

Development of h - p adjoint-based error estimation for LES of reactive flows

Christopher Ngigi
Ph.D. Candidate

Supervisor: Prof. C. P. T. Groth

Doctoral Examination Committee
Meeting I
University of Toronto, Institute for Aerospace Studies

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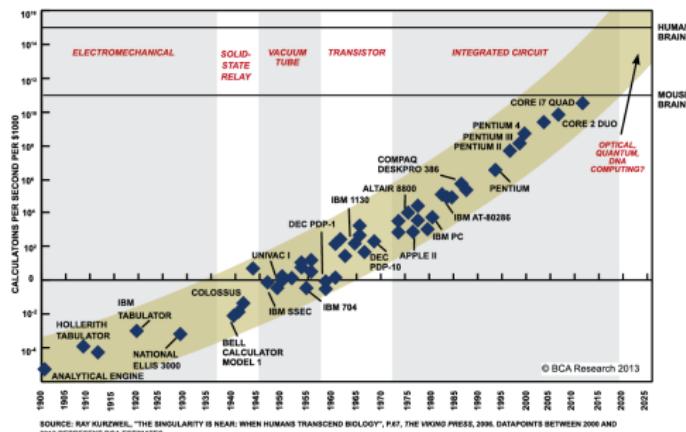
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- (a) Computational Fluid Dynamics (CFD): help reduce time & cost of prototype design for engineering systems
- (b) Corresponding experiments: can be expensive (time, resources) to conduct
- (c) CFD: developed to capture complex phenomena to varying extents of accuracy
- (d) Computing power \approx doubles per 2 years. Costs less to do more?



(a) Moore's Law over the years [BCA Blog] [1]]



Turbulent combustion - Example flame

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[Köhler 2006] [2, 3]

Lifted turbulent Ethylene (C_2H_4) jet flame issuing into a concentric co-flow of air. Zone between flame-base and nozzle may have partial premixing.

- Dimensions: nozzle diameter = 2.0 mm; co-flow air annulus diameter = 140 mm
- Exit fuel Reynolds number: 10×10^3



Turbulent combustion: practical reactive flows almost always involve turbulence.

Simulation techniques:

- DNS resolves all the scales
- LES models sub-filter scales (SFS) while resolving larger scales
- RANS models all turbulent scales

Large eddy simulation (LES): higher accuracy than Reynolds averaged Navier Stokes (RANS) → lower cost (time, resources) than direct numerical simulation (DNS).



Turbulent combustion - LES & DNS comparison

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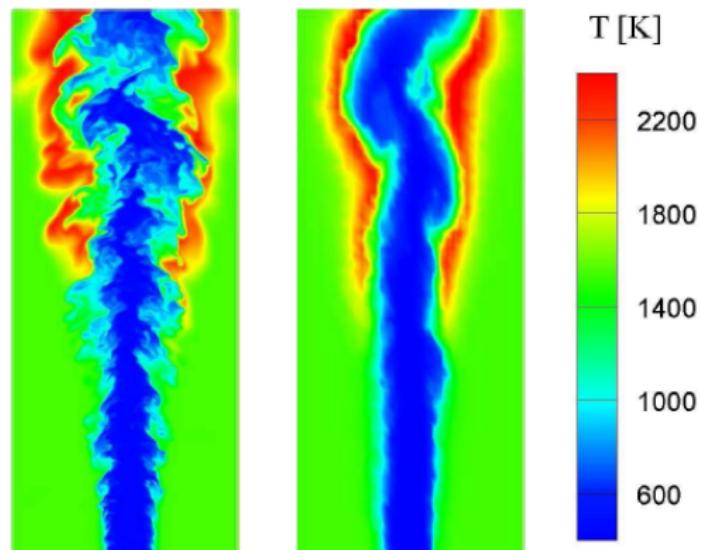
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Comparison of DNS and LES by Yang, Pope and Chen [2013][4]: turbulent Ethylene jet flame in co-flow



(a) temperature in x - y plane: DNS (L), LES/PDF (R)



Turbulent combustion - LES & DNS comparison cont'd

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Numerical setup of Yang, Pope and Chen [2013]:

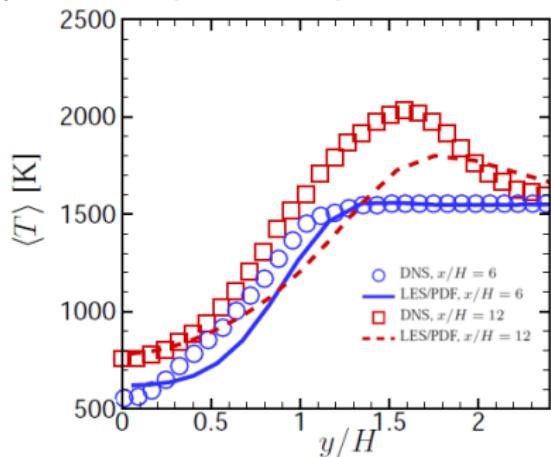
$H = 2 \text{ mm}$ is the jet width, computational domain = cuboid

DNS:

- (1) Grid points = 1.3×10^9 .
- (2) Computational cost $\approx 14 \times 10^6$ CPU hours. (10^6 hrs = 114 years)

LES:

- (1) Grid points $\approx 83 \times 10^3$.
- (2) Cost not specified - expected to be several orders of magnitude *lower*.



Mean temperature results:

- (a) agreement between LES and DNS at $x = 6H$.
- (b) LES values lower than DNS for $x = 12H$.
- (c) Anticipate improvement for increased LES mesh resolution.

(a) Mean temperature: DNS and LES



Overview of error in CFD

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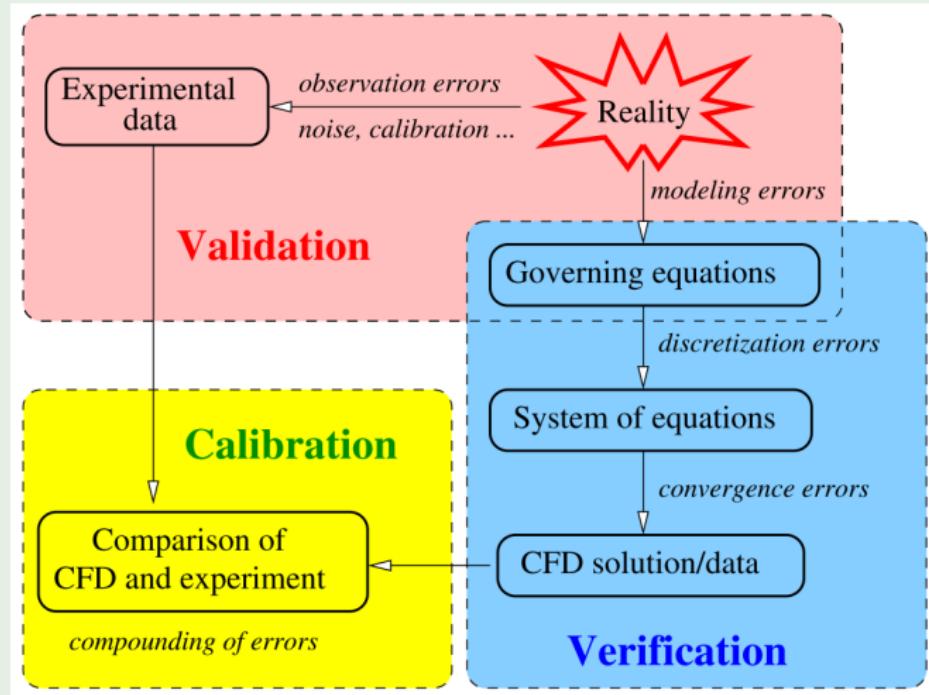
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Likely sources of error



(a) Some sources of error [Fidkowski 2012]



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We can broadly classify the two key sources of error in CFD as follows:

1. Numerical error:

- (a) Solution error - between exact value and the CFD-obtained value
- (b) Truncation error - actual governing equations PDEs and the discretized form of the numerical scheme
- (c) Convergence error - nature of the iterative technique used

2. Modeling error:

- (a) Sub-filter scale turbulence model: inappropriate model selected
- (b) Combustion and chemistry model
- (c) Aliasing errors - decomposed nonlinear terms in FANS = feedback of frequencies beyond filter bandwidth, = 'fake' stresses
- (d) Commutation errors - exist between filtering and differential operations



Techniques for controlling numerical error

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We can control numerical error in CFD using a combination of:

1. Mesh refinement
2. High-order discretizations (space and time)



Common adaptive mesh refinement (AMR) strategies

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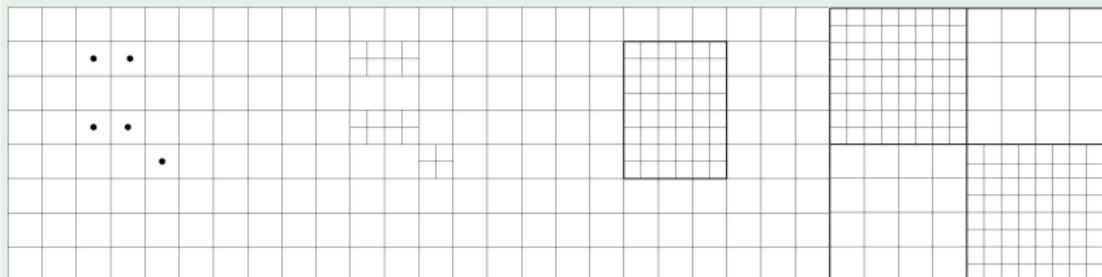
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Key characteristics

[Northrup 2013]



(a) flagged cells

(b) Cell-based refinement

(c) Patch-based refinement (d) Block-based refinement

- Cell based: [Powell et al, 1993] [Berger and Leveque, 1989] [Aftosmis et al, 1998]
Cells refined individually, hierarchy stored in a (very dense) tree
- Patch based: [Berger and Collela, 1989]
Cells are organized into collections of rectangular patches
- Block based: [Groth and co-workers, 1999, 2005, 2011, 2013, 2015] [Berger, 1994]
Entire blocks get refined, much lighter tree structure, efficient domain decomposition

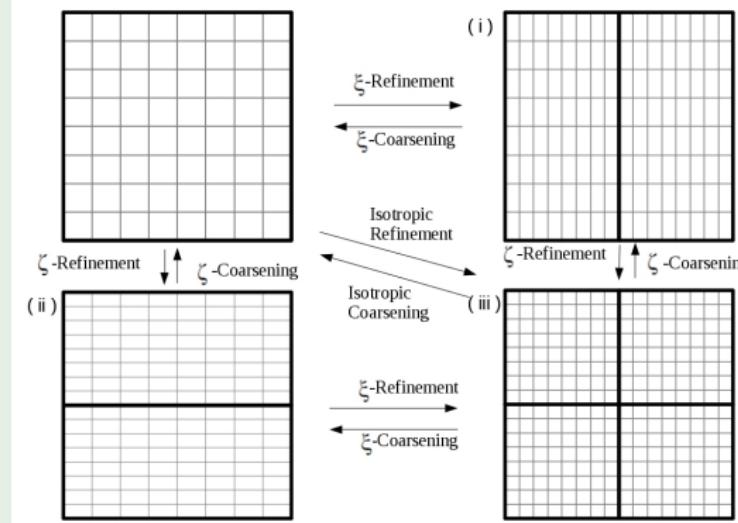
AMR: isotropic and anisotropic

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Key characteristics of AMR

[Groth et al 1999, 2005, 2011, 2013, 2015]

- localized refinement, large variation of scales, easily automatable
- Benefits of AMR: overall large computational cell count savings



(a) Cell refinement strategies on a reference uniform mesh: [Zhang 2011]



Benefits: anisotropic vs isotropic AMR

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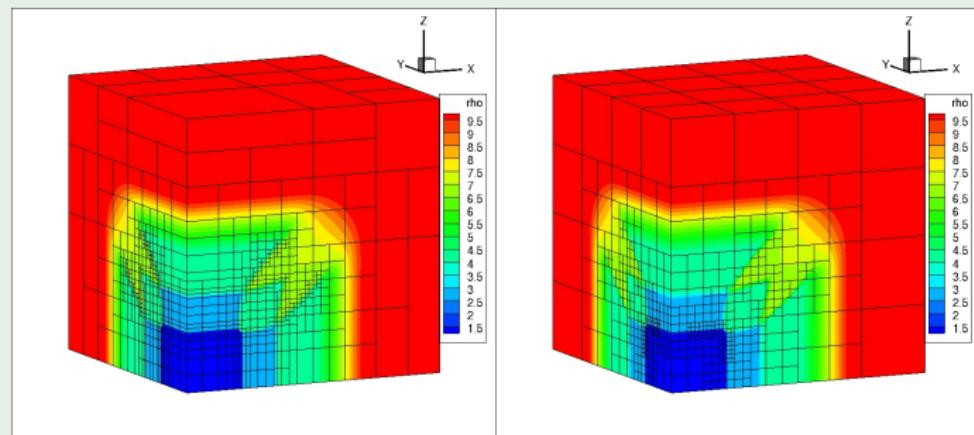
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(a) With Anisotropic AMR. 5522 (8x8x8) blocks (b) With isotropic AMR. 7036 (8x8x8) blocks
[Freret 2015]

- For the above shockcube cases, cell count ratio of anisotropic : isotropic ≈ 0.72 thus reflecting approximately 28% reduction in number of cells
- Williamschen [2013] previously obtained savings of up to 85% reduction in cell count for 3D meshes for inviscid flows



Block-based AMR

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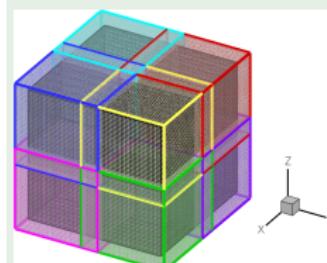
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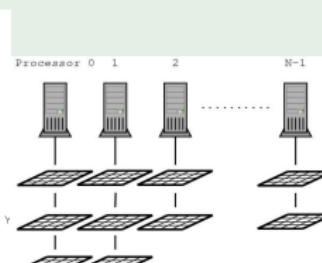
Key characteristics

Block-based AMR: [Groth and co-workers 1999, 2005, 2006, 2010, 2011, 2012, 2013]

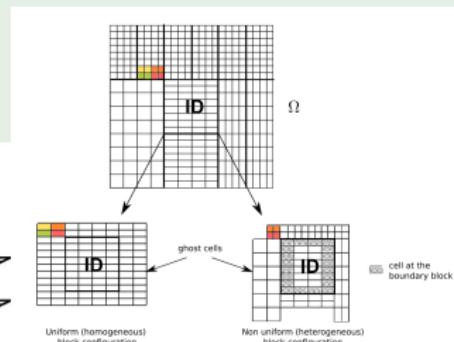
- Entire block gets refined, along with all its cells.
- This approach is much cheaper than individual cell refinement, having a lighter tree structure.
- How the block-based technique works; ghost cells for intercommunication
- Parallelizable, low memory and storage requirements
- New non-uniform approach [Freret 2015]



(a) Ghost cells on 8 blocks
[Rashad 2009]



(b) Parallelization [Northrup
2013]



(c) Non-uniform approach [Freret 2015]



High order FVM and CENO

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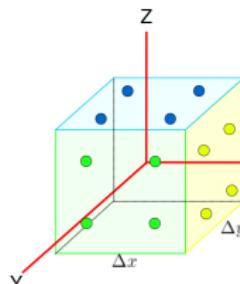
Integral Form of the Governing Equations

$$\frac{d\bar{\mathbf{U}}}{dt} = -\frac{1}{V} \iint_A (\vec{\mathcal{F}}^I - \vec{\mathcal{F}}^V) \cdot \hat{n} dA + \bar{\mathbf{S}}$$

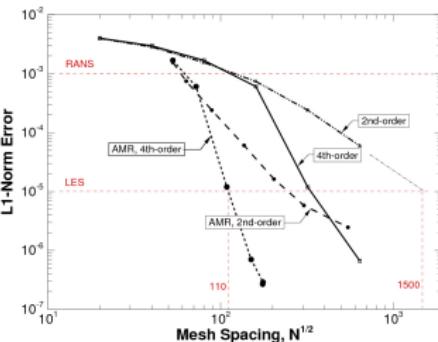
Using a two-dimensional Gauss-Legendre quadrature integration rule:

$$\frac{d\bar{\mathbf{U}}_{ijk}}{dt} = -\frac{1}{V_{ijk}} \sum_{l=1}^{N_f} \sum_{m=1}^{N_{GF}} \left(\omega \left(\vec{\mathcal{F}}^I - \vec{\mathcal{F}}^V \right) \cdot \hat{n} A \right)_{ijk,l,m} + \sum_{n=1}^{N_{GV}} (\omega_n \mathbf{S})_{i,j,k,n} = \bar{\mathbf{R}}_{ijk}(\bar{\mathbf{U}})$$

1. High order: $> 2^{nd}$ order. Error (Governing equations : discretized formulations) \downarrow , ($N_G, N_F \uparrow$, more ω)
2. Quadrature rules \rightarrow reference elements \rightarrow computational elements via mapping functions



(a) Example of quadrature points on the cell faces



(b) L1 error norm, unsteady advection equation. $N =$ total mesh count



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3. Discretized scheme expressed in matrix-vector form.
4. Evaluate the inviscid and viscous fluxes: apply weights to quadrature points
5. Evaluate source vector, which adds effects of turbulence and chemistry in reacting flows
6. Apply appropriate time-marching, e.g. RK4 suitable to high order methods.
7. Other groups researching high order methods in LES: Ihme (Stanford) and Poinsot (CERFACS)

Central essentially non-oscillatory (CENO) scheme implementation:
[Ivan and Groth 2011, 2012]

- Reconstruction: FVM technique is cell centered. May need state values on faces (flux evaluations), quadrature points (high-order)
- CENO uses fixed stencil for reconstruction
- CENO eliminates oscillations typical of shocks & discontinuities
- smoothness indicators used to select between high order (p) or piecewise linear reconstruction



Large Eddy Simulation (LES)

[Piomelli 1999][Ghosal and Moin 1999]

1. LES utilizes a spatial filtering of a given width, $\bar{\Delta}$.
2. Any scales smaller than $\bar{\Delta}$ are modeled \rightarrow sub-filter scales (SFS)
[Smagorinsky 1963][Germano 1991][Piomelli 1991][Ghosal and Moin 1999][Lilly 1992]
3. Scales larger than $\bar{\Delta}$ are fully resolved.
4. Need appropriate balance for accuracy, \rightarrow modeling error.
5. Types of filters broadly categorized into:
 - (a) **Implicit filtering** [Aspden et al 2008]
 - Filter width not fixed: inherently related to grid resolution
 - Difficult to compare results of adapted/refined meshes and control aliasing and commutation errors
 - (b) **Explicit filtering** [Vasilyev et al, 1998] [Deconinck, 2008]
 - Define $\bar{\Delta}$ to fixed for the entire mesh: easily comparable for different meshes
 - Order of truncation errors can be controllable to same order of commutation errors
 - Allows control of aliasing errors



Foundation for error estimation

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Two general types of error estimation:

1. *a priori* error estimators: predict long-term behavior of the errors.
2. *a posteriori* error estimators: use simulation results to derive estimates of solution errors and guide adaptive schemes.

Where calculated error estimates are higher than set threshold:

- Mesh can be locally refined (*h*-refinement) while coarsening the less critical areas to save on computational cost.
- Degree of polynomial representation can be raised (*p*-refinement)

Two main *a posteriori* approaches are:

- (a) gradient-based: [Bibb et al, 2006] [Giles and Pierce, 2000]
- (b) adjoint-based: [Giles and Pierce, 2000][Venditti and Darmofal - 2000,2002][Fidkowski and Darmofal, 2011]



Gradient/physics-based refinement

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Where rates of change of (physical) solution variables are highest, mesh refinement can be applied, or the scheme order can be increased.

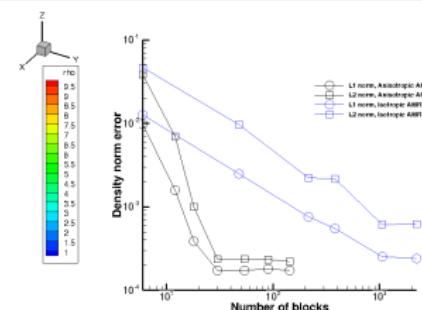
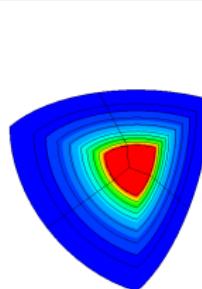
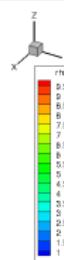
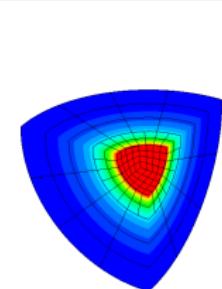
- (a) Thus have enough cells to capture solution changes → represent them as *smoothly* as possible.
- (b) Re-iterate solution to establish new gradients. Changes are again made as necessary.
- (c) Error can be compared to a higher h -refinement solution.
- (d) This is the present utility in the anisotropic and isotropic AMR functionality of the CFFC code used by the CFD and Propulsion group.
- (e) Main disadvantages of the gradient based approach is the lack of continual reduction of solution error with continued h -refinement - see next slide



Limitation of gradient-based mesh refinement

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Physics as criteria for mesh coarsening/refinement - sphere with a bow shock example



(a) With isotropic AMR

(b) With anisotropic AMR

(c) Error in the density norm

[Freret, 2015] and [Williamschen, 2013]

- The graph reveals the asymptotic behavior of the convergence, yet for increased number of cells, there should be continual reduction in the density error norm



Solution adjoint

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- (a) Make error estimation more relevant for engineering applications
- (b) Assess error in predicting an *integral quantity representation* of an engineering output (the functional): e.g. average pressure on a wall.
- (c) Adjoint technique is a sensitivity analysis. Measures rates of change of a design functional to a given change in the input.



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Two main formulations

[Giles and Pierce: 1997,2001][Jameson, 2001][Venditti and Darmofal, 2000, 2002] [Becker and Rannacher: 2001,2003]:

1. *Continuous:*

- Objective function formed to enforce the flow conditions (i.e. primal nonlinear PDEs).
- Applying linear perturbations to primal flow variables
- Then obtains analytical adjoint equations. Apply relevant b.c.s, and discretize

2. *Discrete:*

- Apply linear perturbations to nonlinear discrete residual equations from primal problem
- If adjoint consistent (discrete adjoint = continuous adjoint), no need for b.c. specification → automatically incorporated via the primal residual.
- thus obtain a linear system of equations - only need linear sensitivities of the functional and the Jacobian matrix associated with the primal residual.



Scope & proposed methodology within existing framework

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1. Reducing numerical error: using high order CENO and adjoint based error estimation : $\mathcal{O}(h^p) \rightarrow h$ and p adaptation
2. Combustion modeling:
 - PCM-FPI: allowing detailed chemical kinetics via tabulation of precomputed laminar premixed flames [H. Perez 2011]
3. Favre-filtered Navier Stokes governing equations
4. Large Eddy Simulation:
 - Explicit Filtering [Deconinck 2008]
 - Sub-filter scale (SFS) modeling [H-Perez 2011]
5. High-order finite volume methods: CENO technique - benefits of higher accuracy on a coarse mesh [Groth and Ivan 2013][Ivan 2010][Rashad 2009]
6. AMR
 - Block-based AMR: speed and parallelization [Groth et al 1999]
 - Anisotropic vs Isotropic: how cell count (computational cost) can be reduced [Zhang 2011][Williamschen 2013][Freret 2015]
 - Now the non-uniform vs the uniform block modification [Freret 2015]
7. Develop a framework for the adjoint based error estimation with h - p AMR and CENO



Discrete adjoint

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Discrete Adjoint

$$\left(\frac{\partial \mathbf{R}}{\partial \mathbf{U}} \right)^T \boldsymbol{\Psi} = - \left(\frac{\partial \mathbf{J}}{\partial \mathbf{U}} \right)^T$$

Linear system of equations in the form:

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

where:

- \mathbf{J} = the functional
- \mathbf{R} = the residual
- $\boldsymbol{\Psi}$ = the adjoint vector

Methods to evaluate the matrix $\frac{\partial \mathbf{R}}{\partial \mathbf{U}}$ for the discrete adjoint:

1. Differentiation by hand - using the inbuilt functions within CFFC code [Northrup, 2013]
2. Finite differencing - perturbing the state \mathbf{U} to evaluate \mathbf{R}
3. Automated differentiation - tools that evaluate the differential [Bischof et al: 1992, 1996, 2008]
4. Complex step [Martins, Alonso and Sturdza: 2003]



Error estimation indicators

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A method to reduce discretization errors based on the mesh resolution. [Venditti and Darmofal 2000][Fidkowski and Darmofal 2011]

- (a) Consider 2 levels of mesh resolution: coarse (H) and fine (h). Calculate the state (\mathbf{U}_H) and functional ($\mathbf{J}_H(\mathbf{U}_H)$) on coarse space. Residual, $\mathbf{R}_H(\mathbf{U}_H) = 0$
- (b) To evaluate functional on the fine space, $\mathbf{J}_h(\mathbf{U}_h)$ is expensive. Use prolongation operator ($\mathbf{U}_h^H = I_h^H \mathbf{U}_H$) to inject coarse space state onto fine space state.
- (c) Output error, $\delta\mathbf{J} \equiv \mathbf{J}_H(\mathbf{U}_H) - \mathbf{J}_h(\mathbf{U}_h) \neq 0$
- (d) Expect new residual, $\mathbf{R}_h(\mathbf{U}_h^H) \neq 0$
- (e) Using the definition of the fine space adjoint, the error estimate =
$$\delta\mathbf{J} \approx \mathbf{J}_h(\mathbf{U}_h^H) - \mathbf{J}_h(\mathbf{U}_h) = \boldsymbol{\Psi}_h^T \delta\mathbf{R}_h = -\boldsymbol{\Psi}_h^T \mathbf{R}_h(\mathbf{U}_h^H)$$
- (f) Error estimate is the value of $\delta\mathbf{J}$, and does not need evaluation of \mathbf{U}_h , primal solution on the fine space. Can use this error estimate as a flag for refinement, given some threshold value



Adjoint as basis of mesh refinement: $\mathcal{O}(h)$

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Extensive literature from groups researching **adjoint with AMR**

1. Becker and Rannacher [2001] - An Optimal Control Approach to a Posteriori Error Estimation in Finite Element Methods
2. Fidkowski and Darmofal [2011] - Review of Output-Based Error Estimation and Mesh Adaptation in Computational Fluid Dynamics
3. Hartmann [2006] - Error Estimation and Adjoint-based Adaptation in Aerodynamics
4. Nemeć and Aftosmis [2007] - Adjoint Error Estimation and Adaptive Refinement for Embedded-Boundary Cartesian Meshes
5. Nemeć, Aftosmis, and Wintzer [2008] - Adjoint-Based Adaptive Mesh Refinement for Complex Geometries
6. Hartmann, Held and Leicht [2010] - Adjoint-based error estimation and adaptive mesh refinement for the RANS and k- turbulence model equations
7. Woopen, May and Schütz [2013] - Adjoint-Based Error Estimation and Mesh Adaptation for Hybridized Discontinuous Galerkin Methods
8. Li, Allaneau and Jameson [2011] - Continuous Adjoint Approach for Adaptive Mesh Refinement
9. Diskin and Yamaelev [2011] - Grid Adaptation Using Adjoint-Based Error Minimization



Extension to order increment $\mathcal{O}(h^p)$

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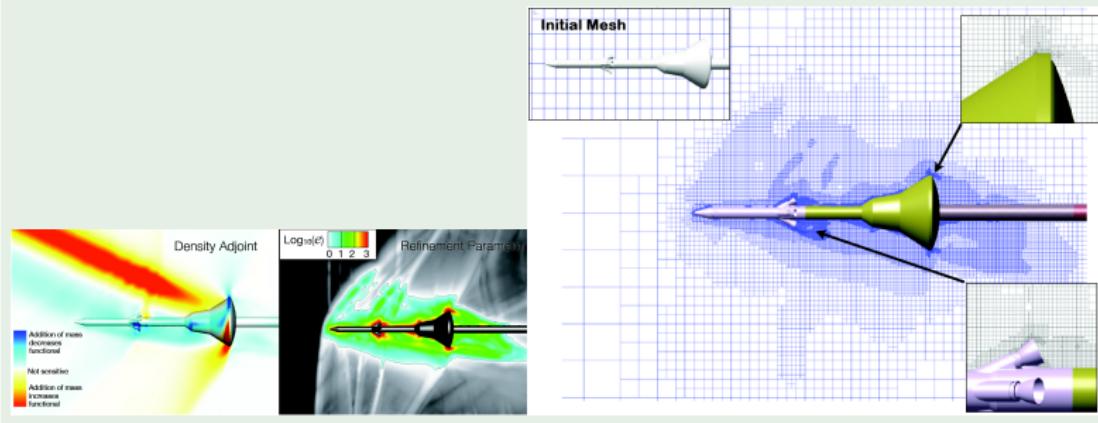
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Launch abort vehicle example, [Nemec et al, 2008]

Functional as a linear combination of normal and axial forces. $M = 1.1$; $\alpha = -25$ deg



(a) Adjoint of density with accompanying error estimate

(b) Final obtained mesh. Initial was 3.7×10^3 cells; final 2×10^6 cells

For p increment, can use error estimates to locally run a higher discretization of the numerical scheme on flagged block.



Poisson problem in 2 and 3 space dimensions

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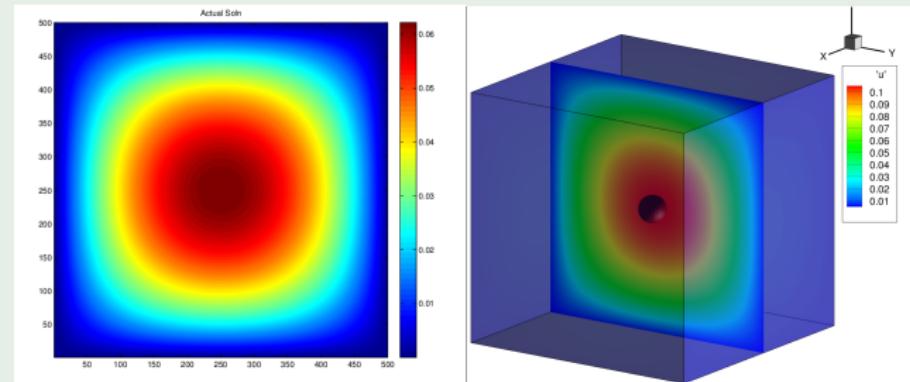
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Creating and solving linear systems in parallel implementation - trilinos and MPI



(a) 2D case on $N^2 = 500^2 = 250,000$ grid points (b) 3D case on $N^3 = 100^3 = 1,000,000$ grid points

Solution contours for Poisson problem

In 2D: $D = [0, 1]^2$, $f(x, y) = 2(x(1-x) + y(1-y))$ is the source term and $u(x, y)$ is the solution to be computed.

$$\text{Using a } 2^{\text{nd}} \text{ order centered finite difference scheme} = -\frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4 * u_{ij}}{h^2} = f_{ij}$$

In 3D: $D = [0, 1]^3$, $f(x, y, z) = 3(x(1-x) + y(1-y) + z(1-z))$ is the source term and $u(x, y, z)$ is the solution to be computed.

Using a 2^{nd} order centered finite difference scheme:

$$-\frac{u_{i+1,j,k-1} + u_{i-1,j,k-1} + u_{i,j+1,k-1} + u_{i,j-1,k-1} + u_{i+1,j,k+1} + u_{i-1,j,k+1} + u_{i,j+1,k+1} + u_{i,j-1,k+1} - 6u_{ijk}}{h^2} = f_{ijk}$$



Performing an LES simulation on SciNET

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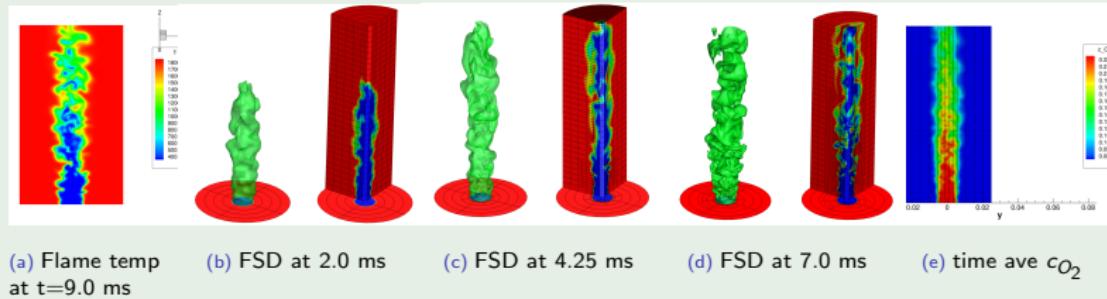
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CFFC code familiarization : LES test case

- on parallel clusters - SciNET. Job scheduling and post-processing results (tecplot)

Turbulent premixed CH_4 flame, $\phi = 0.7$.



(a) Flame temp at $t=9.0$ ms (b) FSD at 2.0 ms (c) FSD at 4.25 ms (d) FSD at 7.0 ms (e) time ave c_{O_2}

*FSD, flame surface density is used to model the flame structure, particularly for turbulent premixed flames.

Computational costs:

- (a) and (e): 800 procs, 3200 (8x8x8) blocks, 1.64×10^6 cells, no refinement, 125×10^3 CPU hrs
- (b) 800 (8x8x8) blocks, 410,000 cells, no refinement
- (c) 5595 (8x8x8) blocks, 2.8 million cells, 3 levels of mesh refinement
- (d) 18531 (8x8x8) blocks, 9.5 million cells, 3 levels of mesh refinement



Solution of adjoint problem for the Euler equations

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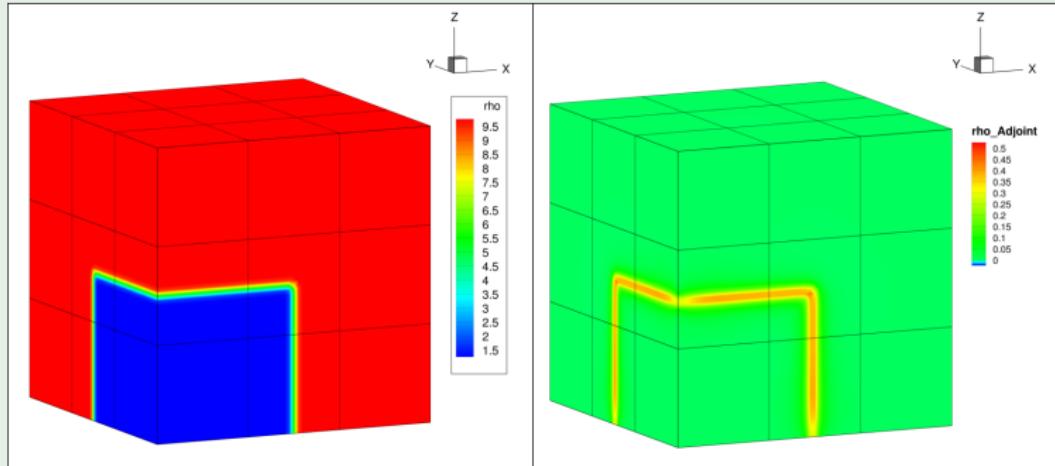
Poisson

LES flame

Adjoint runs

Timeline

Preliminary work with the discrete adjoint - shockcube problem. 27 (20x20x20) blocks



- Adjoint code was written to evaluate Ψ for multiple AMR blocks and multiprocessors
- Shock cube problem: initial conditions $\rightarrow \frac{\rho_R}{\rho_L} = 8$, $\frac{P_R}{P_L} = 10$.
- Selected as functional the average pressure in the shockcube.



Timeline: September 2014 - March 2015

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| Work done to date | |
|---|------------------------|
| Task | Completion Date |
| Literature Review | September-October 2014 |
| Trelis Meshing Software | November 2014 |
| CFFC Group Code Flux Jacobian Analysis | December 2014 |
| Trilinos Package solution for Poisson Problem in serial and parallel configurations | December 2014 |
| Running a current-state LES case of a Turbulent Premixed Methane Flame using PCM-FPI to get a threshold estimation of solution run time | January 2015 |
| Implementing the approximate Adjoint Derivative to the Flux Jacobians testing on Euler Equations | March 2015 |



Future work

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| Projected milestones | |
|--|-----------------|
| Task | Completion Date |
| Extension to Mesh adaptation | May 2015 |
| Application of Adjoint Problem to Navier Stokes | June 2015 |
| Explicit Filters for High Order FVM implementation | October 2015 |
| Coupling of High Order method with Adjoint-based AMR | December 2015 |
| CFD simulation of Cold Flow | January 2016 |
| CFD simulation of Laminar Non-Premixed Flame | February 2016 |
| CFD simulation of Laminar Premixed Flame | February 2016 |
| CFD simulation of Turbulent Non-Premixed Flame | November 2016 |
| CFD simulation of Turbulent Premixed Flame | March 2017 |
| Thesis write-up | September 2017 |



Questions?

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Thank You For Your Attention!



References

References

Backup

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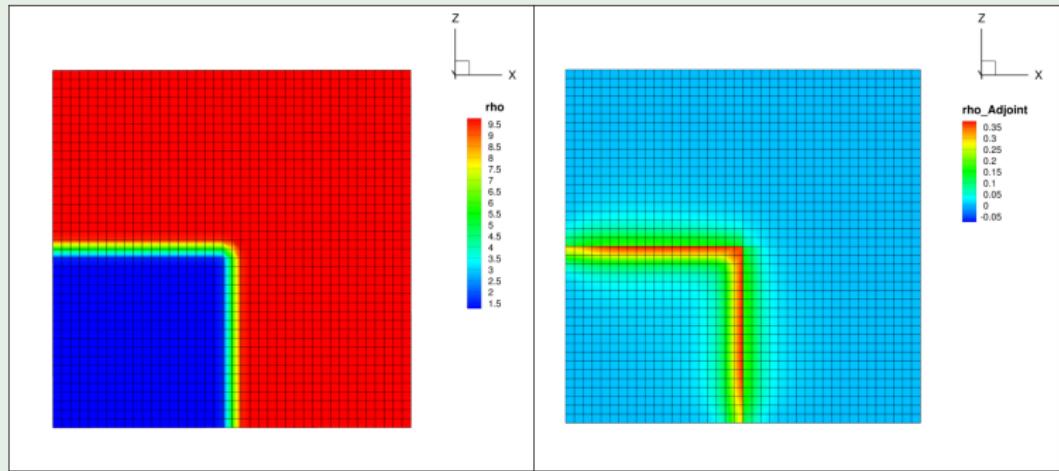
Work on the adjoint: Backup slide

References

Backup

Preliminary work with the discrete adjoint - shockcube problem. 8 (20x20x20) blocks

CFFC = Computational Framework for Fluids and Combustion



(a) ρ at $t = 0$

(b) Ψ_ρ at $t = 0$

- Initial conditions: $\frac{\rho_L}{\rho_R} = 8$, $\frac{P_L}{P_R} = 10$.
- how the code was modified - multiblock and multiproc for uniform blocks
- Selected as functional the average pressure in the shockcube.



Turbulent combustion - LES & DNS comparison cont'd

References

Backup

Numerical setup of Yang, Pope and Chen [2013]:

Boundary conditions (BCs) are inflow/outflow in x and y , while periodic in z .

$H = 2$ mm is the jet width.

Computational domain = cuboid

Streamwise x -, transverse y -, and spanwise z -directions.

- DNS
 - (1) Grid points = 1.3×10^9 .
 - (2) Computational cost $\approx 14 \times 10^6$ CPU hours.
 - (3) $L_x \times L_y \times L_z = 15H \times 20H \times 3H$
- LES
 - (1) Grid points $\approx 8.3 \times 10^3$.
 - (2) Cost not specified - expected to be several orders of magnitude *lower*.
 - (3) $L_x \times L_y \times L_z = 15H \times 30H \times 3H$.
(larger y moves the y -boundary away from central turbulent jet, avoiding the artifact of the Dirichlet boundary condition on entrainment near the jet.)

Mean temperature results show agreement between LES and DNS at $\frac{x}{H} = 6$.

LES values lower than DNS for $\frac{x}{H} = 12$.

Anticipate this to improve for increased LES mesh resolution.



Turbulent combustion - Example flame

References

Backup

[Köhler 2006] [2, 3]

Lifted turbulent Ethylene (C_2H_4) jet flame issuing into a concentric co-flow of air. Zone between flame-base and nozzle may have partial premixing.

- Fuel temp, air temp and pressure near std.
- Dimensions: nozzle diameter = 2.0 mm; co-flow air annulus diameter = 140 mm
- Exit fuel Reynolds number: 10×10^3
- Air mass flow: 320 g/min
- Mean fuel jet velocity: 44 m/s



Turbulent combustion: practical reactive flows almost always involve turbulence.

Simulation techniques:

- (a) DNS resolves all the scales
- (b) LES models sub-filter scales (SFS) while resolving larger scales
- (c) RANS models all turbulent scales

Large eddy simulation (LES): higher accuracy than Reynolds averaged Navier Stokes (RANS) → lower cost (time, resources) than direct numerical simulation (DNS).



High order finite volume method

References

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Error reduction via p

Integral Form of the Governing Equations

$$\frac{d\bar{\mathbf{U}}}{dt} = -\frac{1}{V} \iint_A (\vec{\mathcal{F}}^I - \vec{\mathcal{F}}^V) \cdot \hat{n} dA + \bar{\mathbf{S}}$$

Using a two-dimensional Gauss quadrature integration rule:

$$\frac{d\bar{\mathbf{U}}_{ijk}}{dt} = -\frac{1}{V_{ijk}} \sum_{l=1}^{N_f} \sum_{m=1}^{N_{GF}} \left(\omega \left(\vec{\mathcal{F}}^I - \vec{\mathcal{F}}^V \right) \cdot \hat{n} A \right)_{ijk,l,m} + \sum_{n=1}^{N_{GV}} (\omega_n \mathbf{S})_{i,j,k,n} = \bar{\mathbf{R}}_{ijk} (\bar{\mathbf{U}})$$

- (a) High order methods generally $> 2^{nd}$ order accuracy, reduce numerical error between PDE governing equations and discretized formulations.
- (b) Discretized scheme expressed in matrix-vector form: [Northrup 2013] GMRES solver.
- (c) High $p \rightarrow$ more quadrature points (N_G with weights ω).
- (d) (Gauss-Legendre) quadrature rules \rightarrow reference elements \rightarrow computational elements via mapping functions
- (e) Evaluate the inviscid and viscous fluxes: apply weights to quadrature points
- (f) Evaluate source vector, which adds effects of turbulence and chemistry in reacting flows
- (g) Apply appropriate time-marching, e.g. RK4 suitable to high order methods.
- (h) Other groups researching high order methods in LES: Ihme (Stanford) and Poinsot (CERFACS)