Chapter 5 Summary.

(1) Joint distribution

$$P((X,Y)\in A)=\sum_{(x,y)\in A} p(x,y) \text{ discrete distributions,}$$

$$P((X,Y)\in A)=\iint_A p(x,y)dxdy \text{ continuous distributions.}$$

(2) Marginals

$$p_X(x)=\sum_y p(x,y),\quad p_Y(y)=\sum_x p(x,y), \text{ discrete distributions,}$$

$$p_X(x)=\int p(x,y)dy,\quad p_Y(y)=\int p(x,y)dx, \text{ continuous distributions.}$$

(3) Conditional distributions $p_{X|Y}(x|y)=\frac{p(x,y)}{p_Y(y)}$.

(4) Two dimensional uniform distribution $f(x,y)=\dfrac{1}{\text{Area}(D)}$ if $(x,y)\in D$.

$$P((X,Y)\in A)=\frac{\text{Area}(A)}{\text{Area}(D)}.$$

(5) Independent Random Variables $p_{X,Y}(x,y)=p_X(x)p_Y(y)$.

(6) Expectation

$$E(h(X,Y))=\sum\sum h(x,y)p(x,y) \text{ discrete distributions,}$$

$$E(h(X,Y))=\iint h(x,y)p(x,y)dxdy \text{ continuous distributions,}$$

(7) Properties of expectation

(a) $E(X+Y)=E(X)+E(Y)$,

(b) $E(aX+b)=aE(X)+b$,

(c) If X and Y are independent $E(XY)=E(X)E(Y)$.

(8) Covariance $\text{Cov}(X,Y)=E(XY)-(EX)(EY)$.

(9) Properties of covariance

(a) $\text{Cov}(X,X)=V(X)$,

(b)$\text{Cov}(aX+b,cY+d)=ac\text{Cov}(X,Y)$,

(c) $\text{Cov}(\sum_i a_j X_i, \sum_j b_j Y_j)=\sum_{ij} a_i b_j \text{Cov}(X_i,Y_j)$,

(d) If X and Y are independent then $\text{Cov}(X,Y)=0$.

(10) Properites of variance $V(\sum_{i=1}^n X_i)=\sum_{i=1}^n V(X_i)+2\sum_{i<j}\text{Cov}(X_i,X_j)$;

If X_i are independent then $V(X_1+X_2+\dots X_n)=V(X_1)+V(X_2)+\dots V(X_n)$.

(11) Central Limit Theorem X_i are independent identically distributed

If $S=X_1+X_2+\dots X_n$ then $S\approx N(E(S),V(S))$. Also $\bar{X}\approx N(E(\bar{X}),V(\bar{X}))$.

(12) Multivariate normal distribution.

(a) If $(X_1,X_2,\dots X_n)$ is multivariate normal then $\sum_{j=1}^n a_j X_j$ is normal. In particular marginals are normal.

(b) Conditionals of normal vectors are normal.

(c) If $(X_1,X_2,\dots X_n)$ is multivariate normal then the components X_i and X_j are independent if and only if they are uncorrelated.