MATLAB Assignment 3

Due October 30th at 11:59 PM EDT on Gradescope

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with one other person (**at most groups of two total**). If you choose to work together, you should simply submit **one copy**, and everyone will be receiving the same grade. **Make sure to include all names when submitting to Gradescope!**

Content of the submission:
You will need to write a script ($.mfile$)for this assignment and then "publish" it in MATLAB as a pdf in order to turn it in on Gradescope. There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.**

1. Use `format rat`. Define

$$A = \begin{bmatrix} 3 & -8 & 22 & -8 & 2 \\ 6 & -8 & -2 & -8 & 44 \\ -3 & 3 & 15 & 3 & -27 \\ -1 & 1 & 6 & 1 & -9 \\ 0 & 5 & -7 & 5 & 25 \\ 4 & 2 & -3 & 2 & 66 \end{bmatrix}.$$

   (a) Use the `rref` command and then determine a basis for the column space and the kernel for matrix A. You can use *disp* or *fprintf* to show your answer. For simplicity, you may express the vectors using parentheses like it is done in class.

   (b) Suppose A is now the **augmented matrix** of a system of equations. What is the solution to the system? If it is unique, state it. If there are infinite solutions, describe it in parametric form. If there are no solutions, explain why.

2. Use `format rat`. Define

$$A = \begin{bmatrix} 5 & -3 & 6 & 7 \\ 0 & 0 & 0 & 1 \\ 7 & -5 & 3 & -7 \\ 4 & 0 & 3 & -7 \end{bmatrix}.$$

   (a) Find a basis for the row space of A. Use *disp* or *fprintf*, and **express the vectors as row vectors using brackets** e.g. $\{[3\ \ 4\ \ 6\ \ 1], [1\ \ 3\ \ 5\ \ 7]\}$

   (b) Find a basis for the column space of A. Use *disp* or *fprintf*. For simplicity, you may express the vectors using parentheses like it is done in class.

(c) Does $\dim(\text{row}(A)) = \dim(\text{col}(A))$? Simply state yes or no.

(d) Does the set $\text{row}(A) = \text{col}(A)$? Use *disp* or *fprintf* to **briefly explain.**

3. Use `format rat`. Consider the set

$$\mathcal{B} = \{(4, 1, -5, 5), (2, 6, 4, -7), (4, 3, 0, 1), (-8, 6, 7, 7)\}$$

Let $\mathbf{v} = (2, -30, 13, -10)$.

(a) Use appropriate Matlab commands to find $\mathbf{u}$ where $[\mathbf{u}]_{\mathcal{B}} = \mathbf{v}$.

(b) Use appropriate Matlab commands to find $\mathbf{w}$ where $\mathbf{w} = [\mathbf{v}]_{\mathcal{B}}$.

Make sure to show your computations, and use *disp* or *fprintf* to briefly explain what you are doing.

4. Use `format rat`. Consider the polynomials (**read carefully**)

$$f_1(x) = 7 - 3x + x^2 + 7x^3 + 2x^4$$

$$f_2(x) = 9 - 3x - 9x^2 - 5x^3 - 6x^4$$

$$f_3(x) = 1 - x + 3x^2 + 4x^3 + 3x^4$$

$$f_4(x) = 5 - 3x - x^2 + x^4$$

that lie in $\mathbb{P}_4$. Let

$$W = \text{Span}(\{f_1(x), f_2(x), f_3(x), f_4(x)\}).$$

(a) Denote each of the 4 vectors $\mathbf{v}_i = [f_i(x)]_{\mathcal{B}}$ to be the coordinate vector of $f_i(x)$ relative to the basis $\mathcal{B} = \{1, x, x^2, x^3, x^4\}$ in $\mathbb{P}_4$. Define these as **column** vectors in Matlab as `v1, v2,`...

(b) Define $A =$ [`v1 v2 v3`...] (matrix whose i-th column is the coordinate vector $\mathbf{v}_i$).

(c) Use appropriate commands with the matrix above to help find a basis `S` for W. Use *disp* or *fprintf* to explicit show what the basis is as polynomials.

(d) Is the set $\{f_1(x), f_2(x), f_3(x), f_4(x)\}$ linearly independent or linearly dependent? If it is linearly dependent, find an explicit dependent relationship. If it is linearly indepedent, use *disp* or *fprintf* to justify your answer. If you write out a combination, you can simply use, for example $f_1(x)$, and not rewrite the entire polynomial out.