Peer-to-Peer and Social Networks

Power law graphs

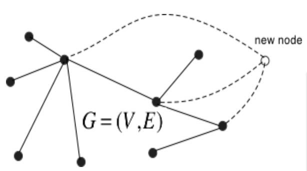
Small world graphs

Preferential attachment

Barabási and Albert showed that when large networks are formed by the rules of preferential attachment , the resulting graph shows a power-law distribution of the node degrees.

We will derive it in the class, so follow the lecture.

Preferential attachment



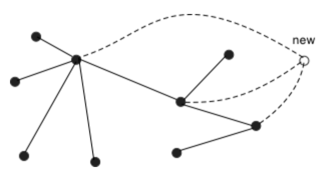
At t = 0, there are no nodes.
At t = 1, one node appears.
Thereafter, each time unit,
a new node is added

Degree of node i = $\delta(i)$

The probability that the new node connects with an existing node i = $C.\ \delta(i)$

Since $\sum_{i \in V} C.\ \delta(i) = 1$ and $\sum_{i \in V} \delta(i) = 2|V|$ so $C = \frac{1}{2t}$

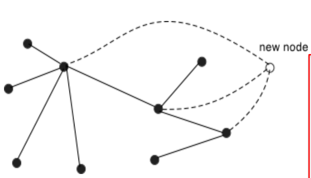
Preferential attachment



$n(k,t)$ = number of nodes with degree k after time step t

$$n(k,t+1) = n(k,t) + n(k-1,t).\frac{k-1}{2t} - n(k,t).\frac{k}{2t}$$

Preferential attachment



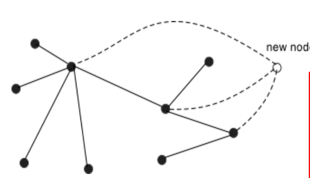
$f(k,t) = \frac{n(k,t)}{|V|}$

is then fraction of nodes with degree k at time t

$$n(k,t+1) = n(k,t) + n(k-1,t).\frac{k-1}{2t} - n(k,t).\frac{k}{2t}$$

$$(t+1).\ f(k,t+1) = t.\ f(k,t) + \frac{1}{2}[(k-1).\ f(k-1,t) - k.\ f(k,t)]$$

Preferential attachment



As $t \longrightarrow \infty$,

$f(k,t+1) \rightarrow f(k,t)$

Call it $f(k)$

$$(t+1).\ f(k,t+1) = t.\ f(k,t) + \frac{1}{2}[(k-1).\ f(k-1,t) - k.\ f(k,t)]$$

$$f(k) = \frac{1}{2}[(k-1).\ f(k-1) - k.\ f(k)]$$

$$f(k) = \frac{k-1}{k+2}.\ f(k-1)$$

Preferential attachment

$$f(k)=\frac{k-1}{k+2}.\,f(k-1)$$

$$\frac{k-1}{k+2}.\frac{k-2}{k+1}.\frac{k-3}{k},\frac{k-4}{k-1}.....\frac{4}{7}.\frac{3}{6}.\frac{2}{5}.\frac{1}{4}\,f(1)$$

$$\frac{3.2.1}{(k+2).(k+1).\,k}.\,f(1)$$

$$f(k)=\frac{4}{k(k+1)(k+2)}$$

$f(k)$ is of the order of $\frac{1}{k^3}$

$$n(1,t+1)=n(1,t)+1\overset{*}{-}n(1,t).\frac{1}{2t}$$

$$(t+1).\,f(1,t+1)=t.\,f(1,t)+1-\frac{f(1,t)}{2}$$

$$t\to\infty,\ f(1,t+1)=f(1,t)=f(1)$$

$$f(1)=\frac{2}{3}$$

* Before time step (t+1), the new node is the only node with degree 0, and its degree will change to 1

Other properties of power law graphs

- Graphs following a power-law distribution $N(k) : k^{-r} (2 < r < 3)$ have a small diameter $d : \ln\ln n$ (n = number of nodes).
- The clustering coefficient decreases as the node degree increases (power law)
- Graphs following a power-law distribution tend to be highly resilient to random edge removal, but quite vulnerable to targeted attacks on the hubs.

The small-world model

Due to **Watts and Strogatz** (1998)

They followed up on Milgram's work and reason about why there is a small degree of separation between individuals in a social network. Research originally inspired by Watt's efforts to understand the synchronization of cricket chirps, which show a high degree of coordination over long ranges, as though the insects are being guided by an invisible conductor.

Disease spreads faster over a small-world network.

Questions not answered by Milgram

Why **six** degrees of separation? Any scientific reason? What properties do these social graphs have? Are there other situations in which this model is applicable?

Time to reverse engineer this.

What are small-world graphs

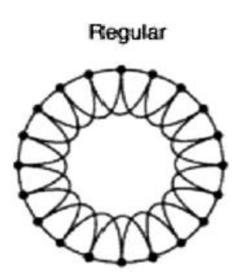
Completely regular

Small-world graphs (n >> k >> ln (n) >>1)

Completely random

n = number of nodes, k= number of neighbors of each node

Completely regular



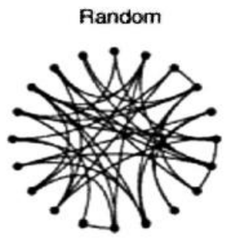
A ring lattice

n=20, k= 4

High clustering coefficient and high diameter L.

C = 3/6 = 1/2, L ~ n/k

Completely random

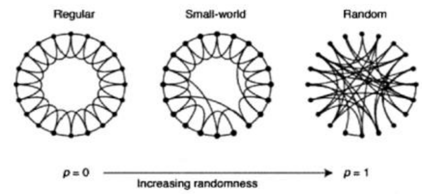


LOW clustering coefficient and LOW diameter.

C ~ k/n, L ~ (log n)/(log k)

Small-world graphs

Start with the regular graph, and with probability *p* rewire each link to a randomly selected node. It results in a graph that has high clustering coefficient but low diameter …



Small-world graphs