$$\int \frac{\left(\sqrt{4\,c^2+1}\ -\ 1\right)}{2\,c^2}\,dy$$

$$\frac{1}{2}\ \frac{\left(\sqrt{4\,c^2+1}-1\right)\,y}{c^2} \tag{1}$$

$$int\left(\frac{\left(\sqrt{\frac{4\,c^2}{R}\,(R\ -\ y)\ +1}\ -\ 1\right)}{2\,c^2},\ y\right)$$

$$\frac{1}{2}\ \frac{-\frac{1}{6}\ \frac{\left(4\,c^2-\frac{4\,c^2\,y}{R}+1\right)^{3/2}\,R}{c^2}-y}{c^2} \tag{2}$$

$$\int\left(\left(1\ -\ \frac{y}{R}\right)\ -\ c^2\cdot\left(1-\frac{y}{R}\right)^2+2\cdot c^4\cdot\left(1\ -\ \frac{y}{R}\right)^3\right)dy$$

$$y-\frac{1}{2}\ \frac{y^2}{R}+\frac{1}{3}\ c^2\left(1-\frac{y}{R}\right)^3\,R-\frac{1}{2}\ c^4\left(1-\frac{y}{R}\right)^4\,R \tag{3}$$