Continuous distributions.

1. *Leaks if a water pipe form a Poisson process with 4 leaks/year on avarage.*

(a) Compute the probability that during the first 6 months there will be no leaks, in the next 6 months there will be three leaks and during the second year there will be 3 leaks.

(b) Let T be the time of the first leak. Compute $P(T \le t)$.

2. *Suppose that X has probability distribution function equal to cx^2, if $x \in [0,1]$ and equal to 0 otherwise.*

(a) Find c. (b) Compute the cumulative distribution function. (c) Compute EX and VX. (d) Compute median, 25th and 75th percentiles.

3. *Let X have the cumulative distribution function $F(x) = \frac{x+x^3}{2}$ if $0 \le x \le 1$, 0 if $x \le 0$ and 1 if $x \ge 1$. Compute (a) probability density function (b) EX (c) VX.*

4. *A passanger arrives at the bus stop. His waiting time T has uniform distribution on $[0,10]$.*

(a) Compute the cumulative distribution function.

(b) Compute $P(2 \le T \le 5)$, $P(T > 3)$.

(c) Compute ET and VT.

5. *Suppose that a random variable X has proability density function $p(x) = e^{-x}$ if $x > 0$ and 0 otherwise. Find the median of X.*

6. *Let Z be standard normal random variable. Find its median, 25th and 75th percentile.*

7. *The weight of a box of appples has normal distribution with mean 30 lbs and standard deviation 2 lbs. Compute the probability that the box weights*

(a) exactly 31 pounds; (b) between 30 and 32 pounds (c) less than 29 pounds.

8. *A coin is tossed 100 times compute approximately the probability that*

(a) there are exactly 50 heads. (b) the number of heads is between 47 and 52 (inclusive).

9. *Let $Y \sim Bin(25,p)$. Approximate $P(Y \le 10)$ without and with continuity correction and compare the results with the binomial table for $p = 0.3, 0.4, 0.5$.*

10. *Let $Y \sim Bin(100, 0.06)$. Approximate $P(Y = 3)$ using Poisson and normal approximation and compare the result with the direct computation using the binomial formula.*

11. *Lifetime T of a certain component has exponetial distribution with mean 10 years.*

(a) Compute the probability that $T > 5$, $T > 10$, $T > 15$.

(b) Compute the probability that componewnt will work for 15 years or more given that it worked for 5 years.

(c) Let S be the lifetime of the component measured in decades, thus $S = T/10$. Compute the distribution of S.

12. *In problem 1 let T_2 and T_3 be the times of the second and third leak respectively. Find their probability density functions.*

13. *Let Z be the standard normal random variable and $X = Z^2$. Find the distribution of X.*

14. *A malfunctioning of a certain engine could be caused by two components. Lifetimes of those components are independent and have exponential distributions with means 2 and 3 years respectively. Find the average lifetime of the engine.*