**中間報告書**

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**学籍番号:** 255113

**名前:** 倉持誠

**指導教員:** 中村祐太

**副指導教員:** 和田淳一郎、中園善行

**論文題目:** 現職教員を考慮した教員の配属マッチングの設計

**研究目的**

本研究は、次の二つの成果を目指す。

1. 教科別カットオフ調整関数に基づく配分ルールの設計

学校×教科ごとの定員（容量制約）と、教員・学校の選好を整合させるために、需要集合とカットオフを反復更新して不動点を構成する配分ルールを提案する。得られる割当は実現可能性・個人合理性・（教科別）公平性を満たし、さらにその集合の中で教員側にとって最良な教師最適性を達成する。

1. 提案ルールの理論的特性の特性付け

上記ルールを、単調性（カットオフ上昇による需要の縮減）、不動点の存在と収束、固定点⇔公平割当の対応、および教師最適性（許容される公正な割当の中での最大性）といった望ましい性質の組で特徴づける。これにより、提案ルールの価値と他方式との理論的差異を明確化する。

**Research Overview**

The main question of this research is whether there exists a voting rule that aggregates voters' preferences on both rankings and approvals, and prevents the incentive to report fewer acceptable alternatives. This question arises from a common problem in scheduling. Typically, people report their available days, and the date with the most approvals is chosen. This method is known as **approval voting** in social choice theory. However, this approach has two major problems:

1. **It fails to reflect refined preferences.** Approval voting only gathers "available" or "unavailable" information. For example, it doesn't consider information like, "I can make both day x and day y, but I would prefer day x."
2. **It can lead to strategic manipulation.** Voters have an incentive to truncate their acceptable alternatives, since this may lead to a better social ranking for them.

For an example of the latter problem, consider a situation where three people—Sam, Tom, and Mia—are trying to schedule a meeting, with the following preferences.

アイコン が含まれている画像

AI 生成コンテンツは誤りを含む可能性があります。

Sam approves of all the alternatives and ranks them as x > y > z. Using approval voting, the meeting would be scheduled on day z because it has the most approvals. However, if Sam strategically reports that only x is available, that is, he **truncates** his approval set, the total score for z decreases.

テキスト, 座る, 準備, 時計 が含まれている画像

AI 生成コンテンツは誤りを含む可能性があります。

As a result, x and z tie for the top spot, and Sam achieves a better social ranking than the original social ranking. Similarly, Tom could also truncate his approval set to only y and obtain a better social ranking. If both Sam and Tom truncate their approval sets at the same time, no date would receive more than one approval, making it impossible to find a date that works for everyone. This is because approval voting provides an incentive for voters to improve the social ranking by truncating their approval set.

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To overcome these two problems, we aim to design a rule that can reflect refined preferences and does not provide voters with the incentive to truncate their approval sets.

We propose the **top-restricted Borda rule**. In this rule, each voter assigns a score to their acceptable alternatives. They start with a score of 1 for the lowest-ranked acceptable alternative and increase the score by 1 for each one they rank higher. The social ranking is determined by the total scores of each alternative. For example, in the table below, voter A accepts both x and y, they give score of 2 to x and score of 1 to y. The aggregated scores yield the social ranking y > x > z.

グラフィカル ユーザー インターフェイス, アプリケーション

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We define **truncation-proofness** as the axiom that requires that any voter cannot improve the social ranking by truncating their approval set. We show that top-restricted Borda rule satisfies truncation-proofness. We also axiomatize the top-restricted Borda rule within the class of approval scoring rules, which assign 0 points to unacceptable alternatives. In addition to truncation-proofness, we introduce **approval monotonicity**, **non-imposition**, and **adjacency symmetry**.

* **Approval monotonicity** ensures that an alternative’s social ranking does not decrease when it receives one additional approval.
* **Non-imposition** ensures that the social welfare function can produce a strict ranking between some pairs of alternatives for at least one evaluation profile.
* **Adjacency symmetry** ensures that adjacent alternatives are treated symmetrically under certain conditions.

This axiomatization provides a rationale for using the top-restricted Borda rule over other voting rules that also satisfy truncation-proofness.

**Progress to Date**

We have addressed the two research objectives:

* We proposed the **top-restricted Borda rule** as a rule that satisfies **truncation-proofness**, and we proved that it does.
* We proved that the **top-restricted Borda rule** is the unique approval scoring rule that satisfies **truncation-proofness**, **approval monotonicity**, **non-imposition**, and **adjacency symmetry**.

**今後の計画**

* 修士論文の執筆と磨き込み
* 容量制約だけに限らず、下限制約・教科横断の結合制約・多様性制約、弱順序（同順位）や確率的配分にも拡張して、公平・個人合理・実現可能なルール全般の中で、提案する教科別カットオフ調整ルールの教師最適性（最良性）と特性付け（唯一性／極大性）をより強固に示す。

**論文の構成**

1. **はじめに**：研究目的・主要な問い・社会的背景（教員配属の現状と課題）・関連研究の整理と本研究の位置づけ。
2. **モデル**：教員・学校・教科の集合、選好と優先順位、容量制約（定員）の定義、需要集合とカットオフ、マッチングと教科別拡張の形式化。
3. **公理／性質**：本研究で用いる性質の提示と直観的説明：実現可能性、個人合理性、公平性、単調性（カットオフ上昇→需要縮減）、固定点の概念、教師最適性。
4. **主要結果**：

4.1 教科別カットオフ調整ルールが上記の性質（実現可能性・個人合理性・公平性・単調性・固定点存在）を満たすことの証明。

4.2 許容されるクラス内での教師最適性および特性付け（必要十分条件の提示／一意性の主張可能範囲の整理）。