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An extrapolative model of house price dynamics[☆]

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ABSTRACT

A model in which homebuyers make a modest approximation leads house prices to display three features present in the data but usually missing from rational models: momentum at one-year horizons, mean reversion at five-year horizons, and excess longer-term volatility relative to fundamentals. Approximating buyers assume that past prices reflect only contemporaneous demand, just like professional economists who use trends in housing prices to infer trends in housing demand. Consistent with survey evidence, this approximation leads buyers to expect increases in the market value of their homes after recent house price increases.

1. Introduction

Metropolitan area housing prices display significant momentum (Case and Shiller, 1989; DiPasquale and Wheaton, 1994), mean reversion (Cutler et al., 1991), and excess variance relative to fundamentals (Glaeser et al., 2014). These features were spectacularly on display during the great housing convulsion that rocked the US, and the world, between 1996 and 2010. Yet these three phenomena characterized house price dynamics even before this episode and continue to do so afterward. Case and Shiller's seminal work on house price momentum was published in 1989, and Glaeser et al. (2014) show mean reversion and excess volatility in house prices between 1980 and 2003. A successful model of house price dynamics therefore must predict momentum, mean reversion, and excess volatility not just during periods of extraordinary turbulence but also at all times.

Models with extrapolative beliefs can readily generate excess volatility and mean reversion (De Long et al., 1990b; Anufriev and Hommes, 2012; Adam et al., 2014; Barberis et al., 2015b). They can even generate momentum, as long as buyers cannot condition on the current asset price (Barberis et al., 2015a). In this paper, we extend the literature on extrapolation and asset prices with a simple micro-foundation that we call the cap rate error and show how institutional features of the housing market imply the momentum that stems from an inability to observe current market-wide housing prices. We also calibrate our model

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and find that it fits housing market fluctuations both quantitatively and qualitatively.

In our model, buyers are partially rational. They correctly value houses conditional upon their mistaken beliefs about the past and future of housing demand. Valuing a home involves forecasting future house prices, as buyers expect to resell at some future point. To forecast prices, buyers use past prices as their primary source of information about demand. But correctly inferring past demand from past prices is difficult because past prices also reflect past beliefs. In a rational economy, all buyers must use the same unintuitive and complex formula to map the price history to current demand. Under some parameters, this formula puts exponentially increasing weights on past prices. This bizarre formula works as long as all past buyers used it as well. It fails wildly when they did not. In our model, they use an approximation that is right on average rather than fathoming the beliefs of past buyers, and that approximation creates the cap rate error.

The cap rate error is the assumption that past prices reflect the flow value of housing (the dividend) divided by a time-invariant capitalization rate. Both academics and real estate practitioners commonly use house prices as a measure of demand for living in an area and use past prices to forecast future growth in housing demand. Because rental housing is often not representative of housing across an area, researchers and professionals alike use the growth in housing prices as a reflection of the growth in the flow value of housing demand, even though that growth could also reflect changing beliefs about future growth.

Naive homebuyers who make the cap rate error correctly calculate the price given their beliefs about demand and demand growth, but they neglect that past buyers, like themselves, used prices to update their beliefs. Instead, naive buyers simply think that past prices provide direct estimates of housing demand. The cap rate error approximation is accurate for deducing the general level of demand from house prices but causes errors when used sequentially by buyers. This approximation seems plausible to us, especially because like many housing economists, we have been guilty of it ourselves.

The cap rate error endogenously generates extrapolative beliefs. Naive homebuyers infer the path of fundamentals from past price changes. They infer, when housing prices grow by 50% over a five-year period, that demand has grown by 50% and expect additional increases in demand and prices as long as demand growth is persistent. Their mistake is that this five-year growth also reflects changes in beliefs about fundamentals; that is, the actual level of demand has grown much less than 50%. As a result, a positive demand shock leads to a wave of sequential upward revisions in beliefs, causing momentum in prices and eventual overshooting. During a boom, naive homebuyers overestimate housing demand and, during a bust, they underestimate it. This misvaluation accords with the empirical evidence offered by Ferreira and Gyourko (2011) that price movements during the recent boom were not closely related to either economic performance or rent growth at the metropolitan area level.

The cap rate error turns rational buyers into extrapolators, but extrapolation is not assumed but rather the result of a more primitive assumption: an inability to infer the beliefs of others. Micro-founding extrapolation is valuable because it provides intellectual foundations for an increasingly common asset pricing assumption and because the micro-foundation tightens the predictions of the model. The level of extrapolation is no longer a free parameter but is implied by the measurable processes that drive the flow benefits of home owning.

We incorporate naive inference into a continuous-time model of house prices. In this model, the growth rate of demand mean reverts and the current growth rate is not directly observable. While individuals observe their own level of demand, they do not observe the current level of market demand directly and must infer that through transaction prices, which are sparse as in real housing markets. Prices load positively on the most recent lagged price, as naive buyers infer the level of demand from this lag, and negatively on the second lag, as a lower second lag signals a higher growth rate. This time series relation leads to short-term momentum and long-term mean reversion.

We then calibrate the model. Many of our parameters come from existing literature and, in some cases, we estimate parameters ourselves. The parameter estimates yield predictions about momentum, mean reversion, and volatility of naive prices that match the data reasonably well. In contrast, simulations of rational prices under the same parameters display none of these features. We explore how these predictions change as parameter values change. The bubble-like features of markets disappear when information is either too good or too bad. If naive buyers have highly accurate direct signals about the state of demand, then momentum, mean reversion, and excess volatility disappear. But these features also disappear if buyers have access to relatively limited data on the number of past housing transactions. The most extreme fluctuations occur when buyers have relatively good data about past prices but limited data on the underlying fundamentals. We also find that high growth-rate persistence is needed to generate mean reversion and excess volatility.

Naive inference also explains the patterns in homebuyer beliefs shown by Case et al. (2012). In their survey, homeowners forecast significant increases in market values after recent price growth. This extrapolation is at odds with full rationality. Rational homeowners realize that demand shocks are fully reflected in house prices almost immediately. In contrast, naive buyers extrapolate because they fail to understand the evolving beliefs of future buyers. This failure leads naive buyers to under-extrapolate future price increases from past ones, a result that (Case et al., 2012) show empirically. Our simulation matches the quantitative extent of extrapolation that appears in their survey.

We compare naive forecasts with those that use linear regressions on past prices, such as the forecast rule studied by Fuster et al. (2011). These regression-based rules require no knowledge of the process by which prices are formed, as they can be implemented simply by running regressions on historical price data. This feature permits an even larger degree of ignorance than naive inference, but it comes at a cost. A large amount of historical data must be used for the regression forecast to be precise. In contrast, naive buyers already know the structural parameters of demand, so they can draw inference with very little data.

Glaeser and Nathanson (2016) simulate a variant of the model in which naive and fully rational buyers compete for housing and the share of housing bought by each type varies over the cycle. The bubble-like features of prices persist even when half of the potential buyers are rational, but only for certain parameters. This result suggests that universally rejecting rationality is not necessary to explain house price dynamics, a point that has been effectively argued by Piazzesi and Schneider (2009). In the simulation, rational individuals comprise a larger share of buyers as prices begin to rise during a cycle, and naive individuals constitute a larger share as prices begin to fall.

While housing bears many similarities to other financial assets such as stocks, important differences motivate key assumptions in our model. Perhaps most important, there is no single, posted price in housing markets that is comparable to the market price of a share of stock. Past price data are available only with a significant lag. Our model suggests that lagged information about prices can be particularly important in explaining why momentum is stronger in housing markets than in other asset markets.

Assumptions about near-rationality in housing can also be particularly palatable because home buying is done almost entirely by nonprofessionals. Short sales are essentially impossible, and buying and holding large amounts of single-family housing can be costly for a single large investor. Our work does not address other key aspects of the housing market, such as the supply of homes for sale or homes being built. It therefore cannot speak to building booms or the movement in turnover and vacancy rates. We hope that future work will remedy this shortcoming.

The cap rate error can be seen as an application of several strands of thought in mainstream behavioral economics. We follow Eyster and Rabin (2010) in assuming that individuals are imperfect at inferring the belief processes of others. We follow the cognitive hierarchy model of Camerer et al. (2004), in that naive homebuyers are similar to what they call "level-2 thinkers," who mistakenly underestimate the strategic capacity of other players. Eyster et al. (2015) model similarly naive investors in financial markets. In their case, traders ignore the information implied by current prices. In our case, traders use an approximation that leads to a misunderstanding of past prices. We also follow the Gabaix (2014) sparse-reasoning theory, in that our buyers err by using an approximation well suited to one environment in a different and less appropriate setting.

All work on semi-rationality is troubled by Tolstoy's corollary: there is only one way to make correct inferences but an uncountable number of ways to get things wrong. Naive inference is one possible model of semi-rationality in housing markets. Many other forms of irrationality can exist, and a rational model could be discovered that reconciles all the facts. Yet, remarkably, this relatively modest deviation from rationality predicts outcomes so much closer to reality than the standard rational model.

The paper proceeds as follows. Section 2 reviews facts about housing markets that guide our model, which we present in Section 3. Section 4 contains theoretical results on the autocorrelations of price changes. We calibrate the model in Section 5 and conclude in Section 6.

2. Four stylized facts about housing markets

Four stylized facts guide our model: positive short-term serial correlation, longer-run mean reversion, excess volatility, and backward-looking price-growth expectations. Estimates of positive serial correlation in price indices find coefficients typically above 0.5 when annual price changes at the metropolitan area level are regressed on the one-year lag of annual changes (Caplin and Leahy, 2011). Some of this momentum can reflect the incorrect coding of the sale date or the smoothing process involved in generating repeat-sales indices, but the bulk of research concludes that much of the serial correlation is not due purely to issues with repeat-sales indices (Ghysels et al., 2013). The longer-run mean reversion of housing prices is also large, with a one-dollar increase in a metropolitan area's prices relative to other metropolitan areas over five years predicting a 30-cent or more decrease over the next five years (Glaeser et al., 2014). Head et al. (2014) find that the volatility of house price changes exceeds that of income changes by a factor of 1.60 to 2.75 at the metropolitan area level.

A number of rational models have tried to fit these facts. Models with neither search nor time-varying interest rates such as Glaeser et al. (2014) fail to deliver either short-run serial correlation or excess price variance. Time-varying interest rates do a better job of explaining price volatility (Campbell et al., 2009; Favilukis et al., 2013) but typically have trouble explaining positive serial correlation. A healthy debate remains about whether easy credit explains the housing price boom of 2000–2006 (Mian and Sufi, 2015; Glaeser et al., 2013).

An alternative rational approach is to follow Wheaton (1990) and assume a search model. Head et al. (2014) and Guren (2015) are two recent examples that use search models to generate positive serial correlation in house prices. Search models are natural in housing as they capture the idea that information dribbles out through decentralized purchases instead of being revealed instantly in a public market price, which is a crucial element in our model. This slow flow increases the level of serial correlation in price changes. But, on its own, search does not generate mean reversion or excess volatility. Rational ignorance leads to small, cautious deviations from priors, not wild swings in beliefs.

We do not claim that a rational model cannot explain all of the stylized facts. It may be possible through a combination of time-varying discount rates and search frictions to fit price data reasonably well. Hedlund (2016), for instance, uses a search model with endogenous credit constraints to explain the patterns of house price dynamics explored in our paper. Yet, surely bounded rationality should not be rejected out of hand given the behavior of many buyers during the 2003 to 2006 period. Any rational model has difficulty matching the survey evidence on homebuyer beliefs during this time (Case et al., 2012). If these surveys are to be believed, then homebuyers in Orange County, California, in 2005 expected prices to go up by 15.2% in each of the next ten years—a fourfold cumulative increase. Case et al. contend that the ten-year expectations reported in their survey displayed excessive optimism, and we calculate that these expectations overstated eventual growth on average by 10.7 percentage points annually in the four counties they study in 2003–2006.[1] In contrast, the one-year expectations in the survey underestimate price momentum. Case et al. estimate a coefficient of 0.23 from regressing the expected one-year price change on one-year lagged price changes, and the empirical momentum in the housing market is above 0.5.[2] Some scholars have argued that these surveys are meaningless, but if the surveys do capture some part of the reality of beliefs, then homebuyers seem unlikely to be fully rational. These expectations are too aggressive relative to past housing experience. We treat the one-year expectations from this survey as data to be matched against the predictions of our model.

The core pricing patterns that we explain exist in many asset markets other than housing (Cutler et al., 1991). A long literature shows momentum (Jegadeesh and Titman, 1993), mean reversion (Poterba and Summers, 1988), and excess volatility (Shiller, 1981) in stock prices and offers nonrational theories of these facts (Barberis et al., 1998; Daniel et al., 1998). While we draw on work in finance, we craft our model to fit the specifics of housing markets, with the prominent exception that we ignore the supply of new housing. This is an important omission, but including supply would considerably complicate the model and seems best left to future work.

Housing markets differ from standard asset markets in at least three major ways. First, selling short stock is far easier than selling short housing. We assume no short selling in our model, but, because short selling is also difficult in many asset markets, this assumption does not make our model irrelevant in other contexts.

Second, buying large amounts of stock is far more efficient than buying large amounts of single-family detached housing, as the dividend flow from owning a house comes from living in that house. Consequently, housing prices cannot usually be run up by the enthusiasm of a small number of buyers with deep pockets. This difference is important, but it also does not mean that our model is relevant only for housing. If anything, the ability of a small number of eager buyers to push up prices in other markets increases, not decreases, the power of the cap rate error.

Third, one unique price of housing does not exist, and knowing, exactly, the current market price of any particular house is usually impossible. We assume limited knowledge of price movements in the recent past, which seems to limit the direct applicability of our model to asset markets in which fundamentals can be shrouded but prices are visible continuously. Perhaps the lower, but significant, amount of momentum found in other asset markets is compatible with a calibration of our model in which the lag with which prices are observed is smaller than the six months we use.

Since De Long et al. (1990a), the noise trader tradition in finance has perturbed rational models by assuming irrationality for a small number of agents and then examining how these agents and their interactions with rational arbitrageurs shape prices. Hong and Stein (1999) present a model in which momentum caused by inattentive investors leads to mean reversion resulting from arbitrageurs, and Barberis et al. (2015b) continue this tradition by examining the impact of modest numbers of extrapolators in formal asset markets.

Piazzesi and Schneider (2009) pioneered this approach in housing, and we agree that many rational individuals likely hold beliefs that do not impact prices during large booms. Burnside et al. (2016) further this approach with a heterogeneous-belief model that explores the effect of social learning on house price dynamics. Yet, we also think that housing markets are easier to understand as being driven by small irrationality from the many instead of major irrationality from the few. Viewing housing markets in 2004–2006 as being dominated by a small number of highly irrational investors seems incorrect. Millions of Americans bought homes during that time period. Polls of homebuyers (Case et al., 2012) suggest that beliefs about high rates of future price appreciation were widespread, and the home buying activity of managers in securitized finance (Cheng et al., 2014) as well as the contemporaneous statements of many economists (Gerardi et al., 2010) indicate that professionals did not expect the price declines that followed. This paper proposes a theory of a universal mistake that produces the key stylized facts of house prices.

3. A model of house price determination

This section presents our model and derives results concerning the beliefs of home buyers.

3.1. Housing market fundamentals

We consider the choice of an individual who is deciding whether or not to purchase a home. This person is matched with one house. If she buys the house, she receives a flow of utility improvement of $D_{i,t}$ relative to her next best alternative. This flow utility can be interpreted as the overall benefit of living in the city relative to a reservation locale, but, in that case, we must also assume that the opportunity to buy in the city is a once-in-a-lifetime chance. Alternatively, the reservation utility could include the opportunity of buying again in the city, but this makes interpretation slightly more difficult. The supply of housing is fixed.[3]

[1] For each county and year, we calculate the annualized nominal increase in the Q1 Federal Housing Finance Agency house price index over the subsequent ten years. The average annualized realized ten-year gain equals 0.3%, and the average annualized expected ten-year gain of survey respondents equals 11.0%. The difference is 10.7%.

[2] The 0.23 is precisely estimated, with a standard error of 0.02. The regression has an R^2 of 0.73 even though only 40 observations are used, pointing to the strength of extrapolation as an explanation for price expectations in the housing market.

[3] The interaction between uncertainty and housing supply has been explored elsewhere. Glaeser et al. (2008) examine the link between belief-based bubbles and housing supply. Nathanson and Zwick (2017) go further and show that in land markets, which can be dominated by small

This overall utility combines an idiosyncratic element a_i with a city-specific component D_t:

$$D_{i,t} = D_t + a_i. \tag{1}$$

These elements include both the labor market returns and utility-related benefits from living in the city. The idiosyncratic component is drawn independently for each individual from a normal distribution with mean zero and standard deviation σ_a. The city-specific component of utility follows

$$dD = gdt + \sigma_D dW^D, \tag{2}$$

where W^D is a standard Wiener process and g is a stochastic process.

In this specification, changes to city-level demand persist over time. Correlation exists between past success and the future population or employment growth of the city. Typically, mean reversion of incomes is evident at the city level, but that pattern has declined over time (Berry and Glaeser, 2005; Ganong and Shoag, 2015). The absence of city-level mean reversion is for convenience, and the model could easily encompass this feature while leaving the results unchanged. The critical assumption is that growth rates shift over time and mean revert, so that

$$dg = -\lambda(g - \bar{g})dt + \sigma_g dW^g, \tag{3}$$

where W^g is a standard Weiner process that is independent from W^D and $\bar{g}$ is the long-run average growth rate. If growth rates were constant, then they would eventually be known, and the learning about growth that is a crucial element in the model would disappear. If growth rates did not mean revert, but instead followed a random walk, then the price dynamics would become too explosive and yield none of the mean reversion in the data.

For simplicity, we set $\bar{g} = 0$ throughout the paper. This restriction has no bearing on our results because they all concern the dynamics of price changes. A nonzero $\bar{g}$ would simply shift all price changes by a constant and thus not affect statistics such as the serial correlation and volatility of price changes. We provide further discussion of this restriction in the Appendix.

The persistence of growth rates is empirically debatable and depends on how demand at the city level is measured. Head et al. (2014) find a correlation of 0.27 between annual income growth and lagged income growth at the city level. In our empirical work, we find a larger correlation of 0.7 using metropolitan area rents as the proxy for demand. This persistence is a necessary feature of our model, and we show what the model implies when the persistence of growth rates is small.

Each transaction involves exactly one buyer and one seller, and the buyer pays a price that makes her indifferent between owning the house or not. The seller's willingness to accept is irrelevant. We do not mean to suggest that this is a realistic model of housing markets, in which most homes have multiple prospective buyers and most buyers consider a number of homes. The role that the bargaining process can play in shaping housing dynamics has been examined elsewhere (Anenberg and Bayer, 2014; Guren, 2015) and we are interested in examining the role of nonstandard beliefs. As such, we have chosen a simple market structure to focus on how the inference process can shape demand volatility and the persistence of price movements.

Individuals remain in their homes unless they receive a shock that forces them to move. These exogenous mobility shocks are Poisson and arrive at a rate μ. Mobility shocks are the easiest way to generate resale and an interest by buyers in future prices. We agree strongly that an endogenous resale model would be more realistic. Moreover, the fixed sale assumption is compatible with the assumption that prices are determined by the buyer's willingness to pay and not by any aspect of the seller.

We define $p_{i,t}$ to be the price of a house transacted at t to buyer i and p_t to be the average price of all houses sold at t. The discounted value of owning the house and the willingness to pay therefore equal

$$p_{i,t} = \mathbf{E}_{i,t}\left[\int_t^T e^{-r(\tau-t)} D_{i,\tau} d\tau + e^{-r(T-t)} p_T | T - t \sim \text{Poisson}(\mu)\right], \tag{4}$$

where r is the discount rate. Price formation depends on each buyer's expectations about future prices and, therefore, on each buyer's expectations of future buyers' expectations. When this expectation forecasting takes a specific form, which is general enough to encompass both the rational and naive rules we specify, prices follow a simple linear structure.

Lemma 1. *Suppose that for all $T \geq t$, $\mathbf{E}_{i,t}\mathbf{E}_T D_T = \mathbf{E}_{i,t} D_T$ and $\mathbf{E}_{i,t}\mathbf{E}_T g_T = \phi_g(T-t)\mathbf{E}_{i,t} g_t$ for some function ϕ_g, where $\mathbf{E}_T$ denotes the average expectation among buyers at T. An equilibrium for the average price of the houses transacted at t is given by*

$$p_t = \frac{1}{r}\left(\frac{r}{r+\mu} D_t^a + \frac{\mu}{r+\mu}\widehat{D}_t\right) + A_g \widehat{g}_t, \tag{5}$$

where D_t^a equals the average flow utility $D_{i,t}$ among buyers at t, $\widehat{D}_t$ and $\widehat{g}_t$ are their average beliefs about the current values of city demand D and its growth rate g, and A_g is a constant.

The proof of Lemma 1 is given in the Appendix along with the proofs of all other lemmas and propositions.[4]

3.2. Information available to buyers

At time t, the buyer knows the current flow utility $D_{i,t}$ that she receives from the house to which she is matched. She also observes a history of transaction prices of houses

numbers of professional buyers, bubbles can appear more readily than in housing markets, in which ownership is far more dispersed.

[4] Lemma 1 assumes away rational bubbles, which are unlikely in housing markets. Rational bubbles predict a positive probability of extreme long-run prices, which with endogenous supply would generate extreme long-run investment and city size. Giglio et al. (2016) compare long-run leases and prices to provide significant evidence that the transversality condition does hold for housing, making rational bubbles impossible. Yet Adam et al. (2011) show how almost-rational bubbles can generate impressive swings in prices and strongly link prices and interest rates.

in the city. She learns the average price $p_{t'}$ every δ units of time before her purchase, that is, for $t' = t - \delta, t - 2\delta$, and so forth. The number of sales in each average equals N. This transaction history corresponds to a price index when N is large or to a list of all transactions when $N = 1$. We derive some analytic results in the limit as $N \to \infty$ but allow for finite N in the quantitative exercises. We denote the set of observed prices by $\Omega_t^p = \{p_{t-m\delta} \mid m \in \mathbb{N}\}$, where $\mathbb{N}$ is the set of positive integers.

The buyer naturally would know her own flow utility $D_{i,t}$ at t. After all, this is the utility the buyer currently receives from living in the city. Knowing past prices is also reasonable, as house prices are readily available from a number of sources. Homebuyers frequently examine the sale prices of similar homes before making a purchase. This practice, called comparable analysis or comps, is the foundation for property appraisals. The lag between the present and the time of the most recently available price introduces sluggishness into the model that generates momentum.

The buyer cannot directly observe the city demand D_t or its growth rate g_t. City-wide demand D_t is an aggregation of private information $D_{i,t}$. Less obvious is why the buyer cannot observe g_t, given that this growth rate directly affects the flow utility the buyer will receive. The intuition here is that the buyer's utility rises and falls with the quality of the city, either though amenities or the labor market. As these fluctuations involve city-wide forces beyond the buyer's control, it is reasonable to suppose that the buyer possesses no private information about the current growth rate. Gao et al. (2015) similarly argue that a core part of house price determination is learning about the amenities and productivity of the neighborhood where the house is located, and Kurlat and Stroebel (2015) present empirical evidence that buyers are not fully informed about the neighborhood of a new house purchase.

In principle, a buyer could obtain information on D_t from looking at various economic indicators about the city. To permit this possibility, we allow buyers to observe noisy signals D_t^s of demand, where $D_t^s = D_t + s_t$ and s_t is an independently drawn normal error with standard deviation σ_s. Buyers have access to this information at the same frequency with which they observe prices. The set of signals known at t equals $\Omega_t^s = \{D_{t-m\delta}^s \mid m \in \mathbb{N}\}$.

In addition to $\{D_{i,t}\}$, Ω_t^p, and Ω_t^s, the buyer at t observes stochastically revealed direct observations of the true state of demand. These observations occur at a set of times $\mathcal{T}$ that are realizations of a continuous-time Poisson process with parameter $\rho > 0$, where ρ is small. That is, given realizations on $\mathcal{T}$ up to some time, the cumulative distribution function for the time Δt until the next realization is $1 - e^{-\rho\Delta t}$. We denote $\mathbf{x}_t = (D_t, g_t)'$ to be the state of demand at t. The buyer at t observes $\Omega_t^{\mathbf{x}} = \{\mathbf{x}_{t'} \mid t' \in \mathcal{T}$ and $t' \le t\}$. This rare revelation of the true state of the world has important consequences for the uniqueness of equilibrium when buyers are rational. As long as $\rho > 0$, with probability one there exists some time in the past when the state of demand was revealed. Rational buyers begin at that state and then infer all of the demand shocks since then. This process of rational inference occurs anytime history has a beginning.

The complete information set for the buyer is

$$\Omega_{i,t} = \{D_{i,t}\} \cup \Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}. \tag{6}$$

3.3. Inference about demand

The buyer's inference problem is to use the data in $\Omega_{i,t}$ to infer the value of market demand D_t and its growth rate g_t. The best a buyer can do is to extract all the data directly observed by buyers before t. In addition to the signals in Ω_t^s, this information includes all individual flow utility $D_{i',t'}$. Due to the normality assumptions, a sufficient statistic for the distribution of buyer flow utility at t' is $D_{t'}^a$, the average flow utility across the N buyers at that time. We denote $\Omega_t^a = \{D_{t-m\delta}^a \mid m \in \mathbb{N}\}$.

When buyers are rational and the rationality of all buyers is common knowledge, observing the infinite history of prices allows the buyers to know the history Ω_t^a.

Proposition 1. *Suppose it is common knowledge among buyers at all times that information sets take the form given in Eq. (6). Then, each buyer can perfectly deduce the history Ω_t^a of average buyer flow utility.*

Given this proposition, inference for the rational buyers involves a standard signal extraction problem. D_t and g_t are inferred from the series of past noisy observations of D in Ω_t^a and Ω_t^s, as well as from the noisy observation of D_t given by private utility $D_{i,t}$.

The proof of Proposition 1 goes as follows. Let $t' < t$ be a time at which buyers at t observe house sales. Conditional on all house prices and demand signals before t', the price at t' is a strictly increasing function of average flow utility $D_{t'}^a$, as higher flow utility directly increases the pricing Eq. (5) and also increases the posterior means $\widehat{D_{t'}}$ and $\widehat{g_{t'}}$. Because the buyer at t observes all prices and demand signals before t', she exactly infers $D_{t'}^a$ from observing the transacted price $p_{t'}$.

As this proof makes clear, deducing Ω_t^a requires a fairly hefty cognitive load on the part of the buyers, as they need to infer everything about beliefs in the past. As such, we introduce a second possibility.

Definition 1. *Naive inference* at t is inference conditional on the belief that $p_{i,t'} = D_{i,t'}/r$ for all i and $t' < t$.

Naive inference makes inference straightforward. Simply multiplying the price $p_{t'}$ by the constant r yields $D_{t'}^a$, and then the inference proceeds as in the rational case. The use of naive inference by buyers is the key assumption of our model.

Naive inference can be motivated as an approximation that results from inattention. Consider the pricing formula delivered by Lemma 1 evaluated at some past time t'. The long-run average of g is zero, so if the expectation $\widehat{g_{t'}}$ is unbiased, it equals zero on average. Similarly, if $\widehat{D_{t'}}$ is unbiased, it equals $D_{t'}^a$ on average. Therefore, as long as the conditions of Lemma 1 hold, the past price can be written as $p_{t'} = D_{t'}^a/r + \xi_{t'}$, where $\xi_{t'}$ is mean zero measurement error. The quantity $rp_{t'}$ is an unbiased estimate for $D_{t'}^a$, and this is the estimate naive buyers use.

The problem with using $rp_{t'}$ to estimate $D^a_{t'}$ is that the measurement error $\xi_{t'}$ is correlated across time. This serial correlation arises from the nonindependence of demand forecasts. If buyers believe that the level or growth rate of demand is higher than its true value or long-run average today, they are likely to believe this tomorrow as well. Proper Bayesians would recognize the dependence across time in the measurement error, but naive buyers do not. Naive inference is good for estimating demand from a single observation of house prices, but it fails when estimating demand using a series of prices. Ignoring the serial correlation in $\xi_{t'}$ is a form of inattention that results from a procedure that is rational in a different context. This sort of inattention is studied in Gabaix (2014). We call this mistake the cap rate error because it arises from believing that the cap rate linking demand and prices is constant at $1/r$.

A complementary interpretation for naive inference is as the result of a simplified problem for individuals who lack the cognitive ability to make inferences about inferences. Buyers avoid this recursion by replacing each prior buyer's expectation by its expected value, i.e., $\widehat{D_{t'}}$ with $D^a_{t'}$ and $\widehat{g_{t'}}$ with zero. In so doing, naive buyers assume that past buyers were simple and used just their private information $\{D_{i,t'}\}$ to draw inferences instead of the full information set $\Omega_{i,t'}$. The idea that economic agents could assign this type of simplicity to others with whom they interact has been extensively explored by Eyster and Rabin (2010, 2014), and naive inference can be seen as an application of their work to financial markets. We focus on the two cases of hyper-rational homebuyers and naive homebuyers.

We now solve for the posteriors on D_t and g_t of rational and naive buyers. The state of demand for the city at t' can be summarized by a 2×1 vector $\mathbf{x}_{t'} = (D_{t'}, g_{t'})'$. Given the laws of motion in Eqs. (2) and (3), this state vector evolves linearly with normal noise. Over δ, the discrete length of time in between sales, this state vector changes according to the rule $\mathbf{x}_{t'+\delta} = \mathbf{F}\mathbf{x}_{t'} + \mathbf{w}_{t'}$, where $\mathbf{F} = \begin{pmatrix} 1 & (1-e^{-\delta\lambda})/\lambda \\ 0 & e^{-\delta\lambda} \end{pmatrix}$ and $\mathbf{w}_{t'}$ is identically distributed mean zero normal noise with covariance matrix $\mathbf{Q}$ that is independent across time and from $\mathbf{x}_{t'}$. The news at t' consists of $D^a_{t'}$ and $D^s_{t'}$. We write this news as $\mathbf{H}_0\mathbf{x}_{t'} + \mathbf{v}_{t'}$, where $\mathbf{H}_0 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{v}_{t'}$ is normal mean zero noise with covariance $\mathbf{R}_0 = \begin{pmatrix} \sigma_a^2/N & 0 \\ 0 & \sigma_s^2 \end{pmatrix}$. At t, the news consists of just $D_{i,\,t}$, which we write as $\mathbf{H}\mathbf{x}_t + \mathbf{v}_t$, where $\mathbf{H} = (1, 0)$. The variance of $\mathbf{v}_t$ is $\mathbf{R} = \sigma_a^2$.

The linear evolution of state variables as well as the normal structure of all noise allows the buyers to use a Kalman filter to derive the optimal posterior $\mathbf{x}_t|\Omega_{i,t}$. The resulting average of the posteriors across buyers at t gives $\widehat{\mathbf{x}_t} = (\widehat{D_t}, \widehat{g_t})'$, the belief terms that appear in the pricing function in Eq. (5). Lemma 2 solves for these posterior averages and for the covariance of each buyer's posterior.

Lemma 2. *Let $\mathbf{x}_t = (D_t, g_t)'$ denote the state of housing demand at t. For both rational and naive buyers, the posterior $\mathbf{x}_t|\Omega_{i,t}$ is a multivariate normal with the same covariance. As $\rho \to 0$, the mean of this posterior for rational buyers converges almost surely to*

$$\widehat{\mathbf{x}_t} = \mathbf{K}D^a_t + (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{F}\times \sum_{m=1}^{\infty} [(\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}]^{m-1}\mathbf{K}_0\left(D^a_{t-m\delta}, D^s_{t-m\delta}\right)',$$

and for naive buyers, the mean converges almost surely to

$$\widehat{\mathbf{x}_t} = \mathbf{K}D^a_t + (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{F} \times \sum_{m=1}^{\infty}[(\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}]^{m-1}\mathbf{K}_0(rp_{t-m\delta}, D^s_{t-m\delta})',$$

where $\mathbf{K}$ *and* $\mathbf{K}_0$ *are matrices that depend on* $\mathbf{F}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{R}_0$, $\mathbf{H}$, *and* $\mathbf{H}_0$. *The covariance of these posteriors converges almost surely to a time-independent matrix* $\mathbf{P}$.

The two types of buyers use the same filters, but naive buyers use $rp_{t'}$ in place of the true value of $D^a_{t'}$ that rational buyers use. A corollary is that naive buyers are overconfident in their estimates of housing demand. The naive posterior $\mathbf{x}_t|\Omega_{i,t}$ is a multivariate normal with covariance $\mathbf{P}$. This matrix $\mathbf{P}$ is the optimal covariance that results from the correct application of the Kalman filter. However, naive buyers do not apply the Kalman filter correctly. They use $rp_{t'}$ in place of $D^a_{t'}$, and as a result, they use some alternate linear filter without realizing it. Because the Kalman filter is optimal among all linear filters, the resulting error covariance is larger than $\mathbf{P}$ and, hence, larger than what naive buyers think it is. Proposition 2 sums up this argument.

Proposition 2. *Naive homebuyers are overconfident in their estimates of housing demand. Let* $\mathbf{P}_n$ *denote the covariance matrix of the naive forecast error* $\mathbf{x}_t - \mathbf{E}(\mathbf{x}_t \mid \Omega_{i,t})$. *Then,* $\mathbf{P}_n > \mathbf{P}$, *where* $\mathbf{P}$ *is the covariance matrix of the naive posterior* $\mathbf{x}_t|\Omega_{i,t}$. *The inequality means that* $\mathbf{P}_n - \mathbf{P}$ *is positive definite.*

The relation $\mathbf{P}_n > \mathbf{P}$ means that naive buyers overestimate the precision of their estimates of the level of demand D_t and its growth rate g_t. Furthermore, naive buyers underestimate the forecast error of any linear combinations of these quantities. They are overconfident in the valuations of their homes, as the pricing Eq. (5) is linear in $\widehat{D_t}$ and $\widehat{g_t}$.

Overconfidence about the accuracy of their beliefs limits naive buyers' attention to noisy signals about market demand. They do not appreciate the imprecision of their inferences from prices, so they demand less information, as the marginal value of information decreases with additional certainty. As a clear example, consider the inference problem when N, the number of house transactions underlying each price, goes to infinity. In this case, $D^a_{t'} \to D_{t'}$ by the law of large numbers, so the history of buyer flow utility Ω^a_t perfectly reveals the history of market demand. As a result, buyers ignore the noisy demand signals Ω^s_t. They believe all such information is already factored into house prices, which they directly observe. This response is optimal for the rational buyers, but it is a mistake for the naive buyers. Their overconfidence leads them to ignore valuable information.

Unlike naive buyers, rational buyers correctly understand the precision of their forecasts, and their posterior has the optimal covariance matrix **P**. They are able to achieve this optimal forecast by inferring the true value of $D_{t'}^a$ from past prices. As described in the proof of Proposition 1, extracting Ω_t^a from Ω_t^p involves knowing exactly how previous buyers form their own expectations of market demand. One of our motivations for introducing naive inference was the complexity of this procedure. To illustrate this claim, we solve directly for the rational posterior on $D_{t-\delta}$ as a function of past prices, which is what buyers directly observe.[5] In general, this expression is complicated. Proposition 3 gives an intuitive form that holds in a special case.

Proposition 3. *Let t_0 be the last time demand was directly observed, and suppose $n \equiv \lfloor (t-t_0)/\delta \rfloor > 1$. When the number N of transactions observed each period goes to infinity, naive buyers ignore information about demand and extrapolate lagged demand from the most recent price:*

$$\mathbf{E}(D_{t-\delta}|\Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}) = rp_{t-\delta}.$$

When $N \to \infty$ and growth rates do not persist ($\lambda \to \infty$), the rational buyer's posterior on the level of demand is a telescoping sum of past prices:

$$\mathbf{E}(D_{t-\delta}|\Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}) = \left(-\frac{\alpha}{1-\alpha}\right)^{n-1} \frac{rp_{t-n\delta} - \alpha_0 D_{t_0}}{1-\alpha_0} + \sum_{m=1}^{n-1} \left(-\frac{\alpha}{1-\alpha}\right)^{m-1} \frac{rp_{t-m\delta}}{1-\alpha},$$

where

$$\alpha = \frac{\mu}{r+\mu} \frac{\sigma_a^2}{\sigma_a^2 + \delta\sigma_D^2}$$

is a constant between zero and one, and $\alpha_0 = \mu[r+\mu]^{-1}\sigma_a^2[\sigma_a^2 + (t-n\delta-t_0)\sigma_D^2]^{-1}$.

When $\lambda \to \infty$, the growth rate is irrelevant and buyers must infer only the level of demand from past prices. Even in this simple case, the rational filter takes a starkly unintuitive form. Every other past price counts negatively toward the rational estimate of current demand. Furthermore, when $\alpha > 1/2$, the weights on past prices grow exponentially. The reason is that when α is near one, very little new information about current flow utility appears in rational prices; that is, prices depend mostly on beliefs.[6] These beliefs depend mostly on the prior price history, so the recursive filter ends up putting more weight on past prices than on recent ones. In contrast, naive buyers simply estimate demand using the most recent price and, in fact, use this rule even when they are inferring the growth rate.

Eyster and Rabin (2014) study settings in which people extract information from observing the sequential actions of others. The correct action is positively correlated with the state of the world, which is only partially known. They show that hyperrationality commonly leads to "anti-imitation." The optimal action depends negatively on the actions of some previous people, even though all people have the same objectives. Proposition 3 provides an example of this phenomenon in the housing market. In this case, the action is the price paid for the house, and the information is the level and growth rate of housing demand.

Their work and Proposition 3 call into question the robustness of rational updating in sequential settings. The divergent nature of the rational filter suggests that it will not work very well if previous prices were not formed by rational filters. If the rational filter lacks this robustness property, then a rational person should not use these rules if even a small chance exists that previous actors are not hyperrational as well. Evaluating the robustness of rational filters is difficult, as all possible inference rules used by others must be considered. This paper takes a small step in this direction by investigating the performance of the rational filter when previous buyers are naive. We perform this exercise in Section 5.

3.4. Price-change forecasts

Up to this point, we have fully specified how buyers infer D_t and g_t using past prices. To close the model and describe how prices are formed, we must specify what buyers at t believe about the expectations of future buyers to whom they expect to resell their houses. These expectations of expectations determine the weight A_g on growth expectations in the pricing formula in Lemma 1. This term A_g is determined by Eq. (4) and, in turn, by the buyer's expectation $\mathbf{E}_{i,t}p_T$ of future prices. This expectation depends on the forecasts of forecasts $\mathbf{E}_{i,t}\widehat{D_T}$ and $\mathbf{E}_{i,t}\widehat{g_T}$.

A natural way to resolve these forecasts is to impose the law of iterated expectations, so that the first equals $\mathbf{E}_{i,t}D_T$ and the second, $\mathbf{E}_{i,t}g_T$. Iterated expectations are consistent with the hyperrationality we attribute to rational buyers in the inference problem. If common knowledge of rationality continues into the future, then future buyers, who have at least as much information as current buyers, should hold beliefs consistent on average with that of present buyers.

Iterated expectations are less consistent with naive inference. Naive buyers believe that past prices are given by $D_{t'}^a/r$, and they reach this conclusion by assuming that other buyers' expectations equal their ex ante averages. A consistent forecast rule would assume that future prices are also given by $p_T = D_{T'}^a/r$, which is equivalent to setting $\mathbf{E}_{i,t}\widehat{D_T} = \mathbf{E}_{i,t}D_T$ and $\mathbf{E}_{i,t}\widehat{g_t} = 0$.

We model buyer forecasts to allow for consistency with both types of inference. A buyer believes that, with probability $1-\phi$, future buyers are as sophisticated as herself, leading the law of iterated expectations to hold. With probability ϕ, future buyers are simple and base their expectations solely on private demand. This case results from buyers thinking that future buyers will be less sophisticated or from our preferred explanation that current buyers choose a simple model of the behavior of others to lessen the cognitive load on themselves. The current buyer's forecasts hence equal $\mathbf{E}_{i,t}\widehat{D_T} = \mathbf{E}_{i,t}D_t$ and $\mathbf{E}_{i,t}\widehat{g_T} =$

[5] Public information observed by buyers at t is only as recent as $t-\delta$, so it is cleaner to separately report $\mathbf{E}(D_{t-\delta})$ here and $\mathbf{E}(g_{t-\delta})$ in Proposition 5. The formula for $\mathbf{E}(D_t)$ combines $\mathbf{E}(D_{t-\delta})$ and $\mathbf{E}(g_{t-\delta})$.

[6] The parameter α captures the dependence of current prices on public information as opposed to $D_{i,t}$. A larger $\mu/(r+\mu)$ leads buyers to care more about resale and, hence, about $\widehat{D_t}$, and a larger $\sigma_a^2/(\sigma_a^2+\delta\sigma_D^2)$ makes prices better signals about D_t than the buyer's idiosyncratic utility $D_{i,t}$.

$(1-\phi)\mathbf{E}_{i,t}g_T = (1-\phi)e^{-\lambda(T-t)}\mathbf{E}_{i,t}g_t$. This forecasting rule satisfies the conditions of Lemma 1. Applying it and solving for A_g yields the following Lemma 3.

Lemma 3. *The weight A_g on the growth-rate expectation in the pricing formula in Lemma 1 is given by $A_g = [r(r+\lambda+\phi\mu)]^{-1}$, where ϕ denotes the perceived probability that future buyers are simple and do not use prices to draw inference and $1-\phi$ is the probability that future buyers are sophisticated enough for the law of iterated expectations to hold.*

One goal of this paper is to make sense of survey evidence concerning expectations about the housing market. Much of the survey evidence on this topic is framed in terms of expectations of price growth. For instance, the Michigan Survey of Consumers asks whether the present is a "good time to buy [housing] for investment" (Burnside et al., 2016). A significant portion of respondents, around 30%, explicitly mention house prices to justify their view (Piazzesi and Schneider, 2009). Case et al. (2012) elicit quantitative estimates of house price growth. They ask: "How much of a change do you expect there to be in the value of your home over the next 12 months?" and "On average over the next ten years how much do you expect the value of your property to change each year?"

To match these surveys, we calculate buyers' expectations of the market value of their house at current and future dates. Using the pricing function in Lemma 1, we calculate the expected market value as

$$\mathbf{E}_t p_T = \frac{\mathbf{E}_t D_T}{r} + \frac{(1-\phi)\mathbf{E}_t g_T}{r(r+\lambda+\phi\mu)}. \tag{7}$$

Larger values of the naivety of buyer forecasts ϕ lead to expectations of higher house price growth. When the law of iterated expectations holds ($\phi = 0$), buyers today believe their information is immediately incorporated into market values. As a result, they forecast very small changes in market values, even when they believe the growth rate is high. In contrast, buyers for whom $\phi = 1$ believe that growth-rate news is never anticipated in market values. They therefore predict continued increases in the market value of their homes when the growth rate is high. Proposition 4 makes these results clear by solving for the expected change in market values as a function of the growth-rate belief.

Proposition 4. *For a given belief about the growth rate, buyers expect greater increases in the market values of their home when they are more naive. The expected growth in the market value of a house equals*

$$\mathbf{E}_t(p_T - p_t) = \frac{r+\phi\lambda+\phi\mu}{r+\lambda+\phi\mu}\,\frac{1-e^{-\lambda(T-t)}}{\lambda}\,\frac{\widehat{g}_t}{r}.$$

Holding $\widehat{g}_t$ constant, this expression increases in ϕ, the perceived probability of selling to a simple buyer in the future.

Rational buyers are much more conservative than naive buyers in forecasting the growth in the market value of their homes. The expected change when $\phi = 0$ equals the change when $\phi = 1$ times $r/(r+\lambda)$, and this fraction is always less than one. The empirical value of this fraction falls significantly below one. For instance, if the annual persistence of growth shocks is 0.3 (the value of income growth persistence at the metro area level), then $\lambda = 1.2$. A value of $r = 0.04$ then leads to $r/(r+\lambda) = 0.03$. Under the parameters we use in Section 5, which assume more persistence in demand growth, the fraction rises to 0.07. This ratio falls well below one unless demand growth is very persistent.

Rational buyers believe public information gets priced into housing immediately. They therefore do not expect much growth in the market value of their homes. In contrast, naive buyers mistakenly believe that information about the growth rate never affects the market value of homes. This belief allows naive buyers to expect significant increases in market values when they perceive the growth rate of fundamentals to be high. According to the survey evidence, expectations of changes to the market value of one's home are large and significant. A model in which all buyers use rational filtering is at odds with this empirical fact.

3.5. Extrapolation

Not only do naive buyers expect significant changes in house prices, but they also extrapolate; that is, they expect particularly large increases after recent increases. The expected increase in prices scales with $\widehat{g}_t$, which itself rises with past price increases, as Proposition 5 shows. For simplicity, we consider the forecasting problem of a buyer before viewing her private information $D_{i,t}$.

Proposition 5. *When the number N of transactions observed each period goes to infinity, naive buyers extrapolate the growth rate from a trailing weighted average of past price changes:*

$$\mathbf{E}(g_{t-\delta} \mid \Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}) = k\sum_{m=1}^{\infty}\kappa^{m-1}\Delta p_{t-m\delta},$$

where k and κ are constants that depend on the structural parameters of the model and $\Delta p_{t-m\delta} = p_{t-m\delta} - p_{t-(m+1)\delta}$.

Proposition 5 is not particularly surprising. The correct Bayesian way of estimating the growth rate of some process is to use the past growth rate, with more weight put on more recent observations when the growth rate changes over time. As naive buyers believe p gives observations of demand, they estimate g using past price changes. Together with Proposition 4, Proposition 5 implies that naive buyers extrapolate past price changes to future ones.

This conclusion is notable because such extrapolation is often assumed in finance papers on expectation formation. For instance, Barberis et al. (2015b) use the above functional form for expected growth rates in their model of how extrapolative expectations affect asset prices. In our framework, this formula arises naturally once we make the assumption that naive buyers equate D and rp because growth rates are persistent.

4. Autocorrelations of naive house price changes

We show in a specific case of the model that naive inference leads prices to obey the equation $\Delta p_t =$

$\beta_1 \Delta p_{t-\delta} - \beta_2 \Delta p_{t-2\delta} + \zeta_t$, where $\Delta p_t = p_t - p_{t-\delta}$ and ζ_t is an AR(1) plus noise that is not negatively correlated with its innovations. As shown by Glaeser and Nathanson (2016), price changes that satisfy this equation display sinusoidal autocorrelations, the first of which is positive, as long as $\beta_1 > 0$ and $(\beta_1/2)^2 < \beta_2 < 1$.

The case we study imposes a number of simplifications. First, the number of home sales is large enough that prices aggregate information completely ($\sigma_a/\sqrt{N} \to 0$). Second, the individual utility noise is so large that buyers ignore their own utility and use only prices when inferring demand ($\sigma_a \to \infty$). Finally, buyers have access only to the two most recent lags of prices. This specification represents a scenario in which buyers learn about demand entirely from recent observations of a housing price index. The restricted information set in this special case is denoted $\Omega'_{i,t} = \{D_{i,t}\} \cup \{p_{t-\delta}\} \cup \{p_{t-2\delta}\}$. A naive buyer's posterior on the lagged level of demand is $\mathbf{E}(D_{t-\delta} \mid \Omega'_{i,t}) = rp_{t-\delta}$, and her estimate of the lagged growth rate equals $\mathbf{E}(g_{t-\delta} \mid \Omega'_{i,t}) = r(p_{t-\delta} - p_{t-2\delta})\lambda e^{-\delta\lambda}/(1-e^{-\delta\lambda})$. The naive buyer simply extrapolates the level of demand from the level of prices and the growth rate from the change in prices.

We now compare our simple rule with an alternative: linear regression based on the past two years of prices. The formulas described provide the weights that a naive Bayesian would put on past prices, which can be derived with no knowledge of prices prior to the previous period. By contrast, estimating a two-period autoregressive model would require a great deal of lagged price data to acquire any degree of precision. If the homebuyer was given the correct regression coefficients, implementation would be easy. But, if the buyer had to run her own regressions, the cognitive load for the average homebuyer would seem to be difficult.

These simple naive formulas lead current prices to depend positively on the first lag of prices and negatively on the second lag. Higher values of $p_{t-\delta}$ increase the buyer's estimate of the level and growth rate of demand, and both of these estimates increase today's price p_t. Conversely, a higher value of $p_{t-2\delta}$ lowers the estimate of the growth rate, thereby negatively impacting p_t. Proposition 6 writes price changes in the form mentioned above, and the Appendix proves that, for certain parameters, the inequalities involving β_1 and β_2 are satisfied.

Proposition 6. *Suppose naive buyers observe only the two most recent house prices. When $\sigma_a \to \infty$ and $\sigma_a/\sqrt{N} \to 0$, one-period house price changes obey the autoregressive equation*

$$\Delta p_t = \left(\frac{(1+e^{-\delta\lambda})\mu}{r+\mu} + \frac{\lambda e^{-2\delta\lambda} r A_g}{1-e^{-\delta\lambda}} \right) \Delta p_{t-\delta} - \left(\frac{e^{-\delta\lambda}\mu}{r+\mu} + \frac{\lambda e^{-2\delta\lambda} r A_g}{1-e^{-\delta\lambda}} \right) \Delta p_{t-2\delta} + \zeta_t,$$

where $\Delta p_t = p_t - p_{t-\delta}$ and ζ_t is an AR(1) plus noise that is not negatively correlated with its innovations.

Naive inference succeeds at conceptually explaining the autocorrelation structure of house prices. To investigate whether naive updating can match these autocorrelations quantitatively, we simulate the richer model exposited in Section 3 using parameters calibrated from housing data.

Table 1
Calibrated parameter values.
We estimate demand and transaction parameters from data on house prices and rents. As shown in the Appendix, the first three autocovariances of annual city-level demand changes uniquely identify λ, σ_g, and σ_D. We take these autocovariances from time series on rents and incomes at the metropolitan area level. The discount rate r comes from Glaeser et al. (2014). The resale probability μ comes from Census data on mobility of owner-occupants. The volatility σ_a is identified from the residual of a hedonic regression of rents on property characteristics and location [public use microdata area (PUMA)] fixed effects. We choose δ to roughly capture the frequency at which housing price and other economic data are released and σ_s to describe the accuracy of local economic indicators (about 10%). Finally, N is determined using the number of owner-occupied houses in a PUMA, together with μ and δ.

Parameter	Value	Role
Demand parameters		
λ	0.51	Demand growth reversion
σ_g	\$230	Volatility of growth shocks
σ_D	\$250	Volatility of demand shocks
r	0.04	Discount rate
Transaction parameters		
σ_a	\$3,120	Volatility of idiosyncratic utility
μ	0.075	Probability of forced sale
δ	0.5	Length of period (years)
N	1,130	Sales observed per period
σ_s	\$1,000	Noise in observations of demand

5. Dynamics of price changes: quantitative results

We now explore the dynamics of house prices and beliefs in quantitative simulations of the model.

5.1. Parameter choices

This subsection calibrates the model using reasonable values of the parameters estimated from housing market data. Substantial uncertainty exists about the true values of the parameters, and they likely vary across space as well. Our approach, therefore, is to show that the model matches empirical house price dynamics using parameters within the range offered by the data. Section 5.7 performs sensitivity analysis with respect to the parameters of which we are most uncertain.

Table 1 lists our estimated parameters, which fall into two groups. The first are identified from data on city-wide demand. The second are identified from data on individual housing transactions. At no point do we use data on the time series of house price changes, which are the data we are trying to explain with our model. Throughout the main exercise, we set $\phi = 1$, meaning that naive buyers use the same model for past and future prices.

5.1.1. Demand parameters

The evolution of city-wide demand D is described by Eqs. (2) and (3) and is governed by three parameters: the persistence λ of growth shocks, their volatility σ_g, and the volatility σ_D of nongrowth demand shocks. These parameters are uniquely determined by the first three autocovariances of annual changes in D, which we denote $\gamma_0 = \mathrm{Var}(\Delta D_t)$, $\gamma_1 = \mathrm{Cov}(\Delta D_t, \Delta D_{t-1})$, and $\gamma_2 =$

$\mathrm{Cov}(\Delta D_t, \Delta D_{t-2})$. Δ denotes the difference over one year. As we show in the Appendix, the ratio γ_2/γ_1 uniquely determines λ. The autocorrelation γ_1/γ_0 then determines the ratio σ_g/σ_D, and γ_0 determines the level of these volatilities.

The literature has used two empirical proxies for housing fundamentals: rents (e.g., Campbell et al., 2009) and local incomes (e.g., Head et al., 2014). Unfortunately, the autocovariances of these series differ significantly from each other, leaving us in the position of choosing between them. We choose intermediate values of these estimates and show that they work fairly well in allowing the model to capture house price dynamics.

The data on rents come from the Bureau of Labor Statistics (BLS), which compiles rental-price indices for 23 metropolitan areas. Campbell et al. (2009) describe these data further and provide the data set we use. Using these data, we compute the standard deviation of annual changes to be $\sqrt{\gamma_0}=$ \$250 and the first and second autocorrelations to be $\gamma_1/\gamma_0 = 0.73$ and $\gamma_2/\gamma_0 = 0.44$, respectively.[7]

The BEA provides income data at the metropolitan area level. An alternate source comes from Home Mortgage Disclosure Act (HMDA) data, which give the median income of new homebuyers in a metropolitan area. To the extent that flow utility D corresponds to that of the marginal homebuyer, HMDA could be more appropriate. Glaeser et al. (2014) describe both data sets. In the BEA data, $\sqrt{\gamma_0}=$ \$1,900, with autocorrelations $\gamma_1/\gamma_0 = 0.30$ and $\gamma_2/\gamma_0 = 0.11$.[8] The HMDA data provide a higher standard deviation of annual changes at $\sqrt{\gamma_0}=$ \$2,700, with autocorrelations $\gamma_1/\gamma_0 = 0.29$ and $\gamma_2/\gamma_0 = 0.09$.

The rent data display much more persistence than the income data. The ratio γ_2/γ_1, which determines the growth persistence λ, equals 0.60 in rents but only about 0.34 in income. Furthermore, γ_1/γ_0, which determines the relative importance of growth shocks, is much higher in the rent data. We combine features from both data sets by setting $\gamma_1/\gamma_0 = 0.3$ and $\gamma_2/\gamma_1 = 0.6$ as our baseline figures. Although this selection is somewhat arbitrary, it falls within the numbers suggested by the data and allows the model to match the dynamics of price changes well.

We adopt the value $\sqrt{\gamma_0}=$\$325, which is much closer to the volatility implied by rents. This value is largely unimportant for the results, as it simply scales the variances in the model and does not affect the autocorrelations of price changes. It allows the model to match the volatility of price changes, but the model can compare the predicted volatility of prices and fundamentals, and this comparison is mostly independent of the assumed value of $\sqrt{\gamma_0}$.

[7] To arrive at γ_0, we convert the rent index provided by the BLS to levels. The standard deviation of annual changes in the index equals 3.2, and the mean of the index is 140. Therefore, the rent index change standard deviation equals 2.3% of the mean level. Median annual rent in the US is \$10,884, so the standard deviation of annual changes equals \$250.

[8] These figures use BEA income data adjusted for state taxes. Without the tax adjustment, the numbers are $\sqrt{\gamma_0}=$ \$2,100, $\gamma_1/\gamma_0 = 0.14$, and $\gamma_2/\gamma_0 = 0.08$.

Finally, we set the discount rate $r = 0.04$, following (Glaeser et al., 2014).

5.1.2. Transaction parameters

The remaining parameters are the flow probability μ of moving, the number N of observed sales each period, the standard deviation σ_a of individual flow utility around the city-wide average, and the standard deviation σ_s of direct signals about demand. We also must determine the length δ of each period.

We identify μ using data on the probability that an owner-occupant sells a house in a given year. The Census, accessed at https://www.census.gov/prod/2000pubs/p23-200.pdf, reports that the five-year mobility rate for owners is 31.2%. Therefore, $1 - e^{-5\mu} = 31.2\%$ and $\mu = 7.5\%$. This figure corresponds to an expected tenancy of 13 years.

To compute σ_a, we use the standard deviation of rents, controlling for location and housing characteristic fixed effects. Rent data at the housing unit level come from the 2000 Census. The rent data provide a snapshot at a given time (2000). In the model, all houses are identical; in the data, they possess different characteristics. We therefore augment Eq. (1) with unit characteristics to arrive at the estimating equation

$$D_i^c = D^c + \mathbf{h}_i \boldsymbol{\beta} + a_i, \tag{8}$$

where c denotes the location and $\mathbf{h}_i$ is a vector of unit characteristics (rooms, bedrooms, plumbing, kitchen, age of the building, number of units in the building, and an indicator for whether the building sits on more than 10 acres of land). We observe D_i^c and $\mathbf{h}_i$, so we estimate Eq. (8) as a fixed effects regression and identify σ_a^2 as the variance of the residual. This procedure assumes that buyers also observe $\mathbf{h}_i$ and know $\boldsymbol{\beta}$. The location identifiers we use are public use microdata areas (PUMAs), the standard location entity used by the Census. Each PUMA contains at least 100,000 people. The value of σ_a we estimate equals \$3,120.

We set the length of each period at half a year ($\delta = 0.5$). This time represents the frequency at which buyers observe home sales and news about demand. As many house price series, such as Case-Shiller and Federal Housing Finance Agency (FHFA), are published at quarterly frequencies, and because this information can take some time to disseminate, $\delta = 0.5$ seems like a natural starting point. To compute the number of observed sales each period, we take the number of owner-occupied homes in the average PUMA, which is 30,800, and multiply it by $1 - e^{-\delta\mu}$, the probability of sale within a unit δ of time. The result is $N =$ 1,130.

The final parameter to choose is σ_s, the noise in the news about fundamentals. We set this value to $\sigma_s =$ \$1,000. As the median annual rent in the United States is \$10,884, this noise equals about 10% of the level. Although this error seems low, it is high enough to make the fundamental news irrelevant in the simulation. Buyers believe they observe D_t^a, the average flow utility of buyers in a given period. This average is also a noisy signal of fundamentals, with standard deviation $\sigma_a/\sqrt{N}$. Given our parameter choices, $\sigma_a/\sqrt{N} =$ \$93. Buyers ignore the news, as it is an order of magnitude more noisy than prices. For news to be relevant, it must have an error rate on the order of 1%. We explore this possibility in Section 5.7.

5.2. Simulation methodology

We simulate the model in the limit as $\rho \to 0$ and measure various statistics about the resulting prices. Each simulation begins with a choice of the initial state vector $(D_0, g_0)'$ and the mean of the priors. Because D is nonstationary, without loss of generality we set the initial value to \$10,000. We pick the initial value of $g_0 \sim \mathcal{N}(0, \sigma_g^2/(2\lambda))$, its stationary distribution. As we show in Lemma 2, in the $\rho \to 0$ limit, the covariance of the buyer posterior after observing any price and news history does not depend on t. We denote it $\mathbf{P}_0$. Motivated by this stationarity, we draw the mean of the prior from a multivariate normal with covariance $\mathbf{P}_0$ and mean $(D_0, g_0)'$, and we set the covariance of the prior to $\mathbf{P}_0$.[9] After seeding the initial values, we iteratively update the prices, states, and beliefs using the formulas in Section 3.3. For the same evolution of fundamentals, we separately keep track of the markets in which all buyers are naive and in which they are all rational. We produce one thousand simulations and analyze the pooled results. In addition to the naive and rational prices, we calculate the prices that would hold if city-wide demand were directly observable. In this Observable specification, prices are given by Eq. (5) with D_t and g_t replacing $\widehat{D_t}$ and $\widehat{g_t}$.

We compare the simulated prices with empirical house price data. Our data set is composed of the annual FHFA house price indices for a panel of the largest 115 metropolitan areas in the United States between 1980 and 2011. To convert the indices into levels, we multiply each city's index by the median house price in the 2000 Census, following Glaeser et al. (2014). We run each simulation for 31 years to align the time horizon in the data and the simulation.

5.3. Price autocorrelations

The autocorrelations of annual price changes are defined by $\mathrm{Corr}(\Delta p_t, \Delta p_{t+k})$, where $\Delta p_t = p_t - p_{t-1}$. These autocorrelations summarize the serial correlation of price changes over time and have been the focus in the literature on the predictability of house prices, starting with Case and Shiller (1989) and appearing more recently in Glaeser et al. (2014) and Head et al. (2014). We calculate these statistics separately in the three simulation specifications and in the FHFA data.

Fig. 1 plots the cumulative sum of the autocorrelations, which equals the average movement in house prices after they initially rise for one year, relative to the initial increase. Empirically, house prices display strong momentum at one- and two-year horizons, followed by mean reversion at longer horizons. The Rational and Observable specifications fail to capture these dynamics. Information is observed with a one-period lag by rational buyers, leading to the modest autocorrelation at a one-year horizon of 0.12. This figure is substantially below the empirical momentum of 0.67, and the remainder of the empirical autocorrelation structure fails to appear in any way in the Observable and Rational specifications.

In contrast, the Naive specification matches the general dynamics in empirical housing prices. It predicts strong momentum over a one-year horizon, with a value of 0.75. At longer horizons, it predicts mean reversion. Mean reversion begins around three years, similarly as in the data. The magnitude of the initial mean reversion is similar to the data, although it is higher in the Naive model and ends faster.

The Naive specification differs from the data by featuring large price increases starting in year 6. The data display mean reversion out to year 10. This discrepancy is partly due to the absence of new housing supply in the model, as new construction adds downward pressure to house prices in the long run. The long-term price increases in the model arise for the additional reason that extrapolation amplifies both the initial bust and the resulting recovery. In Appendix Table A1, we recalculate the autocorrelations when non-extrapolating agents participate in the market alongside naive buyers. We find that the presence of these agents significantly dampens the long-run autocorrelations without a large decrease in the short-run momentum and reversals. This result suggests that a model in which extrapolation influences prices more strongly during booms than busts could fit the data well.

In the Observable and Rational models, prices are close to a random walk. This result is unsurprising in the Observable model, as news gets incorporated into prices immediately. More noteworthy is that this result holds in the Rational model as well. This similarity between the two models suggests that the rational buyers are extremely good at quickly filtering underlying demand from past prices, leading Rational prices to behave similarly to Observable prices.

5.4. Belief dynamics

To explore the boom and bust profile depicted in Fig. 1, we study the evolution of prices after an exogenous demand shock. We decompose the resulting impulse response into three components: the idiosyncratic utility of the buyer, the buyer's belief about the level of city demand, and the buyer's belief about the growth rate. Explicitly,

$$p_t = \underbrace{\frac{1}{r+\mu} D_t^a}_{\substack{\text{Idiosyncratic}\\ \text{utility}}} + \underbrace{\frac{\mu}{r(r+\mu)} \widehat{D_t}}_{\substack{\text{Level}\\ \text{belief}}} + \underbrace{A_g \widehat{g_t}}_{\substack{\text{Growth}\\ \text{belief}}} . \tag{9}$$

To calculate the impulse response, we simulate the model with and without a one-time, one standard deviation shock to the demand increments dW^D and dW^g. We report the average difference between the impulsed and non-impulsed simulations.

[9] We give the naive buyers the same initial mean as the rational buyers. As shown in Proposition 2, the covariance of the naive forecast error exceeds the covariance of the naive stationary posterior. To account for this fact, we experimented with burning in the simulations by discarding the first five years. Doing so did not materially affect the results.

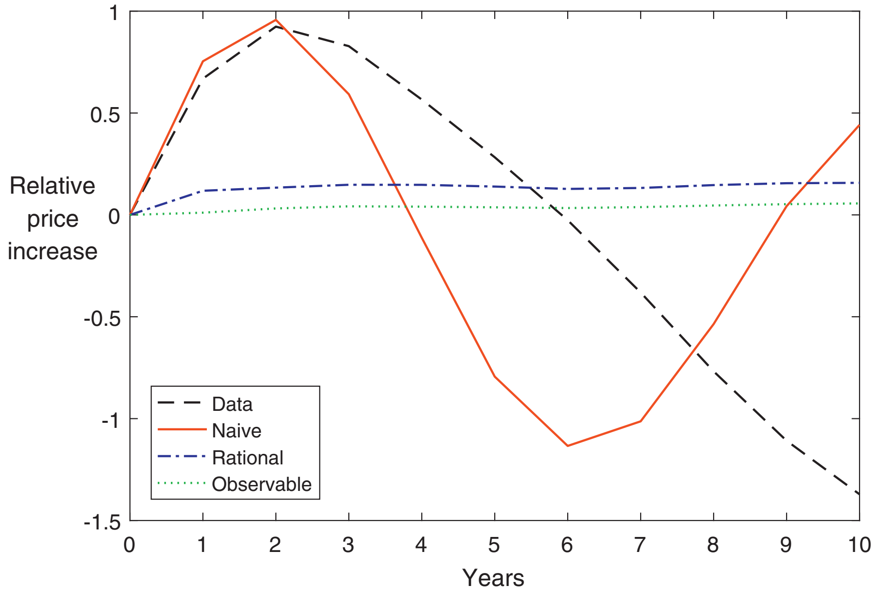

Fig. 1. Cumulative autocorrelations in annual price changes. This figure plots the cumulative autocorrelations $\sum_{j=1}^{k} \text{Corr}(\Delta p_t, \Delta p_{t+j})$, where $\Delta p_{t+j} = p_{t+j} - p_{t+j-1}$. "Observable" denotes the model in which buyers can observe the current state of demand, "Rational" denotes the model in which demand is unobservable but the buyers apply a rational filter, and "Naive" denotes the model in which buyers apply a naive filter. The correlations are estimated in both the data and the model by computing the correlation of all pairwise realizations of each pair of price changes over 31 years. Data come from the Federal Housing Finance Agency house price indices.

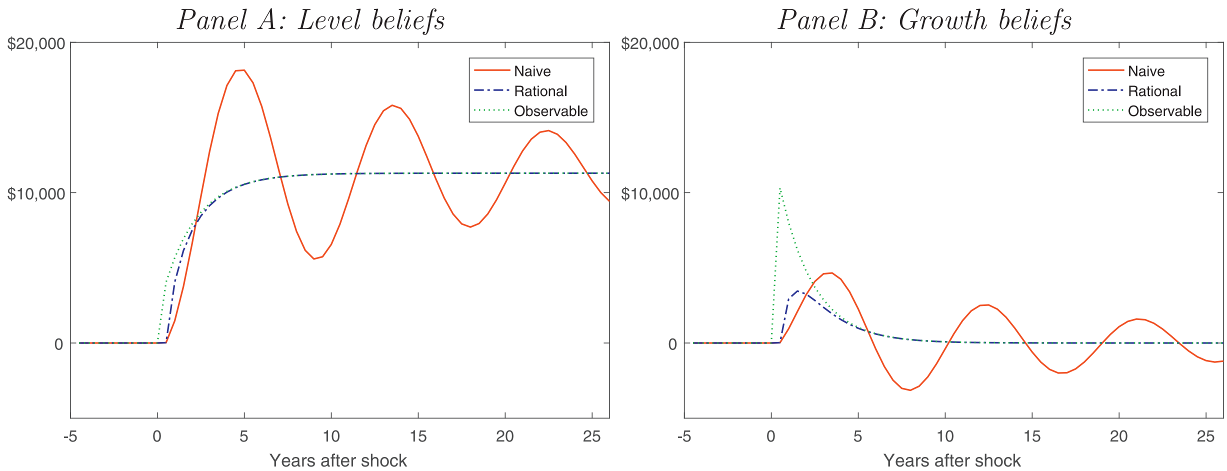

Fig. 2. Evolution of beliefs after a demand shock. We plot impulse responses from a one standard deviation shock to demand. The panels plot the average difference between the simulated model with and without the shock. Panel A is the component of prices related to beliefs about the level of demand D_t, and Panel B is the price component corresponding to beliefs about the growth rate of demand g_t. This decomposition appears in Eq. (9). "Observable" denotes the model in which buyers can observe the current state of demand, "Rational" denotes the model in which demand is unobservable but the buyers apply a rational filter, and "Naive" denotes the model in which buyers apply a naive filter.

Fig. 2 plots the impulse responses for the level belief and growth belief components in Eq. (9). The axes are the same for each subfigure so that the relative importance of the belief components can be easily compared. Relative to Observable and Rational prices, the Naive prices substantially overshoot after a demand shock. Almost all of this overshooting comes from buyers overestimating the level of demand after the shock. They overestimate the demand level because they neglect the growth belief component of prices. Naive buyers erroneously believe that $\widehat{g_{t'}}$, the growth belief of past buyers, never moves around. As they filter demand from past prices, naive buyers overestimate the level of demand when growth rates are high.

The profile of prices in Fig. 2 captures the common accounts of bubbles found in a number of sources, such as Kindleberger and Aliber (2005), Shiller (2005), Pástor and Veronesi (2009), and Glaeser (2013). In this narrative, some fundamentally good shock, such as the discovery of a new technology, leads to increases in asset prices. Then, for some reason, this boom in asset values goes beyond

Table 2
Impact of past price changes on expected appreciation.

This table reports the result from the regression $\mathbf{E}_t(p_{t+1} - p_t) = \sum_{k=0}^{9} \beta_k \Delta p_{t-k} + \xi_t$, estimated using ordinary least squares with the simulated data from each model. The Δ denotes an annual difference, so that $\Delta p_{t-k} = p_{t-k} - p_{t-k-1}$. "Observable" denotes the model in which buyers can observe the current state of demand, "Rational" denotes the model in which demand is unobservable but the buyers apply a rational filter, and "Naive" denotes the model in which buyers apply a naive filter. All β_k for $5 \leq k \leq 9$ round to 0.00 so are excluded from the table.

	Model		
Past price change	Naive	Rational	Observable
Δp_t	0.09	0.01	0.02
Δp_{t-1}	0.15	0.01	0.01
Δp_{t-2}	0.04	0.00	0.01
Δp_{t-3}	0.01	0.00	0.00
Δp_{t-4}	0.00	0.00	0.00

what is justified by fundamentals, leading to an eventual bust. What causes the overshooting is a matter of debate, for which the sources cited, as well as many other papers, offer competing explanations. Our explanation of this phenomenon is that buyers think the initial asset price boom conveys better information than it actually does because the buyers neglect that part of this boom involves revisions to other buyers' beliefs about the growth rate. This filtering error leads to overestimates of fundamentals, which cause an overshooting of prices and an eventual bust as prices return to fundamentals.

5.5. Expected price changes

As discussed in Section 3, homebuyers empirically extrapolate expected increases in the market value of their homes from past price increases. To explore this phenomenon in our model, we regress the expected annual gain in market value, as given by Proposition 4, on the ten most recent lagged annual price changes.

Table 2 displays the results. The naive buyers strongly extrapolate future increases in the market values of their houses from past price increases. The rational buyers, and those who can observe demand directly, do not. We can put these numbers in perspective using empirical survey evidence of homebuyer expectations. Case et al. (2012) regress the reported expected one-year change in home prices on one-year lagged price changes and find a coefficient of 0.23. The naive buyers in our model match this empirical behavior quantitatively. When we regress the expected market value change on only the most recent annual price change, the coefficient equals 0.20. Naive buyers think news about growth rates does not get incorporated into prices. Therefore, when they see recent price increases, they infer a high growth rate, which leads them to expect increases in flow utility and hence prices in the future. According to Table 2, they draw these inferences from price changes using several years of data.

In contrast, the rational buyers hardly extrapolate any increases in the market values of their homes. They (rightly) believe that all available information about growth rates already appears in the current market value of their home. Past price increases fail to convince them of any future appreciation in their house values. Although this behavior is perfectly rational, it is strongly at odds with the survey evidence on expectations.

Rational beliefs about future prices must be correct on average, by definition. Naive beliefs do not have this restriction. To explore the forecasting ability of naive buyers, we plot the expected change in market values and the empirical change after a one-year price increase. The realized change in market values exactly equals the price response in Fig. 1. To compute the expected change, we use Proposition 4 to extend the expected one-year change conditional on a lagged one-year change (which is 0.20) to further years.

Fig. 3 plots the results. After a one-year increase in house prices, naive buyers underestimate the subsequent increase in their home values over the short run. This result can at first seem surprising. Naive buyers extrapolate from prices much more strongly than rational buyers, and rational buyers extrapolate perfectly. One could guess, therefore, that naive buyers overextrapolate. The reason they do not is that they fail to anticipate that future buyers also revise their beliefs upward after a price increase. This result—that naive buyers under-extrapolate in the short run—is essential to the workings of this model. Momentum can exist in price increases only if buyers are continually being surprised by the extent of price increases. If naive buyers fully anticipated price increases, then these anticipations would become priced immediately, negating their realization.

Over longer horizons, naive buyers do overextrapolate. As Fig. 3 shows, these buyers completely fail to anticipate the eventual mean reversion in prices. Naive buyers believe market values follow the path of D_t, the citywide demand. This demand exhibits no mean reversion, as it is a random walk with persistent drift. Actual prices, however, do exhibit mean reversion, because naive buyers overestimate fundamentals after recent price increases.

Our model therefore microfounds the result that homebuyers fail to forecast busts. This phenomenon has recently been explored in a number of papers on natural expectations (Fuster et al., 2010a, 2010b, 2011). This line of research studies consumers who, due to cognitive limitations, forecast macroeconomic variables as truncated AR(p) processes [e.g., using an AR(1) to model an AR(2)]. This forecast restriction can prohibit consumers from forecasting mean reversion in variables such as house prices where it exists, while allowing them to forecast momentum. In contrast to consumers with natural expectations, naive buyers perfectly understand the underlying process for fundamentals. Their naive view that house prices reflect just fundamentals and not beliefs prohibits them from forecasting busts.

5.6. Volatility

Empirically, house prices exhibit excess volatility relative to the movements of underlying fundamentals (Glaeser et al., 2014; Head et al., 2014). We compare the volatility of price changes with that of movements in flow utility in the three specifications of our model. The fundamental in our model is D_t, the citywide flow utility at

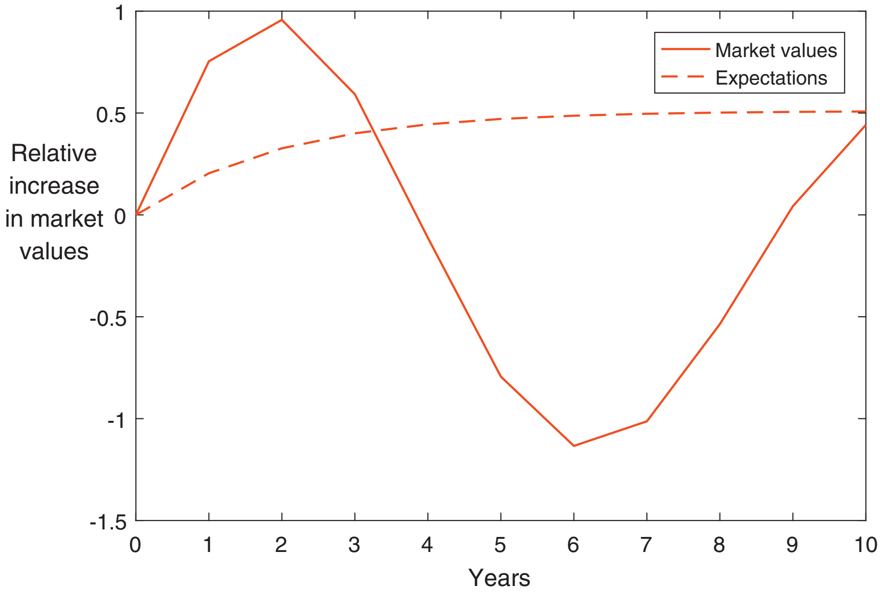

Fig. 3. Price and belief evolution for naive buyers. This figure plots the naive price changes from Fig. 1 along with naive buyers' expected change in market values after a one-year price increase. This expectation is calculated using Proposition 4 and a regression of $\widehat{g_t}$ on a one-year price increase.

Table 3
Volatility of price changes at different horizons.

We calculate volatility as the sample standard deviation of changes over the given time horizon, for all realizations over this horizon and over all simulations we run. "Observable" denotes the model in which buyers can observe the current state of demand, "Rational" denotes the model in which demand is unobservable but the buyers apply a rational filter, and "Naive" denotes the model in which buyers apply a naive filter. The fundamental D_t/r denotes the present value of the current citywide level of flow utility. Empirical volatility is computed using Federal Housing Finance Agency house price data for 115 metropolitan areas from 1980 to 2011.

	Horizon		
Sample	One year	Three years	Five years
Data	\$16,000	\$40,000	\$50,000
Naive simulated prices	\$16,000	\$42,000	\$52,000
Rational simulated prices	\$11,000	\$21,000	\$28,000
Observable simulated prices	\$12,000	\$22,000	\$28,000
Simulated fundamentals (D_t/r)	\$8,000	\$17,000	\$25,000

time t. We must scale D_t so that it is comparable to house prices p. On average, $g_t = 0$, so house prices on average are $p_t = D_t/r$. This result leads us to use D_t/r as our measure of fundamentals at t. We estimate the volatility of a price change over a horizon of k years using the sample standard deviation of k-year price changes over all simulations and k-year price intervals.

Two broad patterns emerge from the results, which appear in Table 3. First, all three simulated price paths display excess volatility relative to fundamentals at the short one-year horizon. This feature has a simple explanation. Because growth is persistent, a rise in fundamentals today conveys news about future fundamentals. Because prices are forward-looking, they move more strongly than current fundamentals and hence exhibit more volatility.

Second, naive prices are more volatile than rational and observable prices, especially at longer horizons. This relation holds because naive buyers overextrapolate fundamentals from demand shocks. This added volatility in naive prices explains the majority of excess volatility at longer horizons. Over five years, the volatility in rational price changes is only 14% higher than fundamentals and the naive volatility is 2.1 times fundamental volatility. Head et al. (2014) calculate that empirical house prices display volatility 2.2 times higher than fundamentals.

5.7. Predictions for alternate parameters

Table 4 explores how the key house price moments change under different parameter values. We focus on three moments: the autocorrelation of annual price changes over a one-year horizon, the autocorrelation over a five-year horizon, and the standard deviation of five-year price changes. These moments encompass the three stylized facts of house prices that motivate this paper: momentum, mean reversion, and excess volatility. To explore the sensitivity of these moments to the parameters, we change a single parameter at a time, holding the ones used in the main analysis constant.[10]

We first adjust λ, the persistence of demand growth. The autocovariance ratio γ_2/γ_1 determines this parameter. The empirical value of this ratio depends on how one measures demand at the city level. Using income yields a ratio of 0.3, smaller than the 0.6 we estimate from BLS rents. In Table 4, we report the ratio γ_2/γ_1 corresponding to the values of λ used. We adjust σ_D for each λ to

[10] In some cases, extreme values of the parameters lead to nonstationarity of naive price changes, causing explosive behavior of the price paths. We do not analyze any such cases in Table 4.

Table 4
Simulated moments for a range of inputs.

"Annual growth persistence" is the ratio of the second and first autocorrelations of annual changes in city-wide demand D. This input uniquely determines λ. As we change λ, we alter σ_D to maintain the volatility of annual demand changes. "Expected tenancy" equals the average time spent in a house before a forced sale. Its inverse equals the moving probability μ. "News noise" gives the standard deviation of error in direct signals about demand. "Houses in the area" determines the number N of sales observed each period. "Utility noise" is the standard deviation of idiosyncratic utility for each buyer. In each row, we simulate the model with naive and rational homebuyers using the parameters of our main analysis (shown in Table 1) with the exception of the parameter of interest studied in that row. "One-year momentum" is the correlation of annual price changes on lagged changes, "Five-year reversion" is the correlation on a five-year lag, and "Five-Year Volatility" is the standard deviation of five-year house price changes.

Input	One-year momentum		Five-year reversion		Five-year volatility	
	Naive	Rational	Naive	Rational	Naive	Rational
Annual growth persistence						
0	0.49	0.15	0.04	0.01	\$16,000	\$18,000
0.3	0.56	0.15	0.00	0.00	\$18,000	\$20,000
0.6	0.75	0.12	−0.68	−0.00	\$51,000	\$28,000
Expected tenancy (years)						
2	0.94	0.03	−0.06	0.00	\$46,000	\$28,000
10	0.80	0.10	−0.81	−0.01	\$63,000	\$28,000
30	0.61	0.16	−0.13	0.00	\$35,000	\$28,000
News noise (σ_s)						
\$10	0.13	0.11	0.00	−0.00	\$28,000	\$28,000
\$100	0.45	0.12	0.01	−0.00	\$30,000	\$28,000
\$1,000	0.75	0.11	−0.67	0.01	\$51,000	\$28,000
\$10,000	0.76	0.12	−0.70	0.00	\$53,000	\$28,000
Houses in the area						
1,000	0.21	0.11	−0.03	−0.01	\$28,000	\$28,000
10,000	0.75	0.13	−0.47	−0.00	\$38,000	\$27,000
50,000	0.75	0.13	−0.70	0.01	\$59,000	\$28,000
Utility noise (σ_a)						
\$100	0.04	0.09	0.01	−0.01	\$26,000	\$28,000
\$1,000	0.69	0.11	−0.52	0.01	\$59,000	\$28,000
\$10,000	0.62	0.17	−0.17	0.01	\$31,000	\$28,000

keep the volatility of demand changes ($\sqrt{\gamma_0}$) constant. According to Table 4, smaller values for the persistence significantly attenuate the mean reversion and excess volatility of naive prices yet leave the momentum largely unchanged. Overshooting occurs in the model when buyers incorrectly attribute price growth from revisions about $\widehat{g}$ to increases in fundamentals. When growth persistence is small, growth shocks are not of much quantitative importance, and growth rate expectations enter into prices only slightly. In terms of Eq. (5), A_g is small. Momentum persists because shocks to dW^D are incorporated slowly into prices when buyers are naive.

The next parameter we consider is tenancy length. This input determines μ, as the expected tenancy equals $1/\mu$. When tenancy length is shorter, buyers expect to resell their houses more quickly and therefore care more about market demand. This change enhances momentum in naive prices but has a non-monotonic affect on mean reversion and volatility.

More precise information about demand moves naive prices closer to rational ones. This result is unsurprising. With perfect news, demand is known and buyers no longer rely on house prices for inference, rendering the two types of buyers identical. News must be very precise to dampen the effects of naive inference. Momentum persists even when the error is on the order of \$100. The reason is that prices already aggregate information about demand well due to the central limit theorem and the large number of observed sales, so news must be very precise to make a difference.

By the same token, naive inference produces stronger mean reversion and volatility when more houses are in the area. The number of houses in the area determines how much weight naive buyers place on past housing prices. When the number of houses is very low, they do not weight the past much and the momentum attenuates. In a sense, this result suggests a non-monotonic relation between information flows and housing fluctuations. With no information, momentum and mean reversion disappear. With good information about fundamentals, the same effect occurs. It is only when good information exists about past market behavior but not about fundamentals that momentum, mean reversion, and volatility become most pronounced.

Finally, the standard deviation of idiosyncratic utility has a non-monotonic affect on the moments of interest. Low values of σ_a mean that buyers know much about the citywide demand from observing their own demand. Hence, in this case, they do not rely on market prices for inference very much, dampening the volatility, momentum, and mean reversion of naive prices. At high values of σ_a, house prices are again not of much use. House prices average away the noise in idiosyncratic utility, but, for a given N, this average is less precise when σ_a is larger. However, momentum and mean reversion exist to some degree even at $\sigma_a = \$10{,}000$, three times higher than the baseline value in Table 1 that is implied by rent data.

In the quantitative analysis so far, we set $\phi = 1$, which means that naive buyers use the same D/r model for prices in the past as they do in the future. As defined in

Table 5
Forecast accuracy.
This table reports the standard deviation of the forecast error in Eq. (10), calculated as the square root of the variance pooled over all simulations and time periods. "Market" denotes the filtering used by buyers to determine the price path, and "Individual" specifies the filtering used by an individual, the error of whose forecasts is evaluated. The parameters used in the simulations are given by Table 1.

Individual	Market	
	Rational	Naive
Rational	\$11,000	$\$(6)10^{32}$
Naive	\$12,000	\$23,000

Section 3.4, ϕ gives the probability that a future sale is to a D/r buyer as opposed to one who is as sophisticated as the current naive buyer, in which case the law of iterated expectations holds. Proposition 4 shows that extrapolation rises with ϕ, and Lemma 3 shows that the sensitivity of prices to $\widehat{g}$ falls with ϕ. Consistent with these results, Glaeser and Nathanson (2016) show in the naive simulations that a lower ϕ amplifies the price cycle but dampens the extrapolation.

5.8. Forecast accuracy

This subsection evaluates the relative accuracy of the naive and rational filters. We consider the accuracy of forecasting the willingness to pay for a home and of forecasting future prices directly.

5.8.1. Willingness to pay

Each buyer's goal is to infer D_t and g_t to minimize the error in valuing her home. One measure of this error is the difference between the price in Eq. (5) under $\widehat{D_t}$ and $\widehat{g_t}$ and that under the true values D_t and g_t:

$$\text{error} = \frac{\mu}{r+\mu}\frac{\widehat{D_t}-D_t}{r} + A_g(\widehat{g_t}-g_t). \tag{10}$$

This error represents the difference between the buyer's willingness to pay using her filter and using the true state of demand. The accuracy of the filter is the standard deviation of this error. We calculate this standard deviation as the square root of the pooled variance across all simulations and time periods of the above error.

We also use this error to evaluate the robustness of the rational and naive filters. As Proposition 3 suggests, the rational filter is sensitive. It performs poorly when applied to price paths produced by buyers using nonrational filters. We show this phenomenon quantitatively here. We calculate the estimates of $\widehat{D_t}$ and $\widehat{g_t}$ obtained using the rational filter applied to naive prices.[11] We also compute the forecast accuracy for the naive filter applied to rational prices.

Table 5 presents the results. The rational equilibrium is twice as accurate as the naive one. When all buyers are rational, the forecast error has standard deviation \$11,000. It rises to \$23,000 when all buyers are naive. The average value of the error across simulations, which is not reported in Table 5, is very small in both equilibria: −\$14 for the rational buyers and \$146 for the naive ones. We consider both errors effectively to be zero given simulation error and the fact that prices are several orders of magnitude larger at around \$250,000. The zero mean error for the naive filter confirms the key motivation for studying this rule of thumb. It is a mean zero approximation. Nonetheless, the equilibrium result when all participants use the naive filter doubles the standard deviation of this mean zero error relative to the rational equilibrium.

The other two entries in Table 5 illustrate the robustness of each filter. When the market uses the rational filter, the marginal cost of using the naive filter is very small in terms of lost accuracy. The standard deviation of the error rises only to \$12,000 from \$11,000. The largest error comes from employing the rational filter when other buyers are in fact naive. In this case, the standard deviation of the error equals practically an infinite number.

These results suggest that the naive filter is much more robust to uncertainty about previous buyers. The equilibrium in which all buyers are rational is extraordinarily fragile. If even a small probability exists that all other buyers are naive, an individual buyer is much better off being naive as well. Furthermore, the cost of naivety is small when others are rational, as the naive buyer essentially free rides off the information aggregation provided by others. A buyer is not limited to these two filters. A truly rational buyer would recognize that all other buyers are naive and would use the optimal filter given that information. Table 5 is meant to illustrate how much optimal behavior depends on knowing the behavior of others.

5.8.2. Future prices

We now study buyer forecasts of future prices. For different horizons τ, we calculate the standard deviation of $p_{t+\tau} - \mathbf{E}_t p_{t+\tau}$ using pooled data across all of our simulations. We conduct this exercise separately for prices from both the naive and the rational equilibria. We report results for both the naive and the rational forecast rules. However, in this case, we allow the rational buyers to fully understand the structure of naive prices and, hence, the rational forecast is the optimal one given the information set at time t. The Appendix provides details of how we compute the rational forecast of naive prices.

We supplement these naive and rational forecasts with those that one would obtain from linear regressions of each $p_{t+\tau}$ on the two most recent lags of prices, $p_{t-\delta}$ and $p_{t-2\delta}$. We estimate the coefficients used in these forecasts with ordinary least squares on the pooled simulated data.[12] Given the large amount of simulated data, the estimated coefficients give excellent approximations for the optimal coefficients in such a forecasting rule. This ordinary least squares (OLS) specification represents a simple forecasting rule that buyers could use to predict prices. Unlike the naive forecast proposed by our paper, the OLS

[11] The rational buyer begins with a prior on $\mathbf{x}_0$, which we seed randomly. At each step, the rational buyer at t extracts $D^a_{t'}$ out of $p_{t'}$ by assuming that $\mathbf{E}(\mathbf{x}_{t'} \mid \Omega^p_{t'} \cup \Omega^a_{t'} \cup \Omega^{\mathbf{x}}_{t'})$ is the same for buyers at t' as it is for herself. Then, using this extracted value for $D^a_{t'}$, the buyer updates her posterior using Lemma 2.

[12] In all specifications, τ/δ is an integer, allowing us to run this regression on the simulated data.

Table 6
Price forecast errors.
The table reports the errors in forecasting prices at one-, five-, and ten-year horizons in the simulated data. The error is calculated as a pooled standard deviation of the differences between the forecasts and the realized prices. The rational rule optimally forecasts prices given knowledge of the complete structure of how prices are formed. The ordinary least squares (OLS) forecast rule predicts future prices by regressing them on two lags of past prices. The errors in this case are the standard deviations of the residuals. The exercise is performed separately using the naive equilibrium prices and using the rational equilibrium prices.

Forecast rule	Naive prices			Rational prices		
	One year	Five years	Ten years	One year	Five years	Ten years
Naive	\$23,000	\$58,000	\$49,000	\$13,000	\$29,000	\$41,000
Rational	\$9,000	\$38,000	\$44,000	\$13,000	\$29,000	\$41,000
OLS	\$16,000	\$50,000	\$47,000	\$14,000	\$29,000	\$41,000

rule does not involve knowing any structural information on how prices are formed; that is, it is model-free.

Table 6 reports the results from this exercise. All of the forecasting rules perform similarly on the rational price paths. Differences appear among the forecast rules when applied to the naive price paths. The rational filter should attain the best accuracy as it is the optimal filter, and the reported error is smallest for that filter. The naive filter performs the worst. The OLS specification performs markedly better at both one- and five-year horizons. As shown in Fig. 3, the naive buyers underestimate both momentum at one-year horizons and reversals at five-year horizons. The optimal linear filter with two lags can correct these mistakes by loading more positively on $p_{t-\delta} - p_{t-2\delta}$ for the one-year forecast and negatively on this difference for the five-year forecast.[13] Thus, the two-lag OLS forecast mitigates error by increasing extrapolation at short horizons and decreasing it at longer horizons.

This two-lag OLS rule could seem to be a better rule of thumb than naive inference. It requires less knowledge, and it results in more precise forecasts. But this rule requires much more data to implement than naive inference because the regression coefficients must be estimated. In our model, naive homebuyers already know the structural parameters governing the evolution of demand and hence can form reasonably accurate forecasts with even limited data.

To make this point quantitatively, Glaeser and Nathanson (2016) re-forecast prices in one year using the OLS model but use only the past three years of price data to estimate the regression coefficients. As a control, Glaeser and Nathanson (2016) forecast one-year prices using the naive model in Section 4 that uses only the last two lags of prices instead of the infinite history. Naive inference then performs significantly better than the OLS, with the standard deviation of the latter's errors rising to \$4,000,000. A more formal analysis would investigate how much data are needed to achieve a given amount of precision for the OLS, but our point here is clear. For the OLS learning to achieve good precision, buyers must be good at running regressions on large amounts of data or they must be given the correct coefficients. Naive inference also assumes that buyers somehow know certain parameters, and thus it is not so obvious that OLS learning is a better model of belief formation than naive inference.

6. Conclusion

Many salient features of house prices—excess volatility, momentum, and mean reversion—can be explained by a model in which homebuyers make a small error in filtering information out of past prices. These naive buyers expect the market value of their home to rise after recent house price increases, and they fail to forecast busts after booms. They are overconfident in their assessments of the housing market.

The model was silent on the implications of this error for transaction volume. Home sales vary strongly with prices over the cycle (Ngai and Sheedy, 2015).[14] One way to incorporate sales volume would be for sellers to post a price at the beginning of each period and for buyers to arrive at the house and decide whether to buy at that price. Naive inference underestimates momentum, so naive sellers can post prices too low during booms and too high during busts. This pricing behavior would lead to a flurry of sales as prices rise and longer time on the market as prices fall. Thus, naive inference can explain the dearth of sales during busts, a phenomenon that has been modeled using loss aversion (Genesove and Mayer, 2001) and credit constraints (Stein, 1995).

Future research can explore the implications of naive inference for consumption. Mian and Sufi (2011) show the explosion of consumption financed by home equity during the 2000–2006 boom. Because naive buyers do not forecast the bust that follows booms, they can overconsume out of house price increases relative to rational buyers. This overconsumption could be important for understanding the extent of leverage homeowners took on during the boom.

Finally, incorporating construction into the model seems important. Supply responses have the potential to temper price increases caused by naive homebuyers.

[13] The estimated forecast rules are $\mathbf{E}_t p_{t+1} = 0.99 p_{t-\delta} + 2.16(p_{t-\delta} - p_{t-2\delta})$ and $\mathbf{E}_t p_{t+5} = 0.99 p_{t-\delta} - 1.43(p_{t-\delta} - p_{2-\delta})$.

[14] As shown in Table 4, naive inference amplifies cycles most when the number of recent transactions is high. In a model that matches the stylized facts of transaction volume, naive inference can be more powerful during booms than busts. We thank Johannes Stroebel for pointing out this observation.

However, if the home builders are also naive, housing supply could amplify rather than attenuate the effects shown in our paper.

Appendix A

A1. Discussion of $\bar{g} = 0$ restriction

This subsection discusses how to extend the cap rate error to the case in which $\bar{g} \neq 0$ and also shows that the results continue to hold to detrended prices and beliefs. We write $D = D\,|_{\bar{g}=0} + \bar{g}t$ and $g = g\,|_{\bar{g}=0} + \bar{g}$. In this extended formulation, naive homebuyers believe that prices obey $p_{i,t'} = D_{i,t'}/r + k_0$, where k_0 is some constant. This formulation is general enough to capture the case in which naive buyers know that $\bar{g}$ should appear in prices. For instance, if the growth rate were constant and equal to $\bar{g}$, then the rational pricing formula would have $\bar{g}/r^2$. But, we do not require any particular value for k_0.

To show that the results extend to detrended prices and beliefs, we posit solutions of the form $p_t = p_t\,|_{\bar{g}=0} + k_1^p + k_2^p t$ and $\widehat{\mathbf{x}}_t = \widehat{\mathbf{x}}_t\,|_{\bar{g}=0} + \mathbf{k}_1^x + \mathbf{k}_2^x t$, where k_1^p and k_2^p are constants and $\mathbf{k}_1^x$ and $\mathbf{k}_2^x$ are constant vectors. The filter in Lemma 2 applies to $\widehat{\mathbf{x}}_t - (\bar{g}t, \bar{g})'$ except with the substitution of $rp_{t-m\delta} - rk_0$ in place of $rp_{t-m\delta}$ in the naive filter. Thus

$$\begin{aligned}\widehat{\mathbf{x}}_t &= (\bar{g}t, \bar{g})' + \mathbf{K}D_t^a + (\mathbf{I} - \mathbf{KH})\mathbf{F} \times \\ &\quad \sum_{m=1}^{\infty} [(\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}]^{m-1}\mathbf{K}_0(rp_{t-m\delta} - rk_0, D_{t-m\delta}^s)' \\ &= \widehat{\mathbf{x}}_t\,|_{\bar{g}=0} + \mathbf{K}\bar{g}t + (\mathbf{I} - \mathbf{KH})\mathbf{F} \times \\ &\quad \sum_{m=1}^{\infty} [(\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}]^{m-1}\mathbf{K}_0(k_1^p + k_2^p t - rk_0, \bar{g}t). \end{aligned} \tag{11}$$

Eq. (11) delivers $\mathbf{k}_1^x$ and $\mathbf{k}_2^x$ as linear functions of k_1^p and k_2^p. We now turn to pricing. It is straightforward to show that Lemma 1 extends to the case in which $\bar{g} \neq 0$ with the simple addition of a term $k_3\bar{g}$ to the right side, where k_3 is a constant. Thus, we can write

$$\begin{aligned} p_t &= \frac{1}{r+\mu}D_t^a + \left(\frac{\mu}{r(r+\mu)}, A_g\right)\widehat{\mathbf{x}}_t + k_3\bar{g} \\ &= p_t\,|_{\bar{g}=0} + \frac{1}{r+\mu}\bar{g}t + \left(\frac{\mu}{r(r+\mu)}, A_g\right)(\mathbf{k}_1^x + \mathbf{k}_2^x t) + k_3\bar{g}. \end{aligned} \tag{12}$$

Using the method of undetermined coefficients, we can solve for k_1^p and k_2^p as a system of two equations in two unknowns, using the early equations for $\mathbf{k}_1^x$ and $\mathbf{k}_2^x$.

A2. Proof of Lemma 1

We posit a pricing function of the form $p_t = A_0 D_t + A_c\widehat{D_t} + A_g\widehat{g_t}$ and show that this type of function is an equilibrium. If the random sale happens at $T > t$, then the realized value to the buyer equals

$$\begin{aligned} &\int_t^T e^{-r(\tau-t)}D_{i,\tau}\,d\tau + e^{-r(T-t)}p_T \\ &= \int_t^T e^{-r(\tau-t)}\left(D_{i,t} + \int_t^\tau g_{\tau'}\,d\tau'\right)d\tau \\ &\quad + e^{-r(T-t)}(A_0 D_T^a + A_c\widehat{D_T} + A_g\widehat{g_T}). \end{aligned} \tag{13}$$

We first solve for the expected value $\mathbf{E}_{i,t}$ of the above quantity. It is a standard result from stochastic calculus that $\mathbf{E}_{i,t}g_\tau = e^{-\lambda(\tau-t)}\mathbf{E}_{i,t}g_t$ for all $\tau > t$, given the specification in Eq. (3). It follows that

$$\begin{aligned} &\mathbf{E}_{i,t}\int_t^T\int_t^\tau e^{-r(\tau-t)}g_{\tau'}\,d\tau'\,d\tau \\ &= \int_t^T\int_t^\tau e^{-r(\tau-t)}e^{-\lambda(\tau'-t)}\mathbf{E}_{i,t}g_t\,d\tau'\,d\tau \\ &= \int_t^T e^{-r(\tau-t)}\frac{1-e^{-\lambda(\tau-t)}}{\lambda}\,d\tau\,\mathbf{E}_{i,t}g_t \\ &= \left(\frac{1-e^{-r(T-t)}}{r\lambda} - \frac{1-e^{-(r+\lambda)(T-t)}}{\lambda(r+\lambda)}\right)\mathbf{E}_{i,t}g_t. \end{aligned} \tag{14}$$

Table A1
House price changes, annual frequencies.
The Δ denotes an annual difference, so that $\Delta p_{t+k} = p_{t+k} - p_{t+k-1}$. "$D/r$" denotes buyers whose valuation for a house equals $D_{i,t}/r$, and "adaptive" denotes buyers for whom $\mathbf{E}D_{i,t+\tau} = D_{i,t}$ and $\mathbf{E}p_{t+\tau} = p_{t-\delta}$. The "Max" specifications involve taking the maximum of these valuations and the naive valuation, and the "Mix" specifications involve the valuations using a weight of 0.85 on the naive ones. The statistics are estimated in both the data and the model by computing the correlation of all pairwise realizations of each pair of price changes in a panel of one hundred cities over 30 years. Data come from the Federal Housing Finance Agency house price indices.

		Naive model				
Moment	Data	Baseline	Max with D/r	Mix with D/r	Max with adaptive	Mix with adaptive
$\text{Corr}(\Delta p_t, \Delta p_{t+1})$	0.67	0.75	0.53	0.67	0.71	0.74
$\text{Corr}(\Delta p_t, \Delta p_{t+2})$	0.26	0.20	0.11	0.13	0.28	0.24
$\text{Corr}(\Delta p_t, \Delta p_{t+3})$	−0.10	−0.37	−0.13	−0.28	−0.03	−0.23
$\text{Corr}(\Delta p_t, \Delta p_{t+4})$	−0.26	−0.70	−0.17	−0.39	−0.17	−0.48
$\text{Corr}(\Delta p_t, \Delta p_{t+5})$	−0.28	−0.68	−0.10	−0.24	−0.17	−0.45
$\text{Corr}(\Delta p_t, \Delta p_{t+6})$	−0.31	−0.34	−0.02	0.00	−0.11	−0.23
$\text{Corr}(\Delta p_t, \Delta p_{t+7})$	−0.35	0.12	0.03	0.16	−0.02	0.05
$\text{Corr}(\Delta p_t, \Delta p_{t+8})$	−0.39	0.48	0.05	0.19	0.04	0.24
$\text{Corr}(\Delta p_t, \Delta p_{t+9})$	−0.34	0.58	0.04	0.10	0.06	0.27
$\text{Corr}(\Delta p_t, \Delta p_{t+10})$	−0.26	0.40	0.02	−0.00	0.05	0.18
$\text{Std}(p_{t+5} - p_t)$	\$51,000	\$52,000	\$30,000	\$33,000	\$30,000	\$38,000

We have $\mathbf{E}_{i,t}\int_t^T e^{-r(\tau-t)}D_{i,t}d\tau = (1-e^{-r(T-t)})D_{i,t}/r$ and $\mathbf{E}_{i,t}D_T^a = \mathbf{E}_{i,t}D_T$ by Eq. (1). This expectation equals

$$\mathbf{E}_{i,t}D_T = \mathbf{E}_{i,t}D_t + \mathbf{E}_{i,t}\int_t^T g_\tau d\tau = \mathbf{E}_{i,t}D_t + \frac{1-e^{-\lambda(T-t)}}{\lambda}\mathbf{E}_{i,t}g_t. \tag{15}$$

Next, note that $\mathbf{E}_{i,t}\widehat{D_T} = \mathbf{E}_{i,t}D_T$ by assumption and that $\mathbf{E}_{i,t}\widehat{g_T} = \phi_g(T-t)\mathbf{E}_{i,t}g_t$. The price $p_{i,t}$ is the expected value of the realized value to the buyer, given that $T-t$ is distributed with probability distribution function $\mu e^{-\mu(T-t)}$:

$$\begin{aligned} p_{i,t} = \int_t^T \mu e^{-\mu(T-t)}\Bigg[&\frac{1-e^{-r(T-t)}}{r}D_{i,t} \\ &+\left(\frac{1-e^{-r(T-t)}}{r\lambda} - \frac{1-e^{-(r+\lambda)(T-t)}}{\lambda(r+\lambda)}\right)\mathbf{E}_{i,t}g_t \\ &+ e^{-r(T-t)}(A_0+A_c)\left(\mathbf{E}_{i,t}D_t + \frac{1-e^{-\lambda(T-t)}}{\lambda}\mathbf{E}_{i,t}g_t\right) \\ &+ e^{-r(T-t)}A_g\phi_g(T-t)\mathbf{E}_{i,t}g_t\Bigg]dT. \end{aligned} \tag{16}$$

Evaluating this integral and collecting terms yields

$$\begin{aligned} p_{i,t} = \frac{r}{r+\mu}\frac{D_{i,t}}{r} + \frac{\mu}{r+\mu}(A_0+A_c)\mathbf{E}_{i,t}D_t \\ + \left(\frac{1+(A_0+A_c)\mu}{(r+\mu)(r+\lambda+\mu)} + \phi^* A_g\right)\mathbf{E}_{i,t}g_t, \end{aligned} \tag{17}$$

where $\phi^* = \int_t^\infty \mu e^{-(\mu+r)(T-t)}\phi_g(T-t)dT$. Taking the average over all buyers i at t and then using the method of undetermined coefficients yields $A_0 = (r/(r+\mu))/r$, $A_c = (\mu/(r+\mu))/r$, and $A_g = 1/(r(r+\lambda+\mu)(1-\phi^*))$. These formulas exactly match Eq. (5).

A3. Proof of Proposition 1

For any two times τ_1 and τ_2, we say a buyer at τ_1 observes τ_2 if $(\tau_1-\tau_2)/\delta \in \mathbb{N}$, the set of positive integers. Let t' be any time that buyers at t observe. Let t_0 denote the maximal member of $\mathcal{T}$ such that $t_0 \le t'$. The time t_0 exists because arbitrarily negative values of $\mathcal{T}$ exist, as $\rho > 0$. We denote $\mathcal{T}'$ to be the set of times at least t_0 that the buyer at t' observes. This set is clearly finite.

We claim that the buyer at t' forms her posterior on $\mathbf{x}_{t'}$ conditional only on the finite set $\Omega'_{i,t'} = \{D_{i,t'}\}\cup\{p_\tau \mid \tau\in\mathcal{T}'\}\cup\{D_\tau^s \mid \tau\in\mathcal{T}'\}\cup\{\mathbf{x}_{t_0}\}$. It is clear from Eqs. (2) and (3) that $\mathbf{x}$ is a Markov process (this statement is proved formally in the proof of Lemma 2). Thus, a rational inference on $\mathbf{x}_{t_1}$ does not use any information from times before t_0, as $\mathbf{x}_{t_0}$ is observed. The totality of $\Omega_{i,t'}$ that occurs no earlier than t_0 is $\Omega'_{i,t'}$.

The state $\mathbf{x}$ evolves linearly with normal noise, and the elements of $\Omega'_{i,t'}$ are all observations of some linear function of a lag of $\mathbf{x}$ with normal noise or they are linear combinations of posteriors on lags of $\mathbf{x}$ and such noisy observations (in the case of prices). Hence, standard Kalman filtering (which we make explicit in the proof of Lemma 2) leads the posterior $\mathbf{x}_{t'} \mid \Omega'_{i,t'}$ to be linear in the observations in $\Omega'_{i,t'}$. The linear weights are common knowledge to all buyers, as they depend only on the parameters governing the noise and evolution of the state. The average posterior $\widehat{\mathbf{x}}_{t'}$ across buyers at t' is thus a linear function of $D_{t'}^a$ and $\mathbf{x}_{t'} \mid \Omega'_{i,t'}\setminus\{D_{i,t'}\}$. As the price at t' follows Eq. (5) in Lemma 1, $p_{t'}$ is a linear combination of $D_{t'}^a$ and $\mathbf{x}_{t'} \mid \Omega'_{i,t'}\setminus\{D_{i,t'}\}$ whose weights are common knowledge. The buyer at t observes $\Omega'_{i,t'}\setminus\{D_{i,t'}\}$ and $p_{t'}$ and, therefore, can perfectly deduce $D_{t'}^a$.

A4. Proof of Lemma 2

We begin by proving that the state variable evolves as described in the text. Consider the evolution of D_t between times $t=0$ and $t=\delta$. We prove that we can write $D_\delta = D_0 + \beta g_0 + w^D$ and $g_\delta = e^{-\delta\lambda}g_0 + w^g$, where w^D and w^g are independent from D_0 and g_0 and have mean zero conditional on data at $t=0$. We also calculate the covariance matrix of $w=(w^D, w^g)'$. From Eqs. (2) and (3), we have $D_\delta = D_0 + \int_0^\delta g_t dt + \sigma_D\int_0^\delta dW_t^D$ and $g_\delta = e^{-\delta\lambda}g_0 + \sigma_g\int_0^\delta e^{-\lambda(\delta-t)}dW_t^g$. We find β as the coefficient from regressing $D_\delta - D_0$ on g_0. This coefficient equals

$$\begin{aligned} \beta &= \frac{\mathrm{Cov}\left(g_0, \int_0^\delta g_t dt + \sigma_D\int_0^\delta dW_t^D\right)}{\mathrm{Var}(g_0)} \\ &= \frac{\mathrm{Cov}\left(g_0, \int_0^\delta g_t dt\right)}{\mathrm{Var}(g_0)} = \int_0^\delta e^{-\lambda t}dt = \frac{1-e^{-\lambda\delta}}{\lambda}, \end{aligned} \tag{18}$$

where we have used the fact that for stochastic processes of the form specified in Eq. (3), $\mathrm{Cov}(g_0, g_t) = e^{-\lambda t}\mathrm{Var}(g_0)$ for all t. The variance of w^D equals the variance of this regression's forecast error, which is $\mathrm{Var}(\int_0^\delta g_t dt + \sigma_D\int_0^\delta dW_t^D) - \beta^2\mathrm{Var}(g_0) = \mathrm{Var}(\int_0^\delta g_t dt) + \delta\sigma_D^2 - \beta^2\mathrm{Var}(g_0)$. We solve for the first variance on the right as

$$\begin{aligned} \mathrm{Cov}\left(\int_0^\delta g_t dt, \int_0^\delta g_s ds\right) &= \int_0^\delta\int_0^\delta e^{-\lambda|t-s|}\mathrm{Var}(g_0)dtds \\ &= \frac{2\left(e^{-\delta\lambda}-1+\delta\lambda\right)}{\lambda^2}\mathrm{Var}(g_0). \end{aligned} \tag{19}$$

Another standard fact about the stochastic process specified in Eq. (3) is that $\mathrm{Var}(g_0) = \sigma_g^2/(2\lambda)$. We use this expression to conclude that

$$\mathrm{Var}(w^D) = \delta\sigma_D^2 + \frac{\sigma_g^2}{2\lambda^3}\left(-3+2\delta\lambda+4e^{-\delta\lambda}-e^{-2\delta\lambda}\right).$$

We turn now to proving the equation $g_\delta = e^{-\lambda\delta}g_0 + w^g$. Another standard fact about the process in Eq. (3) is that for all t, we can write $g_t = e^{-\lambda t}g_0 + \int_0^t \sigma_g e^{-\lambda(t-\tau)}dW_\tau^g$. Substituting $\delta = t$ yields the equation we desire. The variance of w^g equals $\sigma_g^2\int_0^\delta e^{-2\lambda(\delta-t)}dt$, so that

$$\mathrm{Var}(w^g) = \frac{\sigma_g^2}{2\lambda}(1-e^{-2\lambda\delta}). \tag{20}$$

The last task is to calculate the covariance of w^D and w^g, which equals $\mathrm{Cov}(g_\delta - e^{-\delta\lambda}g_0, \sigma_D\int_0^\delta dW_t^D + \int_0^\delta g_t dt - \beta g_0)$. The dW^D term drops out because W^D is independent from W^g. The remainder can be written as the sum of four covariances. The first is $\mathrm{Cov}(g_\delta, \int_0^\delta g_t dt) =$

$\frac{\sigma_g^2}{2\lambda}\int_0^\delta e^{-t\lambda}dt = \frac{\sigma_g^2}{2\lambda^2}(1-e^{-\delta\lambda})$, the next one is $\mathrm{Cov}(g_\delta, -\beta g_0) = -\frac{\sigma_g^2}{2\lambda^2}(e^{-\delta\lambda} - e^{-2\delta\lambda})$, the third covariance equals $\mathrm{Cov}(-e^{-\delta\lambda}g_0, \int_0^\delta g_t dt) = -e^{-\delta\lambda}\frac{\sigma_g^2}{2\lambda}\int_0^\delta e^{-t\lambda}dt = -\frac{\sigma_g^2}{2\lambda^2}(e^{-\delta\lambda} - e^{-2\delta\lambda})$, and the fourth and final covariance is $\mathrm{Cov}(-e^{-\delta\lambda}g_0, -\beta g_0) = \frac{\sigma_g^2}{2\lambda^2}(e^{-\delta\lambda} - e^{-2\delta\lambda})$. The total covariance equals the sum of these four terms:

$$\mathrm{Cov}(w^g, w^D) = \frac{\sigma_g^2}{2\lambda^2}\left(1 - e^{-\delta\lambda}\right)^2. \tag{21}$$

We conclude that $(D_\delta, g_\delta)' = \mathbf{F}(D_0^c, g_0)' + \mathbf{w}$, where $\mathbf{F}$ is the matrix given in the text, and the covariance matrix of w equals

$$\mathbf{Q} = \begin{pmatrix} \delta\sigma_D^2 + \frac{\sigma_g^2}{2\lambda^3}\left(-3 + 2\delta\lambda + 4e^{-\delta\lambda} - e^{-2\delta\lambda}\right) & \frac{\sigma_g^2}{2\lambda^2}\left(1 - e^{-\delta\lambda}\right)^2 \\ \frac{\sigma_g^2}{2\lambda^2}\left(1 - e^{-\delta\lambda}\right)^2 & \frac{\sigma_g^2}{2\lambda}\left(1 - e^{-2\delta\lambda}\right) \end{pmatrix}. \tag{22}$$

The formulas in Lemma 2 result from applying a Kalman filter to the problem specified. For an exposition of Kalman filtering, see, for instance, Hamilton (1994). We solve for the posterior conditional on observing $\{D_{i,t}\} \cup \Omega_t^a \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}$. The naive buyers substitute $rp_{t'}$ for $D_{t'}^a$. As in the proof of Proposition 1, we let t_0 denote the time of the most recent observation of $\mathbf{x}$. The buyer ignores all data that occurs before t_0. Let t_1 be the first time not before t_0 that the buyer at t observes. The buyer forms a posterior on $\mathbf{x}_{t_1}$ from observing just $\mathbf{x}_{t_0}$ as well as $D_{t_1}^a$ and $D_{t_1}^s$. This posterior is a normal distribution with mean $\widehat{\mathbf{x}}_1$ and covariance $\mathbf{P}_1$. As we show below, we do not need to solve for this posterior directly. At each subsequent period, the buyer learns $D_{t_k}^a$ and $D_{t_k}^s$ and iteratively applies the Kalman filter using the formulas $\widehat{\mathbf{x}}_k = \mathbf{K}_k(D_{t_k}^a, D_{t_k}^s)' + (\mathbf{I} - \mathbf{K}_k\mathbf{H}_0)\mathbf{F}\widehat{\mathbf{x}}_{k-1}$ and $\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_0)(\mathbf{F}\mathbf{P}_{k-1}\mathbf{F}' + \mathbf{Q})$, where $\mathbf{K}_k = (\mathbf{F}\mathbf{P}_{k-1}\mathbf{F}' + \mathbf{Q})\mathbf{H}_0'(\mathbf{H}_0(\mathbf{F}\mathbf{P}_{k-1}\mathbf{F}' + \mathbf{Q})\mathbf{H}_0' + \mathbf{R}_0)^{-1}$. Let n be such that $t_n = t - \delta$. Then, the posterior $\mathbf{x}_{t-\delta} \mid \Omega_{i,t} \setminus \{D_{i,t}\}$ is a normal with mean $\widehat{\mathbf{x}}_n$ and covariance $\mathbf{P}_n$. The final posterior $\mathbf{x}_t|\Omega_{i,t}$ updates this posterior based on the information in $\{D_{i,t}\}$ and the evolution of time between $t-\delta$ and t. The mean of this posterior equals $\mathbf{E}(\mathbf{x}_t \mid \Omega_{i,t}) = \mathbf{K}D_{i,t} + (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{F}\mathbf{E}(\mathbf{x}_{t-\delta} \mid \Omega_{i,t} \setminus \{D_{i,t}\})$, where $\mathbf{K} = (\mathbf{F}\mathbf{P}_n\mathbf{F}' + \mathbf{Q})\mathbf{H}'(\mathbf{H}(\mathbf{F}\mathbf{P}_n\mathbf{F}' + \mathbf{Q})\mathbf{H}' + \mathbf{R})^{-1}$. The covariance of the posterior equals $\mathbf{P} = (\mathbf{I} - \mathbf{K}\mathbf{H})(\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})$. Averaging the posterior mean across i gives

$$\begin{aligned} \widehat{\mathbf{x}}_t &= \mathbf{K}D_t^a + (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{F} \\ &\times \left[\sum_{m=1}^{n-1}\left(\prod_{k=1}^{m-1}(\mathbf{I} - \mathbf{K}_{n-k+1}\mathbf{H}_0)\mathbf{F}\right)\mathbf{K}_{n-m+1}(D_{t-m\delta}^a, D_{t-m\delta}^s)' \right. \\ &\left. + \widehat{\mathbf{x}}_1\prod_{k=1}^{n-1}(\mathbf{I} - \mathbf{K}_{n-k+1}\mathbf{H}_0)\mathbf{F}\right]. \end{aligned} \tag{23}$$

We now show that as $\rho \to 0$, this expression converges almost surely to the formula in the lemma. We demonstrate point-wise convergence and, hence, show convergence of the coefficients on the dividend terms. According to Proposition 13.2 of Hamilton (1994), $\lim_{n\to\infty}\mathbf{P}_n = \mathbf{P}_0$, where $\mathbf{P}_0$ is the unique solution to

$$\begin{aligned} \mathbf{P}_0 &= \left(\mathbf{I} - (\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})\mathbf{H}_0'\left(\mathbf{H}_0(\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})\mathbf{H}_0' + \mathbf{R}_0\right)^{-1}\mathbf{H}_0\right) \\ &\times (\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q}). \end{aligned} \tag{24}$$

We can apply this proposition because $\mathbf{Q}$ is strictly positive definite (as it is the covariance matrix variables that are not linear combinations of each other). As written in Hamilton (1994), the proposition requires the eigenvalues of $\mathbf{F}$ to lie inside the unit circle (which they do not, as one is an eigenvalue), but the proof shows that the strict positive definiteness of $\mathbf{Q}$ is sufficient. We define $\mathbf{K}_0 = (\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})\mathbf{H}_0'(\mathbf{H}_0(\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})\mathbf{H}_0' + \mathbf{R}_0)^{-1}$. Note that $\lim_{n\to\infty}\mathbf{K}_n = \mathbf{K}_0$.

We also claim that $\lim_{n\to\infty}\prod_{k=1}^{n}(\mathbf{I} - \mathbf{K}_{n-k+1}\mathbf{H}_0)\mathbf{F} = 0$. A direct computation shows that $\mathbf{F}$ has an eigenvalue less than one in magnitude and an eigenvalue of one with eigenvector $(1, 0)'$. Similarly, for any $\mathbf{K}_k$, $\mathbf{I} - \mathbf{K}_k\mathbf{H}_0$ has an eigenvalue less than one in magnitude (and that is independent of k) and an eigenvalue of one with eigenvector $(0, 1)'$. As the eigenvectors with eigenvalue one are not collinear, and the other eigenvalues are less than one in magnitude, the limit is zero as claimed.

It follows that given an error tolerance ϵ for the coefficients, we can choose n large enough so that the coefficients in the equation for $\widehat{\mathbf{x}}_t$ are within ϵ of those in the formula in the lemma. We first choose n_1 large enough so that $|((\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F})^m - \prod_{k=1}^{m}(\mathbf{I} - \mathbf{K}_k\mathbf{H}_0)\mathbf{F}| < \epsilon$ for $m \geq n_1$ and any valid $\mathbf{K}_k$. As these products both converge to zero, n_1 exists. Then, we choose n_0 such that for $n \geq n_0$, $|((\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F})^m - \prod_{k=1}^{m}(\mathbf{I} - \mathbf{K}_{n-k+1}\mathbf{H}_0)\mathbf{F}| < \epsilon$ for $m \leq n - n_1$. n_0 exists because $\lim_{n\to\infty}\mathbf{K}_n = \mathbf{K}_0$. Thus convergence occurs as long as $n \geq n_0$, which happens with probability at most $1 - e^{-\rho(1+n_0)\delta}$. This probability converges to zero as $\rho \to 0$.

A5. Proof of Proposition 3

In the limit as $N \to \infty$, the noise in $D_{t'}^a$, which is σ_a^2/N, goes to zero. Naive buyers believe that $rp_{t-\delta} = D_{t-\delta}^a = D_{t-\delta}$, so they neglect all information in $\Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}}$ before $t-\delta$, which is all information other than $p_{t-\delta}$ and $D_{t-\delta}^s$. They ignore $D_{t-\delta}^s$, as it provides a noisy signal of $D_{t-\delta}$. This argument proves the naive formula.

We now prove the rational formula. From Lemma 1, $rp_{t'} = D_{t'}r/(r+\mu) + \widehat{D_{t'}}\mu/(r+\mu)$. At $t' = t - n\delta$, the expectation is formed using only $\mathbf{x}_{t_0}$ and $D_{i,t-n\delta}$, as older data are obviated by $\mathbf{x}_{t_0}$. The noise in using D_{t_0} as a measure of $D_{t-n\delta}$ is $(t - n\delta - t_0)\sigma_D^2$, and the noise in using $D_{i,t-n\delta}$ is σ_a^2. Therefore,

$$\begin{aligned} \widehat{D}_{t-n\delta} &= \frac{(t - n\delta - t_0)\sigma_D^2}{\sigma_a^2 + (t - n\delta - t_0)\sigma_D^2}D_{t-n\delta} \\ &+ \frac{\sigma_a^2}{\sigma_a^2 + (t - n\delta - t_0)\sigma_D^2}D_{t_0}. \end{aligned} \tag{25}$$

Prices therefore are given by $rp_{t-n\delta} = (1-\alpha_0)D_{t-n\delta} + \alpha_0 D_{t_0}$, where α_0 is as defined in the text.

At all other times t', from Proposition 1, we know that the rational buyer can infer all observed demand. Therefore, $\mathbf{E}(D_{t'-\delta} \mid \Omega_{t'}^p \cup \Omega_{t'}^a \cup \Omega_{t'}^{\mathbf{x}}) = D_{t'-\delta}$ and there is no noise in this estimate. It follows that the posterior on $D_{t'}$ combines this estimate and $D_{i,t'}$, with weights equal to the relative variance. The variance of the lagged demand estimate is $\delta\sigma_D^2$ and the variance of the idiosyncratic estimate is σ_a^2. Therefore, $rp_{t'} = (1-\alpha)D_{t'} + \alpha\mathbf{E}(D_{t'-\delta} \mid \Omega_{t'}^p \cup \Omega_{t'}^s \cup \Omega_{t'}^{\mathbf{x}})$, where α is as defined in the proposition. Hence,

$$\mathbf{E}(D_{t'-\delta} \mid \Omega_{t'}^p \cup \Omega_{t'}^s \cup \Omega_{t'}^{\mathbf{x}}) = \frac{rp_{t'-\delta} - \alpha\mathbf{E}(D_{t'-2\delta} \mid \Omega_{t'-\delta}^p \cup \Omega_{t'-\delta}^s \cup \Omega_{t'-\delta}^{\mathbf{x}})}{1-\alpha} \tag{26}$$

for $t' > t-(n-1)\delta$. At $t' = t-(n-1)\delta$,

$$\mathbf{E}(D_{t-n\delta} \mid \Omega_{t-(n-1)\delta}^p \cup \Omega_{t-(n-1)\delta}^s \cup \Omega_{t-(n-1)\delta}^{\mathbf{x}}) = \frac{rp_{t-n\delta} - \alpha_0 D_{t_0}}{1-\alpha_0}. \tag{27}$$

Iterating Eq. (26) until Eq. (27) is employed yields the formula in the proposition.

A6. Proof of Lemma 3

Using the terminology from the proof of Lemma 1, $\phi^* = \int_0^\infty \mu e^{-(r+\mu)\tau}(1-\phi)e^{-\lambda\tau}d\tau = (1-\phi)\mu/(r+\mu+\lambda)$. We then use the formula from that same proof that $A_g = [r(r+\lambda+\mu)(1-\phi^*)]^{-1}$ to arrive at the result.

A7. Proof of Proposition 4

Using Eq. (7) and the forecasting rules in the text, we write $\mathbf{E}_t(p_T - p_t)$ as

$$\frac{\mathbf{E}_t(D_T - D_t)}{r} + \frac{(1-\phi)(\mathbf{E}_t g_T - \widehat{g_t})}{r(r+\lambda+\phi\mu)} = \frac{1}{r}\int_t^T \mathbf{E}_t g_\tau d\tau - \frac{(1-\phi)(1-e^{-\lambda(T-t)})\widehat{g_t}}{r(r+\lambda+\phi\mu)}, \tag{28}$$

which reduces to the formula in Proposition 4. This expression increases in ϕ. The derivative of the first fraction is positive when $(r+\lambda+\phi\mu)(\lambda+\mu) > (r+\phi\lambda+\phi\mu)\mu$, which is true because $r+\lambda+\phi\mu > r+\phi\lambda+\phi\mu$ and $\lambda+\mu > \mu$, as $\lambda > 0$.

A8. Proof of Proposition 5

In the limit as $N\to\infty$, the noise in $D_{t'}^a$, which is σ_a^2/N, goes to zero. As a result, $\mathbf{R}_0 \to \begin{pmatrix} 0 & 0 \\ 0 & \sigma_s^2 \end{pmatrix}$. Using the formulas in the proof of Lemma 2, we directly compute $\mathbf{K}_0 = \begin{pmatrix} 1 & 0 \\ k & 0 \end{pmatrix}$, where $k = ((0,1)(\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})(1,0)')/((1,0)(\mathbf{F}\mathbf{P}_0\mathbf{F}' + \mathbf{Q})(1,0)')$. Another direct computation produces

$$[(\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}]^{m-1}\mathbf{K}_0 = \begin{cases} \begin{pmatrix} 1 & 0 \\ k & 0 \end{pmatrix} & \text{if } m = 1 \\ \begin{pmatrix} 0 & 0 \\ -(1-\kappa)\kappa^{m-2}k & 0 \end{pmatrix} & \text{if } m > 1, \end{cases} \tag{29}$$

where $\kappa = e^{-\delta\lambda} - k(1-e^{-\delta\lambda})/\lambda$. The naive formula for $\mathbf{E}(g_{t-\delta} \mid \Omega_t^p \cup \Omega_t^s \cup \Omega_t^{\mathbf{x}})$ follows immediately.

A9. Proof of Proposition 6

We know from Proposition 3 that the posterior on $D_{t-\delta}$ equals $rp_{t-\delta}$. For $g_{t-\delta}$, $D_{i,t}$ is too noisy to provide information. From the equation $\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-\delta} + \mathbf{w}_t$, we have $\Delta D_{t-\delta} = g_{t-2\delta}(1-e^{-\delta\lambda})/\lambda + w_{t-2\delta}^D$. Therefore, as the naive buyer sets $rp_{t'} = D_{t'}$, her posterior on the growth rate is $\mathbf{E}(g_{t-\delta} \mid \Omega'_{i,t}) = r\Delta p_{t-\delta}\lambda e^{-\delta\lambda}/(1-e^{-\delta\lambda})$. To arrive at the contemporaneous estimates, we again use the law of motion for $\mathbf{x}$ and get $\mathbf{E}(D_t \mid \Omega'_{i,t}) = rp_{t-\delta} + e^{-\delta\lambda}r\Delta p_{t-\delta}$ and $\mathbf{E}(g_t \mid \Omega'_{i,t}) = r\Delta p_{t-\delta}\lambda e^{-2\delta\lambda}/(1-e^{-\delta\lambda})$. Substituting these expressions into Eq. (5) and differencing yields the equation in Proposition 6. The growth rate is an AR(1) as $g_t = e^{-\delta\lambda}g_{t-\delta} + w_t^g$, and $\mathrm{Cov}(w_t^g, w_t^D) > 0$ by the formula for $\mathbf{Q}$, so the innovation satisfies the conditions in the proposition. Finally, to show that the condition $(\beta_1/2)^2 < \beta_2 < 1$ holds for some parameters, use $\delta = 1$, $\lambda = 1$, $r = 0.04$, $\mu = 0.075$, and $\phi = 1$, which yields $\beta_1 = 1.08$ and $\beta_2 = 0.43$.

A10. Identification of demand parameters

We observe the following three covariances from the data: $\gamma_0 = \mathrm{Var}(\Delta D_t)$, $\gamma_1 = \mathrm{Cov}(\Delta D_t, \Delta D_{t-1})$, and $\gamma_2 = \mathrm{Cov}(\Delta D_t, \Delta D_{t-2})$. These identify σ_D, σ_g, and λ as follows. Note that $D_{t+1} = D_t + g_t(1-e^{-\lambda})/\lambda + w_t^D$, where w^D is the error defined in the proof of Lemma 2. This equation comes from applying the law of motion for $\mathbf{x}_t$, which is $\mathbf{x}_{t+1} = \mathbf{F}\mathbf{x}_t + \mathbf{w}_t$, where $\mathbf{w}_t$ and $\mathbf{x}_t$ are independent and the covariance matrix of $\mathbf{w}$ is $\mathbf{Q}$. As a result, $\Delta D_t = g_t(1-e^{-\lambda})/\lambda + w_t^D$. It follows that the variance of the yearly price change equals

$$\gamma_0 = \left(\frac{1-e^{-\lambda}}{\lambda}\right)^2 \frac{\sigma_g^2}{2\lambda} + \mathrm{Var}(w_t^D) = \sigma_D^2 + \frac{\sigma_g^2}{\lambda^3}(e^{-\lambda} - 1 + \lambda). \tag{30}$$

The first covariance of price changes equals

$$\gamma_1 = \mathrm{Cov}\left(\frac{1-e^{-\lambda}}{\lambda}g_t + w_t^D, \frac{1-e^{-\lambda}}{\lambda}(e^{-\lambda}g_t + w_t^g) + w_{t+1}^D\right) = \frac{(1-e^{-\lambda})^2\sigma_g^2}{2\lambda^3}. \tag{31}$$

The second covariance equals

$$\gamma_2 = \mathrm{Cov}\left(\frac{1-e^{-\lambda}}{\lambda}g_t + w_t^D, \frac{1-e^{-\lambda}}{\lambda}(e^{-\lambda}(e^{-\lambda}g_t + w_t^g) + w_{t+1}^g) + w_{t+2}^D\right) = \frac{e^{-\lambda}(1-e^{-\lambda})^2\sigma_g^2}{2\lambda^3}. \tag{32}$$

These three equations identify the parameters. Note that $\gamma_2/\gamma_1 = e^{-\lambda}$, so this ratio determines λ. Conditional on λ, γ_1/γ_0 uniquely determines the ratio σ_g/σ_D. Finally, γ_0 pins down the level of these volatilities.

A11. Attenuating the second cycle

We explore a number of changes to the naive equilibrium. We introduce "D/r" buyers who value houses as $D_{i,t}/r$.

These buyers can be thought of as using only their current flow utility as the information set or as buyers using a simple heuristic that abstracts from time-varying growth rates. Our first specification allows prices to be the maximum of the D/r valuation and the naive valuation. The second specification sets prices equal to the weighted average of the two valuations. We use a weight of 0.85 on the naive buyers. The second type of buyer we introduce are buyers with adaptive expectations who assume that $\mathbf{E}p_{t+\tau} = p_{t-\delta}$ for all $\tau \geq 0$. These buyers also ignore time-varying growth rates, so they set $\mathbf{E}D_{i,t+\tau} = D_{i,t}$ for all $\tau \geq 0$. As with the D/r buyers, we explore equilibria in which prices are the maximum with respect to the naive valuation and one in which prices are a weighted average (we use the same 0.85 weight on the naives).

Table A1 displays the resulting autocorrelations of annual prices changes, as well as the standard deviation of five-year price changes. The one-year momentum (the first row) remains high in the four new specifications. At the same time, the magnitudes of the autocorrelations at longer horizons fall. The specifications involving the max operator reduce the long-run autocorrelations more strongly. The volatility of five-year price changes falls in all four specifications.

A12. Rational forecast of naive prices

From Lemma 1, we can write the naive price as $p_t = D^a_t/(r+\mu) + \mathbf{\Lambda}\widehat{\mathbf{x}}_t$, where $\mathbf{\Lambda}$ is a 1-by-2 vector. Denote $\widehat{\mathbf{x}}^f_{t-\delta} = \mathbf{E}(\mathbf{x}_{t-\delta} \mid \Omega_{i,t} \setminus \{D_{i,t}\})$. We show in the proof of Lemma 2 that $\widehat{\mathbf{x}}_t = \mathbf{K}D^a_t + \mathbf{M}\widehat{\mathbf{x}}^f_{t-\delta}$, where $\mathbf{M} = (\mathbf{I} - \mathbf{KH})\mathbf{F}$. We also show (indirectly; a direct proof is straightforward) that $\widehat{\mathbf{x}}^f_t = \mathbf{M}_0\widehat{\mathbf{x}}^f_{t-\delta} + \mathbf{K}_0(rp_{t-\delta}, D^s_{t-\delta})'$, where $\mathbf{M}_0 = (\mathbf{I} - \mathbf{K}_0\mathbf{H}_0)\mathbf{F}$. Combining all these equations produces the recursion $\widehat{\mathbf{x}}^f_t = \mathbf{M}_d\widehat{\mathbf{x}}^f_{t-\delta} + \kappa_a D^a_t + \kappa_s D^s_t$, where $\mathbf{M}_d = \mathbf{M}_0 + r\mathbf{K}_0(1,0)'\mathbf{\Lambda}\mathbf{M}$, $\kappa_a = r\mathbf{K}_0(1,0)'(1/(r+\mu) + \mathbf{\Lambda}\mathbf{K})$, and $\kappa_s = \mathbf{K}_0(0,1)'$. Thus, any $\widehat{\mathbf{x}}^f_{t+\tau}$ can be written recursively in terms of $\widehat{\mathbf{x}}^f_{t-\delta}$ and values of D^a and D^s at t and thereafter. A rational buyer at t who understands the full structure of naive prices can back out the history of D before t as in Proposition 1 and can therefore figure $\widehat{\mathbf{x}}^f_{t-\delta}$ exactly. The rational buyer uses the history of D to forecast future values of D^a and D^s, which is the same as forecasting D due to idiosyncratic noise. To form these forecasts, we use the rational forecasts used earlier as we perform this exercise on the same simulated sequence of D. Then, we can plug the forecast of $\widehat{\mathbf{x}}^f_{t+\tau}$ into the pricing equation to forecast $p_{t+\tau}$.

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