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Computational Physics: Solutions to problem Set 3

1 Euler integration I – damped harmonic oscillator

a)

b) see solution program

c) The harmonic oscillator

$$\partial_t^2 y(t) + \gamma \partial_t y(t) + y(t) = 0 \tag{1}$$

is a homgeneous second order linear ODE, therefore it has two linearly independent solutions. We make the ansatz

$$y(t) = \exp(\lambda t), \tag{2}$$

which leads to the equation:

$$(\lambda^2 + \gamma\lambda + 1)\exp(\lambda t) = 0. \tag{3}$$

This has solutions

$$\lambda_{1/2} = \frac{1}{2}\Big(-\gamma \pm \sqrt{\gamma^2 - 4}\Big). \tag{4}$$

This means, the two linearly independent solutions are usually:

$$y_{1/2}(t) = \exp(\lambda_{1/2} t). \tag{5}$$

The only exception is the critically damped oscillator, where we find only one value for λ. For the critically damped case, the two linearly independent solutions are:

$$y_1^{\text{crit}}(t) = \exp(\lambda^{\text{crit}} t), \qquad y_2^{\text{crit}}(t) = t\exp(\lambda^{\text{crit}} t). \tag{6}$$

For the critically damped case, we furthermore find $\lambda^{\text{crit}} = -1$.

We can now solve the initial value problem, first for the non-critical case:

$$y(t) = a\exp(\lambda_1 t) + b\exp(\lambda_2 t), \tag{7}$$

with conditions

$$y(0) = 1 \qquad \Rightarrow \quad b = 1 - a, \tag{8}$$

$$y'(0) = 0 \qquad \Rightarrow \quad \lambda_1 a = -\lambda_2 b. \tag{9}$$

$$\Rightarrow \quad a = -\frac{\lambda_2}{\lambda_1 + \lambda_2}. \tag{10}$$

Now we find the solution for the critically damped case:

$$y(t) = (a + bt)\exp(-t). \tag{11}$$

$$y(0) = 1 \qquad \Rightarrow \quad a = 1, \tag{12}$$

$$y'(0) = 0 \qquad \Rightarrow \quad -a + b = 0, \tag{13}$$

$$\Rightarrow \quad a = b = 1. \tag{14}$$

d) see solution program