Lecture 4
Radio Wave Propagation: Fading and Multipath

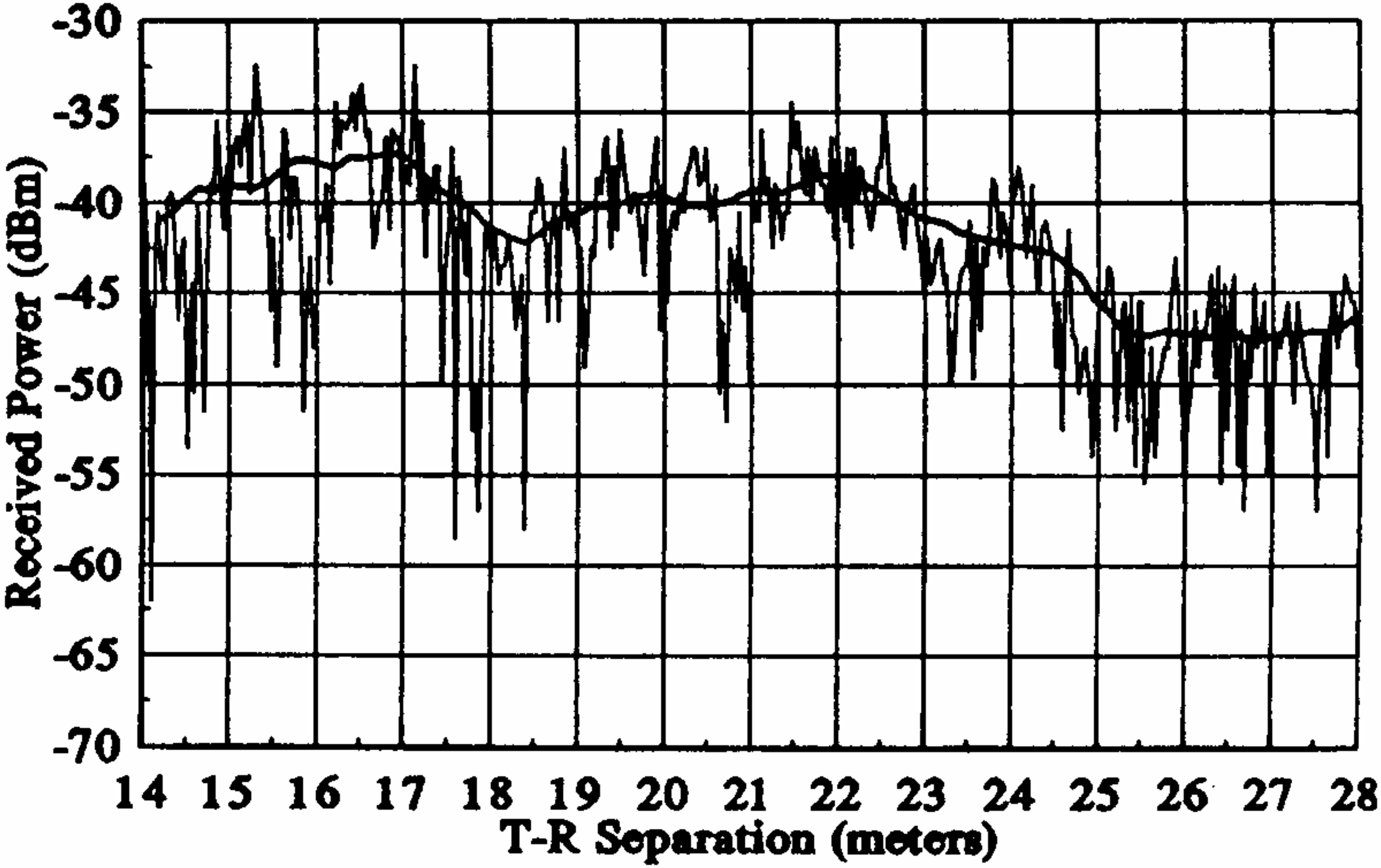
Lecture Aims

- Understand the theory of multipath fading channel
- Know how to calculate the parameters for fading channel
- Learn different types of multipath fading channel
- Some common fading channel models

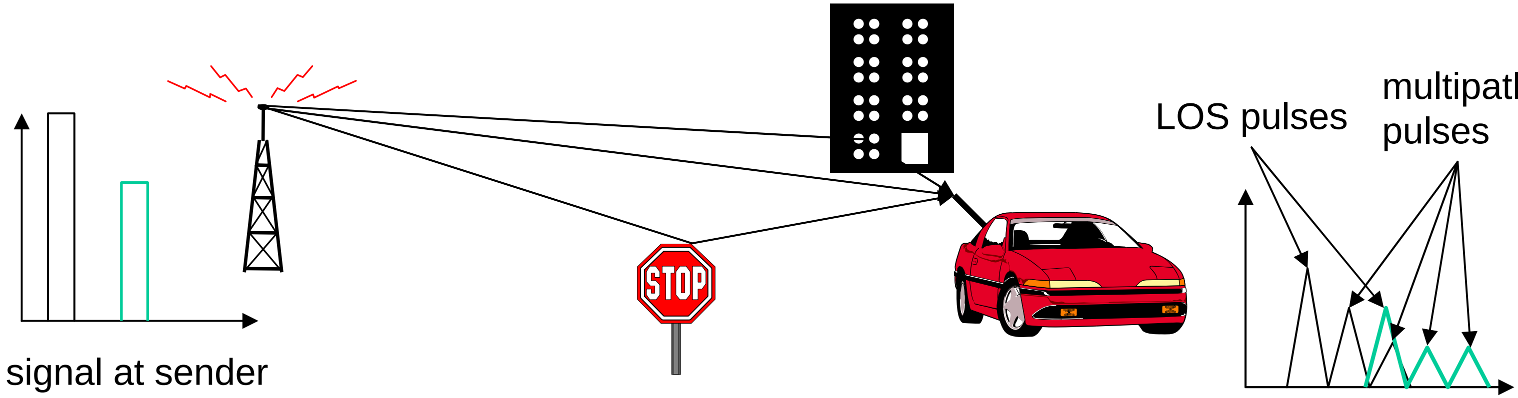
Backgrounds (I)

- Large-scale propagation (Lecture 2)
  - Predicts mean received signal strength at large Tx-Rx distance
    - Hundreds or thousands of meters
  - Path loss, shadowing etc
  - **Importance**
    - Proper site planning
- Small-scale propagation
  - Characterize the rapid fluctuations over short distance or time
  - Fading
  - **Importance**
    - Proper receiver design to handle the rapid fluctuations

Backgrounds (II)



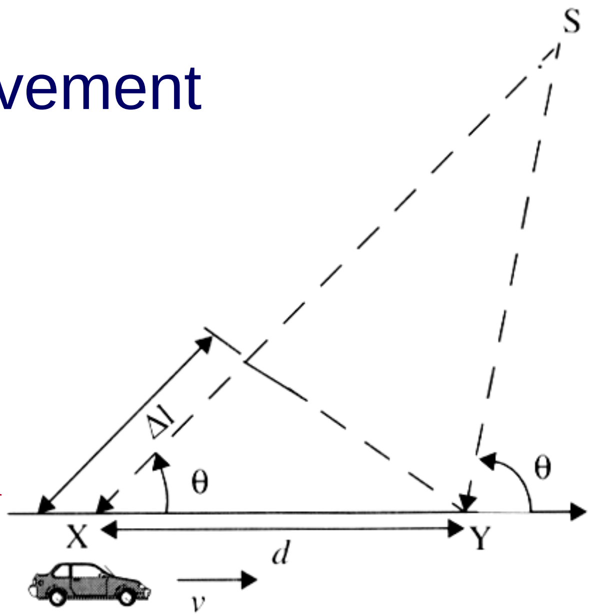
Factors Affecting Fading (I)



- Multipath propagation
  - Signal arrives at Rx through different paths
    - Reflection, Diffraction, Scattering
  - Paths could arrive with different gains, phase, & delays
  - Small dist variation can have large amplitude variation
    - e.g. 2 paths with perfect reflector (Plan earth model)
      - At 900MHz, 0-to-0 within 30cm $\left|\tilde{E}_{TOT}\right| = 2\left|\tilde{E}_{LOS}\right| \sin\left(\frac{\pi \Delta d}{\lambda}\right)$

Factors Affecting Fading (II)

- Speed of mobile/surrounding objects
  - The mobile can be in motion and the environment can also be varying (cars, pedestrians, etc)
    - Induces Doppler shift
- Doppler Shift
  - The change of frequency due to movement
    - Phase change $$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$
    - Frequency change $$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$
    - v = speed of mobile, λ = carrier wavelength
    - +/-ve when moving towards/away the wave

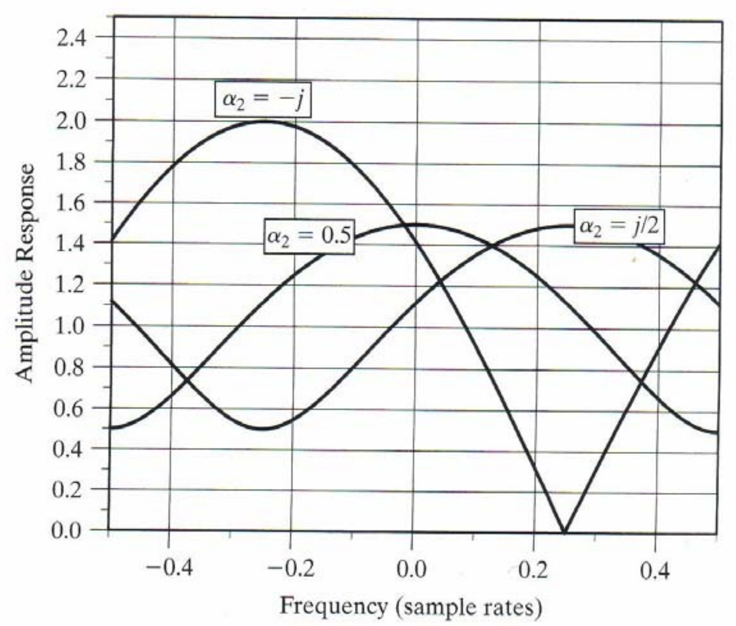


Factors Affecting Fading (III)

- Signal transmission bandwidth
  - If the signal bandwidth is wider than the channel "bandwidth", the received signal will be distorted
  - Consider a 2-ray model
    - Second ray arrive at one symbol period later than the first ray

$$h(\tau) = \delta(\tau) + \alpha_2 \delta(\tau + T_s)$$

$$H(f) = 1 + \alpha_2 e^{-j2\pi f T_s}$$



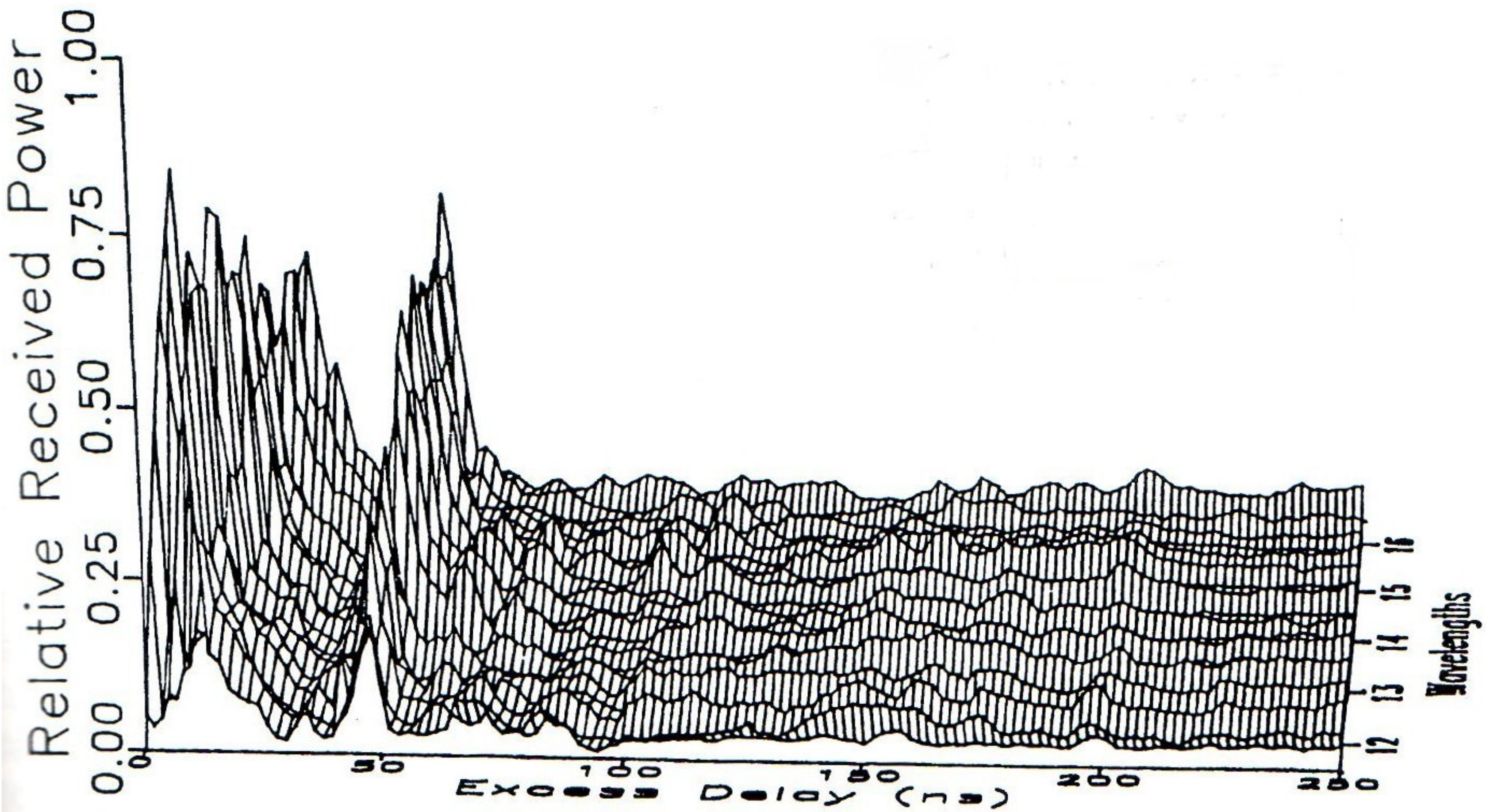
Multipath Channel Impulse Response (I)

- The multipath channel can be modelled as a filter
  - A summation of all multipath
  - Each multipath contain its gain, phase and delay
  - Varies with time and distance
    - If the mobile is moving, distance is also related to time $d = vt$
  - Two time-related variables
    - t is the time variation due to motion
    - τ is the time variation due to multipath delay
      - Excess delay - relative delay compared to the first arriving path

$$\boxed{h_p(t,\tau) = \sum_{i=0}^{N-1} \alpha_i(t,\tau) \exp\left[j\theta_i(t,\tau)\right] \delta\left(\tau - \tau_i(t)\right)}$$

      - $\tau_i(t)$ = i-th path excess delay at time t
      - N = total number of arriving paths

Multipath Channel Impulse Response (II)



Multipath Channel Impulse Response (III)

- Received signal

$$y(t,\tau) = x(t) \otimes h_p(t,\tau)$$

  - $x(t)$ = passband transmitted signal
  - $h_p(t,\tau)$ = channel impulse response due to motion and excess delay

- Baseband equivalent representation
  - Signal processing is done in baseband
  - Need to obtain a baseband representation of the channel $h_b(t,\tau)$
  - Consider the transmitted passband signal $x(t)$
    - Let $s(t)$ be the baseband signal with $s(t) = s_I(t) + js_Q(t)$

$$x(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) = \mathrm{Re}\left\{s(t)e^{j2\pi f_c t}\right\}$$

Multipath Channel Impulse Response (IV)

– Similarly, baseband received signal *r*(*t*)

$$y(t) = \mathrm{Re}\left\{ r(t) e^{j2\pi f_c t} \right\}$$

– Baseband equivalent channel impulse response *h*(*t*, τ)

$$h_p(t,\tau) = \mathrm{Re}\left\{ h(t,\tau) e^{j2\pi f_c t} \right\}$$

– Hence

$$r(t) = \frac{1}{2} s(t) \otimes h(t,\tau)$$

- ½ come from down-conversion from passband to baseband
- To retrieve the in-phase component, multiply with carrier

$$x(t)\cos(2\pi f_c t) = s_I(t)\cos^2(2\pi f_c t) - s_Q(t)\cos(2\pi f_c t)\sin(2\pi f_c t)$$

$$= \frac{1}{2} s_I(t)\left(1 + \cos(4\pi f_c t)\right) - \frac{1}{2} s_Q(t)\sin(4\pi f_c t)$$

- After LPF: $= \frac{1}{2} s_I(t)$

Multipath Channel Impulse Response (V)

- Discrete-time baseband impulse response
  - Divide multipath delay into discrete segments called **excess delay bins**
  - i-th excess delay = $\tau_i = i\Delta\tau \quad \forall i = \{0, \ldots, L-1\}$
    - $\Delta\tau$ = delay bin width; L = maximum resolvable delay path

$$h(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\exp\left[j\left(2\pi f_c \tau_i(t) + \phi_i(t,\tau)\right)\right]\delta\left(\tau - \tau_i(t)\right)$$

$$\boxed{= \sum_{k=0}^{L-1} a(t,k\Delta\tau)\exp\left[j\theta(t,k\Delta\tau)\right]\delta(\tau - k\Delta\tau)}$$

- $\phi_i(t,\tau)$ = i-th path random phase shift at time t
- $a(t,k\Delta\tau)$ = baseband real amplitude of k-th bin at time t
- $\theta(t,k\Delta\tau)$ = phase shift due of k-th bin at time t

Multipath Channel Impulse Response (VI)

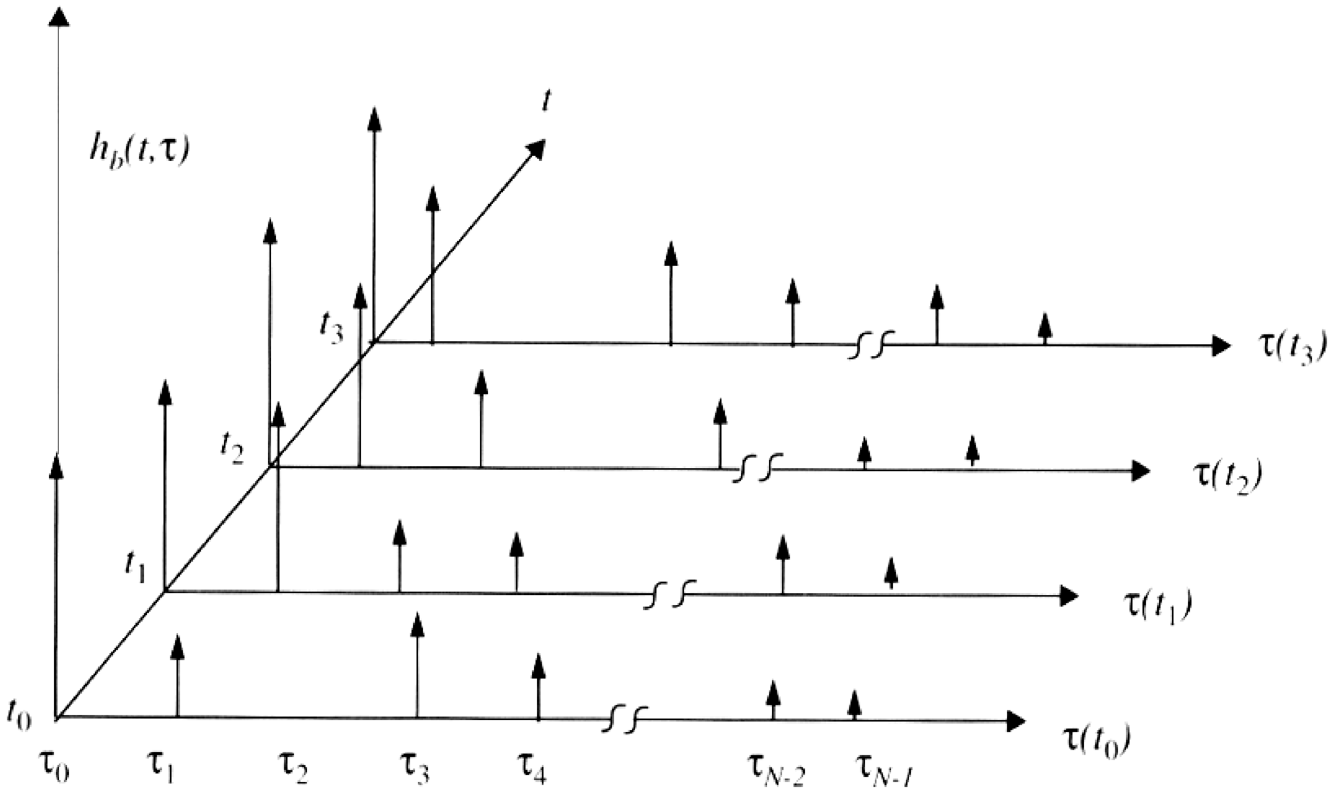
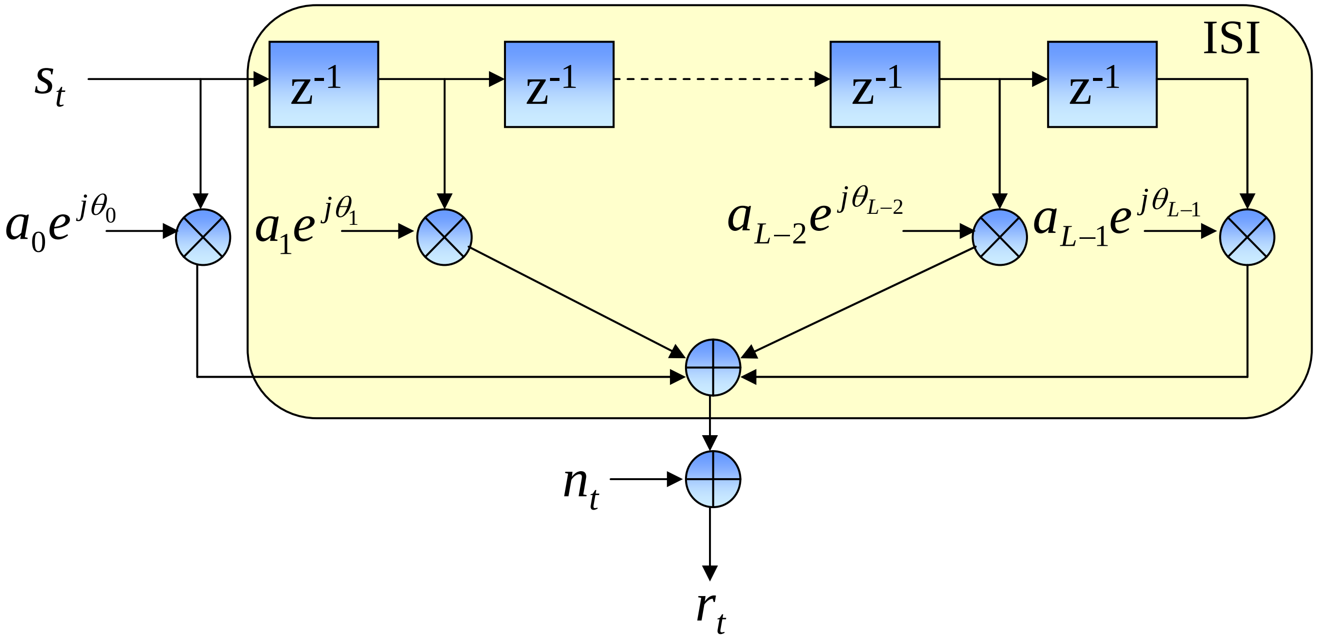


Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

Tap Delay Line Model

- If the excess delay bin equals to the symbol period, the channel can be modelled as a tap delay-line filter
  - Each tap delay is exactly 1 symbol period



- s_t = baseband signal, n_t = noise, r_t = received baseband signal

Problems with Multipath Channel

- Several paths arrive within one delay bin ($\Delta\tau$)
  - These paths cannot be resolved ($N \neq L$)
  - Different gains and phase will be combined
  - Constructive and destructive interference can occur
  - **Fading**
    - The received signal power changes rapidly from bin to bin
    - When a symbol is in deep fade (all bins in that symbol period have small values), it cannot be detected correctly
    - Diversity technique required
  - If the excess delay is longer than one symbol period
    - Inter-symbol interference (ISI) occurs
    - Equalization techniques required

WSSUS Channel

- Assume channel is wide-sense stationary (WSS)
  - The autocorrelation function is dependent on the time difference
    - i.e. Autocorrelation function is the same at any time

$$E\left[h(t_1,\tau_1)h^*(t_2,\tau_2)\right] = R_h(t_1 - t_2;\tau_1,\tau_2)$$

- Assume all paths are uncorrelated
  - Uncorrelated Scattering (US)

$$R_h(t_1 - t_2;\tau_1,\tau_2) = R_h(t_1 - t_2;\tau_1)\delta(\tau_1 - \tau_2)$$

- R_h=0 unless $\tau_1 = \tau_2$

Channel Autocorrelation Function

- Let $\tilde{a}_k(t,\tau) = a(t,k\Delta\tau)\exp[j\theta(t,k\Delta\tau)]$

$$R_h(t_1 - t_2;\tau_1,\tau_2) = E[h(t_1,\tau_1)h^*(t_2,\tau_2)]$$

$$= E\left[\sum_{k=0}^{L-1}\tilde{a}_k(t_1,\tau_1)\delta(\tau_1-\tau_k)\sum_{i=0}^{L-1}\tilde{a}_i^*(t_2,\tau_2)\delta(\tau_2-\tau_i)\right]$$

$$= \sum_{k=0}^{L-1}\sum_{i=0}^{L-1}E[\tilde{a}_k(t_1,\tau_1)\tilde{a}_i^*(t_2,\tau_2)]\delta(\tau_1-\tau_k)\delta(\tau_2-\tau_i)$$

$$= \sum_{k=0}^{L-1}R_a(t_1,t_2;\tau_1)\delta(\tau_1-\tau_k)\delta(\tau_1-\tau_2)$$

  – where $R_a(t_1,t_2;\tau_1) = E[\tilde{a}_k(t_1,\tau_1)\tilde{a}_i^*(t_2,\tau_1)]$ is the autocorrelation function of the discretised channel bin
    - Note that $\tau_1 = \tau_2$, i.e. uncorrelated among different bins

Power Delay Profile

- Characterise the power distribution against the excess delays
  - Average of $|h(t,\tau)|^2$ over time t

$$\boxed{P(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left|h(t,\tau)\right|^2 dt = R_h(0;\tau)}$$

  - For discrete-time model

$$\boxed{P(\tau) = \sum_{k=0}^{L-1} a(t,k\Delta\tau)^2 \delta(\tau - k\Delta\tau) = \sum_{k=0}^{L-1} P_k \delta(\tau - k\Delta\tau)}$$

  - P_k = power at k-th delay bin
- Common delay profiles
  - Uniform: P_k being constant over k
  - Exponential: $P_k = c\exp(-k\Delta\tau/c)$

Time Dispersion Parameters (I)

- Determined from the power delay profile
  - Treat the power delay profile as a probability mass function
  - Calculate the mean, second moment, and standard deviation for it
- Mean excess delay

$$\boxed{\bar{\tau} = E[\tau] = \sum_k prob(k)\tau_k = \sum_k \left(\frac{a_k^2}{\sum_k a_k^2} \right) \tau_k}$$

Second moment

$$\boxed{\overline{\tau^2} = E[\tau^2] = \sum_k prob(k)\tau_k^2 = \sum_k \left(\frac{a_k^2}{\sum_k a_k^2} \right) \tau_k^2}$$

- RMS delay spread

$$\boxed{\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}}$$

- Maximum excess delay (X dB) $\boxed{= \tau_X - \tau_0}$
  - τ_0 = first arriving signal delay
  - τ_X = max excess delay within X dB of the strongest path

Time Dispersion Parameters (II)

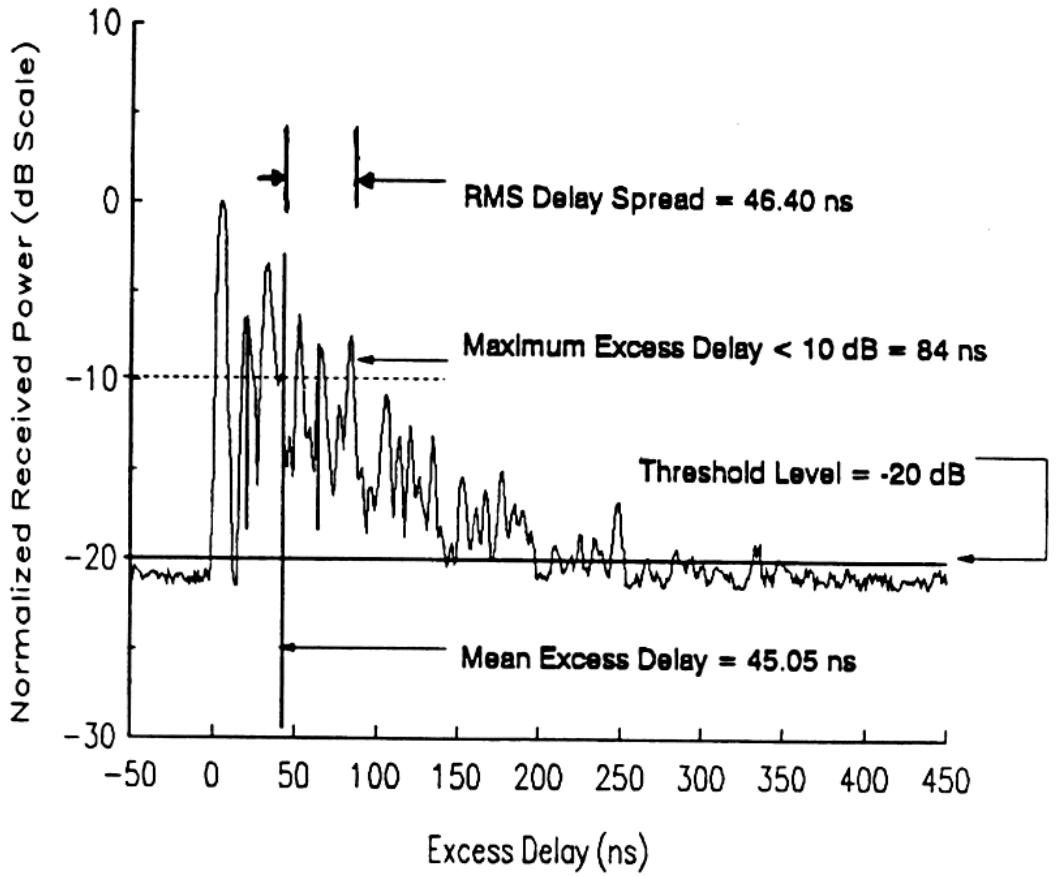


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Time Dispersion Parameters (III)

Table 5.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10–25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200–310 ns	Averaged typical case	[Cox72]
Suburban	910	1960–2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10–50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70–94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

Coherence Bandwidth

- **Coherence bandwidth B_c**
  - Frequencies separated by less than this bandwidth will have their fades highly correlated
    - Flat frequency spectrum within B_c
  - Signals will be affected differently with the frequency separation goes beyond B_c
  - Frequency correlation higher than 0.9 and 0.5

$$R_h\left(f_1 - f_2; \Delta\tau\right) > 0.9$$

$$\boxed{B_c \approx \frac{1}{50\sigma_\tau}}$$

$$R_h\left(f_1 - f_2; \Delta\tau\right) > 0.5$$

$$\boxed{B_c \approx \frac{1}{5\sigma_\tau}}$$

    - RMS delay spread ↑, Coherence bandwidth ↓
    - Frequency correlation ↑, Coherence bandwidth ↓

Doppler Spread and Coherence Time

- Parameters to describe the time varying nature of a channel
- **Doppler spread** B_D
  - Measure of spectral broadening due to time variation
  - $\boxed{B_D = 2f_{d\text{-}max}}$
    - $f_{d\text{-}max}$ = max Doppler shift = v/λ
- **Coherence Time** T_c
  - Time duration that the fading parameters remain fairly constant
  - Coherence time for correlation above 0.5:

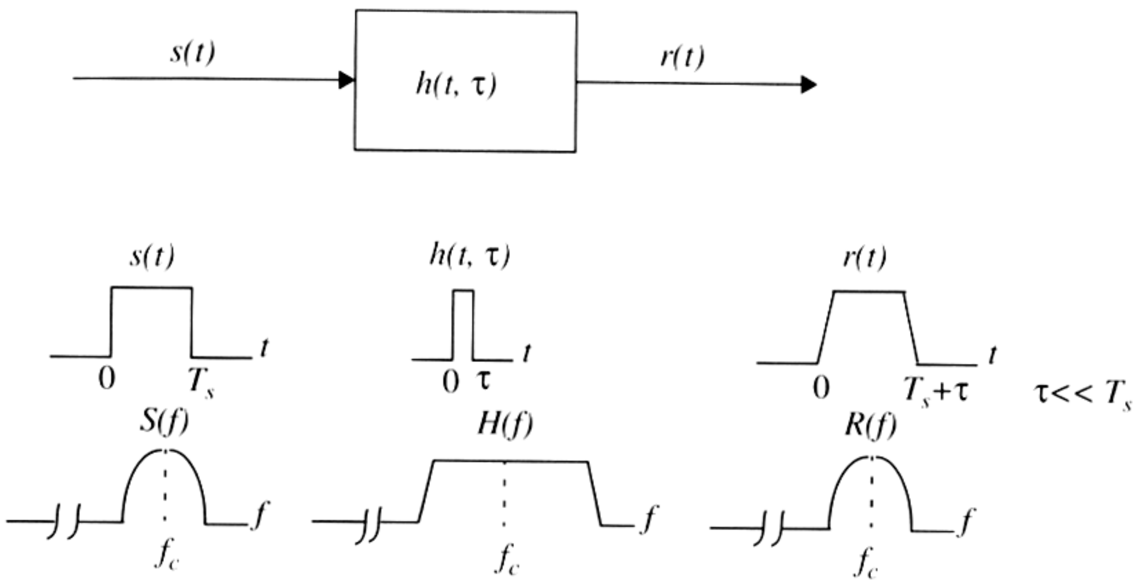
$$R_h(\Delta t;0) = R_a(\Delta t) > 0.5$$

$$\boxed{T_c \approx \frac{9}{16\pi f_{d-\max}}}$$

  - Mobile speed ↑, Doppler spread ↑, Coherence time ↓

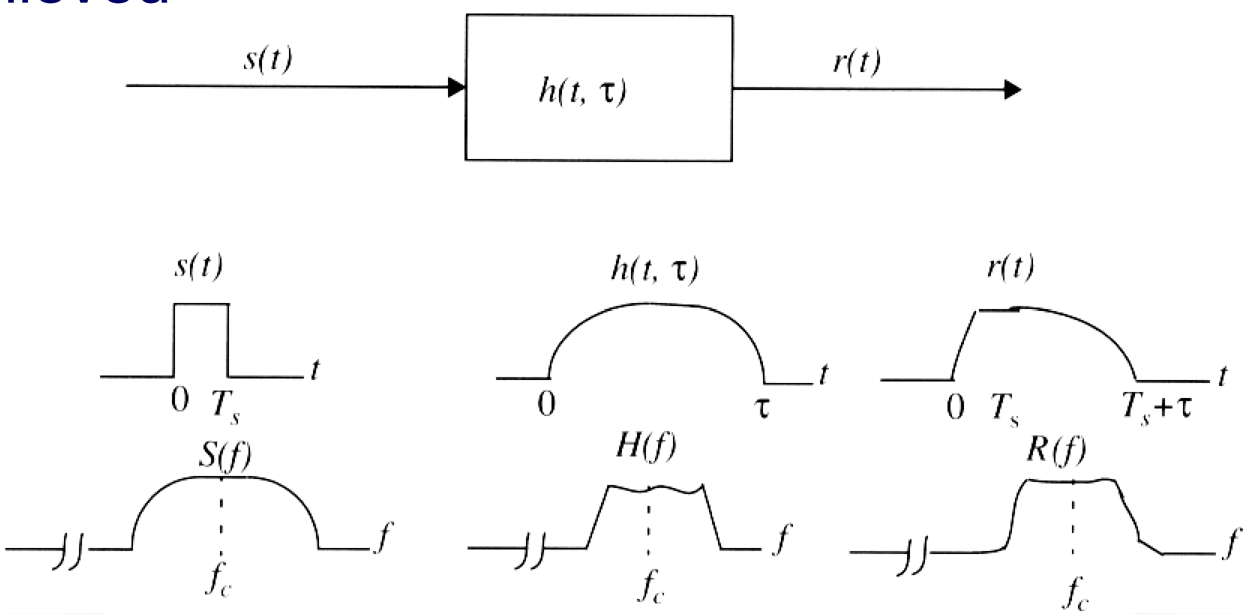
Types of Fading (Delay Spread)

- Flat fading
  - Signal BW B_S < Coherence BW B_C
  - Delay spread σ_τ < Symbol period T_S
  - A deep fade can degrade the system performance significantly



Types of Fading (Delay Spread)

- Frequency selective fading
  - $B_S > B_C$, $\sigma_\tau > T_S$
  - ISI occurs => equalization technique required
  - With proper equalization, frequency diversity can be achieved



Types of Fading (Doppler Spread)

- Fast fading (Time selective fading)
  - T_S > Coherence time T_C , B_S < Doppler spread B_D
  - Channel impulse response changes within the symbol duration
  - Occurs for very low data rates
  - With packetised transmission, fast fading is now commonly referred to rapid channel changes within one packet or frame
  - With proper system design, time diversity can be obtained

Types of Fading (Doppler Spread)

- Slow fading
  - $T_S \ll T_C$, $B_S \gg B_D$
  - Channel changes at a rate much slower than the symbol duration
  - Very common, especially in high data rate applications
    - GSM900 at 200km/h, $T_C \approx 1$ms, User frame = 576.92 μs
- Quasi-static slow fading
  - Channel is static within a frame but varies independently from frame to frame
  - Used in simulation to provide an averaged performance over many channel realisations

Types of Fading

- Fast/slow and frequency flat/selective fading is not mutually exclusive

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Common Channel Models - Rayleigh

- Consider the channel gain at *k*-th bin with N_k arriving paths

$$\tilde{a}_k = a_k e^{j\theta_k} = \sum_{i=0}^{N_k-1} a_{k,i} e^{j\theta_{k,i}} = \sum_{i=0}^{N_k-1} a_{k,i}^I + ja_{k,i}^Q = a_k^I + ja_k^Q$$

  - I and Q is the in-phase and quadrature phase component of channel gain
- Assumptions
  - Infinite arrival paths at the same time
  - All paths have zero mean and similar variance (i.e. no dominant path)
  - All path gains are statistically independent
- By central limit theorem, the I and Q are Gaussian distributed
  - Rayleigh distribution = envelope of the sum of 2 quadrature Gaussian source (*x*, *y*)

Common Channel Models - Rayleigh

- Rayleigh fading
  - A commonly used model for no line-of-sight (N-LOS) channels

$$\tilde{a}_k = a_k^I + j a_k^Q = a_k e^{j\theta_k} \qquad a_k = \sqrt{{a_k^I}^2 + {a_k^Q}^2} \qquad \theta_k = \tan^{-1}\left(\frac{a_k^Q}{a_k^I}\right)$$

    - I & Q component a_k^I and a_k^Q is Gaussian distributed $N(0, \sigma^2)$
    - Magnitude a_k is Rayleigh distributed
    - Phase θ_k is uniformly distributed over 2π
  - Rayleigh distribution pdf

$$p(a_k) = \begin{cases} \dfrac{a_k}{\sigma^2}\exp\left(-\dfrac{a_k^2}{2\sigma^2}\right) & 0 \le a_k \le \infty \\ 0 & a_k < 0 \end{cases}$$

    - $E[a_k^2] = 2\sigma^2$ = average channel power

Rayleigh Fading Channel Simulator

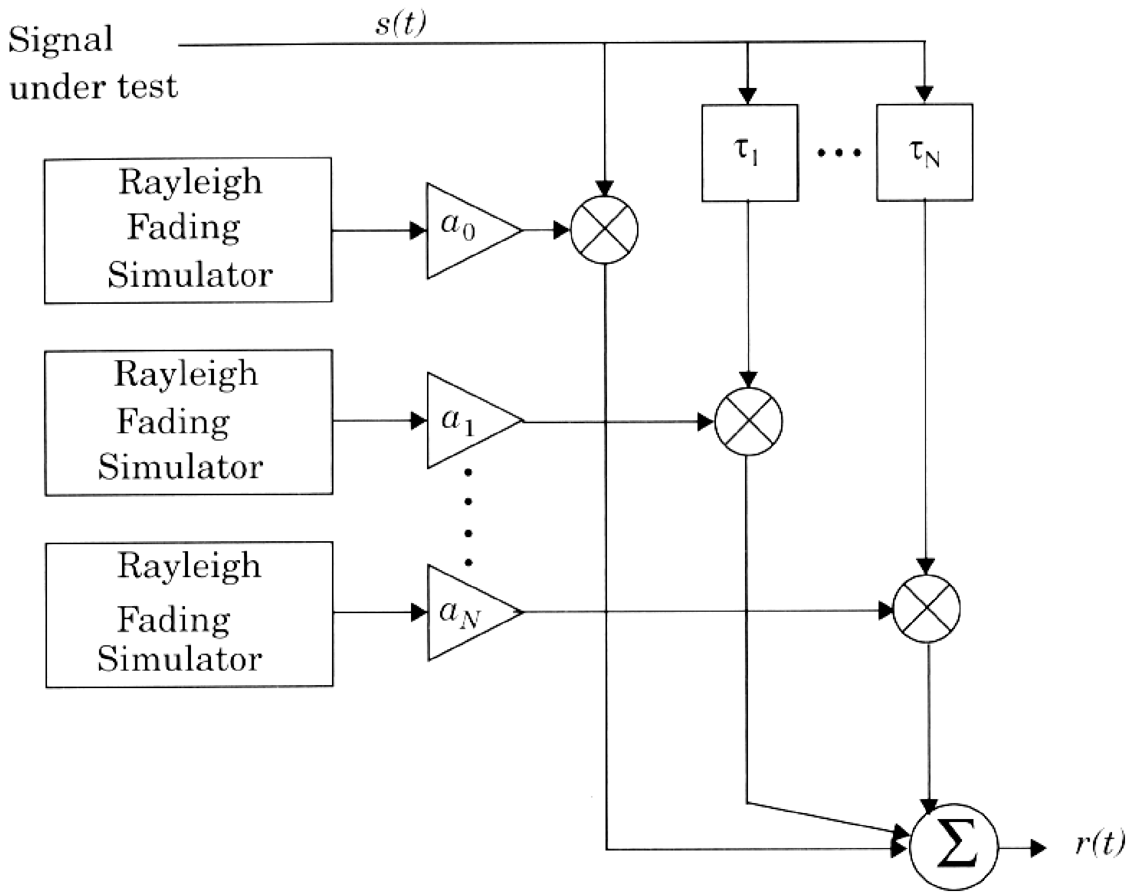


Figure 5.25 A signal may be applied to a Rayleigh fading simulator to determine performance in a wide range of channel conditions. Both flat and frequency selective fading conditions may be simulated, depending on gain and time delay settings.

Common Channel Models - Rician

- Rician fading
  - A channel with a dominant path and numerous “weak” multipath
    - i.e. with line-of-sight path
  - Channel fading statistics is Ricean distributed
    - When the dominant component fades away, the statistics degenerates to Rayleigh
  - PDF

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\dfrac{Ar}{\sigma^2}\right) & A \geq 0, r \geq 0 \\ 0 & r < 0 \end{cases}$$

    - A = peak amplitude of the dominant signal
    - $I_0(.)$ = zero-order Bessel function of the first kind
    - Often described by $K = A^2/2\sigma^2$

Common Channel Models

- Example of Rayleigh and Ricean distribution

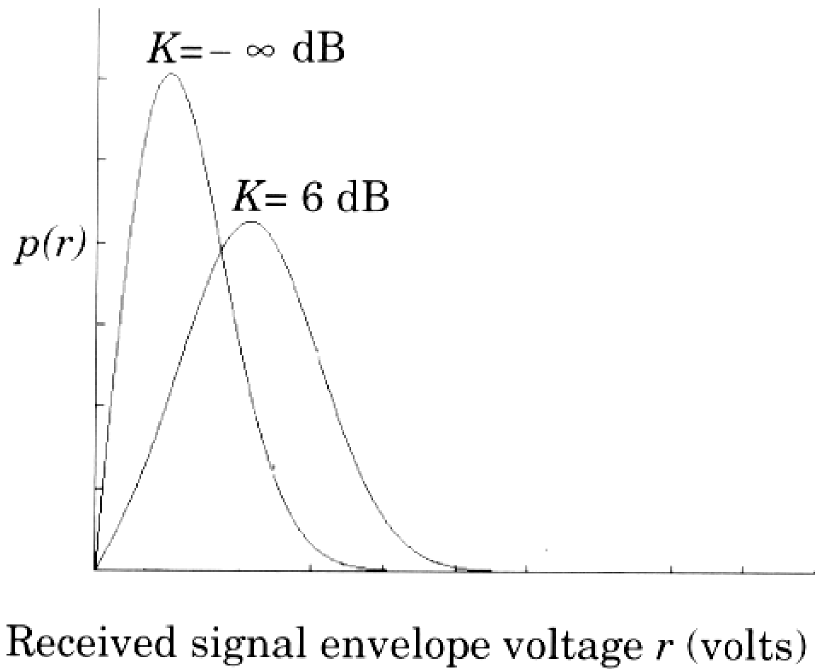


Figure 5.18 Probability density function of Ricean distributions: $K = -\infty$ dB (Rayleigh) and $K = 6$ dB. For K >> 1, the Ricean pdf is approximately Gaussian about the mean.

Common Channel Models – Clarke Model (I)

- Assuming all rays arrive in horizontal direction and at the same time
  - – Channel gain with *N* arriving paths

$$\tilde{a} = \sum_{i=0}^{N-1} a_i e^{j\theta_i}$$

- When the mobile is moving, each ray experience different Doppler shifts

$$\tilde{a}(t) = \sum_{i=0}^{N-1} a_i e^{j(2\pi f_i t + \theta_i)} \qquad f_i = f_{D-\max} \cos\psi_i$$

  - where f_i and ψ_i is the Doppler shift and direction of travel for path *i*
  - $f_{D\text{-}max}$ is the maximum Doppler shift

Clarke Model (II)

MANCHESTER 1824
The University of Manchester

- Consider the channel autocorrelation function

$$R_h(t_1 - t_2; \tau_1, \tau_2) = \sum_{k=0}^{L-1} R_a(t_1, t_2; \tau_1)\delta(\tau_1 - \tau_k)\delta(\tau_1 - \tau_2)$$

  - As all paths arrive at the same time ($\tau_i = \tau_k\ \forall i, k$)
    - τ can be removed and $L=1$
  - Let $t=t_1$, $t_2=t_1+\Delta t$ $\quad R_h(\Delta t;\tau) = R_a(\Delta t)$

$$R_a(\Delta t) = E\left[\tilde{a}(t)\tilde{a}^*(t+\Delta t)\right] = E\left[\sum_{i=0}^{N-1} a_i e^{j(2\pi f_i t + \theta_i)} \sum_{k=0}^{N-1} a_i e^{j(2\pi f_k (t+\Delta t) + \theta_k)}\right]$$

$$= \sum_{i=0}^{N-1} E\left[a_i^2 e^{-j2\pi f_i \Delta t}\right] = \sum_{i=0}^{N-1} E\left[a_i^2\right] E\left[e^{-j2\pi f_i \Delta t}\right]$$

$$= \sum_{i=0}^{N-1} E\left[a_i^2\right] E\left[e^{-j2\pi f_{D-\max} \Delta t \cos\psi_i}\right]$$

Clarke Model (III)

– Computing the expectation for the second term

- Assuming the angle of arrival is uniformly distributed [-π, π]

$$R_a(\Delta t) = \sum_{i=0}^{N-1} E\left[a_i^2\right]\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j2\pi f_{D-\max}\Delta t\cos\psi_i}\, d\psi_i\right)$$

$$= \sum_{i=0}^{N-1} E\left[a_i^2\right] I_0\left(2\pi f_{D-\max}\Delta t\right) = P_{av} I_0\left(2\pi f_{D-\max}\Delta t\right)$$

- where $I_0(x)$ is the zeroth order Bessel function of the first kind

$$I_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{-jx\cos\psi}\, d\psi$$

- P_{av} is the average channel power

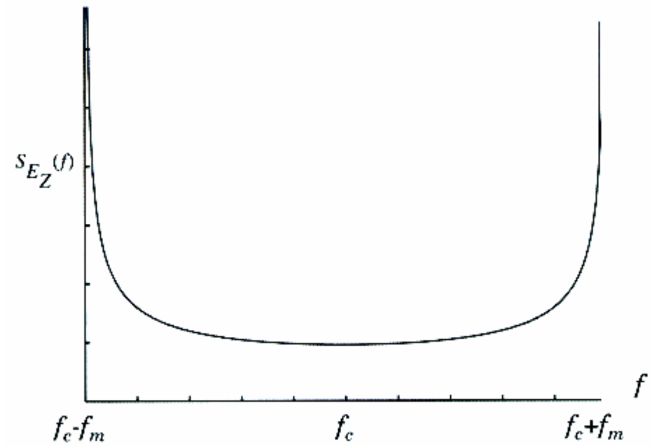
$$P_{av} = \sum_{i=0}^{N-1} E\left[a_i^2\right]$$

Clarke Model (IV)

- Doppler Spectrum
  - The power spectral density (PSD) is the Fourier transform of the autocorrelation function

$$S_h(f) = \Im\{R_h(\Delta t;\tau)\} = \Im\{P_{av} I_0(2\pi f_{D-\max}\Delta t)\}$$

$$= \begin{cases} \dfrac{P_{av}}{\sqrt{1-(f/f_{D-\max})^2}} & |f| < f_{D-\max} \\ 0 & |f| > f_{D-\max} \end{cases}$$



- Significance
  - Convolved with signal spectrum
    - Spectrum will be smeared
      - This is why Doppler spread $B_D = 2f_{d\text{-}max}$
    - Receiver must be able to handle this widened bandwidth
  - Infinity at $f_{D\text{-}max}$ because of uniform arrival assumption

Simulating Doppler Spread

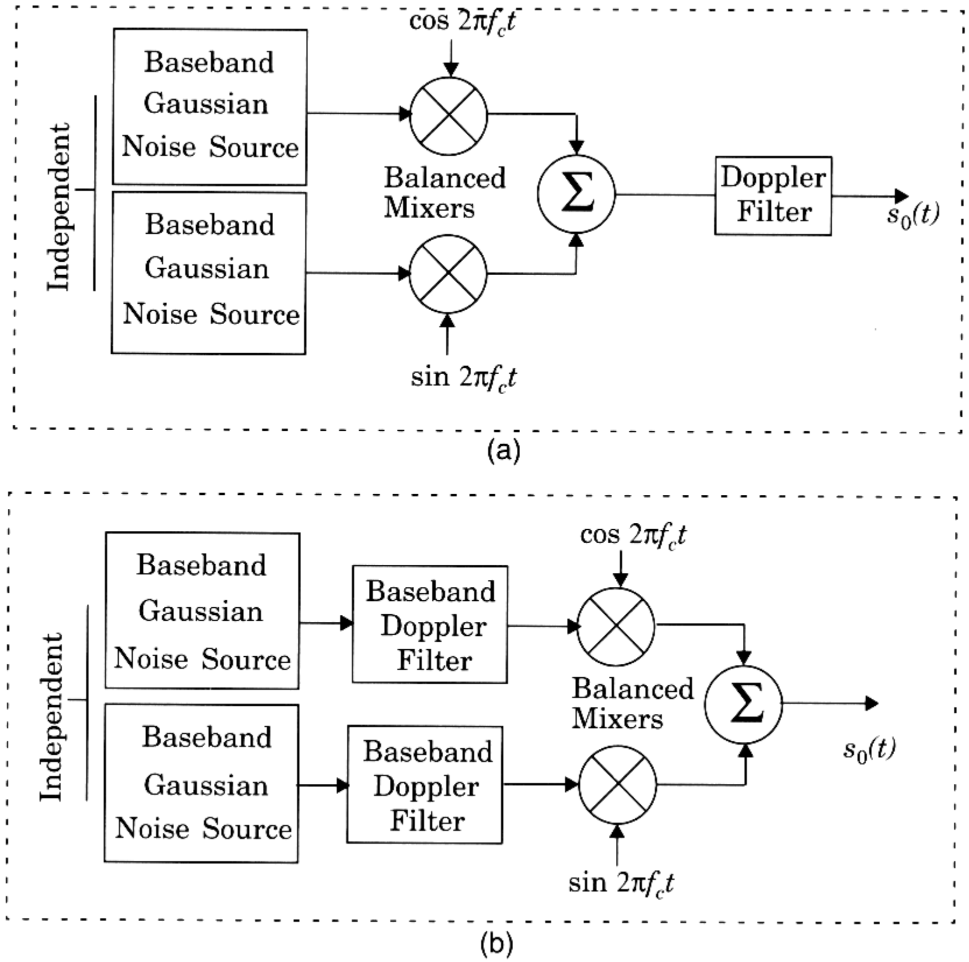


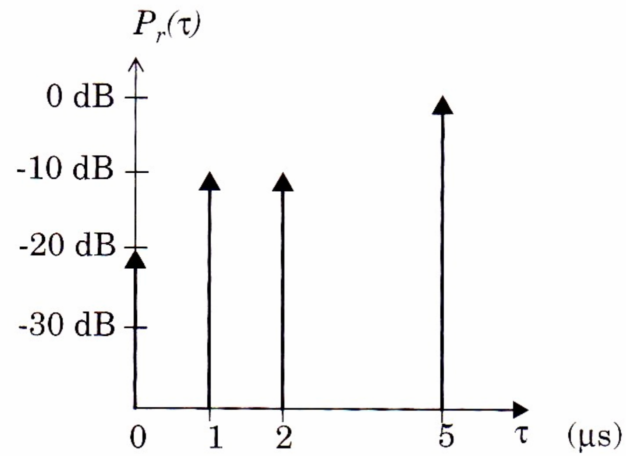
Figure 5.22 Simulator using quadrature amplitude modulation with (a) RF Doppler filter and (b) baseband Doppler filter.

Summary

- Multipath channel impulse response
- Parameters of multipath channel
  - Time dispersion parameters: mean, rms delay spread, max excess delay
  - Coherence bandwidth
  - Coherence time and Doppler spread
- Different types of fading
  - Frequency flat/selective fading
  - Fast/slow fading
- Common fading channel models
  - Rayleigh/Ricean fading
  - Clarke Model

Tutorial Question

- Consider the provided power delay profile
  – Calculate the mean excess delay, rms delay spread, and the max excess delay (10dB) for the power delay profile provided
  – Estimate the 50% coherence bandwidth of the channel
  – Would this channel be suitable for AMPS (30kHz) or GSM (200kHz) service without the use of an equalizer?



- If you are travelling at 100km/h and the system carrier freq is 900MHz
  – What is the maximum Doppler shift?
  – What is the 50% coherence time?
  – What is Doppler Spread?

Tutorial Question – Solution

- Max excess delay (10dB) = 5μs
- Mean excess delay $$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{(0.01+0.1+0.1+1)} = 4.38\mu s$$
- Second moment $$\overline{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)^2}{(0.01+0.1+0.1+1)} = 21.07\mu s^2$$
- RMS delay spread $$\sigma_\tau = \sqrt{21.07-4.38^2} = 1.37\mu s$$
- Coherence bandwidth $$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\times10^{-6})} = 146kHz$$
- AMPS do not need an equalizer (30kHz BW) but GSM does (200kHz BW)

Tutorial Question – Solution

- f = 900MHz
- v = 100km/hr = 100*1000/3600m/s = 27.778m/s
- Maximum Doppler shift

$$f_{D-\max} = \frac{v}{\lambda} = \frac{vf}{c} = \frac{27.778 \times 900 \times 10^{6}}{3 \times 10^{8}} = 83.333Hz$$

- 50% Coherence time

$$T_c = \frac{9}{16\pi f_{D-\max}} = 2.149ms$$

- Doppler Spread B_D = 2*$f_{D\text{-}max}$ = 166.67Hz