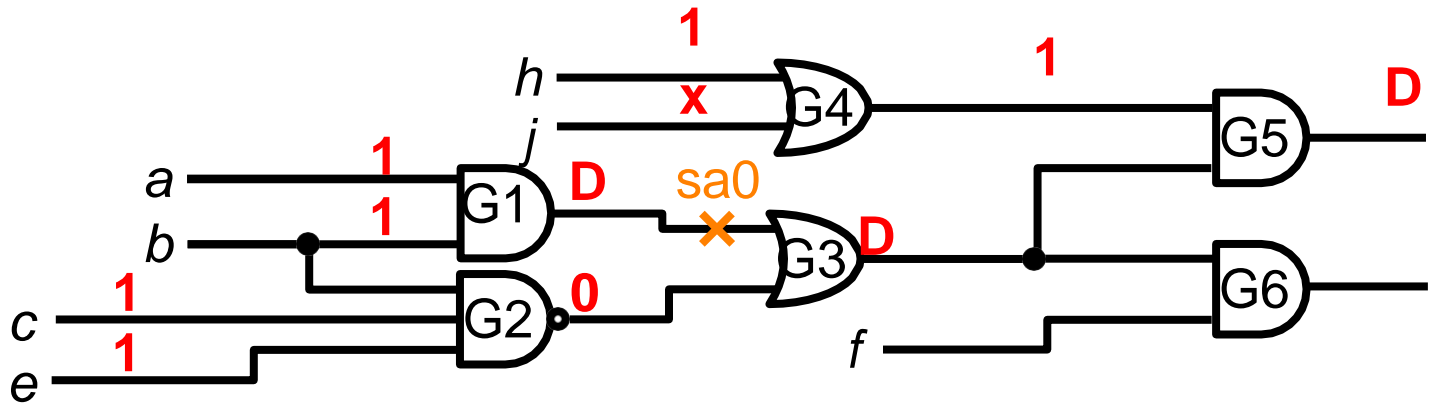
The D-Algorithm

- **The D-algebra**
- A D-algorithm example
- **Types of cubes**
- **Implication and Justification**
- **Flowchart of the D-algorithm**
- **Another example**
- **Problems with the D-Algorithm**

Example



	a	b	c	e	G ₁	G ₂	G ₃	h	j	G ₄	f	G ₅	G ₆	Comments
Initial test cube TC(0) = PDFC for G ₁ stuck-at 0	1	1			D									Fault Activation. Implication: nothing happen.
Prop. D-cube of G ₃ , PD _{G3}					D	0	D							D-frontier: {G3}
TC(1)=TC(0)∩PD _{G3}	1	1			D	0	D							D-Drive through G3
Singular Cover SC _{G2}		1	1	1		0								
7 TC(2)=TC(1)∩SC _{G2}	1	1	1	1	D	0	D							Backward implication



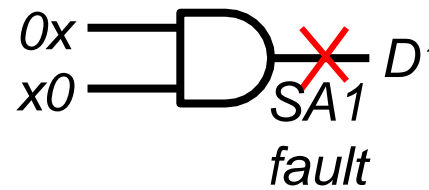
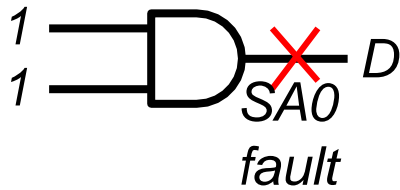
	a	b	c	e	G ₁	G ₂	G ₃	h	j	G ₄	f	G ₅	G ₆	Comments
TC(2)	1	1	1	1	D	0	D							D-frontier: {G ₅ ,G ₆ }
Propagation D-cube of G ₅ , PD _{G₅}							D			1		D		Choose path through G ₅
TC(3)=TC(2)∩PD _{G₅}	1	1	1	1	D	0	D			<u>1</u>		D		D-drive through G ₅ . D reach PO.
Singular Cover SC _{G₄}								1	X	1				Justification J-frontier = {G ₄ }
TC(4)=TC(3)∩SC _{G₄}	1	1	1	1	D	0	D	1	X	1	X	D	X	Done.

D-Algorithm

- The D-algebra
- An D-algorithm example
- Types of cubes
- Implication and Justification
- Flowchart of the D-algorithm
- Another example
- Problems with the D-Algorithm

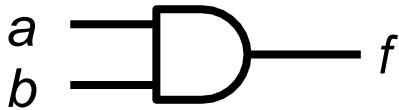
Primitive D-Cubes for a Fault (PDCF)

- Specify minimal input conditions
 - ◆ applied to gate input to produce error at gate output
 - ◆ Used in *fault activation* (more on this later)
- Example: AND gate output stuck-at faults:
 - ◆ Stuck-at-0 fault: 11D
 - ◆ Stuck-at-1 fault: 0xD' and x0D'



Singular Cover (SC)

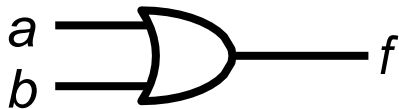
- Minimum gate input assignments for gate output =0 or =1
 - ♦ Used in line *justification* or *implication* (more on this later)



Singular covers of AND

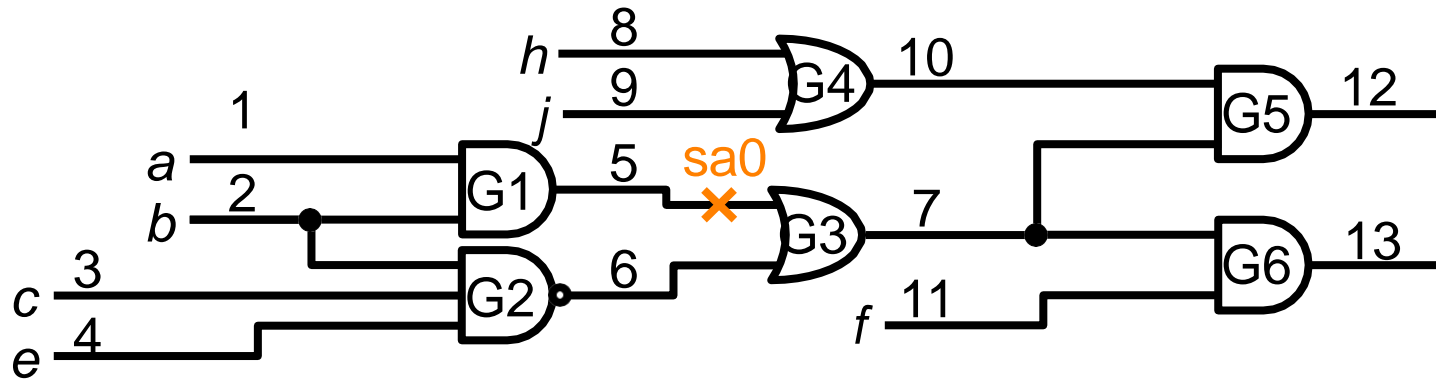
a	b	f
1	1	1
0	x	0
x	0	0

Singular covers of OR



a	b	f
0	0	0
1	X	1
X	1	1

Example



	a	b	c	e	G ₁	G ₂	G ₃	h	j	G ₄	f	G ₅	G ₆
Primitive D-cube for G1 sa0 fault	1	1			D								
Singular covers of G2		0	x	x		1							
		x	0	x		1							
		x	x	0		1							
		1	1	1		0							
Singular covers of G3					1	x	1						
					x	1	1						
					0	0	0						
Singular covers of G4								1	x	1			
								x	1	1			
								0	0	0			
Singular covers of G5							x			0		0	
							0			x		0	
							1			1		1	
Singular covers of G6							x				0		0
							0				x		0
							1				1		1

Propagation D-Cube (PDC)

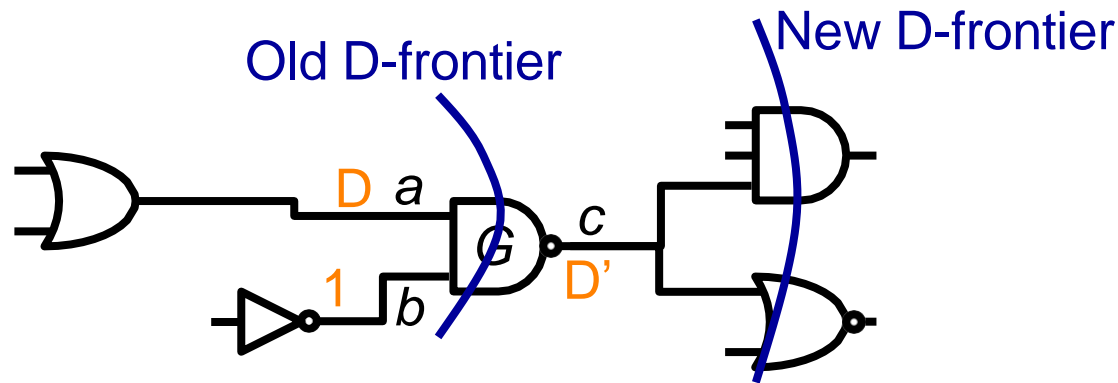
- Minimum gate input assignments required
 - ♦ to propagate a D or D' from gate input(s) to gate output
- Used in *D-Drive* or *implication* (more later)

<i>A</i>	<i>B</i>	<i>AB</i>
D	1	D
1	D	D
D	D	D
D'	1	D'
1	D'	D'
D'	D'	D'

<i>A</i>	<i>B</i>	$(A+B)'$
0	D	D'
D	0	D'
D	D	D'
0	D'	D
D'	0	D
D'	D'	D

D-drive

- D-drive selects an element in D-frontier
 - ◆ and attempts to propagate D or D' from gate input to gate output
 - ◆ Using propagation D-cube



	...	<i>a</i>	<i>b</i>	<i>c</i>	...
Test cube before D-drive	...	D	X	X	
Propagation D cube of gate <i>G</i>	...	D	1	D'	...
Test cube after D-drive	...	D	1	D'	...

The D-Algorithm

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Implication

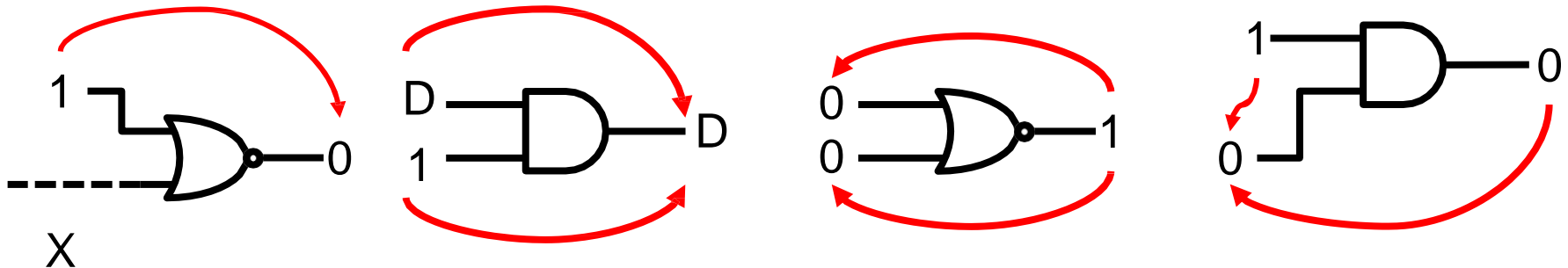
- **Forward Implication**

- ♦ partially (or fully) specified input values *uniquely* determines the output values.

- **Backward Implication**

- ♦ knowing the output values (and some input values) can *uniquely* determine the un-specified input values.

- **Examples**

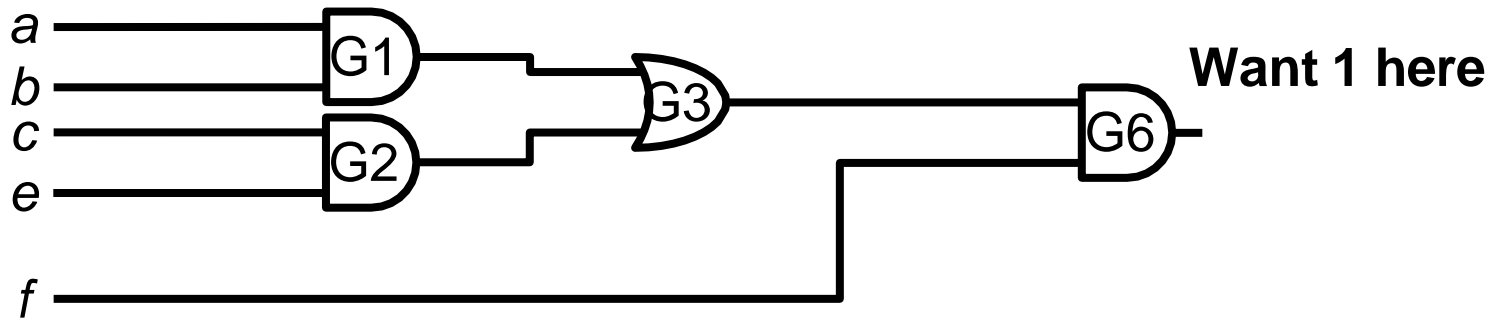


- **Note 1:** Implication means **NO choice**

- **Note 2:** Implication can be done any time a decision is made

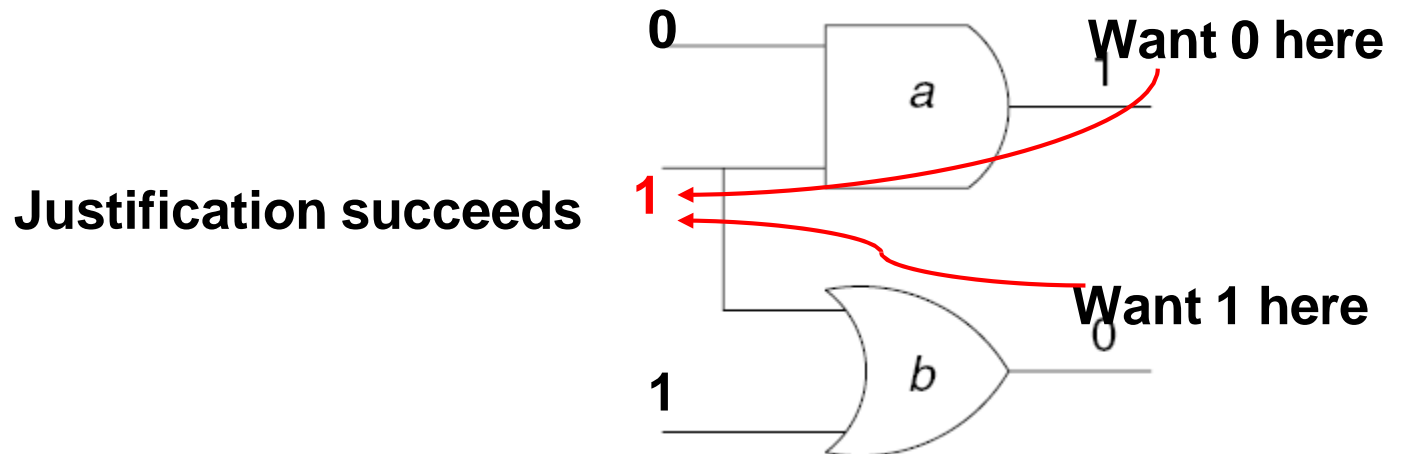
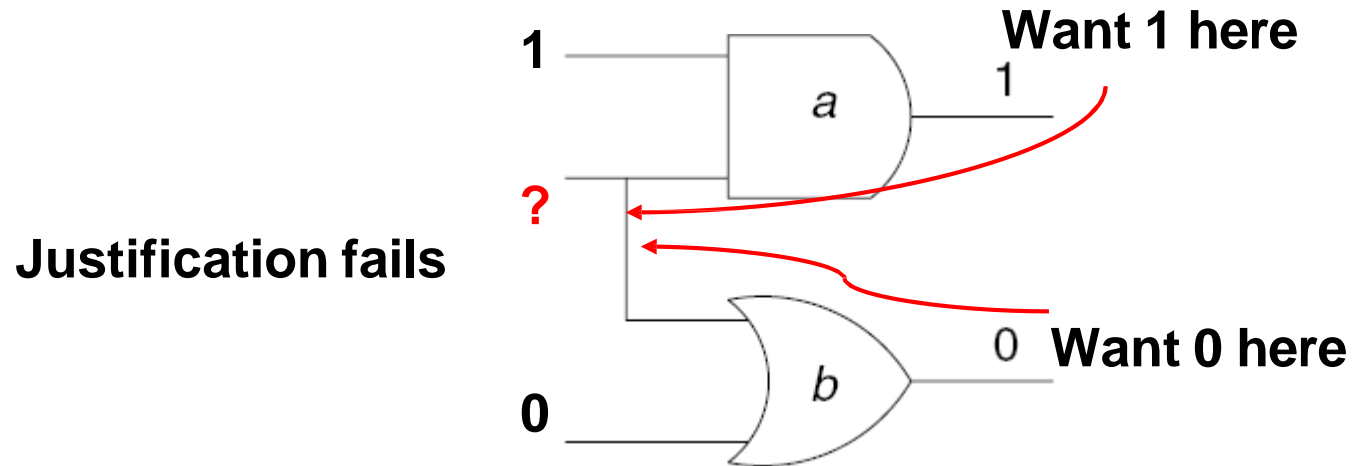
Justification

- Definition: find a valid primary input assignment for desired values
- Justification is easy inside a *fanout-free circuit*
- No decision needed
 - ♦ Always finds an answer
- Example



Justification (2)

- Justification may fail when there are **fanout branches**



The D-Algorithm

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Test Cube (TC)

- A **Test cube** is a partially specified Boolean values for testing a fault
 - In D algorithm, a test cube contains
 - ♦ not only primary inputs, but also internal nodes
 - Notation
 - ♦ $TC(n)$ = test cube at ATPG step n
-
- NOTE: In PODEM, a test cube contains primary inputs only
 - * More details later

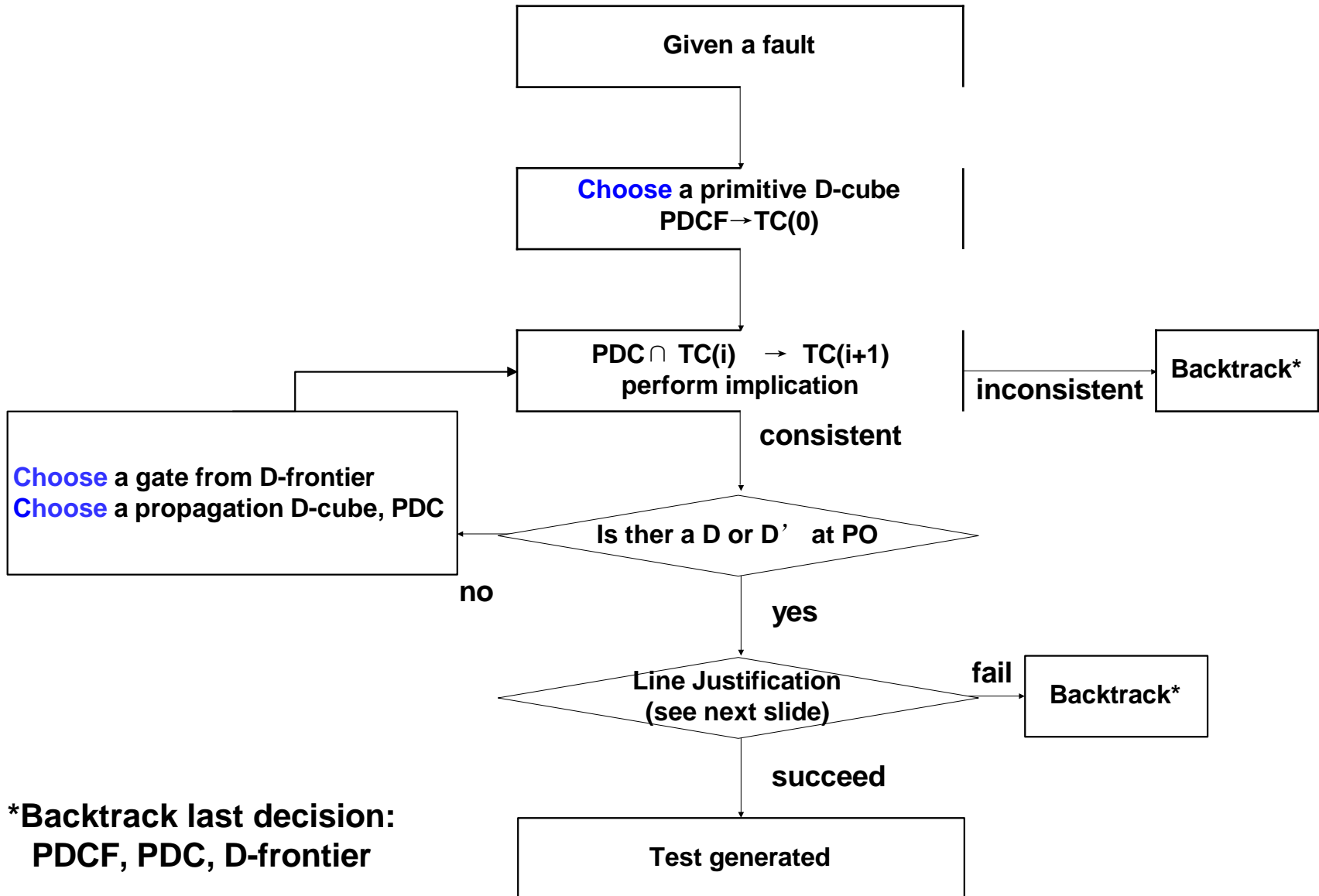
Intersection of Test Cubes

- Two bits has *intersection* if their logic values are not conflicting
- Two test cubes has intersection if there is no confliction in any bit
- Example :
 - ♦ $TC(1) \cap TC(2) = X0X1 \cap 1XXX = 10X1$
 - ♦ $TC(1) \cap TC(2) = 10X1 \cap 0XXX = \text{no intersection}$

Bit 1 \cap Bit 2					
bit2 \ bit1	0	1	X	D	D'
0	0		0		
1		1	1		
X	0	1	X	D	D'
D			D	D	
D'			D'		D'

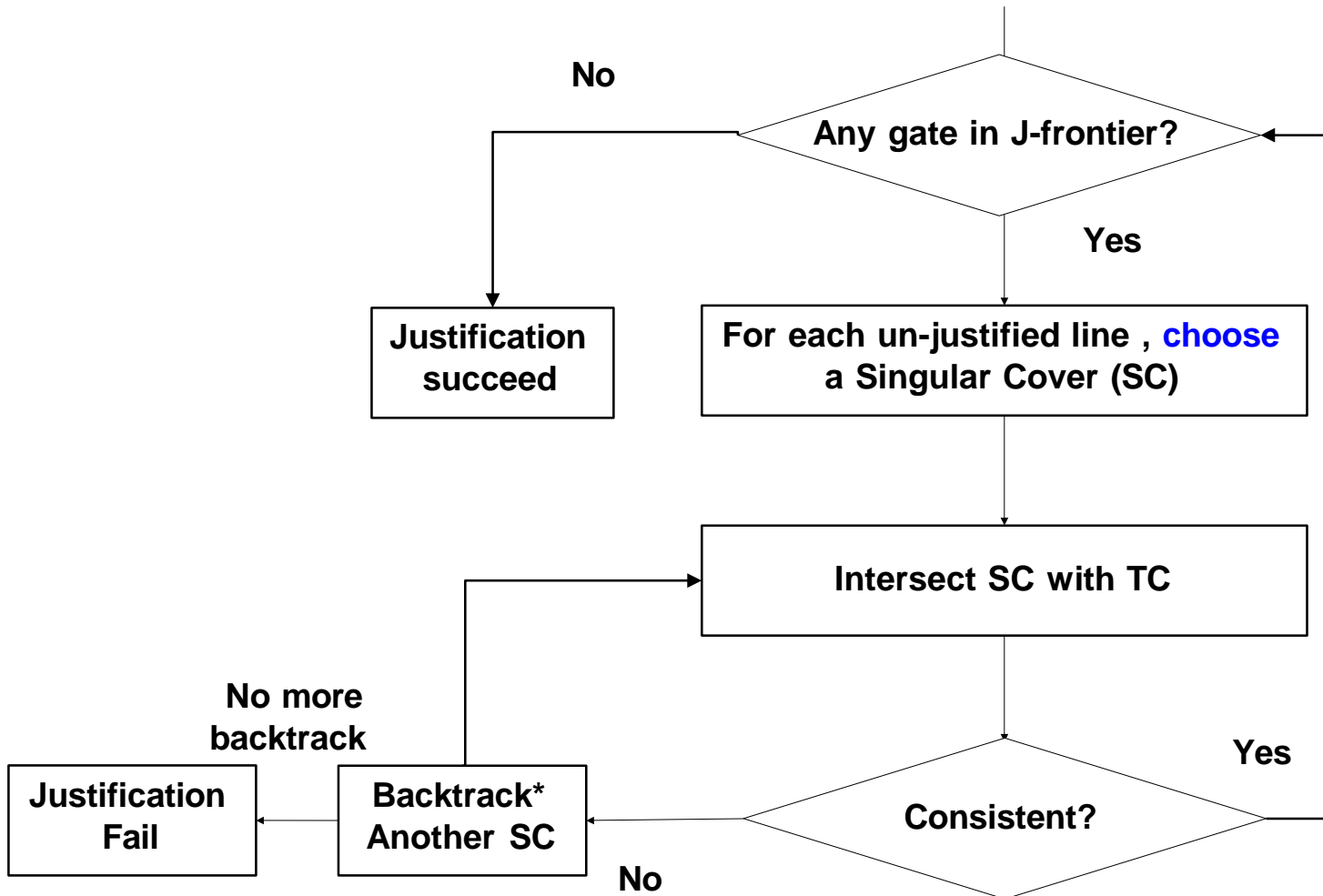
empty box = confliction = no intersection

Flowchart of the D-Algorithm



*Backtrack last decision:
PDCF, PDC, D-frontier

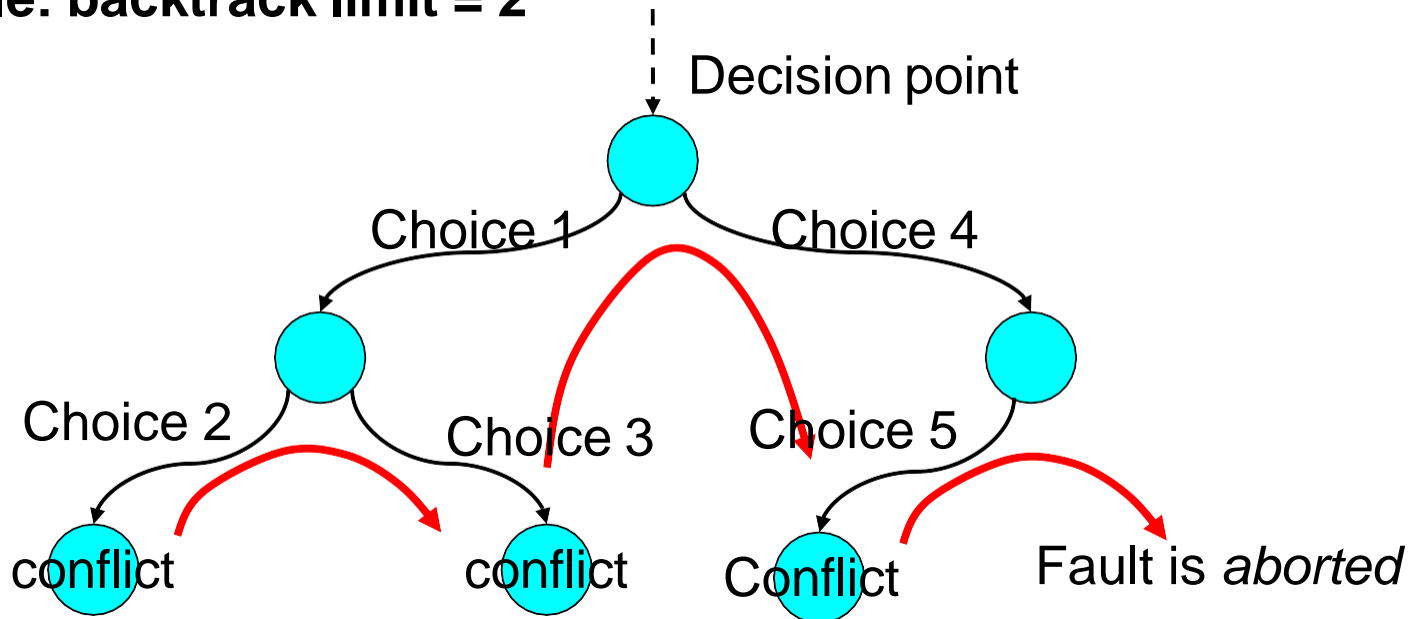
Line Justification



***backtrack last decision: SC**

Backtrack

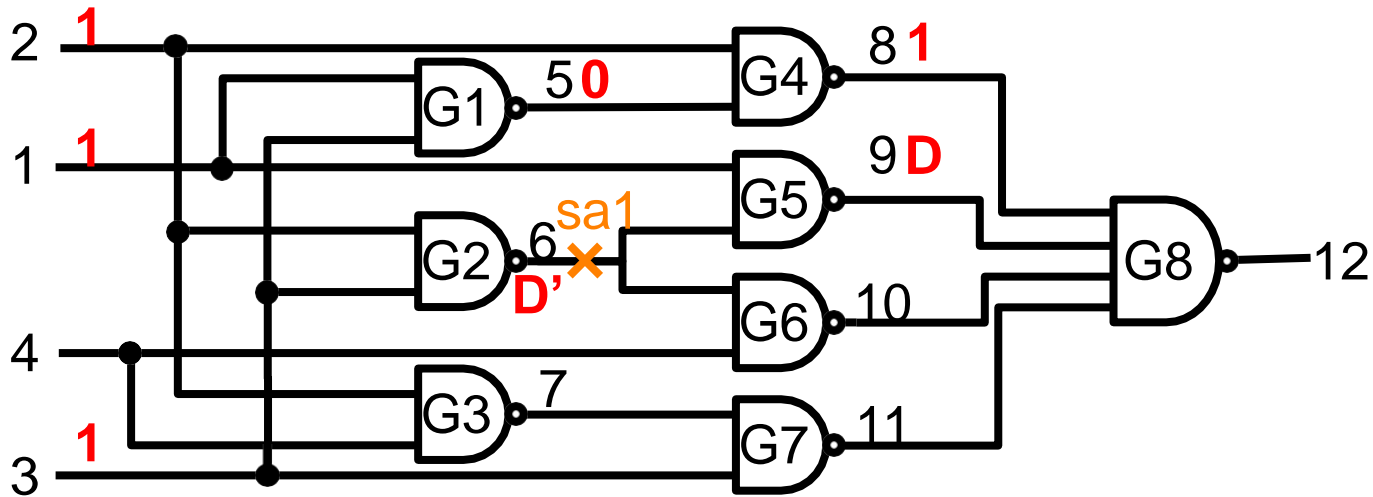
- When conflict, **backtrack** to last decision point and change choice
 - ♦ Choice can be: PDCF, PDC, SC, D-drive gate
- To avoid spending too much time on a fault
 - ♦ Use specify a **backtrack limit**
 - * maximum number of backtracks allowed for a single fault
 - ♦ Fault is **aborted** if backtrack limit is reached
- Example: backtrack limit = 2



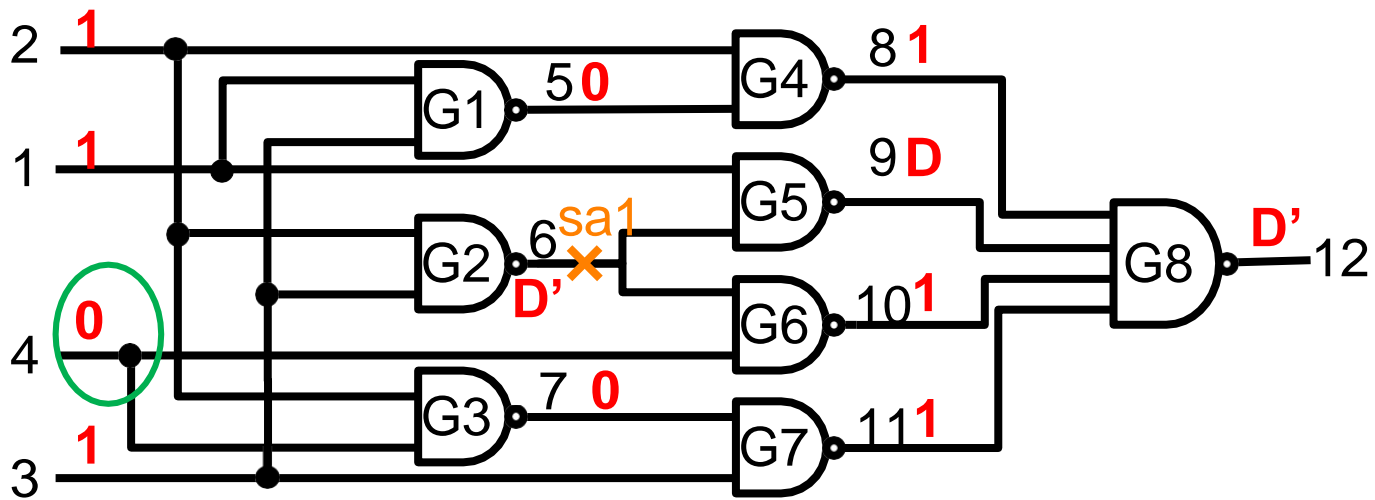
The D-Algorithm

- The D-algebra
- A D-algorithm example
- Types of cubes
- Components of the D-algorithm
- Flowchart of the D-algorithm
- Another example
- Problems with the D-Algorithm

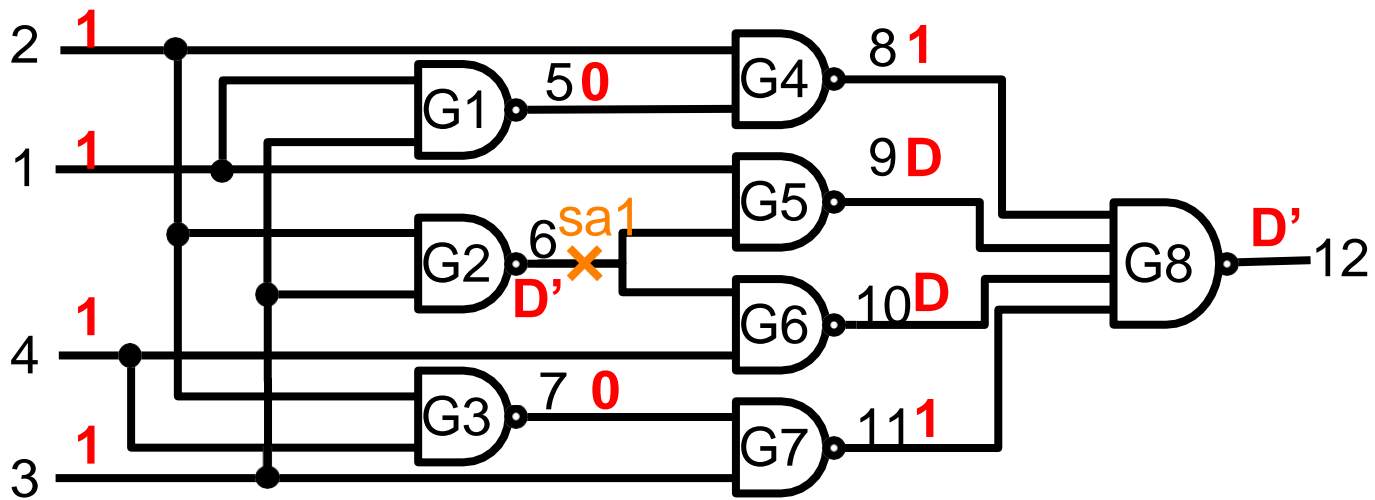
Another D-Algorithm Example



		1	2	3	4	5	6	7	8	9	10	11	12
1	PDCF:TC(0) = PDFC		1	1			D'						
2	Implication: nothing happens		1	1			D'						
3	choose D-frontier G5: PDC TC(1)=TC(0) ∩ PDC	1 1	 1	 1	 1	 	D' D'			D D			
4	Forward Implication: SC _{G1} TC(2)	1 1	 1	1 1	 1	0 0	 D'			 D			
5	Forward Implication: SC _{G4} TC(3)	 1	X 1	 1	 1	0 0	 D'		1 1	 D			



		1	2	3	4	5	6	7	8	9	10	11	12
	TC(3)	1	1	1		0	D'		1	D			
6	choose D-frontier G8: PDC TC(4)	1	1	1		0	D'		1	D	1	1	D'
7	Backward Implication G ₇ : TC(5)	1	1	1		0	D'	0	1	D	1	1	D'
8	Backward Implication G ₆ : SC _{G6} TC(6)	1	1	1	0	0	D'	0	1	D	1	1	D'
9	Backward Implication G ₃ : SC _{G3} Fail. Backtrack step to 6		1		1			0					



		1	2	3	4	5	6	7	8	9	10	11	12
	Backtrack to step 6. TC(3)	1	1	1		0	D'		1	D			
10	Choose D-frontier G6: PDC				1		D'				D		
	TC(4)	1	1	1	1	0	D'		1	D	D		
11	Forward Implication: SC_{G3}		1		1			0					
	TC(5)	1	1	1	1	0	D'	0	1	D	D		
12	Forward Implication: SC_{G7}			1				0				1	
	TC(6)	1	1	1	1	0	D'	0	1	D	D	1	
13	Forward Implication: SC_{G8}								1	D	D	1	D'
	TC(7)	1	1	1	1	0	D'	0	1	D	D	1	D'
14	No justification needed.												
	Test generated.	1	1	1	1	0	D'	0	1	D	D	1	D'

The D-Algorithm

- The D-algebra
- A D-algorithm example
- Types of cubes
- Components of the D-algorithm
- Flowchart of the D-algorithm
- Another example
- Plus & Minus of D-Algorithm

Plus/Minus of D-Algorithm

- + D algorithm is *complete ATPG*
 - ♦ Guarantee to generate a pattern for a testable fault
- Large search space
 - ♦ Assignment of values is allowed for internal signals
 - ♦ Backtracking could occur at each gate
 - ♦ Very large search space

FFT

- **Q1. Why we do justification at the end of D-algorithm**
 - ♦ **Why not immediately after implication?**
- **Q2. Please explain why D-algorithm is complete ATPG**

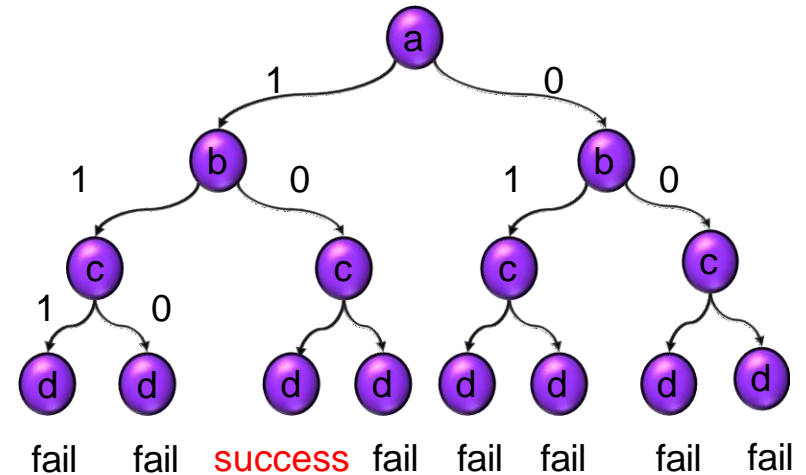
Combinational ATPG

- **Deterministic Test Pattern Generation**

- ◆ Boolean difference approach*
- ◆ Path sensitization method**
- ◆ D-Algorithm [Roth 1966] **
- ◆ PODEM [Goel 1981]**
 - * Idea
 - * Heuristics
 - * Algorithms
 - * Summary
- ◆ FAN [Fujiwara 1983]**
- ◆ SAT-based [Larrabee 1992]*

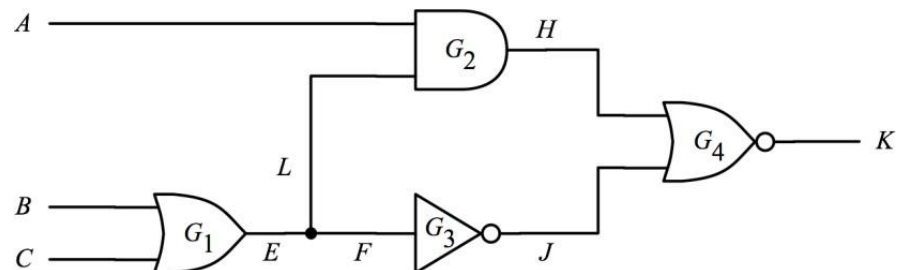
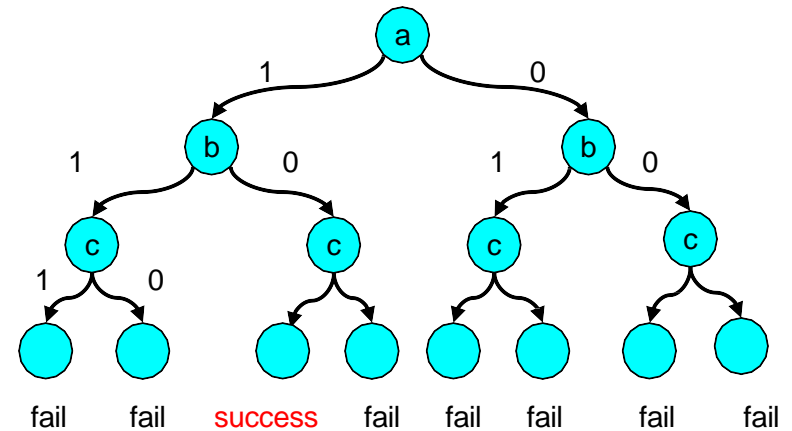
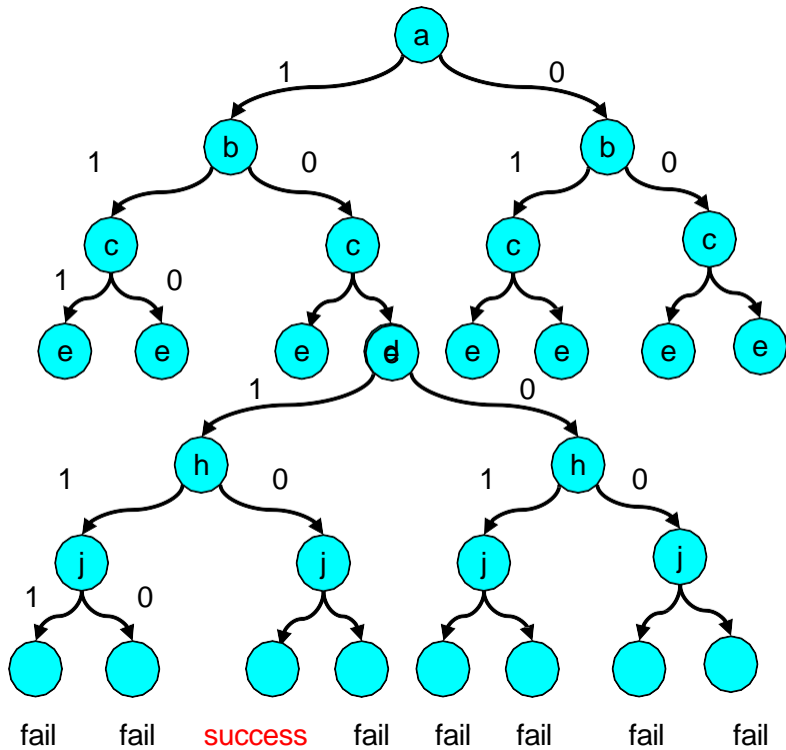
*Boolean-based methods

**path-based methods



$$2^6 \rightarrow 2^3$$

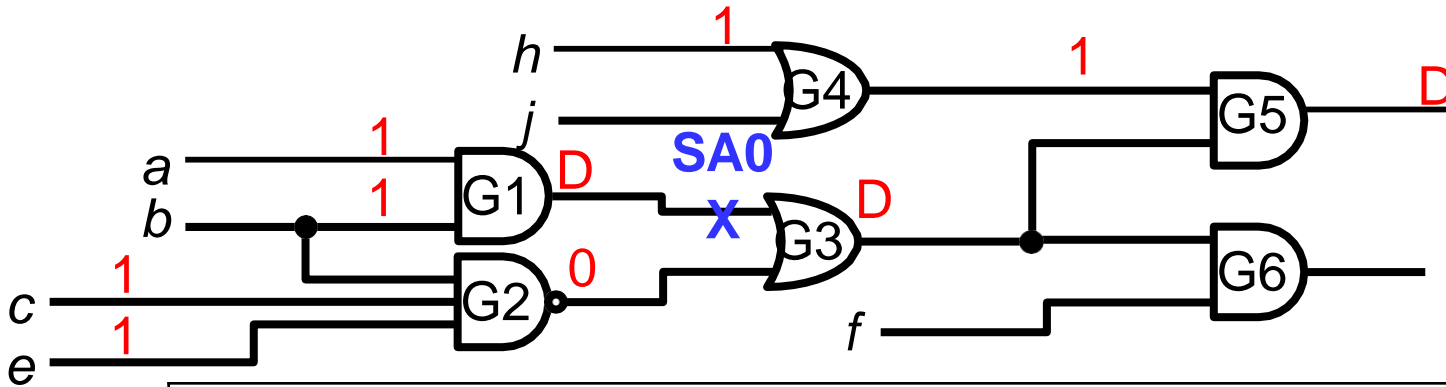
- D-alg. makes decision at **intern nodes**
 - ♦ 6 internal nodes
 - ♦ $2^6=64$ decisions! large
- PODEM makes decision at **PI**
 - ♦ 3 PI
 - ♦ $2^3=8$ decisions, smaller



PODEM [Goel 1981]

- *Path Oriented Decision Making (PODEM)*
- IDEAS:
 1. Only allow assignments to *PI only*
 - ♦ Doesn't assign internal nodes
 - ♦ Greatly reduces search tree
 2. Assigned PI are then *forward implication*
 - ♦ No justification needed (Why ? FFT)
 3. Flip last PI assignment when two conditions:
 - ♦ A. *Fault not activated*
 - ♦ B. *No propagation path* to any output

PODEM Example



Initial objective: $G_1 = 1$

Backtrace to PI: $b = 1$. simulation, objective not achieved

Backtrace to PI: $a = 1$. simulation, objective achieved

Objective: $G_2 = 0$ (propagate through G_3)

Backtrace to PI: $C = 1$. simulation, objective not achieved

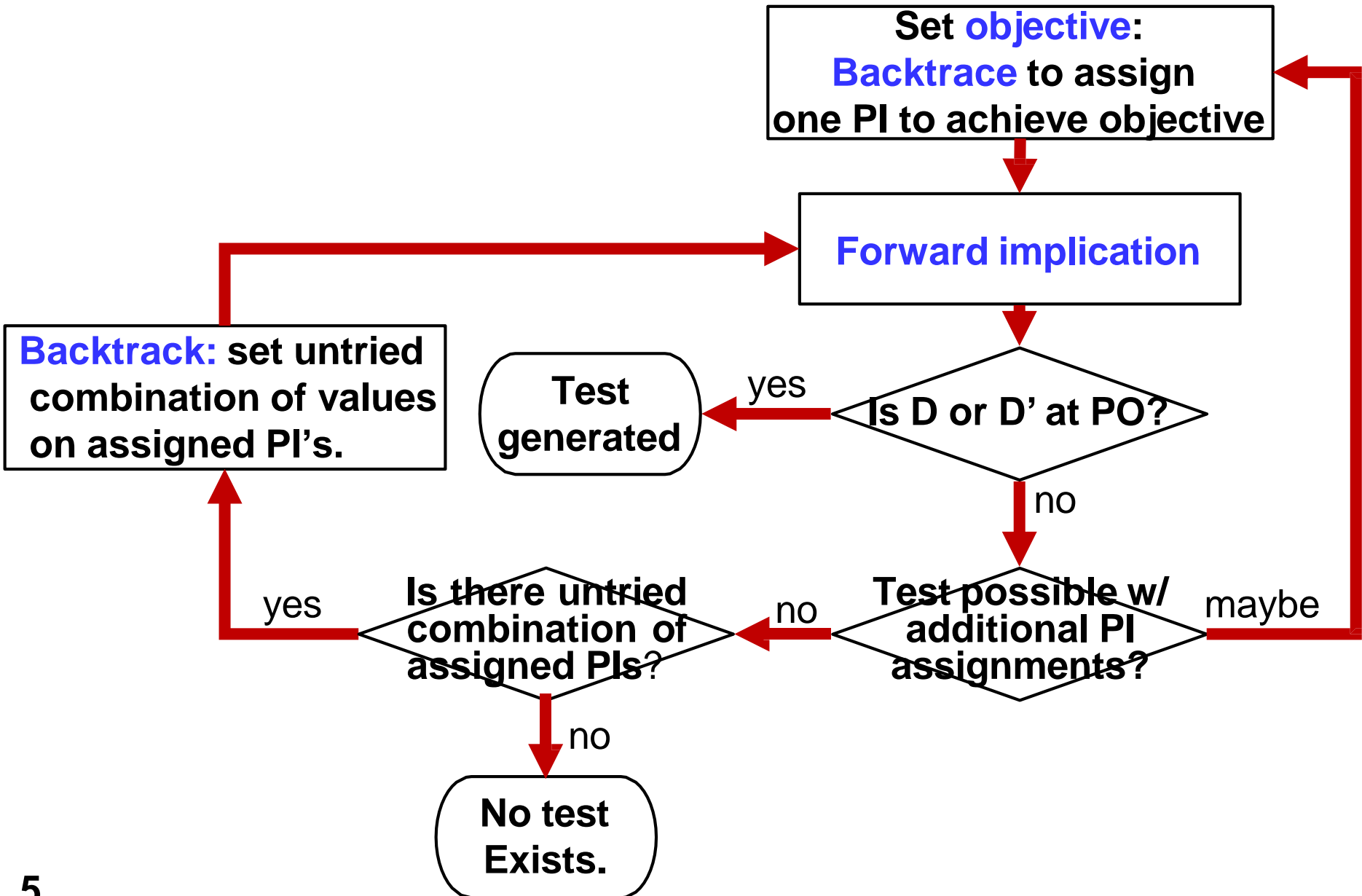
Backtrace to PI: $e = 1$. simulation, objective achieved

Objective: $G_4 = 1$ (we choose to propagate through G_5)

Backtrace to PI: $h = 1$. objective achieved.

Test Generated: $abcehjf=11111XX$

Flowchart of PODEM

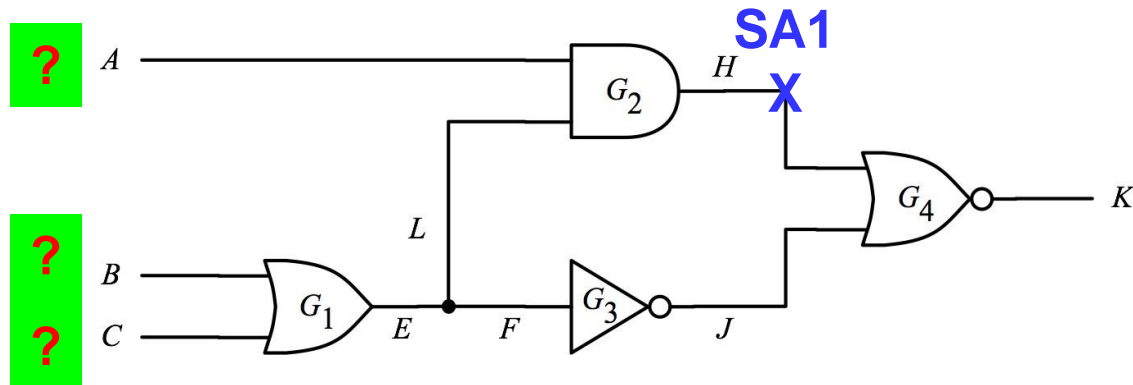


Components of PODEM

- Determine an *objective*
 - ◆ If D/D' has not appeared at the fault site,
 - * set objective to *activate fault*
 - ◆ If D/D' has appeared
 - * set objective to *propagate fault effect*
- Given objective, determine PI value
 - ◆ *Backtrace to PI*
 - ◆ *Backtrack* if conflict occurs
- NOTE: *Backtrace* is different from *backtrack*
 - ◆ Backtrace goes back to primary inputs of a certain signal
 - * In netlist
 - ◆ Backtrack goes back to last decision
 - * in decision tree

Quiz

**Q: Use PODEM to generate a test for H SA 1 fault
please mark your**
1. objective,
2. backtrace

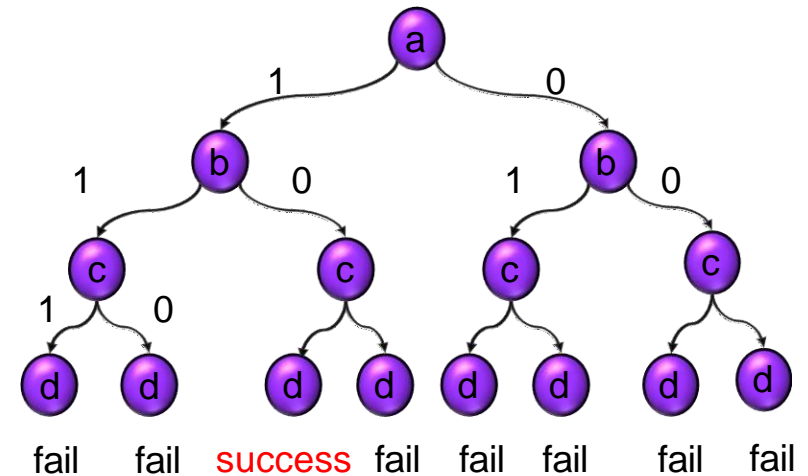


Totally 2 objectives, 2 backtraces

PODEM

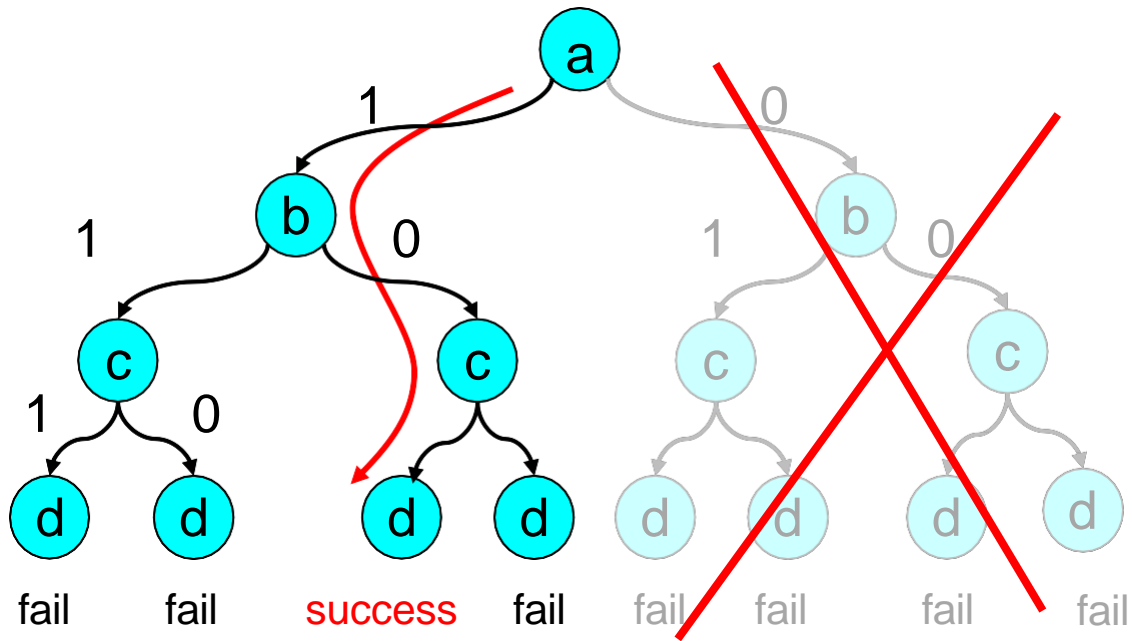
- PODEM [Goel 1981]

- ◆ Idea
- ◆ Heuristics
- ◆ Algorithms
- ◆ Summary



ATPG Decision Tree

- Need smart heuristics to speed up
 - ♦ 1. Prune **impossible sub-trees** ASAP
 - ♦ 2. Find **good assignments** ASAP
- Heuristics are experience-based rules that help correct decision
 - ♦ Note: Heuristics **Do NOT** guaranteed to be correct all the time



Questions to Be Answered

- PODEM proposed **three heuristics** to answer three questions:
 - ◆ Q1: What path to *backtrace*?
 - ◆ Q2: **What input value** to assign?
 - ◆ Q3: What path to **propagate D (D')** to PO?

**Good Heuristic = Simple
and Effective** (most of the time)

Q1: What Path to Backtrace?

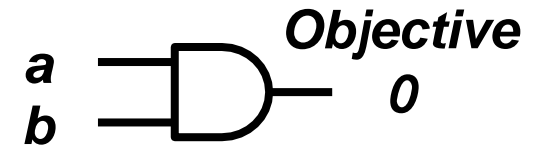
- **Decision Gate:**

- ♦ Only one input can control gate output to objective value

- * OR/NAND with output objective =1

- * AND/NOR with output objective =0

- ♦ choose easiest gate input



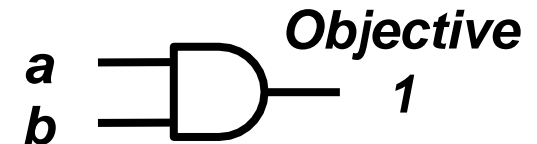
- **ImPLY Gate:**

- ♦ One input can't control gate output to objective value

- * OR/NAND with output objective =0

- * AND/NOR with output objective =1

- ♦ Choose hardest gate input

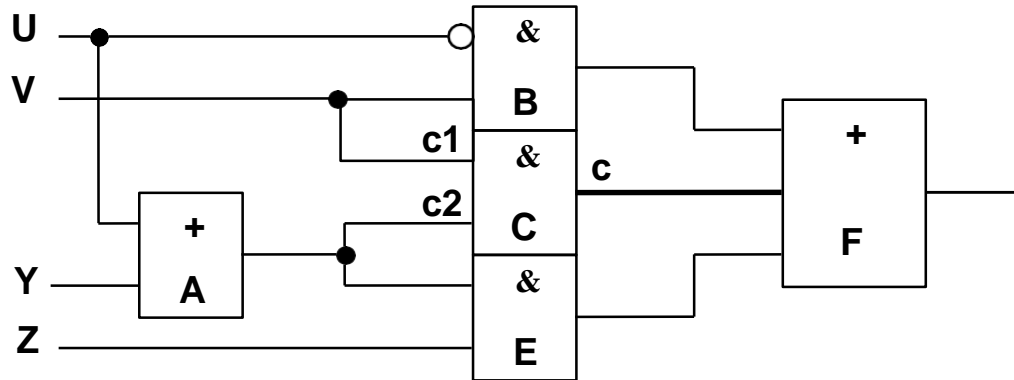


- Why? find out if test exists or not as soon as possible

Heuristic#1: Make Correct Backtrace ASAP

Example

- If objective is $c=0$; backtrace **c1**, then $V=0$ (decision gate)
- If objective is $c=1$; backtrace **c2**, then $U=1$ or $Y=1$ (imply gate)



- Q: how do you know **c1** is easy and **c2** is hard?
 - ◆ Level of **c1** smaller than level of **c2**
 - ◆ (Use other controllability measure, like SCOAP, also fine)

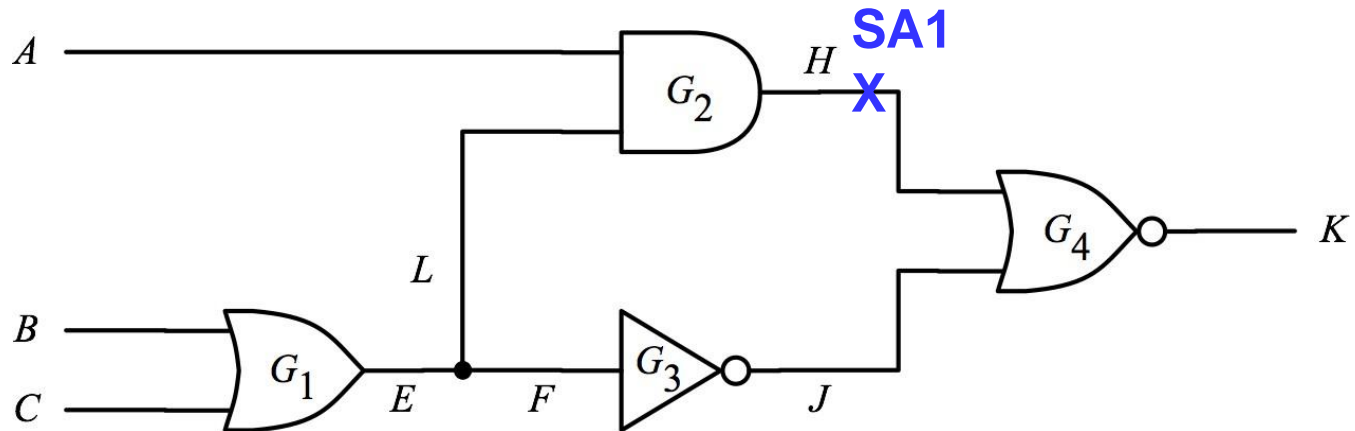
Quiz

Q: Given SCOAP, generate a test for H SA1 fault.

A1: Follow heuristic

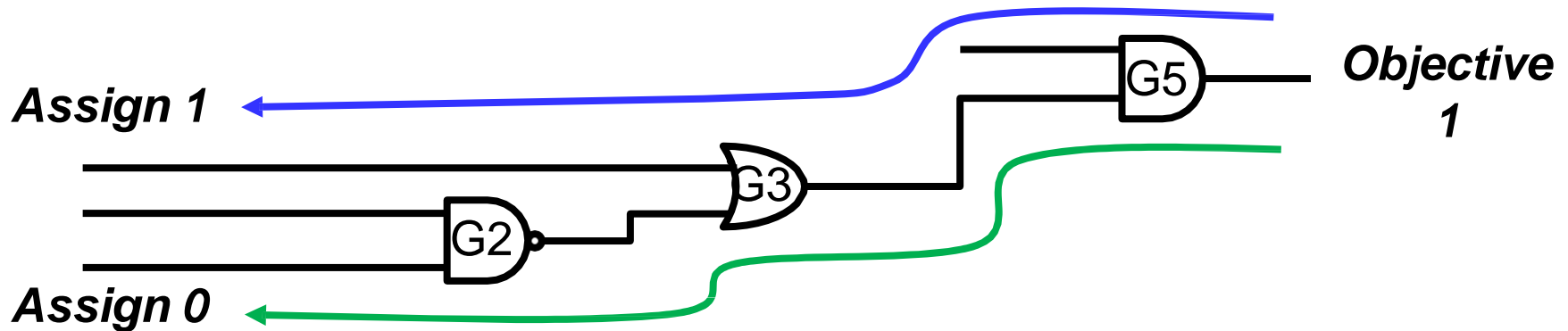
A2: Do not follow heuristic

	A	B	C	E	F	L	H	J	K
CC ⁰	1	1	1	3	3	3	2	3	5
CC ¹	1	1	1	2	2	2	4	4	6
CO	7	6	6	4	4	6	4	3	0



Q2: What Input Values to Assign?

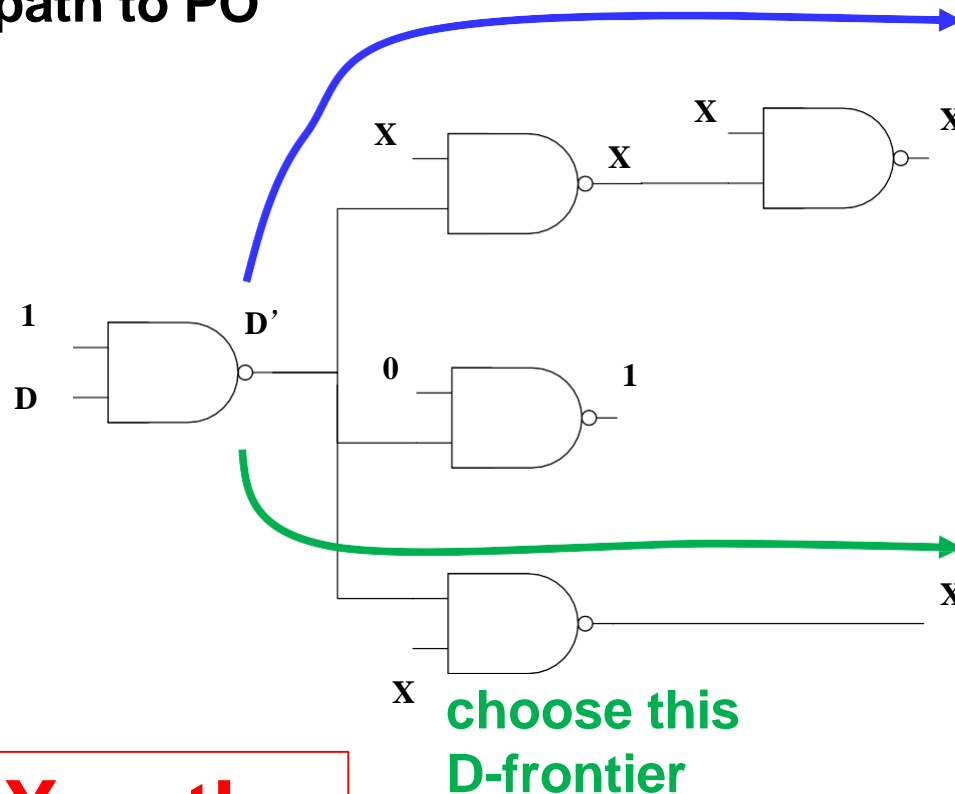
- If **even inversion parity** path
 - ♦ Assign same value as objective
- If **odd inversion parity** path
 - ♦ Assign opposite value to objective
- Why? This assignment is most likely to be correct



Heuristic#2: Inversion Parity for Assignment

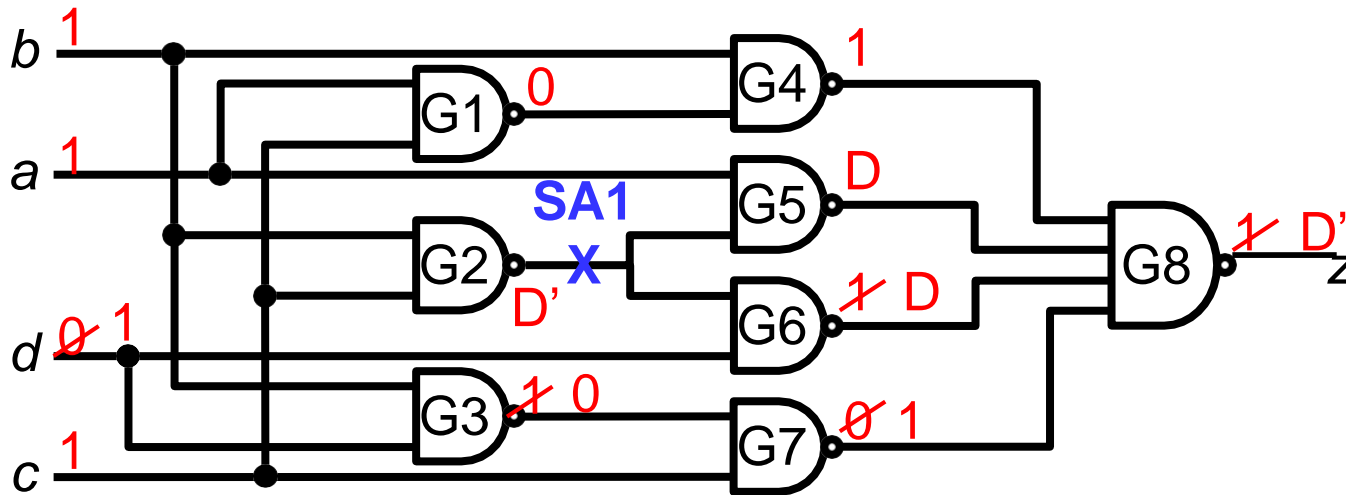
Q3: What Path to Propagate ?

- All gate output of the chosen path must have X values
 - ♦ Called **X-PATH**
- If more than one X-path to choose,
 - ♦ chose shortest X-path to PO
- If X-path disappear,
 - ♦ **backtrack**



Heuristic#3: X-path

PODEM Example w/ Backtrack



Initial objective: (G2, 0)

Backtrace to PI: $b = 1$

Initial objective: (G2, 0)

Backtrace to PI: $c = 1$

Implication: $G2 = D'$

Choose shortest X-path {G5}

Objective $a = 1$

Assign $a=1$

Implication: $G1 = 0, G4 = 1, G5 = D$

Try propagate through G8.

objective: (G6,1)

Backtrace to PI: $d = 0$

Implication: $G3 = 1, G7 = 0, G8 = 1$

X-path disappear!

Backtrack to most recent PI assignment:

$d = 0 \rightarrow d = 1$

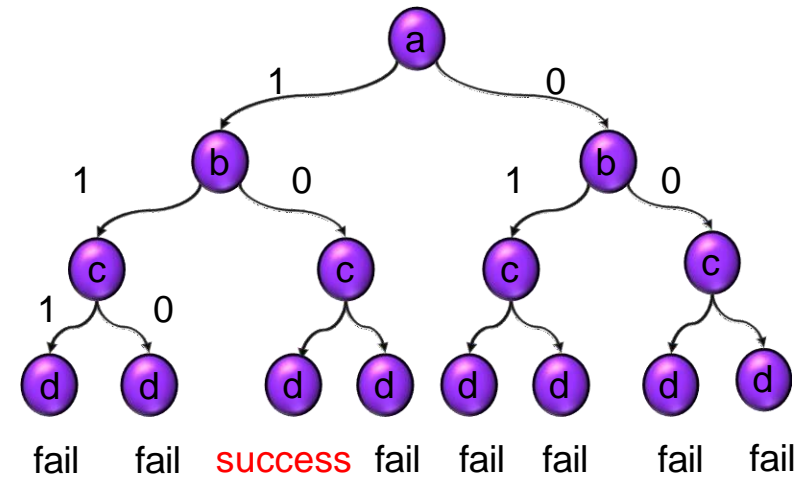
Implication: $G3 = 0, G6 = D, G8 = D'$

Test generated!

PODEM

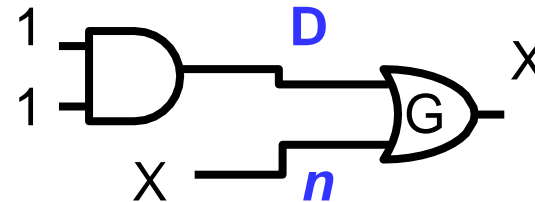
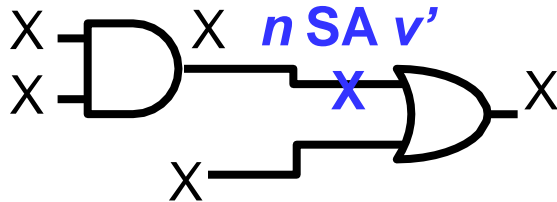
- PODEM [Goel 1981]

- ◆ Idea
- ◆ Heuristics
- ◆ Algorithms
- ◆ Summary



Objective

- Pick an objective: (1) fault activation (2) propagate fault effect



```
Objective (n, v) // target fault: net n stuck-at v'
// if fault has not been activated
1.  if (net n is unknown)
2.    return (n, v);
// else, propagate fault effect
3.  select a gate G from D-frontier on shortest X-path
4.  select an unassigned input n of G
5.  if (gate G has non-controlling value)
6.    v = non-controlling value of G; // AND v=1; OR, v=0
7.  return (n, v); // n is objective net; v is objective value
```

Backtrace

- Translates objective to PI assignment
- **Depth-first search**: recursively calls itself until hits PI

```
Backtrace (n, vs) /* n is objective net; vs is objective y; */  
1. v = vs;  
2. while (n is gate output)  
3.   if (n is NAND or INVERTER or NOR) v = v'; // inversion parity  
4.   if (objective requires setting all inputs) // imply gate  
5.     a = hardest gate input that is still X;  
6.   else // decision gate  
7.     a = easiest gate input that is still X;  
8.   n = a;  
9.   (n, v) = Backtrace (n, v); // recursive call  
   // out of while loop, n is now PI  
10. return (n, v) // assign PI n to value v
```


PODEM

- **Branch and bound** search algorithm

PODEM (*fault* , *v_{fault}*)

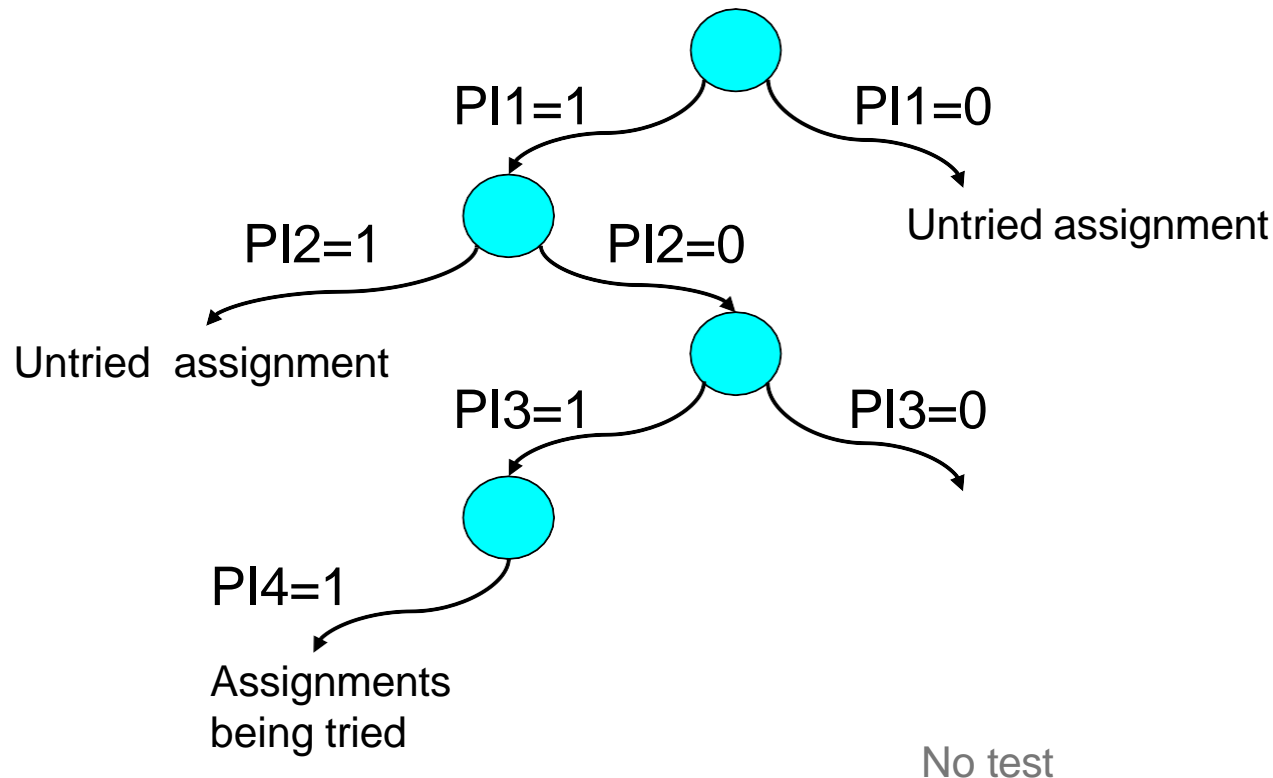
1. if (D or D' at PO) return (SUCCESS)
2. if (test impossible) return (FAILURE)
3. else (*n*, *v_s*) = *Objective* (*fault*, *v_{fault}*);
4. (*pi*, *v*) = *Backtrace* (*n*, *v_s*);
5. *Imply* (*pi*, *v*); // assign pi, forward implication
6. if (*PODEM* (*fault*, *v_{fault}*) == SUCCESS) return (SUCCESS);
 // backtrack
7. *Imply* (*pi*, *v'*);
8. if (*PODEM* (*fault*, *v_{fault}*) == SUCCESS) return (SUCCESS);
9. *Imply* (*pi*, "X"); // release PI as unknown
10. return (FAILURE); // this node is pruned
11. end;

test impossible for 2 reasons:

- (1) target fault cannot be activated
- (2) X-path disappear

Decision Tree of PODEM (1)

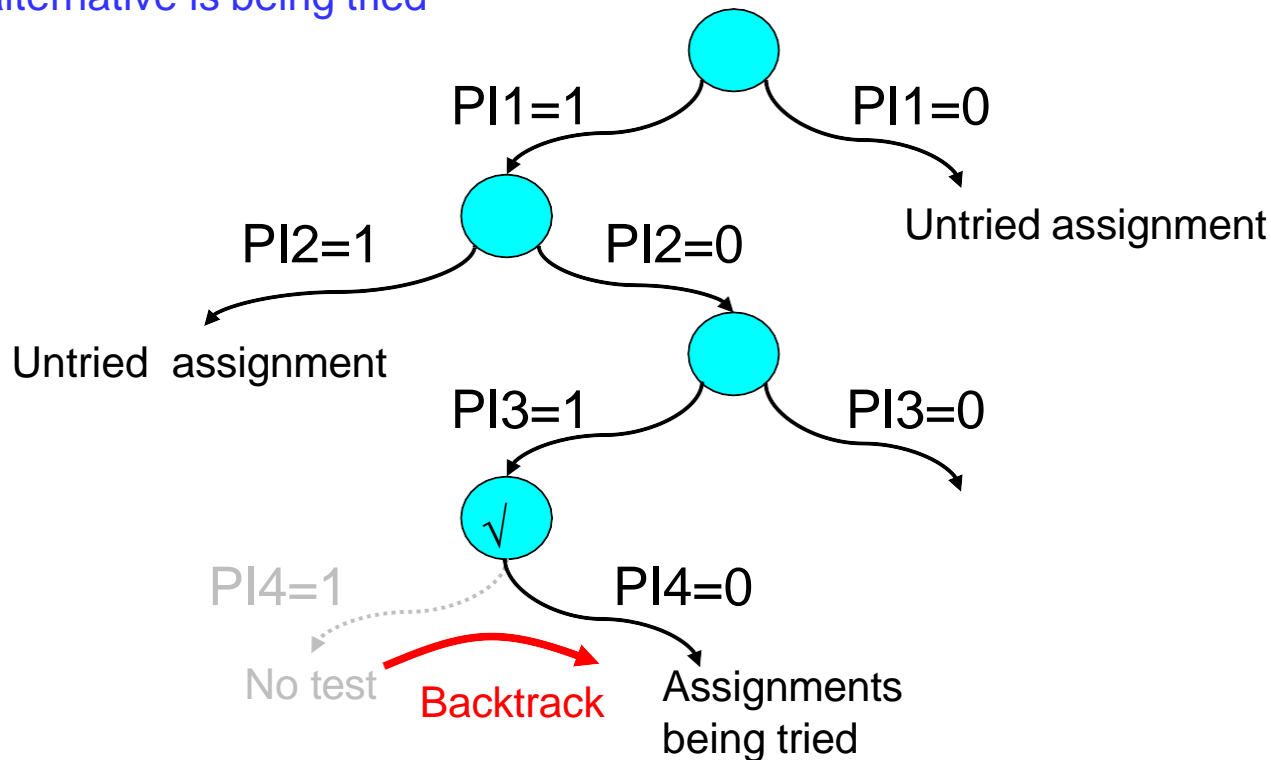
- **Branch and bound** search algorithm
 - ♦ Each decision is a PI



Decision Tree of PODEM (2)

- **Branch and bound** search algorithm
- **Chronological backtrack**: [DPLL satisfiability 1962]
 - ♦ Flip **last PI** that has not been tried

✓ : initial assignment rejected
alternative is being tried

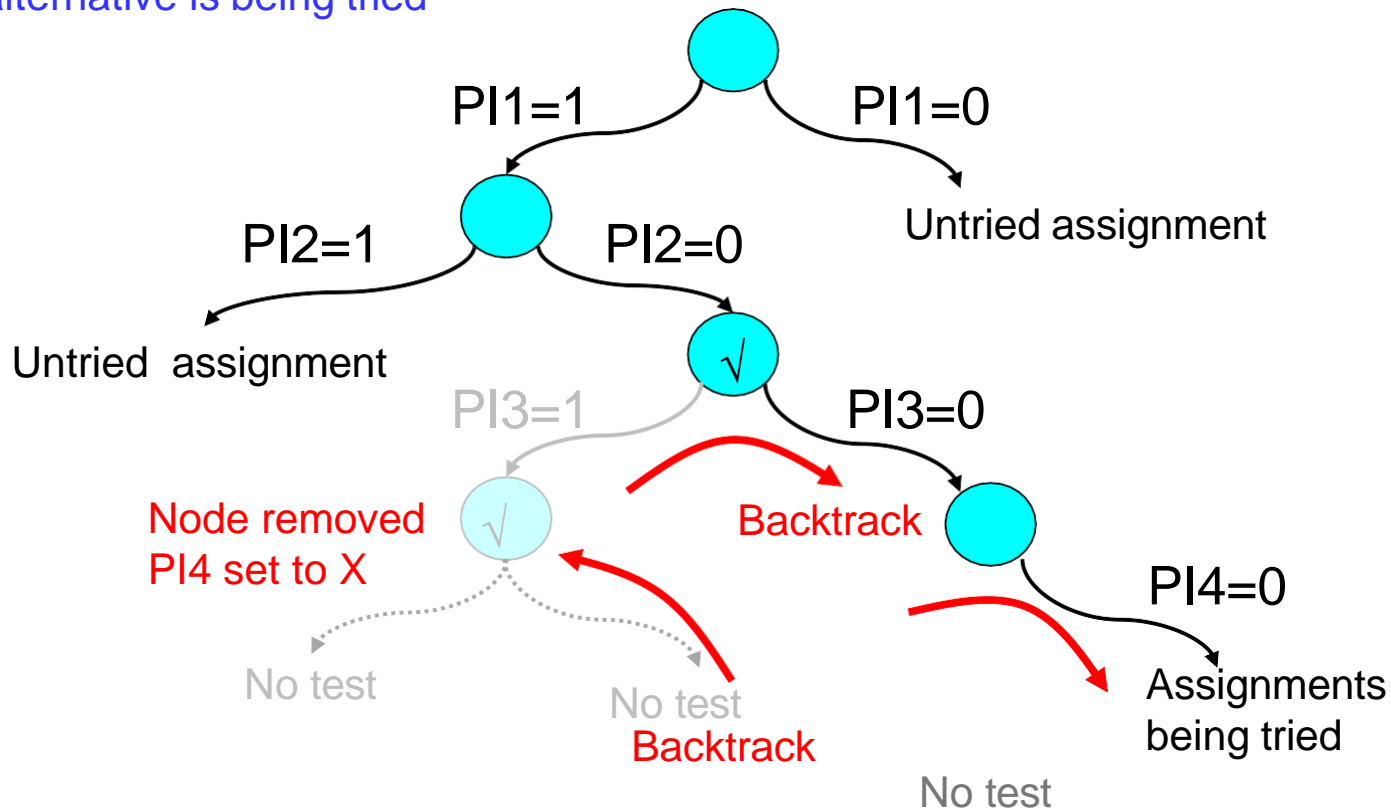


No test

Decision Tree of PODEM (3)

- **Branch and bound** search algorithm
 - ♦ A node is removed when **both** alternatives have been tried
 - ♦ Prune **PI3=1** subtree

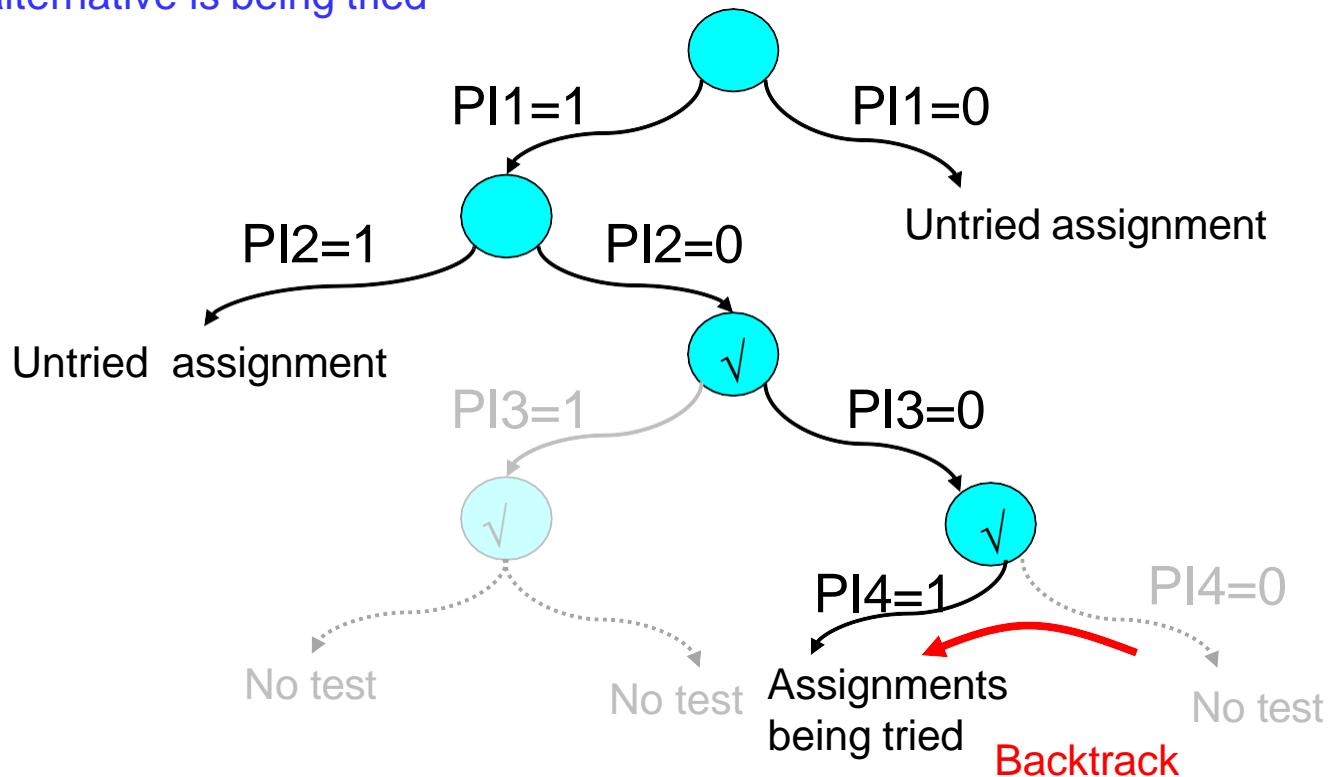
✓ : initial assignment rejected
alternative is being tried



Decision Tree of PODEM (4)

- **Branch and bound** search algorithm

✓ : initial assignment rejected
alternative is being tried

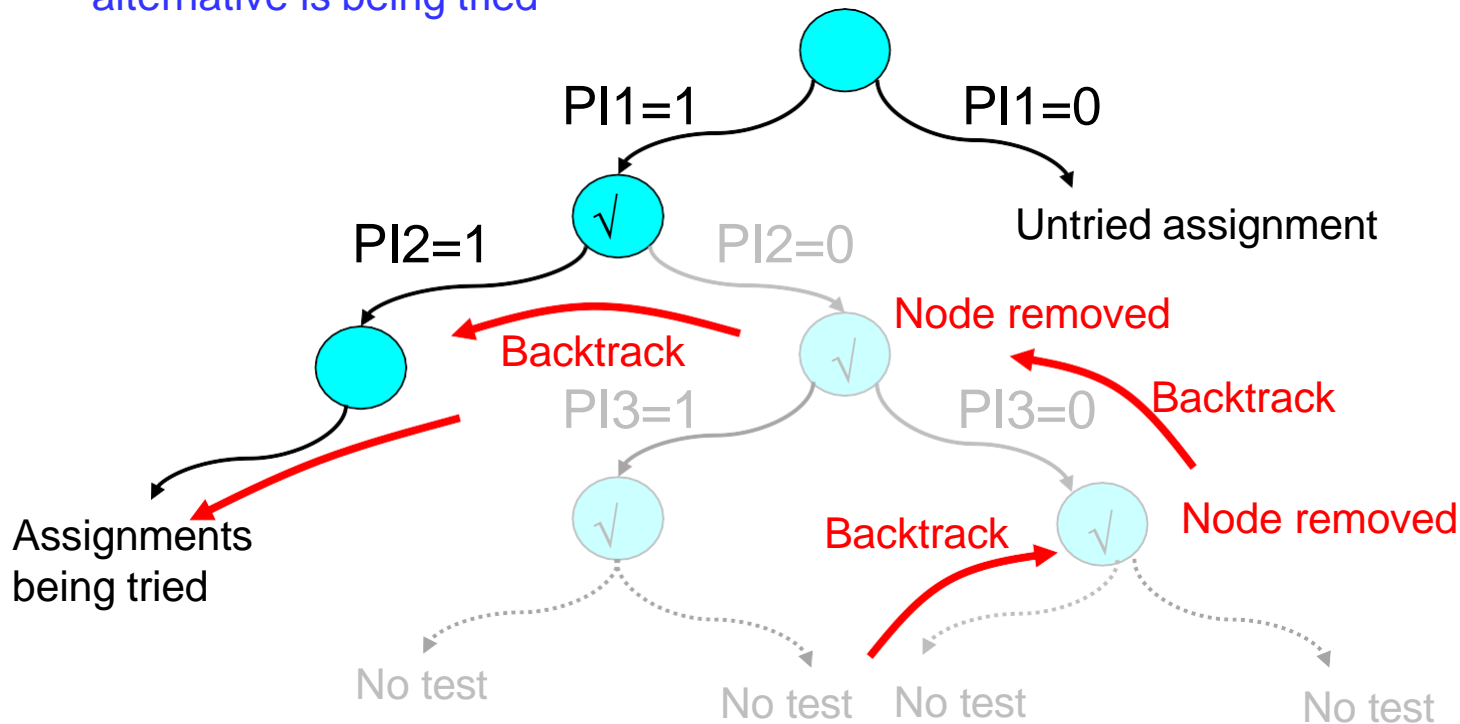


Decision Tree of PODEM (5)

- **Branch and bound** search algorithm

- ♦ Prune $PI3=0$ subtree
- ♦ Prune $PI2=0$ subtree

✓ : initial assignment rejected
alternative is being tried



PODEM: Summary

- **PODEM [Goel 1981]**
 - ◆ **Idea:**
 - * assign PI, not internal nodes
 - * forward implication only, no justification
 - ◆ **Heuristics**
 - * (1) backtrace easy/hard input for decision/imply gate
 - * (2) keep path inversion parity
 - * (3) propagate along X-path
 - ◆ **Algorithms**
 - * branch and bound search

FFT

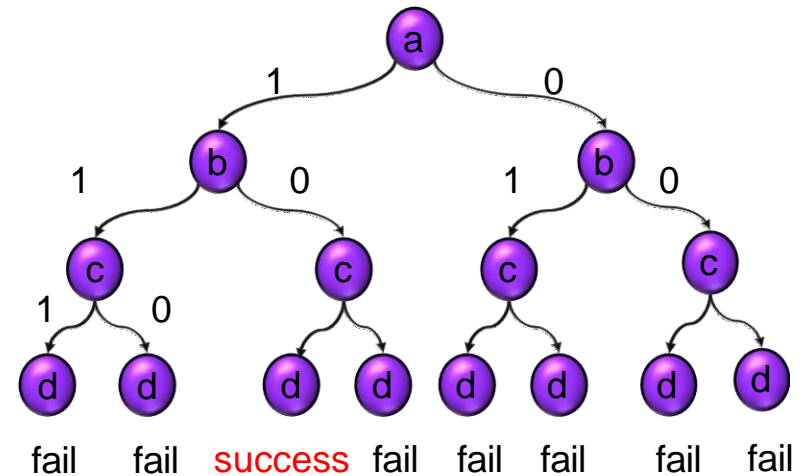
- **Q1: Is PODEM a complete ATPG algorithm?**
- **Q2: Why does PODEM have no justification?**
- **Q3: Please give examples where heuristic #1/#2 gives wrong guess**

Combinational ATPG

- Introduction
- **Deterministic Test Pattern Generation**
 - ◆ Boolean difference approach*
 - ◆ Path sensitization method**
 - ◆ D-Algorithm [Roth 1966] **
 - ◆ PODEM [Goel 1981] **
 - ◆ **FAN [Fujiwara 1983]****
 - ◆ SAT-based [Larrabee 1992] *
- Acceleration Techniques
- Concluding Remarks

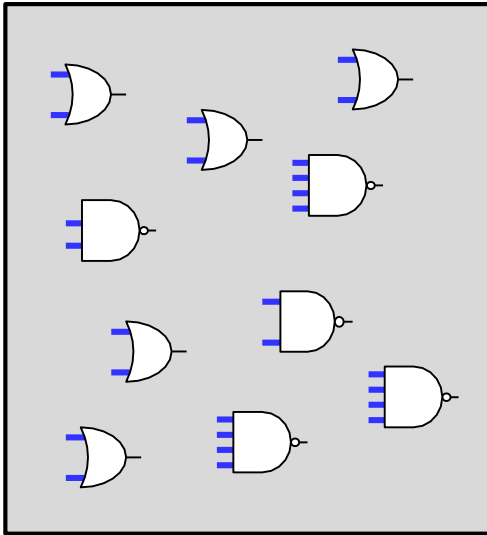
*Boolean-based methods

**path-based methods



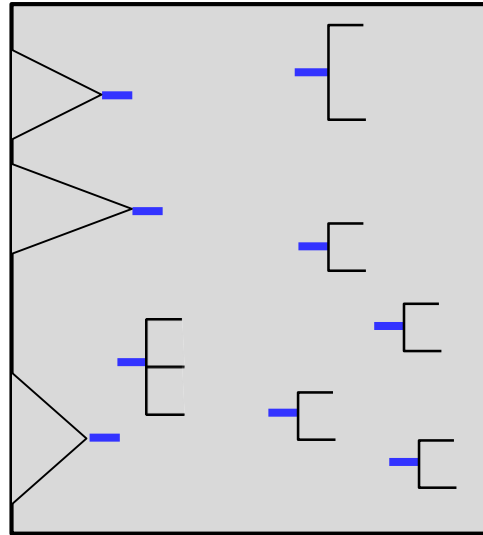
D → PODEM → FAN

- D-algorithm decision at **internal nodes** → too many decisions, slow!
- PODEM decision at **PI** → too little information, mistake-prone!
- FAN decision at **head lines** and **fanout stems** → good trade-off



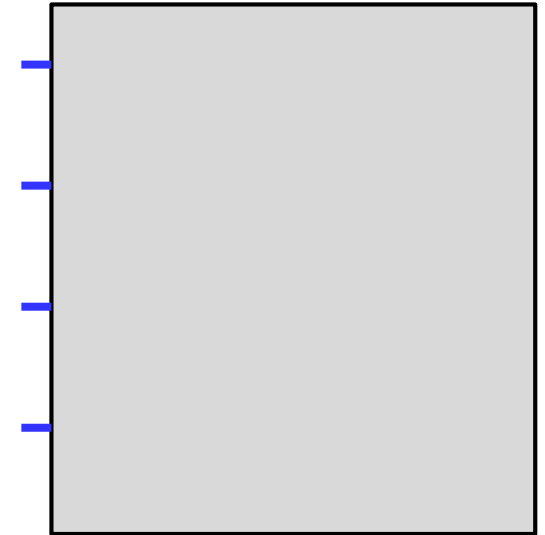
D-alg.

too many decisions



FAN

good trade-off



PODEM

too little information

— decision points

FAN [Fujiwara 1983]

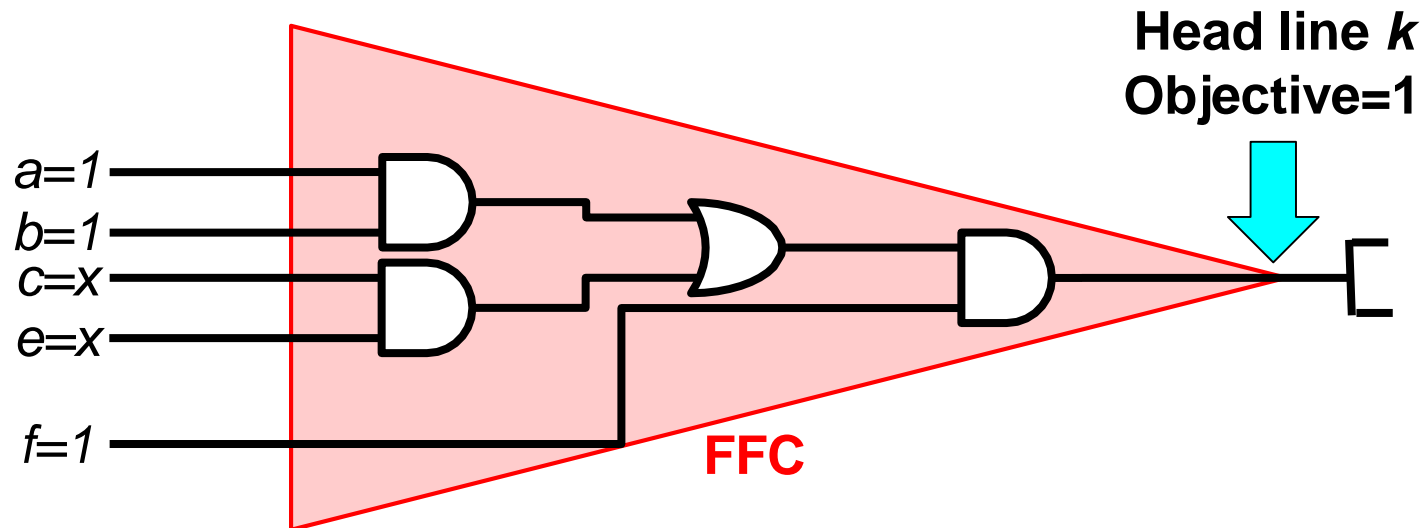
- ***FANout-oriented test generation***
- **Four improvements over PODEM**
 - ◆ **#1. Make decision at head lines or fanout stem**
 - ◆ **#2. Forward/backward Implications**
 - ◆ **#3. Unique sensitization**
 - ◆ **#4. Multiple backtraces**



<https://insights.ubuntu.com>

Justify Head Line Is Easy

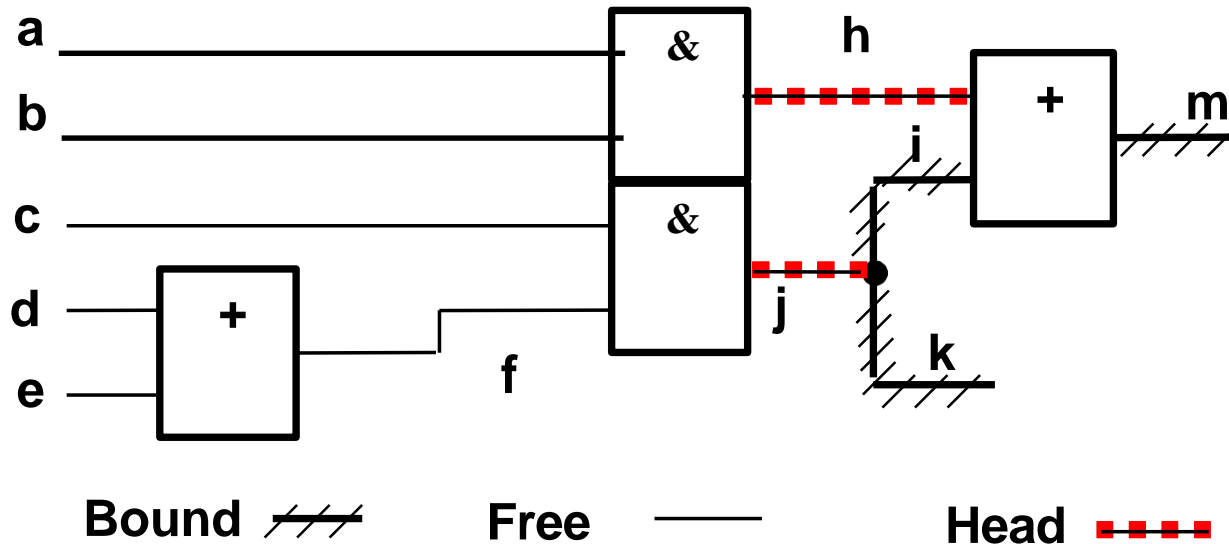
- Justification inside FFC (*fanout-free cone*) is **linear time**
 - ♦ Guaranteed to find an answer
- Example: a, b, c, e, f are PI, *k is head line*
 - ♦ objective $k=1$



**Can Make Decision at Head Line
Instead of PI**

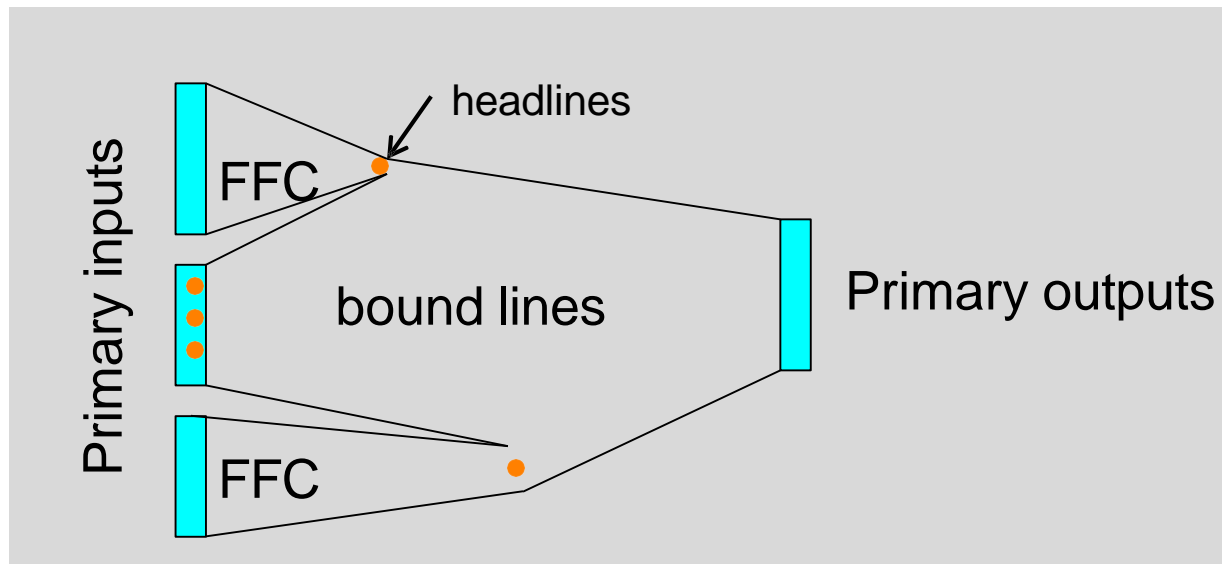
Head Line

- **Bound line**: line fed directly (*i, k*) or indirectly (*m*) by fanout stem
- **Free Line**: line that is not Bound (*a~f, h, j*)
- **Head Line**: free line that is either
 - ♦ Fanout stem (*j*), or
 - ♦ Input to a gate with bound output (*h*)



#1. Make Decision at Head Lines

- FFC can be isolated from rest of circuit by cutting head lines
- Assignment of PI's that feed head lines are
 - ◆ deferred until other objectives have been achieved
 - * Reduce search space

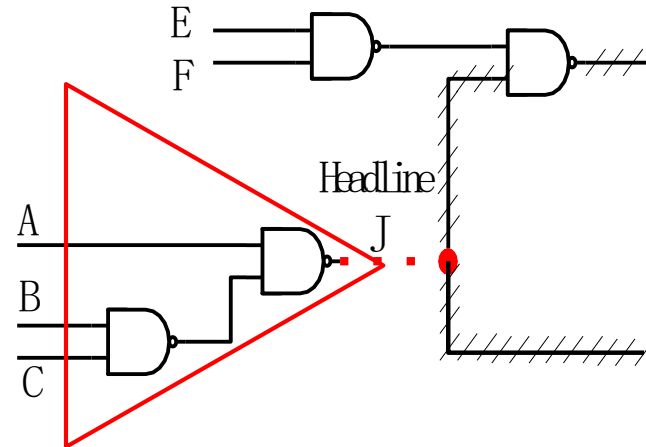


Head Lines Reduces Search Space

Example

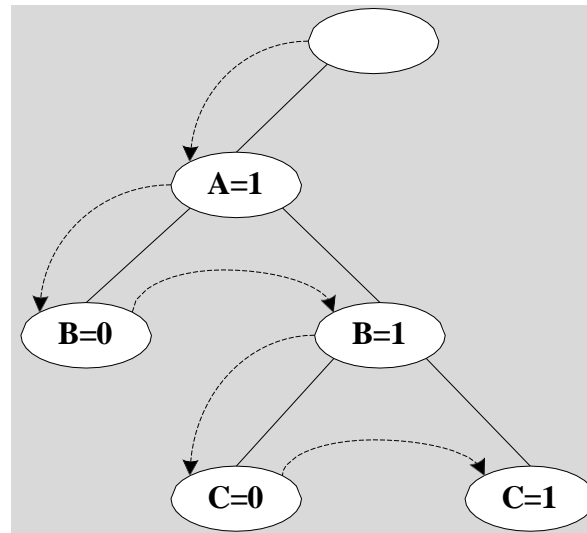
- PODEM decision tree is **big**

- ♦ $A=1, B=0$
 - * No test, backtrack
- ♦ $B=1, C=0$
 - * No test, backtrack
- ♦ $C=1$

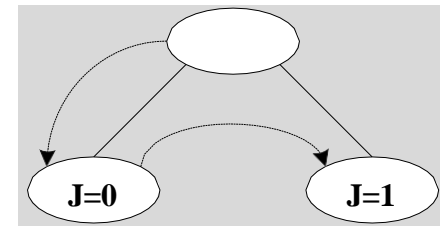


- FAN decision tree is **small**

- ♦ $J=0$
 - * No test, backtrack
- ♦ $J=1$



PODEM

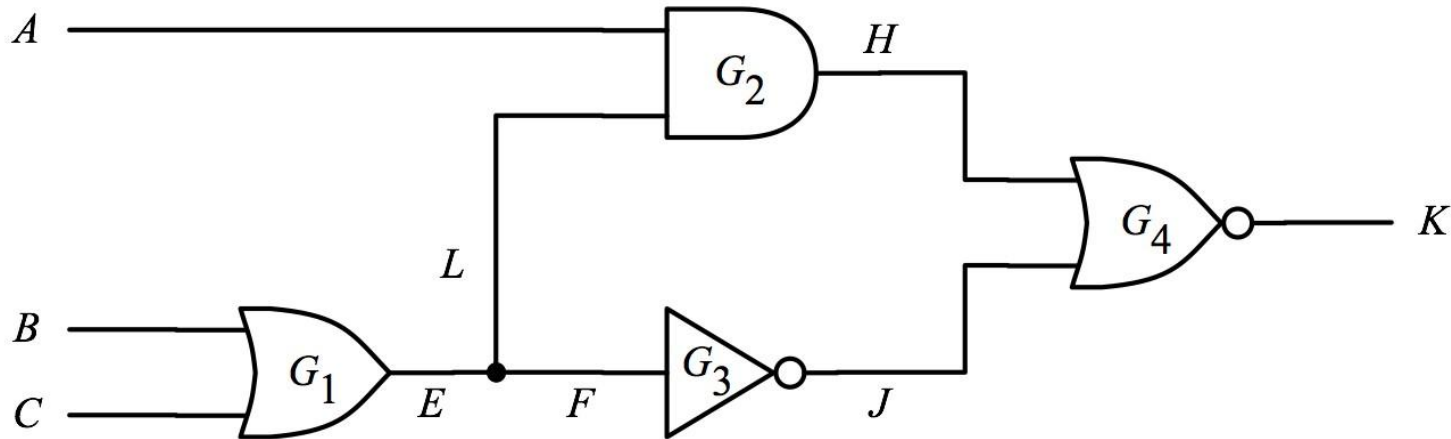


FAN

Quiz

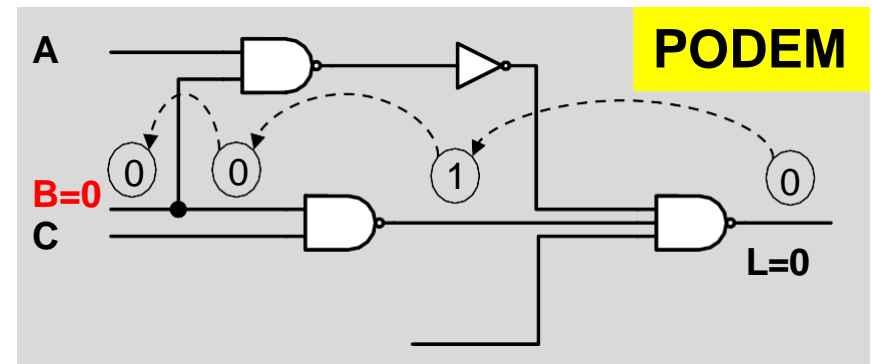
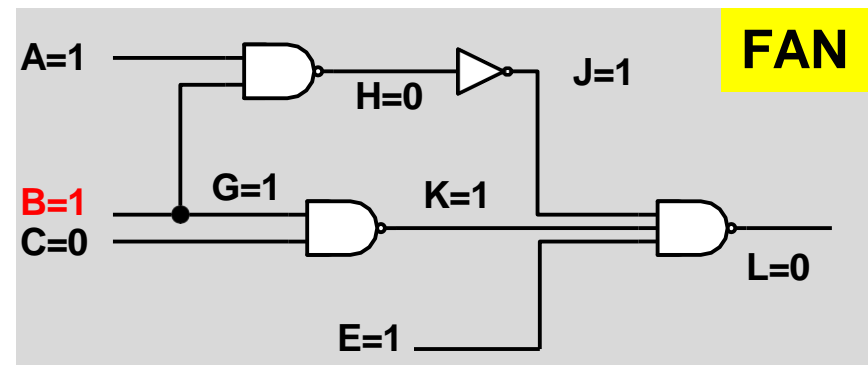
Q: Which are bound lines? free lines? head lines?

A:



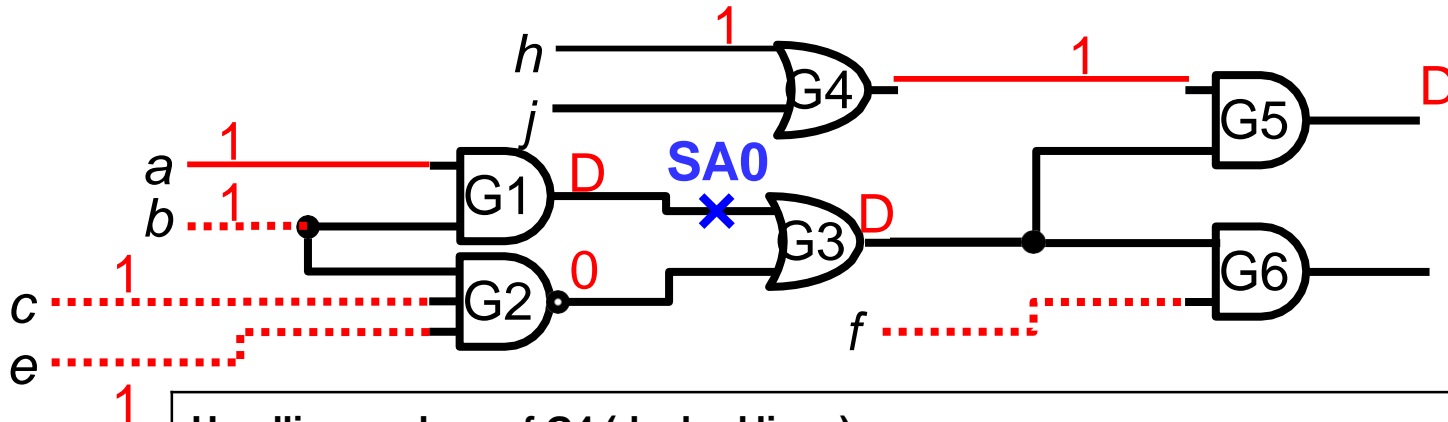
#2. Forward/backward Implication

- PODEM does not assign internal values
 - ♦ **Only forward** implication, **no backward** implication
- FAN assigns internal values when they are uniquely implied
 - ♦ **Both forward and backward** implication
- Example: *L* SA1 fault
- FAN
 - ♦ *Bwd*: $JKE=1$, $H=0$, $A=1$, $B=1$
 - ♦ *Fwd*: $G=1$
 - ♦ *Bwd*: $K=1$, $C=0$
 - ♦ **no backtrack needed**
- PODEM
 - ♦ Backtrace to $B=0$
 - ♦ Forward implication
 - ♦ **Wrong! backtrack**



FAN Example

.....headlines



Headlines: a b c e f G4 (dashed lines)

Initial objective: G1 output =1

Implication: assign a = 1; b=1

Implication

Objective: propagate through G3, objective G2=0

Implication: assign c = 1; e=1

Propagate through G5, objective G4=1

Assign headline G4 = 1

Make decision
at head line

G5=D, Objective achieved.

Justify head line G4 = 1 → h=1

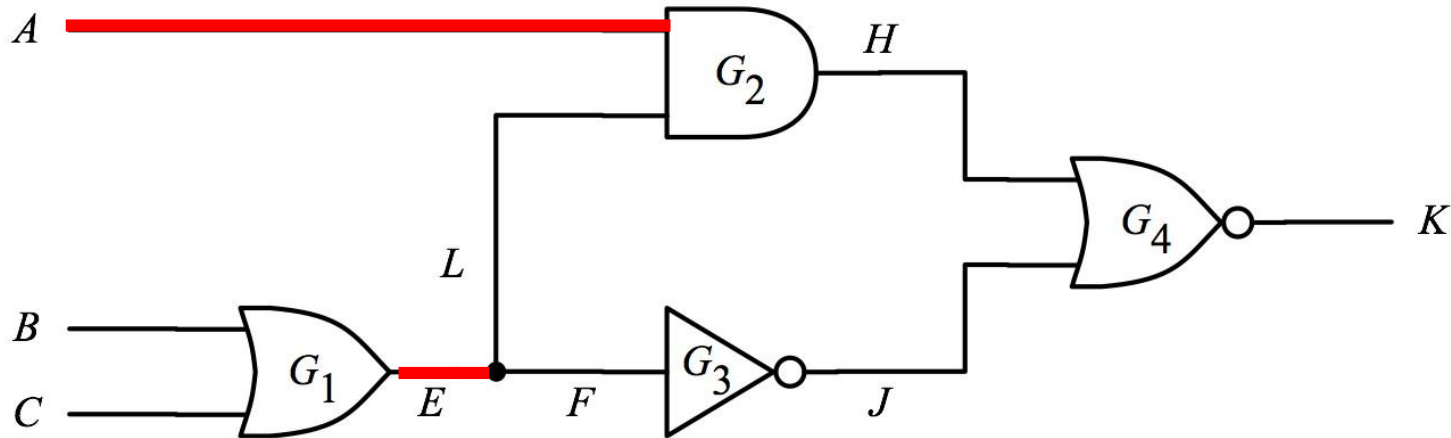
Justify head
lines at end

Test generated abcehjf = 11111xx

Quiz

**Q: If we want $K=1$, apply implication to determine head lines
 $A=?$ $E=?$**

ANS:



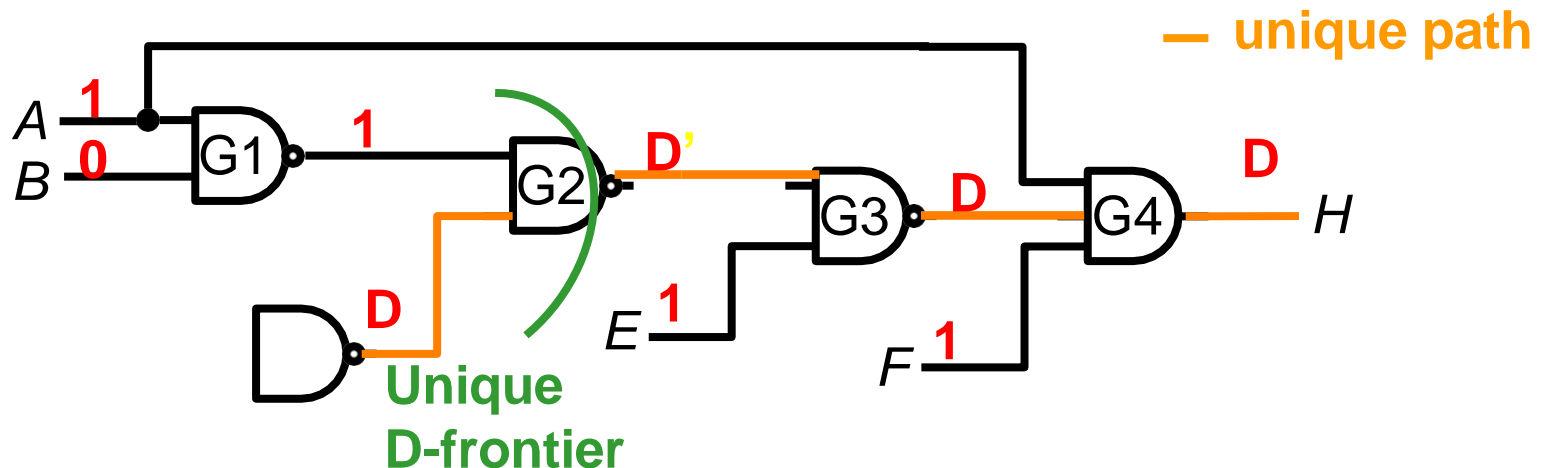
FAN

- **Four improvements over PODEM**
 - ◆ #1. Make decision at head lines or fanout stem
 - ◆ #2. Forward/backward Implications
 - ◆ **#3. Unique sensitization**
 - ◆ **#4. Multiple backtraces**



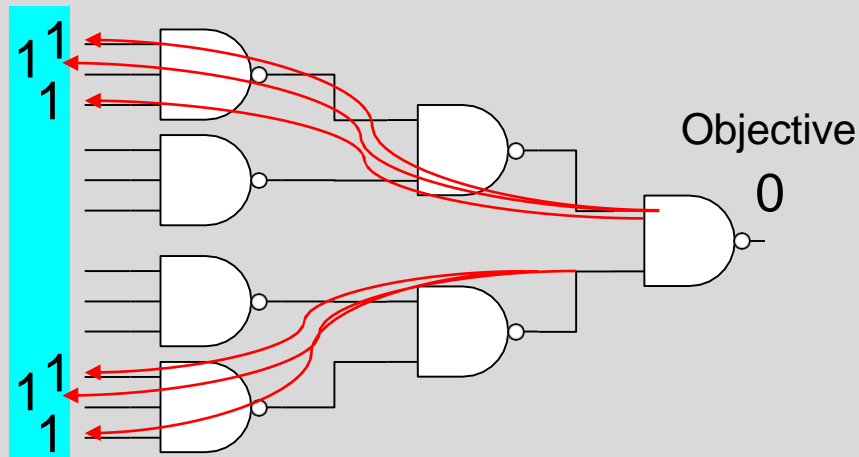
#3. Unique Sensitization

- When there is **only one gate in D-frontier**
 - ♦ if unique path exists, set side inputs to non-controlling values
- Example:
 - ♦ FAN
 - * **G2** is unique D-frontier, only one path to *H*
 - * $G1 = 1, E = 1, F = 1, A = 1 \rightarrow B = 0 \rightarrow$ success!
 - ♦ PODEM
 - * Initial objective: $G1 = 1 \rightarrow$
 - * backtrace to $A = 0 \rightarrow$ X-path disappear!

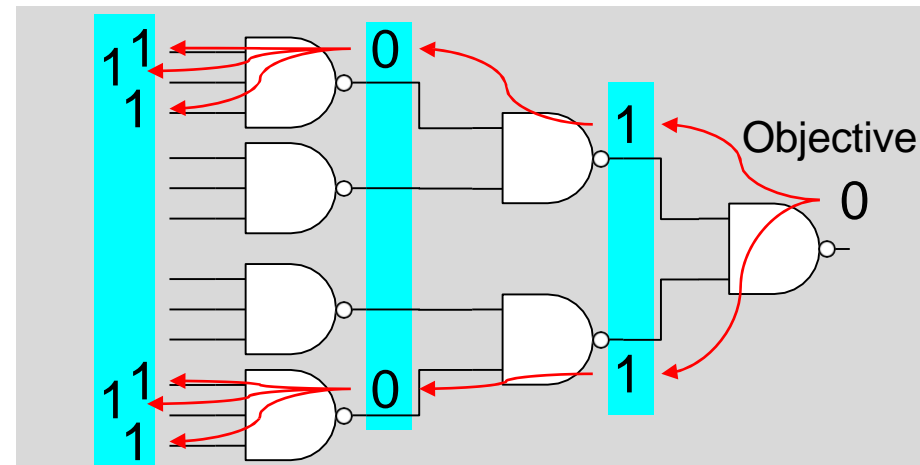


#4. Multiple Backtraces

- PODEM uses **depth-first search (DFS)**
 - ♦ One single backtrace at a time
- FAN uses **breadth first search (BFS)**
 - ♦ Multiple parallel search at a time
- Example
 - ♦ PODEM needs **6 backtraces**
 - ♦ FAN needs only **1 multiple backtrace**



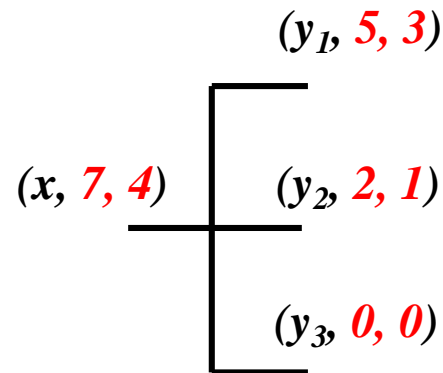
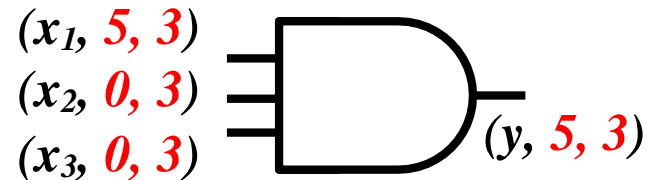
PODEM



FAN

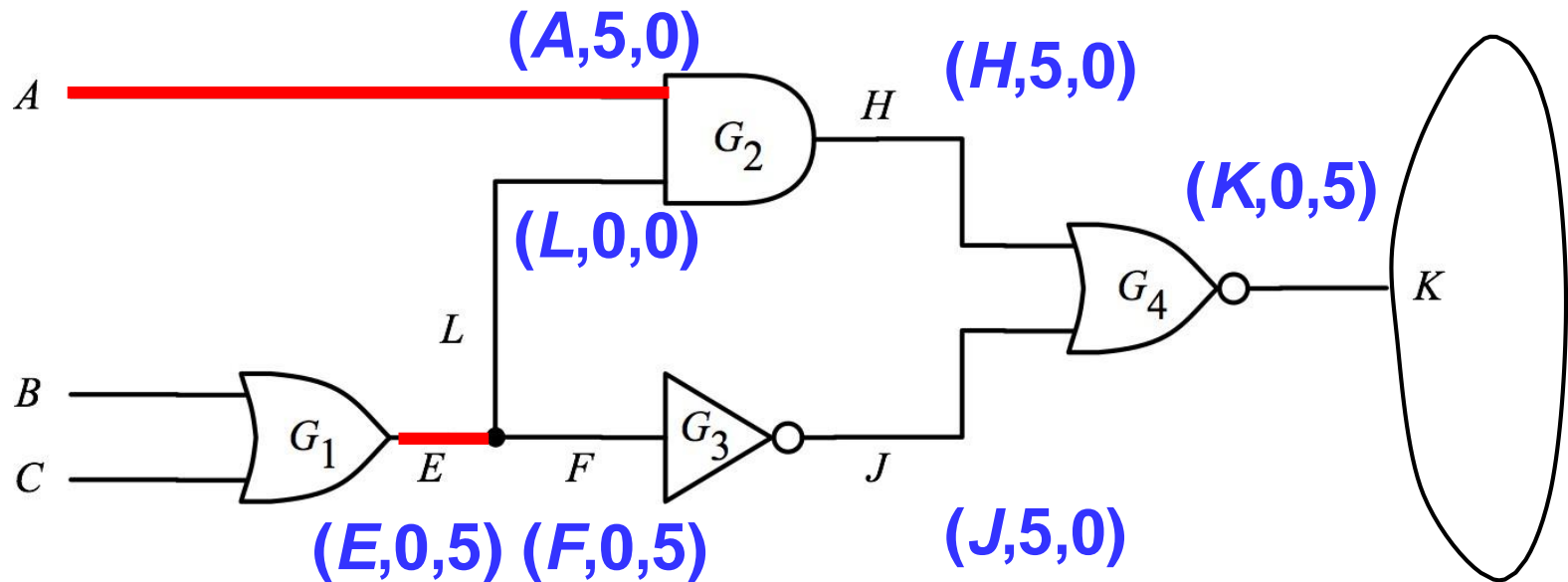
Rules for Multiple Backtraces

- **Objective:** (x, n_0, n_1)
 - ◆ number of backtraced zeros (n_0) and ones (n_1) on signal x
- **For AND gate**
 - ◆ Easiest unspecified input x_1
 - * $(x_1, n_0, n_1) = (y, n_0, n_1)$
 - ◆ Other inputs x_2, x_3
 - * $(x_2, n_0) = 0$
 - * $(x_2, n_1) = (y, n_1)$
- **For fanout Stem**
 - * $(x, n_0) = \text{sum of } (y_i, n_0)$
 - * $(x, n_1) = \text{sum of } (y_i, n_1)$



Example

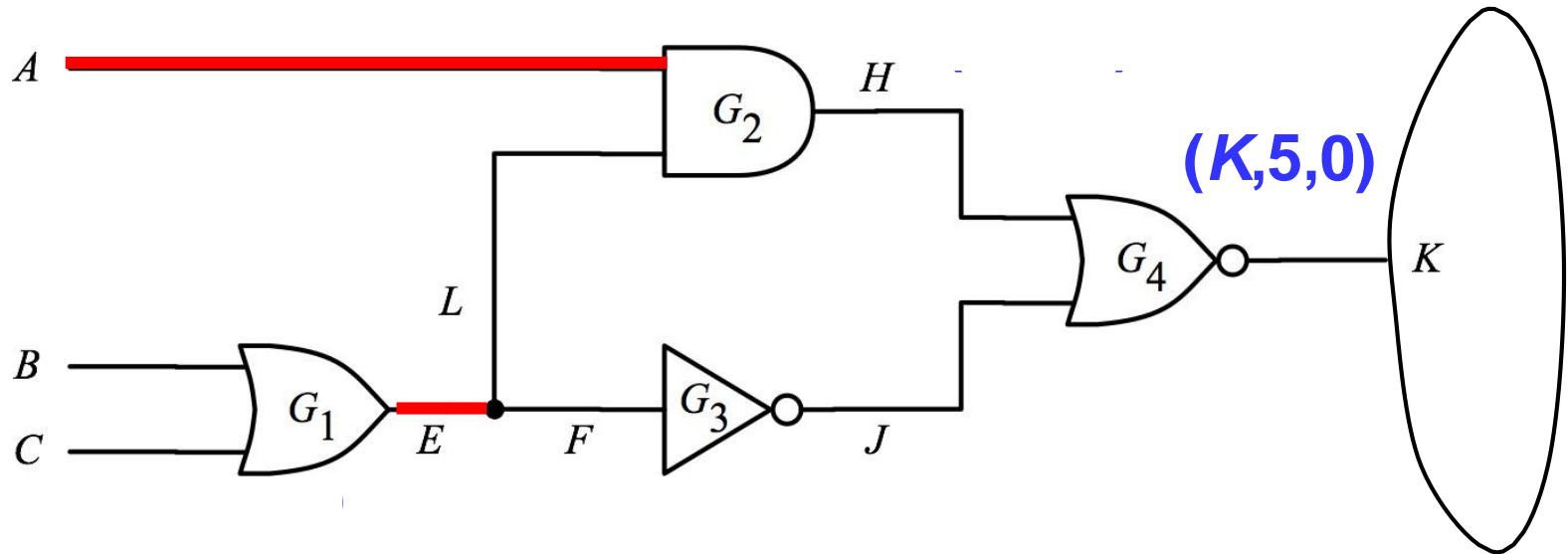
- Starting from $(K,0,5)$, multiple backtrace to head lines A and E
 - So we get two assignments $A=0, E=1$



Quiz

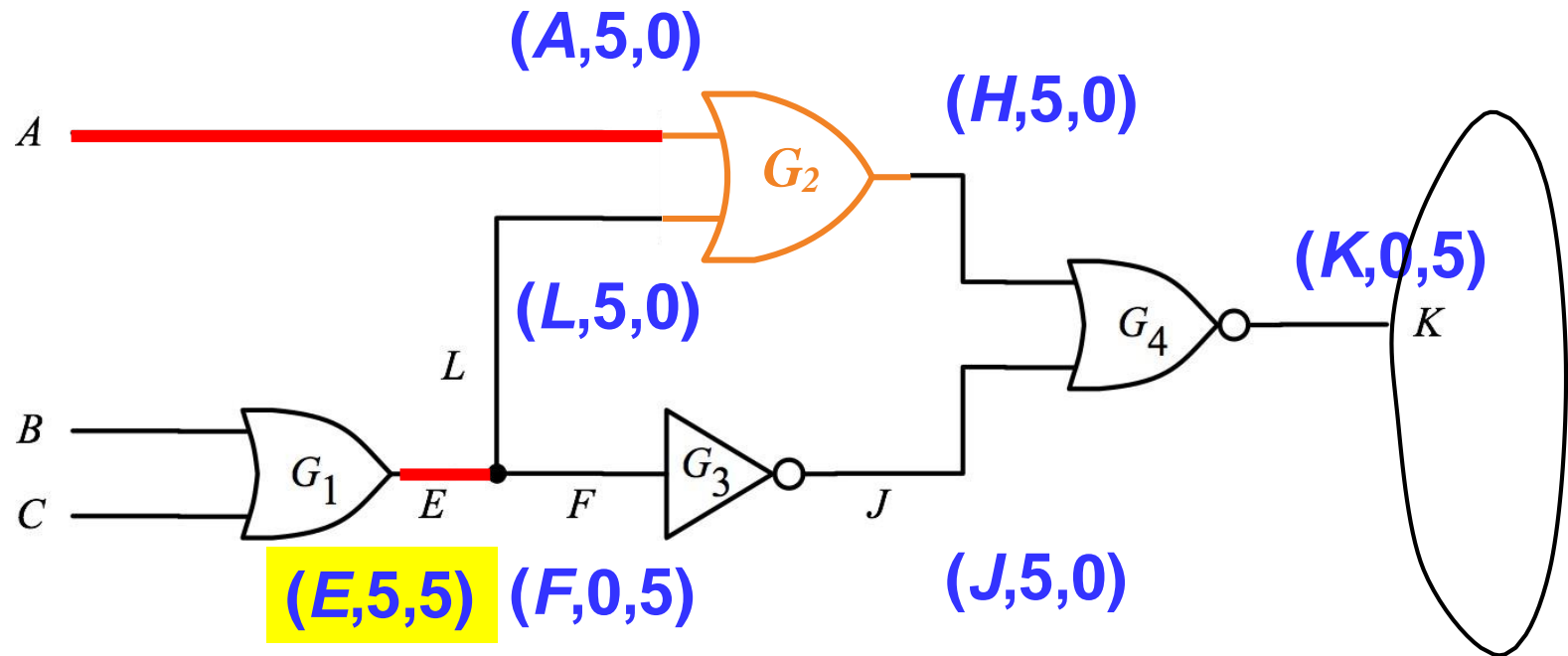
Q: Starting from $(K, 5, 0)$, multiple backtrace to head lines
 $(E, n_0, n_1)=?$ $(A, n_0, n_1)=?$ Suppose H is chosen over J

A:



Multiple Backtrace Conflict

- What if change G_2 to OR gate?
- Want $K=1$, perform multiple backtrace to headlines A and E
 - ♦ conflicting values at E !



- How to handle a conflict ?
 - ♦ assign a value **most requested**, then start next backtrace
 - ♦ will backtrack if it is wrong

Multiple Backtrace (1/2)

```
MultipleBacktrace (Initial_objectives, Fanout_objectives) {  
    Current_objectives = Initial_objectives  
    while (Current_objectives  $\neq \phi$  or Fanout_objectives  $\neq \phi$ ) {  
        dequeue entry  $(k, v_k)$  from Current or Fanout_objectives  
        switch (type of entry) {  
            //  $(k, v_k)$  = want  $v_k$  on signal  $k$   
            1. HEAD_LINE:  
                add  $(k, v_k)$  to Headline_objectives  
            2. FANOUT_BRANCH:  
                 $j = \text{stem}(k)$ ;  
                increment  $n_0$  or  $n_1$  at  $j$  for  $v_k$ ; //sum of  $n_0, n_1$   
                add  $j$  to Fanout_objectives  
            3. GATE: //page 15  
                 $i = \text{inversion of } k$ ;  $c = \text{controlling value of } k$ ;  
                if  $((v_k \oplus i) == c)$  {  
                    select easiest input  $j$  with unknown value  
                    add  $(j, c)$  to Current_objectives;  
                }  
                else {  
                    for every input  $j$  of  $k$  with value  
                    add  $(j, c')$  to Current_objectives; }  
            } // switch  
        }  
    }  
}
```

Multiple Backtrace (2/2)

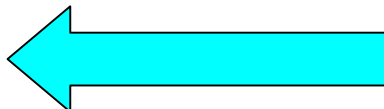
(cont'd from previous page)

*simplified from Fig. 8 of FAN paper

```
if(Fanout_objectives  $\neq \phi$ ) {  
    dequeue highest-level stem ( $k$ ) from Fanout_Objectives  
     $v_k = 0$  or  $1$ , depends on which of ( $n_0, n_1$ ) is larger  
    if there is no conflict on  $k$  {  
        add ( $k, v_k$ ) to Current_objectives } // continue backtrace  
    else { return ( $k, v_k$ ) as the Final_objective } } // stop backtrace  
else { // no fanout objective  
    dequeue ( $k, v_k$ ) from Headline_objectives  
    return ( $k, v_k$ ) as the Final_objective }  
} // while  
} // MultipleBacktrace
```

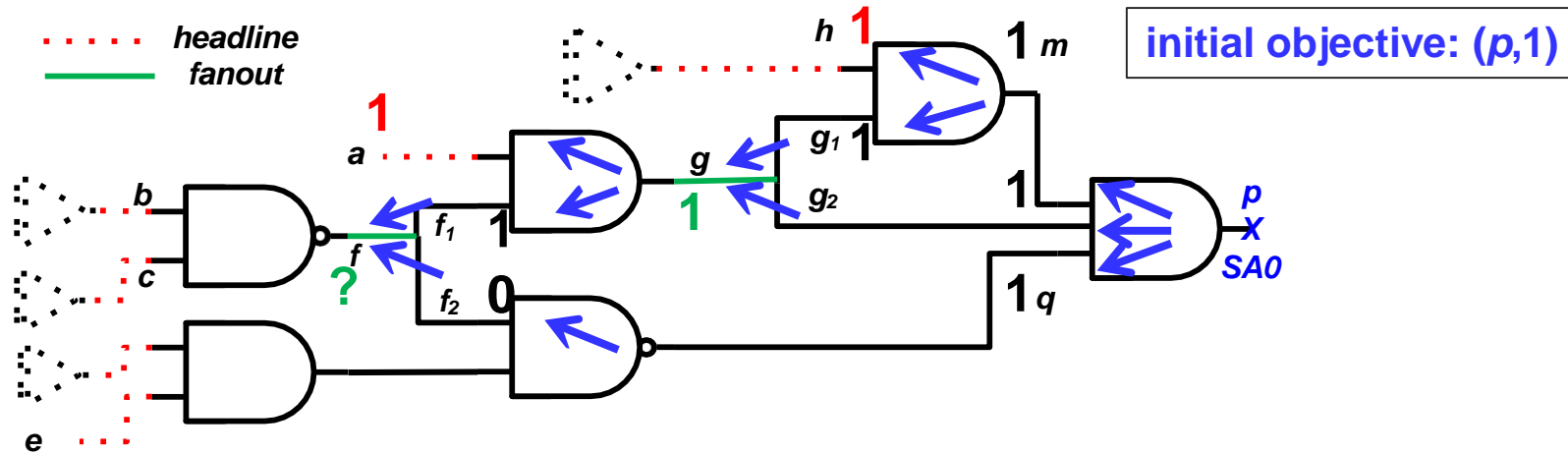
final_objective
(either a fanout or a headline)

MultipleBacktrace



initial_objectives
fanout_objectives

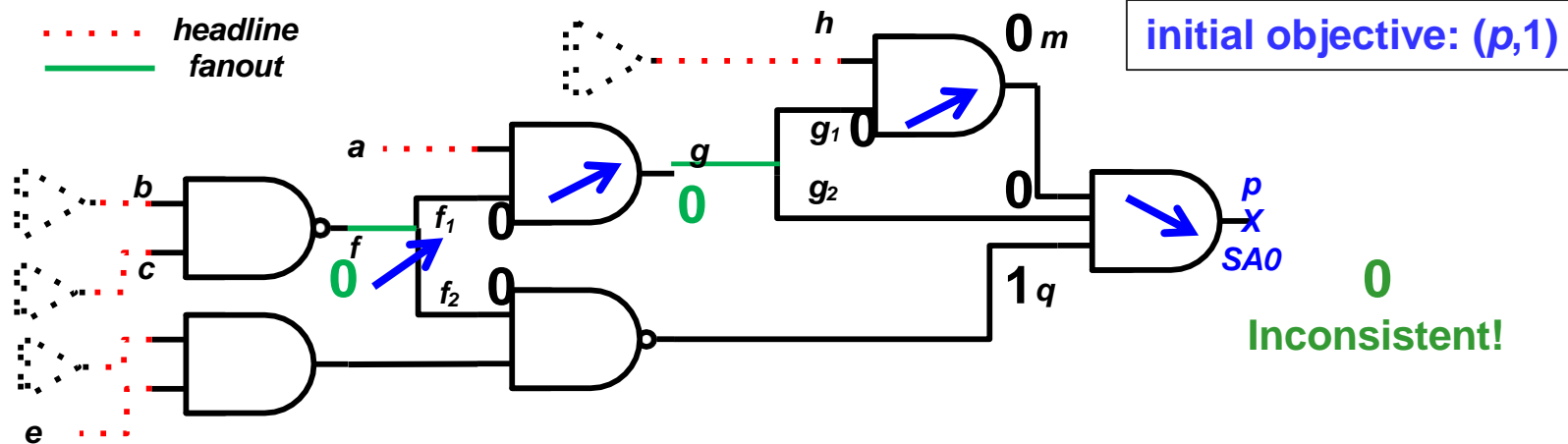
Multiple Backtrace Example (1/3)



Current Obj.	processed entry	Fanout Obj.	Headline Obj.
$(m,1)(g_2,1)(q,1)$	$(m,1)$	-	$(h,1)$
$(g_2,1)(q,1)(g_1,1)$	$(g_2,1)$	$(g,n_1=1)$	$(h,1)$
$(q,1)(g_1,1)$	$(q,1)$	$(g,n_1=1)$	$(h,1)$
$(g_1,1)(f_2,0)$	$(g_1,1)$	$(g,n_1=2)$	$(h,1)$
$(f_2,0)$	$(f_2,0)$	$(g,n_1=2) (f,n_0=1)$	$(h,1)$
-	$(g,1)$ consistent	$(f,n_0=1)$	$(h,1)$
$(g,1)$	$(g,1)$	$(f,n_0=1)$	$(h,1)(a,1)$
$(f_1,1)$	$(f_1,1)$	$(f,n_0=1 n_1=1)$	$(h,1)(a,1)$
-	f conflict!	-	$(h,1)(a,1)$

Final Objective: $f=?$

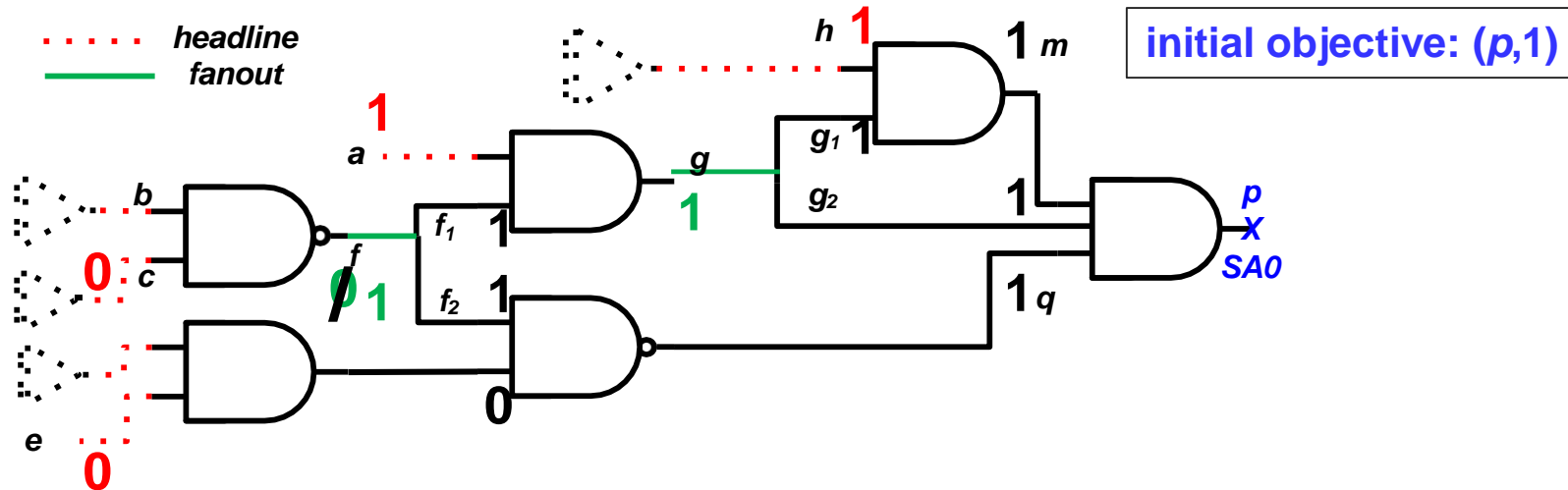
Multiple Backtrace Example (2/3)



because $n_0=n_1=1$, just choose randomly
 suppose we assign $f = 0$
 forward implication
 inconsistent with initial objectives!
 backtrack to $f=1$

**Decision at Fanout Stem
 Detects Inconsistency Earlier**

Multiple Backtrace Example (3/3)



Assign $f=1$

Forward implication, consistent.

Multiple_Backtrace again

This time, headline objectives: $h=1, a=1, e=0, c=0$

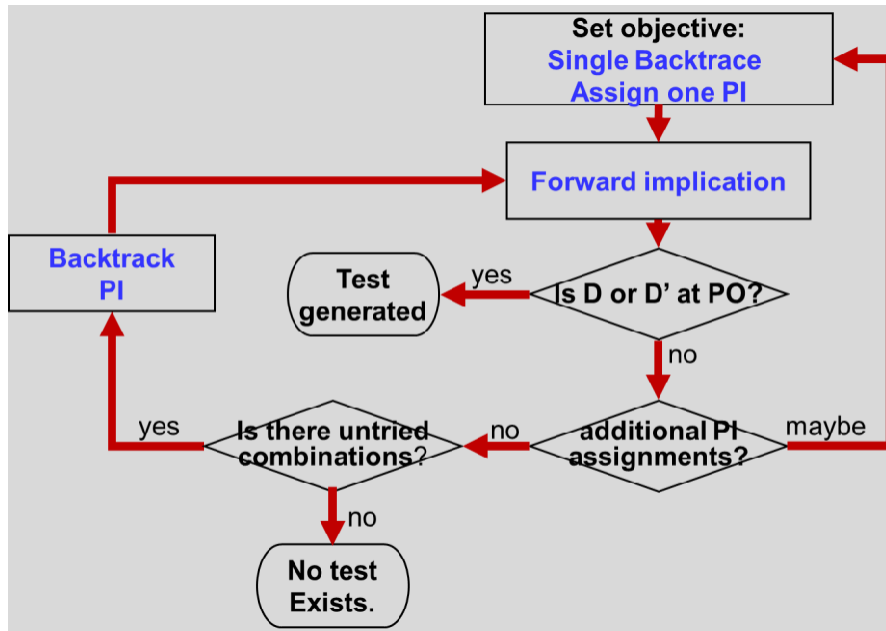
Forward implication

Initial objective achieved!

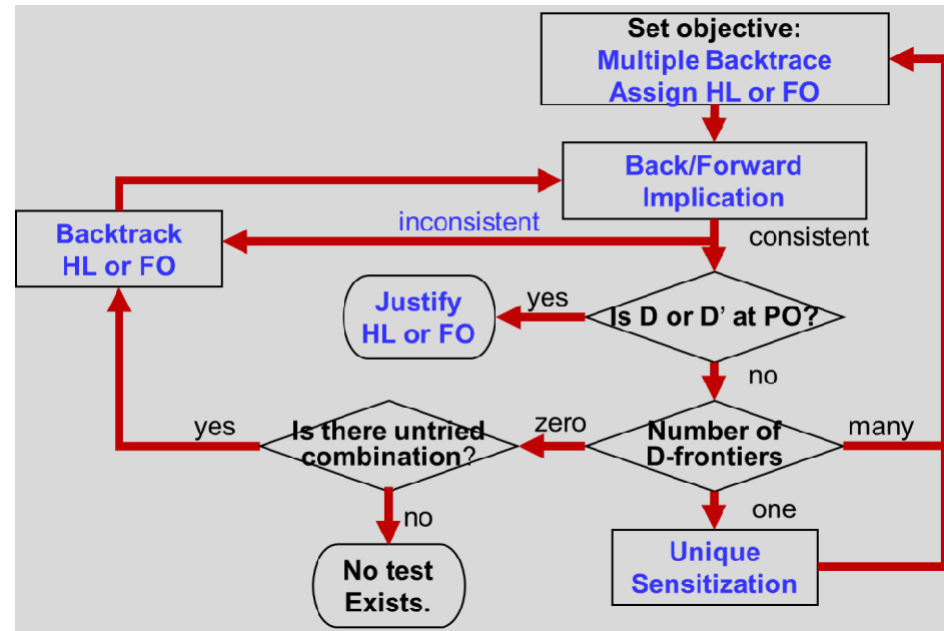
Multiple Backtrace is Fast

PODEM v.s. FAN

PODEM



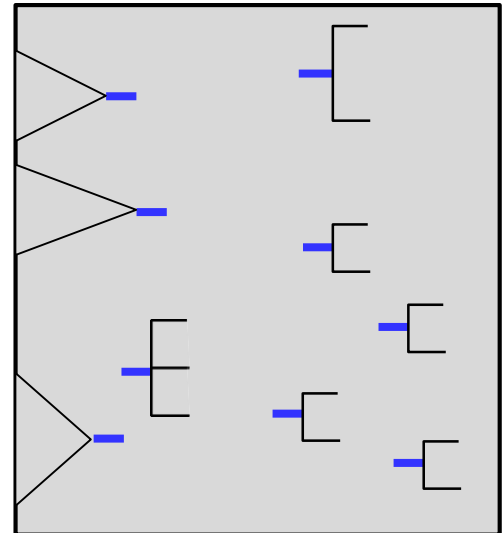
FAN



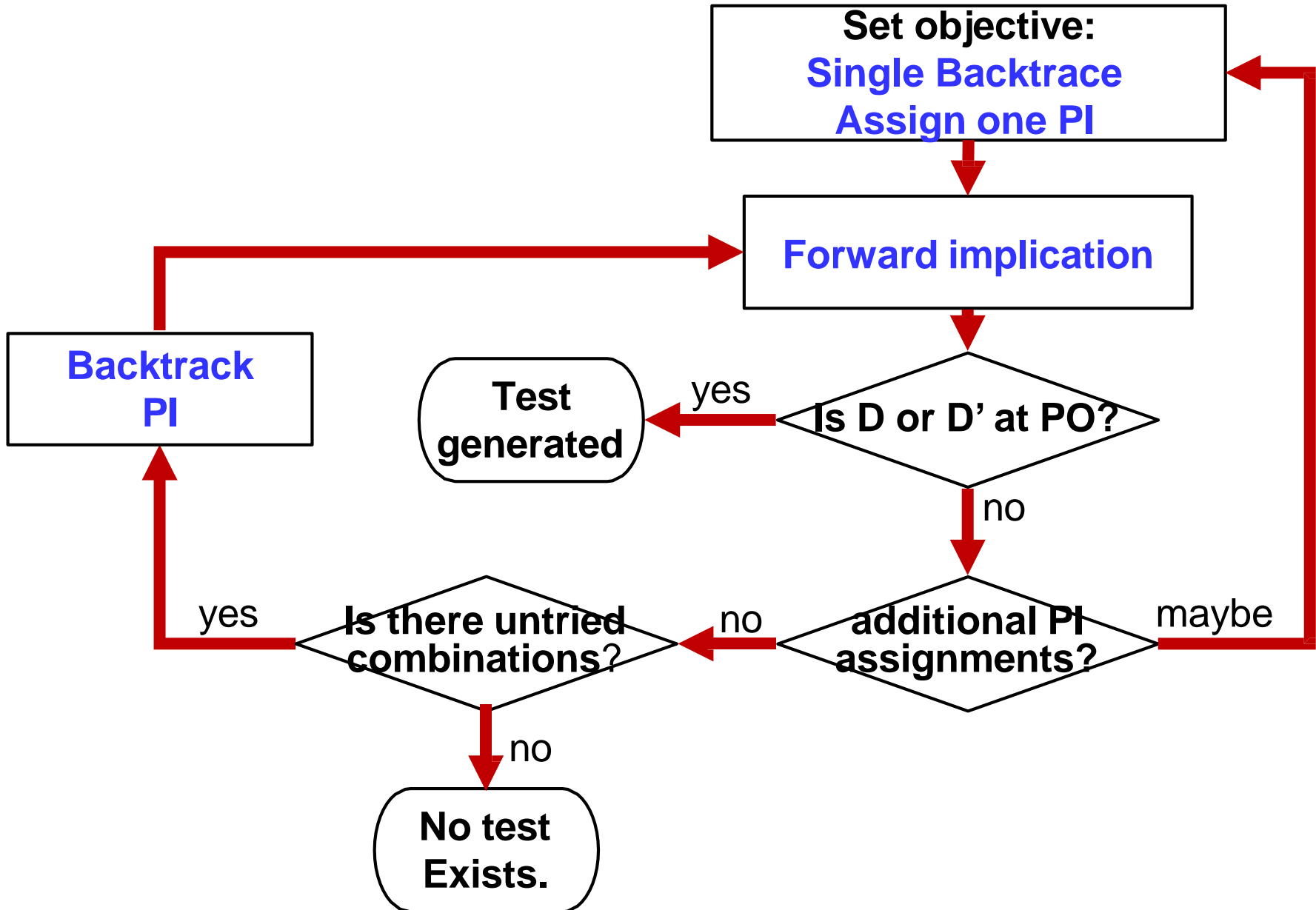
Difference highlighted in Blue

Summary

- 1. Make decision at **head lines** and **fanout stem**
 - ♦ Reduce search space
- 2. Forward/**backward Implications**
 - ♦ More information to make correct decision
- 3. **Unique sensitization**
 - ♦ Unique path to output
- 4. **Multiple backtraces**
 - ♦ BFS to search many paths together

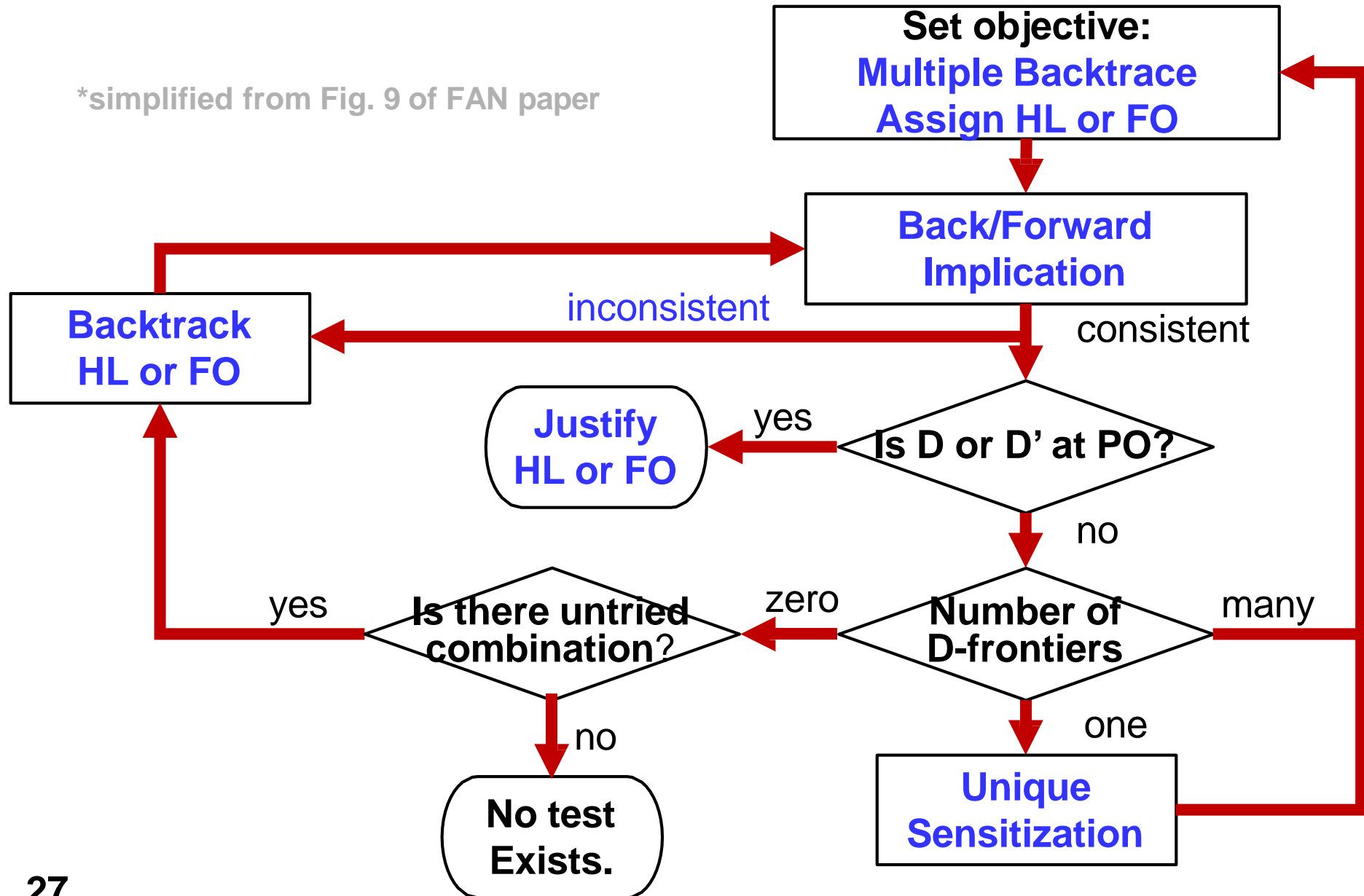


Flowchart of PODEM



Flowchart of FAN

*simplified from Fig. 9 of FAN paper

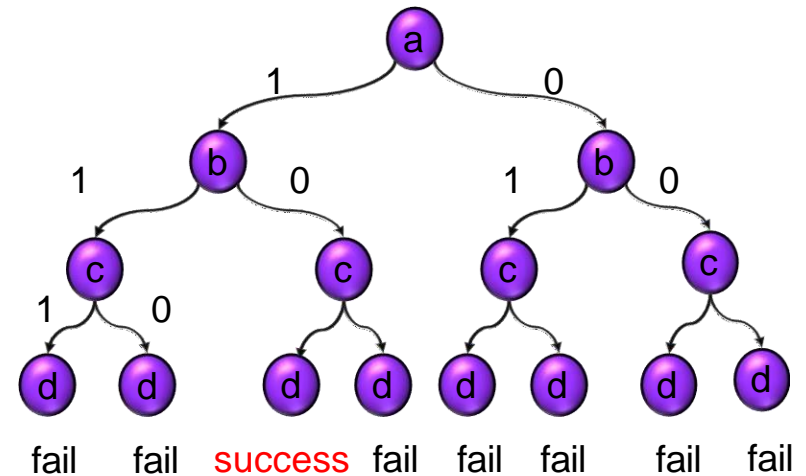


Combinational ATPG

- Introduction
- **Deterministic Test Pattern Generation**
 - ◆ Boolean difference *
 - ◆ Path sensitization **
 - ◆ D-Algorithm [Roth 1966]**
 - ◆ PODEM [Goel 1981]**
 - ◆ FAN [Fujwara 1985]**
 - ◆ **SAT-based [Larrabee 92] ***
- Acceleration techniques
- Concluding Remarks

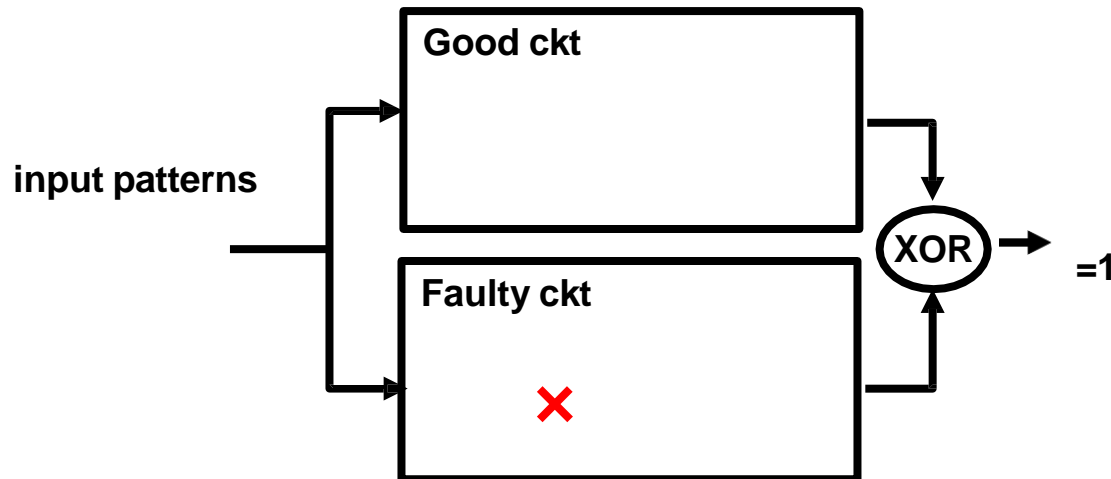
*Boolean-based methods

**path-based methods



SAT-based ATPG

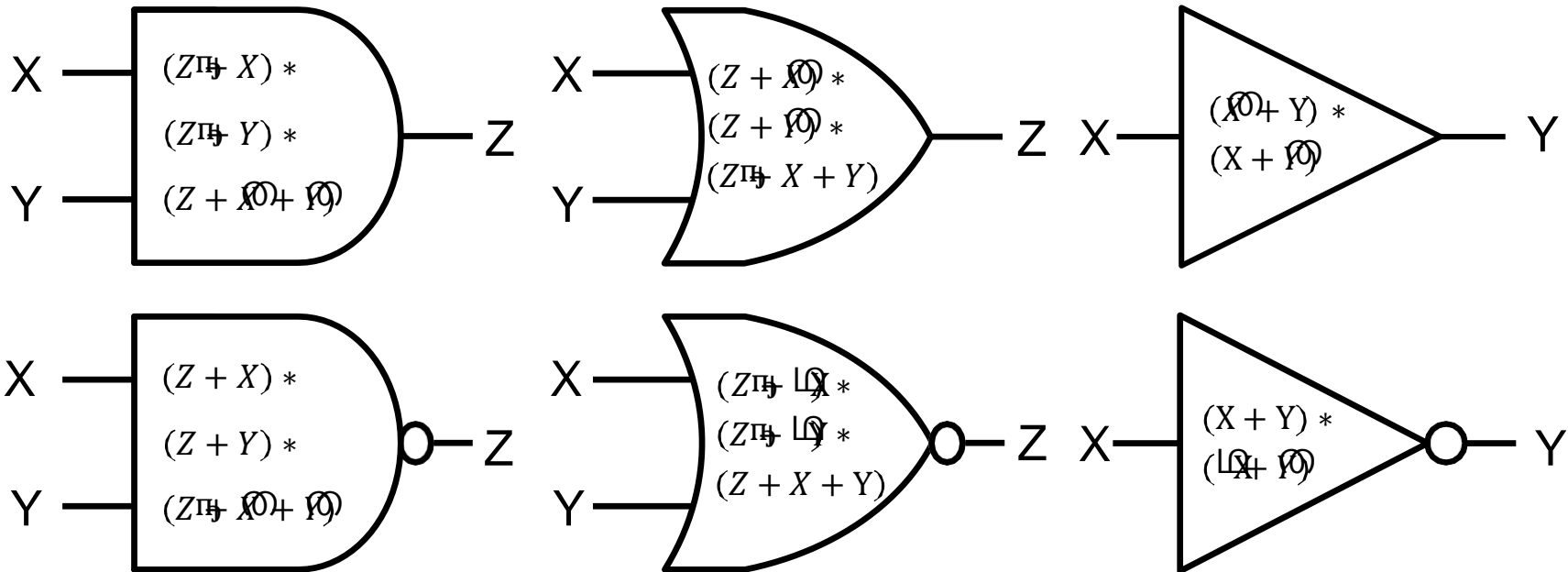
- Idea: convert test generation problem into *satisfiability problem*
 - ♦ Solve for a set of inputs such that
 - ♦ (good output) XOR (faulty output) = 1



this is called a *miter*

Tseitin Transformation [Tseitin 66]

- Converts circuit into **Conjunctive Normal Form (CNF)**
 - ♦ linear time, linear number of clauses and literals

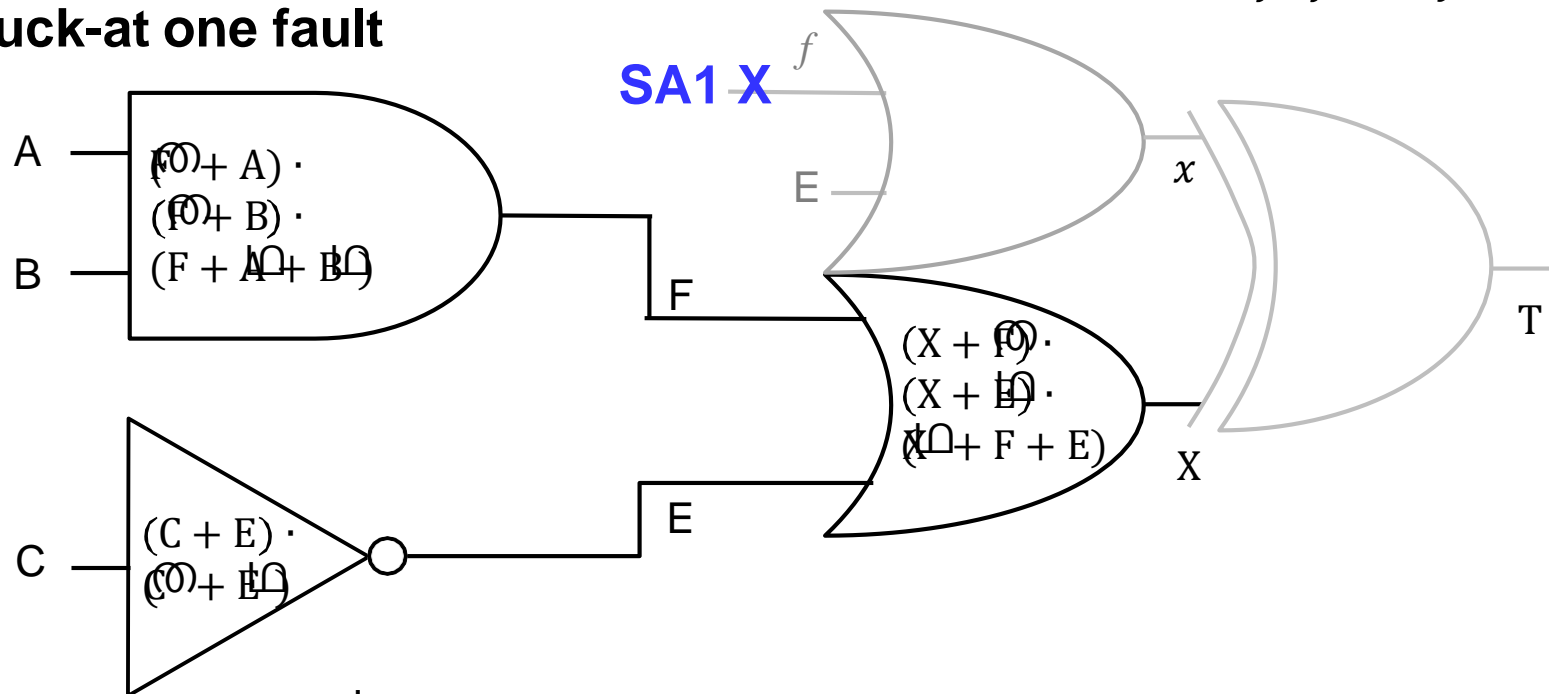


CNF for basic gates

Example (cont'd)

F good
f faulty

- F stuck-at one fault



faulty

$$(x + f) \cdot (x + 1) \cdot (x + f + E) \cdot (f)$$

good

$$\begin{aligned} &\cdot (C + E) \cdot (0 + 1) \\ &\cdot (X + 0) \cdot (X + 1) \cdot (X + F + E) \\ &\cdot (0 + A) \cdot (0 + B) \cdot (F + A + B) \end{aligned}$$

XOR

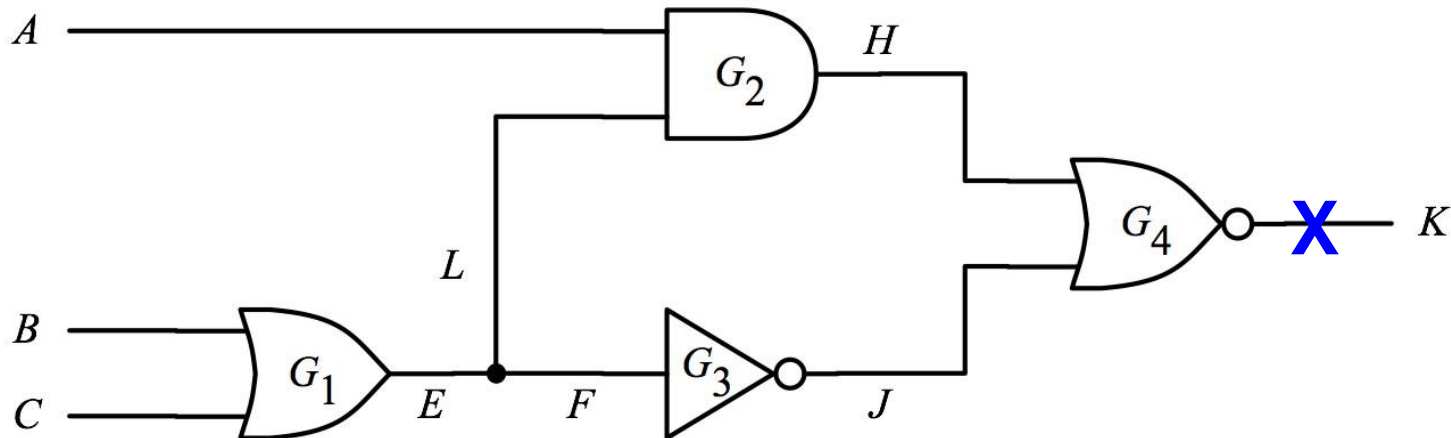
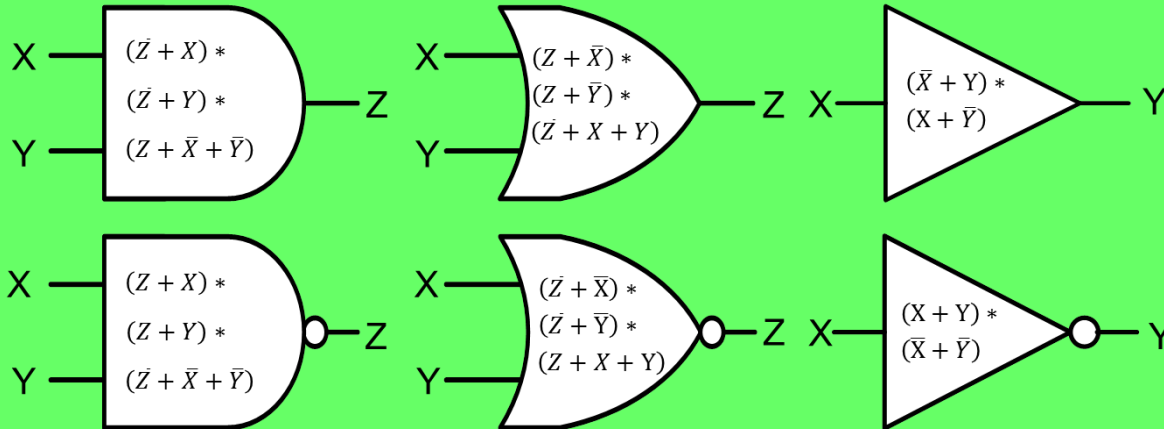
$$\begin{aligned} &\cdot (X + x + T) \cdot (X + x + 1) \cdot T \\ &\cdot (X + x + 1) \cdot (X + x + 1) \cdot T \end{aligned}$$

=1

Quiz

Q: Please write CNF to generate a test for K stuck-at one fault

A:



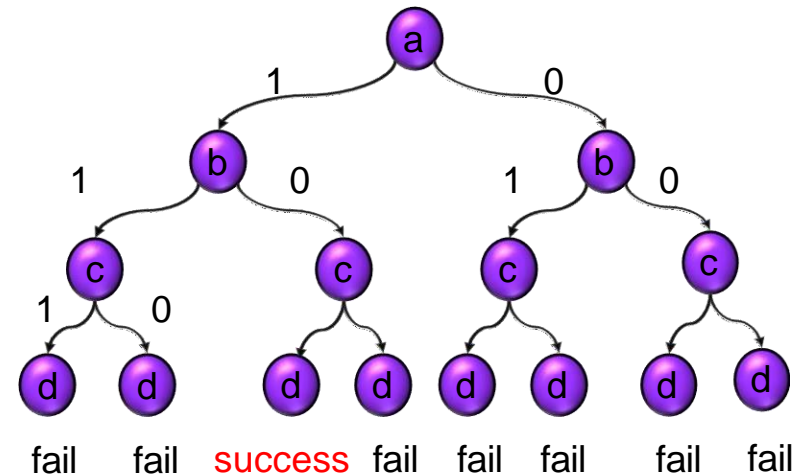
Pros and Cons

- **SAT-based ATPG**
 - + SAT engine is making big progress recently
 - + **proves untestable faults** if CNF is unsatisfiable
 - does not allow **don't cares** in input
 - needs to **redo CNF every time** a target fault is injected
 - does not preserve **circuit structure** information
 - difficult for **multi-valued logic** (such as high impedance)

SAT-based ATPG Not Used in Commercial Tool

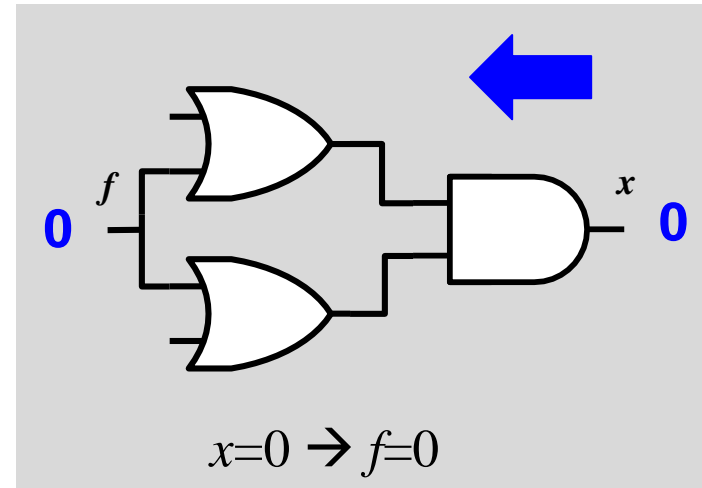
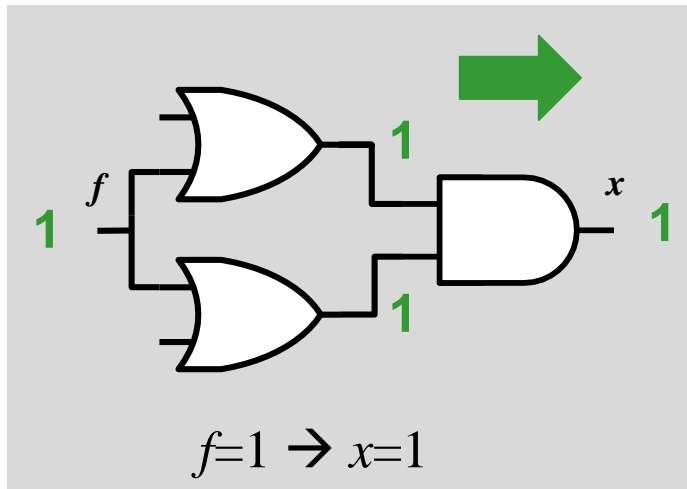
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
- **Acceleration techniques**
 - ♦ **Learning [Schulz 1988]**
 - ♦ **Redundant fault identification [Iyer 1996]**
- Concluding Remarks



Learning

- **Learning** memorizes circuit information to speed up test generation
 - ♦ **Static learning:** performed in preprocess. no test pattern required.
 - ♦ **Dynamic learning:** performed in test generation (not in lecture)
- Static learning example: WWW Fig 4.22
 - ♦ Set $f=1$, implies $x=1$
 - ♦ So we learn $x=0$, implies $f=0$

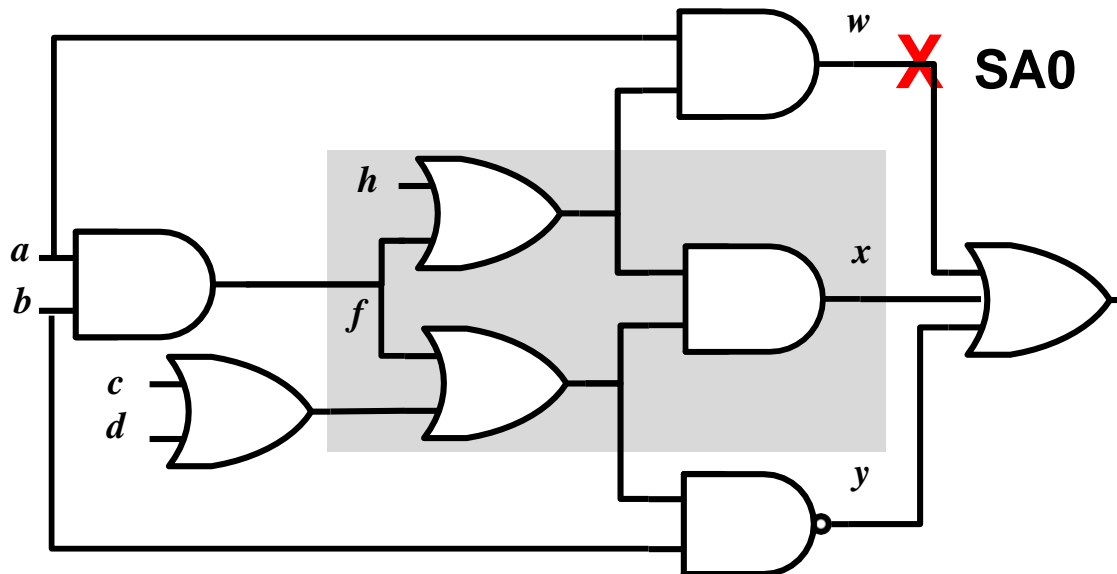


Contrapositive Law: $If\ p \rightarrow q\ then\ q' \rightarrow p'$

Learning Reduces Backtracks (1)

without learning

- Obj : $w=1$. assign $h=1$, $a=1$
- Obj : $x=0$. assign $c=0$, $d=0$, $b=0$
 - ♦ $y=1$ conflict!
- backtrack $b=1$
 - ♦ $x=1$ conflict!
- backtrack $d=1$, conflict!
- backtrack $c=1$, conflict!
- backtrack $a=0$, conflict!
- backtrack $h=0$ NO TEST!



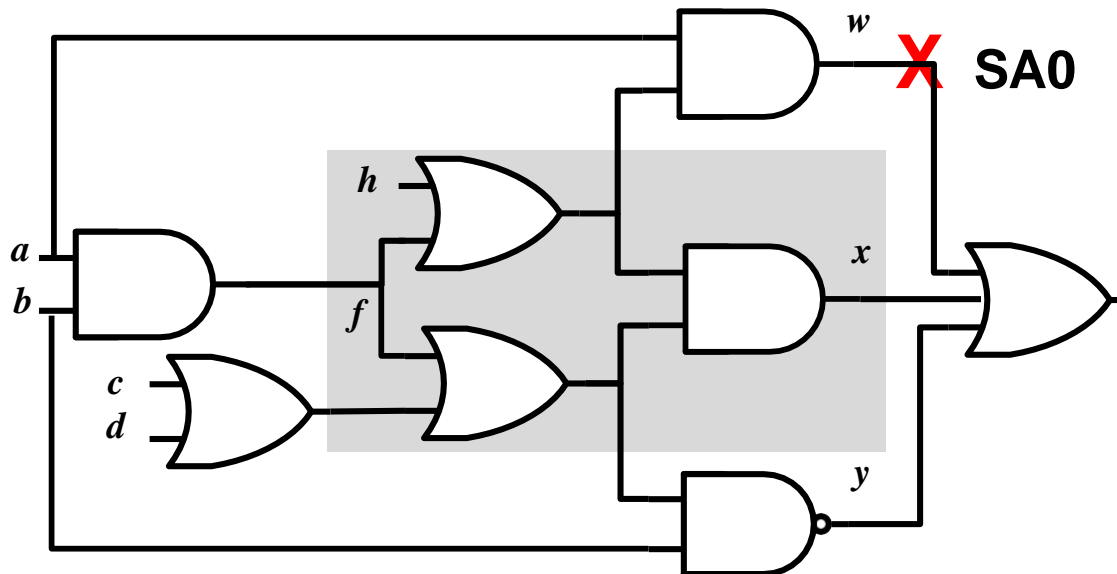
Learning Reduces Backtracks (2)

without learning

- Obj : $w=1$. assign $h=1$, $a=1$
- Obj : $x=0$. assign $c=0$, $d=0$, $b=0$
 - ♦ $y=1$ conflict!
- backtrack $b=1$
 - ♦ $x=1$ conflict!
- backtrack $d=1$, conflict!
- backtrack $c=1$, conflict!
- backtrack $a=0$, conflict!
- backtrack $h=0$ NO TEST!

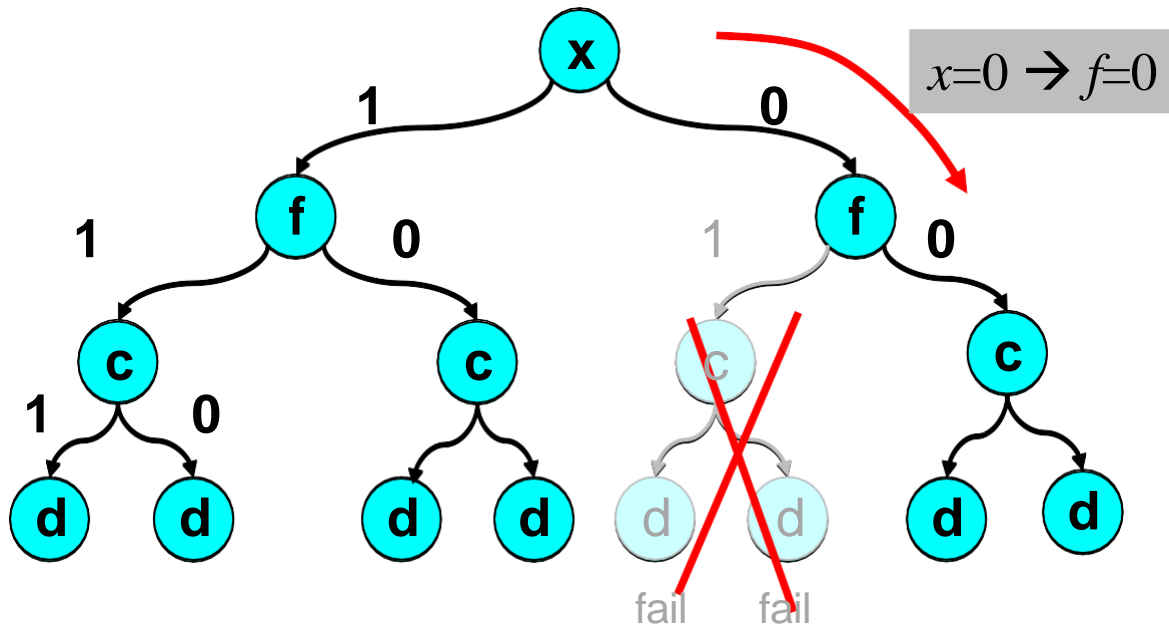
with learning

- Obj : $w=1$. assign $h=1$, $a=1$
- Obj : $x=0 \rightarrow f=0$. assign $b=0$
 - ♦ $y=1$ conflict!
- backtrack $b=1$
 - ♦ $x=1$ conflict!
- backtrack $a=0$, conflict!
- backtrack $h=0$... NO TEST!



Learning Speedup Decision

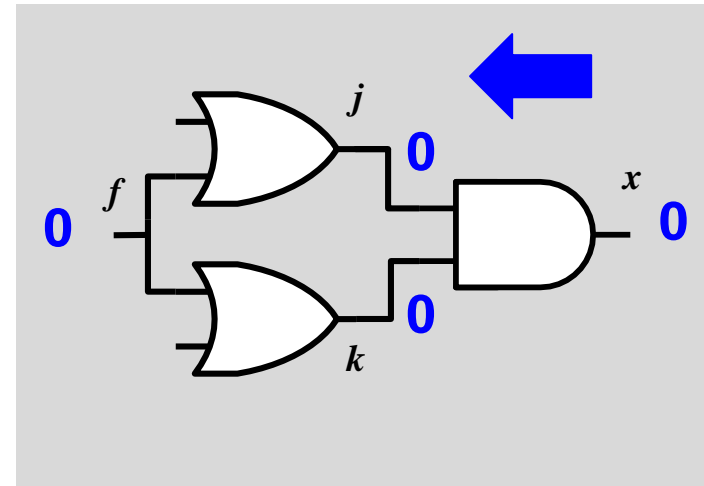
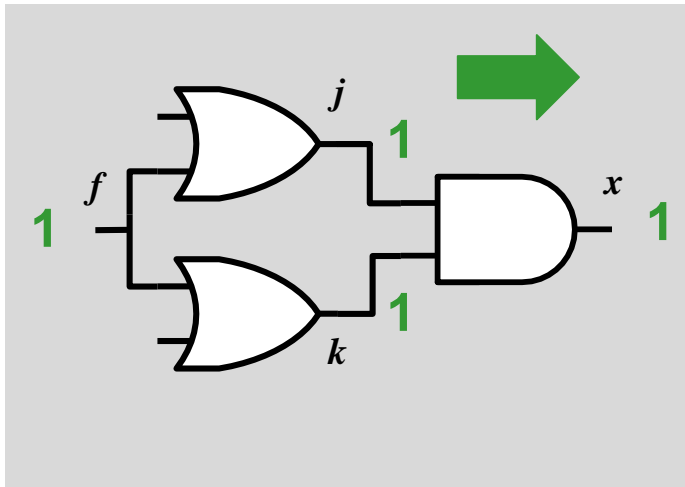
- Learning helps to
 - ♦ 1. Prune impossible sub-trees ASAP
 - ♦ 2. Find necessary assignments ASAP



Learning Trade-off Memory for Run Time

But ... Too Many to Learn!

- $f=1 \rightarrow j=1$ so $j=0 \rightarrow f=0$
 - ♦ Only local implication. not worth learning
- $f=1 \rightarrow k=1$ so $k=0 \rightarrow f=0$
 - ♦ Only local implication. not worth learning
- $f=1 \rightarrow x=1$ so $x=0 \rightarrow f=0$
 - ♦ global implication. **worth learning!**



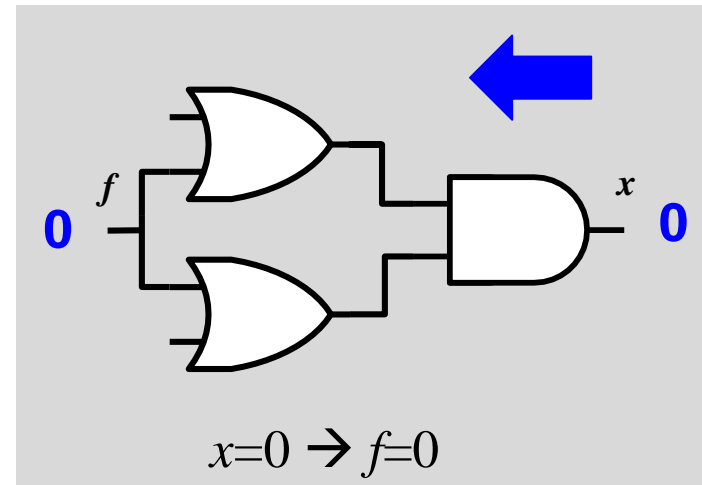
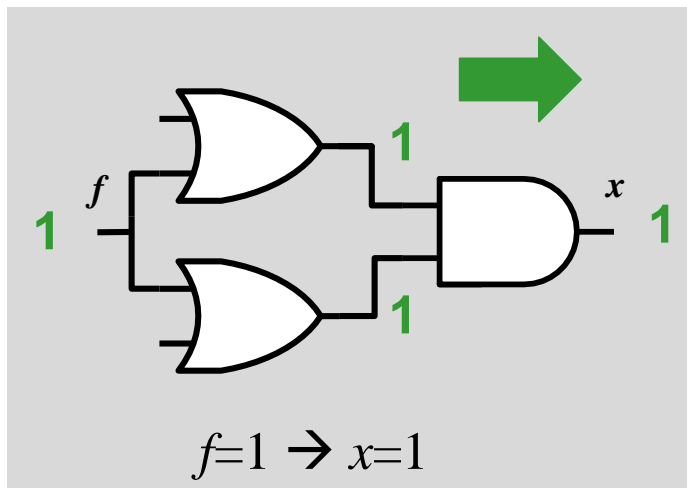
Which Are Worth Learning?

SOCRATES Algorithm [Schulz 1988]

- In preprocess phase, sets **all signals** to 0 and 1
 - ♦ Discovers what other signals are implied
 - ♦ **Two criteria** to select **useful** learning

```
analyze_results(f)
  for every signal x whose value  $\neq$  unknown
    if (all gate inputs to x are non-controlling)
      & (there is forward path from  $f$  to  $x$ )
        save learning result ( $x=v_x' \rightarrow f=v_f'$ )
```

```
static_learning()
  for every signal  $f$ 
    assign_value( $f, 0$ )
    implication()
    analyze_results( $f$ )
    assign_value( $f, 1$ )
    implication()
    analyze_results( $f$ )
```



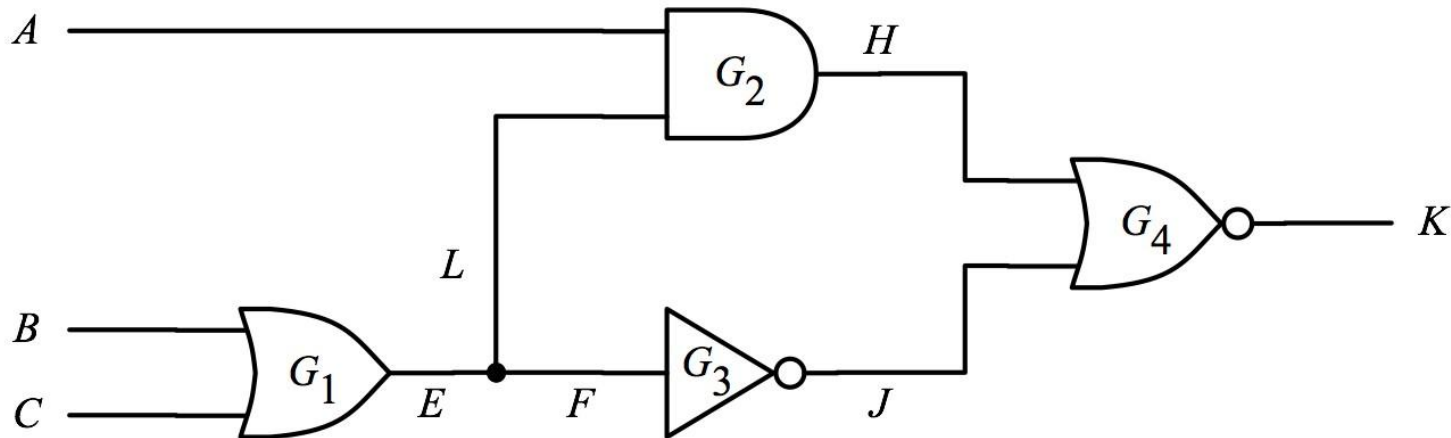
Quiz

Q1: Set $E = 0$. What can you learn about K using contrapositive law?

A:

Q2: (Cont'd) If we want to detect K SA 0 fault, what is value of E ?

A:

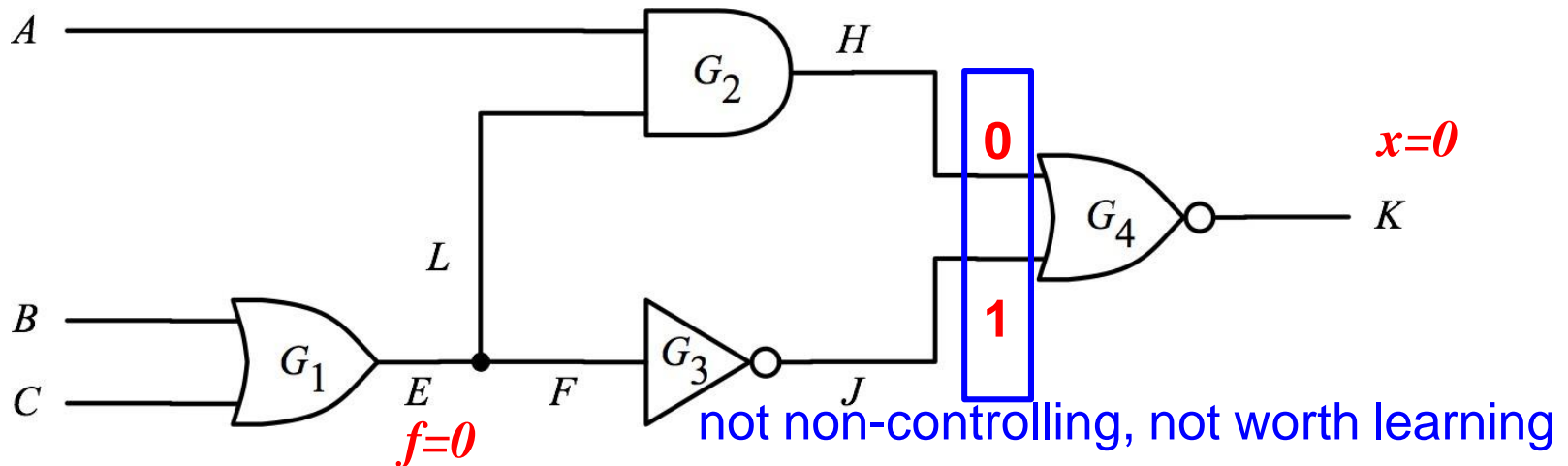


FFT

- Q1: Why SOCRATES requires both inputs are non-controlling?
 - ♦ Hint: use following circuit as example
- Q2: Why forward path from f to x ?

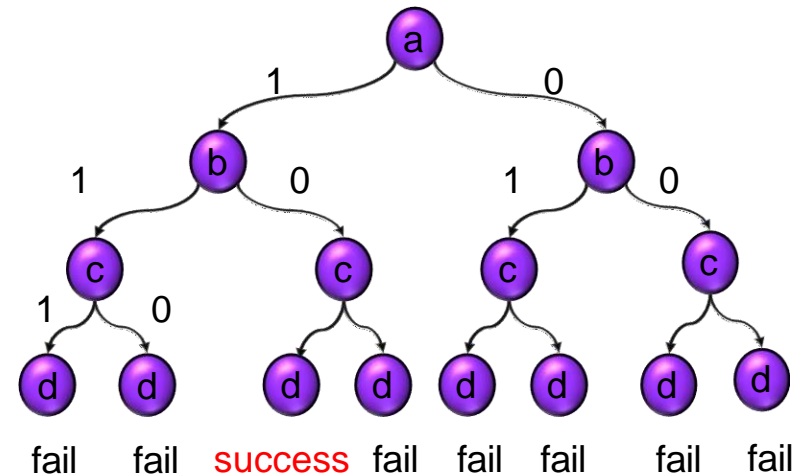
```
analyze_results(f)
  for every signal x whose value  $\neq$  unknown
    if (all gate inputs to x are non-controlling)
      & (there is forward path from f to x)
        save learning result ( $x=v_x' \rightarrow f=v_f'$ )
```

```
static_learning()
  for every signal f
    assign_value(f, 0)
    implication()
    analyze_results(f)
    assign_value(f, 1)
    implication()
    analyze_results(f)
```



Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
- **Acceleration techniques**
 - ♦ Learning [Schulz 1988]
 - ♦ **Redundant fault identification [Iyer 1996]**
- Concluding Remarks

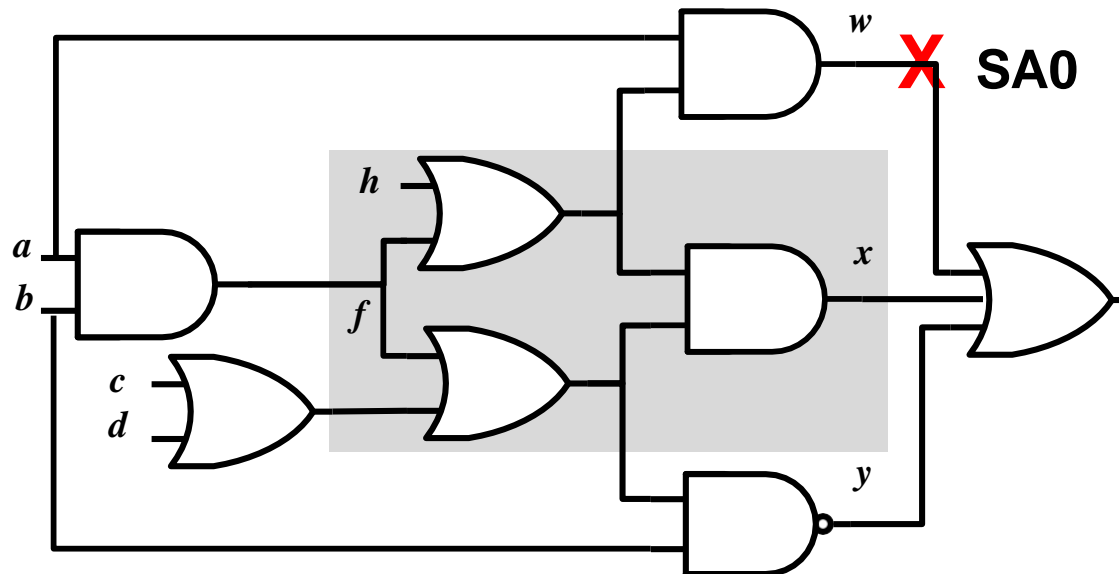


Proving Redundant Fault is Difficult

without learning

- Obj : $w=1$. assign $h=1, a=1$
- Obj : $x=0$. assign $c=0, d=0, b=0$
 - ♦ $y=1$ conflict!
- backtrack $b=1$
 - ♦ $x=1$ conflict!
- backtrack $c=1$, conflict!
- backtrack $d=1$, conflict!
- backtrack $a=0$, conflict!
- backtrack $h=0$ NO TEST!

**Can We Find
Redundant Fault
Faster?**



Redundant Fault Identification

- **Untestable faults** (aka. **redundant faults**)
 - ♦ faults that cannot be **excited**, or
 - ♦ faults that cannot be **propagated**, or
 - ♦ faults that cannot be **simultaneously** excited and propagated
- Why redundant fault identification?
 - ① Speed up ATPG
 - * ATPG spend long time on untestable faults
 - ② Reduce area
 - * Redundant logic can be removed
- To prove redundant fault is **NP-complete**
 - ♦ Quickly identify many redundant fault (not all) is good enough

Redundant Faults are Trouble for ATPG

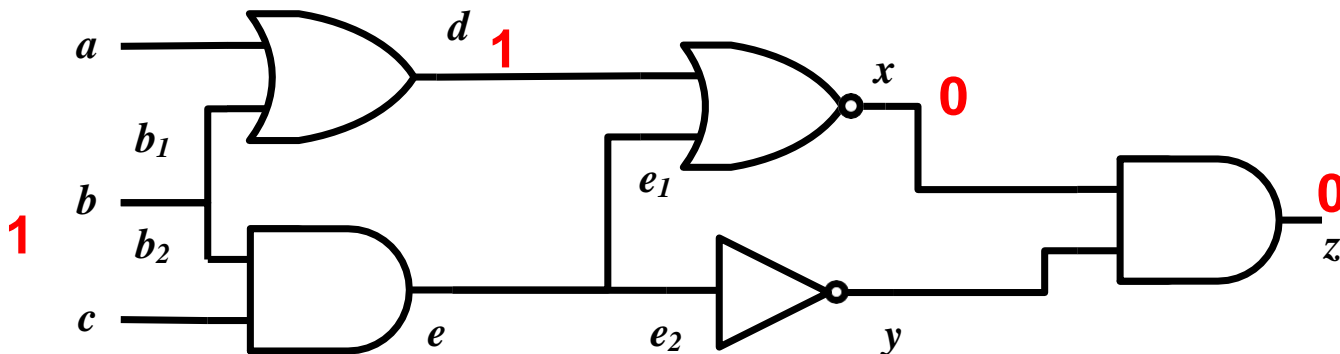
FIRE [Iyer 1996]

- **Fault Independent Redundant Identification (FIRE)**
 - ◆ based on single-line conflict analysis
- **Idea**
 - ◆ **S0** = set of faults untestable when signal $s=0$
 - ◆ **S1** = set of faults untestable when signal $s=1$
 - ◆ intersection **S0** \cap **S1** are untestable faults
- To find unexcitable faults
 - ◆ use **forward implication**
- To find unobservable faults
 - ◆ use **backward tracing**
- Advantage: close to **linear time** complexity (for one signal)
 - ◆ FFT: can we solve NPC problem in linear time?

FIRE Example

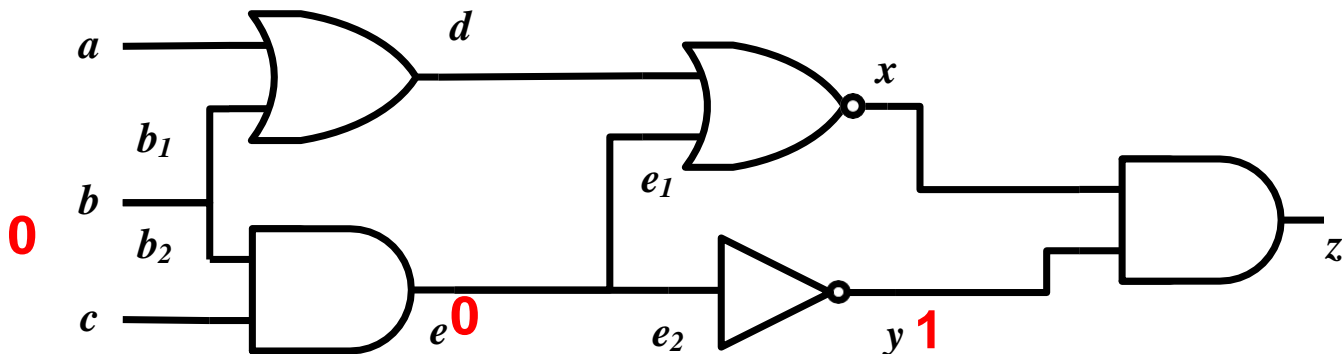
- set $b=1$ implies $\rightarrow \{b=1, b_1=1, b_2=1, d=1, x=0, z=0\}$
 - ♦ Faults unexcitable when $b=1$: $\{b/1, b_1/1, b_2/1, d/1, x/0, z/0\}$
 - ♦ Faults unobservable when $b=1$: $\{a/0, a/1, e_1/0, e_1/1, y/0, y/1, e_2/0, e_2/1, e/0, e/1, b_2/0, b_2/1, c/0, c/1\}$
 - * Q: why $b/0$ not in the list?
 - ♦ Faults untestable when $b=1$: union of above two sets
 - * **S1** = $\{a/0, a/1, b/1, b_1/1, b_2/1, d/1, e_1/0, e_1/1, e_2/0, e_2/1, e/1, e/1, x/0, y/0, y/1, z/0, b_2/0, c/0, c/1\}$

$b/1 = b$ SA1 fault



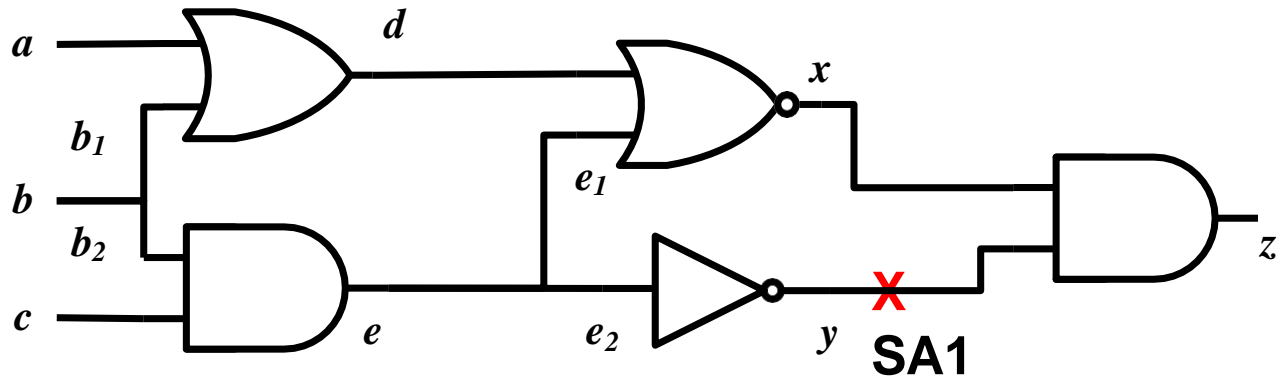
FIRE Example (cont'd)

- set $b=0$ implies $\rightarrow \{b=0, b_1=0, b_2=0, e=0, e_1=0, e_2=0, y=1\}$
 - ◆ Faults unexcitable when $b=0$: $\{b/0, b_1/0, b_2/0, e/0, e_1/0, e_2/0, y/1\}$
 - ◆ Faults unobservable when $b=0$: $\{c/0, c/1\}$
 - ◆ Faults untestable when $b=0$: union of above two sets
 - * $S_0 = \{b/0, b_1/0, b_2/0, c/0, c/1, e/0, e_1/0, e_2/0, y/1\}$
 - * $S_1 = \{a/0, a/1, b/1, b_1/1, b_2/1, d/1, e_1/0, e_1/1, e_2/0, e_2/1, e/1, e/1, x/0, y/0, y/1, z/0, b_2/0, c/0, c/1\}$
- Intersection of S_1 and S_0 are untestable faults
 - ◆ $S_1 \cap S_0 = \{b_2/0, c/0, c/1, e/0, e_1/0, e_2/0, y/1\}$



FFT

- Q1: Use PODAM to find a test for **y/1** fault. Prove y/1 is untestable.
- Q2: Can we use linear time algorithm to solve NPC problem?



Quiz

Q1: Set B=0. Find set of faults undetectable.

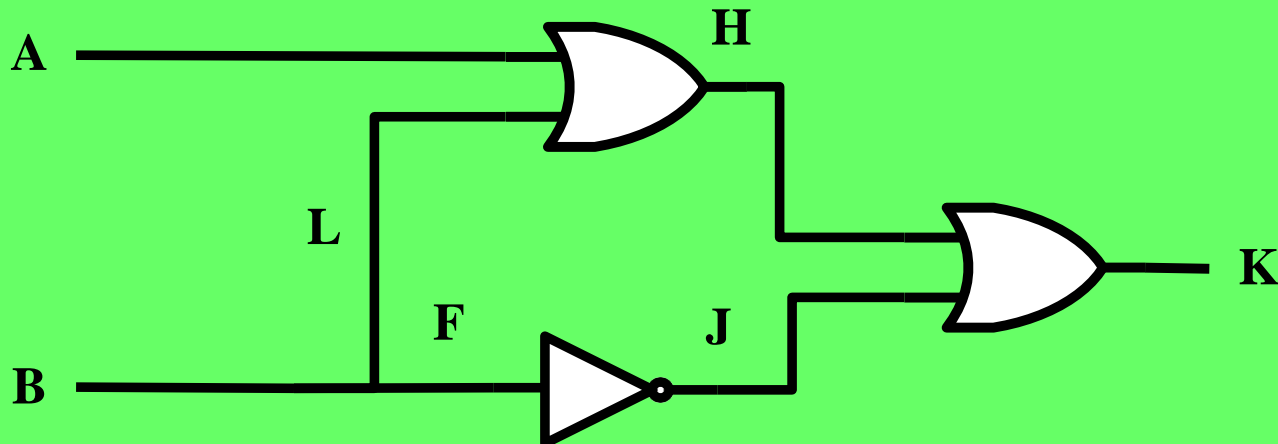
A: faults unexcitable = { }
 faults unobservable = { }

Q2: Set B=1. Find set of faults undetectable.

A: faults unexcitable = { }
 faults unobservable = { }

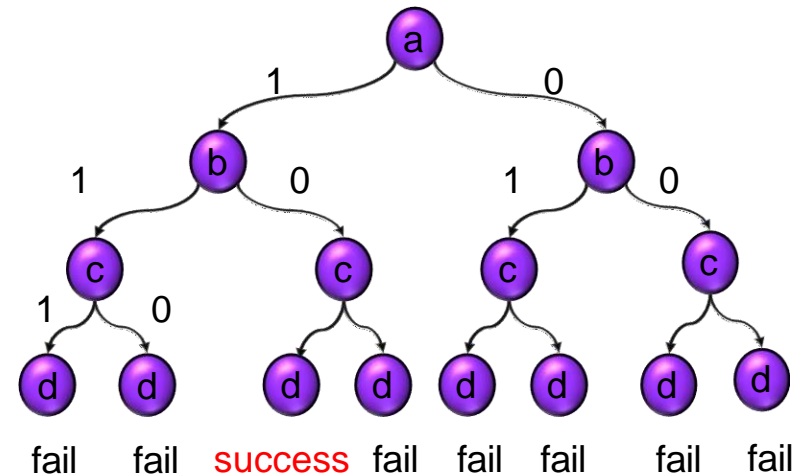
Q3: (cont'd) Which faults are redundant ?

A: $Q1 \cap Q2 = \{ \}$



Combinational ATPG

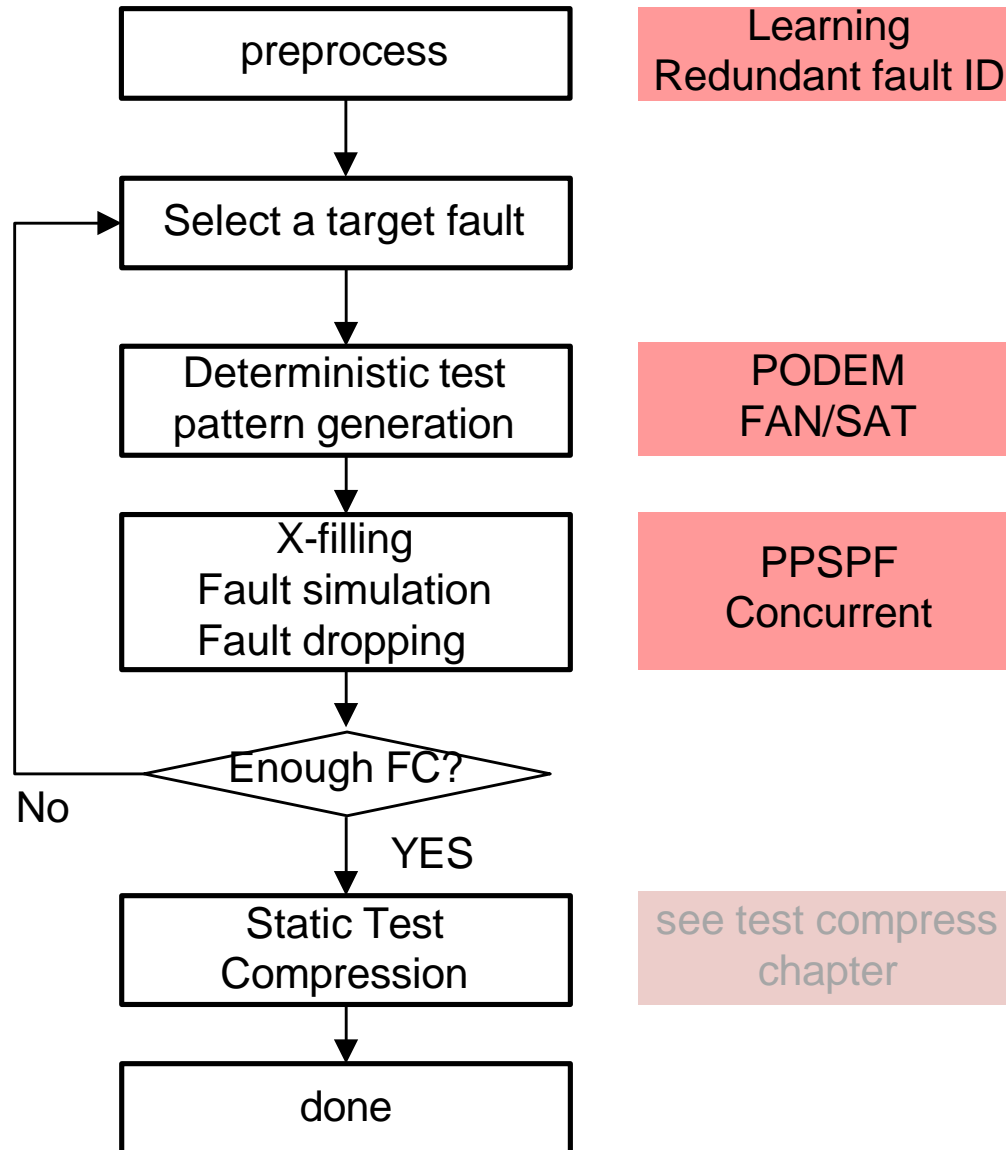
- Introduction
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Summary

- ATPG is NPC. Many acceleration techniques needed
- **Static learning (SOCRATES)** trade off memory for run time
 - ♦ Setting all signals to 0 and 1. imply other signals
 - ♦ Apply contrapositive law: if $p \rightarrow q$ then $q' \rightarrow p'$
- **Redundant fault identification (FIRE)**
 - ♦ Setting a signal to 0 and 1
 - * unexcitable faults \cup unobservable faults
 - ♦ Redundant fault = $S1 \cap S0$
- Many other acceleration techniques
 - ♦ Dominator (TOPS) [Kirkland 87]
 - ♦ Recursive learning [Kunz 92]
 - ♦ Transitive Closure Graph (NNATPG) [Chakradhar 93]
 - ♦ ...

ATPG Review



How to Read ATPG Report?

	Uncollapsed	Collapsed
Total Faults	1234	800
Detected faults	1000	700
Redundant faults	230	98
Aborted faults	4	2
Fault coverage	1000/1234 %	700/800 %
ATPG effectiveness	1230/1234 %	798/800 %
Test Length	328 patterns	
Run Time	10:57	

proven redundant by ATPG

undetected. not sure redundant or not.

$\frac{\text{detected faults}}{\text{total faults}} \times 100\%$

$\frac{\text{detected} + \text{redundant faults}}{\text{total faults}} \times 100\%$

less are better

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Commercial Tools

- **Mentor Graphics**
 - ◆ **Fastscan**
- **Synposys**
 - ◆ **Tetramax**
- **Syntest**
 - ◆ **Turboscan**
- **Cadence**
 - ◆ **Encounter Test**