UEC747: ANTENNA AND WAVE PROPAGATION

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Lecture 11: Antenna Parameters

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2.6 Directive Gain

Directivity - The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \qquad \left(\because U_0 = \frac{P_{rad}}{4\pi} \right)$$

$$D_{\max} = D_0 = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}$$

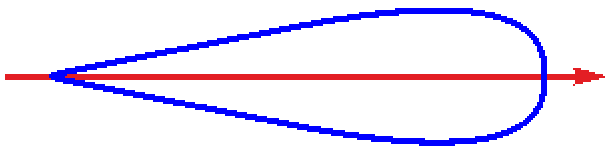
D = directivity (dimensionless)

D_0 = maximum directivity (dimensionless)

U = radiation intensity (W/unit solid angle)

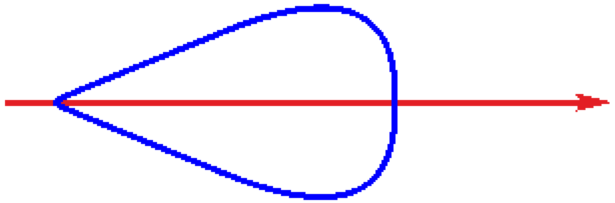
$U_{\max}$ = radiation intensity of isotropic source (W/unit solid angle)

P_{rad} = total radiated power (W)



More gain
narrower beamwidth

Only major
lobe shown



Less gain
wider beamwidth

$$D = \frac{U(\theta,\phi)}{U_o} = \frac{4\pi U(\theta,\phi)}{P_{rad}} \quad (2\text{-}16)$$

$$D_{\max} = D_o = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{rad}} \quad (2\text{-}16a)$$

$$\boxed{D(dB) = 10\log_{10}[D(\text{dimensionless})]}$$

$$D = \text{directivity (dimensionless)}$$

$$D_0 = \text{maximum directivity (dimen.)}$$

$$U = \text{radiation intensity (W/unit solid angle)}$$

$$U_{\max} = \text{maximum radiation intensity}$$

$$U_o = \text{radiation intensity of isotropic}$$

$$P_{rad} = \text{radiated power (W)}$$

Partial Directivities

$$U \simeq \frac{1}{2\eta}\left[\underbrace{\left|E_\theta^\circ(\theta,\phi)\right|^2}_{} + \underbrace{\left|E_\phi^\circ(\theta,\phi)\right|^2}_{}\right]$$

$$U = U_\theta + U_\phi$$

Partial Directivities

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\pi}\left[U_\theta + U_\phi\right]\sin\theta d\theta d\phi$$

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{4\pi\left(U_\theta + U_\phi\right)}{P_{rad}}$$

$$= \underbrace{\frac{4\pi U_\theta}{P_{rad}}} + \underbrace{\frac{4\pi U_\phi}{P_{rad}}}$$

$$D = \quad D_\theta \quad + \quad D_\phi$$

$D_\theta, D_\phi =$ are the partial directivities

$$U = U_\theta + U_\phi$$

$$D_0 = D_\theta + D_\phi \tag{2-17}$$

$$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi} \tag{2-17a}$$

$$D_\phi = \frac{4\pi U\phi}{(P_{rad})_\theta + (P_{rad})_\phi} \tag{2-17b}$$

2.6 Directivity

☺ isotropic source

$$D = 1 \quad \left(\because U, U_{\max}, \textit{ and } U_0 \textit{ are all equal to each other.}\right)$$

Partial directivity of an antenna for a given polarization in a given direction

- That part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions.

$$D_0 = D_\theta + D_\phi$$

$$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi} \quad , \quad D_\phi = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$

U_θ = radiation intensity in a given direction contained in θ field component

U_ϕ = radiation intensity in a given direction contained in ϕ field component

$(P_{rad})_\theta$ = radiated power in all directions contained in θ field component

$(P_{rad})_\phi$ = radiated power in all directions contained in ϕ field component

2.6 Directivity

- Directivity of an isotropic source = 1
(its power is radiated equally well in all directions)

- All other sources, the maximum directivity will always be greater than 1.
(an indication of the directional properties of the antenna as compared with those of an isotropic source)

- Directivity can be smaller than 1; in fact it can be equal to zero.

- The values of directivity will be equal to or greater than zero and equal to or less than the maximum directivity.

$$0 \leq D \leq D_0$$

Example 2.5:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

Solution:

$$P_{rad} = \pi^2 A_0$$

$$U = r^2 W_{rad} = A_0 \sin\theta$$

$$U_{max} = U\big|_{max} = A_0 \sin\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(1)A_0}{\pi^2 A_0} = 1.27 = \underbrace{1.038\ dB}$$

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Example 2.6:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin^2\theta}{r^2}$$

Solution:

$$P_{rad} = \frac{8\pi}{3}$$

$$U = r^2 W_{rad} = A_0 \sin^2\theta$$

$$U_{max} = U\big|_{max} = A_0 \sin^2\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{8\pi/2} = 1.5 = \underbrace{1.761\ dB}$$

$$D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

Three-Dimensional Radiation Patterns

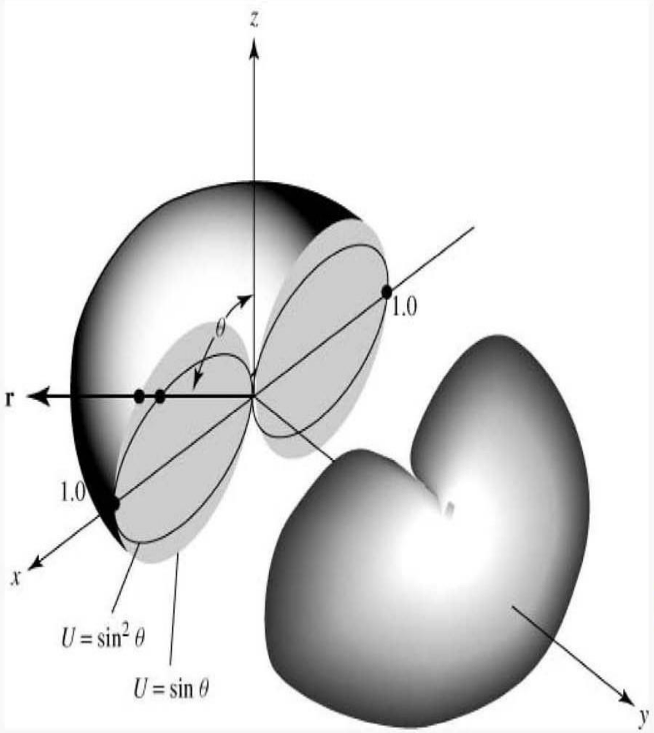


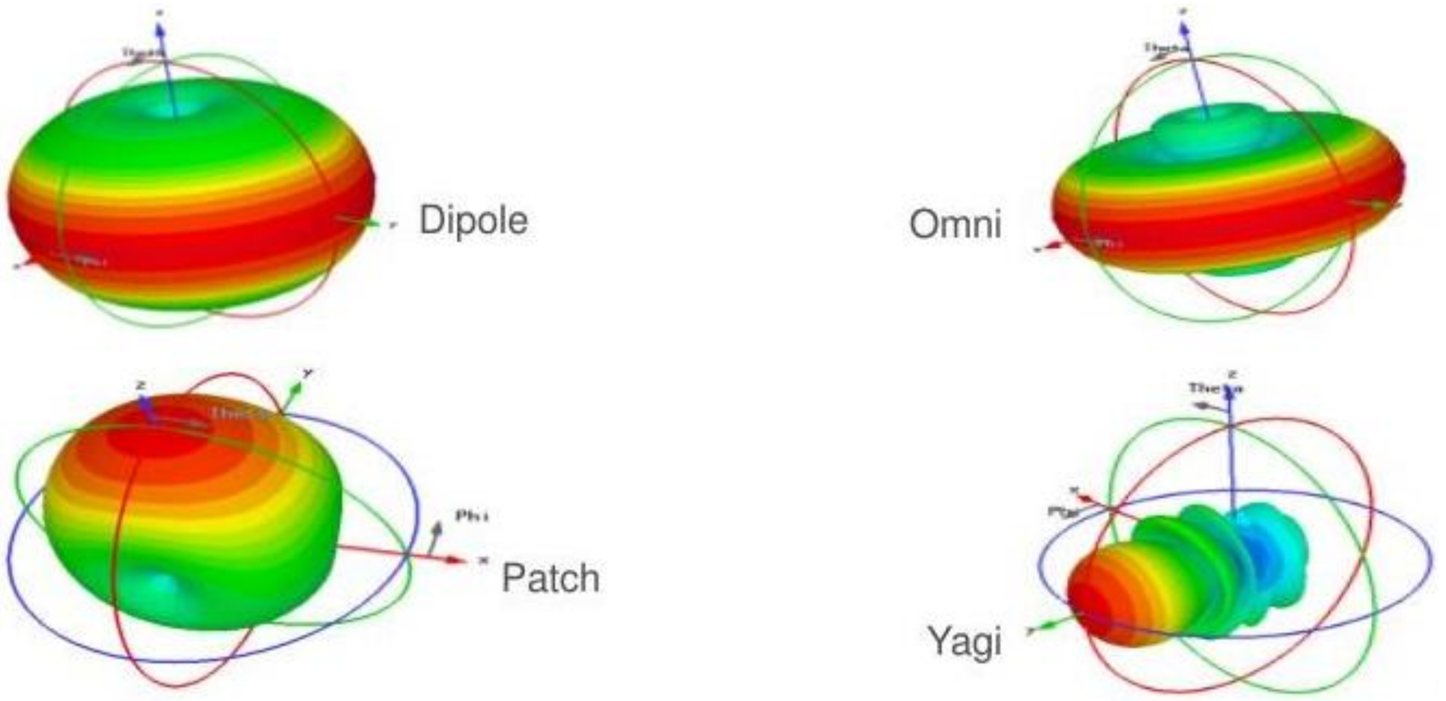
Fig. 2.12

2.6 Directivity

Half-wavelength dipole ($\ell = \lambda/2$)

$$\sin^3\theta \approx \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$

$$D = D_0 \sin^3\theta = 1.67\sin^3\theta$$



$$1.27 \sin\theta\Big| = 1$$

$$\theta = \theta_{\mathrm{o}}$$

$$\sin\theta_o = 1/1.27 = 0.787$$

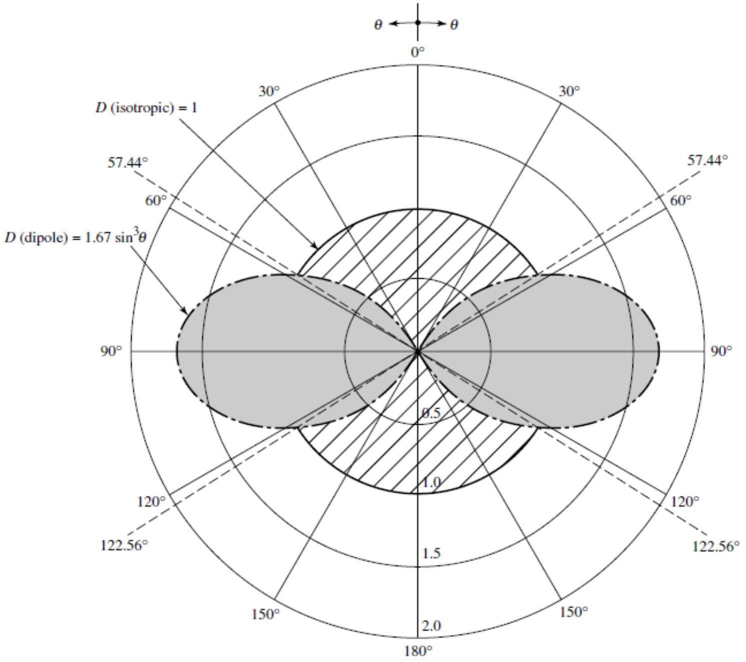
$$\theta_o = \sin^{-1}(0.787) = 51.94°$$

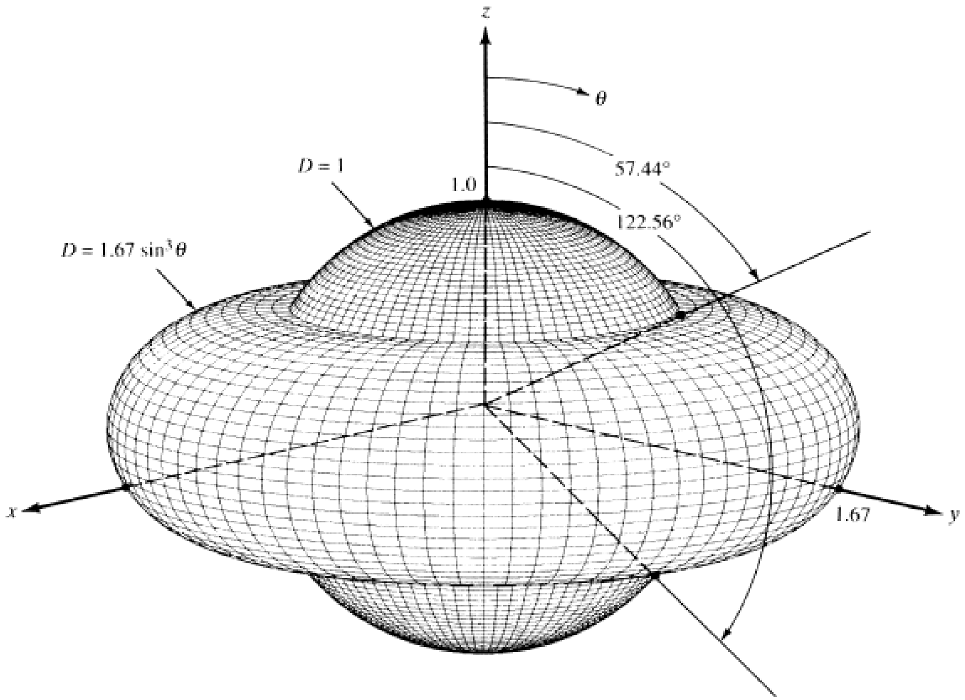
$$1.5 \sin^2\theta_o = 1$$

$$\theta_o = \sin^{-1}(1/1.5)^{1/2} = \sin^{-1}(0.667)^{1/2}$$

$$\theta_o = \sin^{-1}(0.8165) = 54.74°$$

2.6 Directivity





$$D = D_0 \sin^3\theta = 1.67\sin^3\theta \qquad \sin^{-1}\left(\frac{1}{1.67}\right)^{\frac{1}{3}} = 57.44° < \theta < 122.56°$$

The dipole radiator has greater directivity (greater intensity concentration) in those directions than that of an isotropic source.

Summary

1. $U = A_o \quad \Rightarrow P_{rad} = 4\pi A_o$ (isotropic)
2. $U = A_o \sin\theta \quad \Rightarrow P_{rad} = \pi^2 A_o$ (no specific one)
3. $U = A_o \sin^2\theta \quad \Rightarrow P_{rad} = \dfrac{8\pi}{3} A_o$ (infinitesimal dipole)
4. $U = A_o \sin^3\theta \quad \Rightarrow P_{rad} = \dfrac{3\pi^2}{4} A_o$ ($\lambda/2$ dipole)

Summary

1. $D = \dfrac{4\pi U}{P_{rad}} = 1 = D_o$ (isotropic)

2. $D = \dfrac{4}{\pi} \sin\theta = 1.27 \ \sin\theta = D_o \sin\theta$ (no specific one)

3. $D = \dfrac{3}{2} \ \sin^2\theta = 1.5 \ \sin^2\theta = D_o \sin^2\theta$ (infinitesimal dipole)

4. $D = 1.67 \ \sin^3 \ \theta = D_o \sin^3\theta$ (λ/2 dipole)

Far-Field

$$U = B_o F(\theta,\phi) = U_\theta + U_\phi$$

$$\simeq \frac{r^2}{2\eta}\left[\left|E_\theta(r,\theta,\phi)\right|^2 + \left|E_\phi(r,\theta,\phi)\right|^2\right]$$

Far-Field:

$$E_{\theta,\phi}(r,\theta,\phi) \simeq \frac{e^{-jkr}}{r} E^o_{\theta,\phi}(\theta,\phi)$$

$$\boxed{U \simeq \frac{1}{2\eta}\left[\left|E^\circ_\theta(\theta,\phi)\right|^2 + \left|E^\circ_\phi(\theta,\phi)\right|^2\right]} \qquad (2\text{-}19)$$

Summary

1. $D = \dfrac{U(\theta,\phi)}{U_0} = \dfrac{4\pi U(\theta,\phi)}{P_{rad}}$

2. $D_o = D_{\max} = \dfrac{U_{\max}}{U_0} = \dfrac{4\pi U_{\max}}{P_{rad}}$

Far-Field

3. $\underline{W}_{rad} \simeq \hat{a}_r \dfrac{1}{2\eta r^2}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right]$

4. $U = r^2 W_{rad} \simeq \dfrac{1}{2\eta}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right]$

General Formulation of Directivity

$$U(\theta,\phi) = B_o F(\theta,\phi)$$

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

$$D_o = \frac{4\pi U_{\max}}{P_{rad}}$$

$$U_{\max} = U(\theta,\phi)\Big|_{\max} = B_o F_{\max}(\theta_m,\phi_m)$$

$$P_{rad} = \oiint_S U(\theta,\phi)\, d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \sin\theta\, d\theta\, d\phi$$

(2-20)

$$D_0 = 4\pi \frac{B_0 F_{\mathrm{max}}(\theta_{\mathrm{m}},\phi_{\mathrm{m}})}{B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \sin\theta\, d\theta\, d\phi}$$

$$D_0 = \frac{4\pi F_{\mathrm{max}}(\theta_{\mathrm{m}},\phi_{\mathrm{m}})}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \sin\theta\, d\theta\, d\phi} \qquad (2\text{-}22)$$

$$D_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \underbrace{\frac{F(\theta,\phi)}{F_{\max}(\theta_m,\phi_m)}}_{F_n(\theta,\phi)} \sin\theta \, d\theta \, d\phi}$$

$$D_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi) \sin\theta \, d\theta \, d\phi} = \frac{4\pi}{\Omega_A} \qquad (2\text{-}23)$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi) \sin\theta \, d\theta \, d\phi = \text{Beam solid angle} \qquad (2\text{-}24)$$

2.6 Directivity

Ex) $$U = B_0 F(\theta,\phi) \propto \frac{1}{2\eta}\left[\left|E_\theta^0(\theta,\phi)\right|^2 + \left|E_\phi^0(\theta,\phi)\right|^2\right]$$

B_0 - constant

E_θ^0 and E_ϕ^0 - the antenna's far-zone electric-field components

$$U_{\max} = B_0 F(\theta,\phi)\Big|_{\max} = B_0 F_{\max}(\theta,\phi)$$

$$P_{rad} = \iint_{\Omega} U(\theta,\phi)\, d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

using, $$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \quad , \quad D_{\max} = D_0 = \frac{U\big|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}$$

$$D(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi} \qquad D_0 = 4\pi \frac{F(\theta,\phi)\big|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

2.6 Directivity

$$D_0 = 4\pi \frac{F(\theta,\phi)\big|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi}$$

$$D_0 = \frac{4\pi}{\dfrac{\left[\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi\right]}{F(\theta,\phi)\big|_{\max}}} = \frac{4\pi}{\Omega_A}$$ (Ω_A is the beam solid angle)

$$\Omega_A = \frac{1}{F(\theta,\phi)\big|_{\max}} \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta \, d\theta \, d\phi$$

$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)\big|_{\max}}$$

Ω_A - Solid angle through which all the power of the antenna would flow of its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A

Non-Symmetrical Pattern

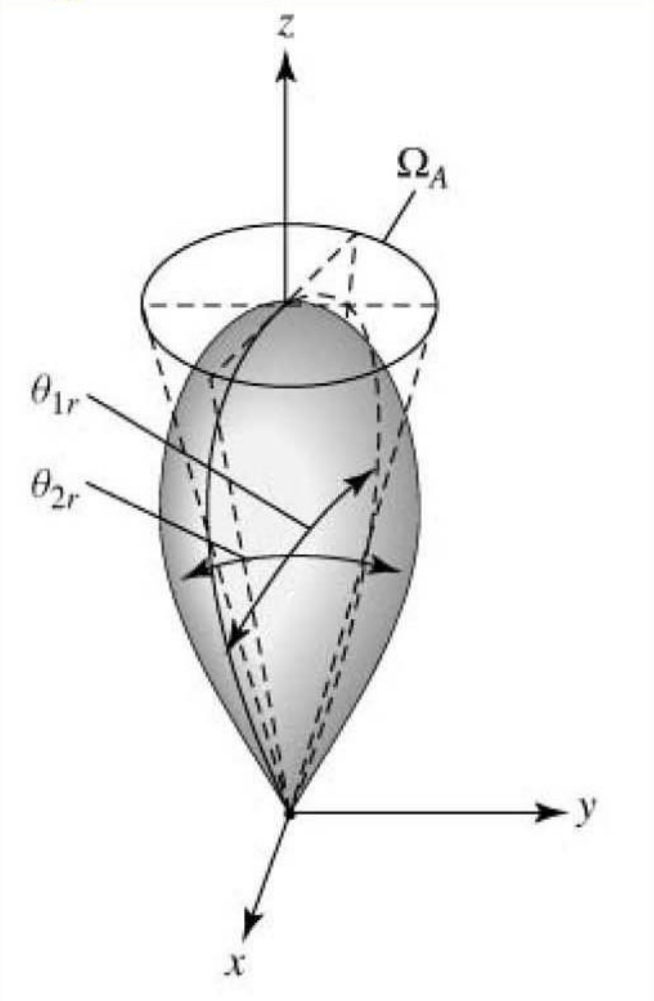


Fig. 2.14(a)

Symmetrical Pattern

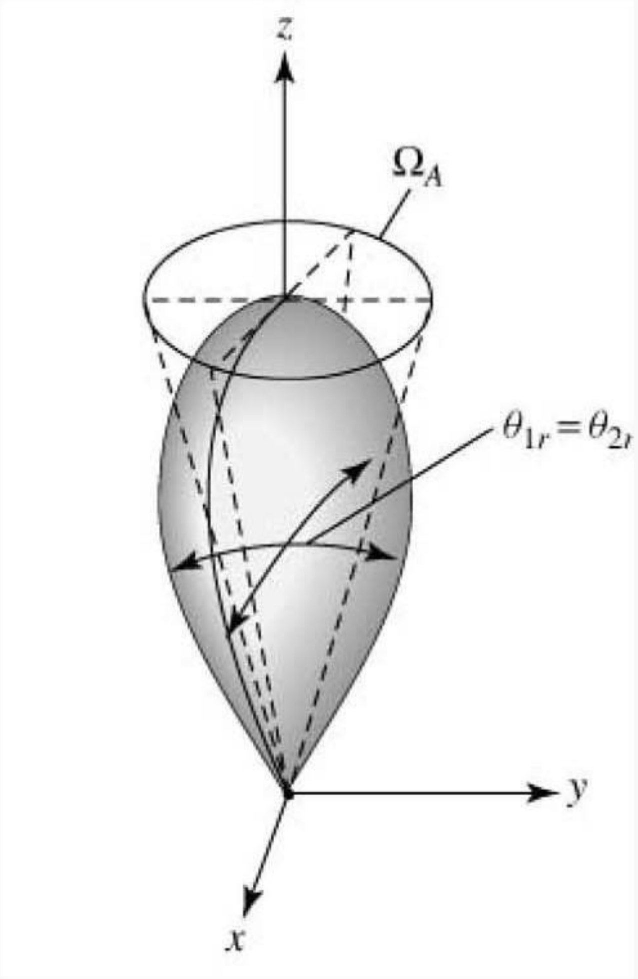


Fig. 2.14(b)

Kraus

$$\boxed{\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\, d\theta\, d\phi \simeq \Theta_{1r}\Theta_{2r}}$$

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}}} \qquad (2\text{-}26)$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi\left(180/\pi\right)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}} \qquad (2\text{-}27)$$

Tai & Pereira

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1} + \frac{1}{D_2}\right)$$ Arithmetic mean (2-29)

$$\boxed{\begin{aligned} D_0 &= \frac{4\pi}{\Omega_A} \simeq \frac{32\ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2} && \text{(2-30a)} \\ D_0 &= \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} && \text{(2-30b)} \end{aligned}}$$

Example 2.7:

$$U = \begin{cases} B_o \cos\theta & \begin{matrix} 0 \le \theta \le \pi/2 \\ 0 \le \phi \le 2\pi \end{matrix} \\ 0 & \begin{matrix} \pi/2 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{matrix} \end{cases}$$

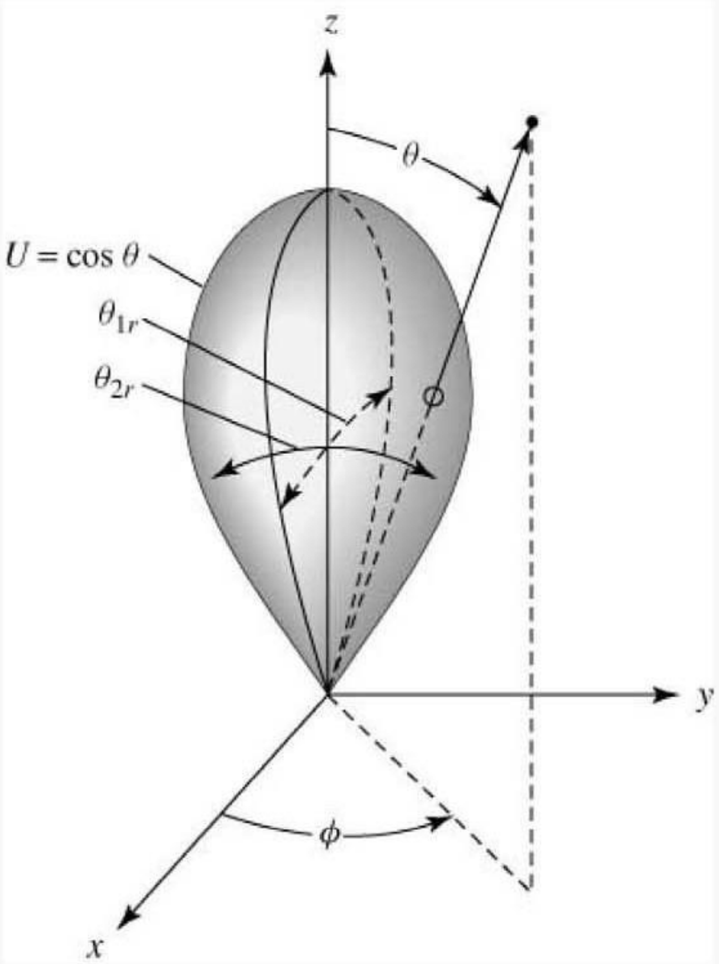
Solution:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta d\theta d\phi = B_o \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi$$

$$= 2\pi B_o \int_0^{\pi/2} \cos\theta \sin\theta d\theta = 2\pi\left(\frac{1}{2}\right) B_o$$

$$P_{rad} = \pi B_o$$

Radiation Intensity Pattern



$$U(\theta,\phi) = \cos\theta$$

$$0 \leq \theta \leq 90^o$$

$$0 \leq \phi \leq 360^o$$

Fig. 2.15

$$D_0(\text{exact}) = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi B_0}{\pi B_0} = 4 = 6.02 \text{ dB}$$

Approximate:

To find the HPBW, you set

$$\cos\theta_h = 0.5 \Rightarrow \theta_h = \cos^{-1}(0.5)$$

$$\theta_h = \frac{\pi}{3} \text{ radians} = 60^\circ$$

Because of the symmetry of the pattern

$$\Theta_1 = \Theta_{2r} = 2\pi/3 \text{ radians} = 120^\circ$$

Using the previous results, we get the following approximate directivities:

$$D_0(\text{Kraus}) \simeq \frac{4\pi}{(2\pi/3)^2} = \frac{9}{\pi} = 2.86 = 4.56 \text{ dB}$$

$$(-28.5\% \text{ Error})$$

$$D_0(\text{T\&P}) \simeq \frac{22.181}{2(2\pi/3)^2} = 2.53 = 4.03 \text{ dB}$$

$$(-36.75\% \text{ Error})$$

Thank you