UNIVERSITY OF THE PHILIPPINES VISAYAS TACLOBAN COLLEGE

PROJECT IN LINEAR ALGEBRA (MATH 114)

IMAGE COMPRESSION USING MODIFIED SINGULAR VALUE DECOMPOSITION

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1 Introduction

To present information in a much greater form, we created images and with the rapid increase of the popularity of images in the Internet, a strong demand has been established by companies who manage these data for better compression algorithms that will reduce the size of stored data in order to accommodate more data. Image compression deals with reducing the size of data required to represent images. Since the advent of the World Wide Web, image compression has provided substantial help in terms of efficiently transmitting data through the Internet. For instance, many websites nowadays use certain amount of images to provide more information to their respective users. In order to load these images in a considerable amount of time, we use image compression. In this research, we aim to use linear algebra in the compression of images. Basically, this study will use a modified form of Singular Value Decomposition or SVD to compress images and thus save storage memory. The proposed approach wants to achieve an effective compression of images without compromising much the quality of an image. The premise for this idea is that, first, an image can be represented into a matrix. Then from this matrix, we can get a reduced matrix by applying the proposed modified SVD. This leads to an image transformed to its reduced matrix which is much smaller and requires less memory to store compared to its original image.

2 Related Literature

Image Compression has been instituted to deliver images with less size with an aim in creating more room for storing other data. During the past years, various image compression algorithms were introduced and continuously been developed to effectively compress images.

Singh et al. [2] has implemented compression of image using SVD. Their technique carries out the compaction according to the compaction of energy concentrated on initial few columns which tend to have the localized content of the matrices. The overall energy of the image is represented by singular values of the image. They found out that SVD can be implemented on matrices of varying sizes and types such as arbitrary, square, reversible and non-reversible matrices. They also found out that SVD reduces the storage requirement of an image.

Tian et al. [4] has proposed three schemes for compressing the images namely direct compression and decomposition scheme, adaptive singular value selection scheme, singular value subtracting one update scheme. They have also studied and analyzed the efficiency of SVD based image compression techniques.

Peters et al. [1] has implemented SVD to compress a micro-array image. These images store huge amounts of DNA information and are also high resolution images which highlights minute details. Because of the high resolution, these images tend to be large in size, which means storage requires a lot of space. So it is very important to reduce the size of the image without compromising the details in the image. They used a very complicated process wherein they first clustered and classified the micro-array images before selecting features. SVD is then used to divide the image into small sub-images then they perform SVD on each sub-image. The method gave a better high peak signal to noise ratio in addition to increasing compression ratio.

As discussed above, many studies have developed variety compression techniques of images. They have commonly used Singular Value Decomposition as their basis underlying compression algorithm to compact images. In this study, we will apply a modified form of SVD to compress images with an aim to keep the most important details of an image, therefore, not compromising the image quality.

3 Statement of the Problem

We are developing an image compression scheme using Singular Value Decomposition as the base matrix for reducing redundant pixel values or information to store lesser data in file storage. In this approach, we will only perform compression on two components of an image to perform faster processing due to high cost of time overhead from computations of singular value decomposition and compute the rank k to determine the level of compression on the image. These are facilitated to achieve a compressed image without affecting much the quality of the image. Then, perform an "Image Compression Turing Test" that will verify the effectiveness of the compression technique based on human visual evaluation.

4 Significance of the Study

The findings of this study will be used to show the comparison between the standard Singular Value Decomposition and the proposed Modified Singular Value Decomposition in establishing a better and faster image compression to create more room for storing other data. In addition, it is hoped that this study will result to a compressed image retaining most of its significant information, without compromising much of its quality by using only the two components of an image for a faster image compression. And also, to propose a new k-computation formula that can be used to automatically compress the image without user intervention of stating the rank k. Lastly, to introduce a new qualitative metric in determining the effectiveness of compression using the "Image Compression Turing Test".

5 Methodology

Singular Value Decomposition works by decomposing the general matrix of the original image. It finds the best approximation of the original matrix dimension using fewer dimensions. It does this by employing SVD to reduce the lower dimension by getting the substructure and ordering it from the most important information to the least. This makes SVD as the method for reduction [3]. SVD is defined as the factorization of the image matrix A where:

$$A = U \sum V^T \tag{1}$$

Where U and V^T are orthogonal matrices and $\sum$ is a diagonal matrix containing singular values that forms the diagonal in decreasing order. U and V matrices serve as the left and right vector respectively. Then given a rank k which determines the number of singular values that will be reduced to compress the decomposed matrix.

The steps for the compression would be:

1. Get the corresponding image matrices of the red, green, and blue layers from input image.

2. Decompose the image matrix of 2 layers, for this case red and green layers [1] to get U$\sum V^T$.

3. Then compute the rank k for reduction of matrix rows using our proposed k-computation formula, where avg is the average of the length and width of the image:

$$avg = \frac{width + height}{2} \tag{2}$$

$$k = (2 * \left(\frac{avg}{100}\right) + 1) * 10 \tag{3}$$

4. Reduce the matrices based on the computed rank k.

5. Merge the factorized matrices with the not decomposed matrix of the blue layer to get the compressed image.

6. Then compute the quantitative metric to determine the percentage of the file size of the compressed image over the uncompressed image using the compression ratio shown below, where C is the compression ratio:

$$C = 100 * 3 * \left(\frac{k * (height + width) + k}{height * width * channels}\right) \tag{4}$$

7. Lastly, perform the qualitative metric to test the effectiveness of the compression algorithm using the "Image Compression Turing Test". This test was motivated by how human intelligence deduce something by just looking at an image. Through the years, this helped in aiding computer vision systems understand images just how humans do which is an ongoing and ultimate problem up until now. In line to this, we will use this process to test how effective the modified compression process. This test will be administered by letting a group of participants choose the original image from the pairs of images containing the original image and the compressed image given an amount of time to test the effectiveness of the compression scheme.

6 Results and Discussion

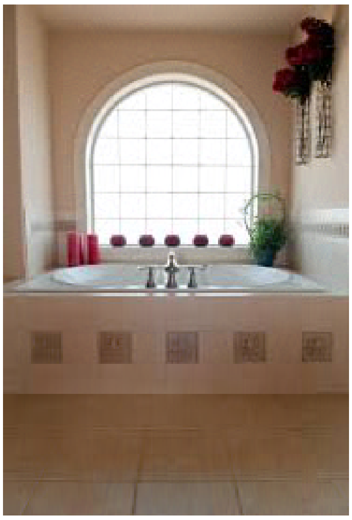
Below are the 24 images used in our compression algorithm. Images on the left and right are the original images and compressed images respectively.

[1]It wouldn't really matter which 2 channels we decompose since it won't affect much the color of the resulting image.

(1) Image A

(2) Compressed Image A

(3) Image B



(4) Compressed Image B

(5) Image C

(6) Compressed Image C

(7) Image D

(8) Compressed Image D

(9) Image E

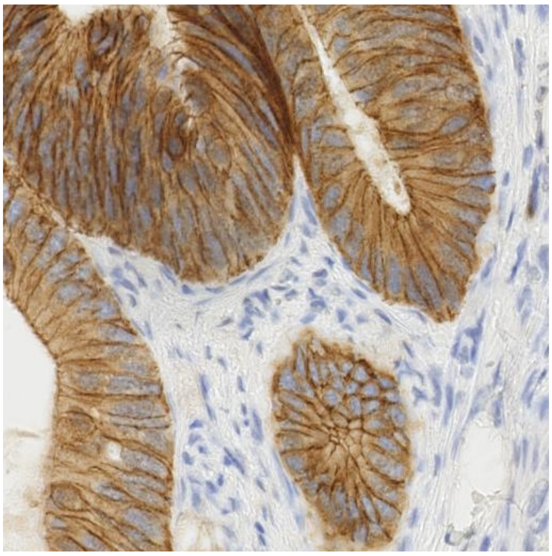
(10) Compressed Image E

(11) Image F

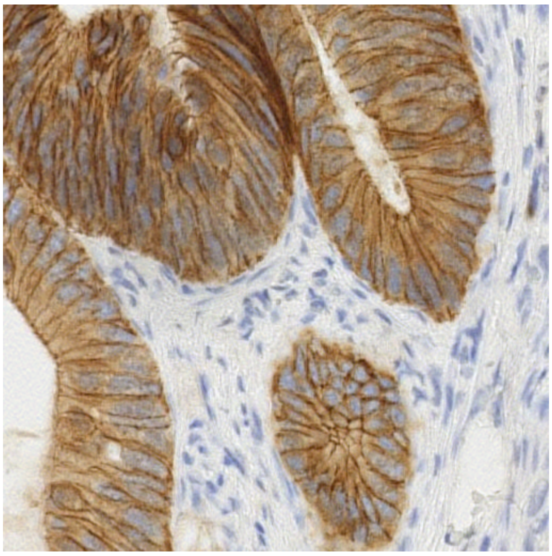
(12) Compressed Image F

(13) Image G

(14) Compressed Image G



(15) Image H



(16) Compressed Image H

(17) Image I

(18) Compressed Image I


(19) Image J


(20) Compressed Image J


(21) Image K


(22) Compressed Image K


(23) Image L


(24) Compressed Image L

(25) Image M

(26) Compressed Image M

(27) Image N

(28) Compressed Image N

(29) Image O

(30) Compressed Image O

(31) Image P

(32) Compressed Image P

(33) Image Q

(34) Compressed Image Q

(35) Image R

(36) Compressed Image R

(37) Image S

(38) Compressed Image S

(39) Image T

(40) Compressed Image T

(41) Image U

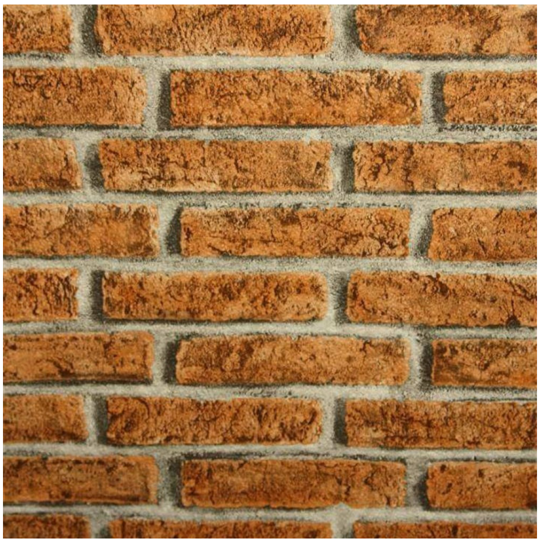
(42) Compressed Image U

(43) Image V

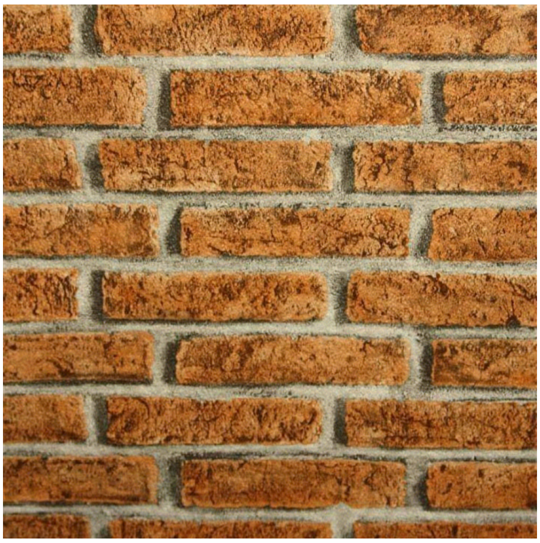
(44) Compressed Image V

(45) Image W

(46) Compressed Image W



(47) Image X



(48) Compressed Image X

To determine the effectiveness of the proposed compression method, we facilitated two kinds of metrics:

1. The first metric, which is a quantitative metric, is used to determine the trend of the *compression ratio* relative to the image resolution of the original image. *Table 1* illustrates

the corresponding resolution of the original image with their computed value of k and another table with the computed *compression ratio*.

2. The second metric, which is a qualitative metric, is done through a Turing test. We facilitated a survey to ten (10) respondents in which we showed twenty-four (24) sets of two (2) images (original and compressed), with different resolutions, and asked the respondents to determine which is the original and uncompressed image.

As shown in *Table 1*, the optimal value of k increases as the image resolution of the original image also increases. This shows that an image requires a larger value of k relative to how high the image resolution of the image. This is because high resolution images has more significant details compared to low quality images which explains the behavior of the value k.

Image Resolutions	Computed k
30000	18
50246	55
50505	55
135300	85
240000	110
262144	112
273280	116
390000	135
712500	180
786432	189
872000	197
1000000	210

Table 1. Image Resolutions and their corresponding value k

In *Table 2*, we can observe that as the image resolution of the original image increases, the *compression ratio* of the image decreases. This shows that the smaller the image, the greater the number of less important information in that image, thus resulting to a higher *compression ratio*.

Image Resolutions	Compression Ratio
30000	35.1
50246	33.13
50505	33.32
135300	31.5
240000	30.59
262144	29.2
273280	30.22
390000	28.87
712500	28.65
786432	28.73
872000	28.21
1000000	28.01

Table 2. Image Resolutions and their corresponding *compression ratio*

The results can be graphically expressed by the following graphs:

Figure 1. Compression Ratio

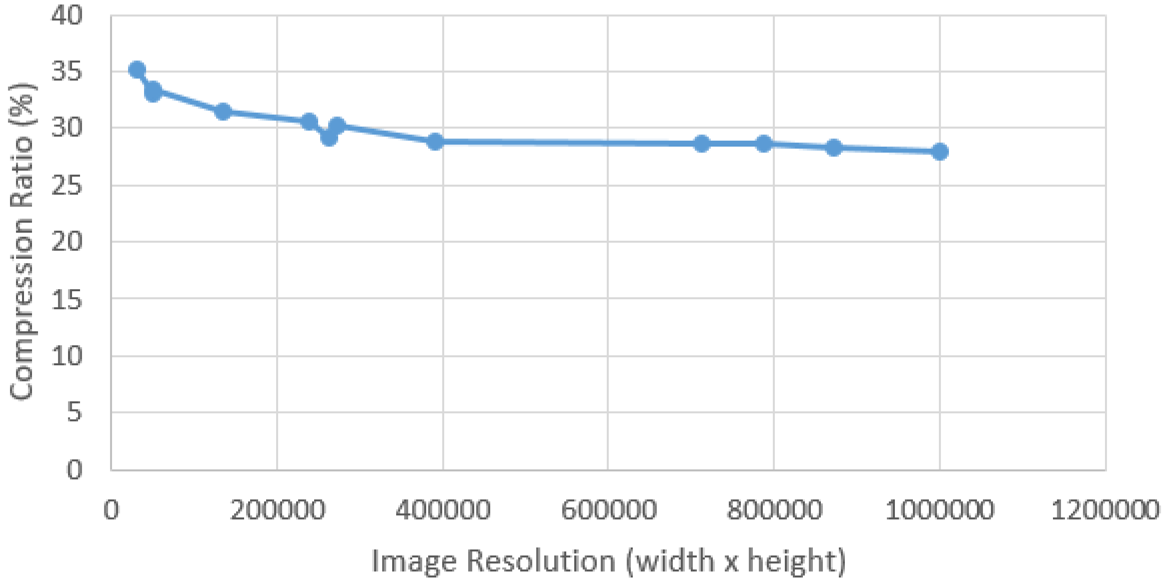
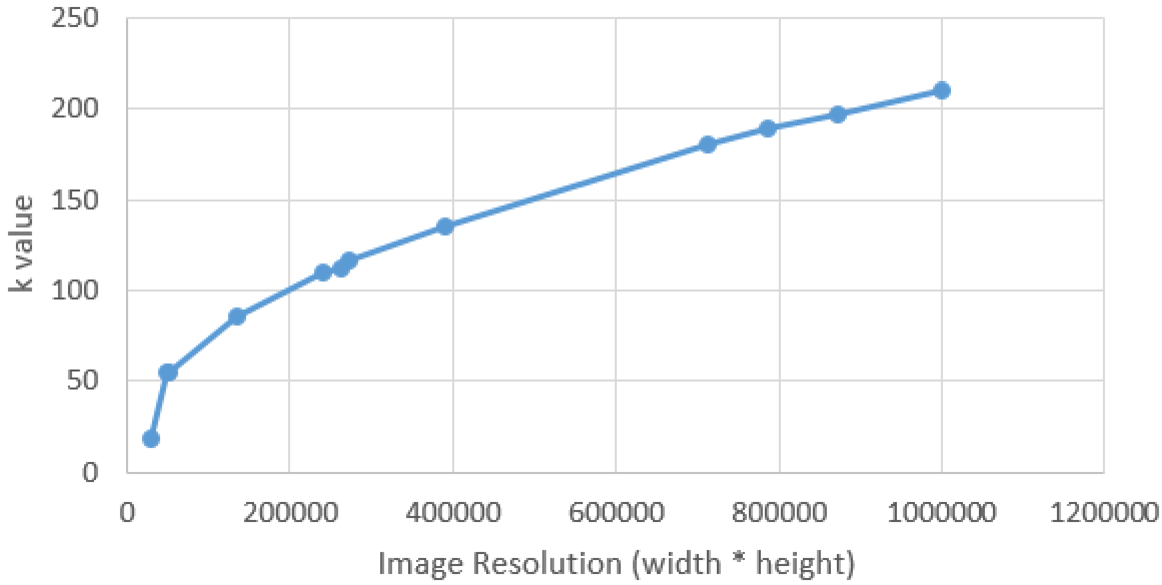
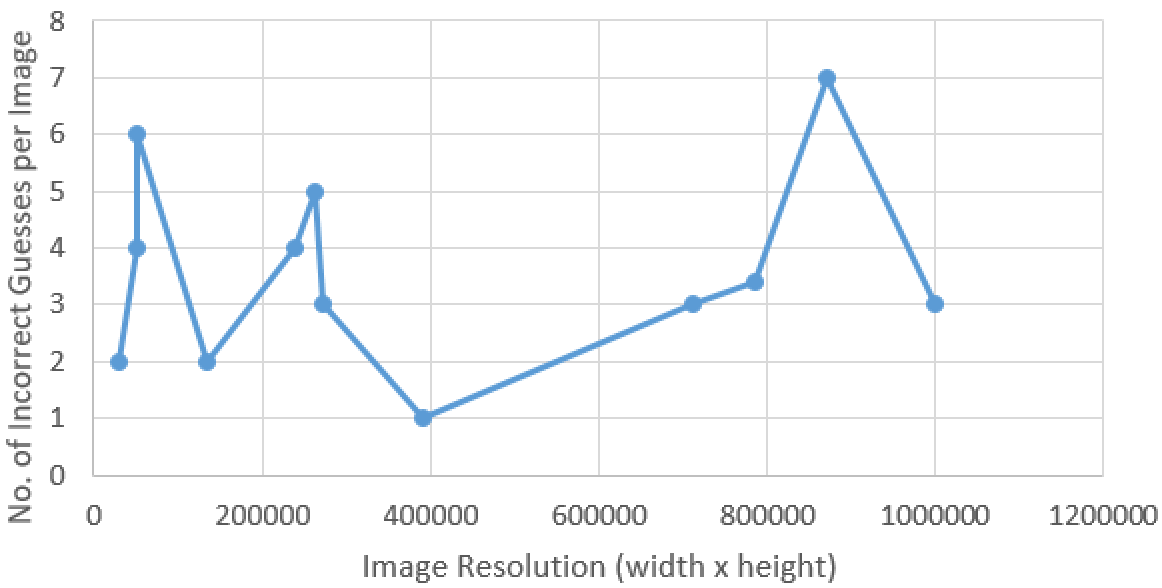


Figure 2. k-computation



As shown in *Figure 3* below, there is a considerable portion of incorrect guesses made by the respondents when we facilitated the Turing Test which means that the images processed by our proposed compression algorithm is relatively close to the quality of the original image thus people can not distinguished which image is the original or the compressed one. This shows that our proposed compression scheme can be effectively used to compress images.

Figure 3. Accuracy Chart



7 Conclusion and Recommendation

From the above results, we have achieved a sufficiently good image compression algorithm that produces a compressed image that do not affect much the image quality. The compressed images, for the most part, can be hard to distinguish from the original image, making it less obvious that the images are in fact compressed. However, there are cases that the images are obviously compressed because of the small presence of noise-like pigments. This is because there are parts where less important informations were removed and have become obvious due to the surrounding colors. Lastly, the use of "Image Compression Turing Test" as a qualitative metric has a considerable great potential to be widely accepted or can be improved by further researches.

A recommendation for future studies would be to use a larger dataset and to have more participants in the survey to achieve a more accurate statistics of the correct and incorrect guesses in distinguishing the original image from the compressed image. And also, to improve the k-computation formula for it has great potential to use as a tool to create a much better image compression scheme.

References

[1] Thomas J Peters, Renata Smolikova-Wachowiak, and Mark P Wachowiak. Microarray image compression using a variation of singular value decomposition. In *Engineering in Medicine and Biology Society, 2007. EMBS 2007. 29th Annual International Conference of the IEEE*, pages 1176–1179. IEEE, 2007.

[2] Satish Kumar Singh and Shishir Kumar. A framework to design novel svd based color image compression. In *Computer Modeling and Simulation, 2009. EMS'09. Third UKSim European Symposium on*, pages 235–240. IEEE, 2009.

[3] HR Swathi, Shah Sohini, G Gopichand, et al. Image compression using singular value decomposition. In *IOP Conference Series: Materials Science and Engineering*, volume 263, page 042082. IOP Publishing, 2017.

[4] Mei Tian, Si-Wei Luo, and Ling-Zhi Liao. An investigation into using singular value decomposition as a method of image compression. In *Machine Learning and Cybernetics, 2005. Proceedings of 2005 International Conference on*, volume 8, pages 5200–5204. IEEE, 2005.