MATLAB Assignment 4

Due December 7th at 11:59 PM EDT on Gradescope

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with one other person (**at most groups of two total**). If you choose to work together, you should simply submit **one copy**, and everyone will be receiving the same grade. **Make sure to include all names when submitting to Gradescope!**

Content of the submission:
You will need to write a script ($.mfile$)for this assignment and then "publish" it in MATLAB as a pdf in order to turn it in on Gradescope. There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.**

1. **Use the `short` format for this problem.**
   (a) Input the following matrix in Matlab: $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 5 & 1 & 3 \\ 0 & 2 & -3 & 5 \\ 1 & 2 & 1 & 0 \end{bmatrix}$.
   (b) Compute `[P,D]=eig(A)`, which creates a diagonal matrix D whose diagonal entries are the eigenvalues of A, and P is a matrix whose columns correspond to eigenvectors of the eigenvalues.
   (c) Based on the matrix D above, determine whether$A$ is diagonalizable or not. Use *disp* or *fprintf* to briefly explain your answer.

2. **Use `short` format**.
   (a) Define the vectors $(2, 3, -3, -6)$, $(6, -1, 4, 1)$, $(0, 5, -3, 6)$, $(-4, 5, -2, 4)$ as **column vectors**. Label them `u1`, `u2`, `u3`, `u4`.
   (b) Define `A=[u1 u2 u3 u4]`
   (c) We now apply the Gram-Schmidt process to the vectors. Input

```
v1=u1
v2=u2-dot(u2,v1)/dot(v1,v1)*v1  %continue defining v3 and v4 similarly
```

   Continue to define `v3` and `v4` by the Gram-Schmidt process.

(d) From the vectors in the previous part, create an **orthonormal basis** by dividing by the magnitude. You can use the `norm` command to compute the norms. Denote these vectors as `w1`, `w2`,...

(e) Define `Q=[w1 w2 w3 w4]`.

(f) Define $R = Q^T A$, and check that $A = QR$.

(g) Input

```
[Q1,R1] = qr(A,0)
```

to find a QR-factorization immediately. Note these factors are possisbly different from those computed above. This is because an orthonormal basis is not unique (think about how the first vector you defined could have been any of the 4).

3. **Use `rat` format.**

(a) Define $A = \begin{bmatrix} 3 & 6 & -7 \\ 4 & -4 & 1 \\ 7 & -6 & 3 \end{bmatrix}$, and let it be the matrix representation of a linear transformation.

(b) Suppose $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \right\}$. Define the three vectors in $\mathcal{B}$ by `v1`, `v2`, `v3` in Matlab (as column vectors).

(c) Use Matlab to compute the $\mathcal{B}$-matrix for the transformation. Denote this matrix by `C`.

4. **Use `rat` format.** Let

$$S_1 = \{(5, -2, 1, -5, 0), (-3, 1, 6, 4, 2), (-6, 1, 0, 4, 2)\}$$

(a) Let W denote the SPAN of the set S_1. Thus, W is a subspace of $\mathbb{R}^5$. Use the Gram-Schmidt process to find an orthogonal basis for W. Denote the vectors by `v1`,`v2`,`v3`.

(b) Define $\mathbf{y} = (7, -9, 0, 3, 2)$ (as a column vector).

(c) Express y as a sum of two vectors, one in W, denoted by `z1`, and the other in $W^{\perp}$, denoted by `z2`.

(d) What point in W is closest to $\mathbf{y}$?