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Computational Physics: Problem Set 6

Discussion: 22/03/2023

$\boxed{1}$ <u>2d finite differences</u>

Consider the 2d wave equation

$$(\partial_t^2 - \Delta)u(x, y; t) = 0.$$

This describes for example the dynamics of a drum skin, or a thin plate that is either under tension or slightly curved.

a) Discretize a rectangular 2d domain by representing the data as a matrix $u(x, y) \approx u_{ij}$, where $x = hi$ and $y = hj$. Using the usual finite-difference approximation

$$u''(x) \approx h^{-2}[u(x + h) - 2u(x) + u(x - h)],$$

for the second derivative and by noticing $\Delta = \partial_x^2 + \partial_y^2$, derive a sencil for the Laplacian.

b) Implement the stencil

$$\Delta u(x, y) \approx h^{-2}[u(x + h, y) + u(x - h, y) + u(x, y + h) + u(x, y - h) - 4u(x, y)].$$

Using the same approach for time-stepping as in the first class-room problem in the 6th lecture (the "naive" finite-difference method), implement a 2d finite difference code on the square domain $-0.5 \leq x \leq 0.5$ and $-0.5 \leq y \leq 0.5$ (use e.g. 50 discretization points per direction). Test with the initial condition

$$u(x, y; 0) = \cos(\pi x) \cos(\pi y).$$

Note: This is not the only viable stencil for the Laplacian.

c) Try other initial conditions like a 2d Gaussian

$$u(x, y; 0) = \exp[-50(x^2 + y^2)].$$

d) After every time step, set the function u explicitly to be zero outside a circle with radius 0.5 to achieve a different shape for the "drum". This is an opportunity to play around with weird shapes, for example the shape of a guitar bottom.

Students are encouraged to solve these problems in groups. They should be able to informally present their solution in the problem class on 22/03/2023 at the white board (no PowerPoint slides to be prepared).