ASSIGNMENT 1 - DSAA
KRITIKA PRAKASH
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P1 $x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases}$

$h[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$

$y[n] = x[n] * h[n]$

$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

Since $x[k]$ has a non-zero value only from $k=0$ through $k=9$,

$y[n] = \sum_{k=-\infty}^{-1} 0 \cdot h[n-k] + \sum_{k=0}^{9} 1 \cdot h[n-k] + \sum_{k=10}^{\infty} 0 \cdot h[n-k]$

$= \sum_{k=0}^{9} h[n-k].$

Now, given, $y[4] = 5$ and $y[14] = 0$

$y[14] = \sum_{k=0}^{9} h[14-k] = h[14] + h[13] + .. h[5] = 0$

$\Rightarrow h[5] = h[6] = \ldots = h[14] = 0$

$y[4] = \sum_{k=0}^{9} h[4-k] = h[4] + h[3] + h[2] + .. \quad h[-4] = 5$

Since impulse response of negative indices is 0,

$h[4] + h[3] + h[2] + h[1] + h[0] = 5$

$\Rightarrow h[4] = h[3] = h[2] = h[1] = h[0] = 1.$

$\Rightarrow N = 5$ as $h[n] = 1, 0 \leq n \leq 5.$ and 0 otherwise.

$\therefore \boxed{N=5}.$

P3 $x[n] = [\ldots 0 0 1 \underset{\uparrow}{1} 1 0 0 \ldots]$

$y[n] = [\ldots 0 1 2 \underset{\uparrow}{3} 2 1 0 \ldots]$

1. Convolution

$z[n] = x[n] * y[n]$

$\Rightarrow z[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot y[n-k].$

Since $x[k]$ has a non-zero value only from $k=-1$ through $k=1$,

$z[n] = \sum_{k=-\infty}^{-2} 0 \cdot y[n-k] + \sum_{k=-1}^{1} 1 \cdot y[n-k] + \sum_{k=2}^{\infty} 0 \cdot y[n-k]$

$$\Rightarrow z[n] = \sum_{k=-1}^{1} y[n-k]$$

$$= y[n+1] + y[n] + y[n-1].$$

So,

$z[-3] = y[-2] + y[-3] + y[-4] = 1+0+0 = 1$

$z[-2] = y[-1] + y[-2] + y[-3] = 2+1+0 = 3$

$z[-1] = y[0] + y[-1] + y[-2] = 3+2+1 = 6$

$z[0] = y[1] + y[0] + y[-1] = 2+3+2 = 7$

$z[1] = y[2] + y[1] + y[0] = 1+2+3 = 6$

$z[2] = y[3] + y[2] + y[1] = 0+1+2 = 3$

$z[3] = y[4] + y[3] + y[2] = 0+0+1 = 1.$

Otherwise, for $n < -3$ or $n > 3$,

$z[n] = 0.$

$$\therefore z[n] = \{..0\ 1\ 3\ 6\ \underset{\uparrow}{7}\ 6\ 3\ 1\ 0..\}$$

(P4) $y[n] = x[n] - y^2[n-1] + y[n-1]$

$\Rightarrow \quad x[n] = y[n] + y^2[n-1] - y[n-1]$

$$y[n] \xrightarrow{T^{-1}} x[n] \quad \equiv \quad x[n] \xrightarrow{T} y[n]$$

$x[n] = T^{-1}(y[n])$.

If T^{-1} is linear then so is T.

$\Rightarrow \quad T^{-1}(\alpha \cdot y[n])$ should be equal to $(\alpha \cdot x[n])$.

$\alpha\, x[n] = \alpha \cdot y[n] + \alpha\, y^2[n-1] - \alpha \cdot y[n-1]$.

but $T^{-1}(\alpha \cdot y[n]) = \alpha \cdot y[n] + \alpha^2 y^2[n-1]$

$- \alpha\, y[n-1]$

Since $x[n] \neq T^{-1}(\alpha\, y[n])$,

T^{-1} is not-linear.

$\Rightarrow$ T is not-linear!

It is also not linear because $y[n] \,!= 0$ when $x[n] = 0$.

If T^{-1} is time-invariant then so is T.

Delay in input should be equivalent to delay in output for time invariance.

If $y[n] \xrightarrow{T^{-1}} x[n]$ then $y[n-n_0] \xrightarrow{T^{-1}} x[n-n_0]$

$f(y(t-T)) = x(t-T)$.

$T^{-1}(y[n+n_0]) = y[n+n_0] + y^2[n+n_0-1] - y[n+n_0-1]$

$x[n+n_0] = y[n+n_0] + y^2[n+n_0-1] - y[n+n_0-1]$

$\therefore$ T^{-1} is time-invariant.

$\Rightarrow$ T is time-invariant.

Now, as $n \longrightarrow \infty$

$n \equiv n-1.$

$\Rightarrow \quad y[n] = x[n] - y^2[n-1] + y[n-1].$

$\Rightarrow \quad \cancel{y[n]} = x[n] - y^2[n] + \cancel{y[n]}$

$\Rightarrow \quad y^2[n] = x[n].$

$\Rightarrow \quad y^2[n] = \alpha \cdot \mu[n]$

$\Rightarrow \quad y[n] = \sqrt{\alpha}\, \mu[n]$

$\Rightarrow \quad \boxed{y[n] \longrightarrow \sqrt{\alpha}}$. as $n \longrightarrow \infty$ as $\mu[n \to \infty] = 1$

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(P5) Input size = $W \times H$

\# filters = N

Filter size = $F \times F$

Step size = S

Zero padding = Z.

Let $P = \min(Z, (F-1)/2)$.

~~Then permissible limits for~~

$\boxed{\text{If } (Z >= (F-1)/2)}$

Permissible limits for centre of filter are such that the no. of centres the filter can take

$$= \left(\left\lfloor \frac{W}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{H}{S} \right\rfloor + 1\right)$$

∴ Dimension of output of convolution

$$= \left(\left\lfloor \frac{W}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{H}{S} \right\rfloor + 1\right)$$

→ per filter, $\boxed{\text{per channel}}$

⇒ No. of convolutions = $N \times \text{channels} \times \left(\left\lfloor \frac{W}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{H}{S} \right\rfloor + 1\right)$.

$\boxed{\text{Else If } (Z < (F-1)/2)}$

Permissible limits for centre of fiter are such that no. of centres the filter can take

$$= \left(\left\lfloor \frac{W+2Z-F}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{H+2Z-F}{S} \right\rfloor + 1\right)$$

∴ = Dimension of output of convolution (per filter, per channel).

⇒ No. of convolutions = $N \times \text{channels} \times \left(\left\lfloor \frac{W+2Z-F}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{H+2Z-F}{S} \right\rfloor + 1\right)$

(6) # Additions and multiplications

$\boxed{\text{If } Z >= (F-1)/2}$

$$\#\text{ additions} = (F^2-1)\cdot\left(\left\lfloor \frac{W}{S} \right\rfloor + 1\right)\cdot\left(\left\lfloor \frac{H}{S} \right\rfloor + 1\right).$$

per channel per filter

$$\#\text{ multiplications} = F^2\cdot\left(\left\lfloor \frac{W}{S} \right\rfloor + 1\right)\cdot\left(\left\lfloor \frac{H}{S} \right\rfloor + 1\right).$$

per channel per filter

$\boxed{\text{If } Z < (F-1)/2}$

$$\#\text{ additions} = (F^2-1)\left(\left\lfloor \frac{W+2Z-F}{S} \right\rfloor + 1\right)\cdot\left(\left\lfloor \frac{H+2Z-F}{S} \right\rfloor + 1\right).$$

per channel per filter

$$\#\text{ multiplications} = F^2\cdot\left(\left\lfloor \frac{W+2Z-F}{S} \right\rfloor + 1\right)\left(\left\lfloor \frac{H+2Z-F}{S} \right\rfloor + 1\right)$$

per channel per filter

Assumptions: F is odd.
Filters are applied iteratively to the original image.

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