Activity 1.1 : Neural Networks

Objective(s):

This activity aims to demonstrate the concepts of neural networks

Intended Learning Outcomes (ILOs):

- Demonstrate how to use activation function in neural networks
- Demonstrate how to apply feedforward and backpropagation in neural networks

Resources:

- Jupyter Notebook

Procedure:

Import the libraries

In [3]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Define and plot an activation function

Sigmoid function:

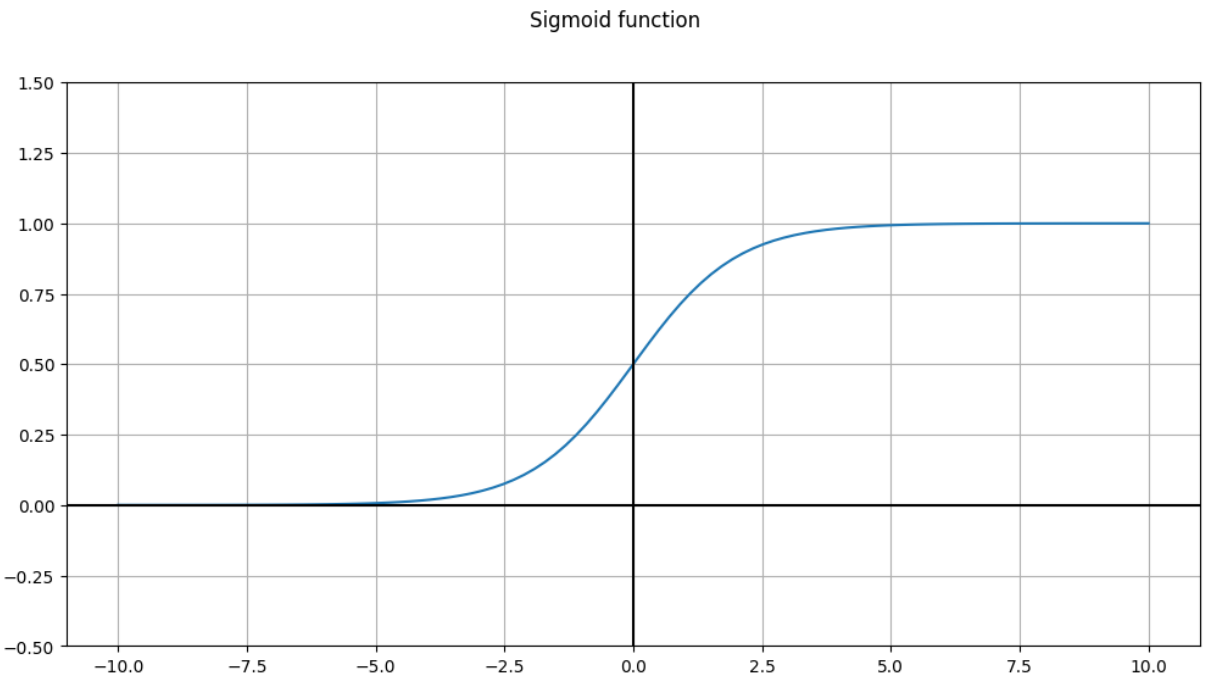
$$ \sigma = \frac{1}{1 + e^{-x}} $$

σ ranges from (0, 1). When the input x is negative, σ is close to 0. When x is positive, σ is close to 1. At $x=0$, $\sigma=0.5$

In [4]:
```
## create a sigmoid function
def sigmoid(x):
    """Sigmoid function"""
    return 1.0 / (1.0 + np.exp(-x))
```

In [5]:
```
# Plot the sigmoid function
vals = np.linspace(-10, 10, num=100, dtype=np.float32)
activation = sigmoid(vals)
fig = plt.figure(figsize=(12,6))
fig.suptitle('Sigmoid function')
plt.plot(vals, activation)
plt.grid(True, which='both')
plt.axhline(y=0, color='k')
plt.axvline(x=0, color='k')
```

```
plt.yticks()
plt.ylim([-0.5, 1.5]);
```



Choose any activation function and create a method to define that function.

In [6]:
```
def hyp_tan(x):
  """Hyperbolic-Tangent Function"""
  return
```

Plot the activation function

In [7]:
```
#type your code here
```

Neurons as boolean logic gates

OR Gate

OR gate truth table

Input		Output
0	0	0
0	1	1
1	0	1
1	1	1

A neuron that uses the sigmoid activation function outputs a value between (0, 1). This naturally leads us to think about boolean values.

By limiting the inputs of x_1 and x_2 to be in $\left\{0, 1\right\}$, we can simulate the effect of logic gates with our neuron. The goal is to find the weights , such that it returns an output close to 0 or 1 depending on the inputs.

What numbers for the weights would we need to fill in for this gate to output OR logic? Observe from the plot above that $\sigma(z)$ is close to 0 when z is largely negative (around -10 or less), and is close to 1 when z is largely positive (around +10 or greater).

$$ z = w_1 x_1 + w_2 x_2 + b $$

Let's think this through:

- When x_1 and x_2 are both 0, the only value affecting z is b. Because we want the result for (0, 0) to be close to zero, b should be negative (at least -10)
- If either x_1 or x_2 is 1, we want the output to be close to 1. That means the weights associated with x_1 and x_2 should be enough to offset b to the point of causing z to be at least 10.
- Let's give b a value of -10. How big do we need w_1 and w_2 to be?
  - At least +20
- So let's try out $w_1=20$, $w_2=20$, and $b=-10$!

In [8]:
```
def logic_gate(w1, w2, b):
    # Helper to create logic gate functions
    # Plug in values for weight_a, weight_b, and bias
    return lambda x1, x2: sigmoid(w1 * x1 + w2 * x2 + b)

def test(gate):
    # Helper function to test out our weight functions.
    for a, b in (0, 0), (0, 1), (1, 0), (1, 1):
        print("{}, {}: {}".format(a, b, np.round(gate(a, b))))
```

In [9]:
```
or_gate = logic_gate(20, 20, -10)
test(or_gate)
```

```
0, 0: 0.0
0, 1: 1.0
1, 0: 1.0
1, 1: 1.0
```

OR gate truth table

Input		Output
0	0	0
0	1	1
1	0	1
1	1	1

Try finding the appropriate weight values for each truth table.

AND Gate

AND gate truth table

Input		Output
0	0	0
0	1	0
1	0	0
1	1	1

Try to figure out what values for the neurons would make this function as an AND gate.

In [10]:
```
# Fill in the w1, w2, and b parameters such that the truth table matches
w1 = 4
w2 = 4
b = -4
and_gate = logic_gate(w1, w2, b)

test(and_gate)
```

```
0, 0: 0.0
0, 1: 0.0
1, 0: 0.0
1, 1: 1.0
```

Do the same for the NOR gate and the NAND gate.

In [11]:
```
#NOR gate
w1 = -4
w2 = -4
b = 4
nor_gate = logic_gate(w1, w2, b)

test(nor_gate)
```

```
0, 0: 1.0
0, 1: 0.0
1, 0: 0.0
1, 1: 0.0
```

In [12]:
```
#NAND gate
w1 = -2
w2 = -2
b = 4
nand_gate = logic_gate(w1, w2, b)

test(nand_gate)
```

```
0, 0: 1.0
0, 1: 1.0
1, 0: 1.0
1, 1: 0.0
```

Limitation of single neuron

Here's the truth table for XOR:

XOR (Exclusive Or) Gate

XOR gate truth table

Input		Output
0	0	0
0	1	1
1	0	1
1	1	0

Now the question is, can you create a set of weights such that a single neuron can output this property?

It turns out that you cannot. Single neurons can't correlate inputs, so it's just confused. So individual neurons are out. Can we still use neurons to somehow form an XOR gate?

In [13]:
```
# Make sure you have or_gate, nand_gate, and and_gate working from above!
def xor_gate(a, b):
    c = or_gate(a, b)
    d = nand_gate(a, b)
    return and_gate(c, d)
test(xor_gate)
```

```
0, 0: 0.0
0, 1: 1.0
1, 0: 1.0
1, 1: 1.0
```

Feedforward Networks

The feed-forward computation of a neural network can be thought of as matrix calculations and activation functions. We will do some actual computations with matrices to see this in action.

Exercise

Provided below are the following:

- Three weight matrices `W_1`, `W_2` and `W_3` representing the weights in each layer. The convention for these matrices is that each $W_{i,j}$ gives the weight from neuron i in the previous (left) layer to neuron j in the next (right) layer.
- A vector `x_in` representing a single input and a matrix `x_mat_in` representing 7 different inputs.
- Two functions: `soft_max_vec` and `soft_max_mat` which apply the soft_max function to a single vector, and row-wise to a matrix.

The goals for this exercise are:

1. For input `x_in` calculate the inputs and outputs to each layer (assuming sigmoid activations for the middle two layers and soft_max output for the final layer.
2. Write a function that does the entire neural network calculation for a single input
3. Write a function that does the entire neural network calculation for a matrix of inputs, where each row is a single input.
4. Test your functions on `x_in` and `x_mat_in`.

This illustrates what happens in a NN during one single forward pass. Roughly speaking, after this forward pass, it remains to compare the output of the network to the known truth values, compute the gradient of the loss function and adjust the weight matrices `W_1`, `W_2` and `W_3` accordingly, and iterate. Hopefully this process will result in better weight matrices and our loss will be smaller afterwards

In [14]:
```
W_1 = np.array([[2,-1,1,4],[-1,2,-3,1],[3,-2,-1,5]])
W_2 = np.array([[3,1,-2,1],[-2,4,1,-4],[-1,-3,2,-5],[3,1,1,1]])
W_3 = np.array([[-1,3,-2],[1,-1,-3],[3,-2,2],[1,2,1]])
x_in = np.array([.5,.8,.2])
x_mat_in = np.array([[.5,.8,.2],[.1,.9,.6],[.2,.2,.3],[.6,.1,.9],[.5,.5,.4],[.9,.1,

def soft_max_vec(vec):
    return np.exp(vec)/(np.sum(np.exp(vec)))

def soft_max_mat(mat):
    return np.exp(mat)/(np.sum(np.exp(mat),axis=1).reshape(-1,1))

print('the matrix W_1\n')
print(W_1)
print('-'*30)
print('vector input x_in\n')
print(x_in)
print ('-'*30)
print('matrix input x_mat_in -- starts with the vector `x_in`\n')
print(x_mat_in)
```

```
the matrix W_1

[[ 2 -1  1  4]
 [-1  2 -3  1]
 [ 3 -2 -1  5]]
------------------------------
vector input x_in

[0.5 0.8 0.2]
------------------------------
matrix input x_mat_in -- starts with the vector `x_in`

[[0.5 0.8 0.2]
 [0.1 0.9 0.6]
 [0.2 0.2 0.3]
 [0.6 0.1 0.9]
 [0.5 0.5 0.4]
 [0.9 0.1 0.9]
 [0.1 0.8 0.7]]
```

Exercise

1. Get the product of array x_in and W_1 (z2)
2. Apply sigmoid function to z2 that results to a2
3. Get the product of a2 and z2 (z3)
4. Apply sigmoid function to z3 that results to a3
5. Get the product of a3 and z3 that results to z4

In [15]:
```
#type your code here
#1 getting the product of array x_in and W_1 (z2)
a = x_in
b = W_1
z2 = np.dot(a,b)

print("z2: ",z2)
```

```
z2:  [ 0.8  0.7 -2.1  3.8]
```

In [16]:
```
#applying sigmoid function to z2 that results to a2
a2 = sigmoid(z2)

print("a2: ",a2)
```

```
a2:  [0.68997448 0.66818777 0.10909682 0.97811873]
```

In [17]:
```
#getting the product of a2 and z2
c = a2
d = z2
z3 = np.dot(c, d)

print("z3: ",z3)
```

```
z3:  4.507458871351723
```

In [18]:
```
#applying the sigmoid function to z3 to get a3

a3 = sigmoid(z3)

print("a3: ",a3)
```

```
a3:  0.9890938122523221
```

In [19]:
```
#getting the product of a3 and z3 to get z4

e = a3
f = z3
z4 = np.dot(e, f)

print("z4: ",z4)
```

```
z4:  4.458299678635824
```

In [20]:
```
def soft_max_vec(vec):
    return np.exp(vec)/(np.sum(np.exp(vec)))

def soft_max_mat(mat):
    return np.exp(mat)/(np.sum(np.exp(mat),axis=1).reshape(-1,1))
```

7. Apply soft_max_vec function to z4 that results to y_out

In [21]:
```
#type your code here
y_out = soft_max_vec(z4)

print("y_out: ", y_out)
```

```
y_out:  1.0
```

In [22]:
```
## A one-line function to do the entire neural net computation

def nn_comp_vec(x):
    return soft_max_vec(sigmoid(sigmoid(np.dot(x,W_1)).dot(W_2)).dot(W_3))

def nn_comp_mat(x):
    return soft_max_mat(sigmoid(sigmoid(np.dot(x,W_1)).dot(W_2)).dot(W_3))
```

In [23]:
```
nn_comp_vec(x_in)
```

Out[23]:
```
array([0.72780576, 0.26927918, 0.00291506])
```

In [24]:
```
nn_comp_mat(x_mat_in)
```

Out[24]:
```
array([[0.72780576, 0.26927918, 0.00291506],
       [0.62054212, 0.37682531, 0.00263257],
       [0.69267581, 0.30361576, 0.00370844],
       [0.36618794, 0.63016955, 0.00364252],
       [0.57199769, 0.4251982 , 0.00280411],
       [0.38373781, 0.61163804, 0.00462415],
       [0.52510443, 0.4725011 , 0.00239447]])
```

Backpropagation

The backpropagation in this part will be used to train a multi-layer perceptron (with a single hidden layer). Different patterns will be used and the demonstration on how the weights will converge. The different parameters such as learning rate, number of iterations, and number of data points will be demonstrated

In [25]:
```
#Preliminaries
from __future__ import division, print_function
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Fill out the code below so that it creates a multi-layer perceptron with a single hidden layer (with 4 nodes) and trains it via back-propagation. Specifically your code should:

1. Initialize the weights to random values between -1 and 1
2. Perform the feed-forward computation
3. Compute the loss function
4. Calculate the gradients for all the weights via back-propagation
5. Update the weight matrices (using a learning_rate parameter)
6. Execute steps 2-5 for a fixed number of iterations
7. Plot the accuracies and log loss and observe how they change over time

Once your code is running, try it for the different patterns below.

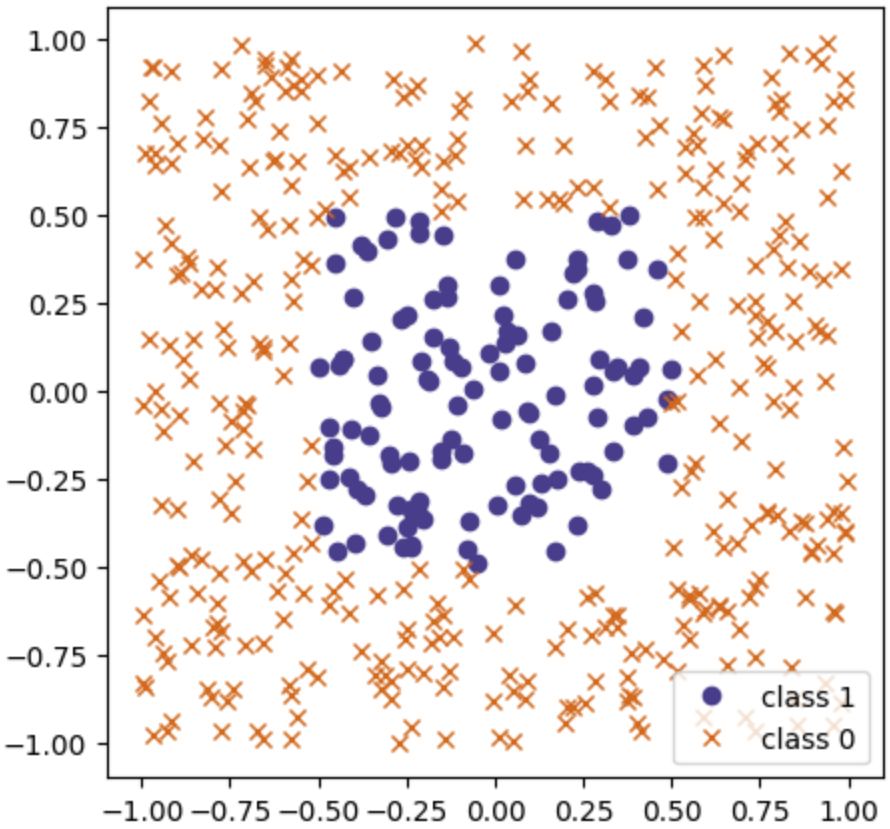
- Which patterns was the neural network able to learn quickly and which took longer?
- What learning rates and numbers of iterations worked well?

In [26]:
```
## This code below generates two x values and a y value according to different patt
## It also creates a "bias" term (a vector of 1s)
## The goal is then to learn the mapping from x to y using a neural network via bac

num_obs = 500
x_mat_1 = np.random.uniform(-1,1,size = (num_obs,2))
x_mat_bias = np.ones((num_obs,1))
x_mat_full = np.concatenate( (x_mat_1,x_mat_bias), axis=1)

# PICK ONE PATTERN BELOW and comment out the rest.

# # Circle pattern
# y = (np.sqrt(x_mat_full[:,0]**2 + x_mat_full[:,1]**2)<.75).astype(int)

# # Diamond Pattern
#y = ((np.abs(x_mat_full[:,0]) + np.abs(x_mat_full[:,1]))<1).astype(int)

# # Centered square
y = ((np.maximum(np.abs(x_mat_full[:,0]), np.abs(x_mat_full[:,1])))<.5).astype(int)
```

```
# # Thick Right Angle pattern
#y = (((np.maximum((x_mat_full[:,0]), (x_mat_full[:,1])))<.5) & ((np.maximum((x_mat

# # Thin right angle pattern
# y = (((np.maximum((x_mat_full[:,0]), (x_mat_full[:,1])))<.5) & ((np.maximum((x_ma


print('shape of x_mat_full is {}'.format(x_mat_full.shape))
print('shape of y is {}'.format(y.shape))

fig, ax = plt.subplots(figsize=(5, 5))
ax.plot(x_mat_full[y==1, 0],x_mat_full[y==1, 1], 'ro', label='class 1', color='dark
ax.plot(x_mat_full[y==0, 0],x_mat_full[y==0, 1], 'bx', label='class 0', color='choc
# ax.grid(True)
ax.legend(loc='best')
ax.axis('equal');
```

```
shape of x_mat_full is (500, 3)
shape of y is (500,)
```

```
<ipython-input-26-46ce47f749bb>:32: UserWarning: color is redundantly defined by the
'color' keyword argument and the fmt string "ro" (-> color='r'). The keyword argumen
t will take precedence.
  ax.plot(x_mat_full[y==1, 0],x_mat_full[y==1, 1], 'ro', label='class 1', color='dar
kslateblue')
<ipython-input-26-46ce47f749bb>:33: UserWarning: color is redundantly defined by the
'color' keyword argument and the fmt string "bx" (-> color='b'). The keyword argumen
t will take precedence.
  ax.plot(x_mat_full[y==0, 0],x_mat_full[y==0, 1], 'bx', label='class 0', color='cho
colate')
```



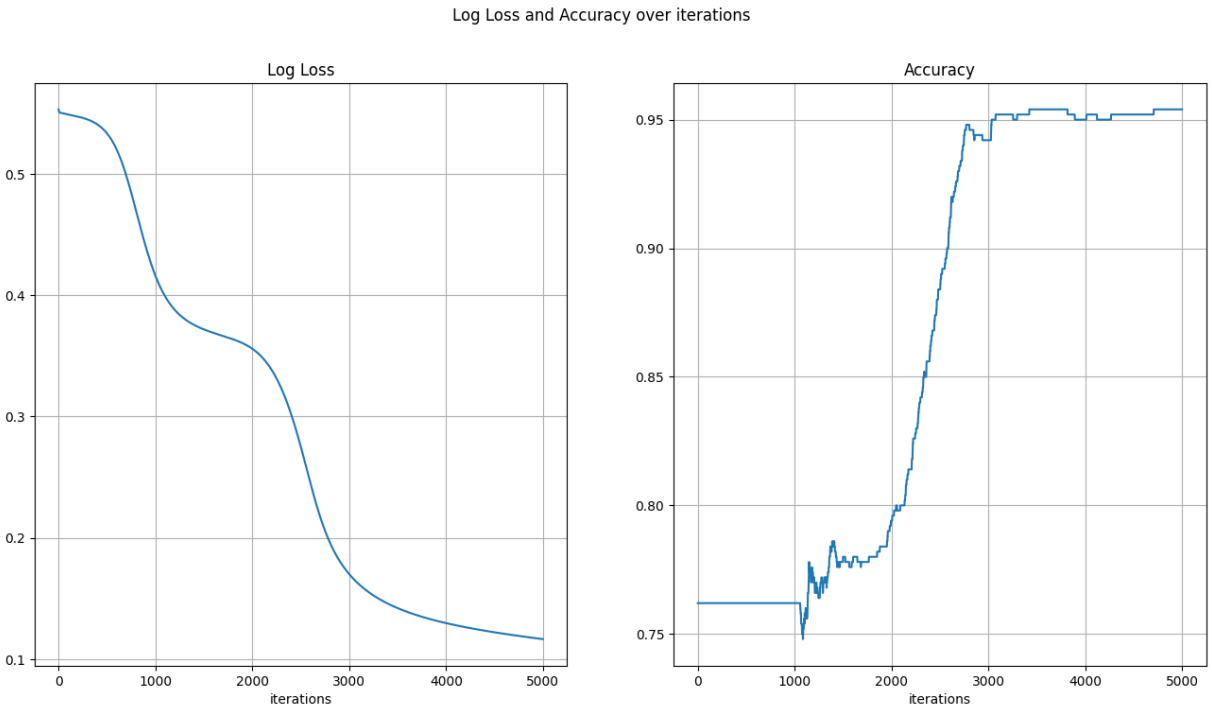
In [27]:
```
def sigmoid(x):
    """
    Sigmoid function
    """
    return 1.0 / (1.0 + np.exp(-x))


def loss_fn(y_true, y_pred, eps=1e-16):
    """
    Loss function we would like to optimize (minimize)
    We are using Logarithmic Loss
    http://scikit-learn.org/stable/modules/model_evaluation.html#log-loss
    """
    y_pred = np.maximum(y_pred,eps)
    y_pred = np.minimum(y_pred,(1-eps))
    return -(np.sum(y_true * np.log(y_pred)) + np.sum((1-y_true)*np.log(1-y_pred)))


def forward_pass(W1, W2):
    """
    Does a forward computation of the neural network
    Takes the input `x_mat` (global variable) and produces the output `y_pred`
    Also produces the gradient of the log loss function
    """
    global x_mat
    global y
    global num_
    # First, compute the new predictions `y_pred`
    z_2 = np.dot(x_mat, W_1)
    a_2 = sigmoid(z_2)
    z_3 = np.dot(a_2, W_2)
    y_pred = sigmoid(z_3).reshape((len(x_mat),))
    # Now compute the gradient
    J_z_3_grad = -y + y_pred
    J_W_2_grad = np.dot(J_z_3_grad, a_2)
    a_2_z_2_grad = sigmoid(z_2)*(1-sigmoid(z_2))
    J_W_1_grad = (np.dot((J_z_3_grad).reshape(-1,1), W_2.reshape(-1,1).T)*a_2_z_2_g
    gradient = (J_W_1_grad, J_W_2_grad)

    # return
    return y_pred, gradient


def plot_loss_accuracy(loss_vals, accuracies):
    fig = plt.figure(figsize=(16, 8))
    fig.suptitle('Log Loss and Accuracy over iterations')

    ax = fig.add_subplot(1, 2, 1)
    ax.plot(loss_vals)
    ax.grid(True)
    ax.set(xlabel='iterations', title='Log Loss')

    ax = fig.add_subplot(1, 2, 2)
    ax.plot(accuracies)
```

```
    ax.grid(True)
    ax.set(xlabel='iterations', title='Accuracy');
```

Complete the pseudocode below

In [34]:
```
#### Initialize the network parameters

np.random.seed(1241)

W_1 = np.random.uniform(-1,1,size=(3,4))
W_2 = np.random.uniform(-1,1,size=(4))
num_iter = 5000
learning_rate = .001
x_mat = x_mat_full


loss_vals, accuracies = [], []
for i in range(num_iter):
    ### Do a forward computation, and get the gradient
    y_pred, (J_W_1_grad, J_W_2_grad) = forward_pass(W_1, W_2)

    ## Update the weight matrices
    W_1 = W_1 - learning_rate*J_W_1_grad
    W_2 = W_2 - learning_rate*J_W_2_grad

    ### Compute the loss and accuracy
    curr_loss = loss_fn(y,y_pred)
    loss_vals.append(curr_loss)
    acc = np.sum((y_pred>=.5) == y)/num_obs
    accuracies.append(acc)

    ## Print the loss and accuracy for every 200th iteration
    if((i%200) == 0):
        print('iteration {}, log loss is {:.4f}, accuracy is {}'.format(
            i, curr_loss, acc
        ))

plot_loss_accuracy(loss_vals, accuracies)
```

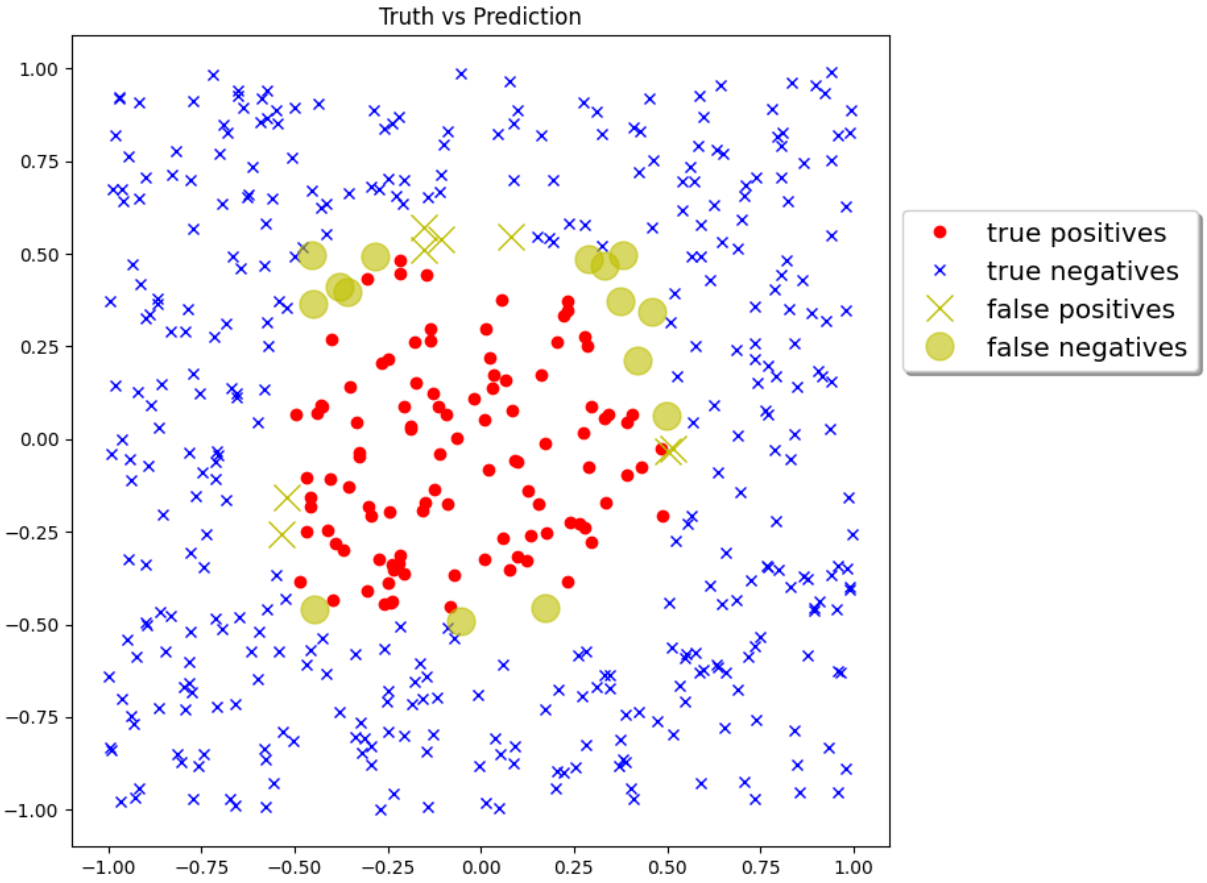
```
iteration 0, log loss is 0.5530, accuracy is 0.762
iteration 200, log loss is 0.5471, accuracy is 0.762
iteration 400, log loss is 0.5406, accuracy is 0.762
iteration 600, log loss is 0.5198, accuracy is 0.762
iteration 800, log loss is 0.4692, accuracy is 0.762
iteration 1000, log loss is 0.4157, accuracy is 0.762
iteration 1200, log loss is 0.3877, accuracy is 0.77
iteration 1400, log loss is 0.3753, accuracy is 0.784
iteration 1600, log loss is 0.3688, accuracy is 0.778
iteration 1800, log loss is 0.3636, accuracy is 0.78
iteration 2000, log loss is 0.3557, accuracy is 0.794
iteration 2200, log loss is 0.3382, accuracy is 0.814
iteration 2400, log loss is 0.3019, accuracy is 0.862
iteration 2600, log loss is 0.2449, accuracy is 0.91
iteration 2800, log loss is 0.1961, accuracy is 0.948
iteration 3000, log loss is 0.1700, accuracy is 0.942
iteration 3200, log loss is 0.1552, accuracy is 0.952
iteration 3400, log loss is 0.1457, accuracy is 0.952
iteration 3600, log loss is 0.1389, accuracy is 0.954
iteration 3800, log loss is 0.1337, accuracy is 0.954
iteration 4000, log loss is 0.1297, accuracy is 0.95
iteration 4200, log loss is 0.1263, accuracy is 0.95
iteration 4400, log loss is 0.1234, accuracy is 0.952
iteration 4600, log loss is 0.1209, accuracy is 0.952
iteration 4800, log loss is 0.1186, accuracy is 0.954
```



Plot the predicted answers, with mistakes in yellow

In [35]:
```
pred1 = (y_pred>=.5)
pred0 = (y_pred<.5)

fig, ax = plt.subplots(figsize=(8, 8))
# true predictions
ax.plot(x_mat[pred1 & (y==1),0],x_mat[pred1 & (y==1),1], 'ro', label='true positive
```

```
ax.plot(x_mat[pred0 & (y==0),0],x_mat[pred0 & (y==0),1], 'bx', label='true negative
# false predictions
ax.plot(x_mat[pred1 & (y==0),0],x_mat[pred1 & (y==0),1], 'yx', label='false positiv
ax.plot(x_mat[pred0 & (y==1),0],x_mat[pred0 & (y==1),1], 'yo', label='false negativ
ax.set(title='Truth vs Prediction')
ax.legend(bbox_to_anchor=(1, 0.8), fancybox=True, shadow=True, fontsize='x-large');
```



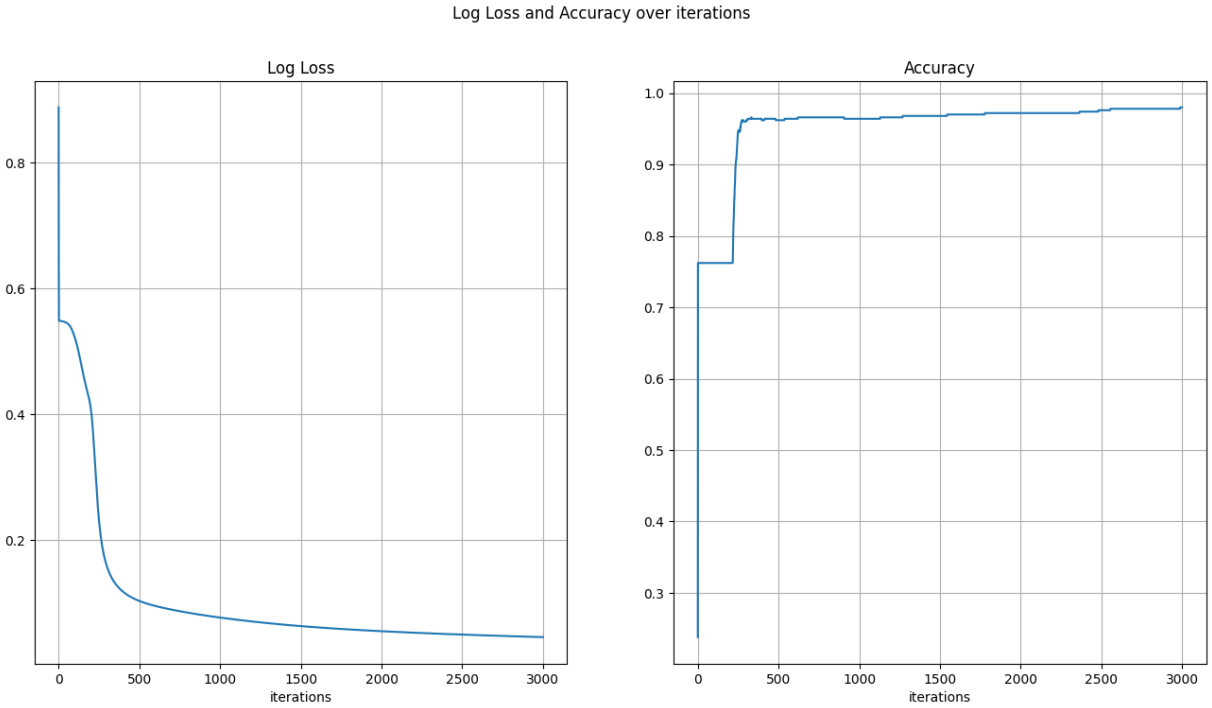
Supplementary Activity

1. Use a different weights , input and activation function
2. Apply feedforward and backpropagation
3. Plot the loss and accuracy for every 300th iteration

In [36]:
```
np.random.seed(2461)

#using different weights, input and activation function
W_1 = np.random.uniform(-1,1,size=(3,4))
W_2 = np.random.uniform(-1,1,size=(4))
num_iter = 3000
learning_rate = .01
x_mat = x_mat_full


loss_vals, accuracies = [], []
for i in range(num_iter):
    ### Applying feedforward pass
```

```
    y_pred, (J_W_1_grad, J_W_2_grad) = forward_pass(W_1, W_2)

    #Doing backpropagation

    ## Update the weight matrices
    W_1 = W_1 - learning_rate*J_W_1_grad
    W_2 = W_2 - learning_rate*J_W_2_grad

    ### Compute the loss and accuracy
    curr_loss = loss_fn(y,y_pred)
    loss_vals.append(curr_loss)
    acc = np.sum((y_pred>=.5) == y)/num_obs
    accuracies.append(acc)

    ## Plotting the loss and accuracy for every 300th iteration
    if((i%300) == 0):
        print('iteration {}, log loss is {:.4f}, accuracy is {}'.format(
            i, curr_loss, acc
        ))

plot_loss_accuracy(loss_vals, accuracies)
```

```
iteration 0, log loss is 0.8877, accuracy is 0.238
iteration 300, log loss is 0.1579, accuracy is 0.962
iteration 600, log loss is 0.0958, accuracy is 0.964
iteration 900, log loss is 0.0811, accuracy is 0.966
iteration 1200, log loss is 0.0712, accuracy is 0.966
iteration 1500, log loss is 0.0639, accuracy is 0.968
iteration 1800, log loss is 0.0586, accuracy is 0.972
iteration 2100, log loss is 0.0546, accuracy is 0.972
iteration 2400, log loss is 0.0513, accuracy is 0.974
iteration 2700, log loss is 0.0487, accuracy is 0.978
```



Conclusion

type your conclusion here

- To conclude, this hands on activity is about how to use neural networks and how does it works. In this activity I learned about input functions and activation functions where there are 4 different activation functions which are sigmoid, step function, hyperbolic tangent function, rectified linear unit function, and leaky rectified linear unit funciton. Also, I learned about perceptron model and how it is used in deep learning where there are 4 main steps which are receive inputs, weights the input, sum the inputs, and generate outputs also to consider the bias of the model. Lastly, I learned about neural networks such as feed-forward network, back propagation,

https://colab.research.google.com/drive/1ufN05DRIPz3TxEZItE9MXOxc7XDDQsmD?usp=sharing