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Computational Physics: Problem Set 5

Discussion: 15/03/2023

1 Convergence analysis of FDTD

Consider the 1d wave equation

$$(\partial_t^2 - \partial_x^2)p(x,t) = 0.$$

a) Take an FDTD implementation (e.g. the one uploaded after the lecture) and modify it such that the spatial parameter h can be changed. Express the time step dt as a multiple of h.

Note: This requires h to appear somewhere in the update equation.

b) Discretize the interval $[-0.5, 0.5]$ with N_x points and initialize with the function $\cos(\pi x)$. Find the analytical time-evolution of this function. Let the program run to $t_{\max} = 10$, plot how the error $||p(t) - p_{\text{ref}}(t)||$ evolves over time. Plot how the norm $||p(t)||$ evolves. Repeat for different time steps dt while keeping $t_{\max}$ constant (same number of oscillations in every program run). What can you say about conservation of energy, stability and convergence with respcet to dt?

c) Fix the time step with respect to h (e.g. $dt = 0.9h$) and run the program for a number of spatial discretizations. What can you say about convergence with respect to h?

d) What is the main source of the observed error?

Students are encouraged to solve these problems in groups. They should be able to informally present their solution in the problem class on 15/03/2023 at the white board (no PowerPoint slides to be prepared).