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Some practice problems for exam 1

1. Solve the following systems by row reducing the augmented matrix. If there is a unique solution, state what it is. If there is no solution, explain why. If there are infinite solutions, express your solution in parametric vector form.

   (a)

$$\begin{aligned} x + y + z &= 3 \\ y + 2z &= -5 \\ x + 2y + 4z &= -4. \end{aligned}$$

   (b)

$$\begin{aligned} x_1 - 3x_2 + 2x_3 - 5x_4 &= 3 \\ 2x_1 - 6x_2 + x_3 - 7x_4 &= 2 \\ x_1 - 3x_2 - 4x_3 + x_4 &= -5. \end{aligned}$$

2. For the following UNRELATED statements, determine if the statement is true or false. If it is TRUE, simply state TRUE. If it is FALSE, **provide an explicit counterexample** i.e. an explicit example that shows the statement is false.

   (a) Suppose A is a 3×4 matrix that is the standard matrix of a linear transformation. Then the transformation is always onto.

   (b) Suppose A is a 3×4 matrix that is the standard matrix of a linear transformation. Then the transformation is never one-to-one.

   (c) Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be vectors in $\mathbb{R}^n$. If $n \geq 3$, then the set of vectors must be linearly independent.

   (d) Let A be $m \times n$. If $m > n$, then the set of ROW vectors (each row represents 1 vector) must be linearly dependent.

   (e) If A is the matrix representation of a linear transformation and we know the columns of A form a linearly independent set, then the transformation is always onto.

3. Determine if $(0, 10, 8)$ lies in

$$\text{Span}(\{(-1,2,3),(1,3,1),(1,8,5)\}).$$

   If it does lie in the span, find an explicit linear combination.

   Is the set $\text{Span}(\{(-1,2,3),(1,3,1),(1,8,5)\})$ linearly independent? What about the set $(\{(-1,2,3),(1,3,1),(1,8,5)\})$?

4. Suppose $\mathbf{v}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_m$. If we add another vector $\mathbf{v}_{m+1}$, will $\mathbf{v}$ sometimes, always, or never be in $\text{Span}(\{\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_m, \mathbf{v}_{m+1}\})$?

5. Matlab assignment 2, problems #1 and #4.

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a transformation where $T(x_1, x_2, x_3) = (x_1x_3, x_2)$. Show T is NOT a linear transformation.

7. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a transformation where $T(x_1, x_2, x_3) = (x_1 + x_3, x_2)$. Find the standard matrix representation of the transformation.

8. Suppose you have 5 vectors in $\mathbb{R}^7$, and none are the zero vector, and every vector is different. You create a matrix where each column is one of those vectors. Treating this matrix as the standard matrix of a linear transformation, is the transformation sometimes, always, or never one-to-one?

9. Let

$$A = \begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 6 & -3 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 & 0 \\ 0 & 4 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 0 & -1 \\ 0 & 4 & 4 \end{bmatrix}.$$

Compute the following, if it exists. If it does not, just write DNE.

(a) CC^T

(b) $(B + A)^2$

(c) AB

(d) CB^T

(e) A^TC

10. If the matrix of a linear transformation is given by

$$\begin{bmatrix} 1 & -1 & 4 \\ -2 & 0 & 2 \\ -3 & 4 & -8 \end{bmatrix}.$$

Is the transformation one-to-one/injective? Onto/surjective? Justify.