**Master's Thesis Progress Report**

**Date of Submission:** 2025/09/

**Student ID:** 255113

**Name:** Moeno Yajima

**Supervisor:** Prof. Yuta Nakamura

**Sub-supervisor:** Prof. Junichiro Wada, Prof. Qingyuan Zui

**Thesis Title:** "Truncation-Proof Voting Rules with Ranking and Approval Input"

**Research Objectives**

This research aims to achieve two main results:

* **Designing a Voting Rule that Satisfies Truncation Proofness:** We introduce a form of strategy proofness called "truncation-proofness," which means voters have no incentive to report fewer acceptable alternatives. We design a voting rule that satisfies this property.
* **Axiomatizing the Designed Voting Rule:** We provide an axiomatic characterization of the proposed voting rule, using several desirable axioms in addition to truncation-proofness. This axiomatization demonstrates the value of the proposed rule.

**Research Overview**

The main question of this research is whether there exists a voting rule that aggregates voters' preferences on both rankings and approvals, and prevents the incentive to report fewer acceptable alternatives. This question arises from a common problem in scheduling. Typically, people report their available days, and the date with the most approvals is chosen. This method is known as **approval voting** in social choice theory. However, this approach has two major problems:

1. **It fails to reflect refined preferences.** Approval voting only gathers "available" or "unavailable" information. For example, it doesn't consider information like, "I can make both day x and day y, but I would prefer day x."
2. **It can lead to strategic manipulation.** Voters have an incentive to truncate their acceptable alternatives, since this may lead to a better social ranking for them.

For an example of the latter problem, consider a situation where three people—Sam, Tom, and Mia—are trying to schedule a meeting, with the following preferences.

アイコン が含まれている画像

AI 生成コンテンツは誤りを含む可能性があります。

Sam approves of all the alternatives and ranks them as x > y > z. Using approval voting, the meeting would be scheduled on day z because it has the most approvals. However, if Sam strategically reports that only x is available, that is, he **truncates** his approval set, the total score for z decreases.

テキスト, 座る, 準備, 時計 が含まれている画像

AI 生成コンテンツは誤りを含む可能性があります。

As a result, x and z tie for the top spot, and Sam achieves a better social ranking than the original social ranking. Similarly, Tom could also truncate his approval set to only y and obtain a better social ranking. If both Sam and Tom truncate their approval sets at the same time, no date would receive more than one approval, making it impossible to find a date that works for everyone. This is because approval voting provides an incentive for voters to improve the social ranking by truncating their approval set.

矢印 が含まれている画像

AI 生成コンテンツは誤りを含む可能性があります。

To overcome these two problems, we aim to design a rule that can reflect refined preferences and does not provide voters with the incentive to truncate their approval sets.

We propose the **top-restricted Borda rule**. In this rule, each voter assigns a score to their acceptable alternatives. They start with a score of 1 for the lowest-ranked acceptable alternative and increase the score by 1 for each one they rank higher. The social ranking is determined by the total scores of each alternative. For example, in the table below, voter A accepts both x and y, they give score of 2 to x and score of 1 to y. The aggregated scores yield the social ranking y > x > z.

グラフィカル ユーザー インターフェイス, アプリケーション

AI 生成コンテンツは誤りを含む可能性があります。

We define **truncation-proofness** as the axiom that requires that any voter cannot improve the social ranking by truncating their approval set. We show that top-restricted Borda rule satisfies truncation-proofness. We also axiomatize the top-restricted Borda rule within the class of approval scoring rules, which assign 0 points to unacceptable alternatives. In addition to truncation-proofness, we introduce **approval monotonicity**, **non-imposition**, and **adjacency symmetry**.

* **Approval monotonicity** ensures that an alternative’s social ranking does not decrease when it receives one additional approval.
* **Non-imposition** ensures that the social welfare function can produce a strict ranking between some pairs of alternatives for at least one evaluation profile.
* **Adjacency symmetry** ensures that adjacent alternatives are treated symmetrically under certain conditions.

This axiomatization provides a rationale for using the top-restricted Borda rule over other voting rules that also satisfy truncation-proofness.

**Progress to Date**

We have addressed the two research objectives:

* We proposed the **top-restricted Borda rule** as a rule that satisfies **truncation-proofness**, and we proved that it does.
* We proved that the **top-restricted Borda rule** is the unique approval scoring rule that satisfies **truncation-proofness**, **approval monotonicity**, **non-imposition**, and **adjacency symmetry**.

**Future plans**

* We will write and refine the master’s thesis.
* We will aim for axiomatization across **all** rules, not just the class of approval scoring rules. This would provide a stronger basis for claiming that the **top-restricted Borda rule** should be preferred over any other rule.

**Thesis Structure**

1. **Introduction:** Research objectives, main questions, and a review of prior research.
2. **Model:** Definitions of individual preferences and voting rules.
3. **Axioms:** Introduction of the axioms used in this research and their intuitive meanings.
4. **Main Result**
   1. The top-restricted Borda rule satisfies the axioms.
   2. Proof that the top-restricted Borda rule is the unique approval scoring rule satisfying the axioms.
5. **Independence of the Axioms:** A demonstration that the introduced axioms are mutually independent.