Word embeddings
LSA, Word2Vec & Glove

Dr. Reda Bouadjenek
Data-Driven Decision Making Lab (D3M)
UNIVERSITY OF TORONTO

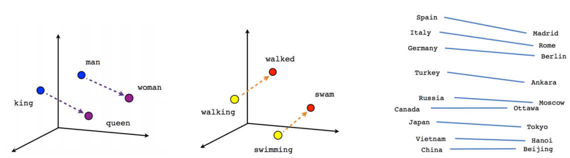
Vector Embedding of Words

- A word is represented as a vector.
- Word embeddings depend on a notion of ***word similarity***.
  - Similarity is computed using cosine.
- A very useful definition is paradigmatic similarity:
  - ***Similar words*** occur in ***similar contexts***. They are ***exchangeable***.
  - Yesterday { POTUS / The President / Trump } called a press conference.
  - "POTUS: President of the United States."

Vector Embedding of Words

Traditional Method - Bag of Words Model	Word Embeddings
▪ Uses one hot encoding. ▪ Each word in the vocabulary is represented by one bit position in a HUGE vector. ▪ For example, if we have a vocabulary of 10000 words, and "Hello" is the 4th word in the dictionary, it would be represented by: 0 0 0 1 0 0 . . . . 0 0 0 0 ▪ Context information is not utilized.	▪ Stores each word in as a point in space, where it is represented by a vector of fixed number of dimensions (generally 300) . ▪ Unsupervised, built just by reading huge corpus. ▪ For example, "Hello" might be represented as : [0.4, -0.11, 0.55, 0.3 . . . 0.1, 0.02]. ▪ Dimensions are basically projections along different axes, more of a mathematical concept.

Example



Male-Female   Verb tense   Country-Capital

- vector[Queen] ≈ vector[King] - vector[Man] + vector[Woman]
- vector[Paris] ≈ vector[France] - vector[Italy] + vector[Rome]
  - This can be interpreted as "France is to Paris as Italy is to Rome".

Working with vectors

- Finding the most similar words to $\overrightarrow{dog}$.
  - Compute the similarity from word $\overrightarrow{dog}$ to all other words.
  - This is a single matrix-vector product: $W \cdot \overrightarrow{dog}$
    - W is the word embedding matrix of |***V***| rows and ***d*** columns.
    - Result is a |V| sized vector of similarities.
    - Take the indices of the k-highest values.

Working with vectors

- Similarity to a group of words
  - "Find me words most similar to cat, dog and cow".
  - Calculate the pairwise similarities and sum them:

$$W \cdot \overrightarrow{cat} + W \cdot \overrightarrow{dog} + W \cdot \overrightarrow{cow}$$

  - Now find the indices of the highest values as before.
  - Matrix-vector products are wasteful. Better option:

$$W \cdot (\overrightarrow{cat} + \overrightarrow{dog} + \overrightarrow{cow})$$

Applications of Word Vectors

- Word Similarity
- Machine Translation
- Part-of-Speech and Named Entity Recognition
- Relation Extraction
- Sentiment Analysis
- Co-reference Resolution
  - Chaining entity mentions across multiple documents - can we find and unify the multiple contexts in which mentions occurs?
- Clustering
  - Words in the same class naturally occur in similar contexts, and this feature vector can directly be used with any conventional clustering algorithms (K-Means, agglomerative, etc). Human doesn't have to waste time hand-picking useful word features to cluster on.
- Semantic Analysis of Documents
  - Build word distributions for various topics, etc.

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Vector Embedding of Words

- Three main methods described in the talk :
  - Latent Semantic Analysis/Indexing (1988)
    - Term weighting-based model
    - Consider occurrences of terms at document level.
  - Word2vec (2013)
    - Prediction-based model.
    - Consider occurrences of terms at context level.
  - GloVe (2014)
    - Count-based model.
    - Consider occurrences of terms at context level.

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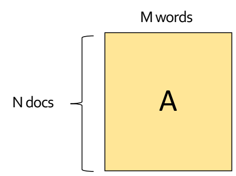
Latent Semantic Analysis

Deerwester, Scott, Susan T. Dumais, George W. Furnas, Thomas K. Landauer, and Richard Harshman. "Indexing by latent semantic analysis." Journal of the American society for information science 41, no. 6 (1990): 391-407.

Embedding: Latent Semantic Analysis

- Latent semantic analysis studies documents in Bag-Of-Words model (1988).
  - i.e. given a matrix ***A*** encoding some documents: A_{ij} is the count* of word ***j*** in document ***i***. Most entries are 0.



* Often tf-idf or other "squashing" functions of the count are used.

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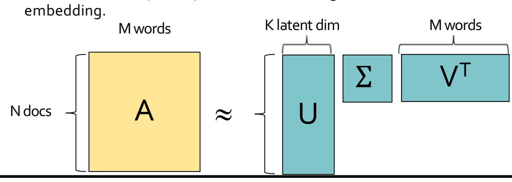
Embedding: Latent Semantic Analysis

- Low rank SVD decomposition:

$$A_{[m\times n]} = U_{[m\times r]}\Sigma_{[r\times r]}(V_{[n\times r]})^T$$

  - U : document-to-concept similarities matrix (orthogonal matrix).
  - V : word-to-concept similarities matrix (orthogonal matrix).
  - Σ : strength of each concept.
- Then given a word ***w*** (column of ***A***):
  - $\varsigma = w^T \times U$ is the embedding (encoding) of the word ***w*** in the latent space.
  - $w \approx U \times \varsigma^T = U \times (w^T \times U)^T$ is the decoding of the word ***w*** from its embedding.



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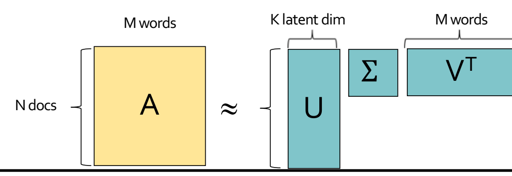
Embedding: Latent Semantic Analysis

- $w \approx U \times \varsigma^T = U \times (w^T \times U)^T$ is the decoding of the word ***w*** from its embedding.
  - An SVD factorization gives the **best possible reconstructions** of the a word ***w*** from its embedding.
- Note:
  - The problem with this method, is that we may end up with matrices having billions of rows and columns, which makes **SVD computationally expensive and restrictive.**



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Word2vec

word2vec: Local contexts

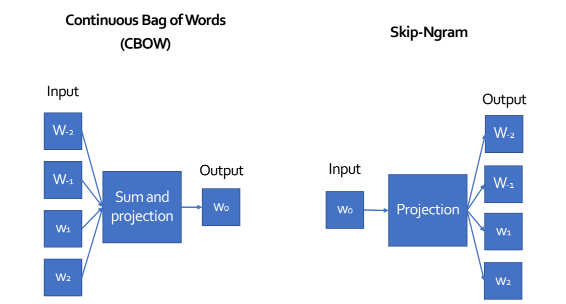
- Instead of entire documents, ***Word2vec*** uses words ***k*** positions away from each center word.
- Example for ***k=3***:
  - "It was a bright cold day in April, and the clocks were striking".
  - Center word: red (also called focus word).
  - Context words: blue (also called target words).
- Word2vec considers all words as center words, and all their context words.

Word2vec: Data generation (window size = 2)

- Example: d1 = "king brave man" , d2 = "queen beautiful women"

word	Word one hot encoding	neighbor	Neighbor one hot encoding
king	[1,0,0,0,0,0]	brave	[0,1,0,0,0,0]
king	[1,0,0,0,0,0]	man	[0,0,1,0,0,0]
brave	[0,1,0,0,0,0]	king	[1,0,0,0,0,0]
brave	[0,1,0,0,0,0]	man	[0,0,1,0,0,0]
man	[0,0,1,0,0,0]	king	[1,0,0,0,0,0]
man	[0,0,1,0,0,0]	brave	[0,1,0,0,0,0]
queen	[0,0,0,1,0,0]	beautiful	[0,0,0,0,1,0]
queen	[0,0,0,1,0,0]	women	[0,0,0,0,0,1]
beautiful	[0,0,0,0,1,0]	queen	[0,0,0,1,0,0]
beautiful	[0,0,0,0,1,0]	women	[0,0,0,0,0,1]
woman	[0,0,0,0,0,1]	queen	[0,0,0,1,0,0]
woman	[0,0,0,0,0,1]	beautiful	[0,0,0,0,1,0]

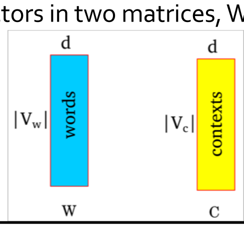
Word2vec: main context representation models



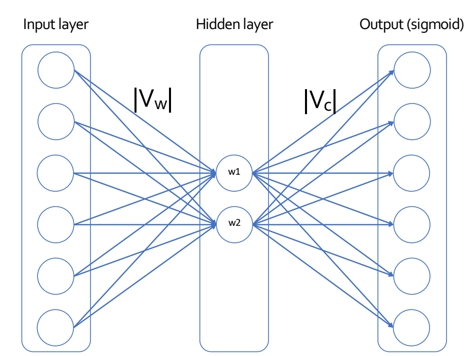
- Will focus on Skip-Ngram model

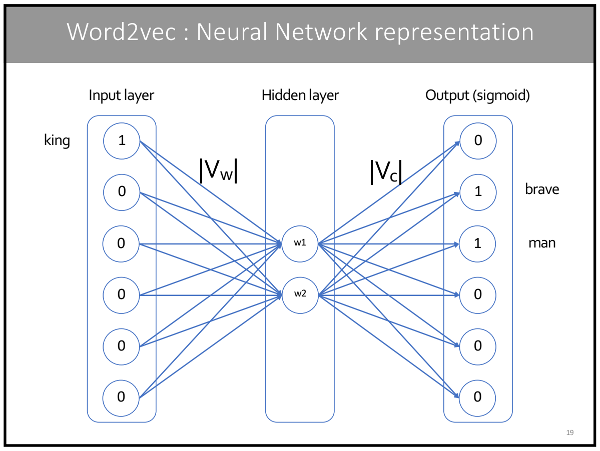
How does word2vec work?

- Represent each word as a ***d*** dimensional vector.
- Represent each context as a ***d*** dimensional vector.
- Initialize all vectors to random weights.
- Arrange vectors in two matrices, W and C.



Word2vec : Neural Network representation




Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
king
|Vw|
|Vc|
w1
w2
brave
man
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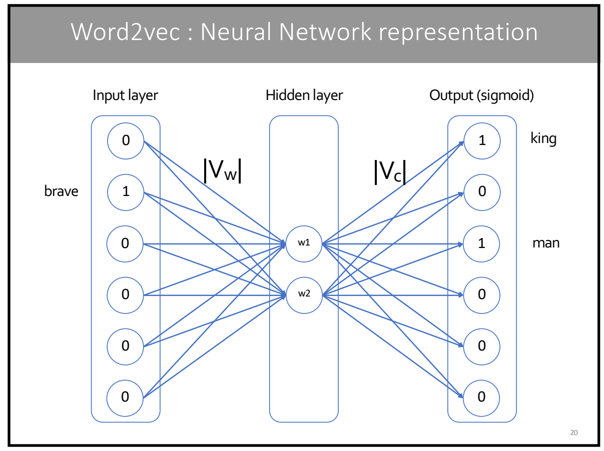
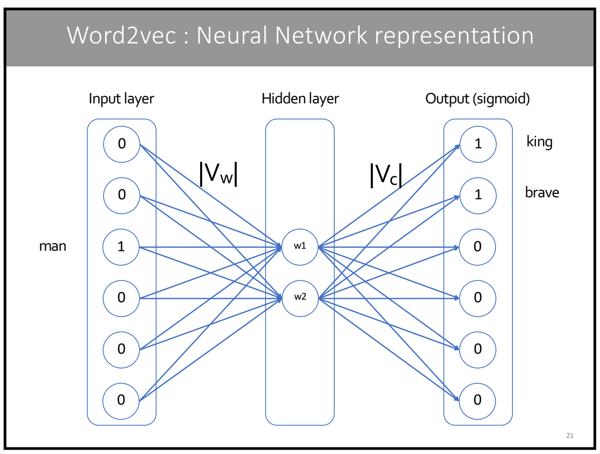
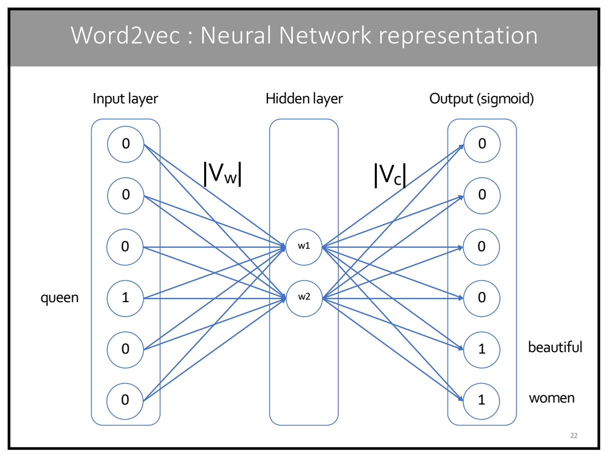
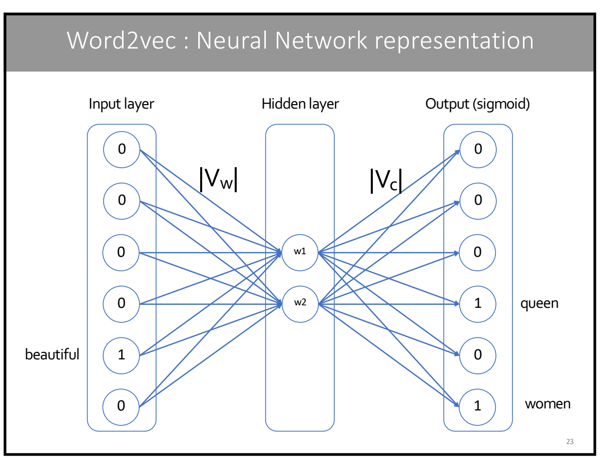
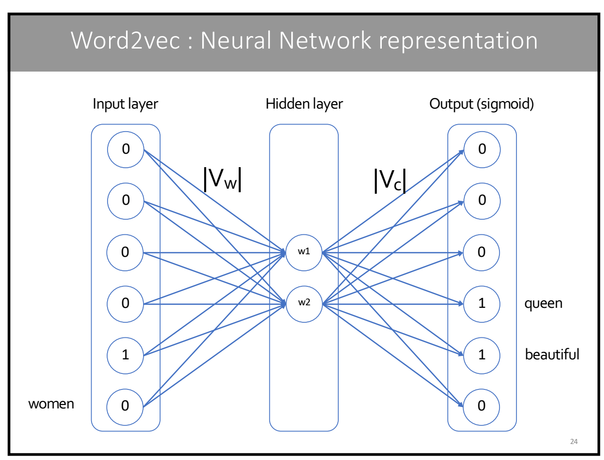

Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
brave
|Vw|
|Vc|
w1
w2
king
man
20


Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
man
|Vw|
|Vc|
w1
w2
king
brave
21


Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
queen
|Vw|
|Vc|
w1
w2
beautiful
women
22


Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
beautiful
|Vw|
|Vc|
w1
w2
queen
women
23


Word2vec : Neural Network representation
Input layer
Hidden layer
Output (sigmoid)
women
|Vw|
|Vc|
w1
w2
queen
beautiful
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Skip-Ngram: Training method

- The prediction problem is modeled using soft-max:

$$p(c|w;\theta) = \frac{\exp(v_c \cdot v_w)}{\sum_{\grave{c}\in C} \exp(v_{\grave{c}} \cdot v_w)}$$

  - Predict context words(s) ***c***
  - From focus word ***w***
  - Looks like logistic regression!
    - v_w are features and the evidence is v_c
- The objective function (in log space):

$$\underset{\theta}{\text{argmax}} \sum_{(w,c)\in D} \log p(c|w;\theta) = \sum_{(w,c)\in D} \left[\log \exp(v_c \cdot v_w) - \log \sum_{\grave{c}\in C} \exp(v_{\grave{c}} \cdot v_w)\right]$$

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Skip-Ngram: Negative sampling

- The objective function (in log space):

$$\underset{\theta}{\text{argmax}} \sum_{(w,c)\in D} \log p(c|w;\theta) = \sum_{(w,c)\in D} \left[\log \exp(v_c \cdot v_w) - \log \sum_{\grave{c}\in C} \exp(v_{\grave{c}} \cdot v_w)\right]$$

  - While the objective function can be computed optimized, it is computationally expensive
    - $p(c|w;\theta)$ is very expensive to compute due to the summation $\sum_{\grave{c}\in C} \exp(v_{\grave{c}} \cdot v_w)$
- Mikolov et al. proposed the negative-sampling approach as a more efficient way of deriving word embeddings:

$$\underset{\theta}{\text{argmax}} \sum_{(w,c)\in D} \log \sigma(v_c \cdot v_w) + \sum_{(w,c)\in \acute{D}} \log \sigma(-v_c \cdot v_w)$$

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Skip-Ngram: Example

- While more text:
  - Extract a word window:

```
A springer is[ a   cow  or  heifer  close  to  calving ]
               c1  c2   c3  w       c4     c5  c6
```

  - Try setting the vector values such that:
    - $\sigma(w \cdot c_1) + \sigma(w \cdot c_2) + \sigma(w \cdot c_3) + \sigma(w \cdot c_4) + \sigma(w \cdot c_5) + \sigma(w \cdot c_6)$ is high!
  - Create a corrupt example by choosing a random word $\acute{w}$

```
[ a   cow  or  comet  close  to  calving ]
  c1  c2   c3  w'     c4     c5  c6
```

  - Try setting the vector values such that:
    - $\sigma(\acute{w} \cdot c_1) + \sigma(\acute{w} \cdot c_2) + \sigma(\acute{w} \cdot c_3) + \sigma(\acute{w} \cdot c_4) + \sigma(\acute{w} \cdot c_5) + \sigma(\acute{w} \cdot c_6)$ is low!

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Skip-Ngram: How to select negative samples?

- Can sample using frequency.
  - Problem: will sample a lot of stop-words.
- Mikolov et al. proposed to sample using:

$$p(w_i) = \frac{f(w_i)^{3/4}}{\sum_j f(w_j)^{3/4}}$$

  - Not theoretically justified, but works well in practice!

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Relations Learned by Word2vec

- A relation is defined by the vector displacement in the first column. For each start word in the other column, the closest displaced word is shown.

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

- "Efficient Estimation of Word Representations in Vector Space" Tomas Mikolov, Kai Chen, Greg Corrado, Jeffrey Dean, Arxiv 2013

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GloVe: Global Vectors for Word Representation

Jeffrey Pennington, Richard Socher, and Christopher D. Manning. 2014. GloVe: Global Vectors for Word Representation.

GloVe: **Global Vectors for Word Representation**

- While word2vec is a predictive model — learning vectors to improve the predictive ability, **GloVe is a count-based model**.
- Count-based models learn vectors by doing dimensionality reduction on a **co-occurrence counts matrix**.
  - Factorize this matrix to yield a lower-dimensional matrix of words and features, where each row yields a vector representation for each word.
  - The counts matrix is preprocessed by normalizing the counts and log-smoothing them.

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GloVe: Training

- The prediction problem is given by:

$$w_i^T \cdot \tilde{w}_j + b_i + \tilde{b}_j = \log X_{i,j}$$

  - b_w and b_c are bias terms.
- The objective function:

$$J = \sum_{i,j=1}^{V} f(X_{i,j})\,(w_i^T \cdot \tilde{w}_j + b_i + \tilde{b}_j - \log X_{i,j})^2$$

  - $f(X_{i,j})$ is a weighting function to penalize rare co-occurrences.

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GloVe: Training

- The model generates two sets of word vectors, W and $\widetilde{W}$.
- W and $\widetilde{W}$ are equivalent and differ only as a result of their random initializations.
  - The two sets of vectors should perform equivalently.
- Authors proposed to use $\frac{W+\widetilde{W}}{2}$ to get word vectors.

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