Smoother Adaptive Parametric Spectrograms: An application to EEG under general anesthesia

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***Abstract*— High quality spectra are crucial in anesthesia related procedures where Electroencephalogram (EEG) frequency content can drastically help track different brain states. To this end, an adaptive autoregressive model framework to fit non-stationary EEG data using hybrid Kalman Filtering (HKF) is developed. In this setup, a state-space formulation is adopted. Hybridity arises from the fact that the state vector, which includes the autoregressive parameters, evolves in continuous time, while the observation equation is discrete, to account for the fact that the observations, i.e. EEG data, are discrete points in time. As a smoothing constraint, the parameters are modeled to follow a continuous multivariate random walk. As shown in this work, their adaptive estimation by means of HKF and expectation-maximization (EM) algorithm yields smoother estimation of frequency spectra, outperforming other current purely discrete parametric methods as well as various non-parametric approaches. Examples of dynamic EEG data were taken from patients under gradually varying doses of propofol. The suitability of our method for online use in combination with its ability to smoothly track frequency changes in human EEG signals suggests that it can be used for real time brain state tracking under general anesthesia, facilitating the design of closed loop systems for automatic and precise control of brain states.**

I. INTRODUCTION

Non-stationary time-series frequency analysis has been traditionally a thematic that attracted the attention of the scientific community. This is due to its enormous practical benefits that arise from the fact that most signals with a temporal representation encountered in real-life applications are characterized by time-varying statistics. An example of a real-life application is the use of EEG to track brain states under anesthesia. EEG has been extensively used to characterize brain states under various anesthetics [1], [2] and to automatically adjust drug infusion rates using closed loop control systems [3]. The issue of non-stationarity was usually overcome by various assumptions about local stationarity properties over short time intervals and application of techniques suitable only for stationary signals. Even though such assumptions can have practical benefits and often provide satisfying performances, they are not always sufficient. Current techniques used to estimate frequency spectra of non-stationary signals include various non-parametric methods, such as the multitaper approach developed in [4]. Non-parametric approaches suffer from a specific limitation: They cannot yield simultaneously both high temporal and frequency resolutions. Improving time resolution by using a shorter, assumed stationary, data segment results in lower frequency resolution and vice versa. An alternative approach are the so-called parametric methods. Such methods are usually based on time-varying linear predictive models, e.g. autoregressive (AR) models. The use of such models has led to higher resolution of time-varying frequency spectra. The first to use a state-space framework to fit adaptive autoregressive models for EEG analysis was Schlogl et al. [5]. A recursive least squares method was used to fit the model. Fitting of adaptive autoregressive models using the Kalman filter to estimate time-varying spectra of EEG under propofol is done in [6]. In [5] and [6] the observation and process covariance matrices are manually set before the filtering procedure. Khan et al. [7] uses an expectation-maximization approach to estimate the components of the state-space model in subsequent data windows and then runs a Kalman smoother to finely estimate the autoregressive coefficients. In all the aforementioned approaches, observations and states are modeled as discrete variables. Inspired by the work of [8] and [9], where continuous, non-adaptive AR models were fit to discrete samples of EEG recordings, this work adopts an intermediate between discrete and continuous approaches and coupled with an EM routine, develops an algorithm for efficient and smooth spectral estimation of non-stationary signals. Finally, it explores its performance on an EEG dataset under propofol and compares it to that obtained by other widely used non-parametric and discrete parametric methods.

II. METHODS

A. State-Space Formulation

In our model, the autoregressive parameters are modeled to follow a continuous multivariate random walk. They are updated in discrete time steps when a new observation is available, as in the common Kalman Filtering approaches.

State Equation: $\frac{d\mathbf{a}(t)}{dt}=\mathbf{w}(t)$ (1)

Observation Equation: $z_k = \mathbf{H}_k\mathbf{a}_k+v_k$ (2)

$\mathbf{w}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(t))$: Continuous multivariate Gaussian process with $\mathbf{0}$ mean and covariance $\mathbf{Q}(t)$ (symmetric, positive-definite)

$v_k \sim \mathcal{N}(0, R(t))$: Discrete scalar white Gaussian observation noise with 0 mean and variance $R(t)$

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$\mathbf{a}(t) = [a_1(t), a_2(t), \ldots, a_p(t)]^T$: State vector with the continuous time-varying autoregressive coefficients and covariance matrix $\mathbf{P}(t)$.

$\mathbf{H}_k = [z_{k-1}, \ldots, z_{k-p}]$: Observation matrix at time t_k comprising the p past discrete observations

$\mathbf{a}_k = \mathbf{a}(t_k)$: Sampled state vector at time t_k

Let p denote autoregressive model order, z_k the observation at time t_k, $Z = \{z_k\}_{k=1}^N$ the set of the observations, $A = \{\mathbf{a}_k\}_{k=1}^N$ the state vector at all times t_k of the observations z_k and N denote the total number of observations.

B. Hybrid Kalman Filtering

With the above model, the HKF equations become:

Initialization: $\mathbf{a}_{0|0} = aryule(Z, p)$ (3), $\mathbf{P}_{0|0} = \mathbf{I}_p$ (4)

Prediction: $\int\limits_{t_{k-1}}^{t_k} \frac{d\mathbf{a}(t)}{dt} d\tau = \mathbf{a}_{k|k-1}$ (5)

$\frac{d\mathbf{P}(t)}{dt} = \mathbf{Q}\Big|_{\mathbf{P}_{k-1|k-1}}$ (6) $\implies \int\limits_{t_{k-1}}^{t_k} \frac{d\mathbf{P}(t)}{dt} d\tau = \mathbf{P}_{k|k-1}$

In this design of the hybrid Kalman filter, the prediction equations do not involve update from measurements. The update from the measurements is used in the discrete part of the filter. Therefore, the Ricatti variance equation that is usually required to be solved in the purely continuous version of the Kalman filter is avoided here. Indeed, in equation (6) the unknown P does not appear in any quadratic expression. Also, since the derivative of the state follows a random walk, there is not a linear term either, as it would be in the more general case where the state transition matrix would not be zero.

Filtering: $\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + R)^{-1}$ (7)

$\mathbf{a}_{k|k} = \mathbf{a}_{k|k-1} + \mathbf{K}_k(z_k - \mathbf{H}_k\mathbf{a}_{k|k-1})$ (8)

$\mathbf{P}_{k|k} = (\mathbf{I}_p - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k|k-1}$ (9)

$\mathbf{S}_k = \mathbf{P}_{k|k}(\mathbf{P}_{k|k} + \mathbf{Q})^{-1}$ (10)

Smoothing: $\mathbf{a}_{k|N} = \mathbf{a}_{k|k} + \mathbf{S}_k(\mathbf{a}_{k+1|N} - \mathbf{a}_{k+1|k})$ (11)

$\mathbf{P}_{k|N} = \mathbf{P}_{k|k} + \mathbf{S}_k(\mathbf{P}_{k+1|N} - \mathbf{P}_{k+1|k})\mathbf{S}_k^T$ (12)

Note that in case of online use of the model, smoothing can be ignored without significant effects on spectrogram quality.

C. EM & Model Selection

In order to estimate the process noise covariance $\mathbf{Q}$ and the observation noise variance R in a maximum likelihood approach, the following EM algorithm can be applied. The procedure is derived by adapting the approach of [10] for our model in an initial EEG sample of 10 seconds. The filter can then be run forward until the end of the recording.

Let $\Theta = \{\mathbf{Q}, R\}$ be the set of parameters whose maximum likelihood values are searched. The goal is to maximize the log-likelihood $log[P(Z|\Theta)]$. Since access to the states is not provided, what is feasible is to calculate the expected complete log-likelihood $\mathcal{Q} = \mathcal{E}_{A|Z}[logP[(Z, A|\Theta)]]$ in the E-Step of the algorithm and maximize it with respect to Θ in the M-Step in an iterative manner of alternating the 2 aforementioned steps. Overall, the EM algorithm for the proposed model is briefly given below.

Initialization: $\mathbf{Q}_{start} = \mathbf{I}_p$ (13), $R_{start} = R(0)$ (14)

E-Step: $\mathcal{Q} = \mathcal{E}_{A|Z}[log[\prod\limits_{k=2}^{N} P(\mathbf{a}_k|\mathbf{a}_{k-1}) \prod\limits_{k=1}^{N} P(z_k|\mathbf{a}_k, \mathbf{H}_k)]]$ (15)

M-Step: $\mathbf{Q}_{ML} = \frac{1}{N-1}[\sum\limits_{k=2}^{N}[\mathbf{P}_{k|N} + \mathbf{a}_{k|N}\mathbf{a}_{k|N}^T - \mathbf{S}_{k-1}\mathbf{P}_{k|N} - \mathbf{a}_{k|N}\mathbf{a}_{k-1|N}^T]])$ (16)

$R_{ML} = \frac{1}{N}\sum\limits_{k=1}^{N}[{z_k}^2 - 2z_k\mathbf{H}_k\mathbf{a}_{k|N} + \mathbf{H}_k[\mathbf{P}_{k|N} + \mathbf{a}_{k|N}\mathbf{a}_{k|N}^T]\mathbf{H}_k^T]$ (17)

The above 2 steps are repeated until convergence. Convergence is established if the relative increase of the marginal log likelihood $\mathcal{L}$ between 2 iterations is below a certain threshold. $\mathcal{L}$ is obtained as follows:

$$\begin{aligned}
\mathcal{L} &= \prod_{k=1}^{N} p\left(z_k \mid z_{k-1}, \ldots, z_0\right) = \\
&= \prod_{k=1}^{N} \int p\left(z_k \mid \mathbf{a}_k\right) p\left(\mathbf{a}_k \mid z_{k-1}, \ldots, z_0\right) d\mathbf{a}_k = \\
&= \prod_{k=1}^{N} \int \mathcal{N}\left(z_k; \mathbf{H}_k\mathbf{a}_k, R\right) \mathcal{N}\left(\mathbf{a}_k; \mathbf{a}_{k|k-1}, \mathbf{P}_{k|k-1}\right) d\mathbf{a}_k = \\
&= \prod_{k=1}^{N} \mathcal{N}\left(z_k; \mathbf{H}_k\mathbf{a}_{k|k-1}, \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + R\right) = \\
&= -\frac{N}{2}log(2\pi) - \frac{1}{2}\sum_{k=1}^{N} log(R + \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T) - \\
&\frac{1}{2}\sum_{k=1}^{N} \frac{(z_k - \mathbf{H}_k\mathbf{a}_{k|k-1})^2}{R + \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T} \quad (18)
\end{aligned}$$

Model selection is then done using the well-known Akaike Information Criterion (AIC), defined as $AIC(p) = 2p - 2\mathcal{L}_{ML}$, where p is the order of the autoregressive model and $\mathcal{L}_{ML}$ is the maximized marginal log-likelihood for that model. The order that yields the lowest AIC will be chosen.

D. Spectral Estimation & Roughness Metric

Having calculated the optimal estimates for the autoregressive parameters, the spectral density at frequency f and time t is calculated using:

$$S(f, t) = \frac{R}{|1 - \sum\limits_{k=1}^{p} a_k(t) \mathrm{e}^{-\mathrm{i}2\pi k \frac{f}{f_s}}|^2}, \quad 0 \le f \le \frac{f_s}{2} \quad (19)$$

Where f_s is the sampling frequency and $a_k(t)$ is the k-th estimated autoregressive coefficient at time t.

Finally, we define a *roughness* metric of a coefficient $a_k(t)$ between 2 time points t_1 and t_2, $R_k = \int\limits_{t_1}^{t_2} [\frac{d^2 a_k(t)}{dt^2}]^2 dt$

(20) [11]. The lower the metric, the smoother the temporal evolution of the autoregressive coefficients.

III. Results

A. Simulation Results

To validate our hybrid model and to obtain an insight of how hybridity enhances smoothness without restricting adaptability, it was fit along with its discrete counterpart to a simulated noisy sinusoidal wave.

An artificial sinusoidal signal of $T = 30sec$, $z_k = A_k sin(\omega_k \frac{k}{f_s}) + v_k$ was generated at a sampling rate $f_s = 250Hz$. v_k is added white Gaussian noise with zero mean and unit variance. Amplitude evolves as $A_k = 1 + \frac{k}{T f_s}$, $0 \leq \frac{k}{f_s} \leq T$. Let ω_k evolve following the equation below.

$$\omega_k = \begin{cases} 50+2\frac{k}{f_s}, & \text{if } 0\leq\frac{k}{f_s}<10 \\ 80-\frac{k}{f_s}, & \text{if } 10\leq\frac{k}{f_s}<30 \end{cases}$$

Let $c_{k|N}$ denote the roots of the characteristic polynomial of the autoregression, which can be written as $c_{k|N} = r_{k|N}\mathrm{e}^{-\mathrm{i}\omega_{k|N}}$, where $r_{k|N}$ is the modulus and $\omega_{k|N}$ is the phase of each root $c_{k|N}$. The dominant frequency is proportional to the phase of the complex roots: $f_{k|N} = \frac{f_s|\omega_{k|N}|}{2\pi}$. Applying this procedure for a discrete and a hybrid model on the simulated data, using an autoregressive model of order $p = 2$, $R = 1$ and $\mathbf{Q} = 10^{-3}\mathbf{I}$ the following estimations are obtained (Figure 1). Figure 1 is zoomed at the point where the frequency change is most abrupt ($t = 10sec$). A low autoregressive order was used for increased tractability of the poles. The values for the remaining parameters were set empirically.

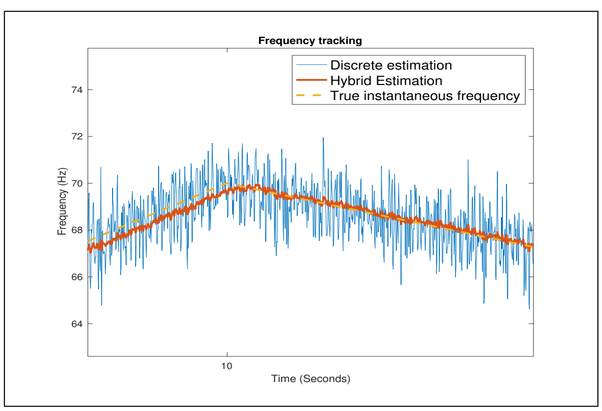

Fig. 1. Instantaneous frequency tracking

Both models are able to track the true frequency evolution, but the hybrid model results in much less fluctuation around the true instantaneous value. Note that a lower covariance matrix could potentially be used for the discrete model, but as it will become apparent in the next section of the paper, such a strategy does not help in efficiently smoothing EEG spectrograms.

B. Spectrograms of EEG under propofol

Spectrograms for propofol were calculated using the hybrid and discrete parametric methods, as well as widely used non-parametric methods including the periodogram, the multitaper spectrogram, the state-space periodogram and the recently developed state-space multitaper spectrogram [12]. For the multitapers, 3 tapers were used at a 2Hz spectral resolution. The time window of assumed stationarity of was 2 seconds. The performance of our model was compared to that of purely discrete parametric methods that have been used before for spectrogram generation [6] and to that of the non-parametric approaches mentioned above. The EEG recordings are part of de-identified data collected from patients at Massachusetts General Hospital (MGH), as a part of a MGH Human Research Committee-approved protocol.

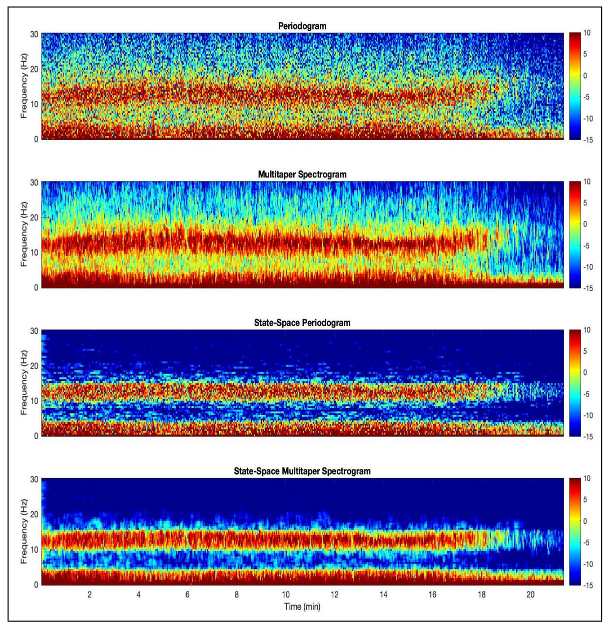

Fig. 2. Non parametric spectrograms of human EEG under propofol

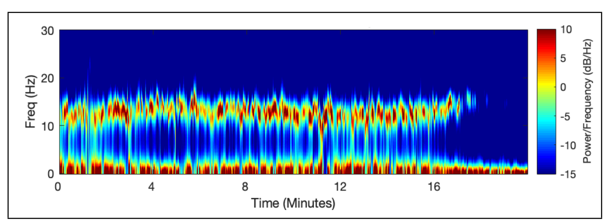

Fig. 3. Discrete parametric spectrogram of human EEG under propofol

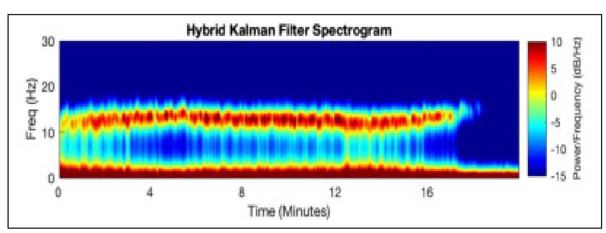

Fig. 4. Hybrid parametric spectrogram of human EEG under propofol

Non-parametric spectrograms (Figure 2) clearly show an α frequency band in addition to the slow oscillations, as expected [1]. The periodogram and the multitaper spectrogram are quite noisy. On the other hand, their state-space counterparts seem to underfit as it is very implausible that the α band is evolving as a completely straight line in an actual EEG experiment.

Autoregressive models of order $p = 14$ were used to calculate the parametric spectrograms. In the purely discrete case (Figure 3), the resulting spectrogram seems able to grasp the subtle changes in frequency but is not able to reduce the underlying noise to the same extent as the hybrid spectrogram (Figure 4). The hybrid model performs better in identifying the slight frequency changes along the α band without introducing extra noise. The performance of the purely discrete model with a lower covariance matrix as a measure to reduce coefficients' variability was tested on the propofol dataset. As shown in Figure 5, its performance deteriorated. This is due to a low covariance matrix that prevents the model from correctly estimating the changes in signal frequency and introduces artifacts in the spectrogram. Exponential smoothing was also tested as a potential smoothing method but led to an even lower quality spectrogram than the one in Figure 5.

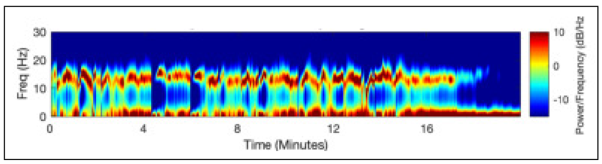

Fig. 5. Discrete parametric spectrogram of human EEG under propofol with lowered $\mathbf{Q}$

Finally, the conclusion drawn by visual inspection for smoother spectrograms of the hybrid model is corroborated by the roughness metrics of Table 1, as defined in (20). The reported final value for each model is the average of the metrics for each individual coefficient taking part in the model.

TABLE I
ROUGHNESS METRICS

	Discrete	Hybrid
Ground Truth	$2.7324 \cdot 10^{-7}$	$6.3428 \cdot 10^{-9}$
Propofol	$3.3380 \cdot 10^{-5}$	$6.5674 \cdot 10^{-7}$

IV. Discussion

The hybrid filter is able to compromise efficiently the underfitting regime of the state-space multitaper periodogram with the overfitting regime of the multitaper spectrogram. In relation to the purely discrete version, the hybrid filter is better able to capture the smoothness in the data, while at the same time this ability does not prevent it to follow sharp frequency changes.

In this hybrid approach, no prior knowledge about the evolution of the parameters in the time intervals between the observations is assumed. As a result, the autoregressive coefficients are assumed to follow a random walk. The proposed model can become even more potent in calculating frequency spectra if some prior knowledge about the parameters is available that will guide a reasonable choice of the state transition matrix. On the other hand, predicting a priori how the parameters are expected to evolve is not a trivial issue and demands a fair amount of research and experience with EEG datasets and effects of anesthetics on the brain. Nevertheless, it is expected that incorporating such prior information can enhance the model's performance even more, since by integrating the state equation, the model will be able to predict values for the parameters even in the time intervals between the observations where no new information is available.

Overall, our method provides another tool for frequency analysis of non-stationary data in the effort of calculating an accurate representation of their spectral content. The suitability of our method for online use, coupled with its ability to provide smoother spectrograms, suggests that it can be used as a monitoring tool for real time brain state tracking under general anesthesia that will further potentiate the design of closed loop systems for automatic and precise control of brain states. The hybrid model will be extended to incorporate non-Gaussian state noise and will be tested on more anesthetic drags in a future paper.

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