N-ELEMENT LINEAR ARRAY UNIFORM SPACING,
NON-UNIFORM AMPLITUDE

Presented by:
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- ***Broadside arrays with uniform spacing and***

Non-uniform amplitude

- ***Binomial and***
- ***Dolph-Tschebyscheff broadside arrays***

Reason

- **A uniform amplitude array yields the smallest half-power beam width. It is followed, in order, by the Dolph-Tschebyscheff and binomial arrays.**

- **In contrast, binomial arrays usually possess the smallest side lobes followed, in order, by the Dolph-Tschebyscheff and uniform arrays.**

- **As a matter of fact, binomial arrays with element spacing equal or less than λ/2 have no side lobes.**

- **It is apparent that the designer must compromise between side lobe level and beam width.**

- **Uniform arrays usually possess the largest directivity.**

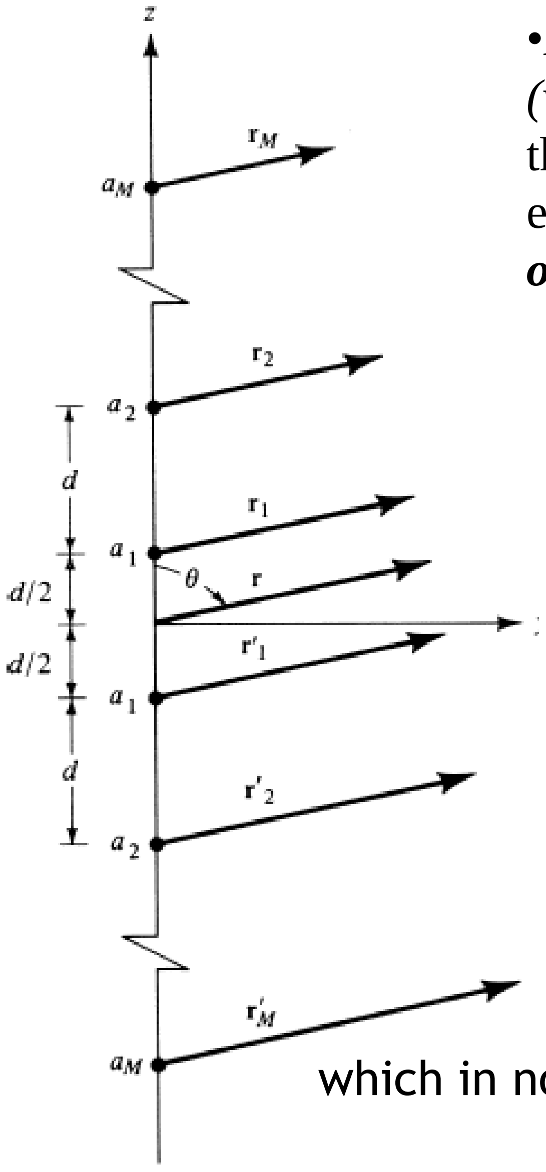
- **super directive (or super gain) antennas possess directivities higher than those of a uniform array .**

- **A certain amount of super directivity is practically**

  possible, super directive arrays usually require very large currents with opposite phases between adjacent elements.

- **Thus the net total current and efficiency of each array are very small compared to the corresponding values of an individual element.**

Array Factor



(a) Even number of elements

- An array of an even number of isotropic elements $2M$ *(where M is an integer) is* positioned symmetrically along the *z-axis, as shown in Figure . The separation* between the elements is *d, and M elements are placed on each side of the* ***origin***.

- Assuming that the amplitude excitation is symmetrical about the origin, the array factor for a non uniform amplitude broadside array

$$\begin{aligned}(\mathrm{AF})_{2M} = a_1 e^{+j(1/2)kd\cos\theta} &+ a_2 e^{+j(3/2)kd\cos\theta} + \ldots \\ &+ a_M e^{+j[(2M-1)/2]kd\cos\theta} \\ &+ a_1 e^{-j(1/2)kd\cos\theta} + a_2 e^{-j(3/2)kd\cos\theta} + \ldots \\ &+ a_M e^{-j[(2M-1)/2]kd\cos\theta}\end{aligned}$$

$$(\mathrm{AF})_{2M} = 2\sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right]$$

which in normalized form is :

$$(\mathrm{AF})_{2M} = \sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right]$$

$N = 2M$ (even)

$$(AF)_{2M} = a_1 e^{j\frac{kd}{2}\cos\theta} + a_2 e^{j\frac{3kd}{2}\cos\theta} + \ldots + a_M e^{j\left(\frac{2M-1}{2}\right)kd\cos\theta}$$

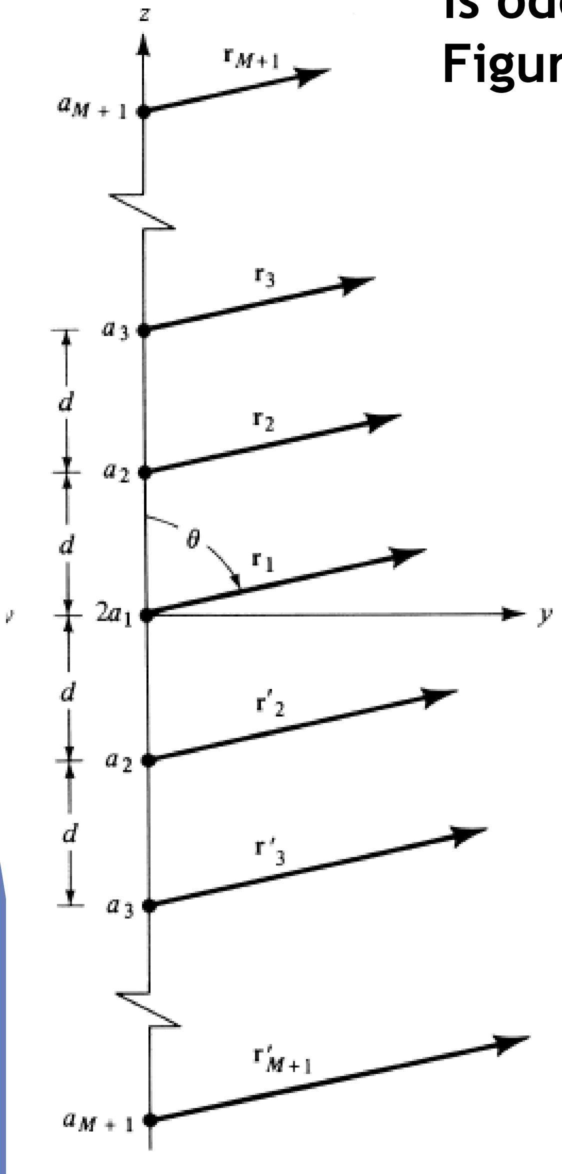
$$+ a_1 e^{-j\frac{kd}{2}\cos\theta} + a_2 e^{-j\frac{3kd}{2}\cos\theta} + \ldots + a_M e^{-j\left(\frac{2M-1}{2}\right)kd\cos\theta}$$

$$2a_1 \cos\left(\frac{kd}{2}\cos\theta\right) \qquad 2a_2 \cos\left(\frac{3kd}{2}\cos\theta\right) \qquad 2a_M \cos\left[\left(\frac{2M-1}{2}\right)kd\cos\theta\right]$$

$$(AF)_{2M} = 2\sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right] \qquad (6\text{-}59)$$

$$(AF)_{2M}\Big|_{\text{norm}} = \sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right] \qquad (6\text{-}59a)$$

If the total number of isotropic elements of the array is odd 2*M* + *1 (where M is* an integer), as shown in Figure , the array factor can be written as:



(b) Odd number of elements

$$(\mathrm{AF})_{2M+1} = 2a_1 + a_2 e^{+jkd\cos\theta} + a_3 e^{j2kd\cos\theta} + \cdots + a_{M+1} e^{jMkd\cos\theta} + a_2 e^{-jkd\cos\theta} + a_3 e^{-j2kd\cos\theta} + \cdots + a_{M+1} e^{-jMkd\cos\theta}$$

$$(\mathrm{AF})_{2M+1} = 2\sum_{n=1}^{M+1} a_n \cos[(n-1)kd\cos\theta]$$

which in normalized form reduces to

$$(\mathrm{AF})_{2M+1} = \sum_{n=1}^{M+1} a_n \cos[(n-1)kd\cos\theta]$$

$N = 2M+1$ (odd)

$$(AF)_{2M+1} = \boxed{2a_1} + \boxed{\begin{array}{l} a_2 e^{jkd\cos\theta} \\ + a_2 e^{-jkd\cos\theta} \end{array}} + \ldots + \boxed{\begin{array}{l} a_{M+1} e^{j\,M\,kd\cos\theta} \\ + a_{M+1} e^{-j\,M\,kd\cos\theta} \end{array}}$$

$$\boxed{2a_1} \quad \boxed{2a_2 \cos(kd\cos\theta)} \quad \boxed{2a_{M+1} \cos(Mkd\cos\theta)}$$

$$(AF)_{2M+1} = 2\sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos\theta\right] \qquad (6\text{-}60)$$

$$(AF)_{2M+1}\Big|_{\text{norm}} = \sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos\theta\right] \qquad (6\text{-}60a)$$

$$\text{Let:}\quad \frac{kd}{2}\cos\theta = \frac{\pi}{\lambda} d\cos\theta = u$$

Normalized Array Factors

$$(AF)_{2M}(even) = \sum_{n=1}^{M} a_n \cos\theta\left[(2n-1)u\right] \quad (6\text{-}61a)$$

$$(AF)_{2M+1}(odd) = \sum_{n=1}^{M+1} a_n \cos\theta\left[2(n-1)u\right] \quad (6\text{-}61b)$$

$$u = \frac{\pi d}{\lambda}\cos\theta \quad (6\text{-}61c)$$

$$(\mathrm{AF})_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos[(2n-1)u]$$

$$(\mathrm{AF})_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u]$$

where

$$u = \frac{\pi d}{\lambda} \cos\theta$$