STAT400. Sample questions for midterm 2.

1. *(a) 40% of lightbulb produced by Shining Beauty company have suboptimal performance. 500 bulbs are chosen for test. Using the normal approximation compute approximately the probability that 210 bulbs or less have suboptimal performance.*

(b) 0.4% of lightbulb produced by Light company have suboptimal performance. 500 bulbs are chosen for test. Using the Poisson approximation, compute approximately the probability that 2 bulbs or less have suboptimal performance.

2. *Let X have cumultive density function*

$$F(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \frac{x^2+x^4}{2} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x \geq 0 \end{cases}.$$

(a) Compute density of X.
(b) Compute EX and VX.
(c) Let $Y = X^3$. Compute EY and VY.

3. *Let X have normal distribution with mean 2 and standard deviation 2.*
(a) Compute 25 and 75th percentiles.
(b) Compute $P(X > 5)$.
(c) Compute $E(X^2)$.

4. *The distribution of accident location on a 100 highway has distribution with density $f(x) = \frac{2x}{10000}$. There are rescue stations at the beginning (milepost 0) and end of the highway (milepost 100). Suppose the third station is built at at milepost a. Let D be the distance from the accident location to the closest rescue station.*
(a) Compute $E(D)$ as a function of the parameter a.
(b) Find the optimal location a to minimize $E(D)$.

5. *Let the distribution of X and Y be given in the following table.*

$X\backslash Y$	1	2	3
0	.05	.10	15
1	.05	.05	.10
2	.20	.05	.25

(a) Compute the marginal distributions of X and Y.
(b) Compute $P(X = Y)$.
(c) Compute $Cov(X, Y)$.

6. *Let (X, Y) have uniform distribution on the trapezoid $0 \leq y \leq 1$, $0 \leq x \leq 1 + y$.*
(a) Compute the marginal distributions of X and Y.
(b) Compute $V(X)$.

(c) Compute $Cov(X, Y)$.

7. *Suppose 10% percent of customers of a certain store pay by American express, 20% pay by Visa, 30% pay by MasterCard, and 40% pay cash. Ten customers visit a store at a particular day.*

(a) Compute the probability that exactly 1 person uses American Express, 2 use Visa, 3 use Mastercard and 4 pay cash.

(b) Compute the probability that exactly 5 people will pay cash.

(c) Let V *be the number of customers who use Visa and* M *be the number of customers who use a Master Card. Compute* $Cov(V, M)$.

8. *Let* X *be independent,* X *have uniform distribution on* $[0, 1]$ *and* Y *have exponential distribution with parameters 1. Let* $Z = X + Y$.

(a) Compute the density of Z.

(b) Compute $Cov(X, Z)$.

(c) Compute $P(3Y > Z)$.

9. *Let* X *have Gamma distribution with parameters* $\alpha = \beta = 3$.

(a) Let $Y = 2X$. *Find the density of* Y.

(b) Let $Z = X^2$. *Find the density of* Z.

(c) Compute EZ *and* VZ.

10. *Let* $S = X_1 + X_2 + \dots X_{162}$ *where* X_j *are independent identically distributed random variables. Suppose that* X_j *have density equal to* $2x$ *if* $0 \le x \le 1$ *and equal to* 0 *otherwise.*

(a) Compute ES.

(b) Compute VS.

(c) Compute approximately $P(S > 110)$.

11. *60 scientists from 30 universities attend a conference. The conference includes a lunch in a cafetria which has 30 tables each suitable for 2 people. The people seated for lunch at random. Let* $X_j = 1$ *if the scientists from university* j *seat togather and* $X_j = 0$ *otherwise.*

(a) Compute $E(X_1)$ *and* $V(X_1)$.

(b) Compute $Cov(X_1, X_2)$.

(c) Let $X = X_1 + X_2 + \dots X_{30}$ *be the number of tables occupied by the people from the same university. Compute* EX *and* VX.

12. *Let* C *and* T *are the number of cars and trucks passing a toll plaza (measured in thousands). Suppose that* (C, T) *is jointly normal,* $EC = 5$, $ET = 6$, $VC = 1$, $VT = 2$ *and their correlation coefficient is* $\rho = \frac{1}{2}$.

(a) Compute $P(C > T)$.

(b) Let $X = 5T + 10T$ *be the total toll collected during the day. Compute* $P(X > 90)$.

(c) Compute $P(X > 90 | T = 6.5)$.