Machine Learning

Learning machine learning

Kazalo

1 Vektorji, Matrike in Norme

1.1 Vektorski prostor

Definicija: Vektorski prostor je četverica $(V, O, +, \cdot)$, kjer je:
$V \ldots$ množica vektorjev
$O \ldots$ obseg skalarjev
$+ \ldots$ dvomestna operacija $+ : V \times V \to V$
$\cdot \ldots$ produkt s skalarjem $\cdot : O \times V \to V$

Kjer:

$$\forall x, y, z \quad \begin{array}{l} x + (y + z) = (x + y) + z \\ x + y = y + x \end{array}$$

$\exists 0 \in V : \; x + 0 = x$
$\exists 1 \in O : \; 1 \cdot x = x$
$\forall x \; \exists y : \; x + y = 0$
$\forall x \in V \; \forall \mu, \lambda \in O : \; \lambda \cdot (\mu \cdot x) = (\lambda \cdot \mu) \cdot x$
$\forall x, y \; \forall \lambda : \; \lambda \cdot (x + y) = \lambda \cdot x + \lambda \cdot y$
$\forall x \; \forall \lambda, \mu : \; (\lambda + \mu) \cdot x = \lambda \cdot x + \mu \cdot x$

V je vektorski prostor nad O.

Primeri:
$(\mathbb{R}, \mathbb{R}, +, \cdot)$
$(\mathbb{R}^3, \mathbb{R}, +, \cdot)$
$(\mathbb{R}^{10 \times 10}, \mathbb{R}, +, \cdot)$
$(P_{\leq 7}, \mathbb{C}, +, \cdot)$
$(\mathbb{C}[x], \mathbb{C}, \cdot, \cdot)$
$(\mathbb{R}[x], \mathbb{C}, +, \cdot)//$ $\quad (5x + 4)i \to 5xi + 4i \notin \mathbb{R}[x]$
$(\mathbb{R} \to \mathbb{R}, \mathbb{R}, +, \cdot)$

1.2 Skalarni produkt

Definicija: Skalarni produkt nad realnim vektorskim prostorom $(V, \mathbb{R}, +, \cdot)$ je preslikava $:< \cdot, \cdot >: V \times V \to \mathbb{R}$ in velja:
$\forall x, y : < x, y >=< y, x >$
$\forall x, y, z : < x, y + z >=< x, y > + < x, z >$
$\forall x : < x, x > \geq 0$
$\forall x : < x, x >= 0 \Rightarrow x = 0$
$< x, y >= 0 \Rightarrow$ pravokotna

Primer: najdi skalarni produkt nad $\mathbb{R}^3$

$< (a_1, a_2, a_3), (b_1, b_2, b_3) >= a_1b_1 + a_2b_2 + a_3b_3$

$< v, u >= \sum_{i=1}^{n} v_i u_i$

1.3 Transponiranje

$(A^T)_{ij} = a_{ji}$

1.4 Sled

def: $sl(A) = \sum a_{ii}$

$< A, B >= sl(A^T B)$

$sl(A^T) = sl(A)$

$sl(AB) = sl(BA)$

Dokaz:

$$sl(AB) = \sum_i (AB)_{ii} = \sum_i \sum_k a_{ik} b_{ki} = \sum_k \sum_i b_{ki} a_{ik} = \sum_k (BA)_{kk} = sl(BA)$$

1.5 Norme

Definicija: Norma na realnem prostoru V je preslikava $|| \cdot || : \ V \to \mathbb{R}$ za katero velja:

$\forall x \in V : \ ||x|| \geq 0$

$\forall x \in V : \ ||x|| = 0 \Rightarrow x = 0$

$||x + y|| \leq ||x|| + ||y||$

$\forall \lambda \in V : \ ||\lambda \cdot x|| = |\lambda| \cdot ||x||$

Primeri:

$||x||_2 = \sqrt{< x, x >} = \sqrt{\sum x_i^2}$

$||x||_1 = \sum |x_i|$

$||x||_p = \sqrt[p]{\sum x_i^p}$

$||x||_\infty = \lim_{p \to \infty} ||x||_p = \max\{|x_i|\}$

Dokaz:

$$\lim_{p \to \infty} \sqrt[p]{x_1^p + \cdots + x_n^p} = \lim_{p \to \infty} \sqrt[p]{\frac{x_1^p + \cdots + x_n^p}{\max\{|x_i|\}^p}} \cdot \max\{|x_i|\} =$$

$$= \max\{|x_i|\} \cdot \lim_{p \to \infty} \sqrt[p]{(\frac{x_1}{m})^p + \cdots + (\frac{x_n}{m})^p} = \max\{|x_i|\}$$