

$$\text{Image} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 9 & 0 \end{bmatrix} \quad \text{Kernel} = \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}$$

1 forward pass, stride = 1

Each receptive field multiplies kernel element-wise and then add together the elements.

$$\begin{array}{ccc} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \\ \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \\ \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & \begin{bmatrix} 7 & 8 \\ 9 & 0 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \end{array}$$

Final output shape is (3,3)

2 backward pass, stride = 1

$$\text{dout} = \begin{bmatrix} d_1 & d_2 & d_3 \\ d_4 & d_5 & d_6 \\ d_7 & d_8 & d_9 \end{bmatrix}$$

$\frac{\partial L}{\partial W}$: multiply downstream gradient elements into corresponding receptive field. Then add all the receptive fields together.

$$\begin{array}{ccc} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \odot d_1 & \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \odot d_2 & \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \odot d_3 \\ \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \odot d_4 & \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} \odot d_5 & \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix} \odot d_6 \\ \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \odot d_7 & \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \odot d_8 & \begin{bmatrix} 7 & 8 \\ 9 & 0 \end{bmatrix} \odot d_9 \end{array}$$

Final output shape (2,2)

$\frac{\partial L}{\partial X}$: multiply downstream gradients into corresponding receptive field. Map receptive fields back into Image, overlapping elements are summed.

$$\begin{array}{ccc} d_1 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_2 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_3 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \\ d_4 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_5 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_6 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \\ d_7 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_8 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} & d_9 \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \end{array}$$

Same shape as X matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

First receptive field mapped back:

$$\begin{bmatrix} d_1 w_1 & d_1 w_2 & 0 & 0 \\ d_1 w_3 & d_1 w_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Second receptive field mapped back:

$$\begin{bmatrix} d_1 w_1 & d_1 w_2 + d_2 w_1 & d_2 w_2 & 0 \\ d_1 w_3 & d_1 w_4 + d_2 w_3 & d_2 w_4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Third receptive field mapped back:

$$\begin{bmatrix} d_1 w_1 & d_1 w_2 + d_2 w_1 & d_2 w_2 + d_3 w_1 & d_3 w_2 \\ d_1 w_3 & d_1 w_4 + d_2 w_3 & d_2 w_4 + d_3 w_3 & d_3 w_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 forward pass, stride = 2

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \quad \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \\ \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \quad \begin{bmatrix} 7 & 8 \\ 9 & 0 \end{bmatrix} \odot \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}$$

1. What is the shape out the output of the convolution?
2. What is the shape of the downstream gradient passed back to this layer?
3. What is the shape of $\frac{\partial L}{\partial W}$?
4. What is the shape of $\frac{\partial L}{\partial X}$?