Robust Video Denoising using Low rank matrix completion

Problem Description:

The paper talks about the problem of video denoising under a mixed noise model.
Most denoising algorithms assume properties about the statistical noise model and perform denoising based on that. Since a mixed noise model is being used here, methods assuming the statistical noise model do not perform as well on these videos. The paper looks at this as a matrix completion problem rather than a denoising problem.

The image is assumed to be a sum of the clean image with noise:

$$f_k = g_k + n_k$$

Where, f_k is the noisy image
g_k is the clean image
And, n_k is the noise

Algorithm:

The algorithm used in this paper for solving the problem of video denoising is fixed point iteration algorithm.
The video contains noise from three different statistical models: Gaussian, Poisson, and Impulse.
In order to denoise the video, first the frames are denoised using a median filtering or adaptive median filtering approach to remove the impulse noise from the image. This is done because the impulse noise affects the accuracy of patch matching.

After removing the impulse noise from the video frames, the output is given to a patch matching routine, with selects a reference patch and finds patches similar to it in the current frame and in the nearby frames. Patch matching is done based on the L1 norm distance between the patches. From each frame, the top 5 most similar patches are extracted. The extracted patches form a $n^2 x\ K$ matrix, where n is the patch size and K is the number of frames. The patch matrix can be written as:

$$P_{j,k} = Q_{j,k} + N_{j,k}$$

Where $P_{j,k}$ is the patch matrix at location j in frame k
$Q_{j,k}$ is the underlying clean patch matrix
$N_{j,k}$ is the noise matrix

From the patch matrix, the reliable set of pixels are identified as Ω. These are pixels whose values do not differ much from the mean of the elements in the same row. To recover the rest of the pixels the following minimization problem is being solved in the paper:

$$min_Q \|Q\|_*$$
$$s.t \ \|Q_\Omega - P_\Omega\|_F^2 < \#(\Omega)\ \sigma^2$$

Solving the above minimization problem gives the denoised patches. To solve this problem, it is converted into it's Lagrangian version and the algorithm shown below is used.

Lagrangian version: $$min_Q \ \frac{1}{2} \|Q_\Omega - P_\Omega\|_F^2 + \mu\|Q\|_*$$

Algorithm 1 Fixed point iteration for solving the minimization (7)

1. Set $Q^{(0)} := 0$.
2. Iterating on k till $\|Q^{(k)} - Q^{(k-1)}\|_F \leq \epsilon$,

$$\begin{cases} R^{(k)} = Q^{(k)} - \tau\mathcal{P}_\Omega(Q^{(k)} - P), \\ Q^{k+1} = D_{\tau\mu}(R^{(k)}), \end{cases} \quad (8)$$

where μ and $1 \leq \tau \leq 2$ are pre-defined parameters, D is the shrinkage operator defined in (4) and $\mathcal{P}_\Omega$ is the projection operator of Ω defined by

$$\mathcal{P}_\Omega(Q)(i,j) = \begin{cases} Q(i,j), & \text{if } (i,j) \in \Omega; \\ 0, & \text{otherwise.} \end{cases}$$

3. Output $Q := Q^{(k)}$.

Here, $D_{\tau\mu}(R^{(k)})$ is the soft shrinkage operator, which is used to solve the objective function mentioned earlier. The soft shrinkage operator is defined as:

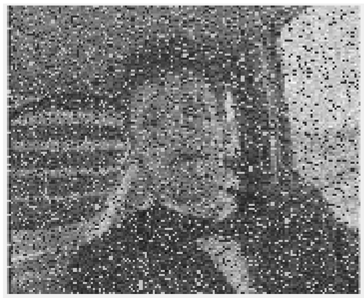
$$D_\tau = U\Sigma_\tau V^T$$

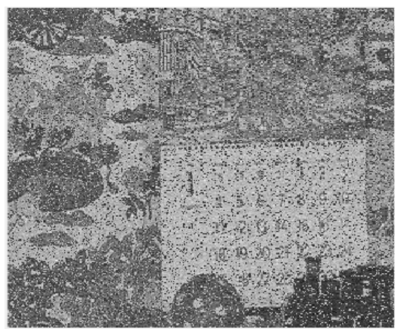
Where, $\Sigma_\tau = diag(\ max(\ \sigma_i - \tau\ ,\ 0\ )\ )$

Outputs:

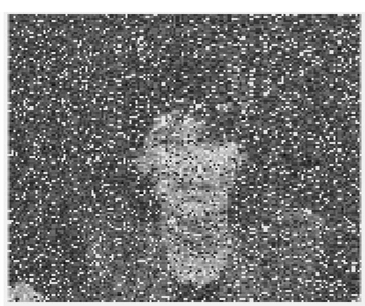
The following are the outputs for denoising, frames corrupted with the following noise statistics:

Gaussian standard deviation: 30
Poisson (K) : 10
Impulse (s) : 20%

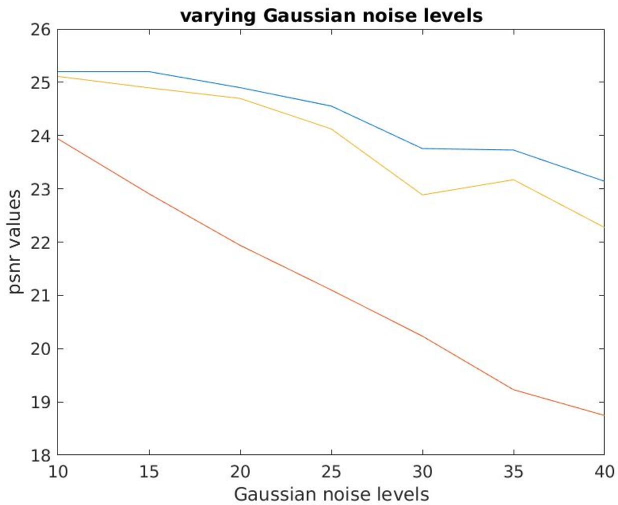



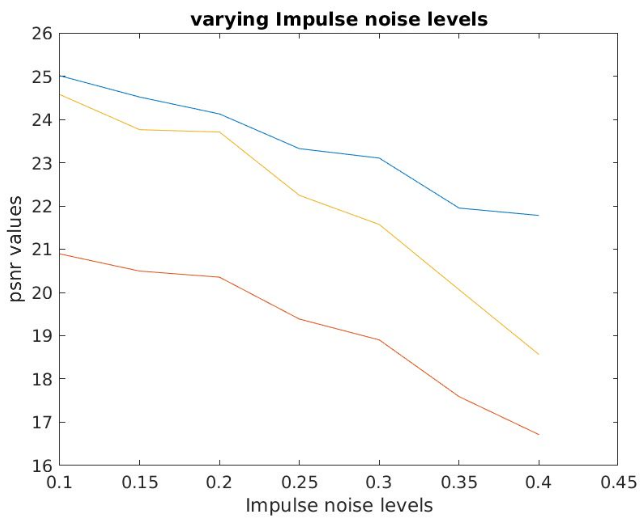


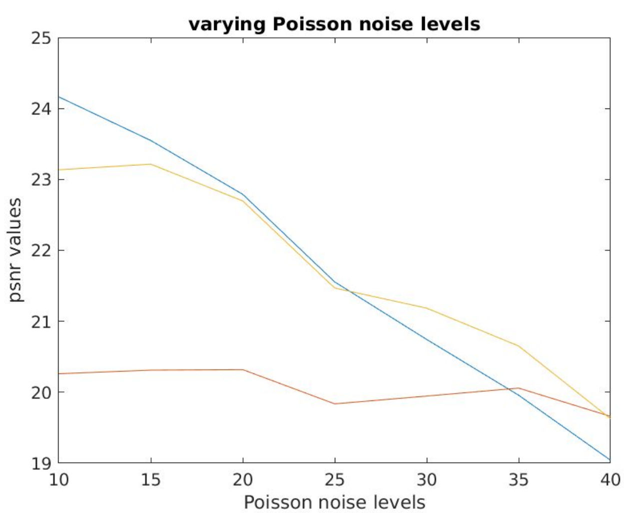



Graphs:

Blue: Matrix Completion
Yellow: Median Filtering
Red: PCA based denoising







Here, we have performed comparison between the PSNR values between median filtering, PCA-based denoising and matrix completion. Here, the PCA-based denoising uses the same patch matrix as matrix completion, with no missing entries unlike in matrix completion. Thus PCA-based denoising is performed on the patches similar to reference patch in the current and the nearby frames.

Conclusion:
Clearly we can see that, matrix completion performs better than median filtering or PCA based denoising when the noise levels are varied. PCA-based denoising assumes a Gaussian model, and median filtering assumes an impulse noise model. Compared to that, matrix completion does not assume the underlying noise model and still performs better than median filtering on variation of Impulse noise levels and better than PCA-based denoising on Gaussian noise model.

Matrix completion is sensitive to poisson noise, and after a certain point, it performs worse than both the other methods. The algorithm is quite fast on single images, but as the number of frames involved increases and if color images are used instead of grayscale, the time taken increases.