Chapter 4 Summary.

(1) Density and cumulative distribution function of continuous distributions

$$F_X(x) = P(X \le x), \quad P(a \le X \le b) = \int_a^b f_X(x)dx, \quad f(x) = F'(x).$$

(2) Median and other percentiles $F_X(m) = 0.5, \quad F_X(x_k) = \dfrac{k}{100}$.

(3) Uniform distribution $f(x) = \dfrac{1}{b-a}$, $EX = \dfrac{b+a}{2}$, $VX = \dfrac{(b-a)^2}{12}$.

(4) Expected value of continuous random variable

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)dx, \quad E(h(X)) = \int_{-\infty}^{\infty} h(x) f_X(x)dx.$$

(5) Variance $V(X) = E(X^2) - (EX)^2$.

(6) Normal distribution $f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{(x-\mu)^2}{2\sigma^2}$

$$X = \mu + \sigma Z \text{ where } Z \sim N(0,1), \quad P(a \le X \le b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right).$$

(7) Binomial approximation for normal distribution $\text{Bin}(n,p) \approx N(np, np(1-p))$. If $X \sim \text{Bin}(n,p)$ then

$$P(X \le k) \approx \Phi\left(\frac{k + 0.5 - np}{\sqrt{np(1-p)}}\right).$$

(8) Exponential distribution $f(x;\lambda) = \lambda e^{-\lambda x}$. $F(x;\lambda) = 1 - e^{-\lambda x}$. $EX = \frac{1}{\lambda}$, $VX = \frac{1}{\lambda^2}$.

(9) Gamma distribution $f(x;\alpha,\beta) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$. $EX = \alpha\beta$, $VX = \alpha\beta^2$.